Currency attack/defense with two-sided private information

Chi-shing CHAN [a], Chor-yiu SIN [b,*] and Yuk-shing CHENG [b]

[a] JCTW Consultancy Firm, 505-511 Hennessy Road, Causeway Bay, Hong Kong

[b] Department of Economics, Hong Kong Baptist University, Renfrew Road, Kowloon Tong, Kowloon, Hong Kong

November 2003

Abstract

A currency attack fails on its own when the speculator suffers from her financial problem. This paper extends the existing models and argues that the monetary authority's willingness to peg and the speculator's cost of attack are private information. Our model thus accounts for the duration of currency attack/defense, and more importantly, allows for failed attack. We employ an asymmetric war of attrition and gauge the time when the speculator stops attacking, or when the monetary authority de-pegs. Comparative static results throw light on the interest rate policy amidst the Exchange Rate Mechanism Crisis and the Asian Currency Crisis.

KEYWORDS Asymmetric war of attrition; Credibility of policymakers; Failed speculative attack; Persistent effect; Two-sided private information

JEL Classification C72, E42, F33

Acknowledgements

This paper is a substantially revised version of Chan, Sin and Cheng (2002a). We are grateful to Kim-Sau Chung for his insightful criticism on the first draft, and to Ivan Pastine for his detailed comments. Thanks also go to participants at the 2002 North American Econometric Society Summer Meeting. Needless to say, all remaining errors are ours.

* Corresponding author: Chor-yiu SIN, Department of Economics, Hong Kong Baptist University, Renfrew Road, Kowloon Tong, Kowloon, Hong Kong. Tel: +852-3411-5200; Fax: +852-3411-5580; Email: cysin@hkbu.edu.hk.

1. Introduction

Since the seminal papers Salent and Henderson (1978) and Krugman (1979), a vast literature, both theoretical and empirical, has been devoted to studying the causes of currency crises. The main theme of this type of balance of payments crises models is clear: An economy is vulnerable to currency attack when there is conflict between maintaining the fixed rate and other economic policy objectives such as a low unemployment rate and a steady growth. The fixed exchange rate system ends when the macroeconomic imbalance exceeds some thresholds such as a minimum level of reserves. On the other hand, introducing some private information on the monetary authority's type or some exogenous shock, Obstfeld (1986, 1996) formalizes the unpredictability of the timing of currency attack. In this type of self-fulfilling currency attack models, the occurrence of a speculative attack depends on the coordination on the particular regime of expectation.

Nevertheless, both types of models are silent about the duration of currency attack/peg and preclude the existence of *failed* currency attacks. In reality, there exists a range of duration in defending the system. The Bank of England exited the Exchange Rate Mechanism (ERM) on September 16, 1992, only a day after an unsuccessful defense by raising the interest rate (Buiter, Corsetti and Pesenti, 1998). In contrast, the Bank of Thailand devalued the baht on July 2, 1997, long after the speculative attack began in 1996 (Corbett and Vines, 1999).

At times a monetary authority can even successfully defend its peg. [1] The Hong Kong linked exchange rate system is one of the prominent examples, as it has survived several major attacks since its inception in October 1983. From 1984 to 1988, contrast to the familiar cases, Hong Kong dollar was speculated to *revalue* for at least four times (Law, 1989). In 1988, the Hong Kong government even threatened to impose "negative interest rate" to deter the mass inflow of capital. [2] Contrarily,

[1] A detailed anecdotal account of successful and failed attacks can be found in Kraay (2003).

[2] Basically, people who deposited a large amount of Hong Kong dollar in the local banks, rather than receiving interest payment, would pay a fee. The proposed policy is similar to the scheme

amidst the Asian Financial Crisis, the direction of attack was reversed. The Hong Kong Monetary Authority raised the interest rate drastically on October 20, 1997 and the Hong Kong government even intervened more than one time in the local stock and futures markets in August 1998 to defend the link (Jao, 2001). Interesting enough, the link is still intact at this moment, at a cost of deflation and sluggish economic growth though.

Sweden is another interesting case often cited. The Riksbank raised the interest rate several times from August to September in 1992. The speculator left the market momentarily. However, this successful defense did not last and the speculative pressure came back in mid November. This time the Riksbank forwent the peg on November 19 (Drazen, 2000). This case also illustrates that a monetary authority may weigh the benefits and costs of the peg differently, should the economic fundamentals change (Obstfeld and Rogoff, 1995).

Apart from the above cases, the currency attack in United States during the period 1894-1896, that in United Kingdom in 1956, that in Mexico in 1995, and that in South Korea in 1997, also failed. See Grilli (1990) and Boughton (2001) for detailed discussions. Nevertheless, those failures are due to the *unexpected* [3] borrowing from IMF or some other source, while there existed no such borrowing in cases discussed above. On the other hand, Broner (2002) allows for failed "probing" attack in a model where speculators "coordinate". While the coordination among speculators is an important issue, [4] once again Broner (2002) considers a balance of payments crisis and it is our opinion that apart from the amount of reserves, objectives other than currency stability should also be considered.

This paper extends the existing models of currency attack to explain both the range of duration of attack/defense and the possibility of failed attack. We argue that the duration of currency attack/defense depends not only on the economic fundamentals such as high interest rate or the magnitude of devaluation, but also on

---

implemented in Switzerland in the 1970s (Greenwood, 1989). At the end the Hong Kong government shelved the plan, as the US dollar appreciated against other major currencies at the end of 1988.

[3] If there are *expected* borrowing which can successfully defend the currency, the speculator would not have been started an attack. See also our model in Section 3.

the two-sided private information kept by the two parties: (1) the (monetary) authority, (2) the speculator. First, we follow the lines in Obstfeld (1996) and assume the authority faces a trade-off between different targets such as output growth and currency/price stability; and this information is kept private. Norman Lamont, the British ex-Chancellors of the Exchequer in-charging of exchange rate policy during the 1992 ERM crisis, said in his autobiography:

"There has been much speculation about my own attitude to the ERM, enlivened by myths such as the story that I sang in my bath on 16 September when we finally ended our membership of it. As has been stated, I accepted the policy when I became Chancellor. *It was not my preferred policy*, but I had no reason to think it would become unworkable." (p.208, Lamont, 1999)

On the other hand, we assume the cost of attack, which includes not only the interest cost but also the speculator's other opportunities to invest, is a piece of information private to the speculator. Eichengreen and Mathieson in an IMF study have the following finding:

"…… it has been suggested, hedge funds precipitated major movements in asset prices, either through the sheer volume of their own transactions or via the tendency of other market participants to follow their lead. Yet for all this attention, *little concrete information is available about the extent of hedge funds' activities*." (p.2, Eichengreen and Mathieson, 1998)

This paper starts with a brief literature review in the next section. The theoretical model can be found in Section 3. In Section 4, we impose some additional assumptions that render an analytical solution. To the best of our knowledge, this is new under the topic of *asymmetric* war of attrition. The dynamic equilibrium is *unique* and thus contrasts with the self-fulfilling currency attack models, which are often built on multiple equilibria. The comparative static results of this formulation are also derived. Policy implications with special attention to the 1992-93 ERM crisis and the 1997-98 Asian Currency Crisis are presented. The persistent effect and the

[4] We will come back to this point in Section 5.

credibility of policymakers, in the context of interest rate policy, will be discussed. We conclude in Section 5. All technical proofs are relegated to the Appendix. Throughout, $x \vee y$ denotes the maximum of x and y; while $x \wedge y$ denotes the minimum of them.

2. A brief literature review

As one can see in Sections 3 and 4 below, our model is a variant of war of attrition with private information in both players. Based on the game-theoretical work developed in Riley (1980) and Fudenberg and Tirole (1986), this type of models has been applied to many economic issues. They include the exit time of a duopolist (Fudenberg and Tirole, 1986) and the delay in stabilization (Alesina and Drazen, 1991). As in all these applications, each player joins the "war" with the hope that she is facing a weak opponent. As time passes, only strong players remain. Each antagonistic player decides the exit time when the (expected) marginal cost of staying on equals the (expected) marginal gain, given the fact that the longer the opponent stays in the game, the stronger the opponent is. In our context, the monetary authority determines the time to de-peg and the speculator determines the time to stop from attacking. [5] Allsopp (2000) also uses a war of attrition to explain the duration of currency crisis. The players in her model are two governments ("Germany" and "United Kingdom" in the 1992-93 ERM crisis), which are treated *symmetrically*. The governments bargain over the changes in their domestic policy after the attack on their peg. While her model is related to the center-periphery model of monetary coordination (see, for instance, Buiter et al., 1998), it does not allow for any *failed* attack.

As far as private information is concerned, our model goes one step beyond the self-fulfilling currency attack models. We do not simply incorporate the monetary authority's evaluation of cost and benefit of the peg but also the speculators'

[5] For only technical reasons, throughout the paper, we assume that the monetary authority determines to de-peg and that the speculator determines to stop from attacking at some *finite* point of time, should their counter party stays on.

evaluation of her cost and benefit to attack. [6] The latter assumption is justified on the ground that people (including those in the monetary authority) in general do not have much information about the speculator's credit line as well as her potential investment opportunities (see Bensaid and Jeanne, 1997).[7] In other words, while the monetary authority signals its willingness to peg, the speculator signals her willingness to attack. As a result, depending on their relative willingness, it may not be the case that the monetary authority finally concedes and thus our model allows for failed attack. Our result contrasts with those in a recent paper Pastine (2002), in which the monetary authority (and only the monetary authority) introduces uncertainty into the speculator's decision by playing a mixed strategy. Moreover, while our paper was inspired by the ideas in Drazen (2000) which models the time-to-time attack/peg, introducing two-sided private information in our model dispenses with the unspecified exogenous shock, as well as the depletion of reserves.

This paper also contrasts with the existing literature on time of devaluation. While Flood and Marion (1997) and Klein and Marion (1997) ignore any currency attack or simply assumes a capital control, this paper also concerns the duration of attack. On the other hand, although Bensaid and Jeanne (1997), Ozkan and Sutherland (1998), and Drazen (2000) discuss, among many other things, the duration of attack, they only considered one-sided (the authority's) private information and/or some exogenous shock. For instance, some of the conclusions in Bensaid and Jeanne (1997) hinge on the arrival of good news or rising interest rate (p.1475). Interesting enough, they also note, "The currency crisis can stop by itself if for example speculators are financially exhausted" (p.1474). In a sense, our model endogenizes the speculator's time to terminate the attack under the assumption of her financial situation being a piece of private information.

3. Currency attack/defense as an asymmetric war of attrition

[6] To avoid confusion, throughout, we use an "it" for the monetary authority, and a "she" for the representative speculator.

