Option Pricing under NIG Distribution

– The Empirical Analysis of Nikkei 225 –

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Introduction -1-

- The distributions of assets returns have
  - fatter tails than Normal distribution(Excess Kurtosis)
  - asymmetry (Negative Skewness)

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- GH, Hyperbolic and NIG distributions
  - Barndorff-Nielsen(1995), Eberlein and Keller (1995) and Prause(1999) etc.
  - As for the research using those distributions in Japanese market, it has not yet been studied so much.

Introduction -2-

- The aim of this report is to conduct an empirical study using the NIG distritution in the Japanese option market.
  - we use the Nikkei 225 call option data
  - compare the model assuming the underlying asset returns follow the NIG distribution with Black-Scholes model.
- Option pricing models:

  1 : The Black-Scholes model $\Rightarrow$ log returns are Normally distributed

  2 : The NIG model $\Rightarrow$ log returns follow NIG distribution

Price Process

We denote the underlying asset price at time t by $S(t)$ and consider the price process of the form

$$S(t) = S(0)\mathrm{e}^{X(t)},\ t \geq 0 \tag{1}$$

Assumption ($X = \{X(t)\}_{t\geq 0}$)

1. $X(0)$ is 0 with probability one
2. X has independent increments
3. X has stationary(time homogeneous) increments
4. X is stochastically continuous

From now on, we will use the NIG distribution but before it we describe the NIG distribution.

The NIG density of X(1)

$$f_{NIG}(x;\alpha,\beta,\delta,\mu) = f_{GH}(x;-\frac{1}{2},\alpha,\beta,\delta,\mu)$$
$$= \frac{\alpha\delta}{\pi}\exp\left\{\delta\sqrt{\alpha^2-\beta^2}+\beta\,(x-\mu)\right\}\frac{K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\sqrt{\delta^2+(x-\mu)^2}}$$

where K_1 is the modified Bessel function of the third kind with index 1

The parameters satisfy $\mu \in \mathrm{R}$, $\delta > 0$ and $|\beta| \leq \alpha$

$\alpha \cdots$ the steepness around the peak(the tail fatness)

$\beta \cdots$ the degree of the asymmetry

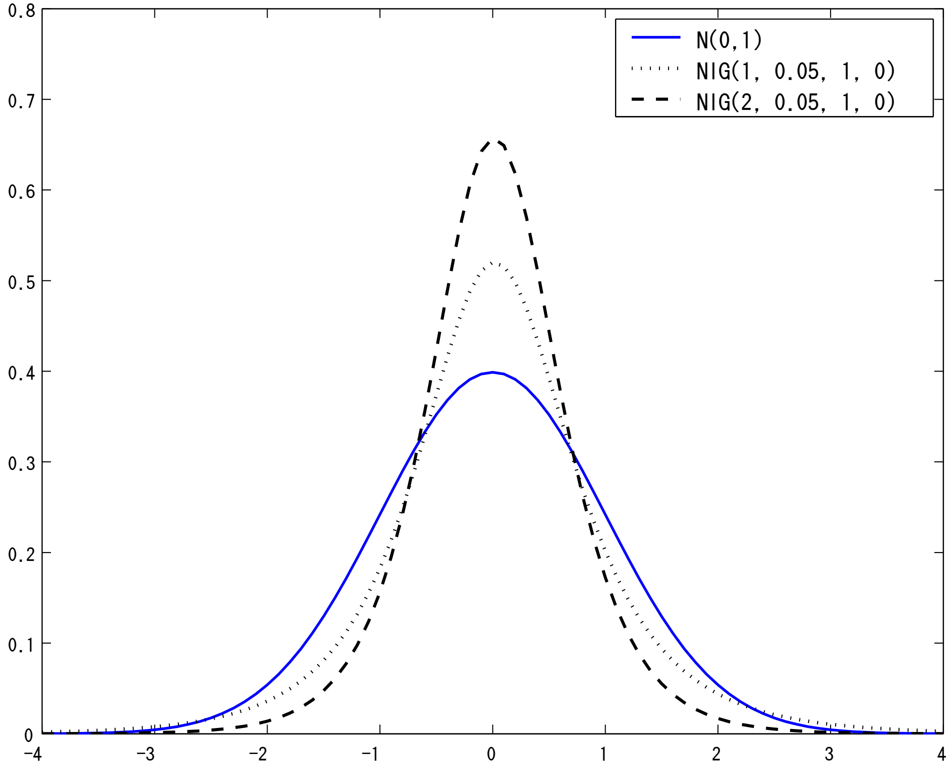
$\delta \cdots$ scale

$\mu \cdots$ location

Fig. 1: The effects of the parameters changes

α

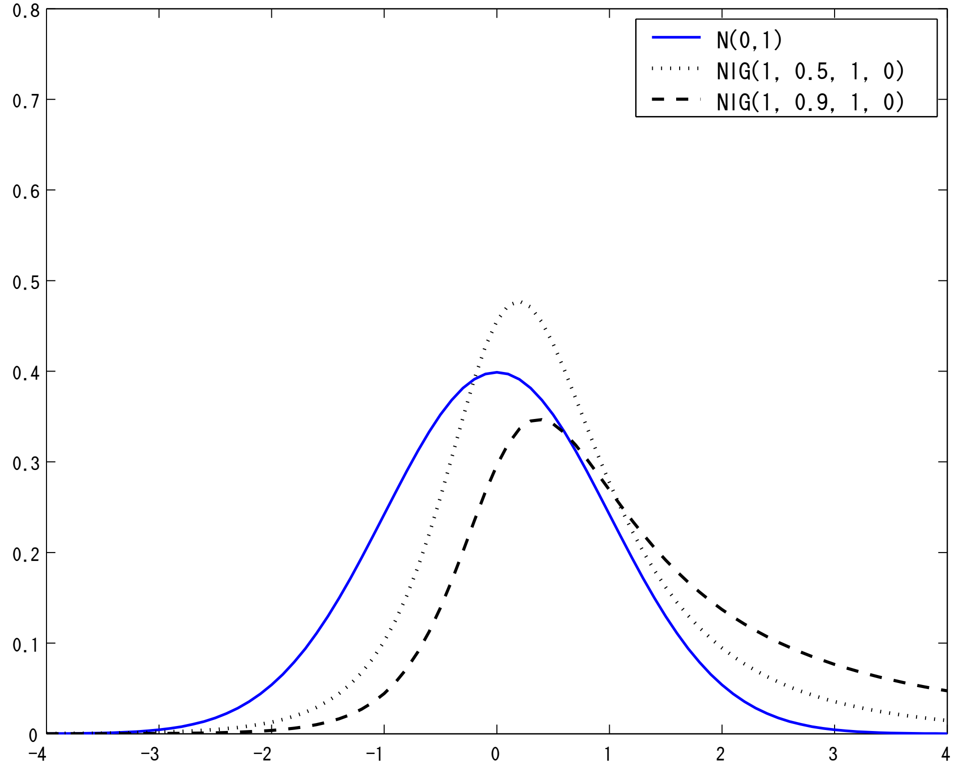
NIG(1, 0.05, 1, 0) $\rightarrow$ NIG(2, 0.05, 1, 0)



