Collusion with Internal Contracting†

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Abstract

In this paper, an infinitely-repeated Bertrand game is considered. The model has a two-tier relationship; two firms make a self-enforced collusive agreement and each firm writes a law-enforced contract to its privately-informed agent. The main finding is that in optimal collusion, interaction between intra-firm (internal) contracting and inter-firm collusion may be exploited; inter-firm collusion may enhance the efficiency of internal contract, and conversely, internal contracting may facilitate collusion.

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1. Introduction

In this paper we explore a simple interaction between intra-firm (internal) contracting and inter-firm collusion, considering an infinitely-repeated Bertrand game. The model has two firms (principals), where each firm has a single agent. The model thus has a two-tier relationship; each firm and its agent write a single-period, law-enforced contract, and firms make a self-enforced agreement. In each period, each agent privately observes its cost type. The cost type is high or low and i.i.d. across agents and time. Each agent reports a cost type to its firm. The firm then makes cost announcements to the rival firm, sets prices, and allocates the market in a state-dependent way.

In the model, firms and agents have the following incentive problems. First, the colluding firms intend to communicate truthfully, so that production is carried out by the lowest-cost firm (productive efficiency). Since each firm privately observes its agent's report, a high-cost firm has an incentive to *understate* the reported cost in the hope of increasing its market share. To elicit truthful communications, firms may use a collusive scheme that promises the high-cost firm future rewards (in the form of high continuation value) at the expense of the current-period profit and promises the low-cost firm the current-period rewards at the expense of future profit.[1] Second, a low-cost agent has an incentive to *overstate* the observed cost in the hope of receiving the greater transfer payment (for a given level of production). To elicit truthful reports from agents, firms may use a contract that grants information rents to the low-cost agent.

The main concern of this paper is to show that these contrasting incentives, observed in collusion and contract, can work to the firms' advantage.[2] Consider first the effect of collusion on

[1]This argument is adopted from Athey and Bagwell [4]. A related idea is found in Atkeson and Lucas [3], who explore the efficient allocation of consumption to many consumers, each of whom is subject to private taste shocks in each period.

[2]The contrasting incentives between informed and uninformed parties differ from the "countervailing incentives"

internal contract. If the colluding firms coordinate to allocate the market by the criterion of productive efficiency, then a low-cost agent who reports high costs will be paid nothing (because of no production) when the other firm announces low cost. Collusion may thus soften the agent's incentive to overstate costs. In other words, given optimal collusion, it is less costly to induce agents to be truthful in terms of information rents. Consider next the effect of internal contract on collusion. If a contract specifies that an agent receives a large payment when the agent reports high cost and the firm requests a large output, then a high-cost firm that understates costs may suffer a large wage expense. Internal contracting may thus soften the firm's incentive to understate costs. In other words, given such a simple contract, it is less costly to facilitate firms' truthful communications in terms of future rewards.

This interaction between collusion and internal contract may be exploited if two conditions are satisfied. First, the continuation value in the future must be large enough for the high-cost firm to be truthful today. The current-period contracting relaxes, however, the rewarding constraint imposed on continuation values (i.e., restriction on the equilibrium value set). Since the high-cost firm suffers a large wage expense when it lies, the level of future rewards required for the firm to be honest today can be reduced.[3] Second, the internal contract must be enforceable. The contract of this nature is contingent on the "hard-to-verify" information: a pair of the two-tier cost reports,

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faced only by an informed party, as seen in Lewis and Sappington [15], Spiegel and Spulber [24] and others.

[3]The continuation values in the equilibrium payoff play the role of side-payments in a legalized cartel. The models with legalized cartel (e.g., Roberts [20], Cramton and Palfrey [6], and Kihlstrom and Vives [12]) show that communication helps firms to identify the most efficient firm, and side-payments provide firms with truth-telling incentives. Our analysis, in its relation to literature on repeated procurement auctions, may describe the case in which (i) two collusive bidders play a knockout auction, prior to actual bidding, to find who will be a lowest-cost supplier (e.g., McAfee and McMillan [16]), and (ii) each bidder suffers some costs, were it to lie; thus the side-payments, required for bidders to be honest, is reduced.

made by each firm and its agent. An internal contract can be, however, easily enforced, if the contract specifies that a high-cost agent earns a large payment when the agent produces more than a *predetermined* level of output, and if the compensations to agents take a linear form with the level of output.

The equilibrium concept here is confined to the class of perfect public equilibrium (PPE). As in the public monitoring models, we find the set of PPE values by using the recursive structure explored by Abreu, Pearce and Stacchetti [1, 2]; after any history, the set of continuation values is equal to the equilibrium value set of the repeated game. Our analysis of "asymmetric" strategies (asymmetric PPE) builds on Athey and Bagwell [4]. As in their paper, if the high-cost firm gives up producing today, then it is rewarded by a high continuation value, which is delivered through market-share favors in the future. Athey and Bagwell characterize the optimal collusion that obtains "first-best" profit, but do not treat the internal incentive problem. The distinct feature of our analysis is that the interaction between collusion and internal contract is exploited; the optimal inter-firm collusion enhances the efficiency of internal contract, and conversely, internal contracting facilitates the optimal inter-firm collusion; for a wide range of parameters, the truthful communication between firms is elicited as internal contracting relaxes the restriction on the equilibrium value set. The colluding firms with a simple internal contracting thus replicate the performance of the firm under a binding inter-firm contract (e.g., a merging contract or a side-payment contract in legalized cartels).

Our analysis of "symmetric" PPE is related to the work done by Athey, Bagwell and Sanchirico [5]. Considering collusion when a continuum of firm (cost) types is assumed, they focus on symmetric strategies where continuation values are assigned to the 45 degree line on the payoff space. This symmetrization imposes a restriction on the value set, and prohibits firms' transfers in the form of continuation values, which might otherwise be facilitated by asymmetric plays; thus, it is more costly to sort firms by their types. Athey, Bagwell and Sanchirico find that wasteful continuation values (prices wars) are not used and price efficiency is achieved, but productive inefficiency obtains. This paper finds, by contrast, that when two discrete firm types are assumed, productive efficiency (i.e., optimal collusion) can also be approximately achieved by symmetric strategies, arguing that the negative effect of the value set being restricted can be alleviated if each firm is bounded by a contract that makes it very costly to tell a lie.

The role of internal contracting, described in this paper, is related to the strategic effect of managerial incentive contracts, as seen in the existing literature.[4] There is a broad analogy between our analysis and the work done by Fershtman and Judd [7] or by Fershtman, Judd and Kalai [8]. They show that a firm may compete more effectively in a Cournot oligopoly game, or collude more effectively with the other firm, if its manager enters this game and is bounded by a wage contract. Likewise, we here show that firms with internal incentive problem may collude more effectively if each firm is bounded by an internal contract that penalizes the firm were it to lie and increase its market share.

Information in this paper is "soft"; it is subject to distortion when transferred. In this sense this paper differs from the existing "information sharing" literature, which studies the issue of whether firms have incentives to share private information in an oligopolistic relationship but ignores whether firms have the incentive to manipulate their private information.[5] This paper is rather close to Ziv [26], who shows that oligopolistic rivals may choose to exchange transfer payments to induce the firm to announce cost type truthfully. Our analysis finds, however, a

[4]Fershtman and Judd [7], Fershtman, Judd and Kalai [8], Katz [11], Reitman [19], Sklivas [22] and Spagnolo [23] study strategic wage schemes of game-playing agents in a static or repeated-game setting.

[5]See, for example, Gal-Or [10], Novshek and Sonnenschein [18], Shapiro [21], Vives [25] and others.

truth-telling incentive mechanism without depending on direct transfer payments.

The rest of this paper is organized as follows. Section 2 introduces the model. Section 3 investigates the incentive constraints that strategies must satisfy, to be implementable as a equilibrium play. Section 4 characterizes the self-enforcing collusive scheme by establishing the equilibrium value set. Section 5 provides conclusions.

2. The Model

We consider a Bertrand game in which two firms sell a homogeneous good. Each firm hires a single agent, letting the agent produce the good. Whereas prices and quantities are publicly observed, the realized unit costs are privately observed by the agent. In each period costs are independently drawn from the identical common-knowledge distribution with discrete support $\{\theta_L, \theta_H\}$, where $\theta_L < \theta_H$. The probabilities of a firm drawing θ_L and θ_H are μ and $1-\mu$, respectively. For technical simplicity, for now $\mu > 1/2$ is assumed. As for the market demand, a unit mass of consumers is assumed to be homogeneous with valuation of the good ρ. It also is assumed that $\rho > \theta_H$, so that a firm drawing θ_H has an incentive to increase its market share when price is ρ.

2.1. Benchmark: The Second-Best Contract

This subsection examines, as a benchmark, the behavior of a monopoly firm, which has two privately-informed agents. When the firm wants to produce a quantity q, each agent $i \in \{1, 2\}$ produces q^i and $q^1 + q^2 = q \leq 1$, where $0 \leq q^i \leq 1$. The associated notations, p and t, denote the price and the money transfer afforded to the agent. The pair (θ_j, θ_k), indexed by (j, k), denotes the state in which agent 1 reports θ_j and agent 2 reports θ_k. The state space is $\Omega \equiv \Theta^1 \times \Theta^2$, where $\Theta^i = \{L, H\}$ for $i = 1, 2$. The quantity that agent i produces in state (j, k) is denoted by q^i_{jk}. Let

$\mu_L \equiv \mu$ and $\mu_H \equiv 1-\mu$, and define $\bar{q}^1_j \equiv \sum_k \mu_k \cdot q^1_{jk}$ and $\bar{q}^2_k \equiv \sum_j \mu_j \cdot q^2_{jk}$ for j and $k \in \{L,H\}$. The second-best contracting solves the problem:

$$\max_{(t^1_{jk},q^1_{jk}),(t^2_{jk},q^2_{jk})} \sum_{j\in\{L,H\}} \sum_{k\in\{L,H\}} \mu_j \mu_k \left[p_{jk}(q^1_{jk}+q^2_{jk}) - t^1_{jk} - t^2_{jk}\right] \tag{1}$$

subject to incentive compatibility (IC) and individual rationality (IR) constraints for agent 1:

$$t^1_{Lk} - \theta_L q^1_{Lk} \geq t^1_{Hk} - \theta_L q^1_{Hk} \tag{IC-A$^1_{Lk}$}$$

$$t^1_{Hk} - \theta_H q^1_{Hk} \geq t^1_{Lk} - \theta_H q^1_{Lk} \tag{IC-A$^1_{Hk}$}$$

$$\sum_{k\in\{L,H\}} \mu_k \left(t^1_{Lk} - \theta_L q^1_{Lk}\right) \geq 0 \tag{IR-A^1_L}$$

$$\sum_{k\in\{L,H\}} \mu_k \left(t^1_{Hk} - \theta_H q^1_{Hk}\right) \geq 0 \tag{IR-A^1_H}$$

and for agent 2.[6] In this context the contract can be equivalently implemented in Bayesian or in dominant strategy if the expected output is strictly decreasing in cost type (i.e., $\bar{q}^i_L > \bar{q}^i_H$), this being satisfied in the solution.[7] Hence, there is no loss of generality in looking for the optimal contract within the set of dominant strategy implementation. As in the standard mechanism design program, optimality implies that there are two binding constraints: the incentive constraint of the "low-cost agent" (IC-A$^1_{Lk}$ and IC-A$^2_{jL}$) and the participation constraint of the "high-cost agent" (IR-A^1_H and IR-A^2_H). This paper considers the case in which the high-cost agent is willing to participate in all states of nature. In this case the *ex post* participation constraints are binding; i.e., $t^1_{Hk} - \theta_H q^1_{Hk} = 0$ for $k \in \{L,H\}$. The firm then offers the following contract: If agent 1 reports

[6] This subsection assumes that no communication limit between principal and agent is present, and thus there is no scope for a hierachical design for communication channels which deters the two agents from making a collusive cost report. Laffont and Martimort [13], for example, study a hierachical design when there is a communication limit between principal and agents.

