Testing the home market effect in a multi-country world: The theory*

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Abstract

We extend the two-country model by Krugman (1980) to a multi-country set-up and show that the 'home-market effect' highlighted with two countries does not readily extend to such a more general setting. In particular, we prove that the most important result, namely the disproportionate causation from demand to supply, generalizes only under the fairly implausible assumption of pairwise symmetric trade costs between all countries. We argue, therefore, that the implications of product differentiation for the structure of world trade are better characterized in terms of spatial ('accessibility') and non-spatial ('attraction') effects, and we provide a theory-based specification that suggests how to test the home market effect in a more general setting.

Keywords: home market effect; hub effect; market potential; new trade theory; economic geography.

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1 Introduction

What determines the structure of world trade? Two main explanations have been put forth (see, e.g., Helpman, 1990). The first one highlights the role of relative cost differences between countries: a country exports the goods that it is able to produce at relatively lower costs. The uneven international distribution of technology (Ricardian model) and/or relative factor endowments (Heckscher-Ohlin model) would then generate those differences (Dixit and Norman, 1980). The second explanation stresses the role of increasing returns to scale and market structure: a country exports the goods for which it offers a relatively large local demand. Strategic interactions and product differentiation support such outcomes, known as the "home market effect" (henceforth HME; Krugman, 1980). Indeed, while some kind of imperfect competition is needed for a sector to exhibit a HME, both oligopoly and monopolistic competition generally serve the purpose (Feenstra *et al.*, 2000; Head *et al.*, 2002).

As shown by Helpman and Krugman (1985), the two explanations are not incompatible. Yet, the first seems better fit for explaining intersectoral trade between somewhat different countries, whereas the second looks more suited to account for intrasectoral trade between similar countries. In particular, it has been argued that the former would explain North-South trade, whereas the latter would account for North-North trade, together more than 80 per cent of world trade flows. Nonetheless, the relative merits of the two explanations are still debated, as highlighted by recent empirical works (see, e.g., Davis and Weinstein, 1999, 2003). The reason being that relative costs matter also for North-North flows, and product differentiation is relevant also for North-South flows. Overall, however, as pointed out by Helpman (1998):

> "adding product differentiation improves the fit between theory and data. Since the inherent richness of models with product differentiation has not yet been much explored, they also carry the potential of providing even better explanations when subjected to further analysis".

One example of how theory still lags behind empirics is the investigation of the HME by Davis and Weinstein (2003). Their point of departure is the model by Krugman (1980), which portrays a two-country economy with one factor of production (labor) and two sectors. One sector supplies a freely-traded homogeneous good under constant returns to scale and perfect competition. The other one produces a horizontally differentiated good under increasing returns and monopolistic competition à la Dixit and Stiglitz (1977). For each differentiated variety, fixed and marginal input requirements are constant and international trade is hampered by frictional ('iceberg') trade costs. Preferences are Cobb-Douglas across the two goods and symmetric CES between varieties of the differentiated good. Due to the fixed input requirement, in equibrium the larger country supports the production of a more than proportionate number of differentiated varieties, thus being a net exporter of this good (Helpman and Krugman, 1985). What is crucial is that, in a Ricardian or Heckscher-Ohlin world, the HME would not arise. Specifically, when there are trade costs, increases in market size map into a less than proportional increase of industry, since a fraction of the additional demand is satisfied by imports from the rest of the world. All this suggests to compare the predictive power of the two alternative explanations by estimating the impacts of aggregate demand on the output of different sectors. A more than proportional causation from demand to supply would support the HME as a driving force for specialization and trade, whereas a less than proportional causation would support relative cost and/or endowment driven patterns.

The problem with applying the above idea to real data is that Krugman's clear-cut result has been derived in a two-country set-up only and, thus, need not hold in a multi-country world. This point has been recently emphasized by Head and Mayer (2004):

> "How do we construct demand measures in the presence of more than two countries? Indeed *how does one even formulate the home market effect hypothesis*? The ratios and shares of the theoretical formulations neglect third country effects."

As a first interesting solution to this intellectual deadlock, Davis and Weinstein (2003) construct an index (called 'IDIODEM') of the demand facing producers in a certain country that takes into account not only local demand but also some measure of demand from neighbouring contries. Then, by analogy with the two-country case, they conjecture that a larger than one estimate of the elasticity of output to the index would provide evidence in favor of the HME. Albeit empirically appealing and a natural first step, the IDIODEM index is not derived from clear theoretical foundations. Thus, unlike the authors seem to argue, its regression coefficient is hard to interpret.

The aim of the present paper is to present a theory-based analysis of the observable implications of Krugman's model when extended to many countries. In particular, our main objective is twofold.

First, we assess which results survive the extension. We show that the so-called 'dominant market effect' and the 'magnification effect' (see, e.g., Head *et al.*, 2002; Baldwin *et al.*, 2003) remain valid, thus suggesting that several of the underlying mechanisms are quite robust. Yet, we also show that the HME itself may not arise in the general setting. This is due to the fact that, once 'third country effects' are taken into account, an increase in one country's expenditure share may well map into a less than proportionate increase in its output share as other countries 'drain away' some firms. In more extreme cases, an increase in the expenditure share may even lead to a decrease in industry share ('home market effect shadow'), thus suggesting that the output response to increasing demand may be an inappropriate instrument to detect the empirical relevance of product differentiation and, more generally, imperfect competition as determinants of trade flows.

Second, since the 'Davis-Weinstein conjecture' is not generally supported by the extension of Krugman's model to a multi-country set-up, we propose an alternative, theory-based test. In particular, we derive an estimating specification in which the regressand variable is 'industry distribution' whereas the regressors are a spatial aggregate of 'country sizes' and a measure of 'accessibility'. Albeit slightly similar to a gravity specification (and related to the IDIODEM index), the null hypothesis is quite different from the one used in the empirical literature so far. In so doing, we move away from the definition in terms of disproportionate causation from demand to supply, which allows us to circumvent the problems highlighted in the foregoing.

The paper is divided into five sections. The first presents the multi-country extension of the model by Krugman (1980) and describes the equilibrium. The second defines the HME with many countries, both in a static and a dynamic way. The third relates the multi-country HME to the concepts of market potential and market size. The fourth discusses the effects of geography and presents a methodology that allows to test the HME in a multi-country world. The fifth finally concludes.

2 The model

The world economy consists of M countries, indexed by $i = 1, 2, \ldots, M$. Country i hosts an exogenously given mass of L_i consumers, each of them supplying one unit of labor inelastically. Hence, both the world population and the world endowment of labor are given by $L = \sum_{i=1}^{M} L_i$. Labor is the only factor of production and it is assumed to be internationally immobile.

2.1 Preferences and technologies

Preferences are defined over a homogeneous good and a set of varieties of a horizontally differentiated good. The preferences of a typical resident of country i are given by the following utility function:

$$U_i = D_i^{\mu} H_i^{1-\mu} \tag{1}$$

with $0 < \mu < 1$ and

$$D_i = \left[\int_{\omega \in \Omega_i} d_i(\omega)^{(\sigma-1)/\sigma} \, \mathrm{d}\omega \right]^{\sigma/(\sigma-1)}. \tag{2}$$

In the above expressions H_i is the consumption of the homogeneous good, $d_i(\omega)$ is the consumption of variety ω and Ω_i is the set of varieties available in country i. The parameter $\sigma > 1$ measures both the own- and cross-price elasticities of demand for any variety.

The production of the homogeneous good is carried out by perfectly competitive firms under constant returns to scale. The unit labor requirement is set to one by choice of units. Trade in the homogeneous good is free. The production of any variety of the differentiated good takes place under internal increasing returns to scale by a set of monopolistically competitive firms. This set is endogenously determined by free entry and exit. We denote by n_i the mass of firms located in country i and by $N = \sum_i n_i$ the total mass of firms in the world economy. The production technology of each variety requires a fixed and a constant marginal labor requirements labeled F and c respectively. Increasing returns to scale and costless product differentiation yield a one-to-one relation between firms and varieties, so we will use the two terms interchangeably. As to trade barriers, the international trade of any variety incurs 'iceberg' trade costs. Specifically, $\tau_{ij} > 1$ units have to be shipped from country i to country j for one unit to reach its destination.

2.2 Market equilibrium

In the homogeneous sector, perfect competition implies pricing at marginal cost, which, given the normalization of the unit input coefficient, is equal to the wage. Free trade then generates equalization across all countries. More precisely, this is the case as long as some homogeneous production takes place in all countries, which we assume to hold from now on. The formal conditions for this to happen are given in Appendix 1 and require that the expenditure share μ on the manufactured good is not too large (see (48)). We choose the homogeneous good as the numéraire, which implies that not only its price but also the wage equals one in all countries.

Turning to the differentiated sector, the symmetric set-up of the model implies that, in equilibrium, firms differ only by the country where they are located. Accordingly, to simplify notation, we will drop the variety label from now on. Then, the maximization of utility (1) yields the following demand in country j for a variety produced in country i:

$$d_{ij} = \frac{p_{ij}^{-\sigma}}{P_j^{1-\sigma}} \mu E_j, \tag{3}$$

where p_{ij} is the delivered price of the variety, E_j is expenditures in country j, and

$$P_j^{1-\sigma} = \sum_i \left(n_i p_{ij}^{1-\sigma} \right) \tag{4}$$

is the CES price index in country j. Expression (3) reveals the essence of monopolistic competition: firms do not interact directly but through changes in aggregate variables, i.e. P_j.

