Time Series Behaviour of Stock Trading Volume : An Evidence from Indian Stock Market*

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Abstract

In this paper, we have constructed trading volume series for the Indian stock market. The presence of long memory in Indian stock market volume data is tested using maximum likelihood method. The estimation of ARFIMA model, which is the most widely used model to study the long memory, obtained a significant parameter for the order of fractional integration, which could be consistent with the long autocorrelations observed in the trading volume series. The findings that stock trading volume is a long memory process is robust, given different estimating methods, different subsamples and temporal aggregation. Because of the conditional heteroscedasticity in the series, we carried out ARFIMA-GARCH procedures to check whether long persistence were robust in the presence of conditional heteroskedasticity.

Keywords: Trading volume, Detrending, Long memory process, ARFIMA, ARFIMA-GARCH, Periodogram regression

JEL Classification: C1, C22, G10

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1 Introduction

In this paper, we attempt to study the time series dynamics of the stock trading volume, or equivalently stock turnover using recently available data for individual stocks traded on the Bombay Stock Exchange (BSE) and the National Stock Exchange (NSE). Stock turnover has been studied intensively in finance literature because of its 1. use as an incomplete measure of liquidity, 2. its use as a proxy for information arrival and 3. its use as a proxy for heterogeneous belief among investors. Return on stock prices and trading volume are the two prime indicators of trading activity in a stock market. These factors are jointly determined by the same market dynamics and may contain valuable information about a security (Lo and Wang 2001). Yet the finance literature has centered more on prices and much less on quantities. In recent years the potential presence of stochastic long memory in financial variables such as stock return and volatility has been an important research topic, however the results in these studies are often quite conflicting across different tests and also are not robust to minor changess in testing methods. The presence of long memory in asset returns contradicts the weak form of the efficient market hypothesis **EMH**, which states that, conditioning on past returns, future returns are unpredictable.

Another market property that seems to have long-memory properties is stock market trading volume. Stock trading volume (also referred to as turnover ratio) is treated as nonstationary (Gallant, Rossi and Tauchen (1992), Anderson (1996)) and analyzing its long memory parameters is often complicated (Lobato and Velasco, 2000). Lobato and Velasco have examined the long memory properties of trading volume and volatility for the 30 stockes in Dow Jones Industrial index. They found strong evidence of long memory in stock market trading volume and volatility for most of the stocks. The long memory analysis was carried out in the frequency domain by tapering the data. Bollerslev and Jubinski, 1999 analyzed the long memory properties of stock trading volume by linearly detrending the data. They found that fractionally integrated process best describes the long run temporal dependencies in both the volatility and trading volume for individual firms composing the Standards and Poor's 100 composite index. Gang Ma, 2003 has analyzed the daily turnover of the stocks listed in Dow Jones Industrial index by decomposing the stock turnover into a nonlinear deterministic trend and a random error term that is highly persistent. He found that the most significant componen t of the error term is long memory process and the short memory process is surprisingly insignificant.

Long memory processes have been observed in natural phenomena volume data also. Hurst (1951), for example found the prescence of strong persistence in yearly average discharge of Nile river and later on Mandelbrot and Walter (19960 confirmed this finding by employing long memory process. Leland et. al (1994) analyzed the volume of the network traffic and found that the network traffic volume can be explained by long memory process. Our work focuses on Indian stock market turnover series instead of the individual stocks. In this paper, first we have constructed trading volume(turnover) series for the Indian stock market repersented by BSE and NSE. We expect to find long memory in market turnover series because of the fact that long memory can result due to aggregation of certain types of short memory series (see, for example, Granger, 1980).The most widely specification of the long-memory models is the one of ARFIMA (Autoregressive Fractionally Integrated Moving Average) models and in this paper, parametric MLE is employed to estimate ARFIMA models for the Indian stock trading volume. We also carried out robustness tests by employing different methods of estimating long memory parameter, different time aggregation and different subsamples. To account for the temporal dependence of conditional variances of the Indian stock trading volume we have aslo estimated ARFIMA-GARCH (Autoregressive Fractionally Integrated Moving Average-General Autoregressive Conditional Heteroskedasticity) models, see Baillie et al.(1996).

1.1 Indian Stock market and Trading Volume

A large body of literature has documented the behavior of trading volumes in the US stock markets. By contrast, relatively little attention has been devoted to trading volume in the Indian stock markets. Our effort is to fill this gap by first analyzing the time series dynamics of the trading volume in two distinct markets : the BSE and the NSE. To unfold the issues related to trading volume in the Indian stock market, it is appropriate to first review the Indian stock market. Over the last two decades, Indian stock markets have witnessed significant changes in terms of trading environment. The screen based trading introduced has made the price discovery process more efficient. Dematerialization of shares and setting up clearing houses has virtually eliminated the risks involved in trading. Similarly rapid strides were made in settlement procedures, corporate governance standards, introduction of derivative products etc. These reforms have increased the participation of Foreign Institutional Investors (FIIs) and other institutional investors in Indian stock market thus widening the investor base and increasing the turn over, market capitalization of the stock exchanges. The impact of all these reform measures reflects clearly on the continuous improvement found in barometers of stock market development such as the number of listed companies, market capitalization, turnover, liquidity etc. Table 1 presents the trends of selected indicators in the Indian Stock Market.

In 2001, two Indian Stock Exchanges, National Stock Exchange (NSE) and Bombay Stock Exchange (BSE) ranked third and sixth among exchanges all over the world, sorted by the number of transactions. The market capitalization grew rapidly between 1990/91 and 1999/2000. The trading volumes on stock exchanges have been undergoing large growth during the 1990s. The average daily turnover grew from about Rs.150 crores in 1990 to Rs.1200 crores in 2000, peaking at over Rs.20,000 crore (Indian Securities Market, 2002). Moreover, the relative importance of various stock exchanges has undergone change during this decade. The turnover increased at the big exchanges and the small exchanges failed to keep pace with the change. Eventually NSE emerged as the market leader with over 80% total turnover in 2001-02. It would be interesting to assess the stock market developments in terms of these changes that have made impact on frequency and depth of trading. It may be agreed that unless stock trading is widespread and deep as signified by higher volumes, the beneficial impact of well functioning capital market would remain limited to a few scrip where trading is frequent and deep. This would also result in equity markets wherein a small number of scrips attracting large trading interest while large number of scrips show low and infrequent trading volumes. Thus biases due to market thiness and nonsynchronous trading are prevalent to Indian stock market.

In contrast to US and other developed stock markets which are highly efficient in terms of informations, investors in Indian stock market seems to react slowly to new information, thereby motivating our study to analyze the evidence of long-memory in the Indian stock trading volume and this effort is likely to have important implications on the role of trading volume in Indian financial markets. Though the long memory behavior of Indian stock return (see, for example, Ashok Razdan, 2002) has been studied, our effort is the first one to analyze the long memory hypothesis for Indian stock trading volume. We test for the prescence of long-memory in carefully construted stock market turnover series for the two major Indian stock markets: the BSE and the NSE.

The plan of the paper is as follows. Section 2 presents the empirical methodologies (ARFIMA and ARFIMA-GARCH model). The primary objective of this section is to investigate if the volume behaviour in Indian stock market can be characterized by long memory models. In third section we define appropriate measures of trading activity for individual securities and for portfolios and also the time series behaviour of the Indian stock trading volume. In the this section we also present the source of the data for our empirical analysis. Section 4 illustrates the ARFIMA and ARFIMA-GARCH estimation results and also the different tests of robustness. Concluding remarks are presented in the last section.

2 Empirical Methodologies

2.1 ARFIMA Model

The class of models known as Long Memory has recently received great attention in time series models. Roughly speaking, a random process is called a long-memory process if it has an autocorrelation function that is not integrable. The most widely used specification of these models is the one of autoregressive models with fractional order of integration developed by Granger and Joyeux(1979) and Hosking(1981), known as ARFIMA(p,d,q) which can be expressed as:

$$\Phi(L)(1-L)^d y_t = \Theta(L)u_t, \qquad u_t \sim i.i.d(0,\sigma_u^2) \tag{1}$$

where L is the backward-shift operator, $\Phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$, $\Theta(L) = 1 + \theta_1 L + \cdots + \theta_q L^q$ and $(1-L)^d$ is the fractional differencing operator defined by

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)} \tag{2}$$

with $\Gamma(.)$ denoting the gamma function. We assume that the roots of the AR and MA polynomials lie outside the unit circle and that they dont have common roots. Process(1) may possess a long memory such that a shock at time t has a long-lasting influence on the future value of the time series compared with the case of a stationary ARIMA process. The parameter d is allowed to assume any real value and the restriction of d to integer values give rise to standard ARIMA(Autoregressive Integrated Moving Average) model. The stochastic process y_t is both stationary and invertible if all the roots of $\Phi(L)$and $\Theta(L)$lie outside the unit circle and $\mid d \mid \leq 0.5$. The process is nonstationary for $d \geq 0.5$, as it possesses infinite variance. Assuming that $d \in (0, 0.5)$ and $d \neq 0$, Hosking(1981) showed that the correlation function, $\rho(.)$, of an ARFIMA process is proportional to k^{2d-1} as $k \to \infty$. Consequently, the autocorrelations of the ARFIMA process decay hyperbolically to zero as $k \to \infty$ with speed dependent on the size of d(Granger and Joyeux,1979, Hosking 1981). This is in contrary to the case of a stationay ARIMA (i.e., d=0 and y_t is white noise) whose autocorrelation function exponentially converges to zero. For $d \in (0, 0.5)$, $\sum_{k=-n}^{n} \mid \rho(k) \mid$ diverges as $n \to \infty$, and the ARFIMA process is said to exhibit long memory, or long range dependence and autocorrelation functions are positve and decay monotonically and hyperbolically to zero. The process exhibit intermediate memory, or long-range negative dependence for $d \in (-0.5, 0)$ and autocorrelation functions are all negative except $\rho_0 = 1$ and decay monotically and hyperbolically to zero. However, since both the processes with $d \in (0, 0.5)$ and $d \in (-0.5, 0)$ exhibit slower convergence than the stationary ARMA case, we call these series long-range persistent which is consistent with the terminology of Campbell, Lo Mackinlay(1977). For $d \in [0.5, 1)$ the process is mean reverting, no longer covariance stationary, and have infinite variance (see Baille, 1996).[1]

There exists a number of methods to test for long memory. We have estimated ARFIMA models by the Sowell's (1992a) exact maximum likelihood estimation method. This procedure allows for the simultaneous estimation of both the long memory parameter and ARMA parameters. Assuming normality of the error term, the loglikelihood funtion for the sample of T observations is given by

$$L(\gamma; Y_t) = -\frac{t}{2}\ln(2\pi) - \frac{1}{2}\ln \mid \Sigma_t \mid - \frac{1}{2}(Y_t' \Sigma_t^{-1} Y_t) \tag{3}$$

The ML estimator is obtained by maximizing (3) with respect to the parameter vector $\gamma = (\Phi, \Theta, d)$ and is consistent and asymptotically normal. Empirical studies have often estimated the size of d using a semi-parametric approach in the frequency domain (Geweke and Porter-Hudak(GPH) 1983, and Robinson

[1]Many authors refer to a process as long memory process for all d$\neq$ 0

1994). Although, the semi-parameteric estimator of GPH is potentially robust to non-normality, the estimates are adversely biased by the prescence of auto-correlation and Robinson's approach suffers from the drawback of discontinuity asymptotically(see Baillie 1996). Hence the maximum likelihood method is employed. We select the parsimonious ARFIMA model based on Schawrz Information Criteria(AIC).

