Evolutionary Forces in a Banking System with Speculation and System Risk

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Abstract

For an N players coordination games, Tanaka (2000) proved that the notion of N/2 stability defined by Schaffer (1988) is a necessary and sufficient condition for such a long run equilibrium in an evolutionary process with mutations (in the sense of Kandori, et. al. (1993)). We argue that the critical number in Schaffer's stability is not unique in every application, but can vary with variables determined before the coordination games. In our specific model, these variables are the portfolio choices of the banks. We derived a Z* stability condition for the long run equilibrium for the banking system, in which there is no speculative bank run. This critical number of players is a function of the size for risky investment, and varies with total risky investments when there are more than two banks. We use this framework to analyze the effect of speculative behavior on banks' risk taking and the phenomenon of system risk, calculating the probability when more than one banks fail together (system risk).

1

1 Introduction

Since Diamond and Dybvig[1] (1983), the speculative behavior in the banking system has received much attention in the literature. *Speculative run* refers to the case where all depositors demand their money simultaneously, which then forces the bank to liquidate its assets at short notice, which may provoke its failure. This equilibrium, however, is just one of the two in the coordination games where all depositors make their simultaneous decisions to withdraw or not. Like all coordination games, e.g., the stock or exchange markets, the multiplicity and indeterminacy problem has impeded further analysis on the effects of speculative behavior to the banking system, let alone the contagious effects of bank failures, noticed as system risks.

There have been several approaches[2] proposed to resolve this indeterminacy. We focus our attention on the one driven by the *evolutionary force* with the random mutations by Kandori, Mailath, and Rob (1993, hence KMR). In the evolutionary explanation, it is assumed that the time span for depositors' decisions is stretched into long enough periods, and an explicit dynamic process is specified describing

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[1]Diamond and Dybvig (1983) studied a *single* bank *screening* model, with incomplete information about infinitely many depositors' types for early and late consumption. They showed that there are two types of Bayesian equilibria. One, a Pareto-dominant equilibrium, has only depositors with genuine preference for early consumption withdrawing early. The second, a Pareto-dominated speculative bank run equilibrium, has depositors who actually prefer late consumption, fearing withdrawal by others of the same type, also withdrawing early.

[2]One approach is to introduce *sunspots*, which turns the coordination games into a game with incomplete information (see Harsanyi (1973)). This approach does not select either equilibrium, rather, it characterises the range of private information where each equilibrium happens. Recent literature (Goldstein and Pauzner (2003) for example) has applied the approch of global game by Classon and Van Damme (1993). The intution is to introduce private observation of each agent about the incomplete information in the sunspots model. They show that, a risk dominant equilibrium will be selected if agents adopt dominant strategies in the incomplete information games (See also Morris and Shin (2002)).

how depositors adjust their choices over time as they learn (from experience) about the other depositors' choices. "This approach tries to explain how an equilibrium emerges, based on trial-and-error leaning" (KMR, p30). *Unfortunately*, this does not help on equilibrium selection, since both equilibria (due to their strictness) survive the evolutionary force of this sort.

The contribution of KMR is to introduce a random switch[3] (characterized by a Markov chain) into the evolutionary process. This randomness allows transitions (perpetually fluctuates) from one equilibrium to the other in the course of evolution, and an equilibrium is called long term equilibrium, if the system spends most of the time on that equilibrium. Tanaka (2000) showed that the N/2 stability defined by Schaffer (1988) is a necessary and sufficient condition for such a long run equilibrium in the sense of KMR. Temzelides (1997) applied this N/2 stability to the Diamond and Dyvig's model, and showed that a sufficiently high deposit rate suffices to select the no run equilibrium as the long term equilibrium.

The purpose of this paper is to argue that the critical number in Schaffer's stability is not unique in every application, but can vary with variables determined before the coordination games of depositors. In our specific model, the variables are portfolio choices of the banks. We derived a Z* stability condition for the long run equilibrium in the banking system. This critical number of players is a function of the size for risky investment, and varies with total risky investments when there are more than two banks. We use this framework to analyze the effect of speculative behavior on banks' risk taking, and the phenomenon of system risk, calculating the probability when more than one banks fail together.

In the present paper, banks act as portfolio managers[4], essentially developed

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[3]Kim (1996) compares the vaious evolutionary froces with randomness, including Matsui and Matsuyama (1995), Young (1993), KMR (1993), and Foster and Young (1990). KMR assumes a random (formulated by the Markov chain) mutation.