Let x_{ij} be the amount of production by the typical firm in country i. Due to trade costs, the firm has to produce $x_{ij} = d_{ij}\tau_{ij}$ units to satisfy final demand d_{ij}. The typical firm takes (3) into account when maximizing its own profit:

$$\begin{aligned} \Pi_i &= \sum_j \big(p_{ij} d_{ij} - c x_{ij} \big) - F \qquad (5) \\ &= \sum_j \left(p_{ij} - c\tau_{ij} \right) \frac{p_{ij}^{-\sigma}}{P_j^{1-\sigma}} \mu E_j - F. \end{aligned}$$

Profit maximization with respect to p_{ij} then implies that the price per unit delivered is:

$$p_{ij} = \frac{\sigma c}{\sigma - 1} \tau_{ij}. \tag{6}$$

Since, due to free entry and exit, profits have to be zero in equilibrium, (5) and (6) also imply that all firms reach the same scale of operations:

$$x_i = \frac{F(\sigma - 1)}{c}, \tag{7}$$

where $x_i = \sum_j d_{ij}\tau_{ij}$ is total firm production inclusive of the amount of output lost in transit. Hence, we can write the market clearing condition for a typical variety produced in country i as:

$$\sum_j d_{ij}\tau_{ij} = \frac{F(\sigma-1)}{c}. \tag{8}$$

Replacing (3) and (4) into (8), multiplying both sides by p_{ii}, and using (6), we get:

$$\sum_k \frac{\phi_{jk}L_k}{\sum_i n_i\phi_{ik}} = \frac{\sigma F}{\mu}, \qquad j = 1, 2\ldots, M, \tag{9}$$

where $\phi_{ik} \equiv \tau_{ik}^{1-\sigma}$ is a measure of trade freeness valued one when trade is free (i.e. $\tau_{ik} = 1$) and limiting zero when trade is inhibited (i.e. $\tau_{ik} \to \infty$). In (9) we have used the fact that, since profits are zero, in equilibrium expenditures equal labor income ($E_j = L_j$).

Multiplying both sides of (9) by n_i and summing up across countries, we get $N = \mu L/F\sigma$: in equilibrium the world mass of firms is constant and proportional to world population. This allows us to rewrite (9) in terms of shares. In particular, after defining $\theta_i \equiv L_i/L$ and $\lambda_i \equiv n_i/N$, the market clearing condition (9) becomes:

$$\sum_k \frac{\phi_{jk}\theta_k}{\sum_i \lambda_i\phi_{ik}} = 1, \qquad j = 1, 2, \ldots, M. \tag{10}$$

2.3 Matrix notation

An interior equilibrium is characterized by M conditions, given by (10). The firm shares λ_i's are M endogenous unknowns whereas the expenditure shares θ_i as well as the trade freeness measures ϕ_{ij}'s are exogenous parameters. From now on, we set $\phi_{ii} = 1$ meaning that trade is free within countries. We also set $\phi_{ij} = \phi_{ji}$ meaning that trade flows between any given pair of countries face the same trade costs in both directions. Since (10) describes a system of linear equations in the endogenous variables λ_i, we make notation more compact by recasting it in matrix form.

Specifically, let

$$\Phi \equiv \begin{pmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1M} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{M1} & \phi_{M2} & \cdots & \phi_{MM} \end{pmatrix}, \quad \lambda \equiv \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_M \end{pmatrix} \quad \text{and} \quad \theta \equiv \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_M \end{pmatrix}$$

where $\lambda^T \mathbf{1} = \theta^T \mathbf{1} = 1$ (in what follows, $\mathbf{1}$ stands for the vector whose components are all equal to one).

Using these definitions, the M equilibrium conditions (10) can be expressed in matrix notation as follows:

$$\Phi \mathrm{diag}(\Phi\lambda)^{-1}\theta = \mathbf{1}. \tag{11}$$

In order to simplify some of the subsequent developments, problem (11) is best conveniently decomposed into an *outer* and an *inner step.* The outer step consists in finding φ such that

$$\Phi\varphi = \mathbf{1}. \tag{12}$$

Note that this problem depends on the trade cost matrix Φ only and is hence independent of the expenditure distribution θ. In what follows, we assume that distance between countries is measured by the euclidian norm so that Φ is positive definite (see Appendix 2 for more details). Hence, there is a unique $\varphi = \Phi^{-1}\mathbf{1}$ satisfying equation (12). The inner step consists then in finding λ^* such that

$$\mathrm{diag}(\Phi\lambda^*)^{-1}\theta = \varphi. \tag{13}$$

Note that this inner step involves both Φ (directly and indirectly via φ) and θ. Equation (13) can also be expressed as

$$\theta = \mathrm{diag}(\varphi)\Phi\lambda^*. \tag{14}$$

If we denote by f_{ij} the cofactor of ϕ_{ij} and by $|\Phi|$ the determinant of Φ, the last expression (14) can finally be written component by component as

$$\theta_i = \varphi_i \sum_k \phi_{ik}\lambda_k^* = \frac{\sum_j f_{ij}}{|\Phi|} \sum_k \phi_{ik}\lambda_k^*, \tag{15}$$

which is simply the i-th row of expression (14).

2.4 Spatial equilibrium: existence and characterization

In what follows, Δ stands for the unit simplex of $\mathbb{R}^M$ and $\mathrm{ri}(\Delta)$ for its (relative) interior:[1]

$$\mathrm{ri}(\Delta) = \left\{ x \in \mathbb{R}^M, \ \sum_i x_i = 1, \ x_i > 0, \ \forall i \right\}.$$

[1] Note that, since Δ is contained in an $M-1$ dimensional hyperplane, its topological interior is empty in $\mathbb{R}^M$.

We assume that $\theta \in \mathrm{ri}(\Delta)$ so that all countries have at least some expenditure share. Furthermore, we focus on *interior equilibria* only, i.e. equilibria in which $\lambda_i^* > 0$ for all countries $i = 1, 2, \ldots M$. Hence, $\lambda^* \in \mathrm{ri}(\Delta)$.

A necessary condition for an interior solution to exist can be obtained by rewriting (15) as:

$$\theta_i = \varphi_i \sum_j \phi_{ij} \lambda_j^* < \varphi_i \sum_j \lambda_j^* = \varphi_i,$$

where the inequality results from $\phi_{ij} \in (0, 1)$ and where the last equality is due to the fact that the λ_j^*'s sum up to one. This implies that

$$\varphi_i > \theta_i, \quad i = 1, 2, \ldots, M \tag{16}$$

is a *necessary condition* for an interior equilibrium to exist. Provided such an equilibrium exists, the *equilibrium distribution* of firms is given by

$$\lambda^* = (\mathrm{diag}(\varphi)\Phi)^{-1}\,\theta, \tag{17}$$

or, component by component, by

$$\lambda_i^* = \sum_j \frac{f_{ij}}{\sum_k f_{jk}} \theta_j. \tag{18}$$

Since Φ is a symmetric matrix, $f_{ij} = f_{ji}$ holds for all i and j. Observe that (18) shows that the relationship between λ^* and θ is linear for any interior solution. Finally, as shown in Appendix 3, every interior equilibrium is *locally stable* in the sense that no small group of firms has any incentive to deviate from the country where it is located.

It is readily verified that a *necessary and sufficient condition* for the interior equilibrium to exist is that the right hand side of (18) is positive:

$$\sum_j \frac{f_{ij}}{\sum_k f_{jk}} \theta_j > 0, \quad i = 1, 2, \ldots, M. \tag{19}$$

Because $\mu > 0$, we can combine this with condition (48) for factor price equalization in order to get:

$$\theta_i > \mu \sum_j \frac{f_{ij}}{\sum_k f_{jk}} \theta_j > 0, \quad i = 1, 2, \ldots, M. \tag{20}$$

In what follows, we assume that (20) always holds.

Since the sum of the left-hand sides of (19) is 1, inequality (19) guarantees that $\lambda_i^* \in (0,1)$ for all i. Note that this condition (19) depends on both Φ and θ, so that the effects of "geography" (i.e. of Φ) and of "expenditure" (i.e. of θ) cannot be clearly separated. Due to the linearity of the model, any interior equilibrium (18) is unique if it exists. Thus, we have shown the following:

Proposition 1 (existence, uniqueness and stability) *A unique and locally stable interior equilibrium with factor price equalization exists if and only if (20) holds.*

Condition (16), although only necessary and not sufficient, allows to separate partly the impact of "geography" from the impact of "expenditure". Indeed, using (18) we have

$$\frac{\partial \lambda_i^*}{\partial \theta_j}\theta_j = \frac{f_{ij}}{\sum_k f_{kj}}\theta_j = \frac{f_{ij}}{|\Phi|}\frac{\theta_j}{\varphi_j} < \frac{f_{ij}}{|\Phi|}$$

for all indices i and j, where the last inequality results from (16). Therefore, summing across all j we have

$$\sum_j \frac{\partial \lambda_i^*}{\partial \theta_j}\theta_j < \frac{\sum_j f_{ij}}{|\Phi|}, \qquad i = 1, 2, \dots, M$$

and, hence, by definition

$$\lambda_i^*(\theta) < \varphi_i, \qquad i = 1, 2, \dots, M \tag{21}$$

or $\lambda^* < \varphi$ in vector notation. Conditions (16) and (21) can be interpreted as follows. Consider a given geography Φ (hence the φ_i are given). Under autarky (i.e. Φ is equal to the identity matrix I_d), $\lambda^* = \theta$, so that condition (21) reduces to condition (16). Hence, condition (16) is *the least stringent necessary condition on the couple* (θ, Φ) *to be met for an interior equilibrium to arise* (note that the condition $\varphi_i > 0$ involves only Φ and not θ). This is because, once there is some trade (finite trade costs), at least one country i is such that $\lambda_i^* > \theta_i$, so condition (21) is more stringent. Condition (21) captures the trade-off between centrality (low values of φ_i) and expenditures (high values of θ_i). When a country is centrally located, it must have a "disproportionally smaller expenditure share" for an interior equilibrium to be feasible. On the other hand, when a country is remotely located (large value of φ_i), it can have a large expenditure θ_i that may be compatible with an interior equilibrium.