[7] Mookherjee and Reichelstein [17] show that the equivalence between Bayesian and dominant strategy implementations holds if the agents' cost functions satisfy a generalized single crossing property. This property is trivially satisfied in the model having constant unit cost.

high cost, the agent receives

$$t^1_{HL} = \theta_H q^1_{HL} \text{ or } t^1_{HH} = \theta_H q^1_{HH},$$

and if agent 1 reports low cost, the agent receives

$$t^1_{LL} = \theta_L q^1_{LL} + (\theta_H - \theta_L) q^1_{HL} \text{ or } t^1_{LH} = \theta_L q^1_{LH} + (\theta_H - \theta_L) q^1_{HH}.$$

The terms $(\theta_H - \theta_L)q^1_{HL}$ and $(\theta_H - \theta_L)q^1_{HH}$ stand for information rents. The low-cost agent 1 thus earns the expected information rent $(\theta_H - \theta_L)\bar{q}^1_H$. Likewise, the transfers to agent 2 can be obtained.

Since $p_{jk} = \rho$ for all (j, k), the problem then boils down to

$$\max_{(q^1_{jk}, q^2_{jk})} \sum_{j \in \{L,H\}} \sum_{k \in \{L,H\}} \mu_j \mu_k \left[\rho(q^1_{jk} + q^2_{jk}) - C_j \cdot q^1_{jk} - C_k \cdot q^2_{jk} \right], \tag{2}$$

where

$$C_L \equiv \theta_L \text{ and } C_H \equiv \theta_H + \frac{\mu}{1-\mu}(\theta_H - \theta_L).$$

Note that letting a high-cost agent produce the good incurs the virtual cost $C_H > \theta_H$, where C_H rises with the cost gap $\theta_H - \theta_L$ and μ.[8] In the optimum, production is carried out as follows: (i) $q^1_{LH} = q^2_{HL} = 1$, (ii) $q^1_{LL} + q^2_{LL} = 1$, and (iii)

$$q^1_{HH} + q^2_{HH} = \begin{cases} 1 & \text{if } \rho \geq C_H \\ 0 & \text{if } \rho < C_H. \end{cases}$$

We here say that productive efficiency is achieved if production is assigned in this way.[9] If $\rho < C_H$, the firm chooses not to produce the good in state (H, H) (i.e., $q^i_{HH} = 0$). This may occur when $\theta_H - \theta_L$ is large, and thus the firm would suffer large information rents if $q^i_{HH} > 0$. Indeed, achieving

[8] When agents tie, it is assumed that they equally share the production: $q^1_{kk} = q^2_{kk}$ for $k \in \{L, H\}$.

[9] If $r < C_H$, there is a time-inconsistency problem. The *ex ante* virtual cost in the inefficient state is C_H, but the *ex post* cost is θ_H. In state (H, H), *ex ante* it is not profitable to produce the good, but *ex post* it is. In this paper, only the optimal (commitment) solution is considered.

productive efficiency has a twofold benefit. First, productions are carried out only by relatively efficient agents. Second, productive efficiency enhances the efficiency of contract; it is less costly to sort out the low-cost agent, for a given level of production. For instance, in state (L, L), each agent receives only

$$\theta_L q^1_{LL} + (\theta_H - \theta_L) q^1_{HL} = \theta_L q^2_{LL} + (\theta_H - \theta_L) q^2_{LH} = \frac{\theta_L}{2},$$

since $q^1_{HL} = q^2_{LH} = 0$. This is reflected in the lowered virtual costs in states (H, L) and (L, H): $C_H q^1_{HL} + C_L q^2_{HL} = C_L q^1_{LH} + C_H q^2_{LH} = \theta_L$.

Lemma 1. *The firm achieves the optimal per-period profit:*

$$\Pi_M = \begin{cases} \rho - E(\theta) & \text{if } \rho \geq C_H \\ \left[1 - (1-\mu)^2\right](\rho - \theta_L) & \text{if } \theta_H < \rho < C_H, \end{cases}$$

if $p_{jk} = \rho$ for every state (j, k), and productive efficiency is achieved.

2.2. The Game

Consider first the stage game in each period. The relationship between principal and agent lasts for a single period; each firm renews the contract with its agent every period. It is assumed that each agent is induced to report costs only by his or her own IC and IR constraints. The timing of the game in each period is as follows:

1. Agent i privately learns type $\theta^i \in \{L, H\}$.

2. Firm i writes its agent a single-period contract M^i.

3. The agent makes a report $r^i \in \{L, H\}$ to the firm.

4. The firm makes an announcement $a^i \in \{L, H\}$ to its rival firm.

5. The firm makes price and market share proposals, p^i and q^i.

6. The quantity of output is determined, and the money transfers (to agents) that have been requested by M^i are implemented.

Each firm announces $a^i \in A \equiv \{L, H\}$, based on its agent's report r^i. Each firm then sets the price p^i and makes market share proposal q^i. The vectors $\mathbf{p} \equiv (p^1, p^2)$ and $\mathbf{q} \equiv (q^1, q^2)$ jointly determine the market share of each firm i, which is denoted ϕ^i. If $p^i > \rho$, then $\phi^i = 0$. If $p^1 = p^2 \leq \rho$, then $\phi^i = 1/2$ if $q^1 + q^2 \neq 1$, and $\phi^i = q^i$ otherwise. Note that equally priced firms ($p^1 = p^2 \leq \rho$) can communicate each other to allocate market share in a state-dependent way. Market proposals matter only if prices are equal, since market share ϕ^i would be determined by prices otherwise.

Consider the interim stage of the game, assuming that agents truthfully reported their cost types (i.e., $r^i = \theta^i$). Each firm i has a finite inter-firm strategy set S^i:

$$S^i = \left\{\tilde{a}^i \mid \tilde{a}^i : \Theta^i \to A\right\} \times \left\{\tilde{p}^i \mid \tilde{p}^i : \Theta^i \times A \to \Re_+\right\} \times \left\{\tilde{q}^i \mid \tilde{q}^i : \Theta^i \times A \to \Re_+\right\},$$

where $\tilde{a}^i$ is the announcement function, $\tilde{p}^i$ is the pricing function, and $\tilde{q}^i$ is the market share proposal function. The payoff function is defined as $\Pi^i : S \to \Re$, where $S = S^1 \times S^2$. A strategy of firm 1, for example, is denoted

$$s^1\left(\theta^1, a^2\right) = \left\{\tilde{a}^1\left(\theta^1\right), \tilde{p}^1\left(\theta^1, a^2\right), \tilde{q}^1\left(\theta^1, a^2\right)\right\},$$

where $a^2 \in A$. Define $\boldsymbol{\theta} \equiv (\theta^1, \theta^2)$ and $\mathbf{s}(\boldsymbol{\theta}) \equiv (s^1(\theta^1, a^2), s^2(\theta^2, a^1))$. In each period each firm actually receives $\pi^i(\mathbf{s}, \boldsymbol{\theta}; M^i)$, which depends on the realizations of cost types. Since Π^i is an expected value for $\mathbf{s}$, the expected stage-game payoffs are given by $\Pi^i(\mathbf{s}) = E_\theta\left[\pi^i(\mathbf{s}, \boldsymbol{\theta};\ M^i)\right]$.

Consider next the repeated game. This paper restricts attention to perfect public equilibrium (PPE), which requires that strategies be public: Firms' choices at date t may be based on their private information from date t, but only on mutually known information (realized choices) from date $t' < t$ (Fudenberg, Levine, and Maskin [9]). When each firm enters a period of play, it observes only the history of its own cost types and the associated choice functions. Thus, while each firm observes the history of realized choices, it does not observe rival types or rival choice functions. Let H_t be the set of public history up to period t, which is denoted $h_t = \{\mathbf{a}_t, \mathbf{p}_t, \mathbf{q}_t; \mathbf{M}_t\}$. A strategy of firm i in period t is denoted $\sigma_t^i : H_t \rightarrow S^i$, and a sequence of strategies $\{\sigma_t^i\}_{t=1}^{\infty}$ is denoted σ^i. For a given strategy profile $\boldsymbol{\sigma} = (\sigma^1, \sigma^2)$, the expected payoff of firm i is $v^i(\boldsymbol{\sigma}) = E\left[\sum_{t=1}^{\infty} \delta^{t-1} \Pi^i(\sigma_t(h_t))\right]$, where δ is common discount factor and where $h_1 = \varnothing$.

As in the public monitoring models, we find the set of PPE values by using the recursive structure explored by Abreu, Pearce and Stacchetti [1, 2]; each firm's PPE payoff is factored into two components, current-period profit and discounted continuation values, and after any history, the set of continuation values is equal to the equilibrium value set. The inter-firm analysis of this paper builds on Athey and Bagwell [4]. Following their work, we exploit the analogy between recursive structure of the repeated game and static mechanism design approaches, so that we find a mechanism that induces a truthful revelation in the two-tier relationship: internal contract and inter-firm collusion. To detail this, suppose that price, market share and continuation value for firm i in state (j, k) are denoted p_{jk}^i, q_{jk}^i and v_{jk}^i, respectively. Let $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{v}$ be the associated vectors, and let $\mathbf{y} \equiv (\mathbf{p}, \mathbf{q}, \mathbf{v})$ be the policy vector. In equilibrium, following the policy vector, each firm truthfully announces the reported costs. To be implementable as an equilibrium play, the policy vector must satisfy the following incentive constraints: (i) "on-schedule" incentive constraints (on-IC), whereby each firm must truthfully announce its cost and be dissuaded from choosing the policy assigned to a different cost type (being truthful within the equilibrium path), and (ii) "off-schedule" incentive constraints (off-IC), whereby each firm must be deterred from choosing a price and market share that is not assigned to any cost type (non-deviating from the equilibrium path). Letting V be the set of equilibrium values, the design of an optimal collusion is then to find a contract $\mathbf{M} = (M^1, M^2)$ such that (i) each agent is induced to be truthful, and (ii) there exists a policy vector $\mathbf{y} = (\mathbf{p}, \mathbf{q}, \mathbf{v}; \mathbf{M})$ that satisfies the on- and off-schedule incentive constraints of firms, where continuation values (v^1_{jk}, v^2_{jk}) are drawn from V.

