Debt, Deficits, and Deflation in an Economy with Central Bank Independence*

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Abstract

This paper studies a simple monetary growth model with debt and deficits to investigate the effects of various polices on output and inflation when the central bank is 'tough.' In sharp contrast to the vast literature which implicitly or explicitly assumes fiscal dominance regime, in which the fiscal authority commits to the primary deficits, an increase in government expenditures or the government's indebtedness *reduces* output and inflation: fiscal stimulus aiming at raising output and inflation will do just the opposite. Fighting deflation in a world with central bank independence requires fiscal discipline.

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1 Introduction

Inflation has been one of the main concerns of macroeconomics and macroeconomic policy. The reason is because until recently the world economy has been fighting inflation. The literature on inflation has become one of the largest in economics. Many theoretical and empirical works on inflation are conducted and the literature is still growing.

In the 80's the theory of monetary policy found the link between budget deficits and inflation. Sargent and Wallace (1981) and Miller (1983a, b), among others, showed that higher budget deficits are inflationary because for the government, inflation tax is the easiest way to collect revenue. An immediate policy implication of the result is that for price stability fiscal discipline is critical. The link between budget deficits and inflation is now understood and popularized. Perhaps most importantly, central banks in many industrial nations are now independent; their primal goal is price stability and they do not have to raise inflation tax to finance government expenditures. This paper argues that the theory of monetary policy needs to incorporate this new institutional arrangement.

From the theoretical perspective, Sargent and Wallace's (1981) contribution is to introduce the consolidated government budget constraint that connects fiscal and monetary policies. An important aspect of their model is that the fiscal authority is assumed to commit to the size of its deficits. In effect, it implies that it is the monetary authority that has to maintain fiscal solvency. Thus, if the total deficit increases, then absence of bond seigniorage, the central bank has to raise currency seigniorage by printing money more aggressively. Thus, higher deficits lead to higher inflation.

A policy regime in which the fiscal authority commits to its deficits is sometimes called fiscal dominance (FD). Alternatively, a regime in which the fiscal authority must adjust its deficits to maintain fiscal solvency is called monetary dominance (MD).[1] As is clear, that higher deficits lead to higher inflation is almost immediate if one assumes fiscal dominance. When Sargent and

[1] Closely related, but not identical, concepts are non-Ricardian and Ricardian regimes, which are discussed in Canzoneri *et al.* (2001).

Wallace published their paper, the world economy was still fighting inflation. Sargent and Wallace and their followers were correct in pointing out that fiscal profligacy is the source of inflation. Since then the theory of monetary policy adopts Sargent and Wallace's (1981) tradition and emphasizes the consolidated government budget constraint in determining inflation and the price level. The literature has evolved into the so-called fiscal theory of the price level. In the literature on fiscal theory of the price level, which is popularized by Woodford (1994, 2001), the fiscal authority is assumed to commits to its deficits, and the market ensures fiscal solvency by adjusting the price level. Thus, in that literature the government budget with deficits effectively determines the price level.

The last decade has witnessed the era of low inflation and even deflation. In many developed economies there is an independent central bank and the world economy seems to have defeat inflation. A sensible question here is: is FD regime still plausible? Bohn (1998) finds that the U.S. primary surplus is increasing in the debt-GDP ratio: fiscal authority takes corrective actions. Models that explicitly or implicitly assume FD are best regarded as describing economies without an independent central bank. I argue that a model must incorporate MD in one form or another to describe the era of low inflation and deflation.

The aim of this paper is to study the effects of monetary and fiscal policies in a simple monetary growth model with a "tough" central bank. The central bank in this paper is independent and "tough" in that the level of fiscal deficits adjusts so as to maintain fiscal solvency. In particular, this paper assumes that taxes are endogenous.

This paper studies an overlapping generations model with productive capital, money, bonds, taxes, and government spending. In order to introduce multiple assets (capital, money and bonds) into the model, I incorporate a cash-in-advance constraint into the Diamond (1965) economy. The modeling strategy is closely related to Woodford (1986), Crettez *et al.* (1999), and Michel and Wigniolle (2003). In particular, the model developed in this paper is a hybrid of the neoclassical growth model of Diamond (1965) and Lucas and Stokey's (1983) cash-in-advance monetary economy. The Lucas-Stokey model is known as a popular framework for monetary analysis, and is adopted, for example, by Chari *et al.* (1991) and Woodford (1994).

Throughout the paper it is assumed that the monetary authority conducts its policy via open market operations to keep the nominal interest rate constant over time. The fiscal authority is assumed to commit to government expenditures and the total government liabilities (i.e., money plus bonds held by public). These are the policy parameters in this paper.

The model has typically two steady-state equilibria, and the high-capital steady state is stable. At that equilibrium raising government spending or the government's indebtedness will reduce output and inflation. This implies that fiscal stimulus aiming at raising output and inflation is doomed to fail. This, rather surprising result contrasts sharply with the vast literature that explicitly or implicitly assumes FD. Indeed, many general equilibrium monetary models assume constant deficits. Those models imply that an increase in government expenditures must be financed by inflation tax. Thus the famous doctrine – budget deficits lead to inflation – obtains in many monetary models. If output and inflation are positively related then budget deficits increase output, and vice versa. The relation between output and inflation depends on how money is modeled and how the monetary policy rule is specified. Under MD, in contrast, an increase in government spending or the government's indebtedness will simply raise the needs of revenue through taxes, because the central bank does not have the responsibility to maintain fiscal solvency. Since taxes increase, output and inflation are lowered.

As an extension, an endogenous growth version of the model is considered to investigate the effects of fiscal and monetary policies on output growth rates. It is shown that there are typically two balanced growth equilibria, which are characterized by different welfare levels. The high-growth equilibrium is stable, but there are infinitely many rational expectations paths toward the equilibrium: it is locally indeterminate. At the high-growth equilibrium, an increase in government expenditures or the government's indebtedness will reduce the output growth rate and welfare.