2.2 ARFIMA-GARCH Model

To complete the examination of long memory in trading volume, our next step is to model their stochastic volatility along with fractional integration. The changing variance indicating conditional heteroskedasticity of volume data have motivated us to extend our model to ARFIMA-GARCH model. ARFIMA-GARCH model allows conditional heteroskedicity (Fung et al 1994), which could be the explanation of nonperiodic cycles thereby making an ARFIMA-GARCH model more flexible than a GARCH model in terms of capturing irregular behaviour. Hence, we try to verify whether conditional heteroskedasticity could induce long term persistence. The joint estimation allows us to efficiently test with the control of heteroskedasticity effect, if the order of integration will still be significant. Furthermore, previous experience of GARCH modelling is utilized (see Vougas, 2000) and a parsimonious GARCH(1,1) model is employed inorder to anlayze the existence of possible long-range dependence arising due to conditional heteroskedasticity. For this, u_t(the error in equation 1) is specified to have conditional variance ,σ_t^2, given by

$$\sigma_t^2 = \omega + \alpha_1 u_(t-1)^2 + \beta_1 \sigma_(t-1)^2 \tag{4}$$

and

$$u_t = \theta^{-1}(L)\phi(L)(1-d)^d y_t \tag{5}$$

Maximum Likelihood estimation is based on maximising $n^{-1}\sum_{t=1}^{n} l_t$ with

$$l_t = -\frac{1}{2}\ln\sigma_t^2 - \frac{1}{2}(\frac{u_t}{\sigma_t})^2 \tag{6}$$

3 Definitions, Construction and Time Series Behaviour of Turnover Series

3.1 Definitions and Construction of Turnover Series

There are numerous ways to measure trading volume. Some studies use the total number of shares traded per period as a measure of volume while other study use individual and aggregate turnover as a measure of trading volume (see Smidt (1990), Lebaron (1992), Campbell, Grossman and Wang (1993), Stickel and Verrechia (1994)). Also the total numbers of trades (Conrad, Hameed and Niden (1994)) and the number of trading days per year (James and Edmister (1983)) have been used as measures of trading activity. Lo and Wang, 2000(hereafter LW 2000) conclude that various measures of trading activity such as share turnover(share volume divided by outstanding shares), dollar turnover(dollar traded volume divided by market capitalization), equal-weighted turnover, value-weighted turnover and share-weighted turnover for a two-asset portfolio are identical . In our analysis, we will focus on turnover as a measure of trading activity throughout this paper. Since the number of shares outstanding and the number of shares traded have both grown steadily(see Table 1) over the period of our study, the use of turnover helps to reduce the low-frequency variation in the series (Campbell, Lo and Wang, 1993). Use of the number shares traded directly would require controlling for events such as stock splits, right issues and stock dividends. As the occurrence of such events increase the number of outstanding shares, without adjustments, trading volume will become noncomparable before and after the event occurrence (Saatcioglu k. and Starks T. Laura, 1998).

Let us consider an economy defined on a set of discrete dates: $t = 0 \cdots T$. Let us also assume that there are J risky assets in the economy, which we call stocks and that there are I investors indexed by $i = 1 \cdots I$ in the economy. Each stock pays a stream of dividends over time.Let D_{jt} denote the dividend of stock j at date t, $j = 1 \cdots J$. We define the total number of shares outstanding as N_{jt} and P_{jt} as the ex-dividend price at date t for each stock. For each investor i, let S^i_{jt} denote the number of shares of stock j he holds at date t. Let $\mathbf{P_t} = [P_{1t} \cdots P_{jt}]$ and $\mathbf{S_t} = [S_{1t} \cdots S_{jt}]$ denote the vector of stock prices and shares held in a given portfolio. Finally, let X_{jt} be the total number of shares of security j traded at time t or in other words share volume which is given as

$$X_{jt} = \frac{1}{2} \mid S^i_{jt} - S^i_{jt-1} \mid$$

where the coefficient corrects for the double counting when summing the shares traded over all investors. Without loss of generality, we assume that the total number of shares outstanding is one for each stock, that is, $N_{jt} = N_j, j = 1 \cdots \jmath$. For our analysis in the subsequent sections, we will use the following definitions :

Individual turnover : Let X_{jt} is the share volume of security j at date t and N_j is the total number of shares outstanding of stock j. Then individual turnover τo_{jt} of stock j at date t is defined as :

$$\tau \mathbf{o_j} = \frac{X_{jt}}{N_j}$$

Since investors trade portfolio or baskets of stocks, we now propose a measure of portfolio based trading volume.

Portfolio Turnover: Consider any portfolio P defined by the vector of shares held $S^p_t = [S_1 t^p \cdots S^p_{jt}]$ with non negativity holdings in all stocks ,that is ,$S^p_{jt} \geq 0$ for all j, and strictly positive market value ,that is, $S^{\acute{p}}_t P_t \geq 0$,let $\omega^p_{jt} = \frac{S^p_{jt} P_{jt}}{S^p_t P_t}$ be the fraction invested in stock j,$j = 1 \cdots \jmath$. Then the **portfolio turnover** is defined as:

$$\tau o^p_t = \sum_{j=1}^{j} \omega^p_{jt} \tau o_{jt}$$

Using this definition, we can also construct value-weighted and equal-weighted indexes as :

$$\tau o^{Vw}_t = \sum_{j=1}^{j} \omega^{Vw}_{jt} \tau o_{jt}$$

and

$$\tau o^{Ew}_t = \sum_{j=1}^{j} \omega^{Ew}_{jt} \tau o_{jt}$$

Since we have chosen turnover as a measure of trading volume for individual securities, we sum turnover across dates to obtain time-aggregated turnover (see LW 2000). If the turnover for stock j at time t is given by τo_{jt} , the time aggregated turnover between t-1 to t+q ,for any q$\geq$ 0 is given by

$$\tau o_{jt}(q) = \tau o_{jt} + \tau o_{jt+1} + \cdots \tau o_{jt+q}$$

3.2 Time-Series Behaviour of Trading Volume Data

Having defined turnover as the measure of trading acitivity, we use the PROWESS Database to construct daily turnover series for inidividual BSE and NSE stocks. As in LW 2000, we confine our analysis to daily turnovers and weekly turnovers. We aggregate the daily turnovers using the time aggregation procedure to form weekly turnovers. Thus our weekly turnover is the sum of five consecutive daily turnovers starting when the market is open. However for our long-memory analysis we have used the daily turnover series for the whole sample period. Instead of focusing on the behavior of the time series of individual stocks' volume data set, we focus on value weighted and equal-weighted turnover indexes as defined in the previous section. The portfolio turnover is the weighted average of the individual stock turnovers for the stocks that comprise the portfolio. We have classified the period of study into various sub-periods and carried out the empirical analysis on each of the sub-periods also.

3.2.1 Trends

Figure 1 and 3 graphically display the time series of equally-weighted and value weighted turnover for our BSE and NSE portfolio respectively. As documented in LW 2000 and in many other studies, aggregate turnover series seems to be non-stationary, exhibiting a significant time trend and time-varying volatilities. The value weighted turnover has increased dramatically since the mid 1990s through mid 2001, with a drop-off following the policy change related to settlement, followed by a slow increase again after mid 2001. The growth from 1995 through mid 2001 may be partly due to technological innovations which have lowered transaction costs. The year 2001-02 started in the backdrop of market turbulence. The volumes declined in the first quarter of 2001-02 following decisions affecting several structural changes in the market that included a shift to rolling settlement (initially in respect of major securities on T+5 basis in July 2001, later for all securities) and withdrawal of deferred products. Such changes are usually accompanied by fall in volume initially (Indian Securities Market, 2002). However as the market gains experience, the trading volume are expected to return to their normal level which can be seen from the figure. Equal weighted turnover behaves somewhat differently, in comparision to value weighted series, the equal weighted series have not grown so dramatically suggesting that smaller-capitalization companies can have high turnover.

In addition, throughout the sample period the variance of turnover seems to increase with its level. To give a more visual information we have also measured turnovers in logs rather than absolute units (see figure 2 and 4) since taking log helps to reduce variance of the turnover series and also it removes the low-frequency variations from variance. Table 2, 3 and 6, 7 reports various summary statistics for the two series over the sample period as well as the subperiods for BSE and NSE portfolio respectively. Over the entire sample the average daily turnover for the equal and value weighted indexes is 0.00099 and 0.00299 for the BSE and 0.00191 and 0.00461 for the NSE. From the tables it can be seen that the coefficients of variations for the turnover series are less than the coefficients of variation for the return series, implying that turnover is not so variable as returns, relative to means. As can be seen from the table 2 & 6, the empirical distributions of the turnover are not normal. The tables also reports the percentiles of the empirical distributions of turnovers and returns documenting that both turnovers and returns are not normal. This is also supported by the Jaque-Bera test of goodness of fit to a normal distirbution.