3. Incentive Constraints with Internal Contracting

In the optimal collusive scheme, firms set the prices $p^1 = p^2 = \rho$, and assign production efficiently. The internal contract is a pair $M^i = \left\{t^i(a^1, a^2; r^i), q^i(a^1, a^2)\right\}$. The transfer is a function of the agent's internal report (r^i) and the firms' external announcements (a^1 and a^2), while market allocations are contingent on firms' announcements. It is assumed that the pair of transfers and quantities schedules $\{t^i(\cdot), q^i(\cdot)\}$ is publicly known by the two firms at the time of making their choices of announcements, and that contracts cannot be secretly renegotiated, due to the high transaction costs. The problem here is that since the schedules $\{t^i(\cdot), q^i(\cdot)\}$ are contingent on the two-tier reports, made by agents and firms, the contract is enforceable only in very stringent conditions, as described in the following assumption. In later analysis, however, this assumption will be relaxed.

Assumption 1. *Agent i is able to verify the following information: (i) Whether its firm distorts his reported information, (ii) the other firm's announcement of costs, and (iii) firms' price and market share schedules in all states (j, k).*

3.1. On-schedule Incentive Constraints

As was previously argued, we here consider a contract that supports the optimal collusion and yet is extremely hard to enforce. In later analysis, we will show that the contract can be easily modified to be an enforceable contract. Consider the following contract. When agent 1 reports low cost and produces q^1, then the agent receives

$$t^1\left(q^1;L\right)=\theta_L q^1+R_L, \tag{3}$$

where $R_L\equiv(\theta_H-\theta_L)\bar{q}^1_H$ is the expected information rent, which is required to dissuade the low-cost agent from overstating costs. It is assumed that the realized production level $\left(q^1\right)$ is verifiable. When agent 1 reports high cost and produces q^1, then the agent receives

$$t^1\left(q^1;H\right)=\begin{cases}\theta_H q^1 & \text{if } q^1=q^1_{Hk}\\ \theta_H q^1+\alpha\cdot G^1 & \text{if } q^1=q^1_{Lk},\end{cases} \tag{4}$$

where $0\le\alpha\le1$ and $G^1\equiv\sum_k\mu_k\left[(p_{Lk}-\theta_H)q^1_{Lk}-\left(p_{Hk}-\theta_H\right)q^1_{Hk}\right]$, which is the expected current-period gain that firm 1 could have by understating its cost type. For firm 2, $t^2(q^2;L)$ and $t^2(q^2;H)$ are analogously defined. Thus, if firm i gains G^i by understating, it suffers an extra expense $\alpha\cdot G^i$. The level of α hereafter acts as a contractual parameter. Our analysis here abstracts from resorting to an immediate solution, $\alpha=\infty$, in order to show that a contract of this nature can be easily modified to a more realistic contract in later analysis. When in particular the compensation to agents takes a linear form (with respect to quantities), firms can construct a simple, enforceable contract, without causing any efficiency loss, for a wide range of parameters.

We next examine the firms' on-schedule incentive constraints. To this end, define $\bar{v}^1_j\equiv\sum_k\mu_k\cdot v^1_{jk}$ and $\bar{v}^2_k\equiv\sum_j\mu_j\cdot v^2_{jk}$. If firm 1 announces cost type $\hat{\jmath}$, given that its agent reports cost type j, then the current-period profit of firm 1 in the interim stage is

$$\Pi^1(\hat{\jmath},j)=\sum_{k\in\{L,H\}}\mu_k\left[p_{\hat{\jmath}k}\cdot q^1_{\hat{\jmath}k}-t^1\left(q^1_{\hat{\jmath}k};j\right)\right],$$

and firm 2's current-period profit is similarly expressed. The interim-stage profit of firm i is then given by

$$U^i(\hat{\jmath},j)=\Pi^i(\hat{\jmath},j)+\delta\bar{v}^i_{\hat{\jmath}}.$$

Thus the on-schedule incentive constraints of firm i are

$$U^i(H,H) \geq U^i(L,H) \qquad \text{(on-IC-P}^i_H\text{)}$$

$$U^i(L,L) \geq U^i(H,L). \qquad \text{(on-IC-P}^i_L\text{)}$$

The following lemma shows that under the contracts of costly distortion, firms are truthful when expected output schedules are strictly monotone (i.e., $\bar{q}^i_L > \bar{q}^i_H$). When firms are truthful ($a^i = r^i$), agents in turn are induced to be truthful ($r^i = \theta^i$) by the transfers in (3) and (4).

Lemma 2. *When the downward on-schedule incentive constraints of firms (on-IC-P^i_H) are binding and $\bar{q}^i_L > \bar{q}^i_H$, the upward on-schedule incentive constraints of firms (on-IC-P^i_L) are slack, given the contracts described by (3) and (4).*[10]

Given the contracts, the high-cost firm i gains

$$U^i(H,H) = \Pi^i(H,H) + \delta\bar{v}^i_H,$$

when it is truthful. When firm i understates, it earns

$$U^i(L,H) = \Pi^i(H,H) + (1-\alpha)G^i + \delta\bar{v}^i_L,$$

after giving αG^i to its agent. If the downward incentive constraint of a firm (on-IC-P^i_H) is binding, then

$$\delta(\bar{v}^i_H - \bar{v}^i_L) = (1-\alpha)G^i. \qquad (5)$$

The RHS of (5) represents the net current gain from understating. The equation describes the relationship between the current gain from understating and the future reward in the form of continuation values. It implies that a higher (discounted) continuation value must be attributed

[10]By focusing on the binding downward IC (on-IC-P^i_H), we will find a minimum level of future rewards for the high-cost firm to be honest today.

to the high-cost firm for it to be honest today. When prices $p^i_{jk} = \rho$, the RHS is

$$(1-\alpha)\, G^i = (1-\alpha)\,(\rho - \theta_H)\left(\bar{q}^i_L - \bar{q}^i_H\right). \tag{6}$$

Note that given that productive efficiency is achieved, the current gains from understating rises with $\rho - \theta_H$.

If the downward IC of firm i (on-IC-P^i_H) is binding, the profit of the low-cost firm in the interim stage is then given by

$$U^i\,(L, L) = U^i(H, H) + (\theta_H - \theta_L)\left(\bar{q}^i_L - \bar{q}^i_H\right) + \alpha(\rho - \theta_H)\left(\bar{q}^i_L - \bar{q}^i_H\right). \tag{7}$$

The second term on the RHS of (7) is net information rents, after $(\theta_H - \theta_L)\bar{q}^i_H$ has been given to the low-cost agent i. This rent for the agent is in part extracted as profit when the market is allocated in accordance with productive efficiency (i.e., when $q^1_{HL} = q^2_{LH} = 0$). The last term on the RHS reflects leakage from the (gross) current gain that the high-cost firm could accrue by understating.

Lemma 3. *If $p^i_{jk} = \rho$, and if the downward IC of firm i (on-IC-P^i_H) is binding, then the ex ante expected utility of firm i along the equilibrium path is*

$$U^i = (\rho - \theta_H)\,\bar{q}^i_H + \delta\bar{v}^i_H + \mu(\theta_H - \theta_L)\left(\bar{q}^i_L - \bar{q}^i_H\right) + \alpha\mu(\rho - \theta_H)\left(\bar{q}^i_L - \bar{q}^i_H\right). \tag{8}$$

3.2. Off-Schedule Incentive Constraints

To implement the optimal collusion, firms must be deterred from charging a price not assigned to any cost type. Following Athey and Bagwell [4], our analysis considers two types of off-schedule deviations that firms can undertake: (i) a firm can slightly undercut the price and capture the entire market after communicating with the other firm, and (ii) a firm can overstate or understate at the communication and then undercut the price.[11]

The first *ex post* deviation of price undercutting will not be profitable for the low-cost firm (e.g., firm 1) if

$$\delta\left(v_{Lk}^{1}-\underline{v}\right)\geq\rho-\theta_{L}-R_{L}-\left(\rho q_{Lk}^{1}-\theta_{L}-R_{L}\right), \qquad (\text{off-IC-P}_{Lk}^{1})$$

where $\underline{v}\equiv\Pi_{N}^{i}/(1-\delta)$, which is the value of infinite repetition of a static Nash game.[12] Likewise, for firm 2, off-IC-P_{Lk}^{2} can be expressed. The off-schedule incentive constraints for the high-cost firm are

$$\delta\left(v_{Hk}^{1}-\underline{v}\right)\geq\rho-\theta_{H}-\left(\rho-\theta_{H}\right)q_{Hk}^{1}, \qquad (\text{off-IC-P}_{Hk}^{1})$$

and off-IC-P_{Hk}^{2} is analogous.

Consider next the second type of off-schedule deviations. The low-cost firm, for example, can announce high cost at the communication and then slightly undercut price. Given the contract in (3) and (4), however, this firm would gain nothing by overstating at the communication if it is tempted to undercut price afterwards. The low-cost firm will be deterred from undertaking the

[11]A firm can deviate by increasing the market share above the proposed level while maintaining the collusive price. This deviation can effectively be replaced, however, by the deviation of undercutting the price slightly.

[12]In a static Bertrand game, market is allocated as in Benchmark (monopoly) case: $q_{LH}^{1}=q_{HL}^{2}=1$ and $q_{LL}^{i}=q_{HH}^{i}=1/2$. In this case, an enforceable contract such that

$$\begin{aligned} t^{i}(q^{i};H) &= \theta_{H}q^{i} \text{ and} \\ t^{i}(q^{i};L) &= \theta_{L}q^{i}+\frac{(1-\mu)(\theta_{H}-\theta_{L})}{2} \end{aligned}$$

is optimal, since given the production schedule, it minimizes the expected transfers, as the contract in Benchmark does. As for pricing, the high-cost firm sets the price equal to cost θ_H, having zero profit. The low-cost firm uses a mixed strategy, having the expected profit:

$$(1-\mu)\left[\theta_{H}-\theta_{L}-\frac{\theta_{H}-\theta_{L}}{2}\right]=\frac{(1-\mu)(\theta_{H}-\theta_{L})}{2},$$

which is obtained by slightly undercutting the price of the high-cost firm. The *ex ante* expected profit is then $\Pi_{N}^{i}=\mu(1-\mu)(\theta_{H}-\theta_{L})/2$.

second type of off-schedule deviation if

$$\delta \sum_{k\in\{L,H\}} \mu_k \left(v^1_{Lk} - \underline{v}\right) \geq \sum_{k\in\{L,H\}} \mu_k \left[\rho - \theta_L - R_L - \left(\rho q^1_{Lk} - R_L\right)\right].$$

The high-cost firm will not understate and undercut price if

$$\delta \sum_{k\in\{L,H\}} \mu_k \left(v^1_{Hk} - \underline{v}\right) \geq \sum_{k\in\{L,H\}} \mu_k \left[\rho - \theta_H - \left(\rho q^1_{Hk} - t^1_{Hk}\right)\right].$$

An investigation shows that this second type of off-schedule is implied by off-IC-P^1_{jk}.

