Official Interventions and Occasional Violations of Uncovered Interest Parity in the Dollar–DM Market [1]

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Abstract

This paper presents a model of exchange rate determination in which the forward premium anomaly emerges as the result of unanticipated central bank interventions in the foreign exchange market. Deviations from uncovered interest parity (UIP) therefore represent neither unexploited profit opportunities nor compensation for bearing risk. In simulations, the model generates a forward premium anomaly and matches several other notable features of US-German data. Additional empirical support is obtained from an analysis of Fed and Bundesbank interventions in the dollar–DM market where it is found that the forward premium anomaly intensifies during those times when a central bank intervenes.

Keywords: Forward premium anomaly, foreign exchange intervention.

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Introduction

This paper investigates an asset pricing anomaly in international finance known as the forward premium anomaly. That is, the empirical finding that the forward premium (or the interest rate differential) is negatively correlated with the future exchange rate return.[1] Although substantial research has been devoted to studying the forward premium anomaly, a satisfactory understanding of the phenomenon has remained elusive. The approach taken in this paper is that the forward premium anomaly emerges as the result of unanticipated central bank interventions in the foreign exchange market. Deviations from uncovered interest parity (UIP) therefore represent neither unexploited profit opportunities nor compensation for bearing risk.

There are several reasons why this is a sensible line of inquiry. First, theories of the risk premium fare poorly when confronted by the data. Empirical investigations of asset-pricing models that explain the anomaly in terms of a time-varying risk premium typically find that the covariance between the exchange rate return and consumption growth (in intertemporal asset pricing models) or the market portfolio (as in the CAPM) is insignificant and much too small to explain the data. Quasi-rational theories such as those that emphasize noise trader risk may explain the forward premium anomaly but do not lend themselves toward straightforward identification and testing. Second, there is evidence that any profits predicted by the forward premium anomaly are not economically significant. Here, we note that in studies of survey expectations, that the median expectation from the survey implies a subjective risk premium of zero so that either the implied profit opportunities are ignored by traders, the risk to return tradeoff is sufficiently unattractive to exploit, or that traders do not perceive that the anomaly exists. Third, with the benefit of larger data sets, there is fragmentary econometric evidence that UIP holds at long horizons.[2]

[1]The equivalence between the interest differential and the forward premium is derived from the covered interest parity condition.

[2]There is an extensive literature that attempts to understand the deviations from UIP. Engel (1996), Froot and Thaler (1990), Hodrick (1987), and Lewis(1995) survey much of this literature. The findings from survey expectations were first established by Froot and Frankel (1989). More recently, Mark and Wu (1998) and Jeanne and Rose (2002) have studied the role of noise-trader risk in generating the forward premium anomaly. Chinn and Merideth (1998) and Alexius (2001) report

The model we present begins with the continuous-time version of UIP, which is a first-order stochastic differential equation. The solution of this differential equation gives the log exchange rate as an exact and nonlinear function of the interest differential. To this basic framework, we introduce Krugman (1992) style central bank interventions which occur at the margins of an informal exchange rate band. If market participants were fully rational and had common and credible knowledge of the central bank's intervention rule, then UIP would hold continuously and also in discretized observations. But this seems an unrealistic description of the dollar-DM market both in light of the forward premium anomaly and also because neither the Fed nor the Bundesbank have announced exchange rate targets for the dollar–DM rate. Instead, intervention plans are formulated in secret and conducted irregularly so that the intervention rules or an exchange rate band while known to central banks, are unknown to market participants.[3] Thus, when an intervention does occur, it creates an ephemeral but unexpected shift in the stochastic process that governs the interest differential. UIP is violated only during these instants when market participants have in mind the wrong stochastic process driving the interest differential. The time series is then composed of a mixture of observations mostly drawn from the UIP urn and some drawn from an urn where UIP does not hold. OLS regressions of the future depreciation on the interest differential detects these violations by returning negative slope coefficient estimates.