The impact of geography is clear from the following proposition, the proof of which is relegated to Appendix 4.

Proposition 2 (Magnification Effect) *Consider a given expenditure distribution $\theta \in \mathrm{ri}(\Delta)$. When trade is sufficiently restricted, there always exists an interior equilibrium, whereas when trade becomes sufficiently free, such an equilibrium never exists.*

Proposition 2 shows that *freer trade* leads to a more uneven spatial distribution of the differentiated sector. This is sometimes called "magnification effect" (see Head *et al.*, 2002; Ottaviano and Thisse, 2004).

3 Defining the multi-country HME

The idea that market size matters for the location of industry dates at least back to the 'early days of gravity theory' (see, e.g., Harris, 1954; Tinbergen, 1962). During the 1980s, new trade theory re-discovered the importance of market size for explaining the pattern of industry location and trade. Although the concept of HME has been widely used in both theory and applications since then, *we still lack a clear and general definition of what exactly a HME is in a multi-country context.* In Krugman's (1980, p. 955) own words, in sectors characterized by Dixit-Stiglitz monopolistic competition "countries will tend to export those kinds of products for which they have relatively large domestic demand". This property is neatly implied by two-country models. Indeed, Helpman and Krugman (1985) show that, in a two-country economy, *the larger country hosts a more than proportional share of the monopolistically competitive industry.* Given preferences that are homothetic and identical across countries, such a pattern of production makes the larger country a net exporter of the differentiated good.

The disproportional positive causation from demand to supply has become the standard definition of the HME (see, e.g., Head *et al.*, 2002). Thus, in identifying the multi-country HME, we adopt such definition and we generalize it from both a *static* (i.e., cross-sectional) and a *dynamic* (i.e., time-series) point of view.

3.1 Static definition

Assume that countries i and j host an industry share that is proportional to their expenditure share, which can be expressed as follows:

$$\lambda_i^* = k_i\theta_i \qquad \text{and} \qquad \lambda_j^* = k_j\theta_j,$$

where k_i and k_j are positive coefficients. In the presence of a HME, the disproportionate positive causation from demand to supply requires that

$k_i \geq k_j$ whenever $\theta_i \geq \theta_j$. Hence,

$$\frac{\lambda_i^*}{\theta_i} = k_i, \ \frac{\lambda_j^*}{\theta_j} = k_j \quad \text{and} \quad k_i \geq k_j \quad \Rightarrow \quad \frac{\lambda_i^*}{\theta_i} \geq \frac{\lambda_j^*}{\theta_j}.$$

This suggests the following definition:

Definition 1 (Static Home Market Effect) *The monopolistically competitive industry of country i exhibits a Static Home Market Effect (henceforth, SHME) at the expenditure distribution $\theta \in \mathrm{ri}(\Delta)$ if and only if*

$$\frac{\lambda_i^*}{\theta_i} \geq \frac{\lambda_j^*}{\theta_j}, \quad \forall j = 1, \ldots, M \quad \text{such that} \quad \theta_i \geq \theta_j, \tag{22}$$

where the inequality in (22) is strict if and only if $\theta_i > \theta_j$.

In what follows, we say that the global economy exhibits a SHME if condition (22) holds for all countries $i = 1, 2, \ldots M$. Assuming, without loss of generality, that $\theta_1 \geq \theta_2 \geq \ldots \geq \theta_M$, this will be the case when

$$\frac{\lambda_1^*}{\theta_1} \geq \frac{\lambda_2^*}{\theta_2} \geq \cdots \geq \frac{\lambda_M^*}{\theta_M}. \tag{23}$$

Stated differently, under a SHME there is no 'industrial leap-frogging' in the global economy, in the sense that smaller countries always host a relatively smaller share of the monopolistically competitive industry. This implies that the ordering in terms of industry shares respects the 'natural' ordering in terms of countries' economic sizes. Note that conditions (22) and (23) do not rely on changes in expenditure shares and, therefore, can be observed at any given moment in time. Thus, provided we possess some convenient measure of θ and λ, (22) and (23) can be checked with the help of cross-sectional data only. This explains why we refer to it as the *static* HME.

3.2 Dynamic definition

A *dynamic* definition of the HME is often presented as an alternative in the literature dealing with two countries only.[2] It builds on the observation that *changes in expenditure shares map into more than proportional changes in industry shares*. While the SHME relates to the *cross-sectional disproportionality* between two countries at the same time, the dynamic home market

[2] Note that the 'dynamic' definition is also frequently used in the empirical literature. For example, Davis and Weinstein (2003, p. 7) define the HME as "a more than one-for-one movement of production in response to ideosyncratic demand".

effect relates to the *time-series disproportionality* between two periods in the same country.

Head *et al.* (2002) have shown that the static and dynamic definitions are equivalent in the symmetric 2×2-setting, thus making the choice immaterial in this case. Not surprisingly, things are no longer that simple in the multi-country world with an arbitrary trade cost matrix.

We may derive the dynamic definition in a way analogous to that used in the previous section. Assume that country i hosts an industry share at period t that is proportional to its expenditure share, which can be expressed as $(\lambda_i^*)^t = k^t \theta_i^t$. Assume that in the following period $t+1$, all θ_j's have changed such that

$$\theta_i^{t+1} - \theta_i^t > 0 \qquad \text{and} \qquad \sum_j \left(\theta_j^{t+1} - \theta_j^t\right) = 0,$$

so that the new industry share is given by $(\lambda_i^*)^{t+1} = k^{t+1}\theta_i^{t+1}$. In the presence of a dynamic HME, the disproportionate positive causation from demand to supply requires that $k^{t+1} > k^t$ whenever $\theta_i^{t+1} > \theta_i^t$. Hence,

$$\frac{(\lambda_i^*)^{t+1}}{\theta_i^{t+1}} = k^{t+1}, \quad \frac{(\lambda_i^*)^t}{\theta_i^t} = k^t \quad \text{and} \quad k^{t+1} > k^t \quad \Rightarrow \quad \frac{(\lambda_i^*)^{t+1}}{\theta_i^{t+1}} > \frac{(\lambda_i^*)^t}{\theta_i^t}.$$

Switching to differential notation, the last condition can be expressed as

$$\frac{\lambda_i^* + \mathrm{d}\lambda_i^*}{\theta_i + \mathrm{d}\theta_i} > \frac{\lambda_i^*}{\theta_i} \quad \Rightarrow \quad \frac{\mathrm{d}\lambda_i^*}{\mathrm{d}\theta_i^*}\frac{\theta_i}{\lambda_i^*} > 1. \tag{24}$$

This suggests the following definition:

Definition 2 (Dynamic Home Market Effect) *The monopolistically competitive industry of country i exhibits a Dynamic Home Market Effect (henceforth, DHME) at the distribution $\theta \in \mathrm{ri}(\Delta)$ and for the perturbation $\mathrm{d}\theta$ if and only if*

$$\frac{\mathrm{d}\lambda_i^*}{\mathrm{d}\theta_i}\frac{\theta_i}{\lambda_i^*} > 1, \tag{25}$$

where $\mathrm{d}\theta$ is a small variation satisfying $\mathrm{d}\theta_i > 0$ and $\sum_j \mathrm{d}\theta_j = 0$.