Lemma 4. *If $p^i_{jk} = \rho$, then off-schedule incentive constraints boil down to off-IC-P^1_{jk}.*

4. Characterization of Perfect Public Equilibrium

In this section, we find the contract (the level of α) that supports the self-generating set of optimal PPE values along the equilibrium path, and find a critical discount factor above which no firm undercut the collusive price ρ. Consider a line segment Z_α:

$$\begin{aligned} Z_\alpha = \quad & \left\{\left(u^1, u^2\right) \subset \Re^2_+ \mid \exists\gamma \in [0,1] \text{ such that} \right. \\ & \left. u^1 = x_\alpha + \gamma\left(X_\alpha - x_\alpha\right) \text{ and } u^2 = X_\alpha - \gamma\left(X_\alpha - x_\alpha\right)\right\}. \end{aligned}$$

This segment corresponds to the contract that specifies α, and has slope of -1 with two endpoints (x_α, X_α) and (X_α, x_α), where $X_\alpha > x_\alpha$. We here establish Z_α as a self-generating set of PPE values, and investigate whether Z_α achieves the Pareto-frontier value set $\mathbf{U}_M$, defined as

$$\mathbf{U}_M \equiv \left\{\left(u^1, u^2\right) \subset \Re^2_+ : u^1 + u^2 = \frac{\Pi_M}{1-\delta}\right\}.$$

Assuming that any off-schedule deviation leads to permanent Nash reversions, then $Z_\alpha \cup \mathbf{U}_N$ is a self-generating set of PPE values, where $\mathbf{U}_N$ is defined as

$$\mathbf{U}_N \equiv \left\{\left(u^1, u^2\right) \subset \Re^2_+ : u^i = u^i_N = \frac{\Pi^i_N}{1-\delta}\right\}.$$

4.1. Contracting along the Equilibrium Path

In this subsection, we consider on-schedule incentive constraints, ignoring off-schedule incentive constraints. Consider first the case of $\rho \geq C_H$, in which $q^1_{HH} + q^2_{HH} = 1$. We here find the level of α under which $\mathbf{y}_\alpha \equiv (\mathbf{p}, \mathbf{q}, \mathbf{v};\alpha)$ can establish the segment Z_α. Every point in the set Z_α has the corresponding assignments of q^i_{LL} and q^i_{HH}.[13] At an endpoint (x_α, X_α), for example, firm 1 receives a small value x_α, being assigned to a "disadvantaged" market shares such that q^1_{LL} and q^1_{HH} are close or equal to zero. Attention is thus on how to induce firm 1 to be honest at the endpoint (x_α, X_α) when in particular it draws high cost. Since the binding on-IC-P^i_H implies on-IC-P^i_L, we focus on the binding on-IC-P^i_H, which implies that

$$v^1_{HL} - v^1_{LH} = \frac{(1-\alpha)(\rho - \theta_H)}{\delta}. \tag{9}$$

This equation describes the level of future rewards, required to dissuade firm 1 from producing today when firm 2 announces high cost at the endpoint (x_α, X_α).[14] It implies that the self-generating segment must be sufficiently long for the high-cost firm to be rewarded by a high continuation value v^1_{HL} within the self-generating segment; the value X_α must exceed x_α by at least the RHS of (9). Suppose that the notation $d(\mathbf{y}_\alpha;\alpha)$ denotes $X_\alpha - x_\alpha$ when this differential is induced by $\mathbf{y}_\alpha$ for a given α. Thus on-schedule incentive constraints imply that there is an "additional" constraint:

$$d(\mathbf{y}_\alpha;\alpha) \geq \frac{(1-\alpha)(\rho - \theta_H)}{\delta}. \tag{add-IC}$$

The LHS represents the differential $X_\alpha - x_\alpha$, generated by $\mathbf{y}_\alpha$, and the RHS stands for the required level of future rewards. The following Lemma shows that if α is greater than a certain level, then there exists a policy vector $\mathbf{y}_\alpha$ that generates a sufficiently long Z_α. To gain some intuition, suppose

[13] Note that $p^i_{jk} = \rho$ and $q^1_{LH} = q^2_{HL} = 1$.

[14] Eq. (9) is derived in the Appendix.

that the policy vector $\mathbf{y}_\alpha$ assigns $q^2_{LL} = q^2_{HH} = 1$ so as to establish the endpoint (x_α, X_α). If α is larger, then a smaller future reward (RHS) is required, whereas a larger differential $X_\alpha - x_\alpha$ (LHS) can be generated; if α is larger, utility for firm 2 can be significantly increased by raising q^2_{LL} or q^2_{HH}, but utility for firm 1 (x_α) is smaller since future rewards to firm 1 in the form of continuation value v^1_{HL} is smaller.

If a policy vector $\mathbf{y}_\alpha$ provides truth-telling incentives at an endpoint (x_α, X_α), then we also can find an analogous policy vector $\mathbf{y}'_\alpha$ that provides truth-telling incentives at the other endpoint (X_α, x_α). Then, the remainder of the segment can be constructed by a convex combination of two policy vectors. The reason is that given the pricing schedule $p^i_{jk} = \rho$, utilities and on-schedule constraints are linear in market share and continuation values, for a given level of α. For here and later use, define

$$\alpha^*(\delta) \equiv \frac{1 - \delta + \delta(2\mu - 1)(1 - \mu\lambda)}{1 - \delta + \delta 2\mu^2}, \tag{10}$$

where $\lambda \equiv (\theta_H - \theta_L)/(\rho - \theta_H)$.

Lemma 5. *(i) For all $\{(\delta, \alpha) : \alpha \geq \alpha^*(\delta)\}$, there exists a policy vector in which both on-IC-P^i_H and add-IC are binding. (ii) For all $\{(\delta, \alpha) : \alpha^*(\delta) < \alpha < 1\}$, there exists a policy vector in which on-IC-P^i_H is binding and add-IC is slack.*

Corollary 1. *(i) The locus $\alpha = \alpha^*(\delta)$ is downward-sloping, and (ii) the rise in λ shifts down the locus.*

An example of the locus $\alpha = \alpha^*(\delta)$ is illustrated in Fig. 1. The locus $a = \alpha^*(\delta)$ is downward-sloping, since more patient firms are more willing to wait for the future rewards rather than capture the current gains by understating. This locus also shifts down as λ rises for the following reasons. When $\rho - \theta_H$ is large, the current gains from understating are large, and thus the corresponding

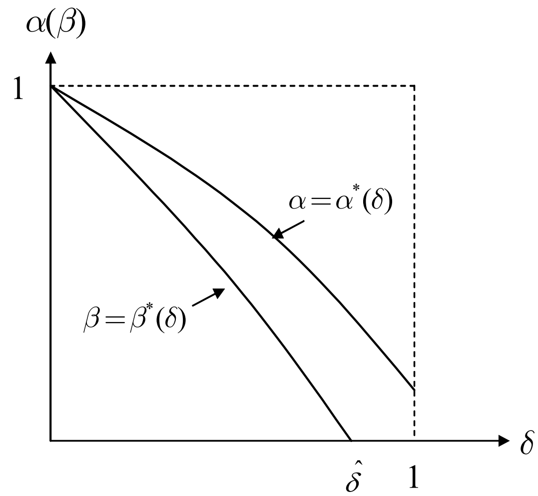

Figure 1: Loci $\alpha = \alpha^*(\delta)$ and $\beta = \beta^*(\delta)$.

future rewards are required to be large. On the other hand, when $\theta_H - \theta_L$ is large, the rewarding through market share favors is large; the *ex ante* expected utility can be significantly increased by raising market allocations (e.g., $q^i_{LL} = 1$). Thus, the rise in λ lowers the locus for a given δ.

Due to Lemma 5, we can immediately obtain the following result: For all $\{(\delta, \alpha) : \alpha \geq \alpha^*(\delta)\}$, there exists a policy vector that satisfies all the on-schedule incentive constraints.

Lemma 6. *Assume that $\rho \geq C_H$. If off-schedule incentive constraints are satisfied, then for all $\{(\delta, \alpha) : \alpha \geq \alpha^*(\delta)\}$, there exists a set $Z_\alpha \subset \mathbf{U}_M$ such that $Z_\alpha \cup \mathbf{U}_N$ is a self-generating set of PPE values.*

Consider next the case of $\rho < C_H$ in which $q^1_{HH} + q^2_{HH} = 0$. When the cost gap is larger, since future market share favor is more rewarding, firms are less tempted to understate costs. The analysis is analogous to the previous analysis. Consider the following contract:

$$t^i\left(q^i;L\right) = \theta_L q^i \text{ and}$$
$$t^i\left(q^i;H\right) = \theta_H q^i + \beta\left(\rho-\theta_H\right)q^i, \quad (12)$$

where $0 \leq \beta \leq 1$. Given this contract, the *ex ante* expected utility of each firm along the equilibrium path has the same form as in (8), where $\bar{q}_H^i = 0$ and β replaces α. The binding on-IC-P_H^i implies that

$$v_{HL}^1 - v_{LH}^1 = \frac{(1-\beta)(2-\mu)(\rho-\theta_H)}{\delta}. \quad (13)$$

Adopting the same procedure as in the previous analysis yields

$$\beta^*(\delta) \equiv \begin{cases} \frac{(2-\mu)(1-\delta+\mu\delta)-\mu^2\delta\lambda}{2-\mu+(3\mu-2)\delta} & \text{if } 0 \leq \delta \leq \hat{\delta} \\ 0 & \text{if } \hat{\delta} < \delta \leq 1, \end{cases} \quad (14)$$

where $\hat{\delta} \equiv (2-\mu)/[(2-\mu)(1-\mu)+\mu^2\lambda]$. The following Corollary characterizes the locus $\beta = \beta^*(\delta)$.

Corollary 2. *(i) The locus $\beta = \beta^*(\delta)$ is decreasing in $\delta \in [0, \hat{\delta}]$. (ii) The rise in λ lowers $\beta = \beta^*(\delta)$ for $\delta \in (0, \hat{\delta}]$. (iii) If λ is sufficiently large such that $\lambda \geq (2-\mu)/\mu$, then for $\delta \in (\hat{\delta}, 1]$, $\beta^*(\delta) = 0$.*

A locus $\beta = \beta^*(\delta)$ is illustrated in Fig. 1. If $\lambda \geq (2-\mu)/\mu$, the high-cost firm can be induced to be honest by a high continuation value within the self-generating segment, even if $\beta = 0$.

Lemma 7. *Assume that $\rho < C_H$. If off-schedule incentive constraints are satisfied, then for all $\{(\delta,\beta) : \beta \geq \beta^*(\delta)\}$, then there exists a set $Z_\beta \subset \mathbf{U}_M$ such that $Z_\beta \cup \mathbf{U}_N$ is a self-generating set of PPE values.*

4.2. Optimal PPE with "Hard-to-Enforce" Contracting

As of yet, our analysis has been confined to the equilibrium path. In this subsection, we identify a discount factor above which both on- and off-schedule incentives are satisfied. We first consider the policy vector in which the constraint add-IC is binding, given the parameter range $\{(\delta, \alpha) : \alpha = \alpha^* (\delta)\}$. Lemma 5 implies that there exists a policy vector $\mathbf{y}^*$ such that

$$d(\mathbf{y}^*; \alpha = \alpha^*(\delta)) = \frac{(1 - \alpha^*(\delta)) (\rho - \theta_H)}{\delta}.$$

Let δ^* and Z^* denote the associated critical discount factor and self-generating segment, respectively. To find δ^*, it suffices to check the off-schedule incentive constraint of a disadvantaged firm at an endpoint of Z^*, since there the firm is more tempted to undercut the price than at any other point of the segment. Consider, for example, off-schedule incentive constraints for firm 1. At the endpoint, firm 1 is assigned to $q^1_{LL} = q^1_{HH} = 0$ by the policy vector $\mathbf{y}^*$. In state (L, H), the firm gains nothing if it undercuts the price; it is supposed to capture the entire market, following the equilibrium path. In states (H, L) and (H, H), the current-period gains from a deviation are the same: $r - \theta_H$. But in state (H, L), the firm is less tempted to undercut the price, since it will be rewarded by a high continuation value v^1_{HL} for giving up producing today. Hence, off-schedule incentive constraints boil down to off-IC-P$^i_{LL}$ and off-IC-P$^i_{HH}$. If critical discount factors, associated with the two constraints, are denoted δ^*_{LL} and δ^*_{HH}, respectively, then $\delta^* = \max\{\delta^*_{LL},\ \delta^*_{HH}\}$. The following proposition summarizes the result.