[7] After the Asian financial crisis, many Asian monetary authorities urged more disclosures of the private financial institutions' activities in the international capital markets. See, for instance, Yam (1999).

In this section, we consider a continuous time model in which time is denoted as t, where $t \geq 0$. There are two players: One is the representative speculator (henceforth speculator) (S) while the other is the monetary authority (henceforth the authority) (A). At the beginning of the game when $t = 0$, the speculator starts a currency attack while the authority defends. At each instant $t > 0$ the speculator decides whether she continues to attack or stays out of the foreign exchange market. Similarly, the authority decides whether it continues to defend or lets the currency float (or manages the currency float). [8]

The speculator bears a constant flow of cost c_S if she stays on. This cost may include but does not confine to the interest cost of short selling. Similarly, the authority bears some economic and/or social costs in defending the peg (which may or may not be due to high interest rate), which is denoted as c_A. On the other hand, if the speculator terminates the attack before the authority de-pegs, the authority's gain will be π_A while by normalization, the payoff of the speculator is 0. Note that while c_A is interpreted as the economic and/or social cost of defending the peg, π_A is interpreted as the economic and/or social benefit of currency stability when there is no attack. Similarly, if the authority forgoes the peg first, the speculator's gain from devaluation will be π_S while the payoff of the authority is 0.

The speculator chooses the "exit" time t_S while the authority chooses the "exit" time t_A, to maximize the expected net present value with a discount rate ρ. The payoff for each player differs in different scenarios.

<u>Scenario I: The speculator stops attacking *first*</u>

If the speculator stops attacking the currency at time t_S (while the authority maintains the peg), the speculator's payoff is:

$$\int_0^{t_S} - c_S e^{-\rho t} dt = -\frac{(1-e^{-\rho t_S})}{\rho} c_S. \quad (1)$$

[8] This implies that once the speculator terminates the attack, and/or the authority de-pegs, a new game

And the authority's payoff is:

$$\int_0^{t_S} -c_A e^{-\rho t} dt \ + \int_{t_S}^{\infty} \pi_A \ e^{-\rho t} dt \ = \frac{e^{-\rho t_S}}{\rho} (\pi_A + c_A) - \frac{c_A}{\rho}. \tag{2}$$

Scenario II: The authority stops defending *first*

In a similar token, if the authority de-pegs at time t_A (while the speculator keeps attacking), the speculator's payoff is:

$$\int_0^{t_A} -c_S e^{-\rho t} dt + \int_{t_A}^{\infty} \pi_S \ e^{-\rho t} dt \ = \frac{e^{-\rho t_A}}{\rho} (\pi_S + c_S) - \frac{c_S}{\rho}. \tag{3}$$

And the authority's payoff is:

$$\int_0^{t_A} -c_A e^{-\rho t} dt = -\frac{(1 - e^{-\rho t_A})}{\rho} c_A. \tag{4}$$

As argued in the previous sections, while c_S (the cost of speculation) is the speculator's private information, π_A (the benefit of currency stability) is the authority's private information. The authority has a prior belief about $1/c_S$ (the reciprocal of c_S), which is represented by a density function $f_S(.)$ defined on a support $[1/\overline{c}_S, \infty)$. $\overline{c}_S > 0$ is the cost which is high enough such that the speculator does not attack. See Assumption (2) below. Similarly, the speculator has a belief about π_A, which is represented by a density function $f_A(.)$ defined on a support $[\underline{\pi}_A, \infty)$, where we assume $\underline{\pi}_A > 0$.

The speculator's strategy is represented by an *optimal time* function T_S: $[1/\overline{c}_S, \infty) \to [0, \infty)$, which states that for each possible value of $1/c_S$, the time when the speculator stops from attacking (while the authority maintains the peg). Similarly, the authority's strategy can be represented by another *optimal time* function T_A: $[\underline{\pi}_A, \infty) \to [0, \infty)$, which states that for each possible value of π_A, the time when the authority

starts.

de-pegs (while the speculator keeps attacking). [9]

Formulating the speculator's objective function

Consider the speculator's problem first. Recall that the authority's strategy is denoted as $T_A(.)$, that the speculator's type is characterized by the *reciprocal* of its cost $1/c_S$, and t_S is the time when the speculator *plans* to stop attacking. The objective function of the speculator is:

$$V_S(t_S, T_A(.), \frac{1}{c_S}) = Prob_A(x|T_A(x) \geq t_S)\ [-\ \frac{(1-e^{-\rho t_S})}{\rho} c_S]$$

$$+ \int_{\{x|T_A(x)<t_S\}} [\frac{e^{-\rho T_A(x)}}{\rho}(\pi_S + c_S) - \frac{c_S}{\rho}] f_A(x)dx. \qquad (5)$$

Note that in the above expression, the first term is the speculator's expected payoff if the authority defends the currency until or beyond t_S; whereas the second term is the speculator's expected payoff if the authority abandons the peg before t_S.

Formulating the authority's objective function

Similarly the objective function of the authority is:

$$V_A(t_A, T_S(.), \pi_A) = Prob_S(x|T_S(x) \geq t_A)\ [-\ \frac{(1-e^{-\rho t_A})}{\rho} c_A]$$

$$+ \int_{\{x|T_S(x)<t_A\}} [\frac{e^{-\rho T_S(x)}}{\rho}(\pi_A + c_A) - \frac{c_A}{\rho}] f_S(x)dx. \qquad (6)$$

The equilibrium

Now we are able to define the equilibrium of the model.

[9] For only technical reasons, we assume that both $1/c_S$ and π_A are unbounded above, which contrasts with that in Fudenberg and Tirole (1986). On the other hand, we consider cases in which the optimal t_S and t_A are finite. (See the *illustrative* case in Section 4.) That merely precludes the unlikely case in which one keeps on attacking while the other keeps on defending.

Definition: $[T_S(\frac{1}{c_S}),\ T_A(\pi_A)]$ is a Bayesian Nash equilibrium (*BNE*), if for all $\frac{1}{c_S} \in [\frac{1}{\bar{c}_S}, \infty)$, for all $\pi_A \in [\underline{\pi}_A, \infty)$, and for all $t \geq 0$,

$$V_S(T_S(\frac{1}{c_S}),\ T_A(.), \frac{1}{c_S}) \geq V_S(t,\ T_A(.), \frac{1}{c_S})$$

and $\quad V_A(T_A(\pi_A),\ T_S(.),\ \pi_A) \geq V_A(t,\ T_S(.),\ \pi_A)$. □

The following assumptions hold throughout the rest of the paper.

Assumption (1): $f_S(.)$ and $f_A(.)$ are continuous and strictly greater than zero on $(\frac{1}{\bar{c}_S}, \infty)$ and $(\underline{\pi}_A, \infty)$ respectively. □

Assumption (2): $0 < \rho < 1$. $T_S(\frac{1}{\bar{c}_S}) = 0$ and $T_A(\pi_A) = \bar{T}$ for all $\pi_A \geq \bar{\pi}_A \in [\underline{\pi}_A, \infty)$. Each of ρ, $\bar{T}$ and $\bar{\pi}_A$ is a piece of common knowledge. □

Assumption (1) is auxiliary and it allows the first order conditions of $V_S(t_S, T_A(.), 1/c_S)$ and $V_A(t_A, T_S(.), \pi_A)$ to exist. Assumption (2) specifies the boundary conditions for solving the differential equations, as one will see in Lemmas 1 and 2. Economically speaking, if the cost of attacking (c_S) is too high, the speculator would have stayed out of the market at the outset, and thus the optimal t_S is 0. On the other hand, there exists a time $\bar{T}$ such that even for an authority at the high end (of gain from currency stability), it will de-peg at $\bar{T}$. As a result, one may interpret $\bar{T}$ as the time when the authority runs out of reserves.

To aptly characterize the equilibrium, we need a lemma that resembles Lemma (1) in Fudenberg and Tirole (1986).

Lemma 1: Consider a *BNE* $[T_S(1/c_S),\ T_A(\pi_A)]$. Suppose in addition to Assumptions (1) and (2), $T_S(.)$ and $T_A(.)$ are strictly increasing and continuously differentiable on $(1/\bar{c}_S, \infty)$ and $(\underline{\pi}_A, \infty)$ respectively. Then:

(a) For $i = S, A$, there exists an inverse function $\Gamma_i(t)$ such that $T_i[\Gamma_i(t)] = t$. $\Gamma_S(t)$ and $\Gamma_A(t)$ are also strictly increasing and continuously differentiable on $(0, \sup_{c_S \in (0, \bar{c}_S)} T_S(1/c_S))$ and $(0 \vee \inf_{\pi_A \in (\underline{\pi}_A, \infty)} T_A(\pi_A), \overline{T})$ respectively.

(b) For $t \in (0 \vee \inf_{\pi_A \in (\underline{\pi}_A, \infty)} T_A(\pi_A), \sup_{c_S \in (0, \bar{c}_S)} T_S(1/c_S) \wedge \overline{T})$, the differential equations for $\Gamma_S(t)$ and $\Gamma_A(t)$ are given by:

$$\frac{f_A(\Gamma_A(t))}{1-F_A(\Gamma_A(t))} \frac{d\Gamma_A(t)}{dt} \frac{\Gamma_S(t)}{\rho} = \frac{1}{\pi_S}, \tag{7}$$

$$\frac{f_S(\Gamma_S(t))}{1-F_S(\Gamma_S(t))} \frac{d\Gamma_S(t)}{dt} \frac{\Gamma_A(t)}{\rho} = c_A. \tag{8}$$ □

The inverse functions $\Gamma_i(.)$'s in Lemma 1, as one can see in the proof (and the proofs of other propositions), much facilitate our discussion. The main thrust of Lemma 1 is the necessary first order conditions in Equations (7) and (8). Equation (7) can be interpreted as follows. The cost of continuing to attack between t and $(t+dt)$ is $e^{-\rho t} c_S dt$. On the other hand, at time t, the (conditional) probability that the authority de-pegs between t and $(t+dt)$ is $\frac{f_A(\Gamma_A(t))}{1-F_A(\Gamma_A(t))} \frac{d\Gamma_A(t)}{dt} dt$ and should it de-pegs, the speculator's gain is $e^{-\rho t}\pi_S/\rho$. Equating the marginal cost with the expected marginal gain, canceling out $e^{-\rho t}$ and dt, and replacing $1/c_S$ by $\Gamma_S(t)$, Equation (7) results. A similar interpretation can be applied to Equation (8). See the proof of Lemma 1 for details.