$\alpha \uparrow \Longrightarrow$ leptokurtic

β

NIG(1, 0.5, 1, 0) $\rightarrow$ NIG(1, 0.9, 1, 0)



$\beta \uparrow \Longrightarrow$ skewed

Goodness of fit to the Nikkei 225

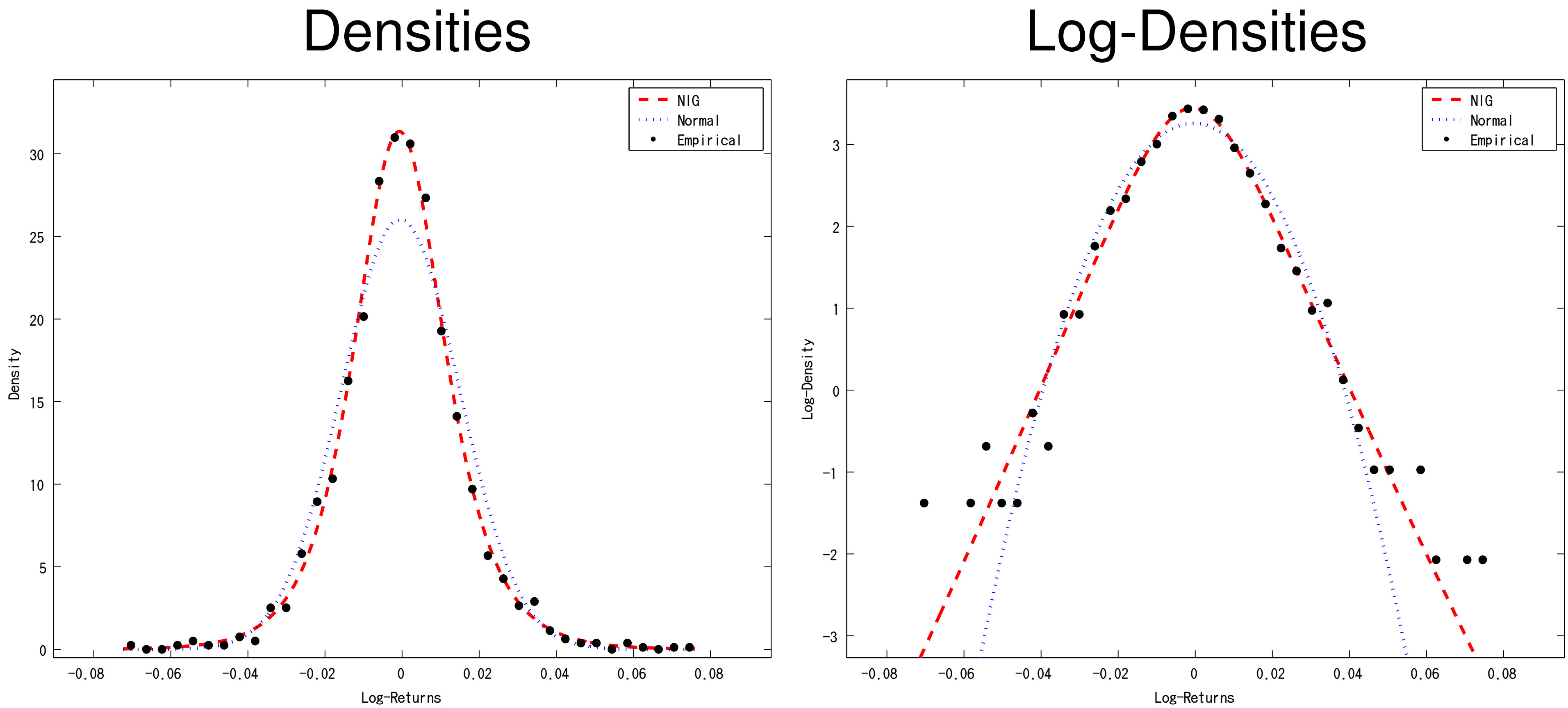
- Goodness of fit of Normal and NIG distributions to the empirically observed returns i.e. the returns of the Nikkei 225

  Estimate parameters of Normal and NIG by using the returns from January 5 1995 to December 30 2002

Table 1: Parameters estimates by ML method

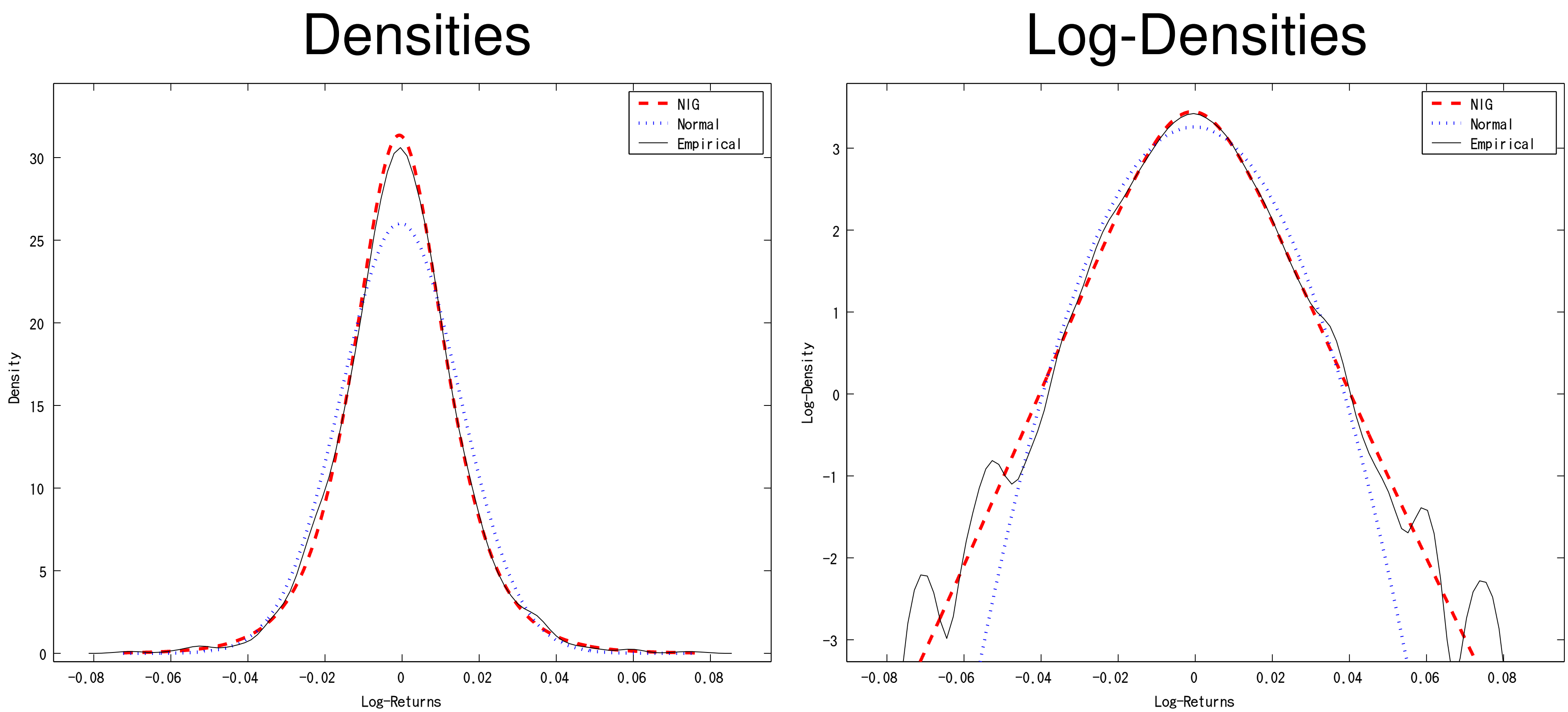
Type	Parameters
Normal	$\hat{\mu} = -0.0004, \ \ \hat{\sigma} = 0.0153$
NIG	$\hat{\alpha} = 82.3726, \ \hat{\beta} = 2.3852, \ \hat{\delta} = 0.0193, \ \hat{\mu} = -0.0010$