Proposition 1. *Assume that $\rho \geq C_H$. Then for $\{(\delta, \alpha) : \delta \in (\delta^*, 1]$ and $\alpha = \alpha^*(\delta)\}$, there exists a set $Z^* \subset \mathbf{U}_M$ such that $Z^* \cup \mathbf{U}_N$ is a self-generating set of PPE values.*

Example. When $\rho = 4$, $\theta_H = 2$, $\theta_L = 1$ and $\mu = 0.6$, then $\delta^*_{LL} \approx 0.729$ and $\delta^*_{HH} \approx 0.678$. The critical discount factor $\delta^* \approx 0.729$. Thus, for $\delta \in (0.729, 1]$ and $\alpha = (1 - 0.86\delta) / (1 - 0.28\delta)$, there exists a self-generating segment Z^* in which $u^1 + u^2 = 2.6/(1 - \delta)$.

The result is analogous for $\rho < C_H$. Interpretations of the findings are as follows. First, there is an internal contract under which firms with $\delta > \delta^*$ can replicate the performance of the firm under an inter-firm merging contract. In other words, there is an internal contract that can mimic an inter-firm merging contract in terms of its performance. Second, moderately patient firms may achieve the optimal merged profit for a wide range of parameters. Even to moderately patient firms, undertaking off-schedule deviations may not be profitable, due to the twofold benefit: productions are carried out by relatively efficient agents, and efficient agents are sorted out in the least costly way.

Until now our analysis has identified δ^* that corresponds to the parameter range $\{(\delta, \alpha) : \alpha = \alpha^*(\delta)\}$. Consider next the parameter range $\{(\delta, \alpha) : \alpha > \alpha^*(\delta)\}$. In Lemma 5, it is shown that there exists a policy vector $\mathbf{y}_\alpha$ in which $d(\mathbf{y}_\alpha; \alpha) = (1 - \alpha)(\rho - \theta_H)/\delta$ for a larger α; thus, firms may be able to establish a "shortened segment."[15] In particular, shortening the segment may relax the off-schedule incentive constraint of the disadvantaged firm at an endpoint by softening the incentive to undercut the price; thus some firms with $\delta < \delta^*$ may be able to establish a self-generating set $Z_\alpha \subset \mathbf{U}_M$. The following proposition shows that there is a boundary of (δ, α), above which firms are able to achieve the optimal collusion.

Proposition 2. *Assume that $\rho \geq C_H$. Then there exists a boundary $\{(\delta, \alpha) : f(\delta, \alpha) = \kappa$, where $\kappa \in \Re_+\}$ such that for $\{(\delta, \alpha) : f(\delta, \alpha) \geq \kappa\}$, there exists a set $Z_\alpha \subset \mathbf{U}_M$ such that $Z_\alpha \cup \mathbf{U}_N$ is a self-generating set of PPE values.*

For $\rho < C_H$, an analogous result is obtained. A boundary $f(\delta, \alpha) = \kappa$ is depicted in Fig. 2. Note that for $\delta \in [\delta^*, 1]$, the boundary is the locus $\alpha = \alpha^*(\delta)$. The boundary cannot be upward-sloping. The reason is that if for a given δ, the segment Z_α could be established for α, then the same segment Z_a can be established for $\alpha' > \alpha$; if a longer segment was constructed, the same

[15] Similarly, there exists a policy vector $\mathbf{y}_\beta$ in which $d(\mathbf{y}_\beta; \beta) = (1 - \beta)(2 - \mu)(r - \theta_H)/\delta$.

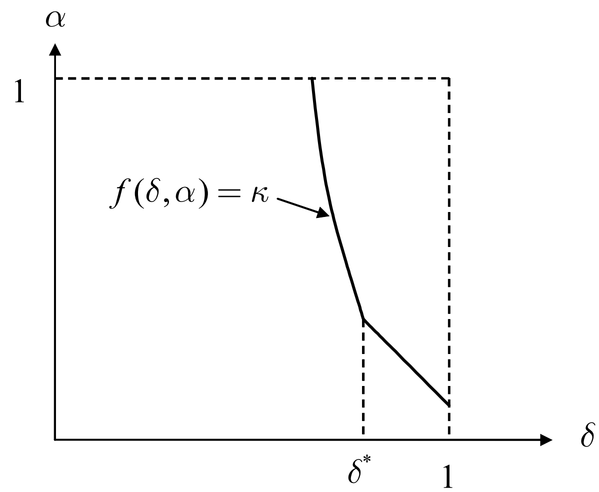

Figure 2: A boundary $f(\delta, \alpha) = \kappa$.

longer segment can be constructed even if the required length is shorter.

4.3. Symmetric PPE with Enforceable Contracting

In this subsection, we will show that if α is sufficiently large, then the Pareto-frontier value set can be approximated by a "symmetric" PPE (SPPE), without depending on Assumption 1. SPPE requires that firms' strategies be symmetric. For example, in an optimal SPPE, $p^i_{jk} = \rho$, $q^1_{LH} = q^2_{HL} = 1$ and $q^1_{kk} = q^2_{kk} = 1/2$. If firms' strategies are symmetric, the equilibrium value set is restricted to the 45 degree line on the payoff space: $\{(u^1, u^2) : u^1 = u^2\}$. This restriction on the equilibrium set negatively affects the firms' payoffs, since it prohibits firms' transfers in the form of continuation values, which might otherwise be facilitated by asymmetric plays; thus, it is more costly to sort firms by their cost types. This restriction, however, makes it easy for firms to write an enforceable contract. Further, even in the presence of this restriction, if $\alpha \to 1$, then SPPE value set can approximate the Pareto-frontier value set, defined as

$$\mathbf{U}_M^S \equiv \left\{ \left(u^1, u^2\right) \subset \Re_+^2 : u^1 = u^2 = \frac{\Pi_M}{2\left(1-\delta\right)} \right\}.$$

Depending on symmetric strategies, we here construct a self-generating set $\{(x,x),(X,X)\}$, where $X > x$.

To construct the point (X,X), consider a symmetric policy vector $\mathbf{y}$ in which price and production are assigned such that $p_{jk}^i = \rho$, $q_{LH}^1 = q_{HL}^2 = 1$, $q_{LL}^i = 1/2$ and $q_{HH}^i = 1/2 - \varepsilon$, and continuation values are assigned such that $v_{LH}^i = v_{HL}^i = v_{HH}^i = X$ and

$$v_{LL}^i = X - \frac{(1-\alpha)(\rho-\theta_H)(1+2(1-\mu)\varepsilon)}{2\delta\mu} = x. \tag{15}$$

The number $\varepsilon > 0$ can be arbitrarily small. The continuation value v_{LL}^i is assigned to a lower value x such that the downward incentive constraints (on-IC-P$_H^i$) are binding: A lower continuation value v_{LL}^i counters the firms' incentive to understate costs today. An additional constraint (add-IC) here is that to elicit truthful communication at the point (X,X), X must be greater than x by $(1-\alpha)(\rho-\theta_H)(1+2(1-\mu)\varepsilon)/\mu(2\delta)$. Given this policy vector, each firm can write an enforceable contract: If agent i reports low cost and produces q^i, the agent receives

$$t^i\left(q^i; L\right) = \theta_L q^i + R_L,$$

where $R_L = (\theta_H - \theta_L)\bar{q}_H^i = (1-\mu)(\theta_H - \theta_L)(1/2 - \varepsilon)$, and if agent i reports high cost and produces q^i, the agent receives

$$t^i\left(q^i; H\right) = \begin{cases} \theta_H q^i & \text{if } 0 \le q^i \le \frac{1}{2} - \varepsilon \\ \theta_H q^i + \alpha G^i & \text{if } \frac{1}{2} - \varepsilon < q^i \le 1, \end{cases}$$

where $G^i = (\rho - \theta_H)(\bar{q}_L^i - \bar{q}_H^i) = (\rho - \theta_H)(1/2 + (1-\mu)\varepsilon)$. This contract can be enforced, since the terms R_L and G^i boil down to constants and the realized productions $\left(q^i\right)$ are assumed to be verifiable.

To construct the other point (x, x), consider a policy vector $\mathbf{y}'$ in which productive efficiency is reduced such that $q^1_{LH} = q^2_{HL} = 1 - \varepsilon$, $q^i_{LL} = 1/2 - \varepsilon$ and $q^i_{HH} = 1/2 - 2\varepsilon$, where $0 \leq \varepsilon < 1/2$, while the same continuation values and price are used as in $\mathbf{y}$. Likewise, given this policy vector, each firm can write an enforceable contract: If agent i reports low cost and produces q^i, the agent receives

$$t^i\left(q^i; L\right) = \theta_L q^i + R_L,$$

where $R_L = (\theta_H - \theta_L)[(1-\mu)/2 + (3\mu - 2)\varepsilon]$, and if agent i reports high cost and produces q^i, the agent receives

$$t^i\left(q^i; H\right) = \begin{cases} \theta_H q^i & \text{if } 0 \leq q^i \leq \frac{1}{2} - 2\varepsilon \\ \theta_H q^i + \alpha G^i & \text{if } \frac{1}{2} - 2\varepsilon < q^i \leq 1, \end{cases}$$

where $G^i = (\rho - \theta_H)\left(1/2 - 2(2\mu - 1)\varepsilon\right)$. The terms R_L and G^i again boil down to constants. Let this contract be denoted M'.

In this symmetric collusive scheme, firms are induced to be truthful as follows: If state (L, L) is realized in the preceding period, each firm offers the contract M' and implements the policy vector $\mathbf{y}'$ by reducing productive efficiency, and otherwise, each firm offers M and implements the policy vector $\mathbf{y}$. When in particular α is closer to 1, the two points (x, x) and (X, X) can be located nearer, and production can be more efficiently assigned as ε is closer to 0. Letting δ_s denote the critical discount factor, the result can be summarized as follows.

