Oil and the G7 business cycle : Friedman's Plucking Markov Switching Approach

Yoon, Jae Ho¹

19th April 2004

POSCO Research Institute, POSRI, 147, Samsung-dong, Gangnam-gu, Seoul 135-878, Korea

Summary

To analyze whether oil price can account for the business cycle asymmetries in the G7, this paper adopts the Friedman's Plucking Markov Switching Model to decompose G7 real GDPs into common permanent components, common transitory components, infrequent Markov Switching negative shock and domestic idiosyncratic components. The findings show that Hamilton's 3 year net oil price increases account for 1973-75, 1980, partially 1990-1991 recessions and LNR oil price increases account for 1973-75, 1980, partially 1960, partially 1970, partially 1990-1991 recessions. These results indicate that oil price shocks have not been a principal determinant of common recessions in the G7 except two major OPEC oil price increases in 1973-1974, 1979-1980.

Keyword : oil, OPEC, G7, GDP, business cycle, Friedman's Plucking Markov Switching, permanent, transitory

¹ Corresponding author Tel.: +82-2-3457-8228; fax: +82-2-3457-8040. E-mail address : jhyoon@posri.re.kr or beowulfkorea@hanmail.net Web Site : http://ie.korea.ac.kr/~supercom/

1. Introduction

One of the issues of the international business cycle is to identify the business cycle fluctuations in economic activities across G7 countries and to find out main reasons for the G7 business cycle fluctuations.

Changes of oil price are considered as one of the most identifiable exogenous shocks to postwar U.S. business cycle fluctuations. In an important paper, Hamilton (1983) observes that all but one of the U.S recessions since World War II were preceded, typically with a lag of around three-fourths of a year by a dramatic increase in the price of crude petroleum and these exogenous oil shocks were a contributing factor in at least some of the U.S. recessions prior to 1972. Raymond and Rich (1997) have investigated the relationship between oil price and the U.S recessions by the two-state Markov Switching Model of Hamilton (1989) with net oil price compared to previous 1 year of Hamilton (1996) for the period 1952-1995. Raymond and Rich (1997) conclude that while the behavior of oil price was a contributing factor to the mean of low-growth phases of output, movements in oil price generally were not a principal determinant in the historical incidence of these phases. Clements and Krolzig (2002) have also investigated the relationship between oil price and the U.S. recessions by the three-state Markov Switching Model of Hamilton (1989) with oil price of Lee, Ni and Ratti (1995) for the period 1952-1999. Clements and Krolzig (2002) conclude that their findings are broadly in line with those of Raymond and Rich (1997): oil price do not appear to be the sole explanation of regime-switching behavior and the asymmetries detected in the U.S. business cycle do not appear to be explicable by oil price.

These papers about economic activities in the U.S suggest that there may exist a relationship between oil price and common recessions in the G7 business cycles. So, the purpose of this paper is two-fold. First, I will try to investigate the relationship between oil price and common recessions in the G7. Second, I will try to find out whether oil price shocks have been a principal determinant of common recessions in the G7 over the past forty years.

Gregory, Head and Raynauld (1997), Kose, Otrok and Whiteman (2003) identified the common fluctuations in the permanent component of macroeconomic aggregates in G7 countries using the linear dynamic factor model. Monfort, Renne, Ruffer and Vitale (2002) also use the linear dynamic factor model with a common permanent component and suggest that there appears to have been an emergence of at least one cyclically coherent group, the major Euro-zone countries.

In order to find out the relationship between oil price and common recessions in the

G7 business cycles, Yoon (2004) propose the generalization of existing the linear dynamic factor models that allow him to decompose G7 GDPs into common permanent components, common transitory components, and domestic idiosyncratic components. Following the asymmetry Friedman's Plucking Markov Switching Model suggested by Kim and Nelson (1999), Yoon (2004) also included Markov switching asymmetry, infrequent shock in the common transitory components. In this paper, I will add exogenous oil price in the Friedman's Plucking Markov Switching Model.

Friedman's Plucking Markov Switching modeling strategy maintains view that output cannot exceed a ceiling level but occasionally is plucked downward by recessions. Further, if the effects of exogenous oil price increases can account for the majority of the downward plucking movements in common part of G7 GDPs, then the estimation procedure may fail to show the probabilities of downward plucking shift with the oil price. And, the effects of exogenous oil price increases can be identified by comparison between probabilities of common recessions without oil price and probabilities of common recessions with oil price.

For the oil price variable, I use the net oil price compared to previous 3 years of Hamilton $(2003)^2$ (Hamilton's 3 year net oil price) and the oil price of Lee, K., Ni, S., Ratti, A.(1995) (LNR oil price) covering 1960:1 – 2001:3 following Hamilton (2003) and Clements and Krolzig (2002).

Section 2 presents the use of Friedman's Plucking Markov Switching model with exogenous oil price. Section 3 explains the data used for empirical research. Section 4 summarizes the empirical results. Section 5 concludes this paper.