Fig. 2: Histograms



The fitted Normal and NIG densities of the returns of Nikkei 225

Fig. 3: The empirical Kernel densities



The fitted Normal and NIG densities of the returns of Nikkei 225

The advantages of the NIG in option pricing

- The advantages of using the NIG distribution:
  - The Bessel function does not appear in the moment generating function:

$$\begin{aligned} M(u,1) &= \mathsf{E}\left[\mathsf{e}^{uX(1)}\right] \\ &= \exp\left\{\mu u + \delta\left(\sqrt{\alpha^2-\beta^2} - \sqrt{\alpha^2-(\beta+u)^2}\right)\right\} \end{aligned}$$

  - The NIG distribution is closed under convolution. In particular, it has reproductivity.

Thus, it is easier to deal with the NIG distribution mathematically.

Using the NIG distribution for the returns process X, it is known that X becomes a process with jumps. So, we are considering the incomplete market in option pricing.

Option pricing in the incomplete market

- Let $f_{NIG}(x;\alpha,\beta,\delta,\mu)$ be the density of $X(1)$

  From the assumption on X and the reproductivity
- The density of $X(t)$ becomes $f_{NIG}(x;\alpha,\beta,t\delta,t\mu)$

The risk neutral Esscher transfom of $f_{NIG}(x;\alpha,\beta,t\delta,t\mu)$ is given by

$$f_{NIG}^{(u^*)}(x,t) = \frac{\mathrm{e}^{u^* X(t)}}{[M(u^*,1)]^t} f_{NIG}(x;\alpha,\beta,t\delta,t\mu) \tag{2}$$

where u^* is the solution of the following equation

$$r = \log \frac{M(1+u,1)}{M(u,1)} \tag{3}$$

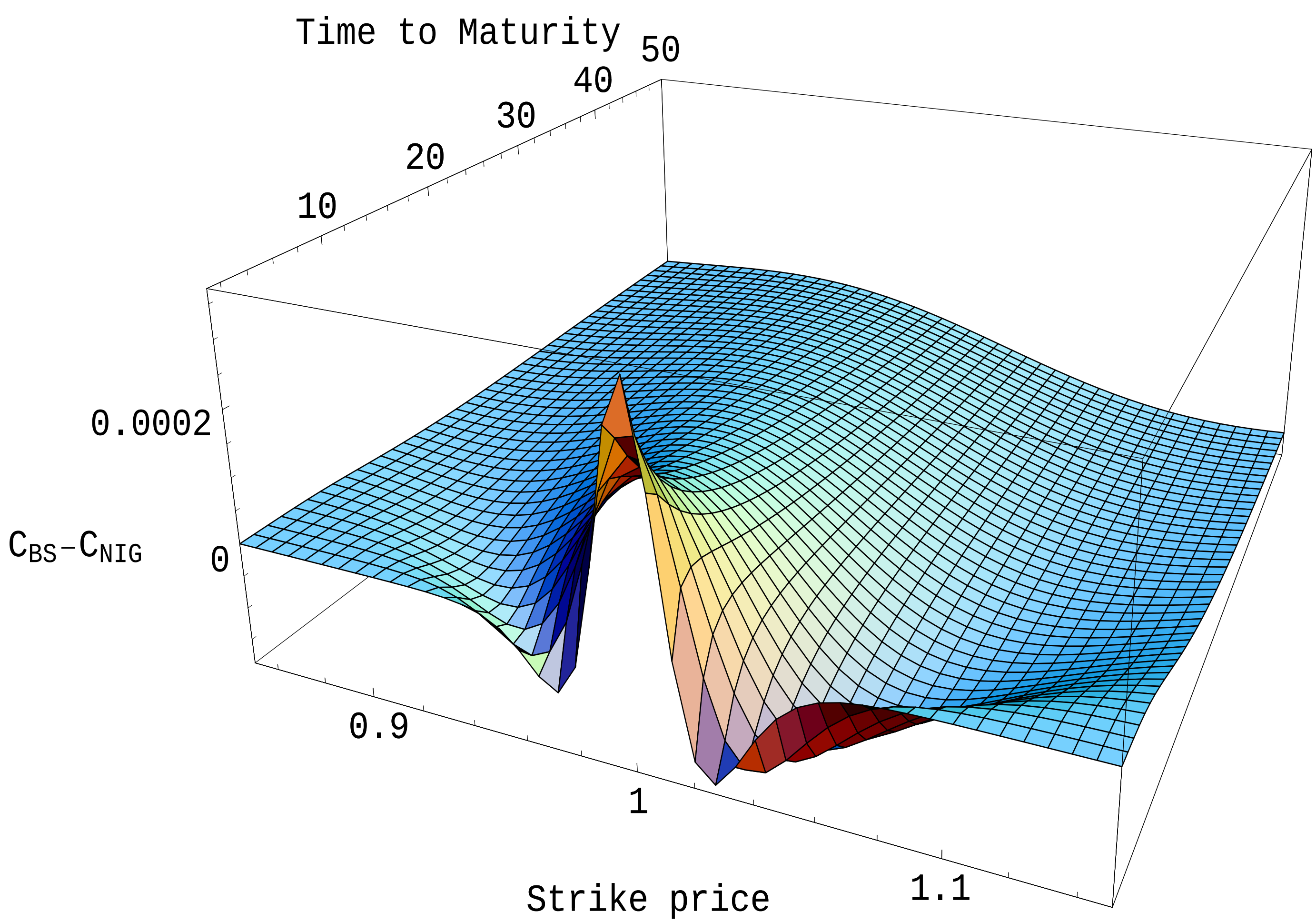
$$= \mu + \delta\left(\sqrt{\alpha^2-(\beta+u)^2} - \sqrt{\alpha^2-(\beta+u+1)^2}\right) \tag{4}$$