The organization of the paper is as follows. Section 2 describes the model. Section 3 characterizes equilibria of the model. Section 4 studies the effects of policies. Section 5 extends the model to consider endogenous growth. Section 6 concludes.

2 The Model

2.1 Environment

The analytical framework considered in this paper is a hybrid of the neoclassical growth model with overlapping generations developed by Diamond (1965) and the cash-in-advance monetary model of Lucas and Stokey (1983). Closely related models are Woodford (1986), Crettez *et al.* (1999), and Michel and Wigniolle (2003).

Consider an economy consisting of an infinite sequence of two-period lived overlapping generations, an initial old generation, and an infinitely-lived government. Let $t = 1, 2, ...$ index time. At each date t, a new generation is born. The population is normalized to one. Each agent is endowed with one unit of labor when young and is retired when old. In addition, the initial old agents are endowed with $K_1 > 0$ units of capital.

There is a single final good produced using the Cobb-Douglas production function $Y_t = AK_t^{\alpha}N_t^{1-\alpha}$ with $A > 1$ and $\alpha \in (0, 1)$, where K_t denotes the capital input, and N_t denotes the labor input. Let $k_t \equiv K_t/N_t$ denote the capital-labor ratio. Then, the intensive production function is $f(k_t) = Ak_t^{\alpha}$. It is easy to see that $f(0) = 0$, $f' > 0 > f''$, and Inada conditions hold. The final good can either be consumed in the period it is produced, or it can be stored to yield capital in the following period. For expositional reasons, capital is assumed to depreciate 100% between periods. The case with incomplete capital depreciation should be considered if one wishes to investigate fluctuations at "business cycle frequencies," which are beyond the scope of the paper.[2]

2.2 Factor Markets

Factor markets are perfectly competitive. Thus, factors of production receive their marginal product. Let r_t and w_t denote the rental rate of capital and the real wage rate. Each young agent supplies his or her labor endowment inelastically in the labor market. Then, profit maximization

[2] Remember that in this paper taxes are adjusted each period to meet fiscal solvency. It does not seem sensible to let taxes to fluctuate at business cycle frequencies. I argue, therefore, that a period in this paper is fairly large and that 100% depreciation is not farfetched.

requires

$$r_t = f'(k_t) = \alpha A k_t^{\alpha-1}, \tag{1}$$

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t) = (1-\alpha) A k_t^{\alpha}. \tag{2}$$

Note that $w'(k) = -kf''(k) > 0$. With the Cobb-Douglas specification, $r_t = \alpha A k_t^{\alpha-1}$and $w_t = (1-\alpha) A k_t^{\alpha}$.

2.3 Consumers

Let c_{1t} (c_{2t}) denote the consumption of the final good by a young (old) agent born at date t. I assume that agents care consumption only when old. This immediately follows that $c_{1t} = 0$ for all t so all income will be saved. This assumption allows one to focus on agents' portfolio choices regarding money versus nonmonetary assets. Following Lucas and Stokey (1983), I assume that consumption goods are divided into two types: "cash goods" and "credit goods." Cash goods must be purchased by cash, so agents wishing to consume cash goods need cash in advance. On the other hand, agents do not need cash to purchase credit goods.[3] Let c_{mt} (c_{nt}) denote the amount of cash (credit) goods consumed when old. Then, $c_{2t} = c_{mt} + c_{nt}$ must hold. It is assumed that the marginal rate of transformation between the two goods is unity so the price is the same for the two goods. The cash-in-advance constraint is therefore

$$p_{t+1} c_{mt} \leq M_t, \tag{3}$$

where p_t denotes the time t price level and M_t denotes the nominal money balance. According to (3), a young agent must set aside cash in advance in order to purchase cash goods when old.

Agents may hold money and non-monetary assets. The non-monetary assets, denoted by Z_t, are assumed to yield the gross nominal return of $I_{t+1} \geq 1$ in the next period. In the asset market the non-monetary assets will be allocated to productive capital and government bonds. The budget constraint for a young agent born at date t is therefore

$$M_t + Z_t + T_t \leq p_t w_t, \tag{4}$$

[3] I follow Lucas and Stokey's (1983) interpretation that at some stores an agent is known to the producer so credit is available, while at other stores the agent is unknown to the seller so cash must be used to make a transaction.

where T_t is tax payment. (4) states that a young agent of generation t receives nominal wage income and allocates all his or her disposable income, $p_t w_t - T_t$, to monetary and non-monetary assets because no one consumes when young. Throughout, I focus on symmetric equilibria in which all agents of the same generation have the same amount of assets. The budget constraint when old is

$$p_{t+1} c_{2t} \leq M_t + I_{t+1} Z_t. \tag{5}$$

The cash-in-advance constraint binds if and only if money is dominated by non-monetary assets in rates of return, or, equivalently, if and only if $I_{t+1} \geq 1$. Under binding cash-in-advance constraint, (3) holds at equality. Then, (3) and (5) imply $c_{mt} = M_t/p_{t+1}$ and $c_{nt} = I_{t+1} Z_t / p_{t+1}$.