2. Friedman's Markov Switching model with exogenous oil price

There has been a large body of research that the economic activity in the U.S has a permanent component which has the persistence of shocks; for example, Nelson and Plosser (1982), Campbell and Mankiw (1987), Watson (1986) and Cochrane (1988), Stock and Watson (1989, 1991). There also has been a large body of research that the economic activity in the U.S has a transitory component which has a smaller persistence of shocks; for example, Clark (1987), Beaudry and Koop (1993).

From the Markov-switching model of Hamilton (1989), many papers have demonstrated that economic activity in the U.S has shown an asymmetrical behavior in the permanent component of real output. This means that if there will be a shock, the

² I obtained the oil price data from <u>http://weber.ucsd.edu/~jhamilto</u>

shock will switch the trend of real output and it will persist; for example, Hamilton (1989), Lam (1990), Chauvet (1998), Kim and Nelson (1998). Many papers have also demonstrated that the economic activity in the U.S has shown an asymmetrical behavior in the transitory component of real output. This means that if there is infrequent shock, the shock will just be temporary and transitory and will have no relation to the trend of real output; for example Kim and Nelson (1999), Kim and Murray (2002), Kim, J. Piger and R. Startz (2002).

These papers about economic activities in the U.S suggest that there may exist unobserved common permanent and transitory components in the G7 business cycles. In order to find out whether the G7 GDPs have common permanent and transitory components like U.S real output, Yoon (2004) propose the simple generalization of existing G7 business cycle models that allow him to decompose G7 GDPs into common permanent components, common transitory components, and domestic idiosyncratic components. Following the plucking asymmetry model suggested by Kim and Nelson (1999), Yoon (2004) also included Markov switching asymmetry, infrequent shock in the common transitory component in this generalization model. In the Friedman's Plucking Markov Switching model, I add exogenous oil price to analyze the relationship between oil price and the common recessions in the G7.

Consider the following unobserved components of economic fluctuations in the log of real GDP (Y_{it}) are decomposed into a deterministic time trend (DT_{it}), a permanent component with unit root (P_{it}), and a transitory component (T_{it}) suggested by Kim and Nelson (1999), Kim and Murray (2002):

$$Y_{it} = DT_{it} + P_{it} + T_{it}$$
(1)
where $DT_{it} = \alpha_i + D_i T$
$$P_{it} = r_i C_t + \zeta_{it}$$

$$T_{it} = \lambda_i X_t + \omega_{it}$$

where C_t and X_t are the international common permanent and common transitory component respectively, and ζ_{it} and ω_{it} are the domestic idiosyncratic components, respectively. The r_i terms are permanent factor loadings and indicate the extent to which each series is affected by the common permanent component, C_t . Similarly, the transitory factor loadings, λ_i , indicate the extent to which each series is affected by the common transitory component, X_t .

To the empirical results, G7 data is integrated, but not co-integrated ³. Thus, I take the first difference, then:

$$\Delta Y_{it} = D_i + r_i \Delta C_t + \lambda_i \Delta X_t + z_{it}$$
(2)
where $z_{it} = \Delta \zeta_{it} + \Delta \omega_{it}$
 $\phi(L) \Delta C_t = \delta + v_t,$
 $\psi(L) X_t = \pi S_t + u_t, \quad \pi \neq 0$
 $v_t \sim iid N (0, 1)$
 $\psi(0, 1)$

z_{it} can be interpreted as a total domestic idiosyncratic component which is unrelated to the two international common components.

Given ΔY_{it} , δ , D_i are not separately identified, I concentrate this parameter out of the likelihood function by writing the model in deviations from means:

$$\begin{split} \Delta y_{it} &= r_i \ \Delta c_t + \ \lambda_i \ \Delta x_t + z_{it} \\ \text{where } \ \Delta y_{it} &= \ \Delta Y_{it} - \ \Delta \hat{Y}_i \\ \Delta c_t &= \phi_1 \ \Delta c_{t-1} + \phi_2 \ \Delta c_{t-2} + v_t \ , \\ x_t &= \psi_1 \ x_{t-1} + \psi_2 \ x_{t-2} + \pi \ S_t + u_t \ , \\ x_t &= \tau_i \ z_{it-1} + e_{it} \ , \end{split}$$
(3)

$$Pr(S_t = 0 | S_{t-1} = 0) = q$$
, $Pr(S_t = 1 | S_{t-1} = 1) = p$

I add oil price $\sum \beta_i Z_{t-i}$ in the Friedman's Plucking Markov Switching model⁴:

$$\begin{split} \Delta y_{it} &= r_i \ \Delta c_t + \ \lambda_i \ \Delta x_t + z_{it} \\ \text{where } \ \Delta y_{it} &= \ \Delta Y_{it} - \ \Delta \hat{Y}_i \\ \Delta c_t &= \phi_1 \ \Delta c_{t-1} + \phi_2 \ \Delta c_{t-2} + v_t , \\ x_t &= \psi_1 \ x_{t-1} + \psi_2 \ x_{t-2} + \pi \ S_t + \ \sum \beta_i \ Z_{t-i} + u_t , \quad \pi \neq 0 \quad u_t \quad \sim \text{iid } N \ (0, 1) \\ z_{it} &= \ \tau_i \ z_{it-1} + e_{it} , \\ \end{split}$$
(4)

$$Pr(S_t = 0 | S_{t-1} = 0) = q$$
, $Pr(S_t = 1 | S_{t-1} = 1) = p$

In this framework, when $\lambda_i = 0$, $\pi = 0$, $\sum \beta_i = 0$, this model is the linear dynamic factor model of Kose, Otrok and Whiteman(2002, 2003) and Monfort, Renne, Ruffer and Vitale(2002) without regional or area model.