[4]It is well known that sufficiently large exogenous shocks can cause a crisis. For example, Allen and Gale (1998) describe a model in which financial crises are caused by exogenous asset-

by Pyle (1971), and Hart and Jaffee (1974). The idea is to assimilate all assets and liabilities of the bank and consider the whole bank itself as an enormous portfolio of these securities. Klein (1974) pointed a major weakness in traditional portfolio theoretic model that ceteris paribus, if a bank wishes to increase its loan/asset ratio it must accept a reduction in the marginal return on loans. That is, *the return is increasing*, but the marginal rate of return is decreasing in the size of investment. The most famous application of this approach is to provide a framework to analyze the risk taking behavior with the rescue policy (e.g., bail out) in case of failures (see Freixas and Rochet (1998)). Here, it serves a more important role in that it provides a channel to explain the *interconnection among banks*: as quantity competition in an industry, the concavity of the marginal value function induces a game among banks' investments. Thus, via their mutual influence on marginal value of the same risky assets, the extent that banks are connected will be determined in the system. In other words, we can use this framework to discuss the system risk problem.

Evidence of bank crises caused by bad portfolio management can be found in Japan. Total commercial loans outstanding as a percentage of GDP was recorded to increase from 73% in 1986 to 97% in 1992. Most of the loans were to the home loan finance companies, to their keiretsu affiliates and to borrowers speculating in real estate property. Foo (2003) observed that the banks are affected by the stock market downturn as the Nikkei's drop shrinks the value of the banks' stock portfolio. The banks depend on a rising stock value to boost capital gains on profits or sell their stock portfolio assets to write off their NPLs. With a falling stock market, the banks have trade off losses to both their NPLs and their stock portfolio. Moreover, the declined stock portfolio erodes the paper gains use to meet the strict Basle international capital adequacy standards of 8% of outstanding loans.

We consider the following two period game, after adding speculative with-

return shocks. Following a large (negative) shock to asset returns, banks are unable to meet their commitments and are forced to default and liquidate assets.

3

drawals among depositors in Klein's portfolio choices. At the beginning of the first stage, banks, receiving[5] a total one unit of fund, determine the proportion to invest in safe asset (high liquidity) or risky asset (low liquidity). Safe asset gives low but certain return, while the risky asset gives high but uncertain returns containing two parts: deterministic part (as Klein) and a random term. A trade off will be: the benefit for investing more risky asset is for high return; while the cost will be the possible negative shock and the penalty for early liquidation. The shock is realized and observed by each agents at the end of the first stage. Observing this shock, agents make decisions whether to withdraw their deposits from the bank, and the payment of the investments are received at the end of stage two, if the banks did not fail. In such a way, the whole banking system are connected.[6].

Our specific results include: first, we propose a Z* stability condition, which is proved to be a necessary and sufficient condition for such a long run equilibrium in the sense of KMR. This critical number of Z* is a function of the total risky investment in the banking system. In the case with two banks, this value could vary across banks. Second, speculative behaviors do not frustrate single bank's risky taking, but rather, encourage the bank to maintain a high enough level of risky investment, to keep the system stay in the equilibrium of no run. This indicates that although the speculative run equilibrium will be eliminated in the long run, the probability of fundamental run will increase with the mere possibility of speculative behavior. Third, the single bank case does not necessarily apply to the case with multiple banks. Symmetric banks can take different level of risks, which induces a different in the probability of bank failures. The probability of joint failures increases, compare to the case without speculation, but the individual probability of bank failures do not necessarily increase.

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[5]Here we do not consider predeposit decision, because consuemrs would not agree to deposit if they knew that a run would take place (see Peck and Shell (2003)).

[6]The timing is similar to the timing in the sunspot literature (see Peck and Shell (2003) and Ennis and Keister (2002).

Temzelides (1997) also studied equilibrium selection by evolutionary process in a banking system where depositors can strategically choose to withdraw prematurely or to stay within a bank. Temzelides showed that by setting a sufficiently high deposit rate, banks can avoid the speculative run result in the post deposit stage by selecting no run equilibrium as the long term equilibrium. The intuition is that evolutionary force will pick up the strategy with high *relative* payoff. By increasing deposit rate, staying within the bank will give relative high payoff. Moreover, the probability of bank run can be decrease to zero hence. This no-run result is criticized to be unrealistic (Peck and Shell), because we do encounter bank failures from time to time. Bank failures do happen in our model, and the difference comes from our setting of a random shock, denoting the sudden change of the fundamentals in the system. Hence, like Temzelides (1997), banks can avoid speculative run in the long run, the probability of fundamental run increases.