It is of interest to note that the DHME requires that the industry share λ_i^* of country i be sufficiently elastic with respect to the expenditure share

θ_i, which clearly captures the idea that changes in expenditure map into disproportionate changes in industry. Differentiating the equilibrium industry share of country i yields

$$\mathrm{d}\lambda_i^* = \sum_j \frac{\partial \lambda_i^*}{\partial \theta_j} \mathrm{d}\theta_j \,,$$

so that (25) can be expressed equivalently as follows:

$$\sum_j \frac{\partial \lambda_i^*}{\partial \theta_j} \frac{\mathrm{d}\theta_j}{\mathrm{d}\theta_i} \frac{\theta_i}{\lambda_i^*} > 1. \tag{26}$$

Condition (26) reveals that the DHME, as defined above, need not hold for some variations $\mathrm{d}\theta$ when trade costs are not pairwise symmetric across all countries. This is because the equilibrium industry shares λ^* are linear in expenditure shares θ, which implies that for any distribution $\theta \in \mathrm{int}(\Delta)$, there exists a variation $\mathrm{d}\theta$ such that (26) is violated.[3]

Proposition 3 ('Third country effects') *For every distribution $\theta \in \mathrm{ri}(\Delta)$, there exists a perturbation $\mathrm{d}\theta$, with $\mathrm{d}\theta_i > 0$ and $\sum_j \mathrm{d}\theta_j = 0$, such that the disproportionate causation from demand to supply does not hold.*

Proof. Because $\lambda_i^* > 0$, $\theta_i > 0$, and $\mathrm{d}\theta_i > 0$, a necessary condition for (25) to hold is that $\mathrm{d}\lambda_i^*$ be strictly positive. However, by linearity,

$$\mathrm{d}\lambda_i^* = \lambda_i^*(\theta + \mathrm{d}\theta) - \lambda_i^*(\theta) = \sum_j c_{ij} \mathrm{d}\theta_j = \sum_{j \neq i} (c_{ij} - c_{ii}) \mathrm{d}\theta_j \tag{27}$$

where the c_{ij} are coefficients as given in (18), and where the last inequality stems from the constraint that the perturbations sum up to one. Two cases may arise: (i) when trade costs are pairwise symmetric across regions, it is easily verified that $c_{ij} = c$ for all $i \neq j$. Hence,

$$\sum_{j \neq i} (c_{ij} - c_{ii}) \mathrm{d}\theta_j = (c_{ii} - c) \mathrm{d}\theta_i,$$

which is positive when $c_{ii} > c$. That this always holds, and that a DHME arises in this case, is shown later (see, e.g., expression (43)); (ii) when trade

[3] Note, however, that (26) may hold in models with non-linear relationships between θ and λ^*. Further, (26) may be useful for empirical purposes, especially because in general all expenditure shares vary between two time periods.

costs are not pairwise symmetric, we can always find perturbations $\mathrm{d}\theta_j$ such that (27) is negative, in which case the DHME does not hold for all perturbations satisfying $\mathrm{d}\theta_i > 0$ and $\sum_j \mathrm{d}\theta_j = 0$. It is sufficient to note that in the general case $\min_j\{c_{ij}\} < \max_j\{c_{ij}\}$ and that at least one $\mathrm{d}\theta_j$, $j \neq i$, must be strictly negative. ■

Proposition 3 shows that the disproportionate causation from demand to supply does not trivially hold in the multi-country setting, which suggests that measuring the HME in this way requires us to be very careful. Indeed, all expenditure shares usually change between two periods in the data, so that a 'HME shadow' may arise, in the sense that *even if country i gains expenditure, it may actually gain a less than proportional industry share if another country j also gains some expenditure.* In some cases, such effect may be so strong that country i simply looses industry, despite its increase in expenditure, as argued in Proposition 3.

To illustrate this result, consider the following example. Assume that trade costs are given by

$$\Phi = \begin{pmatrix} 1 & 1/4 & 1/3 \\ 1/4 & 1 & 1/3 \\ 1/3 & 1/3 & 1 \end{pmatrix}, \tag{28}$$

which, using expression (18), yields the following equilibrium industry distribution:

$$\begin{pmatrix} \lambda_1^* \\ \lambda_2^* \\ \lambda_3^* \end{pmatrix} = \begin{pmatrix} 16/9 & -5/18 & -4/7 \\ -5/18 & 16/9 & -4/7 \\ -1/2 & -1/2 & 15/7 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}.$$

Assume further that all countries have the same initial expenditure share, i.e., that $\theta = (1/3, 1/3, 1/3)$. Then when the variation $\mathrm{d}\theta$ is e.g. given by $\mathrm{d}\theta = (\epsilon, -9\epsilon, 8\epsilon)$, with a sufficiently small $\epsilon > 0$, country 1 gains some expenditure share yet looses some industry share because

$$\sum_j \frac{\partial \lambda_1^*}{\partial \theta_j} \frac{\mathrm{d}\theta_j}{\mathrm{d}\theta_1} \frac{\theta_1}{\lambda_1^*} = \left(\frac{16}{9} \times 1 + \frac{5}{18} \times 9 - \frac{4}{7} \times 8 \right) \frac{1/3}{13/42} \epsilon < 0.$$

Note that, as shown by $\mathrm{d}\theta$, country 3 displays a much stronger "dynamic" HME (it gains eight times as much expenditure than country 1), which can thus drain some industry from country 1, even though this country actually gains some expenditure.

Condition (26) does not necessarily hold in our model, because we have no information a priori on the "perturbation" $\mathrm{d}\theta$. In order to obtain a tractable specification that accounts for the cross-effects, we can restrict ourselves to the case of an *exogenous expenditure shock* $\mathrm{d}E_i$ *for country* i *only.* Indeed, suppose that country i's expenditure changes from E_i to $E_i + \mathrm{d}E_i$. Then θ_i changes from

$$\theta_i = \frac{E_i}{E} \quad \text{to} \quad \theta_i + \mathrm{d}\theta_i = \frac{E_i + \mathrm{d}E_i}{E + \mathrm{d}E_i},$$

whereas θ_j, for $j \neq i$, changes from

$$\theta_j = \frac{E_j}{E} \quad \text{to} \quad \theta_j + \mathrm{d}\theta_j = \frac{E_j}{E + \mathrm{d}E_i}.$$

Stated differently, the expenditure shares of all countries other than i vary in the same way, which greatly simplifies the analysis of (25) and (26). Some straightforward calculations show that in this case we have

$$\frac{\mathrm{d}\theta_j}{\mathrm{d}\theta_i} = \frac{-E_j}{E - E_i} = \frac{-\theta_j}{1 - \theta_i} \qquad \forall\, j \neq i, \tag{29}$$

which allows to rewrite the left-hand side of (25) as follows:

$$\begin{aligned}
\frac{\mathrm{d}\lambda_i^*}{\mathrm{d}\theta_i}\frac{\theta_i}{\lambda_i^*} &= \sum_j \frac{\partial\lambda_i^*}{\partial\theta_j}\frac{\mathrm{d}\theta_j}{\mathrm{d}\theta_i}\frac{\theta_i}{\lambda_i^*} \\
&= \frac{\partial\lambda_i^*}{\partial\theta_i}\frac{\theta_i}{\lambda_i^*} + \sum_{j\neq i}\frac{\partial\lambda_i^*}{\partial\theta_j}\frac{-\theta_j}{1-\theta_i}\frac{\theta_i}{\lambda_i^*} \\
&= \frac{\partial\lambda_i^*}{\partial\theta_i}\frac{\theta_i}{\lambda_i^*} - \frac{\theta_i}{(1-\theta_i)\,\lambda_i^*}\sum_{j\neq i}\frac{\partial\lambda_i^*}{\partial\theta_j}\theta_j \\
&= \frac{\partial\lambda_i^*}{\partial\theta_i}\frac{\theta_i}{\lambda_i^*} - \frac{\theta_i}{1-\theta_i}\left(1 - \frac{\partial\lambda_i^*}{\partial\theta_i}\frac{\theta_i}{\lambda_i^*}\right).
\end{aligned} \tag{30}$$

The first term of the left-hand side of (30) is the direct effect of the change in $\mathrm{d}E_i$, while the second term captures all indirect effects. As shown in Lemma 3 of Appendix 4, the condition

$$\frac{\partial\lambda_i^*}{\partial\theta_i}\frac{\theta_i}{\lambda_i^*} > 1 \tag{31}$$

holds under fairly general assumptions. In what follows, we refer to (31) as the *direct DHME*. Using (31), the second term of the right-hand side of (30) is positive, thus implying that the indirect effect amplifies the direct DHME.[4]

[4] One should further note that, since this impact is stronger for larger values of θ_i, we may say that the direct DHME is amplified within countries exhibiting a greater SHME.

The next proposition establishes that each country exhibits a DHME at any interior equilibrium with respect to an exogenous expenditure shock $\mathrm{d}E_i$.

Proposition 4 *An increase in country i's expenditure (i.e. $\mathrm{d}E_i > 0$ and $\mathrm{d}E_j = 0, \ \forall j \neq i$) leads to a direct DHME and a DHME, as given by (25), in country i.*

Note that the exogenous expenditure shock $\mathrm{d}E_i > 0$ in country i necessarily decreases λ_j^*, because

$$\frac{\partial \lambda_j^*}{\partial E_i} = \frac{\partial \lambda_j^*}{\partial \theta_j}\frac{\partial \theta_j}{\partial \theta_i}\frac{\partial \theta_i}{\partial E_i} < 0. \tag{32}$$

The first terms on the LHS is positive from (31), the second term is negative from (29), whereas the last term is positive because

$$\frac{\partial \theta_i}{\partial E_i} = \left(\frac{E_i + \mathrm{d}E_i}{E + \mathrm{d}E_i} - \frac{E_i}{E}\right)\frac{1}{\mathrm{d}E_i} = \frac{1}{E + \mathrm{d}E_i} > 0.$$

It should be finally noted, however, that Proposition 4 neglects the possible cross-effects when workers move from one country to another or when more than one expenditure shock occurs. As shown previously, in such a case the DHME need not arise as the 'HME shadow' may be too strong.