 ³ A detailed description of test results is provided in the appendix B
 ⁴ A detailed description is provided in the appendix A

3. Data

The G7 data represents quarterly real GDPs for the G7 countries (US, Japan, Germany, France, Italy, UK, Canada) covering 1960:1 – 2002:4, the same data used in Stock & Watson $(2003)^5$. For the empirical results, G7 data are integrated, but not co-integrated. Using the Augmented Dickey-Fuller Test, I fail to reject the unit root null for any of the series. Using the Johansen's tests for co-integration, I fail to reject the null hypothesis that there are no co-integrating vectors.⁶

The choice of the oil price variable is an important issue. There has been a large body of research that the oil price has a clear negative correlation with GDP or GNP in the U.S; for example, Rasche and Tatom(1977, 1981), Hamilton(1983), among others. Nevertheless, there remains controversial when Hooker (1996) concluded that there was a weaker oil price-U.S GDP relationship in data obtained since 1985.

A number of authors have attributed this instability of oil price-U.S GDP relationship to misspecification of the functional form of oil price and suggested the representative oil price variable to analyze the relationship between this representative oil price and U.S GDP; Mork (1989), Lee, K., Ni, S., Ratti, A.(1995), Hamilton(1996), Hamilton (2003).

Raymond and Rich (1997) choose the net oil price compared to previous 1 year of Hamilton (1996) as an alternative to Mork's oil price. Because most of the oil price increases since 1986 have been subsequent to large oil price decreases, Mork's measures that focus solely on positive changes in the price of oil will overstate the significance and magnitude of the price movements. However, the problems with the net oil compared to previous 1 year of Hamilton (1996), are that the oil price surge of 1999 was not followed by a noticeable economic slowing in 2000. Hamilton (2003) explains the reason of this particular situation that the Asian financial crisis was associated with a drop in world oil price of over 50% during 1997-1998 and the price increases in the first half of 1999 had only recovered what was lost in 1997-1998. So, Hamilton (2003) suggests the net oil price compared to previous 3 years of Hamilton (2003) (Hamilton's 3 year net oil price) as the representative oil price instead of the net oil price compared to previous 1 year of Hamilton (1996). After the statistical analyses of different transformed oil price) as the representative oil price. Clements and Krolzig

⁵ Data sources are summarized in Appendix C.

I obtained the G7 GDP data from http://www.wws.princeton.edu/~mwatson/wp.html

⁶ A detailed description of test results is provided in the appendix B

(2002) choose LNR oil price by using the best fit in an autoregressive-distributed lag (ADL) model. Following Hamilton (2003) and Clements and Krolzig (2002), I choose Hamilton's 3 year net oil price and LNR oil price covering 1960:1 - 2001:3 as the representative oil price to investigate the relationship between oil price and the common recessions in the G7 in this paper.

4. Empirical results

The empirical analysis examines quarterly data on G7 real GDP and the representative oil price covering 1960:1 - 2001:3

I estimate the model presented in Section 2, using log differenced data. Furthermore, the differenced data is demeaned by removing the sample mean and the variance is standardized to one. Estimation results are summarized in Table 1.

In Table 1, the model which used Hamilton's 3 year net oil price has negative coefficient a lag of fourths of a year and the model which used LNR oil price has negative coefficient a lag of three, fourths of a year. These results accord quite well with the results of Hamilton (1983).

In order to find out whether oil price increases are statistically significantly correlated with G7 GDP, I compare the log likelihood values of three models in Table1. The LR test statistics for the hypothesis o1=o2=o3=o4 is 8.62 for the model including Hamilton's 3 year net oil price and 13.22 for the model including LNR oil price, respectively. The test statistics, which is distributed asymptotically as $\chi_{(4)}$ under null hypothesis, rejects the former model at a 10 percent, not 5 percent, significance levels and rejects the latter model at the 2 percent significance level. This finding offers the evidence that oil price increases are negatively and statistically correlated with common transitory components in the G7 GDP.

The degree of the downward plucking movements by oil price increases can be measure with the coefficient π . These π estimates increase from -2.387 to -2.283 for the model with Hamilton's 3 year net oil price and increase from -2.387 to -1.986 for the model with LNR oil price. This finding offers the implication that the representative two oil price as additional exogenous variables, account well for the occasional downward plucking movements by common recessions in the G7. In addition, an expected duration⁷ of plucking shock increases from 2.58 quarters to 2.75 with Hamilton's 3 year net oil price and to 2.87 with LNR oil, respectively.