Compared to the literature using Carlsson and van Damme (1993)'s framework, since agents are assumed to make noisy observations about fundamentals. These observations serve as a coordination device for agent beliefs about the true state of the economy. The construction allows for determining a unique equilibrium for each realization of fundamentals in the Diamond and Dybvig model. An excellent paper analyzing this approach is Goldstein and Pauzner (2000).

Owing to its single bank mechanism problem, the problem of system risks is not mentioned in the Diamond and Dybvig model. Chen (1999), Dasgupta (2003) and Rochet and Vives (2002) extends Diamond and Dybvig model to explain contagious bank failures due to bank runs, by considering interbank deposits like those in Rochet and Tirole (1996). There are other interbank linkages like the contractual obligations between banks, OTC derivative and money market transactions (Staub (1998)).

The remainder of the paper is organized as follows. Section 2 gives the model with a representative bank. We characterize the criteria of a Z* stability and

prove that it will induce a long run equilibrium in the sense of KMR. Section 3
extends the model to two banks, and the Z* stability is revisited. We then discuss
the probability of systemic risk and speculative behaviors. Section 4 contains the
concluding remark.

2   The Model

In this section, I describe the environment in a banking system with one bank. I
will turn to the multiple bank case in the next section. Throughout the model,
all agents are assumed risk neutral[7] and services provided by bank(s) are assumed
homogenous.

The model has two periods, $t = 1, 2$. Figure 1 helps illustrating the timing.
Firstly, in the beginning of $t = 1$, there N identical depositors who put[8] their
money in bank 1. This total deposit of one unit is the only source of fund (see
also Acharya,2001, Matutes and Vives (2000) for similar assumptions). The bank
behaves as a portfolio manager[9] of this fund, and invests the borrowed fund in safe
and risky (with less liquidity) asset. Let s be the proportion invested in the safe
asset and r be the proportion invested in the risky asset. With fund constraint,
$s + r = N$. The investment is divisible (like Matutes and Vives (2000)).

Safe asset produces certain but relatively low return per unit of investment,
and to simplify (but will not affect our point), the marginal rate of return is
assumed to be 1. In other words, the investment on safe asset performs more

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[7]Hence we do not consider the wealth effect coming from the increasing degree assumption of
relative risk attitude.

[8]There is no predeposit decision, that is, whether to deposit in a bank like Beck and Shell
(2003).

[9]The theory of portfolion management has helped for banking behavior, essentially developed
by Pyle (1971), and Hart and Jaffee (1974). The idea is to assimilate all assets and liabilities of
the bank into securities of a particualr, and to consider the whole bank itself as asn enromour
portfolio of these securites.

like the preparation for sudden liquidity need. Risky asset, on the other hand, produces a relatively higher but uncertain return. The return consists of two part: deterministic and shock. The deterministic part is a increasing concave function, reflecting the monopoly influence of the bank on the project. Following Klein (1974), we assume this marginal rate of return to be a *concave* function of level of investment. That is, *if all investment lasts* until the end of $t = 2$, denote $R(r) + \varepsilon$ as the marginal return for per unit of investment on risky asset, and $R'(r) > 0$ and $R''(r) < 0$. The shock reflects unexpected effect, which is distributed according to $F(.)$ over $(-\infty, \infty)$, with density function of $f()$. In case of premature withdrawal (which is divisible), there will be a fine of $\lambda > 0$, per unit of investment withdrawn, catching the suspending cost of withdrawal.

Having made its investment decision under uncertainty, we follow Diamond and Dybvig (1983) in assuming that the bank is mutually owned and liquidated in period 2. So, the chief managers of the bank and the depositors, who do not withdraw in period 1, will get a pro rata share of the bank's assets in period 2. Since the determination of share of profit is not our main point, we assume that a proportion α, $0 < \alpha < 1$, of the banks' asset will pay the salaries to the chief manager, and $(1 - \alpha)$ will be splitted to the remaining depositors.

