An Influence of Voters' Preferences on the Stable Parliamentary Seat Share

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Abstract

We explain an influence of voters' preferences on the stable parliament seat share. Under proportional representation, we have the possibility of lock-in into one of alternative political party or, conversely, the feasibility of their long term co-existence. If most voters have strong support for the particular party, the long term co-existence is realized easily. If most voters pay attention to political effectiveness, the lock-in into one of political party is easy to occur. A dominant party is decided depending on early historical event.

1   Introduction

In this paper, we show that the dynamics of a parliament seat share using an adoptive process modeled by urn scheme. In particular, we clarify the condition under which one dominant party or co-existence arises, when voters recognize the political effectiveness as important.

One of the important factors for voters to decide their voting is how a political party's proposal is close to their ideals, which reflect their religion, social class, age

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and so on. As Lipset and Rokkan (1967) shows, in Europe structural conflicts of interests in the society (centre-periphery, state-church, land-industry, and owner-worker) are important fact to construct a party support. In 1950's Britain, for example, the Conservative Party was mainly the party of southern Britain and of the Church of England; the Liberals had much greater strength in the peripheries and among religious Nonconformists.

There exists another important factor to choose a party. In the literature it is often supposed that the winning party has the power to implement any proposal with probability 1. However, voters have room to consider the *likelihood* that a party will implement its proposal, i.e., the political effectiveness. A society is formed by voters who have different ideals. Some of them may be more in favor of the policy that was announced by the defeated party; others, on the contrary, may be in favor of the winning party's proposal. The actions taken by the government are influenced by voters. The influence of voters in favor of a proposal will become bigger if the proportion of seats obtained by the corresponding party. Clearly a party winning 51% of the seats will have more difficulty to carry out its proposal than one winning 80% of the seats. As shown in Duverger(1967), voters for small parties will see that their vote is being 'wasted' and they will switch to supporting a major party.

We consider the case where voters pay attention to two points to choose a party; the closeness between a party's ideology and their ideals, and the political effectiveness. Here, we consider the simple society where there are two parties, say, A and B, and three type voters, say, a, b, and c. Voter a receives positive benefits from A and negative one from B. She always votes for party A tough A has any low seat share. Conversely, voter b obtains negative benefits from A and positive one from B. He votes for party B irrespective of the seat share of B. Voter c gets the same benefits from both parties. Party A and B are indifferent for voter c. Voter c votes for a party with higher seat share. Voter a and b count the closeness to a party strongly and they ignore the political effectiveness. Voter a has the low critical seat share to vote for party A

2

and voter b has the high critical seat share to vote for A. Voter c pays attention to the political effectiveness. He has the moderate critical seat share to vote for party A. We regard a voter's critical seat share as his preference. For purposes of comparison we consider two shapes of distribution of voters' preferences, bimodal and single-peaked.

If voters' preferences are distributed as bimodal, each party has strong supporters who always vote for the support party and the proportion of voters who care the political effectiveness as important is small. As each party has strong supports potentially even if it leaves what initial state, in progress of time, two seat shares become equivalent, i.e., the long term co-existence is realized in the equilibrium. Conversely, if voters' preferences are distributed as single-peaked, most voters are interested in the political effectiveness. Then, a party with higher seat share is more attractive for voters and it gains an even higher seat share. After all, a party with a high initial seat share comes into power, i.e., the lock-in into one of political party is realized.

The dynamics treated in this paper is the generalized urn process discussed in Hill *et al.* (1980), Arthur *et al.* (1983), and Dosi *et al.*(1994). As Dosi *et al.*(1994) mentioned, by specifying the function which characterizes an agent's behavior, it is possible to analyze the stochastic evolution of the share. We define the voters' behavioral patterns and demonstrate how the global forces ruling the dynamics of whole populations can be derived from the individual behavior of voters.

The organization of the rest of the paper is as follows. Section 2 set up the model. Some results on the stable seat share in parliament are presented in Section 3. Section 4 concludes and proposes some issues for a further research.

2   The Model

We consider an influence of voters' Preferences on the stable parliamentary seat share under the proportional representation where there are two political parties, A and B, and contest a seat in the election. At each period $t = 1, 2, \cdots$, only one voter chosen

randomly can obtain the voting opportunity. He votes for A or for B, without the possibility of abstention.