 $^{^{7}}$ With constant transition probabilities, the expected duration of a contraction is 1/(1-p)

Parameters	Without Oil	Hamilton's 3 year Oil	LNR Oil
φ ₁	0.701 (0.375)	0.684 (0.345)	0.730 (0.401)
φ ₂	0.209 (0.356)	0.214 (0.332)	0.184 (0.382)
Ψ_1	1.504 (0.179)	1.375 (0.181)	1.522 (0.136)
Ψ2	-0.562 (0.170)	-0.444 (0.171)	-0.578 (0.129)
r _{us}	0.076 (0.080)	0.082 (0.049)	0.070 (0.040)
r _{japan}	0.314 (0.116)	0.332 (0.113)	0.303 (0.118)
r _{germany}	0.107 (0.054)	0.115 (0.056)	0.102 (0.053)
r _{france}	0.153 (0.061)	0.166 (0.063)	0.148 (0.062)
r _{italy}	0.213 (0.085)	0.232 (0.088)	0.207 (0.087)
r _{uk}	0.068 (0.050)	0.077 (0.055)	0.064 (0.048)
r _{canada}	0.112 (0.052)	0.124 (0.058)	0.108 (0.051)
λ_{us}	0.403 (0.080)	0.458 (0.097)	0.429 (0.075)
λ_{japan}	0.044 (0.054)	0.034 (0.066)	0.054 (0.055)
λ_{germany}	0.013 (0.046)	0.010 (0.057)	0.027 (0.047)
λ_{france}	0.070 (0.037)	0.060 (0.045)	0.071 (0.038)
λ_{italy}	0.097 (0.056)	0.074 (0.067)	0.092 (0.057)
λ_{uk}	0.168 (0.053)	0.178 (0.058)	0.177 (0.052)
λ_{canada}	0.328 (0.070)	0.347 (0.068)	0.335 (0.060)
au us	-0.194 (0.119)	-0.228 (0.136)	-0.252 (0.122)
$ au_{japan}$	-0.148 (0.106)	-0.136 (0.108)	-0.145 (0.105)
$\tau_{germany}$	-0.171 (0.078)	-0.171 (0.078)	-0.170 (0.078)
τ_{france}	-0.470 (0.070)	-0.471 (0.071)	-0.468 (0.070)
τ _{italy}	0.113 (0.083)	0.115 (0.080)	0.117 (0.083)
τ _{uk}	-0.110 (0.080)	-0.113 (0.079)	-0.111 (0.079)
τ _{canada}	-0.091 (0.094)	-0.062(0.091)	-0.073 (0.091)
σ_{us}	0.644 (0.067)	0.603 (0.080)	0.618 (0.069)
σ_{japan}	0.718 (0.057)	0.721 (0.057)	0.722 (0.055)
$\sigma_{germany}$	0.954 (0.054)	0.953 (0.054)	0.955 (0.053)
σ_{france}	0.824 (0.048)	0.822 (0.048)	0.825 (0.048)
σ_{italy}	0.862 (0.051)	0.862 (0.051)	0.865 (0.051)
σ_{uk}	0.932 (0.053)	0.934 (0.053)	0.932 (0.053)
σ_{canada}	0.762 (0.052)	0.772 (0.052)	0.771 (0.050)
π	-2.387 (0.924)	-2.283 (0.953)	-1.986 (0.828)
q	0.941 (0.038)	0.968 (0.035)	0.969 (0.030)
р	0.612 (0.194)	0.636 (0.191)	0.651 (0.200)
01	-	-0.025 (0.029)	-0.227 (0.232)
02	-	-0.036 (0.033)	0.283 (0.255)
03	-	-0.025 (0.028)	-0.554 (0.268)
04 Log Likelihaad	-	-0.051 (0.032)	-0.501 (0.276)
Log Likelinood	-442.54	-438.23	- 433.93

TABLE1: MAXIMUM LIKELIHOOD ESTIMATION OF THE MODEL : G7 $GDP(1960.2 \sim 2001.3)$

Standard errors of the parameters estimates are reported in the parentheses

Although the factor loadings for the common transitory component, λ_{japan} , $\lambda_{germany}$ are insignificant, the inferred probabilities of common transitory recessions without oil price, with Hamilton's 3 year net oil price, with LNR oil price in figure 1 through figure 3 show that Hamilton's 3 year net oil price increases account for 1973-75, 1980, partially 1990-1991 recessions and LNR oil price increases account for 1973-75, 1980, partially 1960, partially 1970, partially 1990-1991 recessions.

Figure 1. Probabilities of common transitory recessions without oil price



Figure 2. Probabilities of common transitory recessions with Hamilton's 3 year oil price



Figure 3. Probabilities of common transitory recessions with LNR oil price





In figure 4, we can clearly compare the oil shocks to the probabilities of common transitory recessions in the G7. From the figure 4, we can infer that oil price shocks have not been a principal determinant of common recessions in the G7 except two major OPEC oil price increases in 1973-1974, 1979-1980.

5. Conclusions

These empirical results suggest a few conclusions. First, the exogenous oil price increases have had impacts on the GDP in the G7. Second, from the probabilities of common transitory recessions with oil price, we can infer that oil price shocks have not been a principal determinant of common recessions in the G7 except two major OPEC oil price increases in 1973-1974, 1979-1980.