From Equations (7) and (8), it is easy to see that $\frac{d\Gamma_S(t)}{dt} > 0$ and $\frac{d\Gamma_A(t)}{dt} > 0$; and by Part (a) of the same lemma, $\frac{dT_S(1/c_S)}{d(1/c_S)} > 0$ and $\frac{dT_A(\pi_A)}{d\pi_A} > 0$. That is, the smaller the c_S is, the longer the speculator stays. On the other hand, the larger the π_A is, the longer the authority stays.

4. Bayesian Nash equilibrium and interest rate policy

In this section, we characterize the *BNE* along the lines in Lemma 1. Instead of arguing the validity of the *general* assumptions imposed in Lemma 1, we specify an exact distribution and solve out the necessary first order conditions (7) and (8) in closed-form. The optimal time functions are then obtained. We proceed to verify the second order conditions and argue that $[T_S(1/c_S), T_A(\pi_A)]$ is a *BNE*. The exact distribution is specified in Assumption (1*).

Assumption (1*): $f_S(\frac{1}{c_S} - \frac{1}{\bar{c}_S})$ is an exponential density with parameter $\frac{1}{m_S}$ and $f_A(\pi_A - \underline{\pi}_A)$ is an exponential density with parameter $\frac{1}{m_A}$. □

The exponential distribution is widely used in many papers on war of attrition, due to the fact that there is no general analytical solution to the differential equations in Lemma 1, should another distribution be used. See, for instance, Riley (1980) which makes a similar claim. The following lemma characterizes the *BNE* under the above specification.

Lemma 2: Suppose Assumption (1*) and Assumption (2) hold. For $t \in (0, \bar{T})$, the *BNE* is characterized by the following two differential equations:

$$\frac{d\Gamma_A(t)}{dt} = \frac{\rho m_A}{\pi_S \Gamma_S(t)}, \tag{9}$$

and
$$\frac{d\Gamma_S(t)}{dt} = \frac{\rho c_A m_S}{\Gamma_A(t)}. \tag{10}$$

The boundary conditions are $\Gamma_S(0) = (1/\bar{c}_S)$ and $\Gamma_A(\bar{T}) = \bar{\pi}_A$. □

Under the assumption of an exponential distribution, the *hazard function* $\frac{f_i(\Gamma_i(t))}{1 - F_i(\Gamma_i(t))} = \frac{1}{m_i}$, which is a constant. This property allows us to solve for $[\Gamma_S(t), \Gamma_A(t)]$ analytically. For $i = S, A$, the mean (of $1/c_S$ or π_A) is m_i while the variance is

m_i^2. Unlike many examples of war of attrition (see, for instance, Alesina and Drazen, 1991, and Riley, 1980), ours can hardly be symmetric. This is because economic fundamentals, which govern m_S, π_S, c_A, m_A and other parameters, have different impacts on the speculator and the authority. To the best of our knowledge, this particular asymmetric war of attrition has not been solved out analytically. Further, as one can see in Propositions 4 and 5, we obtain some interesting comparative static results.

<u>Proposition 3</u>: Suppose Assumption (1*) and Assumption (2) hold. For $t \in [0, \overline{T}]$,

$$\Gamma_S(t) = (\frac{1}{\overline{c}_S})(\frac{\rho P t}{k} + 1)^{\frac{1}{1+R}}, \text{ and} \tag{11}$$

$$\Gamma_A(t) = (k\overline{c}_S)(\frac{\rho P t}{k} + 1)^{\frac{R}{1+R}}; \tag{12}$$

on the other hand,

$$T_S(\frac{1}{c_S}) = \frac{k}{\rho P}\,[(\frac{\overline{c}_S}{c_S})^{1+R} - 1], \text{ for } \frac{1}{c_S} \in [\frac{1}{\overline{c}_S}, \infty), \text{ and} \tag{13}$$

$$T_A(\pi_A) = \frac{k}{\rho P}\,[(\frac{\pi_A}{k\overline{c}_S})^{\frac{1+R}{R}} - 1], \text{ for } \pi_A \in [k\overline{c}_S \vee \underline{\pi}_A, \overline{\pi}_A], \tag{14}$$

where $P \equiv (m_A/\pi_S + c_A m_S)$, $R \equiv (m_A/\pi_S)/(c_A m_S)$. $k > 0$ is implicitly defined in the equation:

$$R\ln(\rho P\overline{T} + k) + \ln k = (1+R)\ln(\frac{\overline{\pi}_A}{\overline{c}_S}). \tag{15}$$ □

From a policymaker's point of view, once the currency attack starts, it is beneficial to shorten the duration. In virtue of the explicit time function $T_S(.)$, the policymaker may want to alter the parameters such that $T_S(.)$ is smaller. To meet this end, we consider the partial derivatives of $T_S(.)$ with respect to the parameters. For completeness, we also derive those of $T_A(.)$. Results are reported in the following two propositions.

<u>Proposition 4</u>: Suppose Assumption (1*) and Assumption (2) hold. Denote $P_1 \equiv m_A/\pi_S$, $P_2 \equiv c_A m_S$. For all $1/c_S \in (1/\overline{c}_S, \infty)$ and for all $\pi_A \in (k\overline{c}_S \vee \underline{\pi}_A, \overline{\pi}_A)$,

(a) (i) $\dfrac{\partial T_A(\pi_A)}{\partial \overline{T}} = \dfrac{Rk}{(\rho P\overline{T}+k)} \; [(\dfrac{\pi_A}{k\overline{c}_S})^{\frac{1+R}{R}} \; \dfrac{1}{R} \; +1] > 0.$

(ii) $\dfrac{\partial T_S(\dfrac{1}{c_S})}{\partial \overline{T}} = - \; \dfrac{Rk}{(\rho P\overline{T}+k)} \; [(\dfrac{\overline{c}_S}{c_S})^{1+R} \; -1] < 0.$

(b) (i) $\dfrac{\partial T_A(\pi_A)}{\partial \overline{\pi}_A} = - \; \dfrac{(\rho P\overline{T}+k)k}{\rho P_2\overline{\pi}_A(\rho P\overline{T}+(1+R)k)} \; [(\dfrac{\pi_A}{k\overline{c}_S})^{\frac{1+R}{R}} \; \dfrac{1}{R} \; +1] < 0.$

(ii) $\dfrac{\partial T_S(\dfrac{1}{c_S})}{\partial \overline{\pi}_A} = \dfrac{(\rho P\overline{T}+k)k}{\rho P_2\overline{\pi}_A(\rho P\overline{T}+(1+R)k)} \; [(\dfrac{\overline{c}_S}{c_S})^{1+R} \; -1] > 0.$

(c) (i) $\dfrac{\partial T_A(\pi_A)}{\partial \overline{c}_S} = \dfrac{(1+R)(\rho P\overline{T}+k)k}{\rho P\overline{c}_S(\rho P\overline{T}+(1+R)k)} [1 - (\dfrac{\pi_A}{\overline{\pi}_A})^{1+\frac{1}{R}} \;] > 0.$

(ii) $\dfrac{\partial T_S(\dfrac{1}{c_S})}{\partial \overline{c}_S} = \dfrac{(1+R)(\rho P\overline{T}+k)k}{\rho P\overline{c}_S(\rho P\overline{T}+(1+R)k)} [(\dfrac{\overline{\pi}_A}{c_S(\rho P\overline{T}+k)})^{1+R} R +1] > 0.$

(d) (i) $\dfrac{\partial T_A(\pi_A)}{\partial P_1} = \dfrac{k\overline{T}}{P(1+R)(\rho\overline{T}+k)} \; [(\dfrac{\pi_A}{k\overline{c}_S})^{\frac{1+R}{R}} \; \dfrac{1}{R} \; +1][\dfrac{\rho P\overline{T}+k}{\Delta} \; +R]$

$$- \; \frac{k}{\rho P^2} \; [(\frac{\pi_A}{k\overline{c}_S})^{\frac{1+R}{R}} -1] - \frac{k}{\rho R P_1^2} \; (\frac{\pi_A}{k\overline{c}_S})^{\frac{1}{R}},$$

(ii) $\dfrac{\partial T_S(\dfrac{1}{c_S})}{\partial P_1} = -\dfrac{k}{\rho P}[(\dfrac{\overline{c}_S}{c_S})^{1+R} -1] \; [\dfrac{1}{P} \; + \dfrac{\rho\overline{T}}{(1+R)(\rho\overline{T}+k)}(\dfrac{\rho P\overline{T}+k}{\Delta} \; +R)]$

$$+\frac{k}{\rho P}(\frac{\overline{c}_S}{c_S})^R(\frac{1+R}{P_2}),$$

with $\Delta > 0$ lies between k and $\rho P\overline{T}+k$.

(e) (i) $\dfrac{\partial T_A(\pi_A)}{\partial P_2} = -\dfrac{kP_1\overline{T}}{P^2(\rho\overline{T}+k)} \; [(\dfrac{\pi_A}{k\overline{c}_S})^{\frac{1+R}{R}} \; \dfrac{1}{R} \; +1][\dfrac{\rho P\overline{T}+k}{\Delta} \; -1]$

$$- \; \frac{k}{\rho P^2}[(\frac{\pi_A}{k\overline{c}_S})^{\frac{1+R}{R}} -1] + \frac{k}{\rho P_1^2} \; (\frac{\pi_A}{k\overline{c}_S})^{\frac{1}{R}}.$$

(ii) $\dfrac{\partial T_S(\dfrac{1}{c_S})}{\partial P_2} = \dfrac{k}{\rho P^2} \; [(\dfrac{\overline{c}_S}{c_S})^{1+R} -1][(\dfrac{\rho P\overline{T}+k}{\Delta} -1)(\dfrac{\rho P_1\overline{T}}{\rho\overline{T}+k}) -1] - \dfrac{kR}{\rho P_2^2}(\dfrac{\overline{c}_S}{c_S})^R.$ □

Interpreting $\overline{T}$ as the time when the authority runs out of reserves, Part (a) of the above proposition is consistent with the results implied by the balance of payments crises. That is, the larger the $\overline{T}$ is, the longer the duration of defense is, and the shorter the duration of attack is. In other words, our model with two-sided private information also suggests that a healthy balance of payments results in a more credible fixed exchange rate. See the illustration in Figure 1. [10]

Figure 1 is here

On the other hand, a larger $\overline{\pi}_A$ means more likely the authority de-pegs before $\overline{T}$. In other words, more likely the authority prefers things (such as output growth or low unemployment) other than stable exchange rate (and/or stable domestic price). We say that the authority is "weaker" in defending the peg. Part (b)(i) shows that the "weaker" the authority is, the sooner it de-pegs. On the other hand, Part (b)(ii) shows that the "weaker" the authority is, the longer the speculator keeps attacking.

In a similar token, a larger $\overline{c}_S$ means more likely the speculator faces a higher constant flow of cost c_S. In other words, the authority will have a higher probability to face a "weaker" opponent and would like to stay longer. Also, the existing speculator signals her "strength" by staying longer. (i) and (ii) of Part (c) confirm these assertions.