Specify the utility function as

$$U(c_{mt}, c_{nt}) = \left[(1-\sigma) c_{mt}^{1-\rho} + \sigma c_{nt}^{1-\rho} \right]^{\frac{1}{1-\rho}}, \tag{6}$$

where $0 < \sigma < 1$ and $\rho > 0$. The elasticity of substitution between c_{mt} and c_{nt} is $1/\rho$. Each young agent chooses c_{mt} and c_{nt} to maximize (6) subject to $c_{mt} = M_t/p_{t+1}$, $c_{nt} = I_{t+1} Z_t / p_{t+1}$, and (4).[4] The first order necessary conditions for the maximization problem require $U_1/U_2 = I_{t+1}$, which gives the money demand function,

$$M_t = \gamma(I_{t+1}) \left[p_t w_t - T_t \right], \tag{7}$$

where

$$\gamma(I) \equiv \left[1 + \left(\frac{\sigma}{1-\sigma} \right)^{\frac{1}{\rho}} I^{\frac{1-\rho}{\rho}} \right]^{-1}. \tag{8}$$

Schreft and Smith (1997, 1998, 2002) use a version of turnpike model to derive money demand. Interestingly, the money demand function obtained in this paper is identical to the one obtained in Schreft and Smith (1997, 1998, 2002): the cash-in-advance model of Lucas and Stokey (1983) and the turnpike model of Schreft and Smith (1997, 1998, 2002) generate the same money demand function, although the structures of the models are quite different.

It is important to check some properties of the money demand function just derived.

[4] Note that when the cash-in-advance constraint binds, the model is effectively a version of the money-in-the-utility-function model.

Lemma 1 *$\gamma(I)$ satisfies (a) $\gamma'(I) < 0$ for $\rho \in (0,1)$ and $\gamma'(I) \geq 1$ for $\rho > 1$, (b) $\lim_{I\to\infty}\gamma(I) = 0$ for $\rho \in (0,1)$ and $\lim_{I\to\infty}\gamma(I) = 1$ for $\rho > 1$, (c) $0 < \gamma(I) < 1$, and (d) $I\gamma'(I)/\gamma(I) = -[1-\gamma(I)](1-\rho)/\rho$.*

Proof. (a) From (8),

$$\gamma'(I) = -\left[1 + \left(\frac{\sigma}{1-\sigma}\right)^{\frac{1}{\rho}} I^{\frac{1-\rho}{\rho}}\right]^{-2} \left(\frac{\sigma}{1-\sigma}\right)^{\frac{1}{\rho}} \frac{1-\rho}{\rho} I^{\frac{1-\rho}{\rho}-1}. \tag{9}$$

It is then easy to check that $\gamma'(I) < 0$ for $0 < \rho < 1$ and $\gamma'(I) \geq 1$ for $\rho > 1$. ■

Lemma 1a gives the condition under which the real money demand is decreasing in the nominal interest rate. As the nominal interest rate increases, the household substitutes away from money, which reduces money demand. An increase in the nominal rates, at the same time, raises earning from bond holding, which raises money demand through income effect. The former dominates the latter if $\rho \in (0,1)$, in which case the elasticity of substitution between c_{mt} and c_{nt} is $1/\rho > 1$. If, on the other hand, $\rho > 1$ holds then the elasticity of substitution is low and the income effect dominates. Schreft and Smith (1998) focus on the case in which $\rho > 1$ holds, but in a different environment.[5]

2.4 Government

The consolidated government budget constraint is

$$G_t + I_t B_{t-1} = T_t + B_t + M_t - M_{t-1} \tag{10}$$

for $t \geq 2$ and $G_1 = T_1 + M_1 + B_1$ for $t = 1$, where it is assumed that $M_0 = B_0 = 0$. The initial stock of money being zero implies that money is supplied in this economy only through the channel of open market operations by the central bank. I assume that the government simply consumes G_t and that it does not affect utility of any generation or the production process at any date. In order to simplify the analysis, I further assume throughout that $G_t/p_t = g \geq 0$ for all t to treat

[5] In Schreft and Smith (1998), ρ measures the degree of risk aversion; $\rho > 1$ in their turnpike environment implies that individuals are highly risk averse and have relatively strong motives for insurance against idiosyncratic risks.

g as a policy parameter. Divide (10) by p_t, and use the Fisher equation, $R_{t+1} \equiv I_{t+1}p_t/p_{t+1}$, to obtain

$$g = \tau_t + b_t - R_t b_{t-1} + m_t - \frac{p_{t-1}}{p_t} m_{t-1}, \tag{11}$$

where $\tau_t = T_t/p_t$, $b_t = B_t/p_t$, $m_t = M_t/p_t$, and the gross real interest rate satisfies the Fisher equation, $I_{t+1} \equiv R_{t+1}p_{t+1}/p_t$.

In order to 'close' the model, fiscal and monetary policies must be specified.

Definition 2 *A policy regime is said to exhibit Fiscal (Monetary) Dominance if the fiscal (monetary) authority moves first at time* 1 *and chooses the time path for its actions without any regard for solvency.*

The vast literature explicitly or implicitly assumes fiscal dominance; in many monetary models the fiscal authority is assumed to commit to its deficits. Bohn (1998) finds that the U. S. primary surplus is an increasing function of the debt-GDP ratio: the government takes corrective actions to maintain solvency. As stated in introduction, the aim of this paper is to investigate the effects of monetary and fiscal policies under monetary dominance. Throughout this paper I focus on a particular form of monetary dominance: the central bank targets the nominal interest rate, and the fiscal authority adjust taxes to maintain solvency. Thus, $I_t = I \geq 1$ for all t. Let $L_t = B_t + M_t$ denote the total government liabilities. It follows that $l_t = b_t + m_t$ where $l_t = L_t/p_t$. Under fiscal dominance, the primary deficit $(g - \tau)$ is constant while the debt (b) is endogenous. Since the primary deficit is endogenous under monetary dominance, I assume that the fiscal authority sets $l_t = \bar{l}$ for all t instead and treat $\bar{l}$ as a policy parameter. Then, (11) becomes

$$g = \tau_t + \bar{l} - R_t \bar{l} + R_t m_{t-1} - \frac{p_{t-1}}{p_t} m_{t-1}, \tag{12}$$