3.2.2 Nonstationarity and Detrending

Table 2 and 6 report the first 12 autocorrelations of turnover and returns and the coressponding Ljung-Box Q-statistics. Unlike returns, turnover is highly persistent, with autocorrelations decaying very slowly. This slow decay suggest some kind of nonstationarity in turnover and this is confirmed by applying the Augmented Dickey Fuller test of nonstationarity to the two turnover series. Indeed LW2000 conducted the unit root tests and found a unit root in the stock turnover series, which is consistent with our findings here. As can be inferred from the comparision of Ljung-Box Q-statistics, the autocorrelation in turnover is substantially higher than in the return which is consistent with the predictions of the bivariate mixture models (see Harris, 1987).

As in the case of many empirical studies involving volume, we also use some kind of detrending methods. We have used the detrending methods like linear, log-linear, first differencing, seasonal deseasonalization[2] (in the spirit of Gallant, Rossi and Tauchen (hereafter GRT), 1994) and kernel regression to induce stationarity and examined the behavior of these series after detrending them by each of the above-stated procedures (see Table 4, 5, 8 and 9). Linear, log-linear, GRT detrending and kernel regression seem to do little to eliminate the persistence in autocorrelations. However, first-differencing and moving average methods seem to eliminate the persistance of autocorrelations. As in LW2000 we found that the residuals from the first difference have high negative autocorrelations. Similar anlaysis has been done with the weekly data and the results are available from the author on request. While analyzing the volume data, it was found that the tails were a bit fatter, and more significantly peak around the mean was higher than predicted by the normal distribution. The presence of fatter tails indicate 'memory' effects which arise due to nonlinear stochastic processes. Actually the information flow to an investor is clustered and its arrival is irregular rather than continuous and smooth in nature. This clustered and/or irregular arrival of 'new' information results in periods of low and high volatility (see Mandelbrot, 1997) which results in 'leptokurtic' distribution instead of normal. This brings in a new view in which reaction of investor or trader to new information is 'nonlinear'. To investigate the validity of this new view the concepts of ARFIMA and ARFIMA-GARCH have been used in the next section to include memory effects .

3.3 Data

In this paper, we have used the data from the Indian Stock market. The data used in the study are based on time series of daily trades data for individual stocks listed in the Bombay Stock Exchange **(BSE)** during the period 1995-2003 and the National Stock Exchange **(NSE)** during the period 1996-2003. We select the stocks based on the number of trading days.The data for the individual stocks listed in BSE and NSE are collected from PROWESS database provided by the 'Center for Monitoring Indian Economy' (CMIE). To study the time-series properties of turnover indexes we compute daily return and turnover series. For our analysis we have taken only those stocks which have traded atleast 75% of the total trading days. Thus we got 591 stocks for the anlaysis of BSE and 656 stocks for the analysis of NSE. We have also constructed the weekly turnover series for the robust analysis. In total we get 2077 and 1892 daily data points for the BSE and NSE respectively

[2]the series is detrended and deseasonalized by regressing the raw series on trend, day of the week dummies and monthly dummies.

and 443 weekly data points for the BSE and 396 weekly data points for the NSE . To account for the changes relating to clearing and settlements of trades in the markets, we have classified the period of study into three subperiods :

- 01/01/1995 to 31/12/1999 refers to the period before the introduction of rolling settlement.
- 01/01/2000 to 01/07/2001 refers to the period when rolling settlement was introduced in phased manner, and
- 02/07/2001 to 29/07/2003 refers to the period when rolling settlement was made compulsory.

For our long-memory analysis we have restricted ourselves to the whole sample period. However, we have focussed on different subperiods also to the check the robustness of our results.

4 Estimation

4.1 Estimation ARFIMA model

A possible explaination of for the failure to reject the unit-root hypothesis in stock trading volume series is due to the restrictiveness of conventional unit-root tests regarding low-frequency dynamic behavior. The conventional unit root tests have no power to distinguish a long memory process with a unit root process because of their inability to capture an order of integration that may not be an integer (Baillie, 1996). Diebold & Rudebush(1991) and Hassler & Wolter(1994) find that ADF tests tend to have low power against the alternative hypothesis of fractional integration. The distinction between I(0) and I(1) processes seems to be far too restrictive as the generation of shocks in an I(0) process occurs at an exponential rate of decay (so that it only captures the short-memory), while for an I(1) process the persistence of shocks is infinite. Hence, in the conditional mean, the ARFIMA specification has been proposed to fill the gap between short and complete persistence, so that the short-run behavior of the time-series is captured by the ARMA parameters, while the fractional differencing parameter allows for modelling the long-run dependence. Lee and Schmidt(1996) show that KPSS test can be used to distinguish short memory and long memory process.

To consider the possibility that the stock trading volume series need not be exactly I(0) or I(1) processes, but they may be integrated of order d, $d \in (0, 1)$ (in which case they will exhibit the long-memory property), henc as our next step we tried to examine the consistent estimation of the long-memory parameter 'd' of the turnover series. Given the evidence of long memory shown by the measurements such as the R/S statistics by Lo, KPSS statistics and Robinson's nonparametric estimation of the order of integration(Table 10), we use robust Sowell's Exact Maximum Likelihood ARFIMA [3]procedures, and estimate a series of specification of ARFIMA(p,d,q) model. The main advantages of this method are that it avoids the small sample bias and arbitrariness of the cut-off parameters of Robinson'smethod and also allows us to control for short memory effects. We have used Schwarz Criteria (SIC) to choose the order of the ARFIMA model and set the maximum number of orders for both AR and MA as 3. The ARFIMA estimators for different orders of the ARMA parameters vary greatly and also vary for BSE and NSE turnover series. Table 11 present the best ARFIMA models for the turnover series. Only ARMA orders selected by SIC are reported here. The d parameter was found to be significant in all the selected models. The ML evidence from the ARFIMA(p,d,q)strongly supports the presence of long-memory in the Indian stock trading volume. It is interesting to note that the trend coefficient was found to be insignificant in case of value weighted turnover series for both BSE and NSE and hence we have excluded the trend term from the analysis of the value-weighted turnover series.

We have also estimated the fractional-integration model using the logarithmic differences of the stock trading volume series in order to ensure that stationarity and invertibility conditions are met. However, the fractional differencing parameters for the differenced turnover series are negative (ranging from -0.25 for BSE to -0.31 for NSE), but are within the range of $d \in (-0.5, 0.5)$. This is not surprising as we have already seen that residuals from the first differencing (see table 4, 5, 8 and 9) have high negative autocorrelations. Hence we conclude that the Indian turnover series is a long-memory series.

[3]The ARFIMA model has been estimated by using ARFIMA package by Marium Ooms and Jurgen Doornik for PcGive.

4.2 Estimation ARFIMA-GARCH model

This subsection considers the possibility that stock trading volume is a long-memory with time dependent heteroscedasticity. This model is used to analyze the relationships between the conditional mean and the variance of a process exhibiting long memory and slow deacy in its level and time-varying volatility. The slow rate of decay of the autocorrelations of trading volume and also the volatile nature of Indian stock trading volume motivates the construction of models with long memory in conditional heteroscedasticity. Table 14 presents results from the estimated ARFIMA-GARCH [4] model for the differenced turnover series. The best model selected by the SIC has been presented in the table. We also observe that the order of integration parameter is significantly larger than the one estimated in the ARFIMA model without the GARCH component and both the GARCH componenets are significant thus providing the evidence that long persistence could be generated by change in the series variance.

4.3 Robust tests

The findings in section 3 suggest that stock turnover is a long memory process. Is that finding robust? We have conducted three tests of robustness.

4.3.1 Robustness test on different estimation method

The fully efficient estimation of any long memory ARFIMA model requires that the model be correctlly specified. Moreover, different criteria may favor different models (Schmidt and Tschernig, 1993). There is a lack of consensus on the most appropriate ARFIMA estimation technique by the MLE method (due to poor performance at high orders or levels of ARMA dynamics, and at low number of observations). As noted by Hauser, Potscher, and Reschenhofer (1999), ARFIMA models are inappropriate for measuring persistence since their cumulative impulse response function is either infinite or zero depending on the parameterization. Hence, we are compelled to utilize two estimation techniques to check the robustness of our results. These include the following semiparametric methods, namely the log periodogram regression of Geweke-Porter-Hudak (henc eforth GPH, 1983) and the Gaussian semiparametric estimation described in Robinson-Henry (GSP,

[4] We have used G@RCH 3.0 by Sebastien Laurent and Jeans-Philippe Peters for OX lanuage for estimating ARFIMA-GARCH models.

1999). Since the estimation are sensitive to the choice of the number of periodogram ordinates, we evaluated d estimates for different values of μ ranging from $T^{0.5}$ to $\frac{T}{2}$(which is the maximum usable value). The inference (see table 13 and 14) drawn regarding the low-frequency dynamics of stock trading volume remains unaltered when these estimates of the fractional differencing parameter are considered. Although the quantitative results vary slightly for different values of μ, the qualitative results do not vary. The analysis shows that trading volume series are best described by mean reverting long-memory-type process which is not surprising since Bollerslev and Jubinski, 1999 also found that daily trading volume for the majority of individual companies in the Standards and Poor's 100 composite index are best described by mean reverting long-memory type process.

4.3.2 Robustness test on Weekly turnover

This test is based on the following facts : if the underlying series is a long memory process, then the series obtianed by time aggregation of that series is still a long memory process [5]. If the underlying generation processes are a short memory process with breaks, their aggregates are either a short memory process or a unit root process. Hence, by comparing the long memory parameters across different time scales we are able to distinguish a true long memory process from the spurios long memory process. Table 16 shows the estimated memory parameter for the weekly stock turnover series. All the memory parameters were significantly different from zero and were within the range of d $\in (-0.5, 0.5)$. This test of robustness strongly supports that stock turnover series is a long memory process.

4.3.3 Robustness test on different subsamples.

This test illustrates the estimations of memory parameters for turnover series for different subsamples. This test has been carried out to determine whether the long memory parameter is stable in different subsamples since structural breaks may result in spurious long memory process. For our analysis we have divided the whole sample into three subsamples which have been dicussed earlier. We apply the ARFIMA model to turnover series in each subsamples to check whether the results are robusts. Table 17 illustrates the estimations of memory parameters

[5]See Taqqu (1975) and Beran and Ocker (2000)

for tuenover series in different subsamples periods. All the memory parameters were statistically different from zero, thereby supporting our evidence that stock turnover is a long memory process.