If $A(B)$ implements its proposal, voter j gets benefits $a_j(b_j)$ from the closeness between $A(B)$'s ideology and $j$fs ideal. Is it the best choice for voter j to vote for A, if $a_j > b_j$? Of course, since he is imperfectly informed about the possibility of enforcement of a proposal, he uses some bounded rational decision rule to make his choice. Party A may be a new and unknown party with a small political power to implement its policies and party B may be a well-known one with a firm political power. Hence, potential voters perceive a different risk in this choice and require different evidence. Voters know that a political party with a low seat share will have more difficulty to implement its proposal than one with a high seat share. We call the likelihood that a party implements its proposal as the political effectiveness. The political effectiveness is one of the important effects for voters to choose a party. We assume that the political effectiveness is an increasing function of seat share as in Stigler(1972). For simplification, we regard the seat share as the likelihood that the corresponding party will realize its proposal.

We assume that voters pay attention to two points to choose a party; the closeness between party's ideology and own ideal, and the political effectiveness. When party A has seat share X, voter j gains an expected benefits Xa_j from party A and gains $(1 - X)b_j$ from B. Voter j has a critical seat share at which he obtains the same expected benefits from party A and B. For each voter j, we can define such a critical seat share $J = b_j(a_j + b_j)^{-1}$, which represents his preference.

We assume that $a_j + b_j > 0$ [1]. This assumption avoids a situation where voters obtain negative benefits from both parties. If a voter gets just negative benefits from both parties, it is natural for her to abstain from voting. The assumption $a_j + b_j > 0$ is consistent with no abstention. However, this assumption does not avoid the situation

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[1] We can have the analogous analysis if $a_j + b_j < 0$. In this case, voter j votes for party A if A's seat share is smaller than j's critical seat share.

that a voter gets positive benefits from a party but she gets negative benefits from another party.

We assume that a_j and b_j distribute over $[-\infty, \infty]$, and J distributes over $[-\infty, \infty]$. A voter votes for party A, if A has a higher seat share than his critical seat share, conversely a voter votes for party B, if A has a lower seat share than her critical seat share. Hence, a voter whose critical seat share is less than zero vote for A regardless of A's seat share, and a voter whose critical seat share is more than one vote for B regardless of B's seat share. They have a favorable party. Voters whose critical seat share is 0.5 vote for a party getting the higher seat share. These voters care the political effectiveness rather than the favor to a party. They dislike getting smaller benefits rather than enduring not to support a favorable party. They prefer to behave strategically.

The two political parties have already got parliamentary seats, say, n_A and n_B at period 0. We shall denote a realized party A's seat share at period t ($t = 0, 1, 2, \cdots$) by X_t. Potential voters know that only one voter can obtain the vote in each period, and a realized seat share is affected by just a new voter's choice. As a new voter at $t + 1$ has a little influence, she just cares the A's seat share at t.

We assume that the density function of the potential voters' preferences is exogenously given. The density function of J is expressed as $p(J)$. A new voter votes for party A, if her critical seat share is lower than the realized A's seat share at previous period. Hence, the probability that a new voter votes for party A at period $t + 1$ is

$$\int_{-\infty}^{X_t} p(J) \, dJ.$$

represented as $P(X_t)$. We denote the number of new seats obtained by party A at period $t + 1$ as $\xi(X_t)$. $\xi(X_t)$ can take on two values:

$$\xi(X_t) = \begin{cases} 1 \text{ with probability } P(X_t) \\ 0 \text{ with probability } 1 - P(X_t). \end{cases}$$

The number of new A's seats is a stochastic variable at period t, and A's seat share

at period $t + 1$ is also a stochastic variable at period t. We represent an expected A's seat share evaluated at period t as x_{t+1}. The dynamics of xt is given by the relation

$$x_{t+1} = \frac{X_t(t + n_A + n_B) + \xi(X_t)}{t + 1 + n_A + n_B}, \quad t \geq 0, \quad X_0 = \frac{n_A}{n_A + n_B}. \tag{1}$$