Beyond the forward premium anomaly, another notable feature of the model is that it generates volatility clustering in the exchange rate excess return in a way that conforms to patterns found in the data. The continuous-time framework provides a basis for the pervasive presence of autoregressive heteroskedastic (ARCH) effects in exchange rate returns data. The source of these ARCH effects is that the innovations to the equilibrium dynamics driving the instantaneous exchange rate return

fragmentary evidence that UIP holds over long horizons. Bansal and Dahlquist (2000) find that the forward premium anomaly is confined to G-7 economies and is absent in emerging market economies.

[3]Lewis (1995) proposes an alternative to the target zone framework by modeling interventions to stabilize the exchange rate around a targeted level where the probability of intervention depends upon and is increasing in the gap between the current exchange rate and the target value.

depends on the interest differential so that variations over time in the size of the differential either magnify or shrink the conditional volatility of the exchange rate returns.[4] Importantly, these ARCH effects are preserved under discretization of the continuous-time process to conform to the sampling intervals of the data. We also show that from the discretization of the model the “big news” representation of Schotman et.al. (1997) can be obtained where the error in the regression of the exchange rate return on the forward premium contains both additive and multiplicative terms.

We provide empirical support for the model along two dimensions. First, in simulations to assess its quantitative ability to match prominent features of the data, the model receives support by generating a forward premium anomaly, by matching the volatility of exchange rate returns and the interest differential (which differ by an order of magnitude), by generating conditional exchange rate volatility that increases with the size of the interest differential, and by generating persistence in the exchange rate and interest differential that corresponds roughly to that found in US-German data. The second line of support comes from a direct examination of the role of Fed and Bundesbank interventions in explaining the dollar–DM market and the forward premium anomaly. This analysis finds that the data are broadly consistent with two key predictions of the model–UIP works better in the absence of foreign exchange intervention and is clearly violated in the direction of the forward premium anomaly when there is intervention, and the likelihood of interventions increases with the magnitude of the interest differential.

The remainder of the paper is organized as follows. The next section presents a set of empirical regularities of the international finance data that we seek to understand. Section 2 presents the model of exchange rate dynamics that we study. The quantitative assessment of the model is carried out by means of set of simulation experiments with parameter values set equal to their simulated method of moments (SMM) estimates. Section 3 discusses SMM estimation of model’s parameters and

[4]The state dependence of the volatility in equilibrium returns is a common feature in general equilibrium continuous time asset pricing e.g., Merton (1990). See also Den Haan and Spear (1998), who present a theory in which volatility clustering in real interest rates are generated by business-cycle dependent financial market frictions.

the results of the simulation experiments are reported in section 4. Further econometric results are discussed in section 5. Section 6 offers some concluding comments. Derivations of analytical results presented in the text are contained in the appendix.

1 Features of the dollar-DM exchange rate and euro deposit differentials

Table 1 presents a list of features of foreign exchange returns and euro currency deposit rates around which we organize our investigation. We let s_t be the log dollar price of the foreign currency and r_t be the corresponding 1-week "US–German" Eurocurrency rate differential.[5] These are weekly observations of the spot exchange rate and weekly Eurocurrency rates for the US and Germany. Observations from 1/2/76 through 12/27/85 are Friday closings reported in the Harris Bank *Weekly Review.* Observations from 1/3/86 through 12/25/98 are Friday quotations from *Datastream.* We ended the sample one year before Germany irrevocably fixed the deutschemark to the euro.[6]

The table begins with some properties of the exchange rate return's conditional error distribution—the residuals from the regression of Δs_{t+1} on r_t. The Lagrange multiplier test for first-order ARCH is highly significant, the skewness coefficient suggests that the error distribution is symmetric and the excess kurtosis coefficient indicates that the error distribution is heavy-tailed relative to the normal distribution.[7]

[5]Interest rates are stated in percent per annum. To conform to this normalization, the log exchange rates are multiplied by 5200.