5 Conclusion

In this study we investigted the nature of persistence in Indian stock trading volume. We first constructed stock market turnover series for the two major Indian stock markets. Unlike return series, we find market turnover series is is nonstationary exhibiting slow decaying autocorrelations. We then used six detrending methods to induce stationarity and examined the behavior of these series after detrending them by each of the six procedures. First- differencing method seems to eliminate the persisitence of autocorrelations. We carried out the stanard integer-order unit-root tests in order to see if the persistence was due to the presence of a unit root. We found that the evidence favoured the prescence of a unit root in each turnover series. However, the dynamics of stock turnover may not be detectable by standard unit-root tests which have low power. Given this state of affairs, we undertook the task of modelling each turnover series as ARFIMA(p,d,q). The results clearly show that the estimated value of d, which is highly significant in every series, lies in the interval $d \in (0, 0.5)$, implying that the turnover series exhibit long-memory.

The conclusion that turnover process is a long memory process provides a consistent and satisfactory description of the dynamics of Indian stock turnover. This conclusion may not be surprising, since in earlier studies it has been found that the stock volatility is a long memory process, and we know that stock volatility is closely related to the stock trading volume. One possible explaination for long memory in stock trading volume is that it simply reflects news arrival. Good(or bad) news may be clustered in time, and thus may result in the persistence of autocorrelations. Time lags in the response of investors to news arrival can cause autocorrelations in trading volume. To be more specific, the stock market can be regarded as a complex system with heterogeneous agents who adopt different time horizons to the news. So when a piece of news arrives at the market, some investors will respond to it immediately, while others may take longer time to respond. As the news penetrate through heterogenous investors, the long term dependence in trading voume is obvious. A different explaination can be made in terms of the execution of large trading volume by institutions, who frequently split their large orders to small pieces, spreading out the execution of the orders over longer periods.

However, it is possible that stock-trading volume may show long memory because of the neglected breaks in the series. Diebold and Inoue (2001) have each shown that regime switching and long memory are intimately linked and that regime switching can also be employed as a means of bridging the gap between stationary ARMA models and non-stationary infinite variance unit root processes. More recently, Granger and Hyung (2004) showed that occasional breaks in data generate slow decaying autocorrelations and other properties of I(d) process where d can be a fraction. Given our data that have structural breaks, non-linear models with level changes can be employed to analyze the dynamics of the stock trading volume, which is the future scope of this paper. By employing the non-linear models we can investigate if the observed evidence of long memory in the Indian stock trading volume is in fact due to nonstationarity during the long period or due to neglected breaks in the series.

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Figure 1: BSE Daily Turnover Series

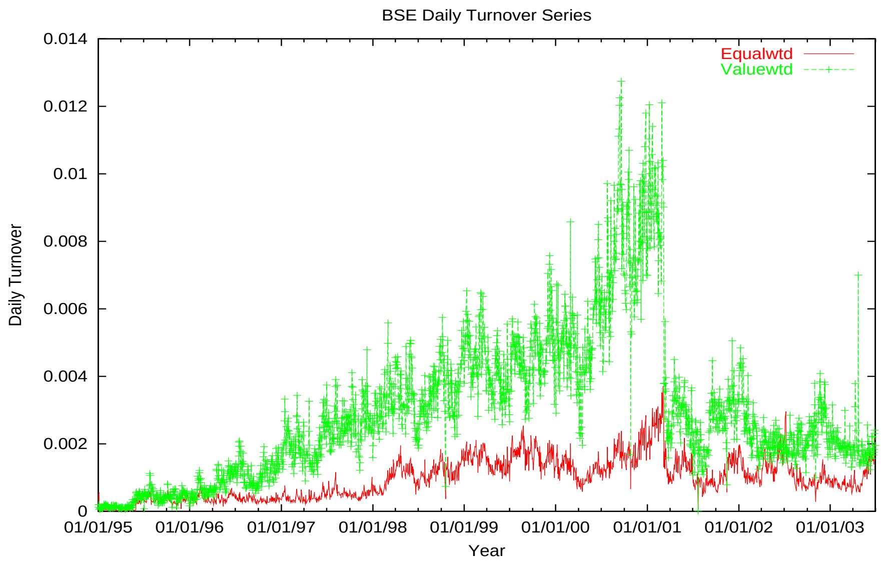

Figure 2: BSE Daily Log(Turnover Series)

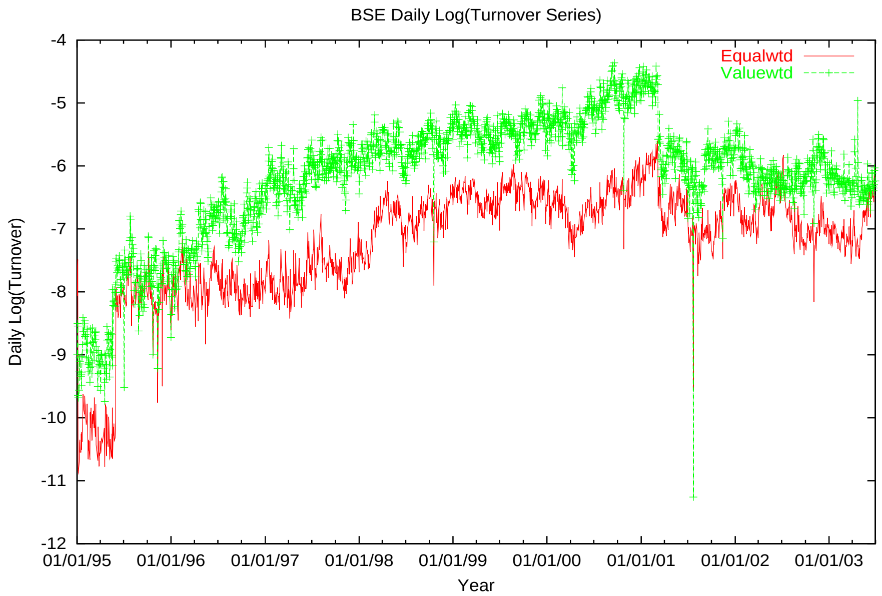

Figure 3: NSE Daily Turnover Series

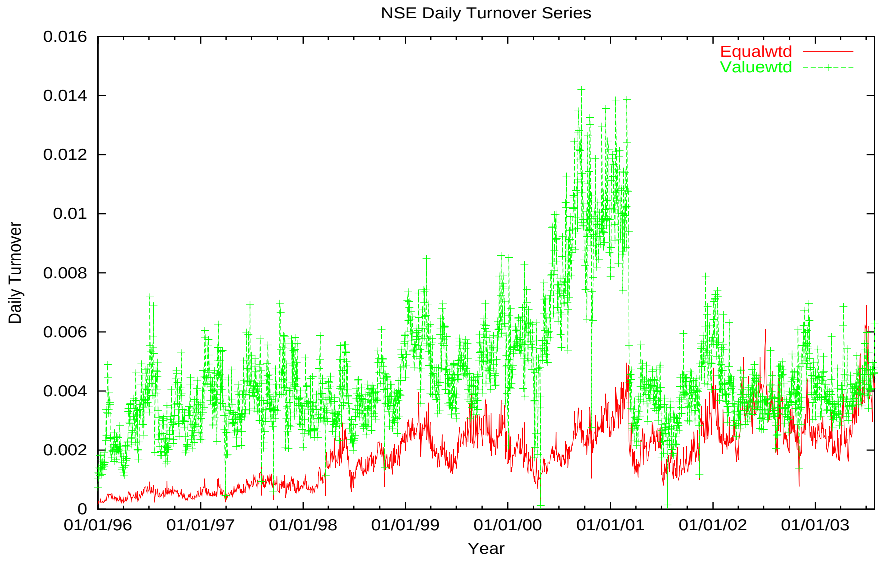

Figure 4: NSE Daily Log(Turnover Series)