An important next step is to find out other reasons for the G7 business cycle fluctuations.

Acknowledgements

The author would like to thank In, S.Y. and Ravi Kavasery for helpful suggestions and comments.

Appendix A

1. Representation

In this section, I discuss representation of the model presented in Section 3. I employ the following state space representation for equations (2)-(4) assuming AR(2) dynamics for the common permanent, common transitory components, and AR(1) dynamics for idiosyncratic component. This model involves unobserved Markov-switching variable S_t in the transitory component and an exogenous variable. The dynamics of Friedman's Plucking Markov Switching model with an exogenous variable can be represented in the following manner:

Measurement Equation : $\Delta y_t = H \xi_t$ Transition Equation : $\xi_t = \alpha_{St} + F \xi_{t-1} + \beta Z_t + V_t$ $E(V_t V_t) = Q$ $Pr(S_t = 0 | S_{t-1} = 0) = q$, $Pr(S_t = 1 | S_{t-1} = 1) = p$, $\pi \neq 0$ for the πS_t

where

$$H = \begin{pmatrix} r_1 & 0 & \lambda_1 & -\lambda_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ r_2 & 0 & \lambda_2 & -\lambda_2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ r_3 & 0 & \lambda_3 & -\lambda_3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ r_4 & 0 & \lambda_4 & -\lambda_4 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ r_5 & 0 & \lambda_5 & -\lambda_5 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ r_6 & 0 & \lambda_6 & -\lambda_6 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ r_7 & 0 & \lambda_7 & -\lambda_7 & 0 & 0 & 0 & 0 & 0 & 1 & - \end{pmatrix}$$

	$\left(\begin{array}{c} \Delta c_t \end{array} \right)$		$\langle 0 \rangle$		$\begin{pmatrix} 0 \end{pmatrix}$	\	$\int V_t$)
	Δc_{t-1}		0		0		0	
	Xt		$\pi \; S_t$		$\sum \beta_i Z_{t-i}$		ut	
	X _{t-1}		0		0		0	
$\xi_t =$	Z _{1t}	$a_{St} =$	0	$\beta Z_t =$	0	$V_{\rm t} =$	e _{1t}	
	Z _{2t}		0		0		e _{2t}	
	Z _{3t}		0		0		e _{3t}	
	Z_{4t}		0		0		e _{4t}	
	Z _{5t}		0		0		e _{5t}	
	Z _{6t}		0		0		e _{6t}	
	Z _{7t}		0))	e _{7t}	J

		ϕ_2	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0	0
	0	0	ψ_{1}	ψ_{2}	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0
F =	0	0	0	0	τ_1	0	0	0	0	0	0
	0	0	0	0	0	τ_2	0	0	0	0	0
	0	0	0	0	0	0	τ_3	0	0	0	0
	0	0	0	0	0	0	0	τ_4	0	0	0
	0	0	0	0	0	0	0	0	τ ₅	0	0
	0	0	0	0	0	0	0	0	0	τ ₆	0
	< 0	0	0	0	0	0	0	0	0	0	$\tau_7 /$

and

	$\int 1$	0	0	0	0	0	0	0	0	0	0 ~
	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
<i>Q</i> =	0	0	0	0	σ^{2}_{1}	0	0	0	0	0	0
	0	0	0	0	0	σ^2_2	0	0	0	0	0
	0	0	0	0	0	0	$\sigma^{2}{}_{3}$	0	0	0	0
	0	0	0	0	0	0	0	σ^{2}_{4}	0	0	0
	0	0	0	0	0	0	0	0	$\sigma^2{}_5$	0	0
	0	0	0	0	0	0	0	0	0	σ^{2}_{6}	0
	< 0	0	0	0	0	0	0	0	0	0	σ^2_7

2. Estimation

Defining S_t and its transitional dynamics as in equations (2)~(4), the above state-space model is an example of that considered by Kim(1994). The following describes Kim's Markov Switching approximate maximum likelihood estimation algorithm. For details of the nature of the approximation and the Bayesian alternative to the estimation procedure, readers are referred to Kim and Nelson(1998). The above state-space model's specific feature is that G7 real GDP's common transitory component follows the Friedman's plucking model by Kim and Nelson(1999), Kim and Murray (2002).

The Kim's Markov Switching approximate maximum likelihood estimation algorithm

is computationally efficient, and experience suggests that the degree of approximation is small ; See Kim(1994) and Kim and Nelson(1998).