In the end of $t = 1$, the shock on the risky asset is realized. Having observed the bank's portfolio decision[10] and the value of realized shock, each depositor determines whether to withdraw from the bank in the beginning of $t = 2$. Here, we concentrate on the *coordination* effect among depositors (see also Temzelides (1997)), without discussing mimicking behavior induced by the assumption of private information about the types of depositors (like the DD model). Each depositor needs to compare the relative returns for early and late withdrawals, which depend on the size of shock as well as the total number of withdrawers. Let z (to be

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[10]That is, we consider full disclosure of the bank's portfolio decision. Partially disclosure like in Davies and McManus (1991), and Matutes and Vives (2000) will be mentioned in our further research.

endogenously determined) be the number of depositors who choose to stay until the end of $t = 2$. The per unit return for withdrawing now is assumed to be 1 if the bank is solvent; if the bank becomes insolvent, it is assumed that each depositor gets the payment from deposit insurance ω, with $\omega \geqslant 0$. In Diamond and Dybvig (1983, p408), ω is assumed to be zero. That is, let $\varepsilon^*(r, z)$ be the level of shock below which the bank will become insolvent. The per unit return for early withdrawal is $u^1(r, z, \varepsilon)$.

$$
\begin{aligned}
u^1(r, z, \varepsilon) &= 1 & if \quad \varepsilon \geqslant \varepsilon^*(r, z), \\
&= \omega^{11} & if \quad \varepsilon < \varepsilon^*(r, z).
\end{aligned}
$$

Here, *insolvency* denotes the case where a bank's equity reaches a non-positve value (Freixas and Rochet (1998, p 248)). Let $\pi(r, z, \varepsilon)$ denote the bank's equity value given that the withdrawing number is z. There are two possible values for $\pi(r, z, \varepsilon)$: if $z \geqslant r$, the overall withdrawal is affordable by the liquidity preparation, then $\pi(r, z, \varepsilon) = (z - r) + r[R(r) + \varepsilon]$; if $z < r$, the bank needs to pay the per unit fines π for convertibility if the investment, and hence $\pi(r, z, \varepsilon) = (r - (1 + \lambda)(r - z))$ $[R(r - (1 + \lambda)(r - z)) + \varepsilon]$. In both cases, $\pi(r, z, \varepsilon)$ is increasing in z and ε. Hence the critical value $\varepsilon^*(r, z)$ is the value such that for $z \geqslant r$, $(z - r) + r[R(r) + \varepsilon] = 0$ and for $z < r$, $(r - (1 + \lambda)(r - z))[R(r - (1 + \lambda)(r - z)) + \varepsilon]. = 0$. $\varepsilon^*(r, z)$ is smaller when $z \geqslant r$. It is important to notice that insolvency involves with two kinds of bank failures: *fundamental run* and *speculative run*. The former happens when $\varepsilon < \varepsilon^*(r, N)$, and the latter happens when $\varepsilon^*(r, N) < \varepsilon < \varepsilon^*(r, 1)$. assumption (continuous, monotonically increasing) guarantees that $\varepsilon^*(r, N)$ and $\varepsilon^*(r, 1)$ exist.

If a depositor does not withdraw prematurely, then she will get a pro rata share of the bank's assets in period 2. Let $u^2(r, z, \varepsilon)$ denote the per unit return.

$$
\begin{aligned}
u^2(r, z, \varepsilon) &= \frac{(1 - \alpha)\pi(r, z, \varepsilon)}{z} & if \quad \varepsilon \geqslant \varepsilon^*(r, z), \\
&= \omega & if \quad \varepsilon < \varepsilon^*(r, z).
\end{aligned}
$$

All returns are realized in the end of $t = 2$.

This game is solved backward. It is a typical n player coordination game, so the following result has been proved by various literature. (footnote here)

Lemma *The states of $z = 0$ and $z = N$ are the only two NE in pure strategy.*

Proof: See the proof for Lemma one in Kim (1996). $\square$

Let $\varphi(r, z, \varepsilon) = u^2(r, z, \varepsilon) - u^1(r, z, \varepsilon)$.

$$
\begin{aligned}
\varphi(r, z, \varepsilon) &= \frac{(1-\alpha)\pi(r, z, \varepsilon)}{z} - 1 & if \quad \varepsilon \geqslant \varepsilon^*(r, z), \\
&= 0 & if \quad \varepsilon < \varepsilon^*(r, z).
\end{aligned}
$$

KMR presented an analysis of long run equilibria of stochastic evolutionary dynamics for 2×2 games. Tanaka (2000) extends their model to an N players game. To incorporate this approach, it is assumed that the depositors take turn (with uncertain order) to make their withdrawing decisions, and if the interval between turns is sufficiently small, the time span for the second period can be stretched to be sufficiently (in KMR's sense) very long. Let the subscript t denote the value for a variable at run t. Then z_t denote the number of agents choosing to stay in run t. A Darwinian deterministic component is defined as: $z_{t+1} = b(z_t)$ Hence $b(z) > z$ when $\varphi(r, z, \varepsilon) > 0$, and $b(z) < z$ when $\varphi(r, z, \varepsilon) < 0$.