4 The impact of market size

In the multi-country setting with a general trade cost matrix, three distinct effects enter the HME: the *market size effect* (attraction), the *hub effect* (accessibility) and the *competition effect* (repulsion). The interplay between attraction and accessibility are central to gravity models and spatial interaction theory (see, e.g., Harris, 1954; Smith, 1975), and have been recently 'rediscovered' in international trade and economic geography (see, e.g., Fujita *et al.*, 1999; Head and Mayer, 2004). One defining characteristic of the general equilibrium models of the latter fields is that the repulsive nature of price competition is now explicitly accounted for.[5]

Following Head and Mayer (2004), we can define the *real market potential* (henceforth RMP) associated with the industry distribution λ as follows:

$$\mathrm{RMP}_i \equiv \sum_j p_{ij} d_{ij} = \sum_j \frac{\phi_{ij}\theta_j}{\sum_k \phi_{kj}\lambda_k}. \tag{33}$$

[5] Given constant elasticity of demand and no strategic interactions among firms, some authors prefer the name 'market crowding effect' to 'competition effect' (see, e.g., Baldwin *et al.*, 2003; Ottaviano and Thisse, 2004).

This can be expressed more concisely in matrix notation as follows:

$$\text{RMP} = \Phi \text{diag}(\Phi\lambda)^{-1}\theta,$$

which is simply the RHS of (11) and, therefore, equal to **1** at any interior equilibrium. Stated differently, any interior equilibrium is such that the real market potential across all regions is equalized.[6] Observe that the numerator of (33) stands for accessibility to consumers' demand, whereas the denominator is accessibility to producers' supply. Whereas the former captures the attractivity of country i, the latter stands for the degree of competition and, hence, reduces the RMP.

Using (33), the *real market potential difference* between countries i and j is given by

$$\text{RMP}_i - \text{RMP}_j = \sum_k \frac{\left(\phi_{ik} - \phi_{jk}\right)\theta_k}{\sum_m \lambda_m \phi_{mk}}. \tag{34}$$

Expression (34) can be used in order to easily highlight the presence of the SHME when trade costs are pairwise symmetric across countries, i.e., when we "sterilize" the hub effect and focus on the market size effect only. Assume that $\phi_{ij} = \phi$, for all $i \neq j$, so that geographical differences between countries no longer matter. In this case, expression (34) boils down to

$$\text{RMP}_i - \text{RMP}_j = \frac{(1-\phi)\theta_i}{\lambda_i + \phi(1-\lambda_i)} - \frac{(1-\phi)\theta_j}{\lambda_j + \phi(1-\lambda_j)},$$

which is equal to zero if and only if

$$\theta_i\left[\lambda_j(1-\phi) + \phi\right] = \theta_j\left[\lambda_i(1-\phi) + \phi\right]. \tag{35}$$

Because $\phi < 1$, when $\theta_i > \theta_j$ condition (35) can only hold when $\lambda_i > \lambda_j$, thus showing that in equilibrium

$$\frac{\theta_j}{\theta_i} = \frac{\lambda_j(1-\phi) + \phi}{\lambda_i(1-\phi) + \phi} \geq \frac{\lambda_j}{\lambda_i}.$$

This reveals the presence of the SHME, as given by Definition 1. Expression (34) further allows us to show the following:

[6] We know that (33) is constant for any (interior) equilibrium distribution λ^*. Hence, firms have no incentive to relocate because the RMP is the same everywhere. Yet, the RMP differs across countries for off-equilibrium distributions. In this case, firms relocate from low to high RMP countries, which is the usual adjustment dynamics used in new economic geography (Fujita *et al.*, 1999; Fujita and Thisse, 2002).

Proposition 5 (Dominant Market Effect) *For every country i, there exists an expenditure share $\theta_i^{\text{sup}} < 1$ such that $\lambda_i^* = 1$ for all $\theta_i \geq \theta_i^{\text{sup}}$.*

Proof. In order for country i to host all monopolistically competitive firms in equilibrium

$$\text{RMP}_i - \text{RMP}_j \geq 0 \qquad \forall j \tag{36}$$

must hold at the distribution $\lambda_i^* = 1$ and $\lambda_j^* = 0$ for $j \neq i$. Stated differently, country i offers a higher RMP than all other countries when $\lambda_i^* = 1$, which implies that no firm has any incentives to change its current location. Some straightforward calculations show that condition (36) is equivalent to

$$\theta_i \geq \max_{j \neq i} \frac{1}{1 - \phi_{ij}} \sum_{k \neq i} \left(\frac{\phi_{jk}}{\phi_{ik}} - 1 \right) \theta_k. \tag{37}$$

Clearly, when $\theta_i = 1$, $\theta_j = 0$ for $j \neq i$, so that condition (37) holds as a strict inequality. The desired result then follows by continuity of both sides of (37) with respect to θ. ■

Proposition 5 shows that a region with a sufficiently large expenditure share attracts the whole mobile industry. In accordance with classical location theory, we will call such a region a *dominant market* (Weber, 1909). Note that expression (37) is highly reminiscent of a well-known result in location theory, namely the *Majority Theorem* (Witzgall, 1964). When country i hosts an expenditure share that is larger than some weighted average of the expenditure shares of the other countries, all mobile firms will agglomerate in country i. When trade costs are pairwise symmetric, condition (37) reduces to

$$\theta_i \geq \frac{1}{\phi} \left(\max_{j \neq i} \theta_j \right).$$

As shown in Appendix 6, the link with Witzgall's Majority Theorem can then be explicitly established, provided a particular metric is used. To the best of our knowledge, this interesting connection between location theory and trade theory has been overlooked until now.

Note also that the pairwise symmetric setting allows to neatly illustrate the magnification effect highlighted in Proposition 2. To see that freer trade always exacerbates the HME and maps into more extreme spatial structures, we compute the equilibrium industry shares (18) when $\phi_{ij} = \phi$ for all $i \neq j$:

$$\lambda_i^* = \frac{1 + (M-2)\,\phi}{1 - \phi} \theta_i - \sum_{j \neq i} \frac{\phi}{1 - \phi} \theta_j$$

or, alternatively,

$$\lambda_i^* - \lambda_j^* = \frac{1 + (M-1)\,\phi}{1-\phi}\,(\theta_i - \theta_j)\,. \tag{38}$$

Since the coefficient of $\theta_i - \theta_j$ is increasing in ϕ, a decrease in trade costs necessarily exacerbates the HME.

5 The impact of geography

The results derived in the previous section show that the assumption of pairwise symmetric trade costs across all countries constitutes a special case, which gives rise to quite particular results. Yet, to the best of our knowledge, this is the only setting that has been investigated in a multi-country framework until now.[7] For instance, although Baldwin *et al.* (2003, p. 333) acknowledge that "the very simple form of (38) is not robust", they do not dig any deeper into the properties of the multi-country HME with asymmetric trade costs. This is most likely due to the fact that the hub effect plays an important role with more general trade cost specifications and significantly complicates the analysis. That several interesting results can nevertheless be established when "geography really matters" is argued in the remainder of this section.

Krugman (1993) highlights the existence of the so-called 'hub effect' with the help of a three-country model. Yet, the HME literature has, to the best of our knowledge, not investigated this issue any further. This is quite puzzling because, as stated by Fujita and Mori (1996, p. 93), "agglomeration economies and the hub effect of transport nodes interplay in the making of major cities". As argued in this section, such a 'neglect' may be due to the fact that the hub effect is very elusive and, therefore, especially hard to define clearly in the multi-country context.

5.1 Defining the hub effect

In what follows, we "sterilize" the market size effect and focus on the hub effect only. Since there are many meaningful ways to define the hub effect, we focus in what follows on both definitions that draw upon the freeness of

[7] This also shows that the two-country case, which is always pairwise symmetric by definition, is very particular and that the results obtained in such setting should be extrapolated with great caution.

trade only, and definitions that combine freeness of trade with expenditure.[8] Let us start with measures that build on the freeness of trade only (i.e., exclusively on the ϕ_{ij}). We know from (16) that

$$\varphi_i > 0, \qquad i = 1, 2, \ldots, M \tag{39}$$

must hold for an interior equilibrium to exist for at least some expenditure distributions θ. The case in which condition (39) does not hold can be seen as a situation in which the freeness of trade is such that the distribution of economic activity is always strongly skewed towards some countries, thus leaving some others empty. This is likely to happen when (i) some countries have a significant locational advantage (e.g., access to the sea) or disadvantage (e.g., being landlocked; see Gallup *et al.*, 1999); or (ii) when some countries have a restrictive trade policy. In such a situation *the expenditure distribution is dominated by the freeness of trade* in the sense that there is no way the industry can be spread across all countries.