Option pricing under the NIG distribution

Taking a European call with strike price K, expiration date $t = \tau$ and riskfree rate r, we can calculate the call option price as follows:

$$\begin{aligned} C_{NIG} &= C(S(\tau), \tau) \\ &= \mathrm{E}^{(u^*)}\left[\mathrm{e}^{-r\tau}\max\big(S(\tau) - K, 0\big)\right] \\ &= S(0)\int_{\log\frac{K}{S(0)}}^{\infty} f_{NIG}^{(u^*+1)}(x,\tau)dx - \mathrm{e}^{-r\tau}K\int_{\log\frac{K}{S(0)}}^{\infty} f_{NIG}^{(u^*)}(x,\tau)dx \end{aligned} \tag{10}$$

Fig. 4: The differences of option prices



The differences of the Black-Scholes prices minus the NIG prices are computed under $r = 0, S(0) = 1$ and $\tau = 1 \sim 50$

The empirical study — About data set

As the Japanese market data, we use

- the closing prices of the Nikkei 225 call option from September 10, 1999 to December 12, 2002.
- the Nikkei 225 Stock Average index

We exclude call options

with trading volume less than 10 units

with greater than 100 days to expiration

The empirical study — parameters estimation

- The Black-Scholes model : Historical volatility $\hat{\sigma}$

  We estimate the historical volatility from returns of 20 days before the option trading day.

- The NIG model : $\hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\mu}$

  We estimate the parameters from returns of 1000 days before the option trading day.

The criterion to compare the pricing performance

To compare the performances of option pricing models, we compute the pricing errors between observed market prices and model prices by

1. Mean absolute error rate(MAER)

$$\frac{1}{M}\sum_{i=1}^{M}\left|\frac{\hat{C}_i - C_i}{C_i}\right| \tag{5}$$

2. Weighted mean absolute error rate

$$\sum_{i=1}^{M}\left|\frac{\hat{C}_i - C_i}{C_i}\right| w_i, \ \ w_i = \frac{V_i}{\sum_{i=1}^{M} V_i} \tag{6}$$

$\hat{C}_i \cdots$ model price, $C_i \cdots$ market price,
$V_i \cdots$ trading volume

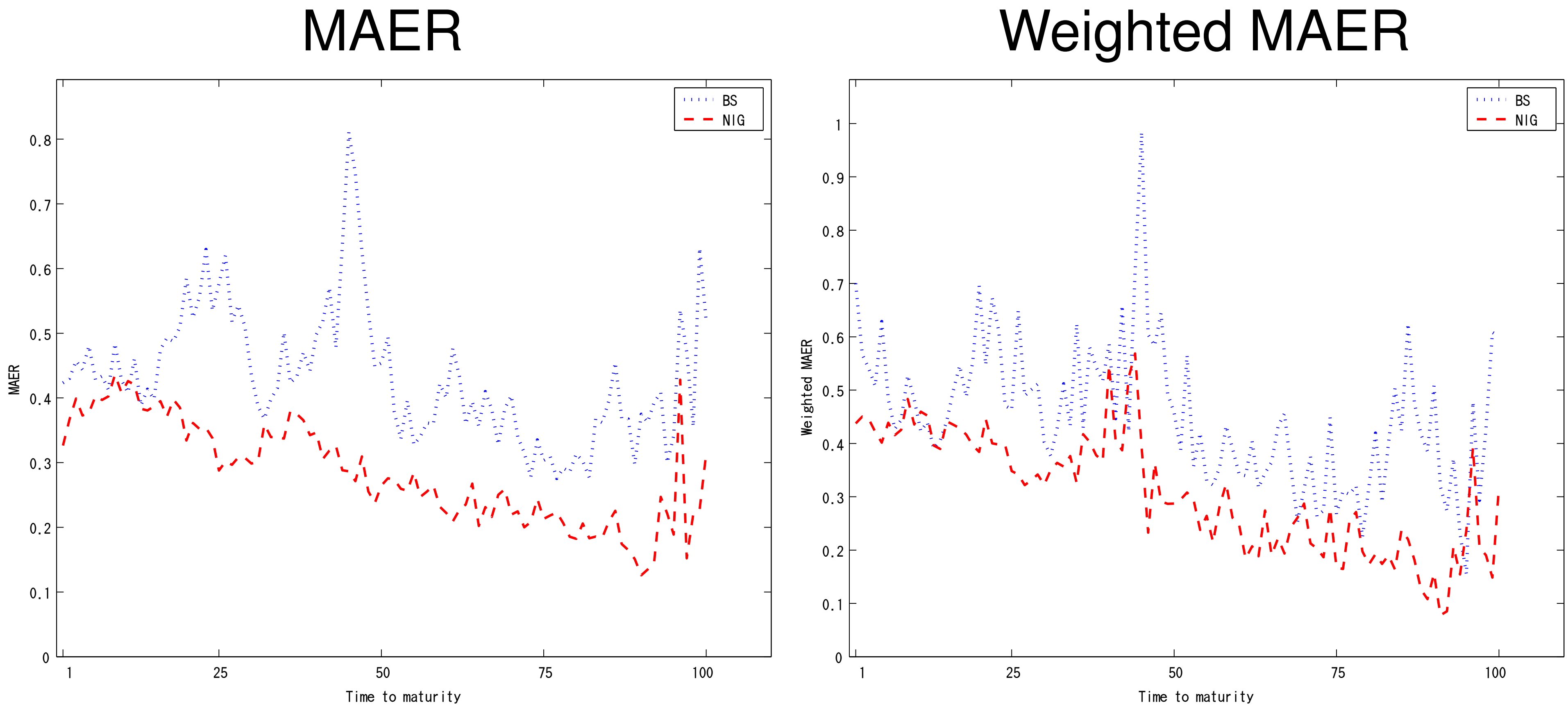
Table 2: MAER and Weighted MAER

Table 2: The pricing errors for BS and NIG models

	BS model	NIG model	sample size
MAER	**0.4625**	**0.3242**	**16063**
Weighted MAER	**0.5108**	**0.4084**	**16063**

- Next, we plot the pricing errors which are calculated from each classified option data according to the term to expiration.