Two related notions on evolutionary stability are defined as follows. Firstly, the notion of finite population Evolutionarily stable strategy (ESS) by Schaffer (1988) is defined as the following: if $\varphi(r, n-1, \varepsilon) > 0$, staying is a finite population ESS; if $\varphi(r, 1, \varepsilon) < 0$, withdrawing is a finite population ESS.

Lemma Both t*he states of $z = 0$ and $z = N$ are equilibrium in ESS.*

Proof: Since both are strict equilibrium, the results are proved in, for example, Weibull (1995). $\square$

Second, Shaffer (1988) further defines M-stabilty of finite population ESSs. Consider a state in which all players chooses not withdrawing. If there are M or

fewer mutant players chooses to withdraw, the average payoff of the players who choose to stay is larger than the average payoff of the mutant players, then staying is called an M- stable (finite population) ESS. Formally, staying is an M-stable ESS if $\varphi(r, z, \varepsilon) > 0$ for $z > M$, and $\varphi(r, z, \varepsilon) < 0$ for $z < M$. When $M = N/2$ we obtain the condition for staying to be an $N/2$ stable ESS[12] as follows:

$$\varphi(r, \frac{N}{2}, \varepsilon) > 0$$

N/2 stability of staying means that, when N/2 (a half of population) or fewer mutant players choose to withdraw, the average payoff of the players who choose to stay is larger than the average payoff the mutant players. Tanaka (2000) showed that N/2 stability of a finite population evolutionarily stable strategy defined by Shaffer (1988) is a necessary and sufficient condition for a long run equilibrium in the sense of KMR. In Temzelides (1997), there is no random shock ε, and the risky investment is replaced with deposit rate. But most importantly, Temzelides replaced z in $\varphi(r, z, \varepsilon)$ with $\frac{N}{2}$ to characterize the conditions of deposit rates for "staying" to be an $\frac{N}{2}$ stable ESS. We argue that the critical value is no longer $\frac{N}{2}$ when r is determined before the beginning of the coordination games. The intuition is easily seen from comparing $u^1(r, z, \varepsilon)$ and $u^2(r, z, \varepsilon)$, where a higher r will increase the advantage for staying, and hence will increase the number of mutants to defect from the equilibrium. Hence, denote $Z^*(r, \varepsilon)$ as the number of agents choosing to stay, given r and ε.

Proposition (1) If $\varphi(r, Z^*(r, \varepsilon), \varepsilon) > 0$, a long run equilibrium is the state z=N, where all agents stay. (2) If $\varphi(r, Z^*(r, \varepsilon), \varepsilon) < 0$, a long run equilibrium is the state z=0, where all agents withdraw. (3) $Z^*(r, \varepsilon)$ decreases with r and ε.

Proof: See KMR's theorem 3 and the theorem in Phode and Stegeman (1996). □

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[12]This condition is for staying to be an N/2 or higher stable ESS. If the following condition holds in addition to this equation, then staying is exactly N/2 stable. $\varphi(r, \frac{N}{2} - 1, \varepsilon) < 0$