Although the use of φ_i to measure the hub effect has its merits, it is clearly too narrow a view. A less restrictive definition of the hub effect may be based on the following observation. If expenditure was equally spread across all countries, every country would host the same share of the industry when $\phi_{ij} = \phi$ for all $i \neq j$. Hence, when $\theta_i = 1/M$ for all i, differences in equilibrium industry shares λ_i^* are purely driven by differences in the ϕ_{ij}'s, i.e. by differences in the countries' respective freeness of trade. Assuming that expenditure is equally split across all countries, the equilibrium industry shares (18) are given as follows:

$$\lambda_i^{\text{hub}} = \frac{1}{M} \sum_j \frac{f_{ij}}{\sum_k f_{jk}} = \frac{1}{M|\Phi|} \sum_j \frac{f_{ij}}{\varphi_j}, \tag{40}$$

which depend on the ϕ_{ij} only. Expression (40) reveals that regions with a low value of φ_j (i.e. regions that offer on average a good access to markets) have a strong impact on country i's industry share, whereas countries with a high value of φ_j (i.e. regions that offer on average a bad access to markets) have a comparatively small impact. Note further that the *sign of the impact* depends on the sign of f_{ij}. Because $f_{ii} > 0$ (see Appendix 2), each country has a positive impact on itself. Stated differently, when country i is

[8] Note that, because $\phi_{ij} = \tau_{ij}^{1-\sigma}$, it is difficult to strictly isolate the impact of geography per se. This is because τ_{ij} may include tariffs and different non-tariff barriers to trade, which are not related to geography per se, and because σ is a parameter of the utility function. Yet, if we consider that trade costs are approximated by distance (see Appendix 1), the 'geographical interpretation' remains plausible.

centrally located (i.e. low value of φ_i), it tends to attract a large share of the industry. Yet, being closely located to other countries with high values of φ_j decreases λ_i significantly when the coefficient f_{ij} is negative. Note that such an interaction, which we could refer to as a 'hub shadow' and which captures the competition effect, may explain why transportation hubs are sufficiently widely spaced in a spatial economy.[9]

Definitions (39) and (40) of the hub effect in terms of the freeness of trade only are slightly at odds with the definition of the hub effect usually used in economic geography. Indeed, according to Baldwin *et al.* (2003, pp. 331), the hub effect is that "superior market access favours the hub as a location of industry...". One should note that "superior market access" depends on both geography (i.e. Φ) and demand (i.e. θ). This is also clear in the three-country cased studied by Krugman (1993, p. 37), who argues that when region 1 is more centrally located such that $\phi_{12} = \phi_{13} > \phi_{23}$, "[if] production had instead been localized at 2, of course, then trade would have flowed along the other sides of the triangle – and 2 would be the hub. So, the position of a hub can be self-fulfilling, determined by history".

Following a well established tradition in gravity and trade theory (see Head and Mayer, 2004, for a survey), the market access net of the competition effect can be measured with the help of the *nominal market potential* (henceforth, NMP). Let $\tilde{d}_{ij} = d_{ij} P_j^{1-\sigma}$ be the demand for country i's varieties in country j when we do not correct for the price index. The NMP of country i is then defined as follows:

$$\mathrm{NMP}_i \equiv \sum_j p_{ij} \tilde{d}_{ij} = \sum_j \phi_{ij} \theta_j. \tag{41}$$

or, in matrix notation, NMP $= \Phi\theta$.[10] All things equal, i.e. by abstracting from price competition, regions that offer a high NMP also host a larger

[9] Fujita and Thisse (2002, p. 363) refer to such a phenomenon as *urban shadow*. The existence of such a shadow yields many counterintuitive results. For example, improving a country's access to the other countries may lead to either inflow or outflow of industry, depending crucially on the access of the other countries.

[10] The NMP, and similar measures, are often used in empirical applications (see, e.g., Davis and Weinstein, 2003). One can show that the nominal market potential belongs to Weibull's (1976) class of "attraction-accessibility measures", of whom the gravity potentials used in spatial interaction theory are a special instance. Although such measures are relatively easy to implement operationally, their main drawback is to abstract from competition effects, which should play a central role in trade theory and economic geography. Indeed, the different trade costs are interrelated through general equilibrium network feedbacks, so that *decreasing trade costs can translate into decreasing real market potential.* Note that such "weird" behavior is reminiscent of the well-known Braess-paradox in transportation science.

share of industry. Though theoretically appealing, the NMP is a rather bad proxy for industry share. In particular, it is easy to find examples in which $\text{NMP}_i < \text{NMP}_j$ but $\lambda_i^* > \lambda_j^*$. In fact, when $\theta = (1/2, 1/4, 1/4)$ in example (28), we get $\text{NMP}_2 < \text{NMP}_3$ but $\lambda_2^* > \lambda_3^*$. This shows that *having good access to demand is not necessarily a locational advantage for the industry, because losses due to competition may more than outweigh the gains due to a higher NMP*. Hence, any general analysis of the hub effect turns out to be complicated, because of indirect feedbacks in the network economy.

Following Head and Mayer (2004), the RMP is equal to NMP adjusted for price competition. We therefore have in equilibrium:

$$\begin{aligned} \text{RMP} &= \Phi \text{diag}(\Phi\lambda^*)^{-1}\theta \\ &= \Phi \text{diag}(\Phi\lambda^*)^{-1}\Phi^{-1}(\text{NMP}) \\ &= \Phi \text{diag}(\text{diag}(\varphi)^{-1}\theta)^{-1}\Phi^{-1}(\text{NMP}) \\ &= \Phi \text{diag}(\varphi_i/\theta_i)\Phi^{-1}(\text{NMP}), \end{aligned}$$

which shows that the competition effect my be captured by $\Phi\text{diag}(\varphi_i/\theta_i)\Phi^{-1}$. If price competition was equalized across regions, $\text{RMP} = k(\text{NMP})$ should hold, where $k > 0$ is an arbitrary constant. In such a case, nominal market potential could be used as a perfect proxy for real market potential. Yet, this clearly only holds when $\text{diag}(\varphi_i/\theta_i) = kI_d$, i.e. when

$$\frac{\varphi_i}{\theta_i} = k > 1, \quad \forall i, \tag{42}$$

where the last inequality is from (16). Note that (42) can be seen as being a "constant competition isocurve". Any increase in country i's accessibility to markets (i.e. a smaller value of φ_i) must be accompanied by a decrease in its expenditure share θ_i in order for the degree of competition in country i to remain the same in equilibrium.

5.2 Disentangling market size and geography

As argued in Section 4, the market size effect can be measured by sterilizing the impact of geography. Using expression (38), we readily have

$$\lambda_i^{\text{size}} = \frac{1 + (M-1)\,\phi}{1-\phi}\theta_i - \frac{\phi}{1-\phi}. \tag{43}$$

The common *average freeness of trade* ϕ across countries can be choosen in many different ways, e.g., as some average weighted by the elasticity of substitution σ:

$$\phi = \left[\frac{1}{M(M-1)} \sum_{\substack{i,j \\ i \neq j}} \tau_{ij} \right]^{1-\sigma}$$

One should note that expression (43) does not depend on θ_j for $j \neq i$. Stated differently, the way the remaining expenditure share $1 - \theta_i$ is distributed across countries does not matter. This shows that the case with symmetric trade costs can be seen as a "country i vs. the rest of the world" scenario.[11] The hub effect can be measured analogously to the market size effect by sterilizing the impact of expenditure, i.e. by expression (40). We may say that the difference $\lambda^{\text{size}} - \theta$ is purely attributable to the market size effect, whereas the difference $\lambda^{\text{hub}} - \theta$ is purely attributable to the hub effect. If these two effects were the only ones at work in the space-economy, the following relation should hold:

$$\lambda^* - \theta = \left(\lambda^{\text{size}} - \theta\right) + \left(\lambda^{\text{hub}} - \theta\right).$$

As one can check, such a decomposition does not hold, which suggests that more effects than the market size effect and the hub effect play a role. Let us hence consider the following decomposition:

$$\lambda^* - \theta = \left(\lambda^{\text{size}} - \theta\right) + \left(\lambda^{\text{hub}} - \theta\right) + \epsilon, \tag{44}$$

where ϵ is the residual which captures potential mis-specifications, the role of factor endowments, and the competition effect.[12] It is the part of the industry share of country i that is not explained by either the market size effect or the hub effect. Note that the decomposition (44) may be used for empirical analysis. Let $i = 1, 2, \ldots, M$ be the subscript for countries and $j = 1, 2, \ldots, N$ be the subscript for industries, which then suggests the following industry-level regression equation:

$$\lambda^*_{ij} = \alpha_i + \beta_j + \text{SIZE}(\phi, \theta_{ij}, M) + \text{HUB}(\Phi_i, M) + \epsilon_{ij},$$

[11] This result is a by-product of the homothetic preferences in the CES specification. Indeed, when preferences are non-homothetic (as, e.g., in the quadratic-linear model by Ottaviano *et al.*, 2002), the equilibrium industry share λ^*_i of country i depends on the whole distribution θ_{-i} of the industry across the remaining countries.