3 Equilibrium

Divide the money market equilibrium condition (7) by p_t to obtain

$$m_t = \gamma(I)\left[w(k_t) - \tau_t\right]. \tag{13}$$

Since all non-monetary assets are allocated to capital and bonds, $Z_t = B_t + p_t K_{t+1}$ holds. The absence of arbitrage opportunity in the asset market requires that capital and bonds yield the same real rate of return, or, $r_{t+1} = R_{t+1}$. From (4) and $Z_t = B_t + p_t K_{t+1}$, one obtains the asset market equilibrium condition:

$$k_{t+1} + b_t + m_t = w(k_t) - \tau_t. \tag{14}$$

Notice $b_t + m_t = \bar{l}$ and rewrite (12) and (14) to obtain

$$\begin{aligned} k_{t+1} + \bar{l} &= w(k_t) - \tau_t, & (15) \\ g &= \tau_t + \bar{l} - f'(k_t)\,\bar{l} + \frac{I-1}{I} f'(k_t)\, m_{t-1}. & (16) \end{aligned}$$

Equations (13) and (15) yields $m_t = \left[k_{t+1} + \bar{l}\right] \gamma(I)$. Use it with (15) to eliminate τ_t and m_t from (16) to finally obtain

$$k_{t+1} = w(k_t) - g + [1 - h(I)]\, k_t f'(k_t) - h(I)\,\bar{l} f'(k_t) \equiv \Omega(k_t), \tag{17}$$

where $h(I) \equiv 1 - \gamma(I) + \gamma(I)/I > 0$. (17) describes the law of motion of k.

Lemma 3 *The function Ω satisfies (a) $\Omega'(k) > 0$ for all k, (b) $\lim_{k\to 0} \Omega(k) = -\infty$, (c) $\lim_{k\to 0} \Omega'(k) = \infty$, (d) $\lim_{k\to\infty} \Omega'(k) = 0$.*

Proof. From (17), $\Omega'(k) = w'(k) + [1 - h(I)][f'(k) + kf''(k)] - h(I)\,\bar{l} f''(k)$. With the Cobb-Douglas production specification, it is easy to show that $\Omega'(k) > 0$ since $f'(k) + kf''(k) = \alpha^2 A k^{\alpha-1} > 0$. ■

The configuration of the function Ω is depicted in figure 1. Lemma 3 implies that the function Ω has two distinct fixed points. Formally,

Lemma 4 *Let $\widetilde{k}$ solve $\Omega'(k) = 1$. Then, (a) the function Ω has two distinct fixed points if $\Omega\left(\widetilde{k}\right) > \widetilde{k}$, (b) the function has one fixed point if $\Omega\left(\widetilde{k}\right) = \widetilde{k}$, and (c) the function has no fixed point if $\Omega\left(\widetilde{k}\right) < \widetilde{k}$.*

From (17), it is easy to see that for sufficiently large g and $\bar{l}$, the Ω locus is below the 45 degree line for all k. Thus, for existence of a stationary equilibrium it is necessary that g and $\bar{l}$ are not too large. In what follows I focus on the case in which Ω has two fixed points. From figure 1 it is easy to check the stability of equilibria.

Proposition 5 *The high-k steady state is stable and the low-k steady state is unstable.*

Let k_l and k_h denote the low-k and high-k steady states. Since the high-k steady state is stable, an economy starting with an initial capital $k_1 \in (k_l, \infty)$ will eventually reach the steady state. If an economy starts with $k_1 \in (k_l, \infty)$, then the economy will shrink over time until the economy collapses. Along the path, the rate of inflation decrease over time.

In addition to (17), an equilibrium sequence must satisfy the restriction that $m_t \leq \bar{l}$, which asserts that the supply of money is bounded by the total supply of bonds issued by the fiscal authority. This restriction is imposed because the central bank cannot conduct open market purchases of bonds more than the stock of bonds. The restriction implies

$$k \leq \frac{1-\gamma(I)}{\gamma(I)}\bar{l} \equiv \overline{k}. \tag{18}$$

This restriction could rule out a candidate steady state equilibrium. Consider figure 1 again. If $k_h < \overline{k}$, then the two candidate steady states are both valid.

If, on the other hand, $k_l < \overline{k} < k_h$, then there arises a catastrophic case in which the high-k steady state is ruled out. In that case, the only equilibrium is $k_t = k_l$ for all t, but this is a knife-edge. From (18) and lemma 1, it is easy to see that such a case arises if (1) the (targeted) debt outstanding, $\bar{l}$, is small, or (2) the nominal interest rate is set low enough so that $\gamma(I)$, which measures the real money demand per disposable income, is large.

From the Fisher equation it is easy to show that a steady state is deflationary if and only if $I < f'(k)$, which reduces to $k < (I/\alpha A)^{1/(\alpha-1)}$ under the Cobb-Douglas specification. Since any stationary equilibrium must satisfy (18), a sufficient condition for a valid steady state to be deflationary is $\overline{k} < (I/\alpha A)^{1/(\alpha-1)}$, or,

$$\bar{l} \leq \frac{\gamma(I)}{1-\gamma(I)}\left(\frac{I}{\alpha A}\right)^{\frac{1}{\alpha-1}}. \tag{19}$$

4 Effects of Policies

4.1 Comparative Statics

In what follows I focus on the case in which $\bar{l}$ is sufficiently large so the two candidate steady states are valid. The primary attention is given to the high-k steady state, since it is the stable equilibrium.

Proposition 6 *(a) k and π are positively related at any steady state. (b) k and τ are negatively related at the high-k steady state.*

Proof. (a) From the Fisher equation $I = f'(k)\pi$ holds. Since I is kept constant, k and π are positively related. (b) (15) implies that at any steady state, $\tau = w(k) - k - \bar{l}$. It follows that $d\tau/dk = w'(k) - 1$. From lemma 3,

$$\Omega'(k) = w'(k) + [1 - h(I)]\left[f'(k) + kf''(k)\right] - h(I)\bar{l}f''(k) < 1 \tag{20}$$

must hold at the high-k steady state. Since (20) implies $w'(k) - 1 < 0$, $d\tau/dk < 0$. ■

According to proposition 6, output and inflation are positively related in the long run, and higher taxes reduce output at the stable equilibrium.