[6]We found that the 1-week maturity Eurocurrency rates spiked upwards towards the end of December for several years without corresponding upward spikes in Eurocurrency rates at either the 30-day or 90-day maturities. The spikes occurred in December of 1980, 1985, 1986, 1987, 1989, 1990, 1991, 1994, 1996 and 1997 for the US and 1976, 1978, 1980, 1981, 1982, 1983, 1984, 1985, 1986, 1987, 1989, 1990, 1991, 1992 and 1996 for Germany. In addition, spikes occur in March 1978, April 1980, February 1981, and April 1988 for Germany. These spikes evidently are real and not data recording errors. We looked at the 1-week rates during a two-day window around those Fridays in which the spikes occurred and found that rates on those days tended to be abnormally high as well which suggests the presence of a "Christmas effect" in 1-week Eurocurrency rates.

[7]LM is TR^2 from a regression of the squared residual regressed on its own lag and is distributed as $\chi^2(1)$ under the hypothesis of no conditional heteroskedasticity. See p.664 of Hamilton (1994).

Table 1: Features of Weekly Data–($/DM rate, US and German Euro deposit rates)

Exchange Rate		
LM test for ARCH(1): $\chi^2(1)$	281.832	
Skewness	0.097	
Excess kurtosis	1.523	
VR(2)	1.001	
VR(12)	1.113	
VR(24)	1.202	
Volatility (Δs_t)	79.063	
1^o-order autocorrelation	0.001	
Interest Differential		
Volatility	3.230	
1^o-autocorrelation	0.985	
12^o-autocorrelation	0.847	
24^o-autocorrelation	0.789	
Joint Features		
$\Delta s_{t+1} = \alpha + \beta r_t + \epsilon_{t+1}$	β	-0.693
	t-ratio ($\beta = 1$)	-2.024
GARCH(1,1)	δ	0.080
$h_{t+1} = \omega + \delta \epsilon_t^2 + \gamma h_t$	(ase)	(0.014)
	γ	0.914
	(ase)	(0.035)
$h_{t+1} = a_0 + a_1 \lvert r_t \rvert + v_t$	a_1	432.300
	(asymptotic t-ratio)	(8.327)
	R^2	0.771

Notes: Log exchange rates multiplied by 5200. Interest rate differential in percent per year.

Variance ratio statistics, computed using exchange rate returns at horizons 2, 12, and 24, provide a summary measure of the autocorrelation function and gauge the persistence in the observations. These variance ratio statistics all lie near or above 1 which indicates the presence of a high degree of persistence in the exchange rate.[8] Exchange rate returns exhibit almost no serial correlation whereas the interest differential is highly serially correlated. Exchange rate return volatility is an order of magnitude greater than interest rate differential volatility.

The lower portion of the table documents the presence of the forward premium anomaly in our sample. UIP predicts a unit slope coefficient in the regression $\Delta s_{t+1} = \alpha + \beta r_t + \epsilon_{t+1}$, whereas the point estimate is negative and significantly less than 1. To take a closer look at the conditional volatility in exchange rate excess returns, we fit the GARCH(1,1) model $\mathrm{E}_t \epsilon_{t+1}^2 = h_t = \omega + \delta \epsilon_{t-1}^2 + \gamma h_{t-1}$ to the regression error. As can be seen, the coefficient estimates of δ and γ are both significant at standard levels. The final aspect of the data that the table addresses is to explore the relationship between the exchange rate conditional variance and the size of the interest differential. We estimate the relation $h_t = a_0 + a_1|r_t| + v_t$ and see that the conditional variance increases with the absolute magnitude of the interest differential $|r_{t-1}|$.[9]

To summarize, our focus is placed on i) the forward premium anomaly, ii) ARCH effects in conditional exchange rate returns, iii) a positive relationship between conditional exchange rate volatility and the magnitude of the interest differential, iv) the exchange rate return is 24 times more volatile than the interest differential, and

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Let μ_j be the $j-$th central moment. Then the skewness coefficient μ_3/σ^3, is zero if the distribution is symmetric. The coefficient of excess kurtosis $(\mu_4/\sigma^4) - 3$, is zero if the distribution is Gaussian. If the underlying distribution is fat-tailed (thin-tailed) relative to the Gaussian distribution, this quantity will be positive (negative).