It is not straightforward to determine the signs of the terms in Parts (d) and (e). That said, succinct investigation on the original expressions for $T_S(1/c_S)$ and $T_A(\pi_A)$ tells us the impacts of changes in the parameter P_1 or P_2. The results are reported in the following proposition, and they are illustrated in Figures 2 – 5(a)(b). [11]

For brevity of notation, for any function $g(x)$, $g(x^+)$ denotes the right-hand limit; while $g(\infty)$ denotes the limit of $g(x)$ when $x \to \infty$.

[10] The actual figures used in this illustration are available upon request to the corresponding author.

Proposition 5: Suppose Assumption (1*) and Assumption (2) hold. Recall that $P_1 = m_A/\pi_S$ and $P_2 = c_A m_S$.

(a) (i) If $P_1^1 > P_1^0$, $T_A^1(\pi_A) > T_A^0(\pi_A)$, for all $\pi_A \in (k\bar{c}_S \vee \underline{\pi}_A, \bar{\pi}_A)$.

(ii) If $P_1^1 > P_1^0$, $T_S^1(\frac{1}{\bar{c}_S}^+) < T_S^0(\frac{1}{\bar{c}_S}^+)$ and $T_S^1(\infty) > T_S^0(\infty)$.

(b) (i) If $P_2^1 > P_2^0$, $T_A^1(\pi_A) < T_A^0(\pi_A)$, for all $\pi_A \in (k\bar{c}_S \vee \underline{\pi}_A, \bar{\pi}_A)$.

(ii) If $P_2^1 > P_2^0$ but $\frac{k^1}{P_2^1} > \frac{k^0}{P_2^0}$, $T_S^1(\frac{1}{\bar{c}_S}^+) > T_S^0(\frac{1}{\bar{c}_S}^+)$ and $T_S^1(\infty) < T_S^0(\infty)$;

if $P_2^1 > P_2^0$ and $\frac{k^1}{P_2^1} < \frac{k^0}{P_2^0}$, $T_S^1(\frac{1}{\bar{c}_S}^+) < T_S^0(\frac{1}{\bar{c}_S}^+)$ and $T_S^1(\infty) < T_S^0(\infty)$. □

Figures 2 - 5(a)(b) are here

Refer to the celebrated analysis in Drazen and Masson (1994). In our context, the "credibility of policymakers" shortens the duration of attack while the "persistent" effect prolongs the durations of attack. Take c_A as an example. When c_A increases and so does P_2, by Part (b) of the above proposition, the duration of attack will be shortened or prolonged, depending on (i) k/P_2 increases or decreases; and if k/P_2 increases, (ii) $1/c_S$ is small or large, a piece of information unknown to the authority. In other words, even with the concrete specification in Assumption (1*), it is unclear if the "credibility of policymakers" dominates the "persistent effect" or the other way round.

We close this section with a discussion on the impacts of interest rate policy on the duration of attack. It should be clear from our discussion below that, unlike many papers on war of attrition, the asymmetry is not only closer to the reality, but also results in interesting implications of interest rate policy.

The above proposition sheds light on the on-going debate about the appropriate interest rate policy during the 1997-98 Asian Currency Crisis. Apparently, the crisis-inflicted Asian economies, such as the ASEAN-4 all

---

[11] See Footnote 10.

experienced a prolonged duration of several weeks of attacks before they finally announced the de-peg of their currencies. In contrast, Hong Kong won the game of defending its currency. This has led analysts to argue that the loose monetary policy of the Asian countries in the early stage of crisis has worsened the situation (see, among others, Corsetti, Pesenti and Roubini, 1999). It is also consistent with the long-holding stance of the IMF, whose rescue programs always include a high interest rate policy. Critics of such policy advice often point to its adverse effects, especially in exacerbating the widespread bankruptcies of banks and corporations, leading to a credit crunch that causes more bankruptcies. Our model adds on the debate by considering how the interest rate tool, which acts as an exogenous parameter in our model, affects the equilibrium. Whether to raise the interest rate is a hard choice because it hurts not only the speculator but also the domestic economy. As shown in Proposition 5, the net effect of an increase in interest rate (and thus an increase in c_A but a decrease in m_S) on the duration of attack may be positive or negative. A high interest rate policy is thus unwarranted in these cases.

This of course does not preclude the possibility that some interest rate policy, such as a one-shot sharp increase in interest rate, is an effective policy. Inspired by the work Lahiri and Végh (2003), Chan, Sin and Cheng (2002b) provides an analysis, with special reference to Hong Kong amidst the Asian Currency Crisis.

5. Concluding comments

A currency attack terminates on its own when the speculator has problems in her financial position. In this paper, we extend the existing models of currency attack to cases with two-sided private information. It is argued that not only the monetary authority's willingness to peg is private information, so is the cost of attack of the speculator. In so doing, we develop a theoretical model that accounts for the duration of currency attack/defense, and more importantly, that allows for failed currency attack. We employ an asymmetric war of attrition and gauge the time when the speculator stops attacking (if she fails) and the time when the monetary authority de-pegs (if it concedes). We derive some comparative static results and thus throw light on the possible interest rate policy in defending against currency attack. Special attention is given to the 1992-93 Exchange Rate Mechanism crises and the 1997-98 Asian currency crises.

The model in this paper is a variant of war of attrition. Unlike many applications in economics, we consider an asymmetric case that is more appropriate in our context. The exact Bayesian Nash equilibrium and the comparative static results are derived under the assumption of an exponential distribution. As in other applications, our equilibrium is subgame perfect.

This paper considers one single *representative* speculator. However, in reality there is generally more than one speculator. Intuitively, the duration of currency attack as well as the failed speculative attack can still be explained once we consider the authority and the speculators have private information on their own benefits and costs in exiting/continuing the defense/attacks in the war of attrition. If the monetary authority is relatively "strong", all speculators may concede first and failed currency attack results. However, there may be multiple equilibria as a speculator's strategy to exit/continue depends on the other speculators' actions. This is similar to the self-fulfilling currency attack models in which a speculator's decision to attack depends on other speculators' actions. As there are diverse beliefs among the speculators, regardless of one-sided or two-sided private information, the strategic interaction and coordination among the speculators become important (Botman and Jager, 2002, Broner, 2002, Chamley, 2003, and Corsetti, Dasgupta, Morris and Shin, 2000). The characterization of the equilibrium/equilibria will thus be complex. We leave this challenging topic to other research.

Appendix: Technical Proofs

Proof of Lemma 1:

(a) is obvious from the assumptions and thus the proof is omitted. For (b), we first consider the speculator's problem. Rewrite $V_S(t_S, \Gamma_A(.), 1/c_S)$ in Equation (5) as:

$$V_S(t, \Gamma_A(.), \frac{1}{c_S}) = Prob_A(x|x \geq \Gamma_A(t))\ [-\ \frac{(1-e^{-\rho t})}{\rho} c_S]$$

$$+ \int_{\{x|x<\Gamma_A(t)\}} [\frac{e^{-\rho t}}{\rho}(\pi_S + c_S) - \frac{c_S}{\rho}] f_A(x)dx. \qquad \text{(A.1)}$$

Differentiate $V_S(t, \Gamma_A(.), 1/c_S)$ with respect to t:

$$\frac{\partial V_S}{\partial t} = (1 - F_A(\Gamma_A(t))(-e^{-\rho t}c_S) - f_A(\Gamma_A(t))\frac{d\Gamma_A(t)}{dt}(-\frac{1-e^{-\rho t}}{\rho}c_S)$$

$$+ f_A(\Gamma_A(t))\frac{d\Gamma_A(t)}{dt}[\frac{e^{-\rho t}}{\rho}(\pi_S + c_S) - \frac{c_S}{\rho}]$$

$$= e^{-\rho t}[f_A(\Gamma_A(t))\frac{d\Gamma_A(t)}{dt}\frac{\pi_S}{\rho} - (1 - F_A(\Gamma_A(t))c_S]. \qquad \text{(A.2)}$$

For the authority's problem, similarly, we can differentiate $V_A(t, \Gamma_S(.), \pi_A)$ with respect to t and get:

$$\frac{\partial V_A}{\partial t} = e^{-\rho t}[f_S(\Gamma_S(t))\frac{d\Gamma_S(t)}{dt}\frac{\pi_A}{\rho} - (1 - F_S(\Gamma_S(t))c_A]. \qquad \text{(A.3)}$$

By (A.2)-(A.3), the first order conditions for the speculator and the authority are respectively:

$$\frac{f_A(\Gamma_A(t))}{1-F_A(\Gamma_A(t))}\frac{d\Gamma_A(t)}{dt}\frac{\pi_S}{\rho} = c_S$$

$$\Leftrightarrow \frac{f_A(\Gamma_A(t))}{1-F_A(\Gamma_A(t))}\frac{d\Gamma_A(t)}{dt}\frac{\frac{1}{c_S}}{\rho} = \frac{1}{\pi_S}, \qquad \text{(A.4)}$$

and $$\frac{f_S(\Gamma_S(t))}{1-F_S(\Gamma_S(t))}\frac{d\Gamma_S(t)}{dt}\frac{\pi_A}{\rho} = c_A. \qquad \text{(A.5)}$$

However, we cannot obtain $\Gamma_S(t)$ and $\Gamma_A(t)$ by simply solving Equations (A.4) and (A.5). It is because the authority (or the speculator), as well as other investigators of the entire game, at most acquires the information on $\Gamma_S(t)$ (or $\Gamma_A(t)$) rather than that on $1/c_S$ (or π_A). In view of this, we replace $1/c_S$ and π_A with $\Gamma_S(t)$ and $\Gamma_A(t)$ respectively. Equations (7) and (8) result. □

Proof of Lemma 2:

Given Assumptions (1*) and (2), and in view of Lemma 1, it suffices to show that:

$$\frac{1-F_A(\Gamma_A(t))}{f_A(\Gamma_A(t))} = m_A \text{ and } \frac{1-F_S(\Gamma_S(t))}{f_S(\Gamma_S(t))} = m_S.$$

The proofs for both equalities are similar. As no ambiguity arises, we suppress the subscripts $_A$ and $_S$. Given Assumption (1*), $f(x) = \frac{1}{m} exp[-\frac{1}{m}(x-\underline{x})]$, where $x \in [\underline{x}, \infty)$.