Fig. 5: The pricing errors at each term to expiration



The pricing errors on each classification data by the same length of time to expiration

The classification by moneyness (S(0)/K)

To look at the pricing performance from a different aspect we classify option data into the following five categories by the size of moneyness S(0)/K:

$$\begin{aligned}
\mathbf{1:} &\quad & \boldsymbol{S(0)/K} &< \mathbf{0.91}, &&\text{Deep-out-of-the-money (DOTM)}\\
\mathbf{2:} &\quad \mathbf{0.91} \le & \boldsymbol{S(0)/K} &< \mathbf{0.97}, &&\text{Out-of-the-money (OTM)}\\
\mathbf{3:} &\quad \mathbf{0.97} \le & \boldsymbol{S(0)/K} &< \mathbf{1.03}, &&\text{At-the-money (ATM)}\\
\mathbf{4:} &\quad \mathbf{1.03} \le & \boldsymbol{S(0)/K} &< \mathbf{1.09}, &&\text{In-the-money (ITM)}\\
\mathbf{5:} &\quad & \mathbf{1.09} &\le \boldsymbol{S(0)/K}, &&\text{Deep-in-the-money (DITM)}
\end{aligned}$$

We use the size of the ratio according to Watanabe(2003).

Table 3: MAER in each category

Table 3: MAER

Moneyness	BS model	NIG model	Sample size
DOTM	**0.8243**	**0.5194**	**6203**
OTM	**0.3929**	**0.3337**	**3996**
ATM	**0.1836**	**0.1583**	**3220**
ITM	**0.0761**	**0.0675**	**1467**
DITM	**0.0390**	**0.0371**	**1177**

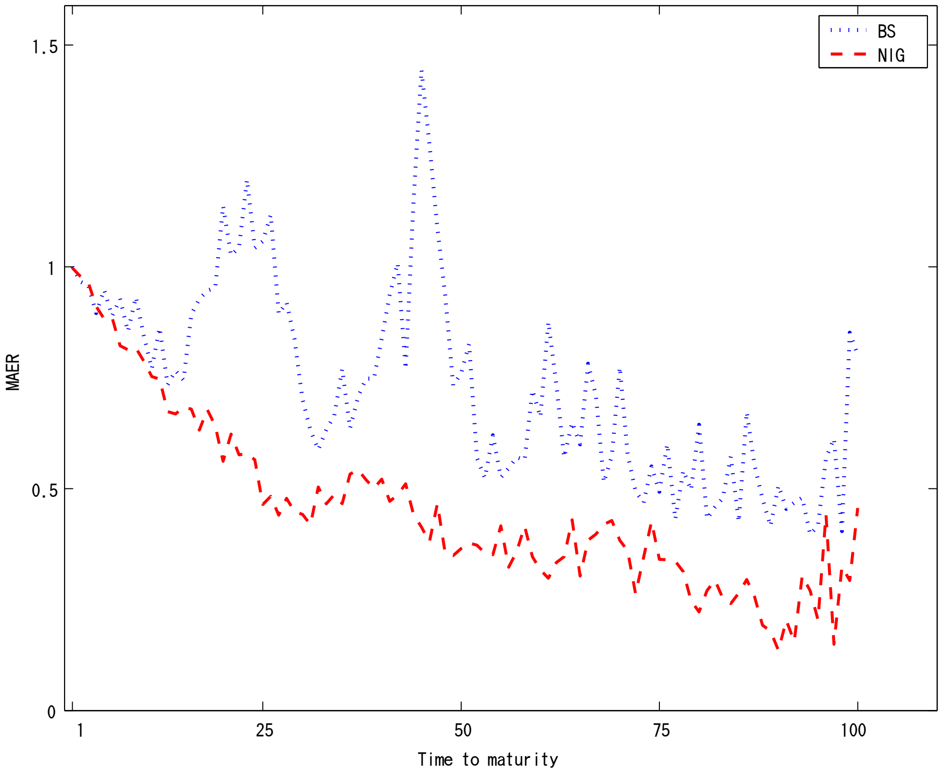
Table 4: Weighted MAER in each category

Table 4: Weighted MAER

Moneyness	BS model	NIG model	Sample size
DOTM	0.7947	0.5998	6203
OTM	0.4701	0.3871	3996
ATM	0.2444	0.2232	3220
ITM	0.0692	0.0681	1467
DITM	0.0409	0.0400	1177

Fig. 6: Pricing errors (DOTM)