[12] The residual is very important if we interpret it as the "price competition" effect. Indeed, the Dixit-Stiglitz CES model has been criticized for abstracting from direct price competition effects (see, e.g., Lai and Trefler, 2002). If the residual is small, the price competition effect is not crucial and DS monopolistic competition offers a fairly good approximation. Yet, if the residual is large, the price competition effect may be important, which thus may suggest that DS monopolistic competition offers a fairly bad approximation.

or the following country-level regression equation:

$$\lambda_i^* = \alpha_i + \mathrm{SIZE}(\phi, \theta_i, M) + \mathrm{HUB}(\Phi_i, M) + \epsilon_i, \tag{45}$$

with $\theta_i = \sum_j \theta_{ij}$, and where α_i and β_j are country and industry fixed effects. Note that equation (45) does not depend on θ_j for $j \neq i$. Because $\lambda^* = \Phi^{-1}\mathrm{diag}(\varphi)^{-1}\theta$, and because

$$\lambda^{\mathrm{size}} = \frac{1+(M-1)\phi}{1-\phi}\theta - \frac{\phi}{1-\phi}\mathbf{1} \quad \text{and} \quad \lambda^{\mathrm{hub}} = \frac{1}{M}\Phi^{-1}\mathrm{diag}(\varphi)^{-1}\mathbf{1},$$

the following decomposition holds:[13]

$$\lambda^* = \frac{1-\phi}{1+(M-1)\phi}\Phi^{-1}\mathrm{diag}(\varphi)^{-1}\lambda^{\mathrm{size}} + \frac{M\phi}{1+(M-1)\phi}\lambda^{\mathrm{hub}}. \tag{46}$$

Note that expression (46) appears to be especially appealing for empirical purposes. Indeed, to see this rewrite it as follows:

$$\lambda^* = \beta_1 W \lambda^{\mathrm{size}} + \beta_2 \lambda^{\mathrm{hub}} \tag{47}$$

Because $W \equiv \Phi^{-1}\mathrm{diag}(\varphi)^{-1}$ depends on the freeness of trade only, it can be interpreted in terms of *spatial weight matrix*, capturing the interactions between regional market sizes. Although our specification is not really a spatial autoregressive one, it has clearly some common features. Further, the theoretical model tells us that $\beta_1 \in (0,1)$, $\beta_2 \in (0,1)$ and $\beta_1 + \beta_2 = 1$ should hold. Stated differently, the equilibrium industry distribution λ^* is a *convex combination of spatially discounted market sizes and the hub effect.* Whereas the first term hence captures the 'gravity part' of the model, the second term has not been really used until now in applied work.

6 Concluding remarks

We have started with what we called the 'Davis-Weinstein conjecture' (Davis and Weinstein, 2003). According to this conjecture, the HME uncovered in

[13] Note that λ^{size} may be rewritten as:

$$\lambda^{\mathrm{size}} = \theta + \frac{M\phi}{1-\phi}\left(\theta - \frac{1}{M}\mathbf{1}\right)$$

which is reminiscent of the estimating equation (3) by Davis and Weinstein (2003, p. 7). The first term stands for the autarky share of industry, whereas the second term captures the idiosyncratic component of local demand. Note, however, that the coefficient capturing the idiosyncratic impact $M\phi/(1-\phi)$, though positive, need not be larger than one in theory.

two-country models may be extended to a multi-country world in a fairly straightforward way. Specifically, with two countries, firms are disproportionately located in the country offering larger local demand. With many countries, the same should happen with respect to some index of local 'effective' demand. Such index should take into account not only local demand but also demands from other countries, possibly weighted by distance.

By developing a multi-country model à la Krugman (1980), we have shown that things are not that simple. In particular, as shown by Proposition 3, it is quite difficult, maybe impossible, to build an index of 'effective' demand whose changes always generate disproportionate responses in output. The reason being that, with many countries, the location of firms is determined by the interaction between spatial ('accessibility') and non-spatial ('attraction') effects, which are crucially influenced by what happens to the entire distribution across all countries ('third country effects'). These conceptual difficulties do, however, not imply the impossibility of assessing the role of product differentiation and market structure in shaping the structure of world trade. We propose, indeed, a new theory-based estimating equation that looks much like a spatial autoregressive model. The next logical step is to take this new specification to the data in order to see whether the results conform to the predictions derived from the theory. We keep this part for future work.

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Appendix 1: Factor price equalization

Factor price equalization requires any $M-1$ dimensional subset of countries to be unable to satisfy world demand for the homogenous good H (see, e.g., Baldwin *et al.*, 2003). Let ℓ_i be the amount of labor employed by a representative firm in country i. For the homogenous production to take place everywhere, the total mass of workers in each country should be greater than total labor requirement in the modern sector, i.e.,

$$L_i > n_i \ell_i \qquad \forall i.$$

Therefore, since $L_i = \theta_i L$ and

$$n_i \ell_i = \lambda_i^* N \left(F + c \sum_j x_{ij} \right) = \lambda_i^* \frac{\mu L}{F\sigma} \left[F + c \frac{F(\sigma-1)}{c} \right] = \lambda_i^* \mu L$$

in equilibrium, the condition for factor price equalization reduces to:

$$\theta_i > \mu \sum_j \frac{f_{ij}}{\sum_k f_{jk}} \theta_j \qquad \forall i. \tag{48}$$

Thus, the manufacturing expenditure share μ must be small enough for the homogenous good to be produced everywhere. In what follows, we assume that condition (48) always holds.

Appendix 2: Positive definiteness of Φ

In order for expressions (17) and (18) to be defined, the trade cost matrix Φ must be invertible. In this appendix, we derive sufficient conditions for this to hold. We especially show that Φ is *positive definite* for the empirically meaningful case in which distance is measured by the euclidian norm. Our first lemma provides a characterization of the iceberg trade cost in terms of the exponential function.

Lemma 1 *Assume that r is a metric. Let $r_{ij} = r(i,j)$ be the distance between countries i and j, and let $\phi(i,j) = \phi_{ij}$ be the associated freeness of trade. When trade costs are of the iceberg form, the relationship*

$$\phi \equiv \mathrm{e}^{-r} \tag{49}$$

must hold.

Proof. Consider three countries i, j and k and let $r_{ik} = r_{ij} + r_{jk}$, where r_{ik} is the distance between i and k. By definition of the iceberg trade cost, if one unit of the good is shipped from country i, only a fraction $1/\tau_{ij}$ arrives in country j, whereas only a fraction $(1/\tau_{ij})(1/\tau_{jk})$ arrives in country k. That is, $\tau_{ik} = \tau_{ij}\tau_{jk}$ holds for any i, j and k. Since trade costs depend on distance, i.e. $\tau_{ik} = \tau(r_{ik})$, it must be that

$$\tau(r_{ij} + r_{jk}) = \tau(r_{ij})\tau(r_{jk}) \qquad \forall r_{ij}, r_{jk}. \tag{50}$$

Fix r_{jk}, differentiate (50) with respect to r_{ij} and evaluate it at $r_{ij} = 0$. This yields the condition $\tau'(r_{jk}) = \tau'(0)\tau(r_{jk})$. Solving this differential equation with the condition $\tau(0) = 1$ yields

$$\tau(r_{jk}) \equiv \tau_{jk} = e^{\tau'(0)r_{jk}}$$

Because $\phi_{jk} = \tau_{jk}^{1-\sigma}$, we finally obtain

$$\phi_{jk} = e^{-\tau_0 r_{jk}},$$

where $\tau_0 = (\sigma - 1)\,\tau'(0) > 0$ which can be normalized to 1 by an appropriate choice of units for the metric r.[14] ■

Observe that (49) ensures that trade costs between any two countries are pairwise symmetric (i.e. $\tau_{ij} = \tau_{ji}$ or $\phi_{ij} = \phi_{ji}$) and that direct trade costs between i and k do not exceed trade costs via a third country j ($\tau_{ik} \leq \tau_{ij}\tau_{jk}$ or $\phi_{ik} \geq \phi_{ij}\phi_{jk}$, due to the triangle inequality of the metric r).

Lemma 1 allows us to establish the following:

Lemma 2 *Assume that r is the euclidian norm and that all countries are distinct. Given (49), Φ is then positive definite.*

[14] Note that if $\tau_{ik} = \tau_{ij} + \tau_{jk}$ for any i, j and k, we have $\tau(r_{ij} + r_{jk}) = \tau(r_{ij}) + \tau(r_{jk})$, which yields the linear trade costs $\tau(r_{jk}) = \tau'(0)r_{jk}$ as in Ottaviano *et al.* (2002).

Proof. See theorems 3' and 6' in Schoenberg (1938). ■

Appendix 3: Local stability of interior equilibria

At every interior equilibrium, the real market potential across countries is equalized:

$$\mathrm{RMP}_i = \sum_j \frac{\phi_{ij}\theta_j}{\sum_k \phi_{kj}\lambda_k^*} = 1, \qquad i = 1, 2, \ldots M. \tag{51}$$

Let

$$g(\lambda) \equiv \mathrm{RMP}_i(\lambda) - \mathrm{RMP}_l(\lambda) = \sum_j \frac{\left(\phi_{ij} - \phi_{lj}\right)\theta_j}{\sum_k \lambda_k \phi_{kj}}$$

be the difference in market potential between countries i and l. Consider the relocation of an (infinitesimal) mass of firms from country l to country i, so that λ_i^* changes to $\lambda_i^* + \mathrm{d}\lambda_i$, whereas λ_l^* changes to $\lambda_l^* - \mathrm{d}\lambda_i$, with $\mathrm{d}\lambda_i > 0$. The first-order Taylor expansion of g is given by

$$g(\lambda + \mathrm{d}\lambda) = g(\lambda) + \sum_j \frac{\partial g}{\partial \lambda_j}(\lambda)\mathrm{d}\lambda_j + \epsilon(\mathrm{d}\lambda), \tag{52}$$

where $\epsilon(\mathrm{d}\lambda)$ is an error term that is negligible when $\|\mathrm{d}\lambda\|$ is small. Because $g(\lambda^*) = 0$ at any interior equilibrium, we have

$$g(\lambda^* + \mathrm{d}\lambda) = -\mathrm{d}\lambda_i \sum_j \frac{\left(\phi_{ij} - \phi_{lj}\right)^2}{\left(\sum_k \phi_{kj}\lambda_k^*\right)^2}\theta_j + \epsilon(\mathrm{d}\lambda).$$

Because

$$\lim_{\mathrm{d}\lambda_i \to 0^+} \frac{\epsilon(\mathrm{d}\lambda)}{\mathrm{d}\lambda_i} = 0$$

since g is differentiable, we see that

$$\lim_{\mathrm{d}\lambda_i \to 0^+} \frac{g(\lambda^* + \mathrm{d}\lambda)}{\mathrm{d}\lambda_i} < 0.$$

The strict inequality holds because countries i and l are distinct such that $\phi_{ij} = \phi_{lj}$ cannot hold for all j. We conclude that any firm migrating from l to i necessarily decreases its real market potential, and hence profits, implying that *any interior equilibrium is locally stable.*

Appendix 4: Proof of Proposition 2

Denote by ϕ_i the i-th column vector of Φ, by ϕ_j^{-1} the j-th column vector of its inverse Φ^{-1}, by $\langle x, y\rangle \equiv x^T y$ the euclidian scalar product, and by $\|x\|$ the euclidian norm of x. Because Φ and Φ^{-1} are symmetric, by definition $\langle \phi_i, \phi_j^{-1}\rangle = 0$ for all $j \neq i$ and $\langle \phi_i, \phi_i^{-1}\rangle = 1$.