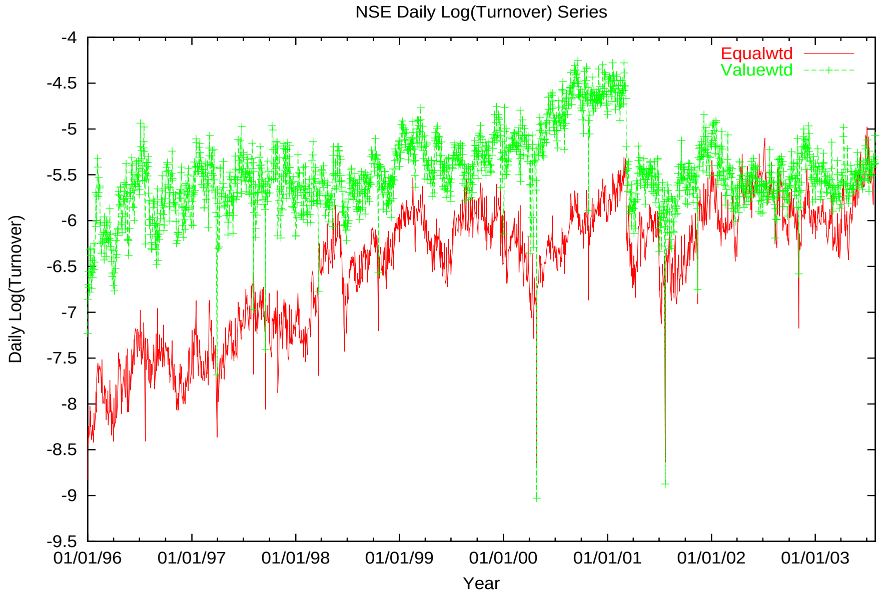

Figure 5: BSE Daily differenced-Turnover Series

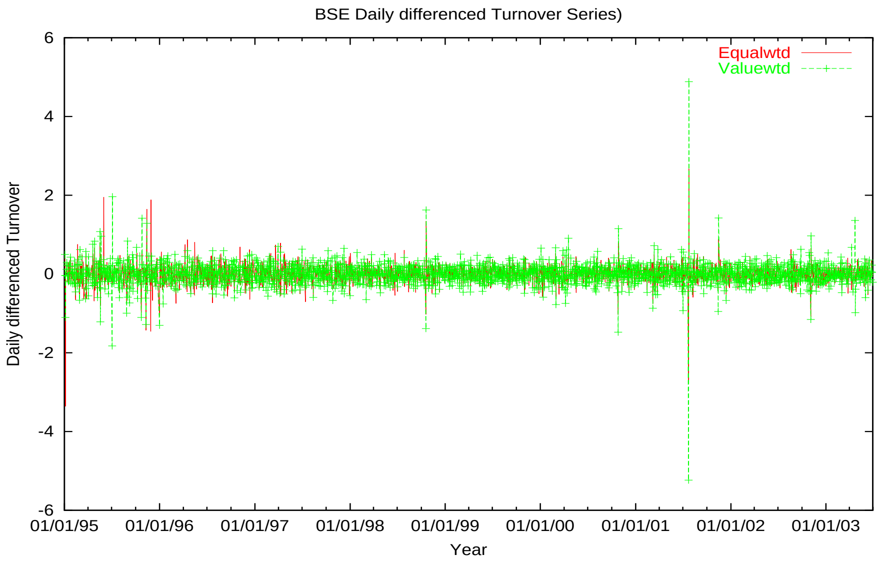

Figure 6: NSE Daily differenced-Turnover Series

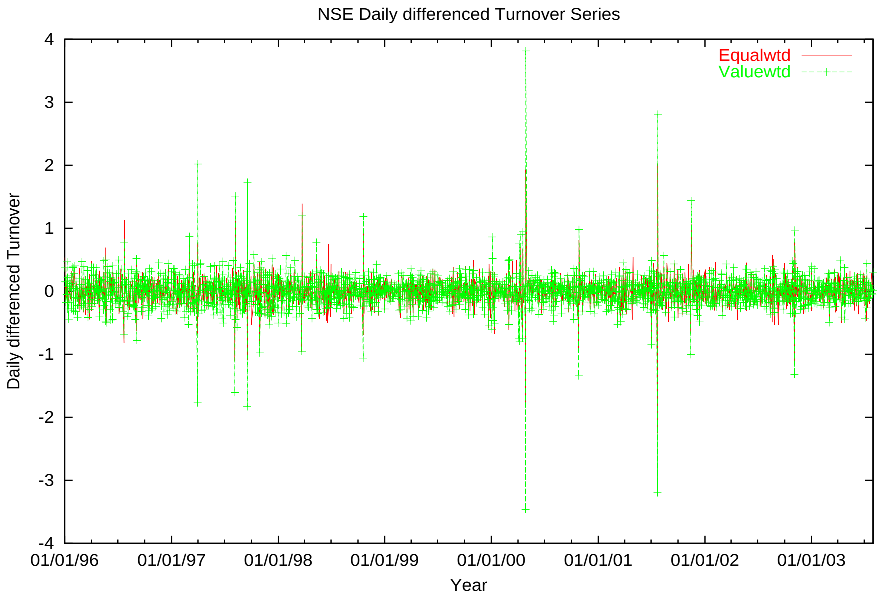

Table 1: **Summary Statistics**

Summary statistics for weekly value-weighted and equal-weighted turnover and return indexes of BSE stocks for January 1995 to June 2003(2074 daily data)and sub-periods

Statistics	τ^{EW}	τ^{VW}	R^{EW}	R^{VW}
Mean	0.00099	0.00299	0.02665	0.09971
Std.dev	0.00061	0.00221	2.16589	1.71008
Coeff. of Var.	0.61909	0.74057	81.27065	17.15117
Skewness	0.60246	1.26182	-6.50962	1.34214
Kurtosis	0.12057	1.82865	120.58778	20.64368
Jarque-Bera	126.7185	39.33913	1271268.18108	37450.14031
ADF Statistics	-3.5947	-2.16480	-13.949†	-10.5242†
Percentiles:				
Minimum	0.00000	0.00000	-42.11756	-7.3844
01%	0.00003	0.00009	-4.22643	-4.3905
05%	0.00019	0.00026	-2.39783	-2.4048
10%	0.00029	0.00050	-1.68121	-1.7259
25%	0.00045	0.00150	-0.84759	-0.8178
Median	0.00093	0.00253	-0.01486	0.0751
75%	0.00141	0.00406	0.89784	0.9607
90%	0.00178	0.00581	1.99251	1.9645
95%	0.00203	0.00755	2.71888	2.7797
99%	0.00271	0.01002	4.71940	4.5736
Maximum	0.00371	0.01274	15.68417	24.3055
Autocorrelations:				
ρ_1	0.93238	0.93686	0.22716	0.13349
ρ_2	0.90996	0.91966	0.10336	0.01530
ρ_3	0.90140	0.91152	0.11249	0.02760
ρ_4	0.89503	0.90121	0.06907	0.00373
ρ_5	0.89497	0.90219	0.09952	-0.00302
ρ_6	0.86856	0.88584	0.16308	-0.04491
ρ_7	0.86041	0.88487	0.03208	0.01459
ρ_8	0.85635	0.88113	0.04775	0.03704
ρ_9	0.85627	0.87206	0.04354	0.05221
ρ_{10}	0.85411	0.87614	0.03283	0.05813
ρ_{11}	0.83722	0.86599	-0.00211	-0.00157
ρ_{12}	0.82628	0.85601	-0.00464	0.00454
Ljung-Box Q_{12}	19128.6920	19856.0506	254.7444	59.4103
	(0.000)	(0.000)	(0.000)	(0.000)

† and ‡ indicates significant values at 0.01 and 0.05 level of significence

Table 2: **Summary Statistics (Contd..)**

Summary statistics for subperiods

Statistics	τ^{EW}	τ^{VW}	R^{EW}	R^{VW}
	Subperiod-I (1202 daily data)			
Mean	0.00077	0.00234	-0.05307	0.10041
Std. developing.	0.00057	1.67373	2.49863	1.52318
Skewness	0.76266	0.38189	-7.45598	0.21903
Kurtosis	-0.57540	-0.88179	114.93074	2.01951
	Subperiod-II (375 daily data)			
Mean	0.00155	0.00600	-0.02466	0.09336
Std. dev.	0.00055	0.00252	1.72843	2.61208
Skewness	1.03957	0.29974	1.90094	2.00760
Kurtosis	0.99328	-0.74742	18.49315	19.15606
	Subperiod-III (497 daily data)			
Mean	0.00109	0.00228	0.25818	0.10279
Std.dev.	0.00040	0.00072	1.47152	1.20817
Skewness	1.18479	1.35376	0.02485	-0.34655
Kurtosis	1.75037	4.18515	2.83960	2.72515

Subperiod-I (01Jan.1995-30Dec.1999)

Subperiod-II (03Jan.2000-29June2001)

Subperiod-III (02July2001-29June2003)

Table 3: **Impact of detrending**

Impact of detrending on the statistical properties of
equal-weighted turnover indexes of BSE for January 1995 to June 2003(2074 daily data)

Statistics	Raw	Linear	Log Linear	First diff.	MA(20)	Kernel	GRT
Mean	0.00099	0.00099	-7.20528	0.00000	1.03242	0.00000	0.00097
Std. dev.	0.00061	0.00036	0.61820	0.00022	0.36262	0.00059	0.00041
Skewness	0.60246	0.00000	-0.00329	-0.35581	8.96021	-0.37051	-0.03497
Kurtosis	0.12057	-1.20000	-1.19736	3.51614	172.59898	0.84614	-1.00003
Jarque-Bera	126.71850	124.44000	123.65695	1111.61561	2577046.69135	109.32325	86.84415
Percentiles:							
Minimum	0.00000	0.00036	-8.27747	-0.00148	0.00000	-0.00180	0.00003
01%	0.00003	0.00037	-8.25610	-0.00064	0.50818	-0.00155	0.00017
05%	0.00019	0.00042	-8.17061	-0.00037	0.66024	-0.00119	0.00034
10%	0.00029	0.00048	-8.06272	-0.00026	0.72592	-0.00086	0.00042
25%	0.00045	0.00067	-7.73904	-0.00010	0.84447	-0.00017	0.00063
Median	0.00093	0.00099	-7.20475	0.00000	0.99930	0.00002	0.00097
75%	0.00141	0.00130	-6.67045	0.00011	1.17598	0.00030	0.00131
90%	0.00178	0.00149	-6.34987	0.00026	1.35448	0.00071	0.00151
95%	0.00203	0.00155	-6.24302	0.00037	1.48261	0.00092	0.00160
99%	0.00271	0.00160	-6.15753	0.00056	1.79932	0.00140	0.00173
Maximum	0.00371	0.00161	-6.13616	0.00105	9.08826	0.00236	0.00181
Autocorrelations:							
ρ_1	0.93238	1.00000	0.99747	-0.33428	0.62139	0.92797	0.98733
ρ_2	0.90996	0.99999	0.99679	-0.10230	0.42651	0.90420	0.97191
ρ_3	0.90140	0.99999	0.99678	-0.01654	0.36012	0.89548	0.97107
ρ_4	0.89503	0.99998	0.99677	-0.04626	0.31147	0.88918	0.98108
ρ_5	0.89497	0.99997	0.99676	0.19482	0.25530	0.88943	0.98362
ρ_6	0.86856	0.99995	0.99674	-0.13500	0.19317	0.86165	0.97143
ρ_7	0.86041	0.99993	0.99672	-0.03011	0.16540	0.85324	0.95983
ρ_8	0.85635	0.99991	0.99670	-0.02967	0.14722	0.84932	0.96087
ρ_9	0.85627	0.99989	0.99668	0.01550	0.09824	0.84960	0.96839
ρ_{10}	0.85411	0.99986	0.99758	0.10887	0.10042	0.84774	0.96783
ρ_{11}	0.83722	0.99983	0.99662	-0.04391	0.04900	0.83007	0.95668
ρ_{12}	0.82628	0.99980	0.99659	-0.05035	0.01800	0.81898	0.94797
Ljung-Box Q_{12}	19128.69200	24986.60560	24785.45680	413.85650	1994.34830	18853.77820	23467.3376

Here six detrending methods have been used: linear, log-linear, first differencing, twenty-lag moving-average, detrending and deseasonalization [in the spirit of GRT(1994)] and nonparametric detrending via kernel regression