We define that $\zeta(X_t) \equiv \xi(X_t) - P(X_t)$. The dynamics of expected X_t becomes

$$x_{t+1} = X_t + \frac{\zeta(X_t) + P(X_t) - X_t}{t + 1 + n_A + n_B}.$$

Since an expected value of $\xi(X_t)$ is $P(X_t)$,

$$E\zeta(X_t) = E(\xi(X_t)) - P(X_t) = 0.$$

Then

$$E(x_{t+1}|X_t) - X_t = \frac{P(X_t) - X_t}{t + 1 + n_A + n_B}.$$

The dynamics (1) shifts on average at time $t \geq 1$ from a point X_t on the value $\{P(X_t) - X_t\}(t + 1 + n_A + n_B)^{-1}$. Consequently, limit points of the sequence $\{X_t\}$ have to belong to the set of zeros of the function $P(X_t) - X_t$. If $P(X_t)$ is equal to X_t, A's seat share keeps X_t from period t on. As shown in Hill *et al.*(1980) and Dosi *et al.*(1994) for an isolated root θ of $P(X) - X$, one can have convergence of X_t to θ with positive or zero probability depending on

$$(P(X) - X)(X - \theta) \leq 0 \tag{2}$$

or

$$(P(X) - X)(X - \theta) \geq 0 \tag{3}$$

in a neighborhood of θ. We call these points stable or unstable, correspondingly.

3 The Stable Seat Share

The dynamics (1) shows that the stable seat share depends on the distribution of voters' preferences. That is, the voters' preferences decide the stable seat share. In

this section, we analyze an influence of voters' preferences on the stable parliamentary seat share. Voters' preferences are shown as a distribution of voters' critical seat shares. For purposes of comparison we consider two shapes of distribution function, bimodal and single-peaked.

If most voters support a favorable party strongly, we may have a bimodal distribution. Strong support means voting a favorable party irrelevant to corresponding party's seat share. That is, most voters have a critical seat share which is less than zero or more than one. Because each party has rigid supporters in the bimodal case, neither party may obtain a monopolistic seat share.

On the other hand, if most voters care the political effectiveness seriously, we may have a single-peaked distribution. It is reasonable for such voters that vote for a party having larger power to implement its proposal. In this case, a representative voterfs critical seat share is around 0.5. As most voters are interested in the political effectiveness, a party which has larger seat share is more attractive for them and will obtain higher seat share. It may be easy to occur that a party getting a low seat share becomes to have a less influence in the political competition.

Figure 1 and 2 around here.

We quote simple distribution functions as examples to show distinct results; a symmetric bimodal distribution $(N[0, 0.3] + N[1, 0.3])/2$ (see Figure 1), and a normal distribution $N[0.5, 0.3]$ (see Figure 2)[2].

Figure 3 and 4 around here

Proposition: The stable seat share is classified into two basic cases according to the voters' preferences:

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[2] $N[\mu, \sigma] = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma^2}(x - \mu)^2\right)$ where μ is average and σ is standard deviation.

(i) If $p(J)$ is $(N[0, 0.3] + N[1, 0.3])/2$, we have just one stable seat share 0.5. The long term co-existence is realized (see Figure 3),

(ii) If $p(J)$ is $N[0, 0.3]$, we have two stable seat share. These are represented as $\hat{X}_L$ and $\hat{X}_R$ shown in Figure 4 ($\hat{X}_L \approx 0.081$ and $\hat{X}_R \approx 0.918$). One party can obtain much larger seat share than the other does. The lock-in into one of political party is occurred.

Proof: First, consider the case where $p(J)$ is bimodal. $P'(X) = p(X) > 0$ and $P''(X) = p'(X)$ is positive if X is less than 0.5 and is negative if X is more than 0.5. Then $P(X)$ is shown in Figure 3. $P(X) - X$ has one root 0.5. As this root satisfies (2), 0.5 turns out to be stable. In the limit we have the ratio between party A and B equals to 0.5:0.5.

Next, consider the case where $p(J)$ is single-peaked. $P'(X) = p(X) > 0$ and $P''(X) = p'(X)$ is negative if X is less than 0.5 and is positive if X is more than 0.5. Then $P(X)$ is shown in Figure 4. $P(X) - X$ has three roots $\hat{X}_L$, 0.5 and $\hat{X}_R$. Satisfying (3), the root 0.5 turns out to be unstable, while $\hat{X}_L$, and $\hat{X}_R$ are stable roots. Consequently, in the limit we can have the situation that the parliament is shared by party A and B in the proportion either $\hat{X}_L : 1 - \hat{X}_L$ or $\hat{X}_R : 1 - \hat{X}_R$. Q.E.D.