Proposition 3. *Assume that $\rho \geq C_H$. If $\alpha \to 1$, then the optimal SPPE can be approximated; for all $\delta > \delta_s$, there exists a set $\{(x, x), (X, X)\}$, such that $\{(x, x), (X, X)\} \cup \mathbf{U}_N$ is a self-generating set of SPPE values, and $X \to \Pi_M/2(1-\delta)$ and $X - x \to 0$.*

This result is similarly obtained for $\rho < C_H$. The results show that the negative effect of symmetrization may be alleviated, if α is sufficiently large. A formal proof for this result is provided in the Appendix. This finding is related to the work by Athey, Bagwell and Sanchirico [5]. Considering collusion when a continuum of firms' types is assumed, they characterize a SPPE where equilibrium set is restricted to $\{(u^1, u^2) : u^1 = u^2\}$.[16] They find that wasteful continuation values (prices wars) are not used and price efficiency is achieved, but productive inefficiency obtains. This paper finds, by contrast, that productive efficiency can be approximately achieved in a SPPE if each firm is bounded by a contract that makes it very costly to lie (to distort the reported information). The reason is that in this discrete-type model, it is less costly to sort firms by their types, since a firm suffers a larger expense when it tells a lie[17]

4.4. Asymmetric PPE with Enforceable Contracting

In this subsection, we will show that the Pareto-frontier value set can be achieved, without requiring firms' strategies to be symmetric and without depending on Assumption 1. Price and production are thus efficiently assigned in this subsection. Consider the following contract offered in the parameter range where $\rho \geq C_H$. If agent i reports low cost and produces q^i, the agent receives

$$t^i\left(q^i;\, L\right) = \theta_L q^i + \frac{(1-\mu)\left(\theta_H - \theta_L\right)}{2}, \tag{15}$$

where the last term of the RHS is the information rent that corresponds to $q^i_{HH} = 1/2$. If agent i reports high cost and produces q^i, the agent receives

$$t^i\left(q^i;\, H\right) = \begin{cases} \theta_H q^i & \text{if } 0 \leq q^i \leq \frac{1}{2} \\ \theta_H q^i + \alpha\left(\rho - \theta_H\right)\left(q^i - \frac{1}{2}\right) & \text{if } \frac{1}{2} < q^i \leq 1. \end{cases} \tag{16}$$

[16] To be specific, the distribution over sellers' types is log-concave.

[17] In this paper, the cost of sorting firms by their types is lowered by the presence of contract. In the context of price discrimination, Lee [14] shows that if a continuum of consumer types is assumed, a firm may be able to discriminate consumers if and only if the firm has a mechanism that reduces the cost of sorting consumers by their types.

The market share q^i_{HH} here is fixed at 1/2. Whereas firms are then less flexible to provide market share favors at state (H, H), a "favored" firm suffers lower information rents than it would when $q^i_{HH} = 1$. Given the contract, if firm i requests the high-cost agent to produce more than a fixed amount of quantity (i.e., 1/2), then it suffers an extra expense, $\alpha(\rho - \theta_H)(q^i - 1/2)$.

If $\rho < C_H$, the contract defined in (12) is enforceable without being modified. In other words, in this parameter range Proposition 1 and 2 hold without depending on Assumption 1. The following Proposition summarizes the result.

Proposition 4. *Assume that $\rho \geq C_H$ and $\mu > 1/3$. For $\{(\delta, \alpha) : \delta \in (\delta^*, 1]$ and $\alpha = \alpha^*(\delta)\}$, where*

$$\alpha^*(\delta) \equiv \frac{1 - \delta + \delta\mu(1 - \mu\lambda)}{1 - \delta + \delta\mu(1 + \mu)}, \tag{17}$$

there exists a set $Z^* \subset \mathbf{U}_M$ such that $Z^* \cup \mathbf{U}_N$ is a self-generating set of PPE values.

Given the simple contract, the previous results that correspond to $\{(\delta, \alpha) : \alpha > \alpha^*(\delta)\}$ can be immediately obtained. The result implies that firms with internal incentive problem may collude effectively, as opposed to the "non-agent" setting, as in Athey and Bagwell [4]. To characterize "first-best" collusion, Athey and Bagwell require λ, defined as $(\theta_H - \theta_L)/(\rho - \theta_H)$, to be not too small. We here show that if each firm is bounded by an internal contract that penalizes the firm were it to lie and increase its market share, achieving the optimal collusion may not depend on such a restriction on λ. Further, the internal contract may be simple and contingent on verifiable information.

5. Conclusions

In this paper, we provided a new perspective on collusive conduct by examining a contrasting incentive problem that collusive firms may face, arguing that internal contracting may facilitate inter-firm optimal collusion, whereas optimal collusion may lead to contractual efficiency. As an extension of the model, we may consider a firm (headquarters) offering a contract to two agents (or two divisions) who privately observe profitability of their own investment opportunities. Assuming that this firm has a budget constraint, it has to approve a large budget to only one project in each period. A speculative finding would be that a simple contracting may facilitate divisions' coordination (i.e., collusion), so that divisions communicate each other and then report headquarters which division will receive a large-budget projects in each period, whereas collusion (e.g., inter-divisional asymmetric plays) may enhance the efficiency of contract. In this way, the relationship between contracting and collusion, facilitated by repeated interactions, awaits further research.

Appendix

On-schedule Incentive Constraints. Given the contract defined in (3) and (4) in the text, if $p^i_{jk} = \rho$, then the interim stage profits of firm i are

$$\begin{aligned} U^i(H,H) &= (\rho - \theta_H)\bar{q}^i_H + \delta\bar{v}^i_H \\ U^i(L,H) &= (\rho - \theta_H)\bar{q}^i_L - \alpha G^i + \delta\bar{v}^i_L \\ U^i(L,L) &= (\rho - \theta_L)\bar{q}^i_L - (\theta_H - \theta_L)\bar{q}^i_H + \delta\bar{v}^i_L \\ U^i(H,L) &= (\rho - \theta_L)\bar{q}^i_H - (\theta_H - \theta_L)\bar{q}^i_H + \delta\bar{v}^i_H. \end{aligned}$$

When incentive constraints are downwardly binding, then

$$\delta(\bar{v}^i_H - \delta\bar{v}^i_L) = (1-\alpha)(\rho - \theta_H)(\bar{q}^i_L - \bar{q}^i_H). \tag{A1}$$

The LHS of (A1) for firm 1 is

$$\delta \left[\mu(v^1_{HL} - v^1_{LL}) + (1-\mu)(v^1_{HH} - v^1_{LH})\right], \tag{A2}$$

and the LHS for firm 2 is

$$\delta \left[\mu(v^2_{LH} - v^2_{LL}) + (1-\mu)(v^2_{HH} - v^2_{HL})\right] = \delta \left[\mu(v^1_{LL} - v^1_{LH}) + (1-\mu)(v^1_{HL} - v^1_{HH})\right]. \tag{A3}$$

To sum up both sides of (A1), we obtain

$$v^1_{HL} - v^1_{LH} = \frac{(1-\alpha)(\rho - \theta_H)}{\delta}. \quad \blacksquare$$

Proof of Lemma 2. The binding downward incentive constraint of firm 1 (on-IC-P^1_H) implies that

$$U^1(H,H) = U^1(L,H) = \sum_{k \in \{L,H\}} \mu_k (p_{Lk} - \theta_H) q^1_{Lk} - \alpha G^1 + \delta \bar{v}^1_L,$$

and

$$U^1(L,L) = \sum_{k \in \{L,H\}} \mu_k \left[(p_{Lk} - \theta_L)\, q^1_{Lk} - (\theta_H - \theta_L)\, q^1_{Hk}\right] + \delta \bar{v}^1_L.$$

From these equations, we obtain

$$U^1(L,L) = U^1(H,H) + (\theta_H - \theta_L)\left(\bar{q}^1_L - \bar{q}^1_H\right) + \alpha G^1.$$

When the low-cost firm overstates, it gains

$$\begin{aligned} U^1(H,L) &= \sum_{k \in \{L,H\}} \mu_k (p_{Hk} - \theta_L) q^1_{Hk} - R_L + \delta \bar{v}^1_H \\ &= \sum_{k \in \{L,H\}} \mu_k (p_{Hk} - \theta_H) q^1_{Hk} + \delta \bar{v}^1_H. \end{aligned}$$

Hence, for $\alpha \in [0,1]$,

$$U^1(L,L) - U^1(H,L) = (\theta_H - \theta_L)\left(\bar{q}^1_L - \bar{q}^1_H\right) + \alpha G^1 > 0.$$

Similarly, the equation holds for firm 2. $\blacksquare$

Proof of Lemma 5. (i) To construct an endpoint (x_α, X_α), consider a policy vector:

$$\mathbf{y}_\alpha \equiv \begin{cases} p^i_{jk} = \rho, \\ q^1_{HH} = q^1_{LL} = q^1_\alpha \ (q^2_{HH} = q^2_{LL} = q^2_\alpha), \\ q^1_{LH} = q^2_{HL} = 1 \ (q^1_{HL} = q^2_{LH} = 0), \\ v^1_{LH} = v^1_{HH} = x_\alpha \ (v^2_{LH} = v^2_{HH} = X_\alpha), \end{cases} \tag{A4}$$

where $q^1_\alpha + q^2_\alpha = 1$. Assume that the downward on-schedule incentive constraints (on-IC-P^i_H) are binding. Then letting $U^1 = x_\alpha$ and $U^2 = X_\alpha$, the policy vector yields

$$x_\alpha = \frac{1}{1-\delta}[(\rho - \theta_H)\left[1 - \mu + \mu\alpha(2\mu - 1)\right] q^1_\alpha + (\theta_H - \theta_L)\mu(2\mu - 1)q^1_\alpha \tag{A5}$$
$$+ (\rho - \theta_H)\mu(1 - \mu\alpha) + (\theta_H - \theta_L)\mu(1 - \mu)]$$

$$X_\alpha = \frac{1}{1-\delta}[(\rho - \theta_H)\left[1 - \mu + \mu\alpha(2\mu - 1)\right] q^2_\alpha + (\theta_H - \theta_L)\mu(2\mu - 1)q^2_\alpha \tag{A6}$$
$$+ (\rho - \theta_H)\mu(1 - \mu)\alpha + (\theta_H - \theta_L)\mu(1 - \mu)].$$

Note that $x_a + X_a = \frac{r - E(\theta)}{1-\delta}$ for all α. The constraint add-IC is binding,

$$d(\mathbf{y}_\alpha; \alpha) = \frac{(1-\alpha)(\rho - \theta_H)}{\delta}, \tag{A7}$$

if and only if the policy vector $\mathbf{y}_\alpha$ assigns

$$q^1_\alpha = \frac{1}{2} - \frac{(1-\alpha)\left(\mu + \frac{1-\delta}{\delta}\right)}{2(1-\mu) + 2\mu(2\mu - 1)(\alpha + \lambda)}, \tag{A8}$$

where $\lambda = \frac{\theta_H - \theta_L}{\rho - \theta_H}$. If we find the differential $X_\alpha - x_\alpha$ from (A5) and (A6) and plug it into (A7), we obtain (A8). The lowest level of α at which (A7) holds is then given by

$$\alpha^*(\delta) \equiv \frac{1 - \delta + \delta(2\mu - 1)(1 - \mu\lambda)}{1 - \delta + \delta 2\mu^2}. \tag{A9}$$

Hence, for $\{(\delta, \alpha) : \alpha \geq \alpha^*(\delta)\}$, there exists a policy vector $\mathbf{y}_\alpha$ in which the constraint add-IC is binding. This is immediate from (A8). For a given δ,

(a) if $\alpha = \alpha^*(\delta)$, then $q^1_\alpha = 0$ (i.e., $q^2_{LL} = q^2_{HH} = 1$),

(b) if $\alpha \in (\alpha^*(\delta), 1)$, then $0 < q^1_\alpha < \frac{1}{2}$ (i.e., $q^1_{LL} = q^1_{HH} \in \left(0, \frac{1}{2}\right)$), and

(c) if $\alpha = 1$, then $q^1_\alpha = \frac{1}{2}$ (i.e., $q^i_{LL} = q^i_{HH} = \frac{1}{2}$).