Let $\widehat{\varepsilon}(r)$ be the lowest level of shocks that $\varphi(r, Z^*(r,\varepsilon), \varepsilon) > 0$. We need to determine the location of $\widehat{\varepsilon}(r)$. Since by definition, $\varepsilon^*(r, z)$ is the level of shocks such that $\pi(r, z, \varepsilon) = 0$, and $\varepsilon^*(r, N) < \varepsilon^*(r, z) < \varepsilon^*(r, 1)$. Because when $\varphi(r, Z^*(r,\varepsilon), \varepsilon) = 0$, we have $\pi(r, Z^*(r,\varepsilon), \varepsilon) = \frac{Z^*(r,\varepsilon)}{(1-\alpha)} > 0$. It is true that $\varepsilon^*(r, Z^*(r,\varepsilon)) < \widehat{\varepsilon}(r)$. We still need to determine whether $\widehat{\varepsilon}(r) \gtreqqless \varepsilon^*(r, 1)$. Since $\widehat{\varepsilon}(r)$ is the level of shocks that will support all depositors to stay in the long run, and $\varepsilon^*(r, 1)$ is the level of shocks that will support all depositors to stay right away. It must be that $\widehat{\varepsilon}(r) < \varepsilon^*(r, 1)$. Moreover, since $\pi(r, z, \varepsilon)$ takes two values depending on the size of r. Let $\widehat{\varepsilon}^1(r)$ and $\widehat{\varepsilon}^2(r)$ be the respective critical value for $r \leqslant Z^*(r, \varepsilon)$ and $r > Z^*(r, \varepsilon)$. Comparing the relative size of $\pi(r, z, \varepsilon)$, it can be shown that $\widehat{\varepsilon}^1(r) < \widehat{\varepsilon}^2(r, \lambda)$, as $R'(r) > 0$. Moreover, $\frac{\partial \widehat{\varepsilon}^1(r)}{\partial r} = \frac{\partial \widehat{\varepsilon}^1(r)}{\partial r} < 0. In summary, when \varepsilon \geqslant \widehat{\varepsilon}^1(r)$ and $\widehat{\varepsilon}^2(r)$ for $r \leqslant Z^*(r, \varepsilon)$ and $r > Z^*(r, \varepsilon)$, respectively, the bank will not encounter bank failure in the long run.

In the beginning of the $t = 1$, the bank determine its portfolio choices on safe and risky assets. That is, let $\pi^k(r, N, \varepsilon) = \max_{r,s} \int_{\widehat{\varepsilon}^k(r)}^{\infty} \{s + r[R(r) + \varepsilon]\} dF(\varepsilon), k = 1, 2$. In the beginning of $t = 1$, the bank $max\{\pi^1(r, N, \varepsilon), \pi^2(r, N, \varepsilon)\}$.

Proposition *(1) The optimal level of risky assets is higher with speculation. (2) The bank will invest less than a half of the deposit on the risky asset.*

Proof: Rearrange $\pi^k(r, N, \varepsilon) = (N - r) + r[R(r) + (1 - F(\widehat{\varepsilon}^k(r))]$. Let $\underline{r}$ and $\overline{r}$ denote the values that maximizes $\pi^1(r, N, \varepsilon)$ and $\pi^2(r, N, \varepsilon)$. The first order condition of maximization is: $1 = [R(r) + (1 - F(\widehat{\varepsilon}^k(r))] + r[R'(r) - f(\widehat{\varepsilon}^k(r))]\frac{\partial \widehat{\varepsilon}^k(r)}{\partial r}$. Denote the RHS of the above equation as $X^k(r)$. The second order condition of maximization is that $\partial X^k(r)/\partial r < 0$. Since $\widehat{\varepsilon}^1(r) < \widehat{\varepsilon}^2(r)$, it can be checked from the first order conditions that $r^1 < r^2$.

Suppose $\pi^1(r, N, \varepsilon) < \pi^2(r, N, \varepsilon)$, that is, by the definition of maximization, $X^2(r^2) = 1 \geqslant X^2(r^1)$, which cannot be true because $\partial X^k(r)/\partial r < 0$ by the second order condition. Hence it must be that $\pi^1(r, N, \varepsilon) \geqslant \pi^2(r, N, \varepsilon)$. $\square$

3   Multiple Banks and System Risk

In this section, we describe the case with two banks, and calculate the probability of joint failures.

The timing of the game is as the single bank case. There are N depositors for each bank, which invests the deposit on the safe and risky assets. Let s_i and r_i be the portfolio choices for bank i, and $s_i + r_i = N$.

The marginal returns for safe assets are assumed to be one, and that of risky assets are a function of two banks' risky investment, plus a random term, representing the effects from unexpected shocks. That is, denote $R(r_1 + r_1) + \varepsilon$ as the marginal return for per unit of investment on risky asset, and $R_i(.) = \frac{R()}{\partial r_i} > 0$ and $R_{ij}(.) < 0$ for $i, j = 1, 2$. The distribution of ε is as the single bank case.