Assume that trade is sufficiently free so that $\phi_i \equiv \mathbf{1} + t_i\xi_i$, where $\xi_i \in \mathbb{R}_+^M$ is a perturbation vector and $t_i \neq 0$ is a coefficient such that $\phi_i > \mathbf{1}$. Note that $\|t_i\xi_i\| \to 0$ when $t_i \to 0$, which implies that the perturbation can always be made sufficiently small with the help of t_i. Note also that it is always possible to choose the M vectors ξ_i such that the M vectors ϕ_i are linearly independent. We know from condition (16) that at any interior equilibrium $\varphi_j \equiv \langle \phi_j^{-1}, \mathbf{1}\rangle > \theta_j$ must hold. Hence,

$$\langle \phi_i, \phi_j^{-1}\rangle = \langle \mathbf{1} + t_i\xi_i, \phi_j^{-1}\rangle = \langle \mathbf{1}, \phi_j^{-1}\rangle + t_i\langle \xi_i, \phi_j^{-1}\rangle = 0$$

which implies that

$$\langle \mathbf{1}, \phi_j^{-1}\rangle = -t_i\langle \xi_i, \phi_j^{-1}\rangle > \theta_j > 0$$

must hold. Because $\theta_j > 0$ is fixed, we can always find $t_i \to 0$ sufficiently small such that this condition is violated (we can choose t_i either positive or negative, depending on the sign of $\langle \boldsymbol{\xi}_i, \phi_j^{-1}\rangle$). We may hence conclude that there is no interior equilibrium no matter the value of $\theta \in \mathrm{ri}(\Delta)$ when trade becomes sufficiently free.

When trade is prohibitive, $\Phi = \Phi^{-1} = I_d$ so that a proportionate equilibrium $\lambda^* = \theta$ prevails from (18). Let $\phi_i = e_i + t_i\mathbf{1}$, where e_i is the i-th vector of the canonical basis of $\mathbb{R}^M$ and where t_i is defined as before. Again, at any interior equilibrium $\varphi_j \equiv \langle \phi_j^{-1}, \mathbf{1}\rangle > \theta_j$ must hold. We have

$$\langle \phi_i, \phi_j^{-1}\rangle = \langle e_i + t_i\mathbf{1}, \phi_j^{-1}\rangle = \langle e_i, \phi_j^{-1}\rangle + t_i\langle \mathbf{1}, \phi_j^{-1}\rangle = 0$$

which implies that

$$\langle \mathbf{1}, \phi_j^{-1}\rangle = -\frac{1}{t_i}\langle e_i, \phi_j^{-1}\rangle > \theta_j > 0$$

must hold. Because $\theta_j > 0$ is fixed, we can always find $t_i \to 0$ sufficiently small such that this condition is satisfied (we can choose t_i either positive or negative, depending on the sign of $\langle e_i, \phi_j^{-1}\rangle$). We may hence conclude that there is always an interior equilibrium no matter the value of $\theta \in \mathrm{ri}(\Delta)$ when trade is sufficiently restricted.

Appendix 5: Existence of the direct DHME

We assume that the distance between countries is measured by the euclidian norm. Hence, by Lemma 2 the trade cost matrix Φ is positive definite. Because the inverse of a positive definite matrix is positive definite, and because each principal minor of a positive definite matrix is strictly positive, we know that

$$f_{ii} > 0, \qquad i = 1, 2, \dots M. \tag{53}$$

This allows us to establish the following.

Lemma 3 *At any interior equilibrium* λ^*,

$$\frac{\partial \lambda_i^*}{\partial \theta_i} \frac{\theta_i}{\lambda_i^*} > 1$$

must hold.

Proof. First, from (18), we know that

$$\frac{\partial \lambda_i^*}{\partial \theta_i} = \frac{f_{ii}}{\sum_j f_{ij}}.$$

Second, from (14) we have

$$\theta_i = \frac{\sum_j f_{ij}}{|\Phi|} \sum_k \phi_{ik} \lambda_k^*.$$

Plugging both expressions into (31) yields

$$\begin{aligned} \frac{\partial \lambda_i^*}{\partial \theta_i} \frac{\theta_i}{\lambda_i^*} &= \frac{f_{ii}}{\sum_j f_{ij}} \frac{1}{\lambda_i^*} \frac{\sum_j f_{ij}}{|\Phi|} \sum_k \phi_{ik} \lambda_k^* \\ &= \frac{f_{ii}}{|\Phi|} \left(1 + \sum_{k \neq i} \phi_{ik} \frac{\lambda_k^*}{\lambda_i^*} \right) \\ &> \frac{f_{ii}}{|\Phi|} \geq 1. \end{aligned} \tag{54}$$

The strict inequality is due to the fact that λ^* is an interior solution, whereas the last inequality is due to *Fischer's inequality* (see, e.g., Horn and Johnson, 1985, p. 478). ■

Appendix 6: A connection with location theory

In this appendix, we offer an alternative interpretation of the condition $\theta_i \geq \phi^{-1} \max_j\{\theta_j\}$ for market i to be dominant when trade costs are pairwise symmetric across countries. Consider the *Fermat-Weber location problem* (Weber, 1909):

$$\min_x \; T(x) \equiv \sum_i \theta_i r(x, a_i)$$

where

$$r(x, a_i) = \begin{cases} \tau & \text{if} \quad x \neq a_i \\ 0 & \text{if} \quad x = a_i \end{cases}$$

is the binary metric (see, e.g., Wesolowsky, 1993, for an overview of the Fermat-Weber problem), and where $\theta \in \mathrm{ri}(\Delta)$.

Lemma 4 *In the Fermat-Weber problem with the binary metric, there exists an index i such that*

$$a_i \in \arg\min_x \; T(x).$$

Proof. When $x \neq a_i$, $i = 1, \ldots M$, we have

$$T(x) = \sum_i \theta_i > T(a_j) = \sum_{i \neq j} \theta_i \qquad \forall j = 1, 2, \ldots M$$

because $\theta_j > 0$ for all j. Hence, x cannot be an optimal solution. ∎

Hence, the optimal location coincides with one of the markets a_i.

Proposition 6 *Consider the Fermat-Weber problem with the binary metric. Then,*

$$\theta_i \geq \max_j \; \{\theta_j\} \quad \Leftrightarrow \quad a_i \in \arg\min_x \; T(x). \tag{55}$$

Proof. Assume that $\theta_i \geq \max_j \; \{\theta_j\}$ and suppose that $a_i \notin \arg\min_x \; T(x)$. Hence, by Lemma 4, there exists $k \neq i$ such that

$$T(a_i) = \sum_{j \neq i} \theta_j > T(a_k) = \sum_{j \neq k} \theta_j$$

which implies that $\theta_k - \theta_i > 0$, a contradiction. Conversely, assume that $a_i \in \arg\min_x \; T(x)$ and that $\theta_i < \max_j \; \{\theta_j\}$. In this case, we have

$$T(a_i) = \sum_{j \neq i} \theta_j \leq T(a_k) = \sum_{j \neq k} \theta_j \qquad k = 1, \ldots M$$

which implies that

$$\theta_k - \theta_i \leq 0 \qquad k = 1, ..., M$$

a contradiction. ■

Because $\phi^{-1} > 1$, we conclude that when i is a dominant market in the HME model, it is also a dominant market in the corresponding Fermat-Weber location problem. This is because

$$\theta_i \geq \phi^{-1} \max_j \{\theta_j\} > \max_j \{\theta_j\}.$$

Note that the reverse does not hold, yet offers a fairly good approximation when trade becomes sufficiently free (i.e. when $\phi \approx 1$). It is of interest to note that because $\phi \equiv \tau^{1-\sigma}$, this also holds when $\sigma \to 1$, i.e. when varieties become independent. In both cases, the result stems from the fact that the distortion due to price competition in segmented markets no longer plays a role when either trade becomes sufficiently free or when varieties become sufficiently independent. In the limit, when there are no more trade costs or when varieties are independent, price competition is the same everywhere so that location decisions are solely driven by considerations of market size, just as in the Fermat-Weber problem.