Table 4: **Impact of detrending**

Impact of detrending on the statistical properties of
value-weighted turnover indexes of BSE for January 1995 to June 2003(2074 daily data)

Statistics	Raw	Linear	Log Linear	First diff.	MA(20)	Kernel	GRT
Mean	0.00299	0.00299	-6.16238	0.00000	1.02506	0.00000	0.00292
Std.dev	0.00221	0.00087	0.56208	0.00079	0.28569	0.00283	0.00112
Skewness	1.26182	0.00000	-0.00329	-0.30386	1.02259	-0.65208	-0.09579
Kurtosis	1.82865	-1.20000	-1.19736	7.67205	5.66314	1.47321	-0.73933
Jarque-Bera	39.33913	124.44000	123.65695	5115.96854	3102.72614	334.53355	50.40877
Percentiles:							
Minimum	0.00000	0.00148	-7.13723	-0.00568	0.00000	-0.01079	0.00007
01%	0.00009	0.00151	-7.11780	-0.00239	0.42205	-0.00836	0.00048
05%	0.00026	0.00163	-7.04007	-0.00113	0.63423	-0.00550	0.00105
10%	0.00050	0.00178	-6.94197	-0.00080	0.71182	-0.00372	0.00143
25%	0.00150	0.00223	-6.64768	-0.00031	0.84823	-0.00107	0.00207
Median	0.00253	0.00299	-6.16189	0.00000	1.00268	0.00021	0.00293
75%	0.00406	0.00374	-5.67611	0.00033	1.17345	0.00154	0.00378
90%	0.00581	0.00419	-5.38463	0.00079	1.34912	0.00292	0.00443
95%	0.00755	0.00434	-5.28747	0.00118	1.49755	0.00421	0.00469
99%	0.01002	0.00446	-5.20975	0.00232	1.90496	0.00643	0.00511
Maximum	0.01274	0.00449	-5.19032	0.00520	3.62618	0.00936	0.00544
Autocorrelations:							
ρ_1	0.93686	1.00000	0.99747	-0.36382	0.53174	0.96113	0.97160
ρ_2	0.91966	0.99999	0.99679	-0.07170	0.40224	0.95053	0.94113
ρ_3	0.91152	0.99999	0.99678	0.01712	0.32067	0.94550	0.94071
ρ_4	0.90121	0.99998	0.99677	-0.08942	0.29057	0.93909	0.96325
ρ_5	0.90219	0.99997	0.99676	0.13725	0.28896	0.93957	0.97180
ρ_6	0.88584	0.99995	0.99674	-0.12174	0.20719	0.92940	0.94525
ρ_7	0.88487	0.99993	0.99672	0.02184	0.15097	0.92866	0.92284
ρ_8	0.88113	0.99991	0.99670	0.04228	0.10531	0.92619	0.92692
ρ_9	0.87206	0.99989	0.99668	-0.10418	0.05308	0.92043	0.94443
ρ_{10}	0.87614	0.99986	0.99758	0.11274	0.07288	0.92278	0.94527
ρ_{11}	0.86599	0.99983	0.99662	-0.00137	-0.00525	0.91635	0.92184
ρ_{12}	0.85601	0.99980	0.99659	-0.04800	-0.05798	0.90999	0.90571
Ljung-Box Q_{12}	19856.05060	24986.60560	24785.45680	431.37560	1654.56270	21733.05410	22171.2343

Here six detrending methods have been used: linear, log-linear, first differencing, twenty-lag moving-average, detrending and deseasonalization [in the spirit of GRT(1994)] and nonparametric detrending via kernel regression

Table 5: **Summary Statistics**

Summary statistics for weekly value-weighted and equal-weighted turnover and return indexes of NSE stocks for January 1996 to July 2003(1881 daily data)and sub-periods

Statistics	τ^{EW}	τ^{VW}	R^{EW}	R^{VW}
Mean	0.00191	0.00461	0.10696	0.11955
Std.dev.	0.00111	0.00215	1.77772	1.65690
Coeff. of Var.	0.58115	0.46518	16.62049	13.86006
Skewness	0.53728	1.59741	0.22924	-0.05245
Kurtosis	0.06611	2.92282	2.57057	2.45539
Jarque-Bera	90.84076	1469.51424	534.36396	473.38178
ADF Statistic	-4.30160	-3.42300	-10.18610 †	-10.24010†
Percentiles:				
Minimum	0.00015	0.00012	-7.95506	-7.05479
01%	0.00026	0.00140	-4.63315	-4.44357
05%	0.00041	0.00214	-2.63208	-2.48639
10%	0.00051	0.00263	-1.82936	-1.73527
25%	0.00089	0.00334	-0.85925	-0.72912
Median	0.00188	0.00407	0.03741	0.10359
75%	0.00261	0.00533	1.03455	0.98471
90%	0.00338	0.00738	2.10932	2.03309
95%	0.00386	0.00964	3.11466	2.78985
99%	0.00475	0.01211	5.13525	4.71525
Maximum	0.00690	0.01421	9.55463	7.83240
Autocorrelations:				
ρ_1	0.93062	0.88812	0.17309	0.12695
ρ_2	0.90973	0.85282	-0.00162	-0.02438
ρ_3	0.90111	0.83455	0.03678	0.02848
ρ_4	0.89245	0.82325	0.07315	0.01021
ρ_5	0.88253	0.81786	0.18395	0.06128
ρ_6	0.86694	0.78958	-0.08510	-0.07270
ρ_7	0.85922	0.78302	-0.02600	0.00991
ρ_8	0.85171	0.77178	0.01874	0.04629
ρ_9	0.84493	0.76386	0.07224	0.06362
ρ_{10}	0.84005	0.76136	0.10728	0.08191
ρ_{11}	0.83015	0.74431	-0.04222	-0.00946
ρ_{12}	0.82261	0.73330	-0.05587	-0.02137
Ljung-Box Q_{12}	17157.74870	14444.89220	189.55710	75.91790
	(0.00000)	(0.00000)	(0.00000)	(0.00000)

† indicates significant at 0.01 level of significance

Table 6: **Summary Statistics (Contd..)**

Summary statistics for subperiods

Statistics	τ^{EW}	τ^{VW}	R^{EW}	R^{VW}
	Subperiod-I (986 daily data)			
Mean	0.00130	0.00393	0.10215	0.14678
Std. dev.	0.00086	0.00132	1.86792	1.64069
Skewness	0.74360	0.37743	0.48404	0.16153
Kurtosis	-0.54089	0.13266	2.67360	2.21824
	Subperiod-II (375 daily data)			
Mean	0.00227	0.00723	-0.07606	0.01561
Std. dev.	0.00076	0.00292	1.92815	2.19540
Skewness	0.62561	0.16904	-0.02701	-0.15044
Kurtosis	0.43376	-0.94352	1.87850	0.87622
	Subperiod-III (520 daily data)			
Mean	0.00280	0.00402	0.24807	0.14286
Std. Dev.	0.00101	0.00102	1.44998	1.16307
Skewness	0.60095	0.63356	-0.16391	-0.32025
Kurtosis	0.64384	1.31570	2.01952	3.64968

Subperiod-II (31Dec.1995-30Dec.1999)

Subperiod-II (03Jan.2000-29June2001)

Subperiod-II (02July2001-31July2003)

Table 7: **Impact of detrending**

Impact of detrending on the statistical properties of
equal-weighted turnover indexes of NSE for January 1996 to July 2003(1881 daily data)

Statistics	Raw	Linear	Log Linear	First diff.	MA(20)	Kernel	GRT
Mean	0.00191	0.00191	-6.47915	0.00000	1.02454	0.00000	0.00189
Std.dev	0.00111	0.02928	0.58641	0.00041	0.23373	0.00075	0.00089
Skewness	0.53728	0.00000	0.00000	-0.25962	0.29675	-0.17401	-0.00157
Kurtosis	0.06611	-1.20000	-1.20000	3.15819	0.90009	0.84410	-1.09984
Jarque-Bera	90.84076	112.86000	112.86000	802.42840	90.13492	65.33547	94.80666
Percentiles:							
Minimum	0.00015	0.00042	-7.49403	-0.00230	0.11635	-0.00269	0.00004
01%	0.00026	0.00045	-7.47374	-0.00124	0.46784	-0.00193	0.00021
05%	0.00041	0.00057	-7.39255	-0.00068	0.68090	-0.00131	0.00053
10%	0.00051	0.00072	-7.29106	-0.00046	0.74708	-0.00099	0.00065
25%	0.00089	0.00116	-6.98659	-0.00017	0.86919	-0.00036	0.00111
Median	0.00188	0.00191	-6.47915	0.00001	1.01438	0.00003	0.00187
75%	0.00261	0.00265	-5.97171	0.00019	1.16489	0.00039	0.00264
90%	0.00338	0.00309	-5.66725	0.00046	1.32746	0.00093	0.00308
95%	0.00386	0.00324	-5.56576	0.00066	1.41486	0.00125	0.00329
99%	0.00475	0.00336	-5.48457	0.00110	1.66822	0.00175	0.00352
Maximum	0.00690	0.00339	-5.46427	0.00188	2.06867	0.00300	0.00364
Autocorrelations:							
ρ_1	0.93062	0.99920	0.99920	-0.34942	0.58350	0.84488	0.99292
ρ_2	0.90973	0.99840	0.99840	-0.08855	0.47524	0.79692	0.98897
ρ_3	0.90111	0.99760	0.99760	0.00012	0.41150	0.77618	0.98834
ρ_4	0.89245	0.99679	0.99679	0.00924	0.37945	0.75655	0.99106
ρ_5	0.88253	0.99599	0.99599	0.04112	0.34052	0.73226	0.99367
ρ_6	0.86694	0.99518	0.99518	-0.05655	0.22780	0.69372	0.98736
ρ_7	0.85922	0.99437	0.99437	-0.00127	0.18697	0.67268	0.98419
ρ_8	0.85171	0.99356	0.99356	-0.00535	0.13816	0.65216	0.98441
ρ_9	0.84493	0.99275	0.99275	-0.01380	0.10302	0.63359	0.98700
ρ_{10}	0.84005	0.99193	0.99193	0.03577	0.08332	0.62015	0.98753
ρ_{11}	0.83015	0.99112	0.99112	-0.01734	0.00375	0.59817	0.98202
ρ_{12}	0.82261	0.99030	0.99030	-0.03873	-0.04069	0.58025	0.97953
Ljung-Box Q_{12}	17157.74870	22437.67380	22437.67380	260.30990	2091.33940	11142.6081	22099.949