It is not difficult to show that:

$$F(x) = 1 - exp[-\frac{1}{m}(x-\underline{x})] \text{ and } \frac{1-F(\Gamma(t))}{f(\Gamma(t))} = m. \ \square$$

<u>Proof of Proposition 3</u>:

Rearranging Equations (9) and (10) in Lemma 2,

$$\Gamma_A(t)d\Gamma_S(t) = (\rho c_A m_S)dt. \tag{A.6}$$

$$\Gamma_S(t)d\Gamma_A(t) = (\rho\frac{1}{\pi_S} m_A)dt. \tag{A.7}$$

Using integration by parts, (A.6) yields

$$\Gamma_A(t)\Gamma_S(t) - \int\Gamma_S(t)d\Gamma_A(t) = (\rho c_A m_S)t + k_1, \text{ where } k_1 \text{ is a constant.} \tag{A.8}$$

By (A.7), $\int\Gamma_S(t)d\Gamma_A(t) = (\rho\frac{1}{\pi_S} m_A)t + k_2$, where k_2 is a constant. (A.9)

Adding (A.8) to (A.9) yields

$$\Gamma_A(t)\Gamma_S(t) = \rho Pt + k,$$

in which $k = k_1 + k_2$ and we recall that $P = (m_A/\pi_S + c_A m_S)$. As $\pi_A > 0$ and $1/c_S > 0$, $k = \Gamma_A(0)\Gamma_S(0) > 0$. Re-write the above equation:

$$\Gamma_A(t) = \frac{\rho Pt + k}{\Gamma_S(t)}. \tag{A.10}$$

Put (A.10) into (A.6) and recall that $R = (m_A/\pi_S)/(c_A m_S)$,

$$(\frac{\rho Pt + k}{\Gamma_S(t)})d\Gamma_S(t) = (\rho c_A m_S)\, dt$$

$$\Rightarrow \frac{1}{\rho c_A m_S}\frac{d\Gamma_S(t)}{\Gamma_S(t)} = \frac{dt}{\rho Pt + k}$$

$$\Rightarrow \rho(\frac{1}{\pi_S} m_A + c_A m_S)\frac{1}{\rho c_A m_S}\frac{d\Gamma_S(t)}{\Gamma_S(t)} = \frac{d(\rho Pt + k)}{\rho Pt + k}$$

$$\Rightarrow\ (1+R)\frac{d\Gamma_S(t)}{\Gamma_S(t)} = \frac{d(\rho Pt+k)}{\rho Pt+k}$$

$\Rightarrow$ $(1+R)\ ln\Gamma_S(t) = ln(\rho Pt + k) + h$, where h is a constant.

Solving, $\Gamma_S(t) = [exp(h)]^{1/(1+R)}[\rho Rt+k]^{1/(1+R)}$. (A.11)

Put (A.11) into (A.10). Solving,

$\Gamma_A(t) = [exp(h)]^{-1/(1+R)}[\rho Rt+k]^{R/(1+R)}$; (A.12)

And the boundary conditions are given by:

$$Rln(\rho P\overline{T} + k) + ln\ k = (1+R)ln(\frac{\overline{\pi}_A}{\overline{c}_S}), \tag{A.13}$$

$$\text{and }\ h = -ln\ k - (1+R)ln\,\overline{c}_S\,. \tag{A.14}$$

However, by (A.14),

$$exp(h) = k^{-1}\ \overline{c}_S^{\,-(1+R)}\,. \tag{A.15}$$

Putting (A.15) into (A.11)-(A.12), it follows that:

$$\Gamma_S(t) = (\frac{1}{\overline{c}_S})(\frac{\rho Pt}{k}+1)^{\frac{1}{1+R}}; \tag{A.16}$$

$$\Gamma_A(t) = (k\overline{c}_S)(\frac{\rho Pt}{k}+1)^{\frac{R}{1+R}}, \tag{A.17}$$

and equivalently,

$$T_S(\frac{1}{c_S}) = \frac{k}{\rho P}\ ((\frac{\overline{c}_S}{c_S})^{1+R}\ -1); \tag{A.18}$$

$$T_A(\pi_A) = \frac{k}{\rho P}\ ((\frac{\pi_A}{k\overline{c}_S})^{\frac{1+R}{R}}\ -1). \tag{A.19}$$

Finally, by (A.16)-(A.17), in view of the fact that $\frac{d\Gamma_S(t)}{dt} > 0$ and $\frac{d\Gamma_A(t)}{dt} > 0$, it is easy to see from (9) and (10) that $\frac{d^2\Gamma_A(t)}{dt^2} < 0$ and $\frac{d^2\Gamma_S(t)}{dt^2} < 0$. In other words, the second order conditions are also satisfied. The proof is thus complete. □

Proof of Proposition 4:

Recall that $P = P_1 + P_2$, $R = \frac{P_1}{P_2}$, $1+R = \frac{P_1 + P_2}{P_2}$, and $\frac{1+R}{R} = \frac{P_1 + P_2}{P_1}$. First of all, by (A.13), tedious algebra shows that

$$\frac{\partial k}{\partial \overline{T}} = - \frac{\rho PRk}{\rho P\overline{T} + k} < 0. \tag{A.20}$$

$$\frac{\partial k}{\partial \overline{\pi}_A} = \frac{(1+R)(\rho P\overline{T} + k)k}{\overline{\pi}_A(\rho P\overline{T} + (1+R)k)} > 0. \tag{A.21}$$

$$\frac{\partial k}{\partial \overline{c}_S} = - \frac{(1+R)(\rho P\overline{T} + k)k}{\overline{c}_S(\rho P\overline{T} + (1+R)k)} < 0. \tag{A.22}$$

Consequently, by (A.18), (A.19) and (A.20),

$$\frac{\partial T_S(.)}{\partial \overline{T}} = - \frac{Rk}{(\rho P\overline{T} + k)} [(\frac{\overline{c}_S}{c_S})^{1+R} - 1] < 0.$$

$$\frac{\partial T_A(.)}{\partial \overline{T}} = \frac{Rk}{(\rho P\overline{T} + k)} [(\frac{\pi_A}{k\overline{c}_S})^{\frac{1+R}{R}} \frac{1}{R} + 1] > 0.$$

Thus Part (a) is proved.

On the other hand, by (A.18), (A.19) and (A.21),

$$\frac{\partial T_S(.)}{\partial \overline{\pi}_A} = \frac{(\rho P\overline{T} + k)k}{\rho P_2\overline{\pi}_A(\rho P\overline{T} + (1+R)k)} [(\frac{\overline{c}_S}{c_S})^{1+R} - 1] > 0.$$

$$\frac{\partial T_A(.)}{\partial \overline{\pi}_A} = - \frac{(\rho P\overline{T} + k)k}{\rho P_2\overline{\pi}_A(\rho P\overline{T} + (1+R)k)} [(\frac{\pi_A}{k\overline{c}_S})^{\frac{1+R}{R}} \frac{1}{R} + 1] < 0.$$

Thus Part (b) is also proved.

On the other hand, by (15)

$$\overline{c}_S^{\,1+R} = \frac{1}{(\rho P\overline{T} + k)^R k} \overline{\pi}_A^{\,1+R}. \tag{A.23}$$

Plug (A.23) into (13),

$$T_S(\frac{1}{c_S}) = \frac{1}{\rho P(\rho P\overline{T} + k)^R} (\frac{\overline{\pi}_A}{c_S})^{1+R} - \frac{k}{\rho P}.$$

Therefore,

$$\frac{\partial T_S(.)}{\partial k} = -\frac{R}{\rho P(\rho P\overline{T}+k)^{1+R}}\,(\frac{\overline{\pi}_A}{c_S})^{1+R} - \frac{1}{\rho P}$$

$$= -\ \frac{1}{\rho P}\ [\,(\frac{\overline{\pi}_A}{c_S\,(\rho P\overline{T}+k)})^{1+R}\,R + 1]. \qquad \text{(A.24)}$$

All in all, by (A.22) and (A.24),

$$\frac{\partial T_S(.)}{\partial \overline{c}_S} = \frac{(1+R)(\rho P\overline{T}+k)k}{\rho P\overline{c}_S\,(\rho P\overline{T}+(1+R)k)}\,[\,(\frac{\overline{\pi}_A}{c_S\,(\rho P\overline{T}+k)})^{1+R}\,R + 1] > 0. \qquad \text{(A.25)}$$

Also by (15),

$$(\frac{1}{\overline{c}_S})^{1+\frac{1}{R}} = (\rho P\,\overline{T} + k)\,k^{\frac{1}{R}}\ \ (\frac{1}{\overline{\pi}_A})^{1+\frac{1}{R}}. \qquad \text{(A.26)}$$

Plug (A.26) into (14),

$$T_A(\pi_A) = \frac{1}{\rho P}\,[\,(\frac{\pi_A}{\overline{\pi}_A})^{1+\frac{1}{R}}\,(\rho P\,\overline{T} + k) - k]. \qquad \text{(A.27)}$$

Therefore,

$$\frac{\partial T_A(.)}{\partial k} = -\frac{1}{\rho P}\,[1 - (\frac{\pi_A}{\overline{\pi}_A})^{1+\frac{1}{R}}\ ]. \qquad \text{(A.28)}$$

All in all, by (A.22) and (A.28),

$$\frac{\partial T_A(.)}{\partial \overline{c}_S} = \frac{(1+R)(\rho P\overline{T}+k)k}{\rho P\overline{c}_S\,(\rho P\overline{T}+(1+R)k)}\,[1 - (\frac{\pi_A}{\overline{\pi}_A})^{1+\frac{1}{R}}\ ] > 0. \qquad \text{(A.29)}$$

By (A.25) and (A.29), Part (c) is also proved.