Conditional on $S_t = j$ and $S_{t-1} = i$, the Kalman filter equations can be written as:

$$\begin{split} \xi^{(i,j)}_{t|t-1} &= \alpha_{St} + F \ \xi^{i}_{t-1|t-1} + \beta \ Z_{t} \\ P^{(i,j)}_{t|t-1} &= F \ P^{i}_{t-1|t-1} \ F' + Q \\ n^{(i,j)}_{t|t-1} &= \Delta \ y_{t} \ - \ H \ \xi^{(i,j)}_{t|t-1} \\ f^{(i,j)}_{t|t-1} &= H \ P^{(i,j)}_{t|t-1} \ H' \\ \xi^{(i,j)}_{t|t} &= \ \xi^{(i,j)}_{t|t-1} + P^{(i,j)}_{t|t-1} \ H' [f^{(i,j)}_{t|t-1}]^{-1} \ n^{(i,j)}_{t|t-1} \\ P^{(i,j)}_{t|t} &= (I - P^{(i,j)}_{t|t-1} \ H' [f^{(i,j)}_{t|t-1}]^{-1}) \ H \ P^{(i,j)}_{t|t-1} \end{split}$$

where Z_t is an exogenous variable. $\xi^{(i,j)}_{t|t-1}$ is an inference on ξ_t based on information up to time t-1, conditional on $S_t = j$ and $S_{t-1} = i$; $\xi^{(i,j)}_{t|t}$ is an inference on ξ_t based on information up to time t, conditional on $S_t = j$ and $S_{t-1} = i$; $P^{(i,j)}_{t|t-1}$, $P^{(i,j)}_{t|t-1}$ are the MSE matrices of $\xi^{(i,j)}_{t|t-1}$ and $\xi^{(i,j)}_{t|t}$ respectively; $n^{(i,j)}_{t|t-1}$ is the conditional forecast error of Δy_t based on information up to time t-1; $f^{(i,j)}_{t|t-1}$ is the conditional variance of $n^{(i,j)}_{t|t-1}$.

As noted by Harrison and Stevens(1976) and Gordon and Smith(1988) each iteration of the Kalman filter produces a 4-fold increase in the number of cases to consider. To render the Kalman filter operational, we need to collapse the 4² posteriors ($\xi^{(i,j)}_{t|t}$ and $P^{(i,j)}_{t|t}$) into 4 at each iteration. Collapsing requires the following approximations suggested by Harrison and Stevens (1976) :

$$\xi^{j}_{t|t} = \frac{\sum_{i=1}^{2} \Pr[\mathbf{S}_{t-1} = i, \mathbf{S}_{t} = j \mid \Omega_{t}] \quad \xi^{(i,j)}_{t|t}}{\Pr[\mathbf{S}_{t} = j \mid \Omega_{t}]}$$

and

$$P_{t|t}^{j} = \frac{\sum_{i=1}^{2} \Pr[S_{t-1} = i, S_{t} = j \mid \Omega_{t}] \{ P_{t|t}^{(i,j)} + (\xi_{t|t}^{j} - \xi_{t|t}^{(i,j)}) (\xi_{t|t}^{j} - \xi_{t|t}^{(i,j)}) \}}{\Pr[S_{t} = j \mid \Omega_{t}]}$$

where Ω_t refers to information available at time t.

In order to obtain the probability terms necessary for collapsing, we needs the following procedure due to Hamilton(1989) :

Step 1 :

At the beginning of the i^{th} iteration, given $Pr[S_{t-1} = i | \Omega_{t-1}]$, we calculate

 $Pr[S_{t-1} = i, S_t = j | \Omega_{t-1}] = Pr[S_t = j | S_{t-1} = i] Pr[S_{t-1} = i | \Omega_{t-1}]$

Step 2 :

Consider the joint density of Δy_{t} , S_{t} , and S_{t-1} :

$$f(\Delta y_{t}, S_{t-1} = i, S_{t} = j | \Omega_{t-1}) = f(\Delta y_{t} | S_{t-1} = i, S_{t} = j, \Omega_{t-1}) \Pr[S_{t-1} = i, S_{t} = j | \Omega_{t-1}]$$

from which the marginal density of Δy_t is obtained by:

$$f(\Delta y_{t} | \Omega_{t-1}) = \sum_{i=1}^{2} \sum_{j=1}^{2} f(\Delta y_{t}, S_{t-1} = i, S_{t} = j | \Omega_{t-1})$$

= $\sum_{i=1}^{2} \sum_{j=1}^{2} f(\Delta y_{t} | S_{t-1} = i, S_{t} = j, \Omega_{t-1}) \Pr[S_{t-1} = i, S_{t} = j | \Omega_{t-1}]$

where the conditional density $f (\Delta y_t | S_{t-1} = i, S_t = j, \Omega_{t-1})$ is obtained via the prediction-error decomposition:

$$f(\Delta \mathbf{y}_{t} | \mathbf{S}_{t-1} = \mathbf{i}, \mathbf{S}_{t} = \mathbf{j}, \ \Omega_{t-1}) = (2 \pi)^{-T/2} |f^{(\mathbf{i},\mathbf{j})}_{t|t-1}|^{-1/2} \exp\{-1/2 n^{(\mathbf{i},\mathbf{j})'}_{t|t-1} f^{(\mathbf{i},\mathbf{j})}_{t|t-1} - n^{(\mathbf{i},\mathbf{j})}_{t|t-1}\}$$

Step 3 :