Lemma 7 *$h'(I) < 0$ holds if $(1-\rho)I < 1$.*

Proof. Since $h(I) \equiv 1 - \gamma(I) + \gamma(I)/I > 0$, it is easy to show that

$$h'(I) = -\frac{I-1}{I}\frac{\gamma(I)}{I}\left[\frac{1}{I-1} + \frac{I\gamma'(I)}{\gamma(I)}\right] = -\frac{I-1}{I}\frac{\gamma(I)}{I}\left[\frac{1}{I-1} - \frac{1-\rho}{\rho}[1-\gamma(I)]\right], \tag{21}$$

where I used Lemma 1 (d) to rewrite the elasticity term above. Since $\gamma(I) \geq 0$ for all I, $(1-\rho)/\rho \geq [1-\gamma(I)](1-\rho)/\rho$ holds for all I. Notice that the assumption $(1-\rho)I < 1$ implies $(I-1)^{-1} > (1-\rho)/\rho$. It follows that

$$\frac{1}{I-1} - \frac{1-\rho}{\rho}[1-\gamma(I)] > 0.$$

Thus, from (21) $h'(I) < 0$ holds. ■

Proposition 8 *At the stable steady state, (a) an increase in g reduces k, (b) an increase in $\bar{l}$ reduces k, (c) an increase in I raises k if $(1-\rho) I < 1$.*

Proof. (a) A steady state is characterized by $k = \Omega(k)$. It is then easy to show that $dk/dg = [1 - \Omega'(k)]^{-1} \partial\Omega(k)/\partial g$. From (17), $\partial\Omega(k)/\partial g < 0$. Thus, an increase in g shifts the Ω locus down. (b) Similarly, $\partial\Omega(k)/\partial\bar{l} < 0$ holds so an increase in $\bar{l}$ shifts the Ω locus down. (c) Use (17) to compute $\partial\Omega(k)/\partial I = -(k+\bar{l})h'(I) f'(k) > 0$ so an increase in I shifts the Ω locus up. ■

It is very important to compare the results with the ones obtained under the conventional fiscal dominance regime, under which $g-\tau$ is constant over time. According to proposition 8a, an increase in government spending reduces k and thereby reduces output and inflation. This contrasts sharply with the result obtained in the vast literature assuming fiscal dominance, in which fiscal stimulus raises inflation.

Proposition 8b asserts that an increase in the government's indebtedness will reduce output and inflation. Since τ and k are negatively related at the stable steady state, the increase in indebtedness raises taxes. The result is consistent with Bohn's (1998) empirical finding that the primary surplus is increasing in debt.

According to proposition 8c, an increase in the targeted nominal interest rate raises output. This result is valid as long as $(1-\rho) I < 1$ is satisfied, which is always met if $\rho > 1$. Proposition 8c immediately follows that nominal interest rates and inflation rates are *positively* related in the long run. Such a relationship is sometimes called the Fisher effects view. There is another important view, the liquidity effects view. The liquidity effects view asserts that interest rates and inflation are *negatively* related because lower nominal interest rates stimulate investment, which has upward pressure on inflation. Empirical studies in Lucas (1980, 1996), McCandless and Weber (1995), and Monnet and Weber (2001) suggest that interest rates and inflation are *positively* related in the long-run.

4.2 Discussions

The government's budget, (11), can be rewritten as

$$g = \tau_t + s_t^b + s_t^m, \tag{22}$$

where $s_t^b \equiv b_t - R_t b_{t-1}$ is the bond seigniorage and $s_t^m = m_t - \frac{p_{t-1}}{p_t} m_{t-1}$ is the currency seigniorage. Note that the bond seigniorage is positive if and only if the economy is dynamically inefficient. Thus, assuming fiscal dominance with $g > \tau$ amounts to say that it is the monetary authority that has to keep the government solvent if the economy is dynamically efficient. Thus, in that case an increase in g forces the monetary authority to increase currency seigniorage, raising inflation and output.

If the economy is dynamically inefficient, then fiscal dominance implies that the government rolls over the debt forever to finance its deficits. However, as Bhattacharya and Kudoh (2002) document, under fiscal dominance, the monetary authority still has the responsibility to raise currency seigniorage even when the bond seigniorage is positive.

This paper assumes monetary dominance, under which an increase in g simply creates the needs for higher taxes. Thus, fiscal stimulus aiming at raising output and inflation will do just the opposite.

4.3 Welfare Implications

Substitute $c_{mt} = M_t/p_{t+1}$ and $c_{nt} = I_{t+1} Z_t / p_{t+1}$ into (6) to obtain

$$U(c_{mt}, c_{nt}) = \left[(1-\sigma) \left(\frac{p_t}{p_{t+1}} m_t \right)^{1-\rho} + \sigma (R_{t+1} z_t)^{1-\rho} \right]^{\frac{1}{1-\rho}}. \tag{23}$$

Use (23) to define steady-state welfare as $U = \left[(1-\sigma)(m/\pi)^{1-\rho} + \sigma (Rz)^{1-\rho} \right]^{\frac{1}{1-\rho}}$. Rewrite it, using (13) and (14), as

$$U = \Theta(I) \left(k + \bar{l} \right) f'(k), \tag{24}$$

where

$$\Theta(I) \equiv \left[(1-\sigma)(\gamma(I)/I)^{1-\rho} + \sigma (1-\gamma(I))^{1-\rho} \right]^{\frac{1}{1-\rho}}. \tag{25}$$