Given banks' portfolio decisions, each depositor observes the realization of the random term and make their decisions to withdraw or stay. The decisions are similar to the single bank case. Denote z_i as the number of depositors who choose to stay until the end of $t = 2$, and $\varepsilon_i^*(r_1, r_2, z_i)$ as the level of shock below which the bank will become insolvent. The per unit return for early withdrawal is $u^1(r_1, r_2, z_i, \varepsilon)$.

$$
\begin{aligned}
u_i^1(r_1, r_2, z_i, \varepsilon) &= 1 & if \quad \varepsilon \geqslant \varepsilon_i^*(r_1, r_2, z_i), \\
&= \omega & if \quad \varepsilon < \varepsilon_i^*(r_1, r_2, z_i).
\end{aligned}
$$

$\varepsilon_i^*(r_1, r_2, z_i)$ is the level of shock that $\pi_i(r_1, r_2, z_i, \varepsilon) = 0$. Depending on the relative size of r_i, there are two possible values: The first is when $z_i \geqslant r_i$, $\pi_i(r_i, \widehat{r}_j, z_i, \varepsilon) = (z_i - r_i) + r_i[R(r_i, \widehat{r}_j) + \varepsilon]$, where for $j \neq i$, $\widehat{r}_j = r_j$ if $z_j \geqslant r_j$, and $\widehat{r}_j = r_j - (1 + \lambda)(r_j - z_j)$ if $z_j < r_j$. The second term is when $z_i < r_i$, $\pi_i(r_i, \widehat{r}_j, z_i, \varepsilon) = (r_i - (1 + \lambda)(r_i - z_i))[R(r_i - (1 + \lambda)(r_i - z_i)) + \varepsilon]$, where for $j \neq i$, $\widehat{r}_j = r_j$ if $z_j \geqslant r_j$, and $\widehat{r}_j = r_j - (1 + \lambda)(r_j - z_j)$ if $z_j < r_j$. Similar to the single bank case, $\varepsilon_i^*(r_1, r_2, z_i)$ is smaller when $z_i \geqslant r_i$. The fundamental bank run happens when $\varepsilon < \varepsilon_i^*(r_1, r_2, z_i)$, which includes fundamental run (i.e., $\varepsilon < \varepsilon_i^*(r_1, r_2, N)$) and possible speculative

run (i.e., $\varepsilon_i^*(r_1, r_2, N) < \varepsilon < \varepsilon_i^*(r_1, r_2, 1)$).

If a depositor does not withdraw prematurely, then she will get a pro rata share of the bank's assets in period 2. Let $u_i^2(r_1, r_2, z_i, \varepsilon)$ denote the per unit return.

$$
\begin{aligned}
u_i^2(r_1, r_2, z_i, \varepsilon) &= \frac{(1-\alpha)\pi_i(r_i, \widehat{r}_j, z_i, \varepsilon)}{z}, && if \quad \varepsilon \geqslant \varepsilon_i^*(r_1, r_2, z_i), \\
&= \omega && if \quad \varepsilon < \varepsilon_i^*(r_1, r_2, z_i).
\end{aligned}
$$

All returns are realized in the end of $t = 2$.

It is easily checked that there will be two NE in each bank. Let $\varphi_i(r_1, r_2, z_i, \varepsilon) = u_i^2(r_1, r_2, z_i, \varepsilon)$- $u_i^1(r_1, r_2, z_i, \varepsilon)$. We next derive the property of Z*- stability condition for these two banks. Denote $Z_i^*(r_1, r_2, \varepsilon)$ as the number of agents choosing to stay in bank i, given r_1, r_2 and ε. Let $\widehat{\varepsilon}_i^1(r_1, r_2)$ and $\widehat{\varepsilon}_i^2(r_1, r_2, \lambda)$ denote the critical values of ε such that $\varphi_i(r_1, r_2, Z_i^*(r_1, r_2, \varepsilon), \varepsilon) = 0$ for $r_i \leqslant Z_i^*(r_1, r_2, \varepsilon)$ and $r_i > Z_i^*(r_1, r_2, \varepsilon)$, respectively. Let $\pi_i^k(r_1, r_2, N, \varepsilon)$ be the corresponding payoff maximums, where $\pi_i^k(r_1, r_2, N, \varepsilon) = \max\limits_{r,s} \int_{\widehat{\varepsilon}^k(r_1, r_2)}^{\infty} \{s + r[R(r_1, r_2) + \varepsilon]\} dF(\varepsilon)$, $k = 1, 2$.

Proposition (1) $Z_i^*(r_1, r_2, \varepsilon)$ is decreasing in r_1, r_2 and ε. (2) $\max\{\pi_i^1(r_1, r_2, N, \varepsilon), \pi_i^2(r_1, r_2, N, \varepsilon)\}$ is decreasing in r_j. (3) r_i and r_j can be different.