Since the continuation values, v^1_{HL} and v^1_{LL}, are not specified in the policy vector, we need to show that given the policy vector $\mathbf{y}_\alpha$, v^1_{HL} and v^1_{LL} are drawn from the self-generating segment: $x_\alpha \leq v^1_{HL} \leq X_\alpha$ and $x_\alpha \leq v^1_{LL} \leq X_\alpha$. Consider first the value v^1_{HL}. Given the policy vector $\mathbf{y}_\alpha$, adding the binding on-schedule incentive constraints of both firms obtains

$$v^1_{HL} = x_\alpha + \frac{(1-\alpha)(\rho - \theta_H)}{\delta}.$$

When q^1_α is chosen as in (A8), the constraint add-IC is binding:

$$X_\alpha - x_\alpha = \frac{(1-\alpha)(\rho - \theta_H)}{\delta}.$$

The continuation value v^1_{HL} is then given by

$$v^1_{HL} = x_\alpha + \frac{(1-\alpha)(\rho - \theta_H)}{\delta} = X_\alpha.$$

Consider next the value v^1_{LL}. The binding on-IC-P^1_H yields

$$v^1_{LL} = x_\alpha + \frac{2\mu - 1}{\mu} \cdot \frac{(1-\alpha)(\rho - \theta_H)\left(1 - q^1_\alpha\right)}{\delta},$$

and thus $x_\alpha < v^1_{LL} < X_\alpha$ for all $\alpha \in [\alpha^*(\delta), 1)$. If $\alpha = 1$, then $q^1_\alpha = q^i_{kk} = \frac{1}{2}$ and $v^i_{jk} = x_\alpha = X_\alpha$.

Hence, the constraint on-IC-P^1_H is binding, since v^1_{LL} is chosen such that this constraint is binding. On the other hand, v^1_{HL} is chosen by adding both sides of on-IC-P^1_H and on-IC-P^2_H, when each of them is binding (adding both sides of (5) in the text). Thus, if on-IC-P^1_H is binding, then on-IC-P^2_H also is binding. Until now, we have seen that for $\{(\delta, \alpha) : \alpha \geq \alpha^*(\delta)\}$, there exists a policy vector $\mathbf{y}_\alpha$ that establishes the endpoint (x_α, X_α). Letting $\mathbf{y}'_\alpha$ denote an analogous policy vector that implements the other endpoint, the remainder of the segment can be established with the convex combination of $\mathbf{y}_\alpha$ and $\mathbf{y}'_\alpha$.

(ii) We next prove the second claim: For the parametric area $A^{on} \equiv \{(\delta, \alpha) : \alpha^*(\delta) < \alpha < 1\}$, there exists a policy vector in which the constraint add-IC is slack. Consider a point $(\hat{\delta}, \hat{\alpha}) \in A^{on}$. Then there exists a point $(\hat{\delta} - \varepsilon, \hat{\alpha}) \in A^{on}$ for a small number $\varepsilon > 0$. The previous result implies that at this point, there exists a policy vector $\mathbf{y}_{\hat{\alpha}}$ such that the associated constraint add-IC is binding:

$$X_{\hat{\alpha}} - x_{\hat{\alpha}} = (1 - \hat{\alpha}) \left(\frac{\rho - \theta_H}{\hat{\delta} - \varepsilon} \right).$$

Thus, given the policy vector $\mathbf{y}_{\hat{\alpha}}$, the constraint add-IC is slack for $\delta = \hat{\delta}$, since

$$X_{\hat{\alpha}} - x_{\hat{\alpha}} > (1 - \hat{\alpha}) \left(\frac{\rho - \theta_H}{\hat{\delta}} \right).$$

It also can be shown that continuation values v^1_{HL} and v^1_{LL} are drawn from the self-enforcing segment; continuation value v^1_{HL} is given by

$$v^1_{HL} = x_{\hat{\alpha}} + (1 - \hat{\alpha}) \left(\frac{\rho - \theta_H}{\hat{\delta}} \right) < X_{\hat{\alpha}},$$

and thus, $x_{\hat{\alpha}} < v^1_{HL} < X_{\hat{\alpha}}$, and it is immediate that $x_\alpha < v^1_{LL} < X_\alpha$. Lastly, for the same reason as above, both on-IC-P^1_H and on-IC-P^2_H are binding. ■

Proof of Proposition 1. Consider first the case in which $\rho \geq C_H$. Finding the policy vector that satisfies all the on-schedule incentive constraints is immediate from the proof of Lemma 5. For $\{(\delta, \alpha) : \alpha = \alpha^*(\delta)\}$, the policy vector that establishes the endpoint (x_α, X_α) is

$$\mathbf{y}^* \equiv \begin{cases} p^i_{jk} = \rho \\ q^1_{HH} = q^1_{LL} = 0 \; (q^2_{HH} = q^2_{LL} = 1) \\ q^1_{LH} = q^2_{HL} = 1 \; (q^1_{HL} = q^2_{LH} = 0) \\ v^1_{LH} = v^1_{HH} = x_\alpha \; (v^2_{LH} = v^2_{HH} = X_\alpha). \end{cases} \tag{A10}$$

Given this policy vector, the constraint add-IC is binding such that

$$d(\mathbf{y}^*; \alpha = \alpha^*(\delta)) = \frac{(1 - \alpha^*(\delta))(\rho - \theta_H)}{\delta}.$$

In other words, given $\mathbf{y}^*$,

$$X_\alpha - x_\alpha = (1-\alpha^*(\delta))\left(\frac{\rho-\theta_H}{\delta}\right).$$

Since the binding on-schedule incentive constraints imply that

$$v^1_{HL} = x_\alpha + (1-\alpha^*(\delta))\left(\frac{\rho-\theta_H}{\delta}\right),$$

we can obtain $v^1_{HL} = X_\alpha$ within the segment. From the binding on-IC-P^1_H,

$$v^1_{LL} = x_\alpha + \frac{2\mu-1}{\mu}\cdot\frac{(1-\alpha^*(\delta))(\rho-\theta_H)}{\delta},$$

we also can obtain v^1_{LL} within the segment: $x_\alpha < v^1_{LL} < X_\alpha$.

The off-schedule incentive constraints, given this policy vector $\mathbf{y}^*$, are

$$\delta(v^1_{LL} - \underline{v}) \geq \rho - \theta_L \qquad \text{(off-IC-P}^1_{LL}\text{)}$$

$$\delta(v^1_{LH} - \underline{v}) \geq \rho - \theta_L \qquad \text{(off-IC-P}^1_{LH}\text{)}$$

$$\delta(v^1_{HL} - \underline{v}) \geq \rho - \theta_H \qquad \text{(off-IC-P}^1_{HL}\text{)}$$

$$\delta(v^1_{HH} - \underline{v}) \geq \rho - \theta_H \qquad \text{(off-IC-P}^1_{HH}\text{)}$$

where $\underline{v} = \frac{\Pi^i_N}{1-\delta}$. Then off-IC-P$^1_{LH}$ is slack and off-IC-P$^1_{HL}$ is implied by off-IC-P$^1_{HH}$. The relevant off-schedule incentive constraints boil down to off-IC-P$^1_{LL}$ and off-IC-P$^1_{HH}$:

$$\delta\left(x_\alpha + \frac{2\mu-1}{\mu}\cdot\frac{(1-\alpha^*(\delta))(\rho-\theta_H)}{\delta} - \underline{v}\right) \geq \rho-\theta_L$$

$$\delta\left(x_\alpha - \underline{v}\right) \geq \rho-\theta_H.$$

When $\alpha = \alpha^*(\delta)$, the policy vector $\mathbf{y}^*$ in (10) yields

$$x_\alpha = \frac{\left[\mu-\mu^2\alpha^*(\delta)\right](\rho-\theta_H)+\mu(1-\mu)(\theta_H-\theta_L)}{1-\delta}$$

$$X_\alpha = \frac{\left[1-\mu+\mu^2\alpha^*(\delta)\right](\rho-\theta_H)+\mu^2(\theta_H-\theta_L)}{1-\delta}.$$

Note that $x_\alpha + X_\alpha = \frac{\rho - E(\theta)}{1-\delta}$. Plugging x_α into the two off-IC yields δ^*_{LL} and δ^*_{HH}. The critical discount factor under the contract in which $\alpha = \alpha^*(\delta)$ is $\delta^* = \max\{\delta^*_{LL}, \delta^*_{HH}\}$.

Consider next the case in which $\rho < C_H$. For $\{(\delta, \beta) : \beta = \beta^*(\delta)\}$, define a policy vector:

$$\mathbf{y}^* \equiv \begin{cases} p^i_{jk} = \rho \\ q^1_{LH} = q^2_{HL} = q^2_{LL} = 1 \\ q^1_{HH} = q^2_{HH} = 0 \\ v^1_{LH} = v^1_{HH} = x_\beta \; (v^2_{LH} = v^2_{HH} = X_\beta). \end{cases} \tag{A11}$$

Given this policy vector, the constraint add-IC is binding such that

$$d(\mathbf{y}^*; \beta = \beta^*(\delta)) = \frac{(1-\beta^*(\delta))(2-\mu)(\rho - \theta_H)}{\delta}.$$

Thus, given the policy vector in (A11),

$$X_\beta - x_\beta = \frac{(1-\beta^*(\delta))(2-\mu)(\rho - \theta_H)}{\delta}.$$

Since the binding on-schedule incentive constraints imply that

$$v^1_{HL} = x_\beta + \frac{(1-\beta^*(\delta))(2-\mu)(\rho - \theta_H)}{\delta},$$

we can obtain $v^1_{HL} = X_\beta$ within the segment. The binding on-IC-P^1_H gives

$$v^1_{LL} = x_\beta + \frac{2\mu - 1 + \mu(1-\mu)}{\mu} \cdot \frac{(1-\beta^*(\delta))(\rho - \theta_H)}{\delta},$$

and thus $x_\alpha < v^1_{LL} < X_\alpha$. Given the policy vector,

$$x_\beta = \frac{[\mu(2-\mu) - \mu\beta^*(\delta)]\,(\rho - \theta_H) + \mu(1-\mu)(\theta_H - \theta_L)}{1-\delta} \tag{A12}$$

$$X_\beta = \frac{\mu\beta^*(\delta)(\rho - \theta_H) + \mu(\theta_H - \theta_L)}{1-\delta}. \tag{A13}$$

Hence, for $\beta = \beta^*(\delta)$, there exists the policy vector $\mathbf{y}^*$ that establishes the endpoint (x_β, X_β) where $x_\beta + X_\beta = \frac{[1-(1-\mu)^2](\rho - \theta_L)}{1-\delta}$. The remainder of the self-generating segment can be implemented by the convex combination of $\mathbf{y}^*$ and $\mathbf{y}^{*\prime}$ that establishes the other endpoint.