If voter's preferences are distributed as bimodal, each party has strong supporters who always vote for the support political party. As each party has strong supports potentially even if it leaves what initial state, in progress of time, a potential situation is realized and two parliament seat shares become equivalent, i.e., the long term co-existence is realized in the equilibrium. On the other hand, if the distribution of votersf preferences is single-peaked, this means that most voters are interested in the likelihood that a party implements its proposal. As most voters pay attention to the political effectiveness, a party with a higher seat share is more attractive for voters and it gains an even higher seat share. After all, a party obtaining a high initial seat share comes

into power, i.e., the lock-in into one of political party is realized. The past shapes, in probability, the future, and this effect self-reinforces along the seat share trajectory. The dominant party is party A, if A has an initial seat share which is more than 0.5, conversely, it is party B, if A has an initial seat share which is less than 0.5. Early historical events decide the future.

In Japanese urban area, it is said that entrants have stronger positions than incumbent in electoral competitions. At the beginning entrants may be able to get a large seat share; however, their seat shares may become smaller gradually. The case of single-peaked distribution shows that a party with relatively high initial seat share like 0.49 cannot make its seat share larger. Entrants are attractive enough for voters to obtain a relatively high seat share in initial, but voters care political effectiveness much, and incumbentsf established capability to implement policies is more attractive for voters. Hence, entrants come to have a less seat share.

4 Conclusion and Further Discussion

In this paper, we show that the stochastic choice process may well bifurcate limit parliament seat share outcomes; lock in and history-dependent selection of a particular party does occur. Especially we focus on an influence of voters' preferences on stable seat share. Voters pay attention to two points to choose a party; the favor to a party and the political effectiveness.

If most voters strictly prefer the particular party, voters' preferences are shown as the bimodal distribution. As most voters always vote for a favorite party irrespective of its seat share and each party has the same strong supports, in progress of time, each party has an equivalent seat share. The long term co-existence is occurred. On the other hands, if strategic voters have a large share, voters' preferences are shown as the single-peaked distribution. Because of the political effectiveness, voters take the seat share into consideration seriously. A party with a high seat share has an even higher

seat share. Another party's power to implement its proposal becomes smaller, and this party is expelled out of politics in the long-run. The lock-in into one of alternative political party is occurred. This distribution generates a mechanism for establishing the dominance of one of the competing parties (and both have positive probability to dominate). A dominant party is determined by early historical event, i.e. an initial seat share.

We may apply our results to explain the stability of party system. Previous works in the literature on the party system have considered that electoral system affects the stability of party system. Duverger(1967) shows that the single-member district system works in favor of major parties and encourage the development of two-party system. Surely in UK and US the single-member district system is used for election and there exists two-party system. On the other hand, in Canada the single-member district system is also used like UK and the election system is not changed, but there exists the fluctuation of party system that the Conservative Party which was the government party in 1993 had just two seats in the Commons (see Shimotomai and Takahashi (1996)). This is an example that the single-member district system does not make two-party system sustainable. Thus, we see that election system has an effect on party system but it is just one factor to affect the sustainability of party system.

In Britain most of the voters vote for a specific party because of the social stratum. We say that Britain in 1950s had a bimodal distribution of voters. Post-war Japan may have another distribution of voters. Murakami (1982) explains the Japanese voting behavior; the Liberal Democratic Party (LDP) was the government party and it had the sufficient power to implement own proposal, then many voters support the LDP. Japanese do not care 'who' gives benefits to them, but care 'how much' a government party allocates benefits to them. That is, Japanese voters care about the political effectiveness rather than the favor to a specific party. In post-war Japan the distribution of voters' preferences may be single-peaked. LDP can establish its political power base at early time, and then it can become a dominant party.