Here six detrending methods have been used: linear, log-linear, first differencing, twenty-lag moving-average, detrending and deseasonalization [in the spirit of GRT(1994)] and nonparametric detrending via kernel regression

Table 8: **Impact of detrending**

Impact of detrending on the statistical properties of
value-weighted turnover indexes of NSE for January 1996 to July 2003(1881 daily data)

Statistics	Raw	Linear	Log Linear	First diff.	MA(20)	Kernel	GRT
Mean	0.00461	0.00461	-5.47343	0.00000	1.01577	0.00000	0.0044
Std.dev.	0.00215	0.00053	0.12737	0.00101	0.24902	0.00001	0.0013
Skewness	1.59741	0.00000	0.00000	-0.27325	0.44879	-0.22157	-0.5324
Kurtosis	2.92282	-1.20000	-1.20000	4.15304	2.47138	1.01378	0.1577
Jarque-Bera	1469.51424	112.86000	112.86000	1374.47034	536.07550	95.94061	90.8246
Percentiles:							
Minimum	0.00012	0.00369	-5.69386	-0.00784	0.02825	-0.00926	0.0001
01%	0.00140	0.00371	-5.68945	-0.00297	0.38500	-0.00778	0.0010
05%	0.00214	0.00379	-5.67182	-0.00161	0.65169	-0.00592	0.0021
10%	0.00263	0.00388	-5.64977	-0.00115	0.73626	-0.00348	0.0028
25%	0.00334	0.00415	-5.58364	-0.00052	0.86106	-0.00142	0.0037
Median	0.00407	0.00461	-5.47343	0.00003	1.00387	0.00020	0.0045
75%	0.00533	0.00507	-5.36321	0.00055	1.15479	0.00149	0.0054
90%	0.00738	0.00535	-5.29708	0.00111	1.32369	0.00323	0.0060
95%	0.00964	0.00544	-5.27504	0.00149	1.43443	0.00508	0.0063
99%	0.01211	0.00551	-5.25740	0.00291	1.70809	0.00737	0.0067
Maximum	0.01421	0.00553	-5.25299	0.00531	2.68913	0.00968	0.0070
Autocorrelations:							
ρ_1	0.88812	0.99920	0.99920	-0.34191	0.57514	0.93713	0.9003
ρ_2	0.85282	0.99840	0.99840	-0.07594	0.43725	0.91525	0.8406
ρ_3	0.83455	0.99760	0.99760	-0.03114	0.35146	0.90263	0.8300
ρ_4	0.82325	0.99679	0.99679	-0.02646	0.32614	0.89402	0.8665
ρ_5	0.81786	0.99599	0.99599	0.10224	0.29256	0.88874	0.9018
ρ_6	0.78958	0.99518	0.99518	-0.09678	0.17748	0.87100	0.8126
ρ_7	0.78302	0.99437	0.99437	0.02086	0.13002	0.86509	0.7647
ρ_8	0.77178	0.99356	0.99356	-0.01452	0.07340	0.85663	0.7651
ρ_9	0.76386	0.99275	0.99275	-0.02431	0.04832	0.84982	0.7993
ρ_{10}	0.76136	0.99193	0.99193	0.06471	0.03224	0.84609	0.8082
ρ_{11}	0.74431	0.99112	0.99112	-0.02676	-0.01982	0.83477	0.7298
ρ_{12}	0.73330	0.99030	0.99030	-0.00319	-0.04922	0.82645	0.6930
Ljung-Box Q_{12}	14444.89220	22437.67380	22437.67380	283.18610	1674.27930	17340.75910	14934.6478

Here six detrending methods have been used: linear, log-linear, first differencing, twenty-lag moving-average, detrending and deseasonalization [in the spirit of GRT(1994)] and nonparametric detrending via kernel regression

Table 9: **Persistence Measures**

Series	Lo's RS	KPSS test	Robinson's d
BSE			
τ^{EW}	4.83558	7.53499	0.48079
τ^{VW}	4.59072	4.82495	0.48670
$Diff\tau^{EW}$	0.75261	0.016178	-0.31202
$Diff\tau^{VW}$	1.39703	0.10703	-0.32838
R^{EW}	1.17186	1.7156	0.21313
R^{VW}	1.29943	0.18393	0.13212
NSE			
τ^{EW}	4.4556	9.52739	0.47872
τ^{VW}	3.86301	2.37948	0.47000
$Diff\tau^{EW}$	1.03525	0.0459782	-0.15842
$Diff\tau^{VW}$	1.1237	0.0386096	-0.36683
R^{EW}	1.08962	0.29697	0.16536
R^{VW}	1.30062	0.05794	0.16068

Table 10: **Selected ARFIMA models for each Turnovers**

parameter	estimated value	standard error	t-value	t-prob
ARFIMA(1,d,1) for Equal weighted Turnover for BSE				
d	0.40729	0.03954	10.30000	0.00000
AR-1	0.95563	0.01212	78.80000	0.00000
MA-1	-0.86026	0.03118	-27.60000	0.00000
Trend	0.00000	0.00000	1.96000	0.05000
ARFIMA(3,d,3) for Value weighted Turnover for BSE				
d	0.46819	0.03000	15.60000	0.00000
AR-1	-0.57957	0.05268	-11.00000	0.00000
AR-2	0.67390	0.03407	19.80000	0.00000
AR-3	0.83734	0.04171	20.10000	0.00000
MA-1	0.59159	0.07203	8.21000	0.00000
MA-2	-0.63933	0.04474	-14.30000	0.00000
MA-3	-0.74150	0.05012	-14.80000	0.00000
ARFIMA(1,d,1) for Equal weighted Turnover for NSE				
d	0.42514	0.04256	9.99000	0.00000
AR-1	0.93918	0.01976	47.50000	0.00000
MA-1	-0.85538	0.04311	-19.80000	0.00000
Trend	0.00000	0.00000	3.24000	0.00100
ARFIMA(1,d,1) for Value weighted Turnover for NSE				
d	0.43781	0.03862	11.30000	0.00000
AR-1	0.96222	0.01760	54.70000	0.00000
MA-1	-0.90405	0.03992	-22.60000	0.00000

ARFIMA(p,d,q) are for the values of p, q in the range (0,3) which minimize the AIC and for which all the parameters are significant. We have not included trend in regression of value weighted turnover series as it was found to be insignificant.

Table 11: **Selected ARFIMA models for each Differenced Turnovers**

parameter	estimated value	standard error	t-value	t-prob
ARFIMA(3,d,0) for Equal weighted Turnover for BSE				
Cst(M)	0.00237	0.00059	4.03400	0.00010
d	-0.21970	0.02335	-9.40800	0.00000
AR(1)	-0.26091	0.02515	-10.37000	0.00000
AR(2)	-0.14223	0.02002	-7.10500	0.00000
AR(3)	-0.05982	0.01717	-3.48500	0.00050
Cst(V)	0.04463	0.00207	21.51000	0.00000
Student(DF)	4.94451	0.37943	13.03000	0.00000
ARFIMA(2,d,3) for Value weighted Turnover for BSE				
Cst(M)	0.00203	0.00048	4.23100	0.00000
d	-0.25745	0.03210	-8.02000	0.00000
AR(1)	0.48138	0.06337	7.59600	0.00000
AR(2)	-0.73079	0.05210	-14.03000	0.00000
MA(1)	-0.69542	0.06884	-10.10000	0.00000
MA(2)	0.74767	0.06190	12.08000	0.00000
MA(3)	-0.18436	0.03093	-5.96000	0.00000
Cst(V)	0.05316	0.00273	19.45000	0.00000
Student(DF)	4.35948	0.30136	14.47000	0.00000
ARFIMA(3,d,2) for Equal weighted Turnover for NSE				
Cst(M)	0.00211	0.00035	5.99100	0.00000
d	-0.30624	0.02315	-13.23000	0.00000
AR(1)	0.29460	0.04741	6.21400	0.00000
AR(2)	-0.78425	0.03488	-22.48000	0.00000
AR(3)	-0.19600	0.02430	-8.06700	0.00000
MA(1)	-0.51274	0.04243	-12.08000	0.00000
MA(2)	0.82267	0.04042	20.35000	0.00000
Cst(V)	0.03571	0.00186	19.21000	0.00000
Student(DF)	4.70926	0.39256	12.00000	0.00000
ARFIMA(3,d,0) for Value weighted Turnover for NSE				
Cst(M)	0.00130	0.00036	3.63900	0.00030
d	-0.30505	0.02719	-11.22000	0.00000
AR(1)	-0.18216	0.03038	-5.99600	0.00000
AR(2)	-0.11949	0.02189	-5.45800	0.00000
AR(3)	-0.08776	0.01743	-5.03400	0.00000
Cst(V)	0.04671	0.00274	17.07000	0.00000
Student(DF)	3.93557	0.25844	15.23000	0.00000

ARFIMA(p,d,q) are for the values of p, q in the range (0,3) which minimize the AIC and for which all the parameters are significant.

Table 12: **Selected ARFIMA-GARCH(1,1) models for each Differenced Turnovers**

parameter	estimated value	standard error	t-value	t-prob
ARFIMA(3,d,0)-GARCH(1,1) for Equal weighted Turnover for BSE				
Cst(M)	0.00207	0.00059	3.49000	0.00050
d-Arfima	-0.21061	0.02556	-8.23900	0.00000
AR(1)	-0.27721	0.02928	-9.46700	0.00000
AR(2)	-0.15598	0.02586	-6.03200	0.00000
AR(3)	-0.06254	0.02127	-2.94000	0.00330
Cst(V)	0.02631	0.00533	4.94000	0.00000
ARCH(Alpha1)	0.16284	0.04093	3.97800	0.00010
GARCH(Beta1)	0.23198	0.13426	1.72800	0.08420
Student(DF)	5.77891	0.45286	12.76000	0.00000
ARFIMA(2,d,3)-GARCH(1,1) for Value weighted Turnover for BSE				
Cst(M)	0.00181	0.00050	3.64200	0.00030
d	-0.24088	0.03527	-6.83000	0.00000
AR(1)	0.47451	0.05283	8.98300	0.00000
AR(2)	-0.79326	0.04494	-17.65000	0.00000
MA(1)	-0.73155	0.06367	-11.49000	0.00000
MA(2)	0.84934	0.05788	14.68000	0.00000
MA(3)	-0.23377	0.03918	-5.96600	0.00000
Cst(V)	0.01437	0.00351	4.09000	0.00000
ARCH(Alpha1)	0.11676	0.03245	3.59800	0.00030
GARCH(Beta1)	0.60016	0.08250	7.27500	0.00000
Student(DF)	5.02901	0.34139	14.73000	0.00000

continued....