MAER



Weighted MAER

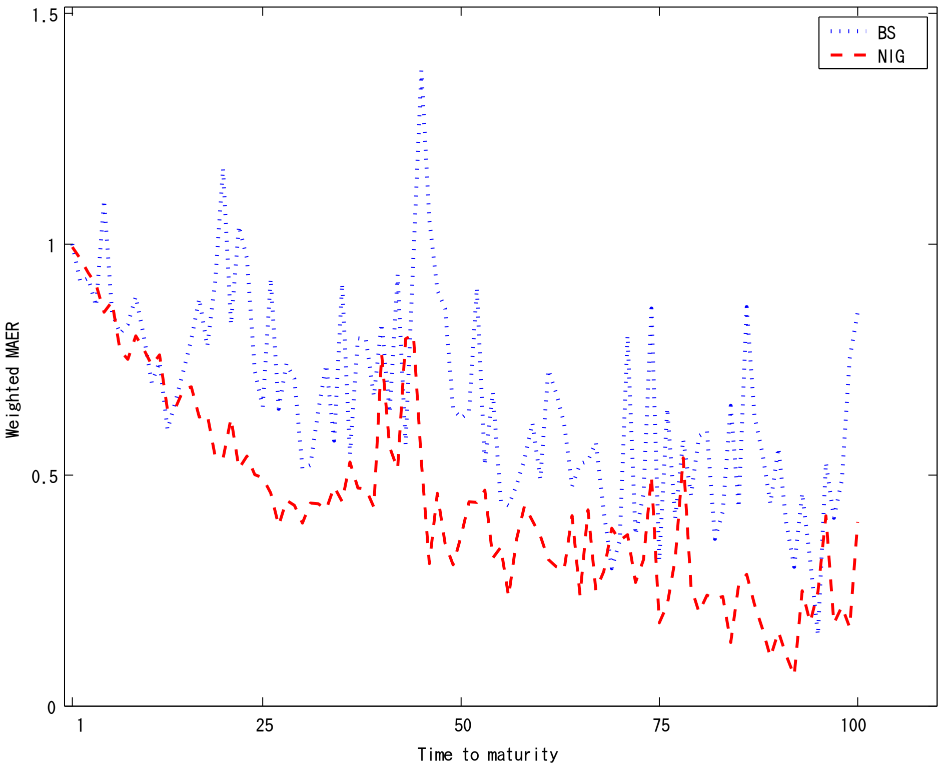
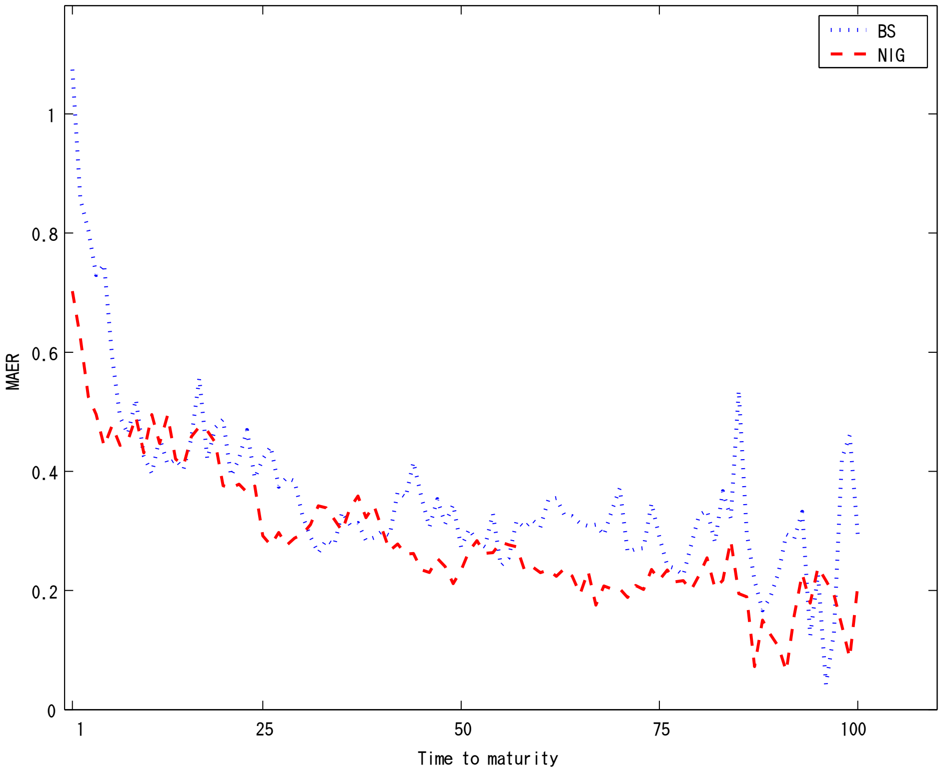


Fig. 7: Pricing errors (OTM)

MAER



Weighted MAER

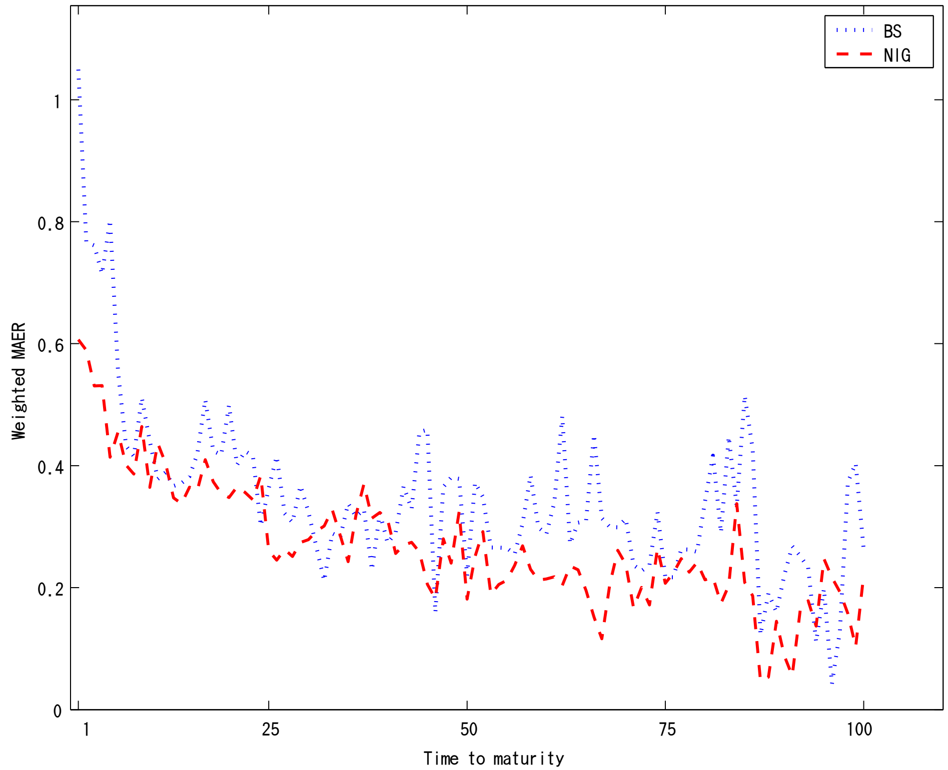
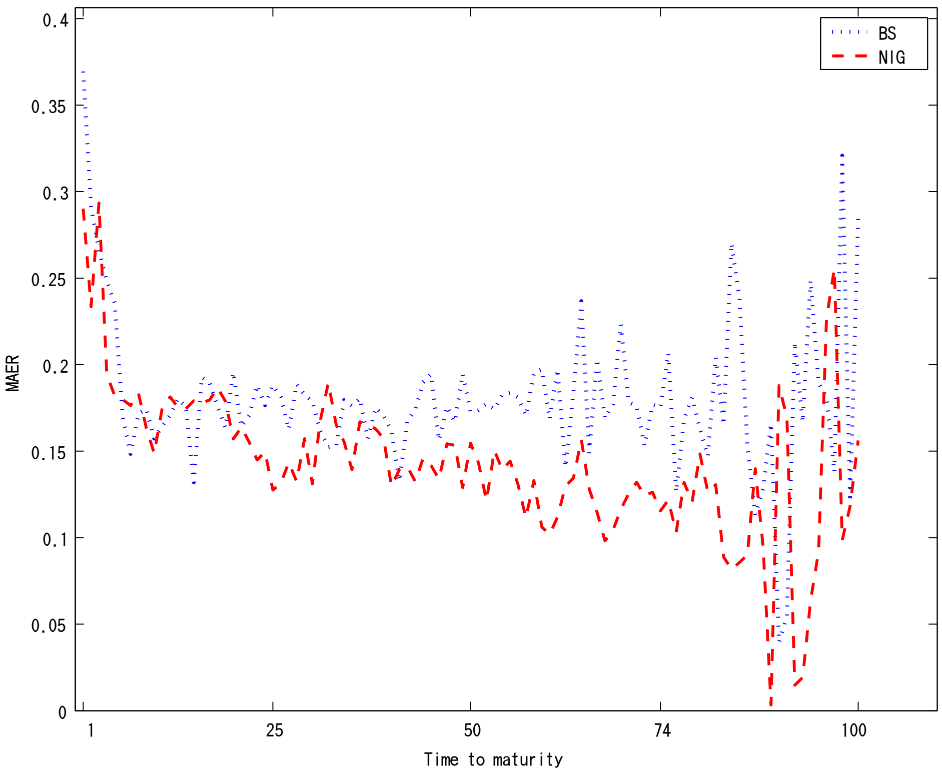