Proposition 9 *(a) Steady-state welfare is maximized at*

$$k = \frac{1-\alpha}{\alpha}\bar{l}. \tag{26}$$

(b) The welfare-maximizing rate of inflation cannot be negative if

$$\bar{l} > \frac{\alpha}{1-\alpha}\left(\alpha A\right)^{\frac{1}{1-\alpha}}. \tag{27}$$

Proof. Remember that $f(k) = Ak^{\alpha}$. (a) From (24) it is easy to show that $dU/dk = 0$ holds at $k = (1-\alpha)\left(k + \bar{l}\right)$. (b) The Fisher equation and (26) imply that the welfare-maximizing value of π is $(\frac{1-\alpha}{\alpha}\bar{l})^{1-\alpha} I/\alpha A$. Thus, $\pi < 1$ holds if and only if

$$I < \frac{\alpha A}{\left(\frac{1-\alpha}{\alpha}\bar{l}\right)^{1-\alpha}}. \tag{28}$$

Notice that I must be greater than one. This implies that if the right-hand-side of (28) is less than one then the inequality cannot be satisfied. Such a case occurs if $\bar{l} > \frac{\alpha}{1-\alpha}\left(\alpha A\right)^{\frac{1}{1-\alpha}}$. ■

Proposition 9a asserts that there is a welfare-maximizing k, and that the welfare-maximizing k depends on the capital share and $\bar{l}$. Proposition 9b implies that if $\bar{l}$ is sufficiently large, then the welfare-maximizing steady state is necessarily inflationary.

Figure 2 illustrates the economy with $\sigma = 0.2$, $\rho = 0.8$, $A = 1.1$, $\alpha = 0.4$, $I = 1.015$, and $\bar{l} = 0.1$. When the government spending is zero, the equilibrium tax is negative. As g increases, tax rises, capital stock and inflation fall, and welfare falls. It is interesting to note that for large values of g, the rate of inflation becomes negative. Under fiscal dominance, an increase in g would raise inflation because tax is assumed to be constant over time or simply ignored; the government needs to rely on inflation tax. In sharp contrast, this model assumes monetary dominance with endogenous taxes. In such an economy, fiscal stimulus aiming at raising output and inflation will do just the opposite. Furthermore, it will reduce steady-state welfare.

5 Endogenous Growth

5.1 Characterization

This section considers a simple endogenous growth model in order to study the effects of monetary and fiscal policies on output growth rates. The production technology employed here is motivated by Romer (1986), Bencivenga and Smith (1991), and Espinosa and Yip (1999). The production function is $Y_t = AK_t^{\alpha} (k_t N_t)^{1-\alpha}$, where k_t is redefined as the *aggregate* capital stock. The aggregate capital stock enters the production function because of externality; the labor productivity rises as the society accumulates capital stock. Note that $k_t = K_t$ holds in equilibrium.

Factor markets are perfectly competitive. Thus, factors of production receive their marginal product. Young agents supply their labor endowment inelastically in the labor market. Thus, $N_t = 1$ in equilibrium. When make decisions, firms take the stock of aggregate capital, k_t, as given. Then factor market equilibrium requires that the gross return on capital and the real wage rate are given by

$$r_t = \alpha AK_t^{\alpha-1} (k_t N_t)^{1-\alpha} = \alpha A, \tag{29}$$

$$w_t = (1-\alpha) AK_t^{\alpha} (k_t N_t)^{-\alpha} k_t = (1-\alpha) Ak_t = (1-\alpha) Y_t. \tag{30}$$

Divide the government's budget constraint, (10), by $p_t Y_t$ to obtain

$$g_t = \tau_t + b_t - \frac{R}{\theta_t} b_{t-1} + m_t - \frac{p_{t-1}}{p_t} \frac{1}{\theta_t} m_{t-1}, \tag{31}$$

where $\theta_t \equiv Y_t / Y_{t-1}$, $g_t \equiv G_t/(p_t Y_t)$, $\tau_t \equiv T_t/(p_t Y_t)$, $b_t \equiv B_t/(p_t Y_t)$, $m_t \equiv M_t/(p_t Y_t)$. Let $l_t \equiv b_t + m_t$. The fiscal authority sets $l_t = \bar{l}$ for all t and adjusts taxes to satisfy the government's budget. Thus, (31) can be rewritten as

$$g = \tau_t + \bar{l} - \frac{R}{\theta_t}\bar{l} + \frac{I-1}{I}\frac{R}{\theta_t} m_{t-1}, \tag{32}$$

$$m_t = \gamma (I) [1 - \alpha - \tau_t], \tag{33}$$

where $m_t \equiv M_t / p_t Y_t$. Substitute (33) into (32) to obtain

$$g = \tau_t + \bar{l} - \frac{R}{\theta_t}\bar{l} + [1 - h (I)] \frac{R}{\theta_t} [1 - \alpha - \tau_{t-1}], \tag{34}$$

Asset market equilibrium requires

$$p_t K_{t+1} + B_t + M_t = p_t w_t - T_t. \tag{35}$$

Divide (35) by $p_t Y_t$ and arrange terms to obtain

$$\frac{\theta_{t+1}}{A} + \bar{l} = 1 - \alpha - \tau_t. \tag{36}$$

Solve (36) for τ_t and substitute it into (34) to finally obtain

$$\theta_{t+1} = (1-\alpha-g)\,A^2 + [1-h\,(I)]\,AR - \frac{h\,(I)\,R\bar{l}A^2}{\theta_t} \equiv \Phi\,(\theta_t)\,. \tag{37}$$

Thus, (37) describes the evolution of the output growth rate of the economy. It is helpful to characterize some basic properties of the function Φ.