Table 13: **Selected ARFIMA-GARCH(1,1) models for each Differenced Turnovers (continued)**

parameter	estimated value	standard error	t-value	t-prob
ARFIMA(3,d,2)-GARCH(1,1) for Equal weighted Turnover for NSE				
Cst(M)	0.00212	0.00036	5.82800	0.00000
d	-0.29303	0.02529	-11.59000	0.00000
AR(1)	0.26314	0.05197	5.06300	0.00000
AR(2)	-0.76975	0.03673	-20.96000	0.00000
AR(3)	-0.21712	0.02948	-7.36500	0.00000
MA(1)	-0.50213	0.04514	-11.12000	0.00000
MA(2)	0.81596	0.04256	19.17000	0.00000
Cst(V)	0.01408	0.00400	3.52400	0.00040
ARCH(Alpha1)	0.14397	0.03996	3.60300	0.00030
GARCH(Beta1)	0.45131	0.13106	3.44300	0.00060
Student(DF)	5.36588	0.43779	12.26000	0.00000
ARFIMA(3,d,2)-GARCH(1,1) for Value weighted Turnover for NSE				
Cst(M)	0.00110	0.00033	3.30900	0.00100
d	-0.31920	0.02722	-11.73000	0.00000
AR(1)	0.31578	0.09221	3.42500	0.00060
AR(2)	-0.68534	0.06520	-10.51000	0.00000
AR(3)	-0.15050	0.03153	-4.77300	0.00000
MA(1)	-0.48817	0.08564	-5.70000	0.00000
MA(2)	0.69408	0.07535	9.21200	0.00000
Cst(V)	0.01817	0.00421	4.31400	0.00000
ARCH(Alpha1)	0.13893	0.03982	3.48900	0.00050
GARCH(Beta1)	0.44369	0.11052	4.01400	0.00010
Student(DF)	4.53223	0.29380	15.43000	0.00000

ARFIMA(p,d,q) are for the values of p, q in the range (0,3) which minimize the AIC and for which all the parameters are significant.

Table 14: **Test of robustness on different estimation methods (BSE)**

	GPH			GSP		
Series	d	SE(d)	p	d	SE(d)	p
			$\mu = T^{0.5}$			
τ^{EW}	0.76070	0.10821	0.0000	0.72910	0.07372	0.0000
τ^{VW}	0.92574	0.10821	0.0000	0.86691	0.07372	0.0000
$Diff\tau^{EW}$	-0.15073	0.10963	0.1691	-0.26525	0.07454	0.0004
$Diff\tau^{VW}$	-0.06765	0.10963	0.5372	-0.03597	0.07454	0.6294
			$\mu = T^{0.55}$			
τ^{EW}	0.64719	0.08741	0.0000	0.71151	0.06108	0.0000
τ^{VW}	0.85739	0.08701	0.0000	0.84880	0.06108	0.0000
$Diff\tau^{EW}$	-0.11637	0.08701	0.1811	-0.19399	0.06108	0.0015
$Diff\tau^{VW}$	-0.09138	0.08701	0.2936	-0.08711	0.06108	0.1539
			$\mu = T^{0.6}$			
τ^{EW}	0.65387	0.07213	0.0000	0.73404	0.05051	0.0000
τ^{VW}	0.86425	0.07025	0.0000	0.84090	0.05051	0.0000
$Diff\tau^{EW}$	-0.07330	0.07025	0.2967	-0.09379	0.05051	0.0633
$Diff\tau^{VW}$	-0.12304	0.07025	0.0799	-0.13259	0.05051	0.0087
			$\mu = T^{0.8}$			
τ^{EW}	0.56227	0.03592	0.0000	0.73603	0.02357	0.0000
τ^{VW}	0.67369	0.03181	0.0000	0.70357	0.02357	0.0000
$Diff\tau^{EW}$	-0.19353	0.03125	0.0000	-0.20667	0.02360	0.0000
$Diff\tau^{VW}$	-0.34520	0.03125	0.0000	-0.30265	0.02360	0.0000
			$\mu = T/2$			
τ^{EW}	0.44947	0.02694	0.0000	0.60646	0.01554	0.0000
τ^{VW}	0.56564	0.02139	0.0000	0.61191	0.01554	0.0000
$Diff\tau^{EW}$	-0.33420	0.02074	0.0000	-0.32996	0.01555	0.0000
$Diff\tau^{VW}$	-0.47297	0.02074	0.0000	-0.40872	0.01555	0.0000

Notes: Estimation methods are log periodogram regression of Geweke-Porter-Hudak(GPH) and Gaussian semiparametric estimation described in Robinson and Henry (GSP). p values are for two-sided testing of d=0.

Table 15: **Test of Robustness on different estimation methods (NSE)**

		GPH			GSP	
Series	d	SE(d)	p	d	SE(d)	p
			$\mu = T^{0.5}$			
τ^{EW}	0.78631	0.11323	0.0000	0.75831	0.07625	0.0000
τ^{VW}	0.78027	0.11264	0.0000	0.75116	0.07625	0.0000
$Diff\tau^{EW}$	-0.34863	0.11264	0.0020	-0.22283	0.07625	0.0035
$Diff\tau^{VW}$	-0.24119	0.11264	0.0322	-0.25983	0.07625	0.0007
			$\mu = T^{0.55}$			
τ^{EW}	0.83976	0.09127	0.0000	0.81447	0.06299	0.0000
τ^{VW}	0.78378	0.10823	0.0000	0.76732	0.07372	0.0000
$Diff\tau^{EW}$	-0.31054	0.09015	0.0006	-0.26826	0.06299	0.0000
$Diff\tau^{VW}$	-0.18857	0.09015	0.0365	-0.20643	0.06299	0.0010
			$\mu = T^{0.6}$			
τ^{EW}	0.86163	0.07521	0.0000	0.90101	0.05213	0.0000
τ^{VW}	0.78155	0.07297	0.0000	0.75085	0.05213	0.0000
$Diff\tau^{EW}$	-0.25654	0.07281	0.0004	-0.22689	0.05213	0.0000
$Diff\tau^{VW}$	-0.24145	0.07281	0.0009	-0.27130	0.05213	0.0000
			$\mu = T^{0.8}$			
τ^{EW}	0.63829	0.03577	0.0000	0.71241	0.02451	0.0000
τ^{VW}	0.61400	0.03315	0.0000	0.62954	0.02451	0.0000
$Diff\tau^{EW}$	-0.37315	0.03289	0.0000	-0.36214	0.02451	0.0000
$Diff\tau^{VW}$	-0.44004	0.03289	0.0000	-0.41665	0.02451	0.0000
			$\mu = T/2$			
τ^{EW}	0.50865	0.02581	0.0000	0.58208	0.01631	0.0000
τ^{VW}	0.53193	0.02353	0.0000	0.53107	0.01631	0.0000
$Diff\tau^{EW}$	-0.49396	0.02311	0.0000	-0.43746	0.01631	0.0000
$Diff\tau^{VW}$	-0.53181	0.02311	0.0000	-0.46692	0.01631	0.0000

Notes: Estimation methods are log periodogram regression of Geweke-Porter-Hudak(GPH) and Gaussian semiparametric estimation described in Robinson and Henry (GSP). p values are for two-sided testing of d=0.

Table 16: **Test of Robustness of Weekly Turnover**

Series	best model	d	SE(d)	t-value	p value
		BSE			
τ^{EW}	ARFIMA(1,d,1)	0.45107	0.06388	7.06000	0.00000
τ^{VW}	ARFIMA(1,d,1)	0.43534	0.09331	4.67	0.00000
		NSE			
τ^{EW}	ARFIMA(2,d,1)	0.46096	0.05507	8.37000	0.00000
τ^{VW}	ARFIMA(1,d,1)	0.43534	0.09331	4.67000	0.00000

ARFIMA results are obtained using Sowell's maximum likelihood estimation method. The ARFIMA(p,d,q) are for the values of p,q in the range (0,3) which minimize the AIC and for which all the parameters are significant.

Table 17: **Test of Robustness of Subperiods**

Series	best model	d	SE(d)	t-value	p value
		Subperiod I			
τ^{EW}BSE	ARFIMA(3,d,3)	0.48256	0.02323	20.80000	0.00000
τ^{VW}BSE	ARFIMA(2,d,1)	0.49752	0.00345	144.00000	0.00000
τ^{EW}NSE	ARFIMA(2,d,2)	0.48038	0.02830	17.00000	0.00000
τ^{VW}NSE	ARFIMA(1,d,1)	0.36167	0.07195	5.03000	0.00000
		Subperiod II			
τ^{EW}BSE	ARFIMA(1,d,1)	0.23975	0.12430	1.93000	0.05500
τ^{VW}BSE	ARFIMA(3,d,3)	0.39520	0.10520	3.76000	0.00000
τ^{EW}NSE	ARFIMA(3,d,1)	0.47523	0.03439	13.80000	0.00000
τ^{VW}NSE	ARFIMA(1,d,1)	0.40023	0.08562	4.67000	0.00000
		Subperiod III			
τ^{EW}BSE	ARFIMA(1,d,1)	0.43637	0.10380	4.21000	0.00000
τ^{VW}BSE	ARFIMA(0,d,0)	0.42206	0.03120	13.50000	0.00000
τ^{EW}NSE	ARFIMA(1,d,1)	0.44964	0.06437	6.98000	0.00000
τ^{VW}NSE	ARFIMA(2,d,2)	0.45455	0.02875	15.80000	0.00000

ARFIMA results are obtained using Sowell's maximum likelihood estimation method. The ARFIMA(p,d,q) are for the values of p,q in the range (0,3) which minimize the AIC and for which all the parameters are significant.