Proof: (1) See the proof in KMR's theorem 3. (2) Since $Z_i^*(r_1, r_2, \varepsilon)$ is decreasing in r_1, r_2 and ε, $\widehat{\varepsilon}_i^1(r_1, r_2)$ decreases with r_j by applying the implicit function theorem on the condition $\varphi_i(r_1, r_2, Z_i^*(r_1, r_2, \varepsilon), \varepsilon) = 0$, and this implies the result. $\square$

Proposition *The probability of system risk is higher with speculation, but the individual probability of bank failure does not necessarily increase.*

Proof: Simply calculate the cumulative values $F(\widehat{\varepsilon}_i^1(r_1^*, r_2^*))$, and $F(\widehat{\varepsilon}_1^1(r_1^*, r_2^*)) \times F(\widehat{\varepsilon}_1^1(r_1^*, r_2^*))$. $\square$

Summer (2003) provides an excellent summary of the existing literature on system risk. The evidence for speculative runs abounds both in Taiwan and in

other countries. In December, 2003, Kaohsiung Business Bank in Taiwan suffered a sudden withdrawal of *3.6 billions* in one month, simply because of a whisper of a rumor for bank run. Ironically, that bank has been taken over by Central Deposit Insurance Corporation in 2002, and hence all depositors are fully insured. In Japan, a speculator example of a bank run occurred in October 1995 where the Hyogo Bank experienced more than the equivalent of \$1billion withdrawals in just one day. In 1991, in Rhode Island in the USA, a perfectly solvent bank was forced to close after the TV channel, CNN, used a picture of this bank to illustrate a story on bank closures, which lead the bank's customers to believe the bank was insolvent, whereas it was not.

Acharya (2001) interpreted system risks in a portfolio framework, where two banks simultaneously choose whether to invest in highly related assets and the returns for each asset is exogenously given and randomly distributed. This, however, summarizes the assets portfolio problems into an odd 2×2 game with two symmetric equilibria (i.e., combination like (highly related, low related) does not have any meaning.

4   Concluding Remarks

For an N players coordination games, Tanaka (2000) proved that the notion of N/2 stability defined by Schaffer (1988) is a necessary and sufficient condition for such a long run equilibrium in an evolutionary process with mutations (in the sense of Kandori, et. al. (1993)). We argue that the critical number in Schaffer's stability is not unique in every application, but can vary with variables determined before the coordination games. In our specific model, these variables are the portfolio choices of the banks. We derived a Z* stability condition for the long run equilibrium for the banking system, in which there is no speculative bank run. This critical number of players is a function of the size for risky investment, and varies with total risky investments when there are more than two banks. We

14

use this framework to analyze the effect of speculative behavior on banks' risk taking and the phenomenon of system risk, calculating the probability when more than one banks fail together (system risk).

We consider the following two period game, after adding speculative withdrawals among depositors in Klein's portfolio choices. At the beginning of the first stage, banks, receiving[13] a total one unit of fund, determine the proportion to invest in safe asset (high liquidity) or risky asset (low liquidity). Safe asset gives low but certain return, while the risky asset gives high but uncertain returns containing two parts: deterministic part (as Klein) and a random term. A trade off will be: the benefit for investing more risky asset is for high return; while the cost will be the possible negative shock and the penalty for early liquidation. The shock is realized and observed by each agents at the end of the first stage. Observing this shock, agents make decisions whether to withdraw their deposits from the bank, and the payment of the investments are received at the end of stage two, if the banks did not fail. In such a way, the whole banking system are connected..

Our specific results include: first, we propose a Z* stability condition, which is proved to be a necessary and sufficient condition for such a long run equilibrium in the sense of KMR. This critical number of Z* is a function of the total risky investment in the banking system. In the case with two banks, this value could vary across banks. Second, speculative behaviors do not frustrate single bank's risky taking, but rather, encourage the bank to maintain a high enough level of risky investment, to keep the system stay in the equilibrium of no run. This indicates that although the speculative run equilibrium will be eliminated in the long run, the probability of fundamental run will increase with the mere possibility of speculative behavior. Third, the single bank case does not necessarily apply to the case with multiple banks. Symmetric banks can take different level of risks, which induces a different in the probability of bank failures. The probability of

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[13]Here we do not consider predeposit decision, because consuemrs would not agree to deposit if they knew that a run would take place (see Peck and Shell (2003)).

joint failures increases, compare to the case without speculation, but the individual probability of bank failures do not necessarily increase.

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