The off-schedule incentive constraints can be reduced to the two constraints:

$$\delta\left(x_\beta + \frac{2\mu - 1 + \mu(1-\mu)}{\mu} \cdot \frac{(1-\beta^*(\delta))(\rho - \theta_H)}{\delta} - \underline{v}\right) \geq \rho - \theta_L \qquad (\text{off-IC-P}^1_{LL})$$

$$\delta\left(x_\beta - \underline{v}\right) \geq \rho - \theta_H. \quad (\text{off-IC-P}^1_{HH})$$

Plugging x_β in (A12) into the two constraints yields δ^*_{LL} and δ^*_{HH} respectively. The critical discount factor is $\delta^* = \max\{\delta^*_{LL}, \delta^*_{HH}\}$. ■

Proof of Proposition 2. Given the policy vector defined in (A4), the relevant off-schedule incentive constraints of firm 1 in an endpoint are

$$\delta\left(x_\alpha + \frac{2\mu - 1}{\mu} \cdot \frac{(1-\alpha)(\rho - \theta_H)}{\delta} - \underline{v}\right) \geq (\rho - \theta_L)\left(1 - q^1_\alpha\right) \tag{A14}$$

$$\delta\left(x_\alpha - \underline{v}\right) \geq (\rho - \theta_H)\left(1 - q^1_\alpha\right), \tag{A15}$$

where x_α and q^1_α are given by (A5) and (A8), respectively. Both x_α and q^1_α rise with $\alpha \in [\alpha^*(\delta), 1]$. Plugging x_α and q^1_α into (A14) and (A15) yields the set of the pair (δ, α), where both off-schedule incentive constraints are satisfied. Then, applying the fact that the boundary cannot be upward-sloping to this set obtains the set $\{(\delta, \alpha) : f(\delta, \alpha) \geq \kappa\}$. ■

Proof of Proposition 3. To construct the point (X, X), consider a symmetric policy vector:

$$\mathbf{y} \equiv \begin{cases} p^i_{jk} = \rho \\ q^1_{LL} = q^2_{LL} = \frac{1}{2} \\ q^1_{HH} = q^2_{HH} = \frac{1}{2} - \varepsilon \\ q^1_{LH} = q^2_{HL} = 1 \\ v^i_{LH} = v^i_{HL} = v^i_{HH} = X, \end{cases} \tag{A16}$$

where $0 < \varepsilon < \frac{1}{2}$. If the downward incentive constraint of each firm (on-IC-P^i_H) is binding, the policy vector gives

$$X = \frac{\rho - E(\theta) - (1-\alpha)\mu(\rho - \theta_H) + 2\alpha\mu(1-\mu)(\rho - \theta_H)\varepsilon - 2\varphi\varepsilon}{2(1-\delta)}, \tag{A17}$$

where $\varphi \equiv (1-\mu)(\rho-\theta_H) - \mu(1-\mu)(\theta_H-\theta_L) > 0$. The continuation value v^i_{LL} is assigned to a lower value:

$$v^i_{LL} = X - \frac{(1-\alpha)(\rho-\theta_H)(1+2(1-\mu)\varepsilon)}{2\delta\mu},$$

so as to soften the firms' incentive to understate costs. This implies that to elicit truthful communication at the point (X, X), X must be greater than x at least by $\frac{(1-\alpha)(\rho-\theta_H)(1+2(1-\mu)\varepsilon)}{2\delta\mu}$. To consider the case in which add-IC is binding at (X, X), let $v^i_{LL} = x$. Then at the point (X, X), on-schedule incentive constraints are satisfied; v^i_{LL} is chosen such that on-IC-P^i_H is binding, and on-IC-P^i_L is slack by Lemma 2.

Given that the point (X, X) is constructed by the policy vector $\mathbf{y}$, we next consider a policy vector $\mathbf{y}'$ to construct the other point (x, x). As in the text, the policy vector $\mathbf{y}'$ thus assigns symmetric market shares such that $q^1_{LH} = q^2_{HL} = 1-\varepsilon$, $q^1_{LL} = q^2_{LL} = \frac{1}{2}-\varepsilon$ and $q^1_{HH} = q^2_{HH} = \frac{1}{2}-2\varepsilon$, where $0 \leq \varepsilon < \frac{1}{2}$. The on-schedule incentive constraints, on-IC-P^i_H and on-IC-P^i_L, are then given by

$$\frac{(1-\alpha)(\rho-\theta_H)\left(\bar{q}^i_L - \bar{q}^i_H\right)}{\delta\mu} \leq \frac{(1-\alpha)(\rho-\theta_H)(1+2(1-\mu)\varepsilon)}{2\delta\mu} \leq \frac{(\rho-\theta_L)\left(\bar{q}^i_L - \bar{q}^i_H\right)}{\delta\mu}.$$

Since $\bar{q}^i_L - \bar{q}^i_H = \frac{1}{2} - 2(2\mu-1)\varepsilon$, both constraints will be satisfied if α is sufficiently large or ε is sufficiently small. We now construct a self-generating value x. The *ex ante* expected payoff at (x, x) is

$$\begin{aligned} U^i &= \mu U^i(L,L) + (1-\mu)\,U^i(H,H) \\ &= \mu(\rho-\theta_L)\,\bar{q}^i_L + (1-\mu)(\rho-\theta_H)\,\bar{q}^i_H - \mu(\theta_H-\theta_L)\,\bar{q}^i_H \\ &\quad +\delta\mu\bar{v}^i_L + \delta(1-\mu)\,\bar{v}^i_H. \end{aligned}$$

Given the policy vector, letting $x = U^i$ obtains the value x:

$$x = \frac{\rho - E(\theta) + (1-\alpha)(\rho-\theta_H)\left(1+\frac{2(1-\mu)(1-\mu^2)\varepsilon}{\mu}\right) - 2\xi\varepsilon}{2(1-\delta)}, \tag{A18}$$

where $\xi \equiv \mu(\rho - \theta_L) - [(1-\mu)(\rho-\theta_H) - \mu(\theta_H - \theta_L)](3\mu - 2) > 0$. Using (A17) and (A18), the binding add-IC constraint $X - x = \frac{1-\alpha}{\mu}\left(\frac{\rho-\theta_H}{2\delta}\right)$ can be rewritten as

$$\varepsilon = \frac{(1-\alpha)(\rho-\theta_H)(1-\delta+\mu(1+\mu)\delta)}{2\mu(\xi-\varphi)\delta - 2(1-\mu)\left(1-\alpha-\delta\mu^2\right)(\rho-\theta_H)},$$

where $\xi - \varphi > 0$. The RHS decreases with α. Hence, if α is sufficiently large, there exists ε (i.e., a policy vector) that satisfies all the on-schedule constraints, on-IC-P^i_H, on-IC-P^i_L and add-IC, at both points (x, x) and (X, X).

The critical discount factor is defined as $\delta_s = \max\{\delta_x, \delta_X\}$, where δ_x and δ_X are given when the respective off-schedule constraints at (x, x) and (X, X) are binding:

$$\begin{aligned} \delta(x - \underline{v}) &\geq \max\left\{(\rho-\theta_L)\left(\frac{1}{2}-\varepsilon\right),\ (\rho-\theta_H)(1-\varepsilon)\right\} \\ \delta(X - \underline{v}) &\geq \max\left\{\frac{\rho-\theta_L}{2},\ \rho-\theta_H\right\}. \end{aligned}$$

In both constraints, the first term of the RHS is what the low-cost firm could earn today in state (L, L) by undercutting the price, and the second term is what the high-cost firm could earn today in state (L, H) or (H, L) by undercutting the price. Lastly, if $\alpha \to 1$, then for $\delta > \delta_s$, there exists $\varepsilon \to 0$ such that $X \to \frac{\rho - E(\theta)}{2(1-\delta)}$ and $X - x \to 0$. ■

Proof of Proposition 4. Consider a policy vector $\mathbf{y}^*$ that implements an endpoint (x_α, X_α), given $\alpha = \alpha^*(\delta)$ defined by (17) in the text.

$$\mathbf{y}^* \equiv \begin{cases} p^i_{jk} = \rho \\ q^i_{HH} = \frac{1}{2},\ q^2_{LL} = 1 \\ v^1_{LH} = v^1_{HH} = x_\alpha\ (v^2_{LH} = v^2_{HH} = X_\alpha). \end{cases} \tag{A19}$$

Given this policy vector, the constraint add-IC is binding such that

$$d(\mathbf{y}^*; \alpha = \alpha^*(\delta)) = \frac{(1-\alpha^*(\delta))(\rho-\theta_H)}{\delta}.$$

In other words, given $\mathbf{y}^*$,

$$X_\alpha - x_\alpha = \frac{(1-\alpha^*(\delta))\,(\rho-\theta_H)}{\delta}.$$

From the binding on-schedule incentive constraints,

$$v_{HL}^1 = x_\alpha + \frac{(1-\alpha^*(\delta))\,(\rho-\theta_H)}{\delta},$$

we obtain $v_{HL}^1 = X_\alpha$. The binding downward on-IC-P^1 yields

$$v_{LL}^1 = x_\alpha + \frac{3\mu-1}{2\mu}\cdot\frac{(1-\alpha^*(\delta))(\rho-\theta_H)}{\delta},$$

and thus if $\mu > \frac{1}{3}$, then $x_\alpha < v_{LL}^1 < X_\alpha$. Given the policy vector in (A19),

$$\begin{aligned} x_\alpha &= \frac{[1+\mu-\mu\,(1+\mu)\,\alpha^*\,(\delta)]\,(\rho-\theta_H)+\mu(1-\mu)(\theta_H-\theta_L)}{2(1-\delta)} \\ X_\alpha &= \frac{[1-\mu+\mu\,(1+\mu)\,\alpha^*\,(\delta)]\,(\rho-\theta_H)+\mu\,(1+\mu)\,(\theta_H-\theta_L)}{2(1-\delta)}. \end{aligned}$$

Notice that $x_\alpha + X_\alpha = \frac{\rho-E(\theta)}{1-\delta}$ and that the length of the self-generating segment equals what it is required, i.e., $X_\alpha - x_\alpha = \frac{(1-\alpha^*(\delta))(\rho-\theta_H)}{\delta}$. Hence, if $\alpha = \alpha^*(\delta)$, there exists the policy vector $\mathbf{y}^*$ that can implement the endpoint (x_α, X_α). The remainder of the self-generating segment can be established by convex combination of $\mathbf{y}^*$ and $\mathbf{y}^{**}$. Given the values at the endpoint, off-schedule incentive constraints are

$$\delta\left(x_\alpha + \frac{3\mu-1}{2\mu}\cdot\frac{(1-\alpha^*(\delta))(\rho-\theta_H)}{\delta} - \underline{v}\right) \geq \rho-\theta_L \qquad (\text{off-IC-P}_{LL}^1)$$

$$\delta\,(x_\alpha - \underline{v}) \geq \frac{\rho-\theta_H}{2}. \qquad (\text{off-IC-P}_{HH}^1)$$

Plugging x_α into the two off-schedule IC's yields δ_{LL}^* and δ_{HH}^* respectively. The critical discount factor is $\delta^* = \max\{\delta_{LL}^*, \delta_{HH}^*\}$; thus, for $\delta > \delta^*$, no firm will undertake an off-schedule deviation.

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