On the other hand, with $\Delta > 0$ lies between k and $\rho P\overline{T} + k$, by (A.13), tedious algebra and a Taylor's expansion yields

$$\frac{\partial k}{\partial P_1} = -\ \frac{k[(\rho P\overline{T}+k)(\ln(\rho P\overline{T}+k) - \ln k) + \rho PR\overline{T}]}{(1+R)P(k+\rho\overline{T})}$$

$$= -\ \frac{k}{(1+R)(k+\rho\overline{T})}\,(\frac{\rho P\overline{T}+k}{\Delta}\ + R)\,\rho\overline{T}\ < 0. \qquad \text{(A.30)}$$

$$\frac{\partial k}{\partial P_2} = \frac{kP_1[(\rho P\overline{T}+k)(\ln(\rho P\overline{T}+k) - \ln k) - \rho P\overline{T}]}{P^2\,(k+\rho\overline{T})}$$

$$= \frac{kP_1}{P(k+\rho\overline{T})}\left(\frac{\rho P\overline{T}+k}{\Delta} - 1\right)\rho\overline{T} > 0. \qquad \text{(A.31)}$$

By (A.18) and (A.30),

$$\frac{\partial T_S(.)}{\partial P_1} = -\frac{k}{\rho P}\left[\left(\frac{\overline{c}_S}{c_S}\right)^{1+R} - 1\right]\left[\frac{1}{P} + \frac{\rho\overline{T}}{(1+R)(\rho\overline{T}+k)}\left(\frac{\rho P\overline{T}+k}{\Delta} + R\right)\right]$$

$$+\frac{k}{\rho P}\left(\frac{\overline{c}_S}{c_S}\right)^R\left(\frac{1+R}{P_2}\right). \qquad \text{(A.32)}$$

Next, by (A.19) and (A.30),

$$\frac{\partial T_A(.)}{\partial P_1} = \frac{k\overline{T}}{P(1+R)(\rho\overline{T}+k)}\left[\left(\frac{\pi_A}{k\overline{c}_S}\right)^{\frac{1+R}{R}}\frac{1}{R} + 1\right]\left[\frac{\rho P\overline{T}+k}{\Delta} + R\right]$$

$$-\frac{k}{\rho P^2}\left[\left(\frac{\pi_A}{k\overline{c}_S}\right)^{\frac{1+R}{R}} - 1\right] - \frac{k}{\rho R P_1^2}\left(\frac{\pi_A}{k\overline{c}_S}\right)^{\frac{1}{R}}. \qquad \text{(A.33)}$$

By (A.18) and (A.31),

$$\frac{\partial T_S(.)}{\partial P_2} = \frac{k}{\rho P^2}\left[\left(\frac{\overline{c}_S}{c_S}\right)^{1+R} - 1\right]\left[\left(\frac{\rho P\overline{T}+k}{\Delta} - 1\right)\left(\frac{\rho P_1\overline{T}}{\rho\overline{T}+k}\right) - 1\right] - \frac{kR}{\rho P_2^2}\left(\frac{\overline{c}_S}{c_S}\right)^R.$$

(A.34)

On the other hand, by (A.19), and (A.31),

$$\frac{\partial T_A(.)}{\partial P_2} = -\frac{kP_1\overline{T}}{P^2(\rho\overline{T}+k)}\left[\left(\frac{\pi_A}{k\overline{c}_S}\right)^{\frac{1+R}{R}}\frac{1}{R} + 1\right]\left[\frac{\rho P\overline{T}+k}{\Delta} - 1\right]$$

$$-\frac{k}{\rho P^2}\left[\left(\frac{\pi_A}{k\overline{c}_S}\right)^{\frac{1+R}{R}} - 1\right] + \frac{k}{\rho P_1^2}\left(\frac{\pi_A}{k\overline{c}_S}\right)^{\frac{1}{R}}. \qquad \text{(A.35)}$$

This completes the proof. □

<u>Proof of Proposition 5</u>:

For any function $g(x)$, denote its derivative as $g^{'}(x)$. Re-write (A.27) as:

$$T_A(\pi_A) = \left(\overline{T} + \frac{k}{\rho P}\right)\left(\frac{\pi_A}{\overline{\pi}_A}\right)^{1+\frac{1}{R}} - \frac{k}{\rho P}.$$

Then $T_A^{'}(\pi_A) = \left(\overline{T} + \frac{k}{\rho P}\right)\left(1+\frac{1}{R}\right)\pi_A^{\frac{1}{R}}\left(\frac{1}{\overline{\pi}_A}\right)^{1+\frac{1}{R}}$ (A.36)

$$= [\overline{T}\,(1+\frac{1}{R}) + \frac{k}{\rho P_1}]\,\pi_A^{\frac{1}{R}}\,(\frac{1}{\overline{\pi}_A})^{1+\frac{1}{R}}. \qquad \text{(A.37)}$$

First consider P_1. By (A.30), when P_1 increases, k and thus $k\overline{c}_S$ decreases. Thus by (14), $T_A(\pi_A)$ cuts the x-axis at a smaller value (or it cuts the minimal y at the same value if the original $k\overline{c}_S$ is smaller than $\underline{\pi}_A$). On the other hand, by (A.36) and (A.30), an increase in P_1 will result in a decrease in $T_A^{'}(\pi_A)$. All in all, $T_A(\pi_A)$ shifts upwards. (See Figure 2 for an illustration.) Thus (a)(i) is proved.

Next consider P_2. By (A.31), when P_2 increases, k and thus $k\overline{c}_S$ increases. Thus by (14), $T_A(\pi_A)$ cuts the x-axis at a larger value (or it cuts the minimal y at the same value if the new $k\overline{c}_S$ is still smaller than $\underline{\pi}_A$). On the other hand, by (A.37) and (A.31), an increase in P_2 will result in an increase in $T_A^{'}(\pi_A)$. All in all, $T_A(\pi_A)$ shifts downwards. (See Figure 4 for an illustration.) Thus (b)(i) is proved.

On the other hand, from (A.18) above:

$$T_S^{'}(\frac{1}{c_S}) = \frac{k}{\rho P}(1+R)(\overline{c}_S)^{1+R}\,(\frac{1}{c_S})^R$$

$$= \frac{k\overline{c}_S}{\rho P_2}\,(\frac{\overline{c}_S}{c_S})^R. \qquad \text{(A.38)}$$

First consider P_1. By (A.30), when P_1 increases, k decreases. On the other hand, R increases. It is not difficult to see from (A.38) that when $\frac{1}{c_S} \to \frac{1}{\overline{c}_S}^{+}$, $T_S^{'}(\frac{1}{c_S})$ decreases; when $\frac{1}{c_S} \to \infty$, $T_S^{'}(\frac{1}{c_S})$ increases. (See Figure 3 for an illustration.) Thus (a)(ii) is proved.

Next consider P_2. By (A.31), when P_2 increases, k increases. On the other hand, R decreases. If the overall k/P_2 increases, it is not difficult to see from (A.38) that, when

$\frac{1}{c_S} \to \frac{1}{\bar{c}_S}^{+}$, $T_S^{'}(\frac{1}{c_S})$ increases; when $\frac{1}{c_S} \to \infty$, $T_S^{'}(\frac{1}{c_S})$ decreases. (See Figure 5(a) for an illustration.) If the overall k/P_2 decreases, it is not difficult to see from (A.38) that when $\frac{1}{c_S} \to \frac{1}{\bar{c}_S}^{+}$, $T_S^{'}(\frac{1}{c_S})$ decreases; when $\frac{1}{c_S} \to \infty$, $T_S^{'}(\frac{1}{c_S})$ also decreases. (See Figure 5(b) for an illustration.) Thus (b)(ii) is proved and the proof is complete.□

References

Alesina, Alberto and Allan Drazen (1991), "Why Are Stabilizations Delayed?" *American Economic Review 81*, 1170-1180.

Allsopp, Louise (2000), "A Model to Explain the Duration of a Currency Crisis," *International Journal of Finance and Economics 5*, 331-337.

Bensaid, Bernard and Olivier Jeanne (1997), "The Instability of Fixed Exchange Rate Systems when Raising the Nominal Interest Rate is Costly," *European Economic Review 41*, 1461-1478.

Botman, Dennis P.J. and Henk Jager (2002), "Coordination of Speculation," *Journal of International Economics 58*, 159-175.

Boughton, James M. (2001), "Northwest of Suez: The 1956 Crisis and the IMF," *IMF Staff Papers 48*, 425-446.

Broner, Fernando A. (2002), "On the Timing of Balance of Payments Crises: Disaggregated Information and Interest Rate Policy," *Mimeograph*, Department of Economics, University of Maryland.

Buiter, Willem H., Corsetti, Giancarlo and Paolo A. Pesenti (1998) *Financial Markets and European Monetary Cooperation: The Lessons of the 1992-93 Exchange Rate Mechanism Crisis.* Cambridge: Cambridge University Press.

Chamley, Christophe (2003), "Dynamic Speculative Attacks," *American Economic Review 93*, 603-621.

Chan, Chi-shing, Chor-yiu Sin and Yuk-shing Cheng (2002a), "Why is Currency Attack/Peg Prolonged," paper presented at the 2002 North American Econometric Society Summer Meeting, June 2002.

Chan, Chi-shing, Chor-yiu Sin and Yuk-shing Cheng (2002b), "Sharp Increase in Interest Rate as a Signal of the Resolve to Defend Currencies," paper presented at the 2002 Hong Kong Economic Association Second Biennial Conference, December 2002.

Corbett, Jenny and David Vines (1999), “The Asian Crisis: Lessons from the Collapse of Financial Systems, Exchange Rates and Macroeconomic Policy,” (with Discussion by Christopher Bliss), Chapter 2, 67-111 in Agenor, Miller, Vines and Weber (eds.) *The Asian Financial Crisis: Causes, Contagion and Consequences*. Cambridge: Cambridge University Press.

Corsetti, Giancarlo, Amil Dasgupta, Stephen Morris and Hyun Song Shin (2000), “Does One Soros Make a Difference? A Theory of Currency Crises with Large and Small Traders,” *Yale Cowles Foundation Discussion Paper 1273,* forthcoming in *Review of Economic Studies*.

Corsetti, Giancarlo, Paolo Resenti and Nouriel Roubini (1999), “What Caused the Asian Currency and Financial Crisis?” *Japan and the World Economy 11*, 305-373.

Drazen, Allan (2000), “Interest Rate and Borrowing Defense against Speculative Attack,” *Carnegie-Rochester Conference Series on Public Policy 53,* 303-348.

Drazen, Allan and Paul R. Masson (1994), “Credibility of Policies versus Credibility of Policymakers,” *The Quarterly Journal of Economics 109*, 735-754.

Eichengreen, Barry and Donald Mathieson (1998), “Hedge Funds and Financial Markets: Implications for Policy,” pp.2-26, Chapter II in IMF Occasional Paper 166 *Hedge Funds and Financial Market Dynamics*. Washington, D.C.: International Monetary Fund.

Flood, Robert P. and Nancy P. Marion (1997), “The Size and Timing of Devaluations in Capital-Controlled Economies,” *Journal of Development Economics 54*, 123-147.

Fudenberg, Drew and Jean Tirole (1986), “A Theory of Exit in Duopoly,” *Econometrica 54*, 943-960.

Greenwood, John G. (1989), “Comment on <<A Speech at Forum on “The Negative Interest Rate Scheme: Is It Necessary?>>,” *Hong Kong Economic Papers 19*, 67-70.

Grilli, Vittorio (1990), “Managing Exchange Rate Crises: Evidence from the 1890s,” *Journal of International Money and Finance 9*, 258-275.

Jao, Yu-ching (2001): *The Asian Financial Crisis and the Ordeal of Hong Kong*. Connecticut: Quorum Books.

Klein, Michael W. and Nancy P. Marion (1997), “Explaining the Duration of Exchange-Rate Pegs,” *Journal of Development Economics 54*, 387-404.