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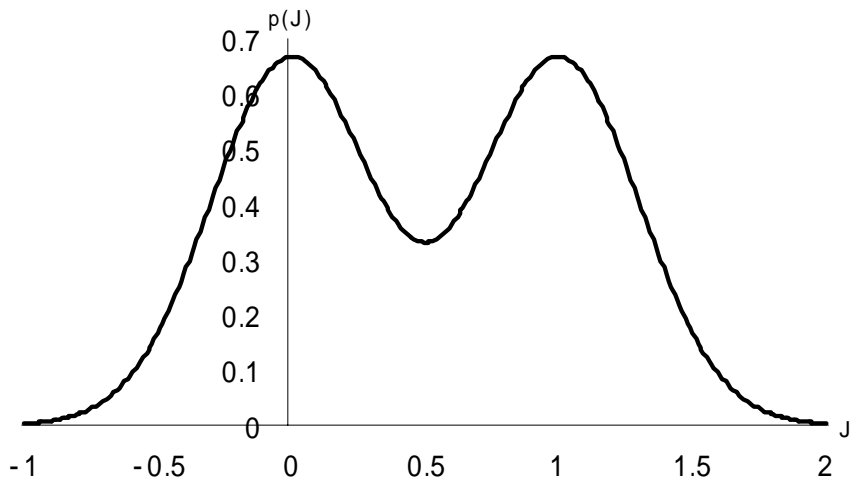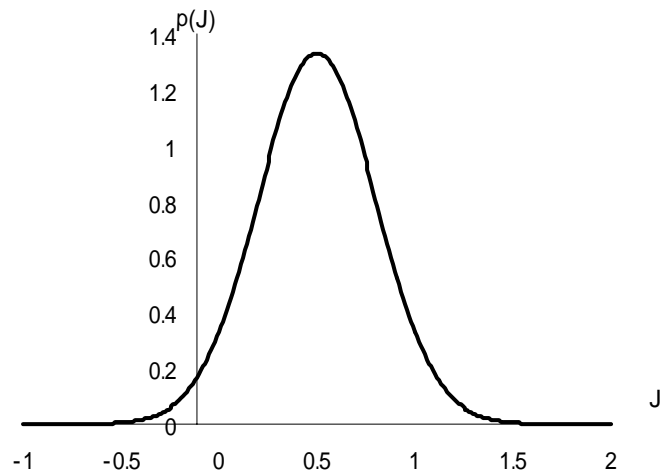[10] Simotomai, N. and N. Takahashi (1996), Senshin Syokoku no Seiji (Politics in the developed countries), Housou Daigaku Kyouiku Shinkoukai (in Japanese).
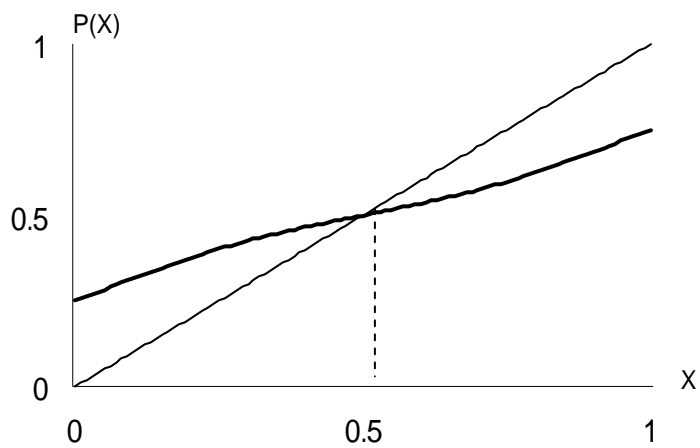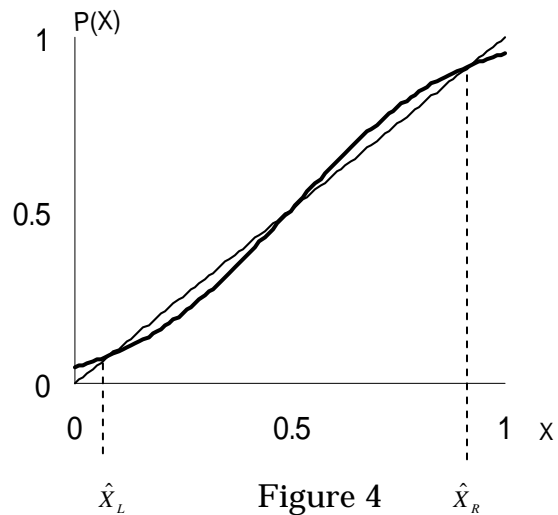
Figure 1

Figure 2

Figure 3

Figure 4