Fig. 8: Pricing errors (ATM)

MAER



Weighted MAER

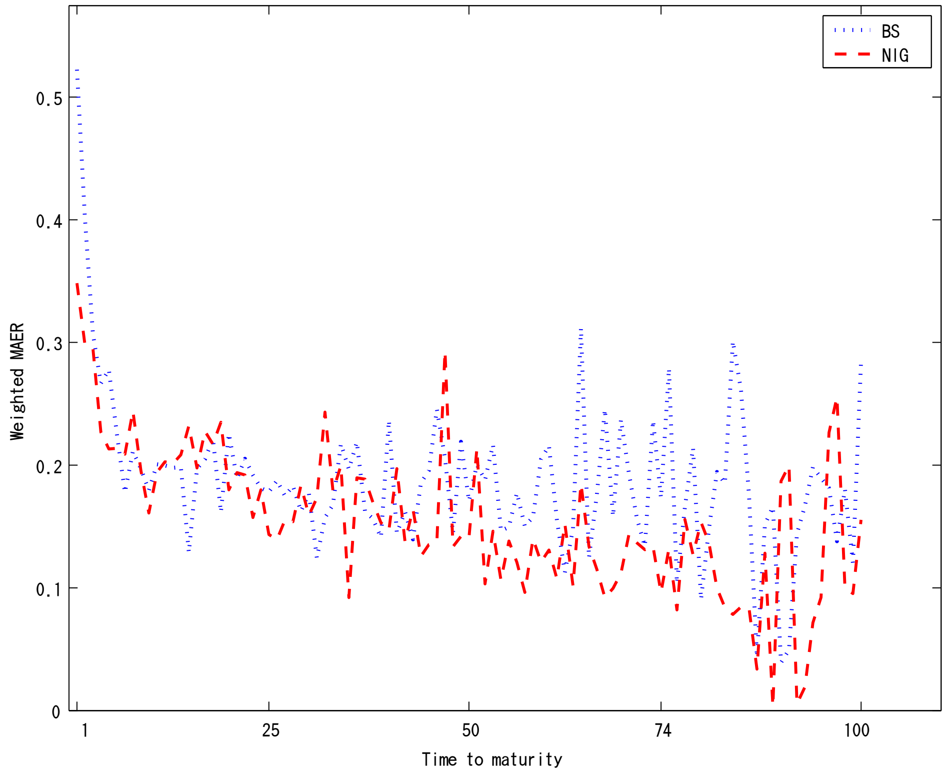
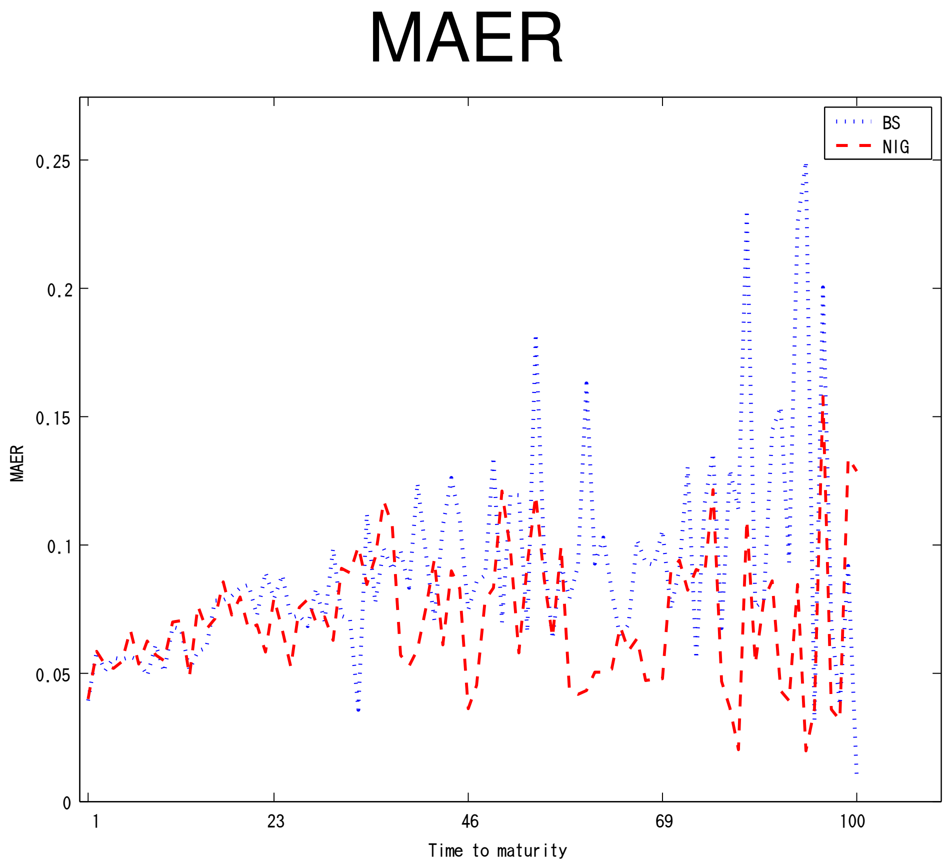


Fig. 9: Pricing errors (ITM)