Kraay, Aart (2003), “Do High Interest Rates Defend Currencies during Speculative Attacks?” *Journal of International Economics 59*, 297-321.

Krugman, Paul R. (1979), “A Model of Balance-of-Payments Crises,” *Journal of Money, Credit and Banking 11*, 311-325.

Lahiri, Amartya and Carlos A. Végh (2003), “Delaying the Inevitable: Interest Rate Defense and Balance of Payments Crises,” *Journal of Political Economy* 111, 404-424.

Lamont, Norman (1999) *In Office*. London: Little, Brown.

Law, Cheung-kwok (1989), “Comment on <<A Speech at Forum on “The Negative Interest Rate Scheme: Is It Necessary?>>,” *Hong Kong Economic Papers 19*, 75-78.

Obstfeld, Maurice (1986), “Rational and Self-Fulfilling Balance-of-Payments Crises,” *American Economic Review Papers and Proceedings 76*, 72-81.

Obstfeld, Maurice (1996), “Models of Currency Crises with Self-fulfilling Features,” *European Economic Review 40*, 1037-1047.

Obstfeld, Maurice and Kenneth Rogoff (1995), “The Mirage of Fixed Exchange Rates,” *Journal of Economic Perspectives 9*, 73-96.

Ozkan, F. Gulcin and Alan Sutherland (1998), “A Currency Crisis Model with Optimizing Policymaker,” *Journal of International Economics 44*, 339-364.

Pastine, Ivan (2002), “Speculation and the Decision to Abandon a Fixed Exchange Rate Regime,” *Journal of International Economics 57*, 197-229.

Riley, John G. (1980), “Strong Evolutionary Equilibrium and the War of Attrition,” *Journal of Theoretical Biology 82*, 383-400.

Salant, Stephen W. and Dale W. Henderson (1978), “Market Anticipations of Government Policies and the Price of Gold,” *Journal of Political Economy 86*, 627-648.

Yam, Joseph (1999), “Building Financial Stability: From Regional to International Co-operation,” *Hong Kong Monetary Authority Quarterly Bulletin May 1999*, 59-64, downloadable from http://www.info.gov.hk/hkma/eng/public/index.htm.