[8]The variance ratio statistic at horizon k, VR(k), is the variance of the k-period change in the log exchange rate relative to k times the variance of the one-period change.

[9]Whether the interest differential is I(1) or I(0) has been heavily tested by testing whether the spot and forward exchange rates are cointegrated. Evans and Lewis (1995) cannot reject that the interest differential is I(1) whereas Baillie and Bollerslev (1989), Choudhry (1999), Corbae *et. al.* (1992), Hai *et. al* (1997), Luintel and Paudyal (1998), Wu and Chen (1998) and Zivot (2000) do reject. Baillie and Bollerslev (1994) conclude that the interest differential has long-memory but is mean reverting with a fractional difference parameter between 1/2 and 1.

iv) both the level of the exchange rate and the interest differential are highly persistent.

2 Occasional UIP violations

In this section, we first present the model in continuous time. Since the data are sampled at discrete time intervals, however, it makes sense also to study the properties of the implied discretized observations which we do in section 2.2.

2.1 Properties of the continuous-time model

Let $r(t)$ be the instantaneous yield differential between domestic and foreign-currency denominated debt instruments with identical default risk and $s(t)$ be the exchange rate at time t. Then in continuous time UIP is the first-order stochastic differential equation

$$\mathrm{E}_t[ds(t)] = r(t)dt, \tag{1}$$

where $\mathrm{E}_t(\cdot)$ is the expectation conditional on information available at instant t and $ds(t)$ is the forward differential of $s(t)$. An explicit solution requires that the process governing the interest differential be known. We adopt an assumption that is standard in the literature on target zones and assume that $r(t)$ follows a regulated Brownian motion. The regulated Brownian motion generates high persistence in the interest differential along with bounded variance. We show below that this simple model adequately captures the features of interest differential presented in Table 1.

Interest differential dynamics

The interest differential is constrained to lie within the reflecting barriers $(\underline{r}, \bar{r})$ where $\underline{r} < \bar{r}$. When $r(t)$ lies strictly within the bands, it evolves according to the Brownian motion,

$$dr(t) = \sigma_r dz(t), \tag{2}$$

where $dz(t)$ is a standard Wiener process and σ_r is the weekly volatility in $dr(t)$. To maintain expositional clarity, we assume that the reflecting barriers are symmetric ($\underline{r} = -\bar{r}$).[10]

This is the Krugman (1992) target zone model with two modifications. First, Krugman's is based on a monetary model of the exchange rate which assumes UIP but relies on several additional and empirically questionable relationships, such as stable money demand functions and purchasing power parity. Second, in our model the monetary authorities intervene by adjusting the interest differential within a band instead of a set of vaguely defined monetary fundamentals. Marginal interventions occur whenever $r(t) = \bar{r}$ or $r(t) = -\bar{r}$ to prevent $r(t)$ from exiting the bands. When $r(t)$ lies in the interior of the bands, we think of the authorities as focusing on domestic objectives so that the interest differential, being subject to many different sources of shocks evolves randomly.[11]

The idea that US-German exchange rate policy is guided by the maintenance of the exchange rate within an informal target zone bears more than a shred of empirical plausibility. Although exchange rate bands for the US dollar during the post Bretton Woods era have never been formally established, both coordinated as well as uncoordinated foreign exchange interventions are frequently engineered by the major central banks, especially during times of unusual dollar strength or weakness. The widespread practice of intervention at least suggests the existence of a set of