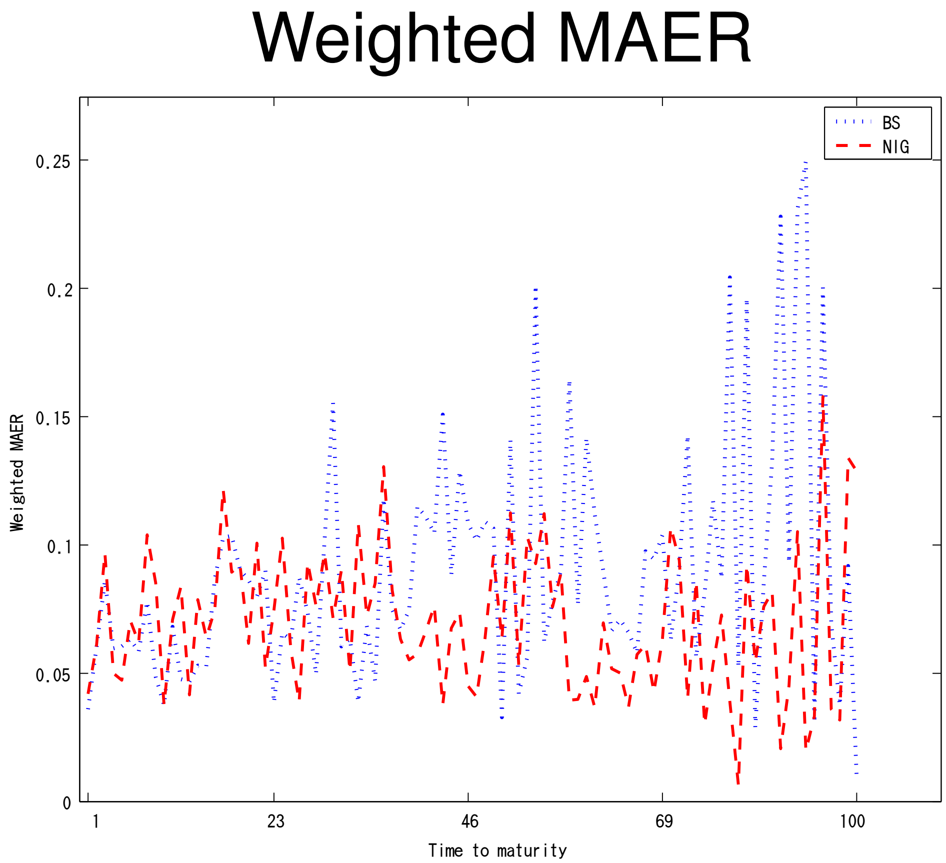
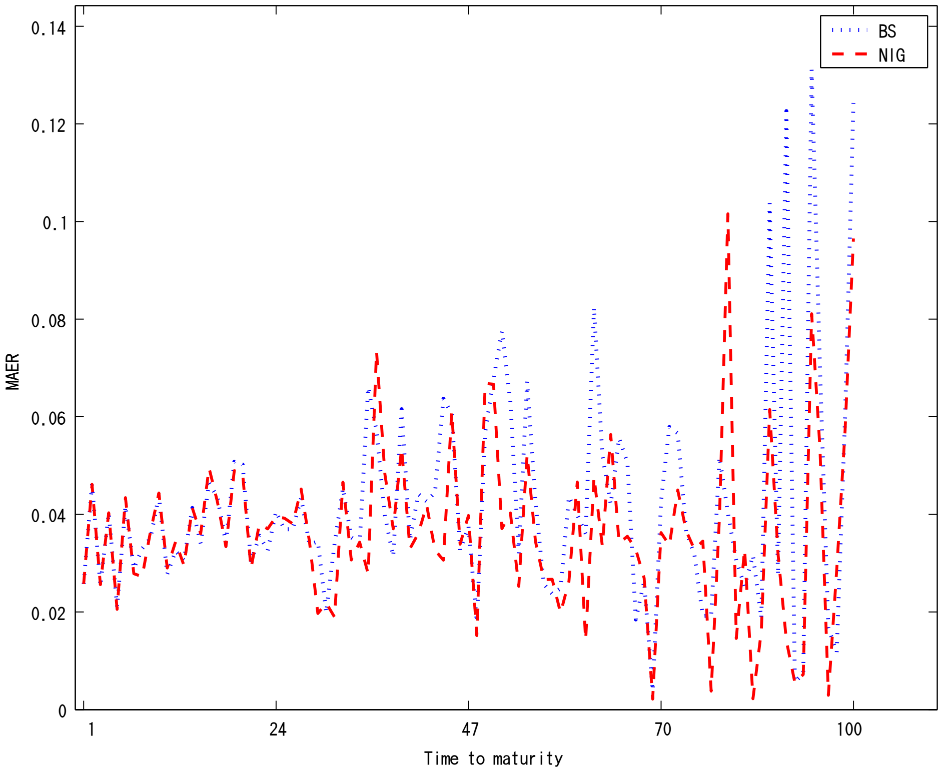
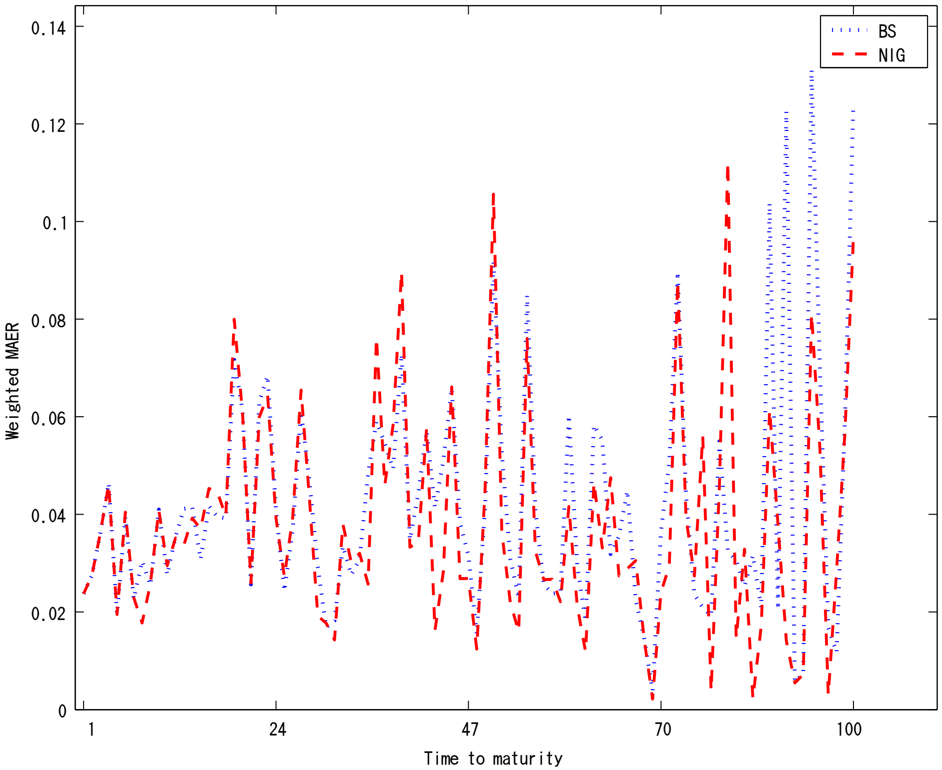


Fig. 10: Pricing errors (DITM)

MAER



Weighted MAER



Summary

- From the empirical evidence on Japanese market, the NIG distribution provides a better fit to the returns of the Nikkei 225 than the Normal distribution.

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  It is appropriate that we use the NIG distribution to describe the features of assets returns.

- As suggested by the results of pricing errors, the NIG model can capture the behavior of the market data more accurately than the Black-Scholes model.