Figure 1: Increase in $\overline{T}$

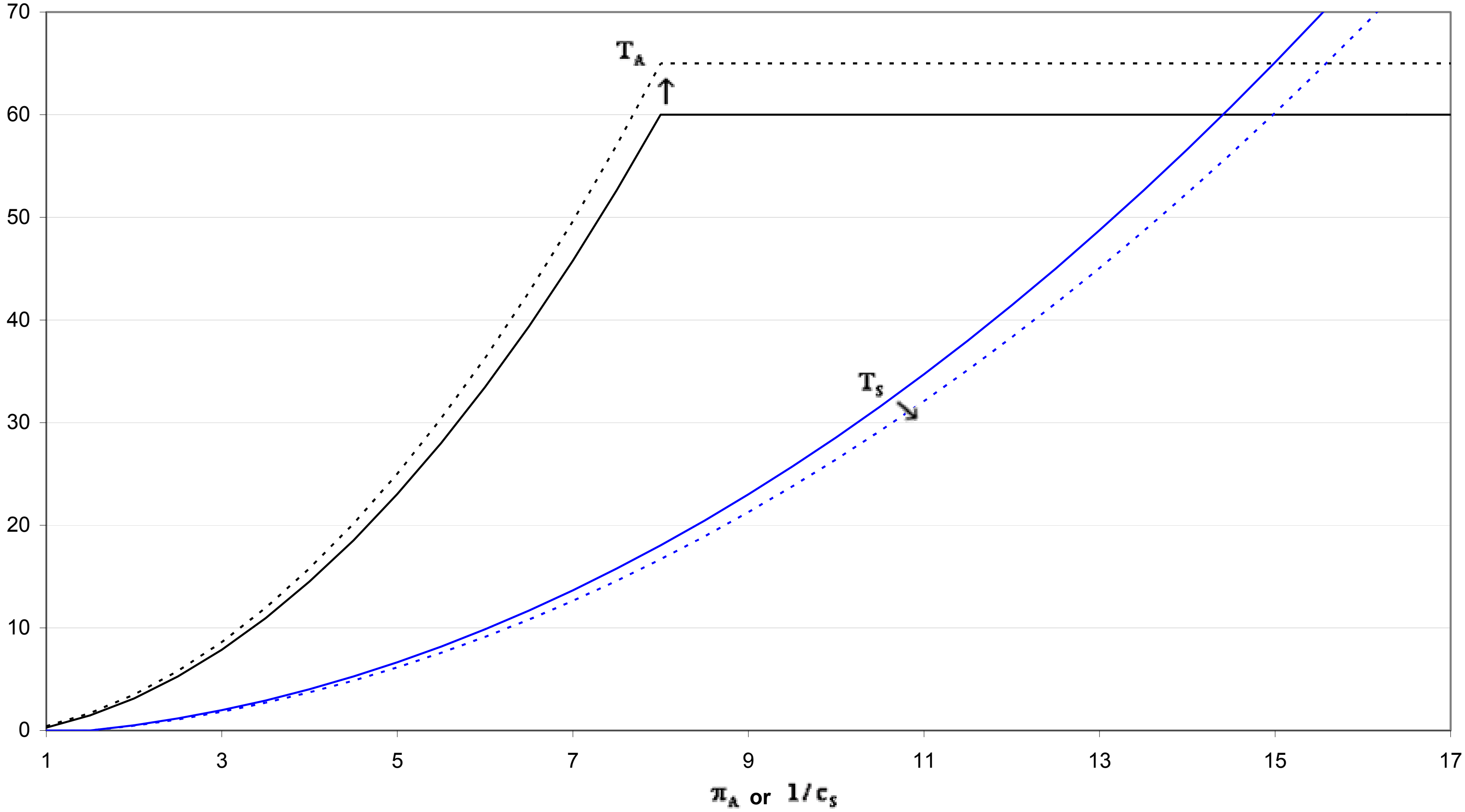

Figure 2: Increase in P_1

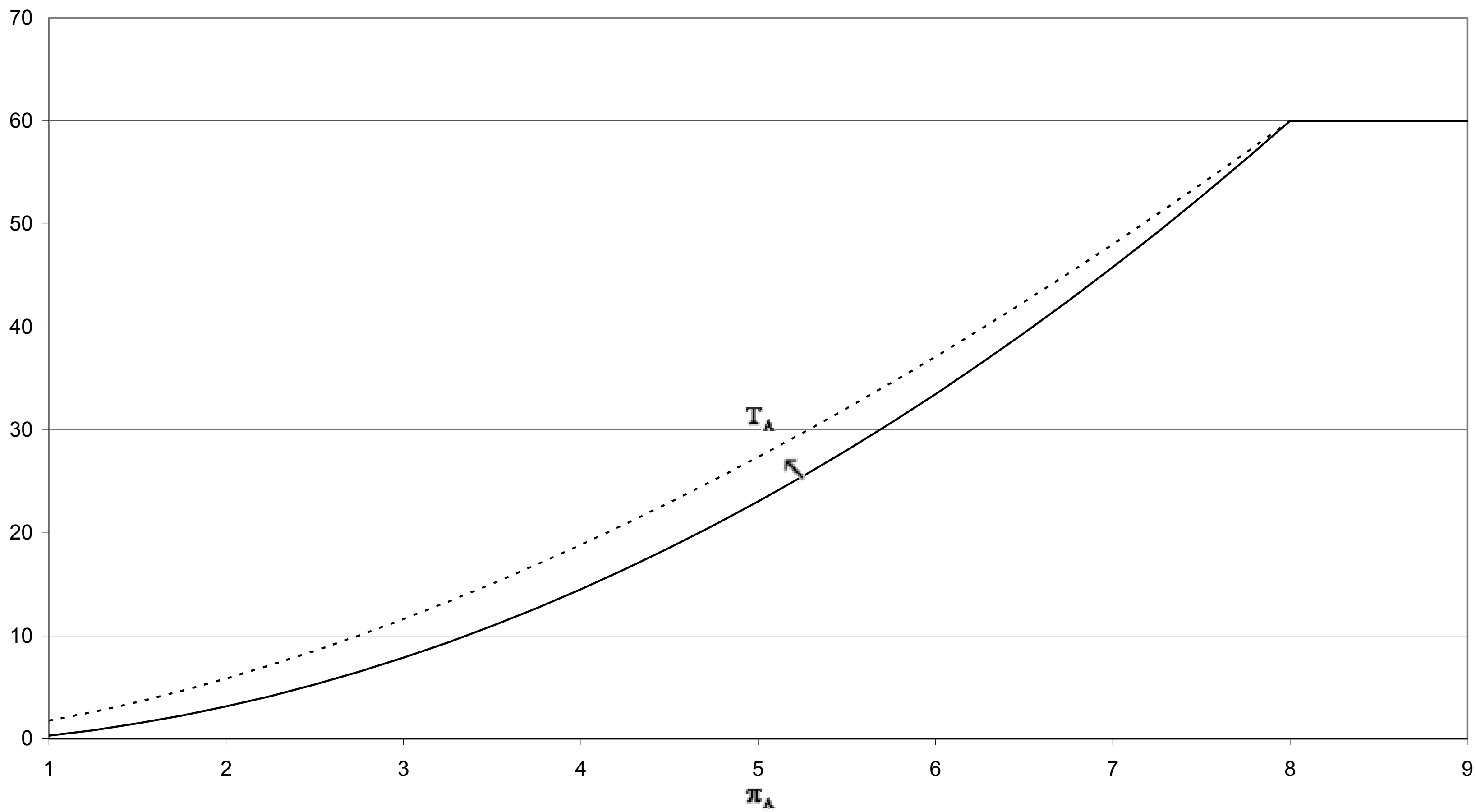

Figure 3: Increase in P_1

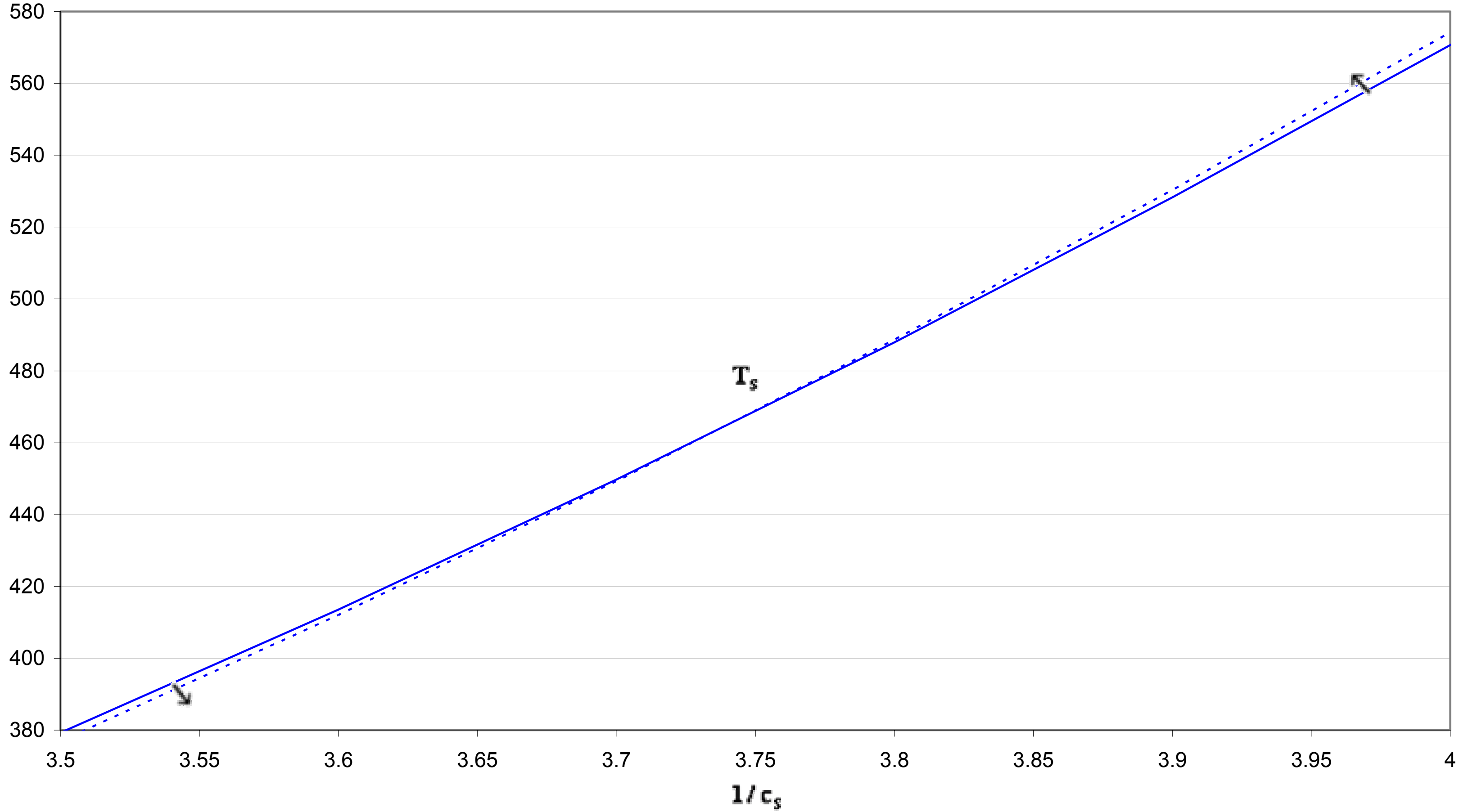

Figure 4: Increase in P_2

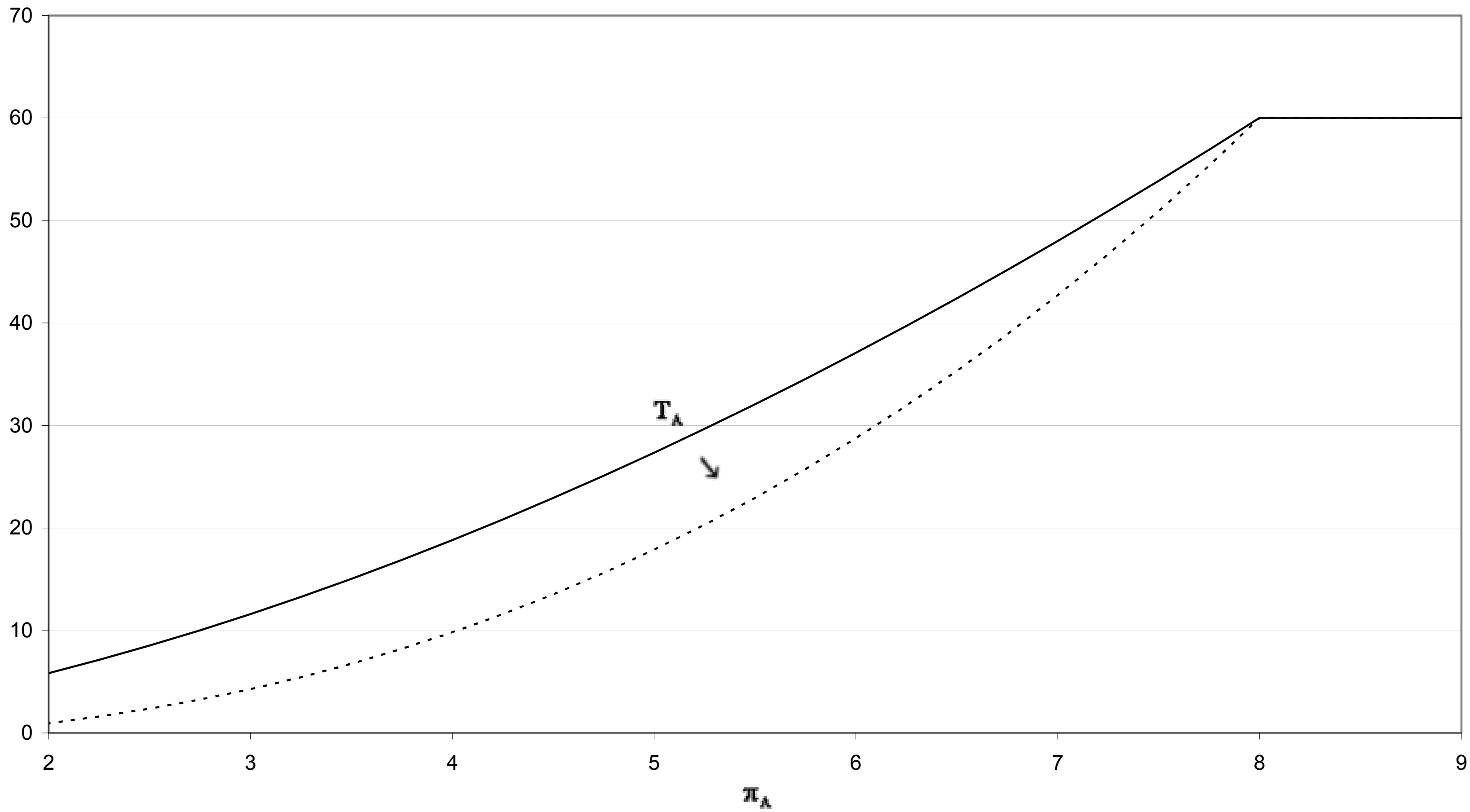

Figure 5(a): Increase in P_2 (Case a)

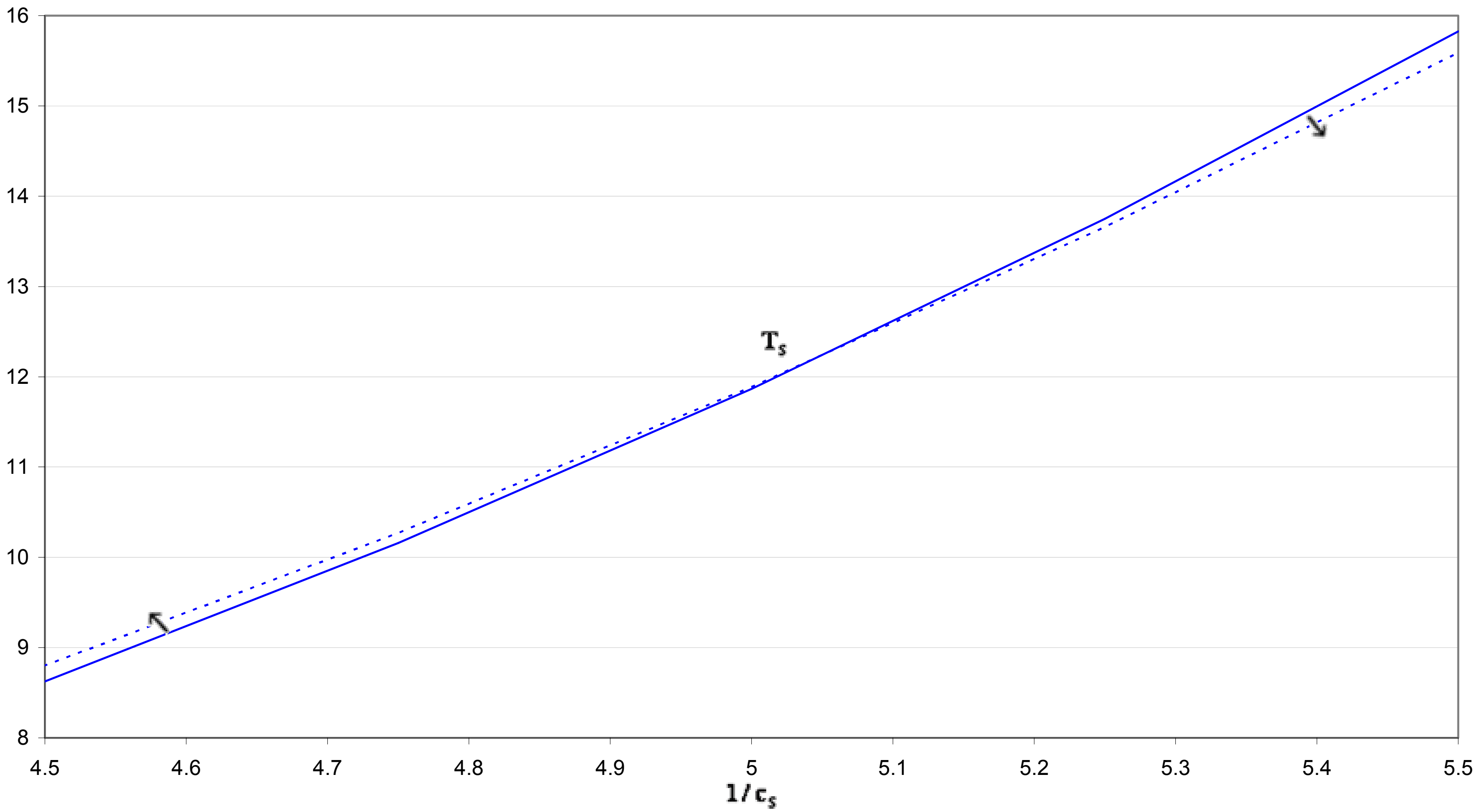

Figure 5(b): Increase in P_2 (Case b)