Lemma 10 *The function Φ satisfies (a) $\Phi'(\theta) > 0$, (b) $\lim_{\theta\to 0}\Phi'(\theta) = \infty$, (c) $\lim_{\theta\to\infty}\Phi'(\theta) = 0$, and (d) $\lim_{\theta\to\infty}\Phi(\theta) = (1-\alpha-g)\,A^2 + [1-h\,(I)]\,AR$.*

Proposition 11 *Let*

$$D \equiv \Big[(1-\alpha-g)\,A^2 + [1-h\,(I)]\,AR\Big]^2 - 4h\,(I)\,R\bar{l}A^2.$$

(a) There exist two distinct balanced growth equilibria if $D > 0$, (b) there exists a unique balanced growth equilibrium if $D = 0$, and (c) there exists no balanced growth equilibrium if $D < 0$.

Proof. Notice that at a balanced growth equilibrium (37) can be written in a quadratic form, with the roots

$$\theta = \frac{(1-\alpha-g)\,A^2 + [1-h\,(I)]\,AR \pm \sqrt{D}}{2}.$$

These roots are real if and only if $D > 0$. ∎

It is easy to see that the condition $D > 0$ is likely to be satisfied for small values of g and $\bar{l}$. In words, a balanced growth equilibrium exists if g and $\bar{l}$ are not too large. In what follows I focus on the case where two balanced growth equilibria exist. From figure 3, it is easy to show the following.

Proposition 12 *The low-growth equilibrium is unstable and locally determinate. The high-growth equilibrium is stable and locally indeterminate.*

The dynamics of $\theta_{t+1} = \Phi(\theta_t)$ is easily checked with figure 3. The low-growth (high-growth) equilibrium is unstable (stable) because the Φ locus cuts the 45 degree line from below (above). A caveat is in order here. In the neoclassical growth model considered in the previous sections, the equilibrium law of motion is given by $k_{t+1} = \Omega(k_t)$. Since k_t is a state variable and cannot jump, a stable steady state is determinate. That is, there is a unique equilibrium path toward the steady state. On the contrary, $\theta_t \equiv Y_t/Y_{t-1} = k_t/k_{t-1}$ is *not* a state variable, so it can jump. Thus, there is an infinitely many rational expectations paths to the stable equilibrium from any given initial condition k_1. In fact, the model's initial condition, K_1, cannot give the initial condition for the map $\theta_{t+1} = \Phi(\theta_t)$. Therefore, the stable equilibrium in this economy suffers the indeterminacy problem.[6]

In addition to the condition given in proposition 11, an equilibrium must satisfy $m_t \leq \bar{l}$. From (33), $1 - \alpha - \tau_t \leq \bar{l}/\gamma(I)$. Combine this with (36) to obtain

$$\theta \leq \frac{1-\gamma(I)}{\gamma(I)} A\bar{l} \equiv \overline{\theta}. \tag{38}$$

Let θ_l and θ_h denote the low-growth and high-growth equilibria. If $\theta_h \leq \overline{\theta}$, then the both equilibria are valid. If $\theta_l \leq \overline{\theta} \leq \theta_h$, then only the low-growth equilibrium is valid. In that case, any path other than $\theta_t = \theta_l$ for all t will eventually violate equilibrium conditions. In that sense, $\theta_t = \theta_l$ for all t is the unique equilibrium path.

5.2 Effects of Policies

Proposition 13 *At the high-growth (low-growth) equilibrium, (a) an increase in g reduces (raises) θ, (b) an increase in $\bar{l}$ reduces (raises) θ, (c) an increase in I raises θ if $(1-\rho)I < 1$.*

Proof. From (37) it is straightforward to show that

$$\frac{\partial \Phi}{\partial I} = -ARh'(I)\left[1 + \frac{A\bar{l}}{\theta}\right].$$

■

[6] In Kudoh (2003), an adaptive learning scheme is used as an equilibrium selection device in an endogenous growth model with multiple assets.

According to proposition 13a, fiscal stimulus reduces the output growth rate at the high-growth equilibrium. The logic is clear. An increase in government expenditures raises taxes, since the total debt per output is fixed. Higher taxes slow down the capital accumulation process and thereby slow down growth. Proposition 13b states that an increase in the committed level of total government liabilities reduces the output growth rate at the high-growth equilibrium. The mechanism behind the result is crowding out of capital, as pointed out by Saint-Paul (1992) and Grossman and Yanagawa (1993).

Since in this AK-type endogenous growth model the real interest rate is constant, under a nominal interest rate peg, the inflation rate satisfies $\pi_t = I/R$ for all t. In words, nominal interest rate pegging in the AK-type endogenous growth model implies an inflation target. Thus an increase in the targeted nominal interest rate raises inflation. According to proposition 13c, an increase in the targeted nominal interest rate is likely to be growth enhancing at the high-growth equilibrium.

5.3 Welfare Implications

(6) implies that at a balanced growth equilibrium

$$U_t = RY_t \left[(1-\sigma)\,(m/I)^{1-\rho} + \sigma z^{1-\rho} \right]^{\frac{1}{1-\rho}} \tag{39}$$

holds. From (33) and (36), $m = \gamma(I)\left[\theta/A + \bar{l}\right]$ and $z = [1-\gamma(I)]\left[\theta/A + \bar{l}\right]$. Rewrite (39) as

$$U_t = \Theta(I)\left[\frac{\theta}{A} + \bar{l}\right]\alpha A Y_t, \tag{40}$$

where $\Theta(I)$ is defined in (25). In contrast to the neoclassical growth model, welfare cannot be defined as the instantaneous utility in steady state because utility depends on the aggregate output, which is time-variant at a balanced growth equilibrium. Thus, following Espinosa and Yip (1999) to define welfare as $W \equiv \sum_{t=1}^{\infty} \beta^{t-1} U_t$, where $\beta < 1$ is the discount factor. Note that at a balanced growth equilibrium the aggregate output evolves according to $Y_{t+1} = \theta Y_t$, which can be solved as $Y_t = \theta^{t-1} Y_1$. Thus,

$$W = \Theta(I)\left[\frac{\theta}{A} + \bar{l}\right]\alpha A Y_1 \sum_{t=1}^{\infty} (\beta\theta)^{t-1}. \tag{41}$$