Once Δy_t is observed at the end of time t, we update the probability terms:

$$Pr[\mathbf{S}_{t-1} = \mathbf{i}, \mathbf{S}_t = \mathbf{j} | \Omega_t] = Pr[\mathbf{S}_{t-1} = \mathbf{i}, \mathbf{S}_t = \mathbf{j} | \Omega_{t-1}, \Delta \mathbf{y}_t]$$

$$= \underbrace{\frac{f(\mathbf{S}_{t-1} = \mathbf{i}, \mathbf{S}_t = \mathbf{j}, \Delta \mathbf{y}_t | \Omega_{t-1})}{f(\Delta \mathbf{y}_t | \Omega_{t-1})}$$

$$= \underbrace{\frac{f(\Delta \mathbf{y}_t | \mathbf{S}_{t-1} = \mathbf{i}, \mathbf{S}_t = \mathbf{j}, \Omega_{t-1}) Pr[\mathbf{S}_{t-1} = \mathbf{i}, \mathbf{S}_t = \mathbf{j} | \Omega_{t-1}]}{f(\Delta \mathbf{y}_t | \Omega_{t-1})}$$

with $\Pr[\mathbf{S}_t = j \mid \Omega_t] = \Sigma_{i=1}^2 \Pr[\mathbf{S}_{t-1} = i, \mathbf{S}_t = j \mid \Omega_t]$

As a byproduct of the above filter in Step 2, we obtain the log likelihood function: $\ln L = \sum \ln(f(\Delta y_t | \Omega_{t-1}))$

which can be maximized with respect to the parameters of the model.

Appendix B

1. Summary Unit Root Tests⁸ for the quarterly G7 real GDP (1960:1 - 2002:4)

Augmented	Dickey Fuller t-Statistic	Critical Value				
		10%	5%	1%		
======================================	-0.78	=============				
Y _{JAPAN}	-1.56					
Y GERMANY	-2.52					
Y FRANCE	-2.45	-3.14	-3.44	-4.02		
Y ITALY	-2.31					
Y _{U.K}	-1.15					
Y _{CANADA}	-1.16					

* reject 10% critical value, ** reject 5% critical value, *** reject 1% critical value

3. Johansen(1991, 1995) Cointegration Tests⁹ for the quarterly G7 real GDP (1960:1 - 2002:4)

Null Hypothesis Te	est Statistic	Critical Value			
		5%	1%		
No Cointegration Vectors	125.6*	124.2	133.6		
At Most One Cointegration Vectors	78.5	94.2	103.2		
At Most Two Cointegration Vectors	47.3	68.5	76.1		
At Most Three Cointegration Vector	rs 30.1	47.2	54.5		
At Most Four Cointegration Vectors	s 15.2	29.7	35.7		
At Most Five Cointegration Vectors	6.8	15.4	20.0		
At Most Six Cointegration Vectors	0.0	3.8	6.7		

* reject 5% critical value ** reject 1% critical value

⁸ 4 lag was chosen for real GDP. Tests for real GDP included a time trend and constant in the test

regression⁸ The test statistic is the Likelihood Ratio statistic and calculated in Eviews using a lag order 4 and each series has a linear trend but the co-integration equation has only intercepts.

Appendix C

4. Sources for GDP Data

I thank Dalsgaard, Elmeskov and Park for sending me the internal OECD series from Dalsgaard, Elmeskov and Park(2002). In the Stock and Watson(2003) p27, Real GDP series were used for each of the G7 countries for the same period 1960:1 – 2002:4. The table below gives the data sources and sample periods for each periods for each data series used. Abbreviations used the source column are (DS) DataStream, (DRI) Data Resources and (E) for an internal OECD series from Dalsgaard, Elmeskov, and Park(2002).

Country	Source	Sample period		
Canada	OECD (DS)	1960:1	1960:4	
	STATISTICS CANADA (DS)	1961:1	2002:4	
France	OECD (DS)	1960:1	1977:4	
	I.N.S.E.E. (DS)	1978:1	2002:4	
Germany	DEUTSCHE BUNDESBANK (DS)	1960:1	2002:4	
Italy	OECD (DS)	1960:1	1969:4	
	ISTITUTO NAZIONALE DI STATISTICA (DS)	1970:1	2002:4	
Japan	OECD (DS)	1960:1	2002:4	
UK	OFFICE FOR NATIONAL STATISTICS (DS)	1960:1	2002:4	
US	Dept. of Commerce (DRI)	1960:1	2002:4	



Figure 5. G7 real GDP : 1960:1 ~ 2002:4

Figure 6. G7 log differenced real GDP from 1960:2 ~ 2002:4



References

Beaudry, Paul, Gary Koop (1993) Do recessions permanently change output? Journal of Monetary Economics 31:149-163

Box and Jenkins (1976) Time Series Analysis : forecasting and control , Holden Day.

Campbell, John Y., Mankiw, N. G. (1987) Are out fluctuations transitory ? Quarterly Journal of Economics 102:857-880

Chauvet, M. (1998) An econometric characterization of business cycle dynamics with factor structure and regime switching. International Economic Review 39:969-996

Clark, Peter K. (1987) The cyclical component of U.S. economic activity. Quarterly Journal of Economics 102:797-814

Clements, M. P., Krolzig, H.-M. (2002) Can oil shocks explain asymmetries in the US Business Cycle? Empirical Economics 27:185-204

Cohrane, John H. (1988) How big is the random walk in GNP? Journal of Political Economy 96:893-923.