[10] Under band symmetry, the unconditional mean of $r(t)$ is 0. The appendix shows how band symmetry can be relaxed. Recent research has exploited similar nonlinear models to study exchange rates [Michael, Nobay and Peel (1997), Kilian and Taylor (2001)]. Since interest differentials and exchange rates are functionally related, it is natural to also consider nonlinear adjustment in the interest differential. We note also that (2) is consistent with individual interest rate dynamics that evolve according to $di(t) = \sigma_1 dz_1(t)$ when $i \in [\underline{i}, \bar{i}]$ and $di^*(t) = \sigma_2 dz_2(t)$ when $i^* \in [\underline{i}^*, \bar{i}^*]$, where $dz_t(t) = \rho dz_1(t) + \sqrt{1-\rho^2}dw(t)$ and $dw(t)$ and $dz_1(t)$ are independent standard Wiener processes. Then we have $dr(t) = di(t) - di^*(t) = \sigma_r dz(t)$ where $\sigma_r = \sqrt{(\sigma_1 - \rho\sigma_2)^2 + \sigma_2^2(1-\rho^2)}$, and $dz(t)$ is a standard Wiener process. If we set $\underline{i} = \underline{i}^* = 0$, then we have $\bar{r} = \bar{i}$ and $\underline{r} = -\bar{i}^*$. In any finite sample, however, we may not have very many realizations of the event $\{i_t = \bar{i} \cap i_t^* = \underline{i}^*\}$ or of the event $\{i_t = \underline{i} \cap i_t^* = \bar{i}^*\}$ so the standard error on the estimate of $\bar{r}$ is likely to be quite large.

[11] The idea that a monetary policy rule that depends on the exchange rate explains violations of UIP was also examined by McCallum (1994). In his analysis, the authorities set the interest differential in response to the currency depreciation rate.

informal bands.[12]

Exchange rate solution

When $r(t)$ lies within the bands, we obtain the family of solutions to (1)

$$s(t) = A + Br(t) + \frac{r^3(t)}{3\sigma_r^2}. \tag{3}$$

A and B are constant coefficients to be determined by auxiliary conditions. A depends on initial conditions and on currency units so without loss of generality, we set $A = 0$. If B is sufficiently negative, the exchange rate will be decreasing in the interest differential, with the result that a strong dollar is associated with high relative US interest rates. Figure 1 shows solutions for alternative values of B. The nonlinear manner in which the exchange rate function bends as the absolute magnitude of the interest differential increases—is qualitatively similar to the Krugman (1992) S-shape relationship between the exchange rate and the 'fundamentals.'

Taking the total differential of (3) using Ito's lemma, the instantaneous change in the log exchange rate is

$$ds(t) = r(t)dt + \left(B + \frac{r^2(t)}{\sigma_r^2}\right)\sigma_r dz(t). \tag{4}$$

Several properties of the model are more transparent in terms of the instantaneous exchange rate return (4) than in levels form (3). First, UIP holds regardless of the value of B as long as the interest differential lies strictly within the bands. This is because $dz(t) \sim N(0, dt)$ so that taking expectations on both sides of (4), gives $\mathrm{E}_t\left[ds(t)\right] = r(t)$.[13] Second, if the intervention rule were completely credible and

[12] See Baillie and Osterberg (2000) for a narrative of Fed and Bundesbank intervention history over the 80s and 90s. Since that time, the interventions have continued. On 22 Sept. 2000, the European Central Bank (ECB), the Federal Reserve, the Bank of Japan (BOJ), Bank of Canada, and the Bank of England engaged in a coordinated intervention to support the euro, the ECB engaged in subsequent purchases of euros on 3 Nov. 2000, on 30 June, 1998 the BOJ intervened to support the yen whereas on 3 April 2000, it intervened to support the dollar.