Suppose the β is sufficiently small so that $\beta\theta < 1$. In that case (41) is well-defined, and can be rewritten as

$$W = \Theta(I)\left[\frac{\theta}{A} + \bar{l}\right]\frac{\alpha A^2 K_1}{1 - \beta\theta}. \tag{42}$$

It is then easy to show that $dW/d\theta > 0$. It follows that the high-growth equilibrium is Pareto-superior to the low-growth equilibrium. Further, since (from proposition 13) an increase in g reduces the output growth rate at the high-growth equilibrium, fiscal stimulus at that equilibrium retards growth and reduces welfare.

6 Conclusion

Models with fiscal dominance are best regarded as describing economies in which the central bank is not independent. A model describing recent industrial nations must assume monetary dominance. One of the main lessens of Sargent and Wallace (1981) and the literature that follows is that in an economy without a tough central bank, fighting *inflation* requires fiscal discipline. The main message of this paper is that, in a world with central bank independence, fighting *deflation* requires fiscal discipline. Empirical question arising from the analysis is whether the relationship between fiscal deficits and inflation depends on the degree of central bank independence.

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Figure 1: Steady state equilibria

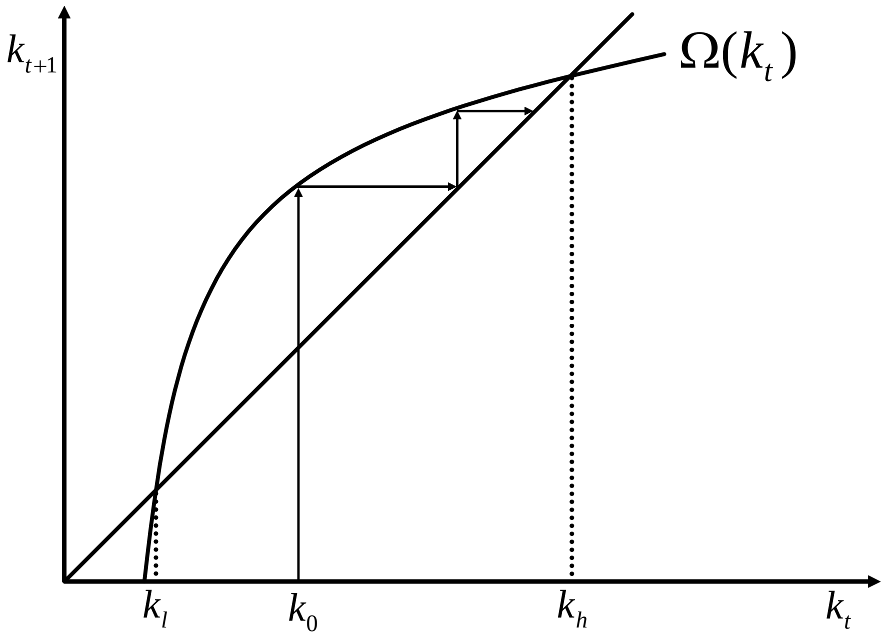

Figure 2: Changes in government spending

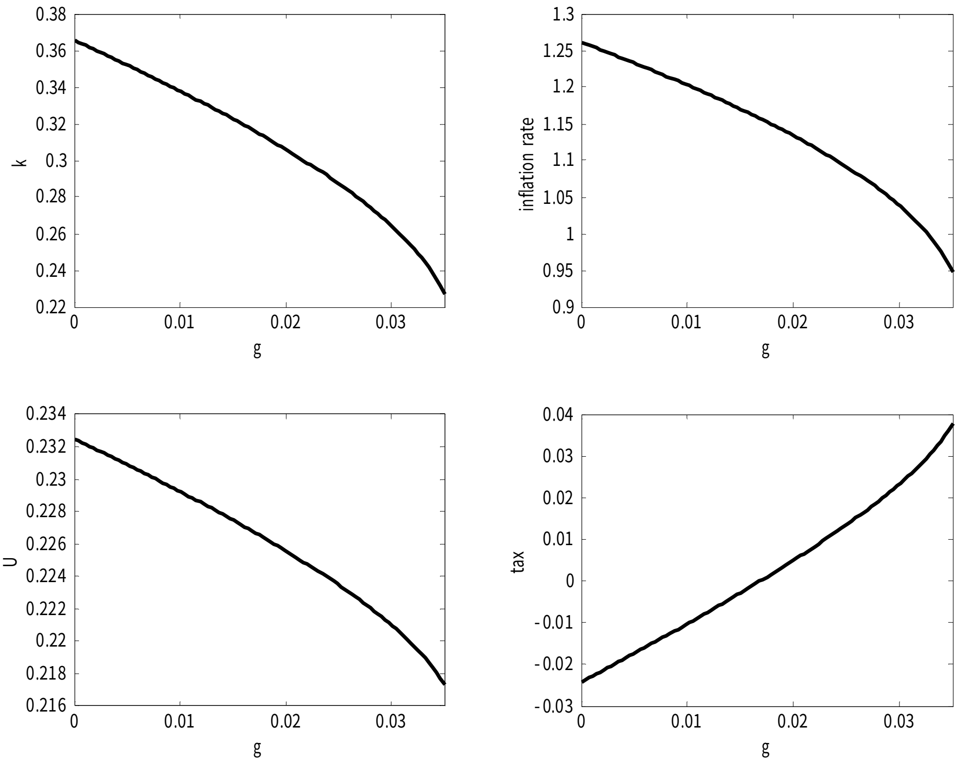

Figure 3: Balanced growth equilibria