Dalsgaard, T., Elmeskov, J., Park, C.Y.(2002) Ongoing Changes in the Business Cycle -Evidence and Causes. OECD Economic Department Working Paper 315

Gregory, Allan W & Head, Allen C & Raynauld, Jacques (1997) Measuring World Business Cycles. International Economic Review 38:677-701.

Hamilton, J.D.(1983) Oil and the Macroeconomy since World War II. Journal of Political Economy 91:228-248

Hamilton, J.D.(1989) A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica 57:357-384

Hamilton, J.D.(1996) This is what happened to the oil price-macroeconomy relationship. Journal of Monetary Economics 38:215-220

Hamilton, J.D.(1994) Time series analysis. Princeton University Press, Princeton

Hamilton, J.D.(2003) What is an oil shock? Journal of Econometrics 113:363-398

Hooker, M. A. (1996) Whatever happened to the oil price-macroeconomy relationship? Journal of Monetary Economics 38:195-213

Hooker, M. A. (1996) This is what happened to the oil price-macroeconomy relationship: Reply. Journal of Monetary Economics 38:221-222

Kim, C.J. (1994) Dynamic factor models with Markov switching. Journal of Econometrics 60:1-22

Kim, C.J., Nelson, C.R. (1998) Business cycle turning points. A new coincident index and tests of durations dependence based on a dynamic factor model with regime switching. Review of Economics and Statistics 80:188-201 Kim, C.J., Nelson, C.R. (1999) State-space models with regime switching: Classical and Gibbs sampling approaches with applications. MIT Press, Cambridge

Kim, C.J., Nelson, C.R.(1999) Friedman's Plucking Model of Business Fluctuations: Tests and Estimates of Permanent and Transitory Component. Journal of Money, Credit, and Banking 31:317-334

Kim, C.J., Murray, C.J.(2002) Permanent and transitory components of recessions. Empirical Economics 27:163-183

Kim, C.J, Piger J., Startz, R. (2002) Permanent and Transitory Components of Business Cycles: Their Relative Importance and Dynamic Relationship. The Federal Reserve Bank of St. Louis Working Paper 2001-017B

Kose, M.A., Otrok ,C., Whiteman, C.H. (2002) Understanding the Evolution of World Business Cycles. presentation.

Kose, M.A., Otrok, C., Whiteman, C.H. (2003) International Business Cycles: World, Region, and Country-Specific Factors. Forthcoming, American Economic Review

Lam, Pok Sang (1990) The Hamilton model with a general autoregressive component: estimation and comparison with other models of economic time series. Journal of Monetary Economics 26:409-432

Lee, K., Ni, S., Ratti, A.(1995) Oil shocks and the macroeconomy: The role of price variability. Energy Journal 16:39-56

Monfort A., J.P.Renne, R. Ruffer, G. Vitale (2002) Is Economic Activity in the G7 Synchronized? Common Shocks vs. Spillover Effects. CEPR Discussion Paper No. 4119

Mork, K. A. (1989) Oil and the Macroeconomy when prices go up and down: An extension of Hamilton's results. Journal of Political Economy 97:703-708

Nelson, Charles R., Plosser, Charles I., (1982) Trends and random walks in macroeconomic time series: Some evidence and implications. Journal of Monetary Economics 10:139-162

Rasche, R.H., Tatom, J.A. (1977) Energy resources and potential GNP. Federal Reserve Bank of St. Louis Review 59:10-24

Rasche, R.H., Tatom, J.A. (1981) Energy price shocks, aggregate supply, and monetary policy: the theory and international evidence. In: Brunner, K., Meltzer, A. H. (eds.), Supply Shocks, Incentives, and National Wealth, Carnegie-Rochester Conference Series on Public Policy, Vol. 14 North Holland, Amsterdam.

Raymond, J. E., Rich, R. W. (1997) Oil and the macroeconomy: A Markov stateswitching approach. Journal of Money, Credit, and Banking 29:193-213

Stock, J.H., Watson, M.W. (1989) New indexes of coincident and leading indicators. In:

Blanchard OJ, Fisher S. (eds) NBER macroeconomics annual. MIT Press, pp351-393

Stock, J.H., Watson, M.W. (1991) A probability model of the coincident and leading indicators. In: Lahiri K, Moore GH(eds.) Leading economic indicators: New approaches and forecasting records. Cambridge University Press, New York, pp63-85

Stock, J.H., Watson, M.W.(2003) Understanding changes in international business cycle dynamics. NBER working paper 9859

Tatom, John A. (1988) Are the macroeconomic effects of oil price changes symmetric? In: Karl Brunner and Allan H. Meltzer(eds.) Stabilization policies and labor markets: Carnegie-Rochester Conferences on Public Policy 28:325-368. Amsterdam, North Holland

Watson, Mark M. (1986) Univariate detrending methods with stochastic trends. Journal of Monetary Economics 18:29-75

Yoon, Jae Ho (2004) Has the G7 business cycle become more synchronized? POSCO Research Institute Working Paper 2004