[13] We can entertain alternative intervention rules suggested in the target-zone literature. For example, suppose participants believe that the authorities will intervene by setting the interest

known to market participants, maintaining UIP at the instant of intervention requires $B = -\bar{r}^2/\sigma_r^2$. This is because at the instant of an intervention, say when the interest differential is at the upper band $\bar{r}$, the distribution of $dz(t)$ becomes right truncated at zero with conditional mean $\mathrm{E}\left[dz(t)|r(t) = \bar{r}\right] \simeq -0.80$. This restriction on B ensures that the composite error term in (4) vanishes when $r(t) = \bar{r}$. Third, the instantaneous conditional variance of the composite error term in (4) is $(B + r^2(t)/\sigma_r^2)^2\sigma_r^2 dt$. Its dependence on the interest differential causes it to vary over time thus giving rise to ARCH effects in exchange rate excess returns. This state-dependent nature of asset returns in continuous-time equilibrium asset pricing models is a common feature of such models [e.g., Merton (1990)] and gives a general theoretical basis for ARCH-effects in asset returns.

We will not, however, assume that market participants perfectly understand and completely believe the intervention policy. Instead, we make the opposing extreme assumption that participants do not anticipate interventions by allowing $B \neq -\bar{r}^2/\sigma_r^2$. By doing so, we posit that market participants are unable to learn the central bank rules so that interventions always take participants by surprise.[14] Only central bankers know the truth and the illegality of trading on such inside information prevents them from exploiting the potentially huge profit opportunities that they themselves create. Thus, conditional on being at the upper band (say), market participants believe ex

differential to 0 when one of the bands is hit, as in Flood and Garber (1991). Then maintaining UIP during instants of intervention gives $B = -\frac{\bar{r}^2}{3\sigma_r^2}$, where the coefficient on r scaled down by 1/3. If the intervention rule lacks full credibility in the sense of Bertola-Caballero (1992), the coefficient is scaled down even further. In this setup, we begin with an initial band $[-\bar{r}, \bar{r}]$ of size $b = 2\bar{r}$. Suppose that when the upper band $\bar{r}$ is touched, there is a probability p that the authorities will realign instead of defending the initial band. $1 - p$ is the probability that they defend the initial band. If realignment occurs, the authorities establish a new band where the old upper band $\bar{r}$ is now the lower band and the new upper band is $\bar{r} + b$ and they place the interest differential in the middle of the new band. If defense takes place, the authorities place the interest differential back at the midpoint of the band as in the Flood-Garber intervention. In this environment, maintenance of UIP during instants of defense or realignments gives $B = \left[\frac{(8p-1)\bar{r}^2}{3\sigma_r^2}\right]$.

[14] An alternative strategy for incorporating this idea would be to build a model of nonsystematic interventions that are sufficiently irregular that agents maintain diffuse priors over the interventions. Dominguez (2003) provides a narrative account of Fed intervention policy and evidence on market discovery of intervention episodes. See also Klein and Lewis (1993) who present a model in which market participants update their prior probabilities about the interventions as Bayesian and learn about the bands over time. An analysis of learning is beyond the scope of this paper.

ante that UIP will hold whereas in truth, it is violated. The deviation from UIP at this instant is

$$\mathrm{E}[ds(t)|r(t)=\bar{r}] - \bar{r}dt = -\left(B + \frac{\bar{r}^2}{\sigma_r^2}\right)\sigma_r(0.8). \tag{5}$$

When central bank interventions are stabilizing, it follows from (5) that in order for deviations from UIP to go in the direction of the forward premium anomaly (where the interest differential is negatively correlated with the future depreciation) it must be the case that $B > -\bar{r}^2/\sigma_r^2$.

2.2 Properties of discretized observations

Since the data are sampled at discrete points in time, it is useful to study a discretization the model to match the theory up with the observations. We begin by integrating (4) to obtain the implied discrete-time depreciation

$$s(1) - s(0) = r(0) + \underbrace{\sigma_r \left[\int_0^1 z(t)dt - z(0)\right]}_{(a)} + \underbrace{\frac{1}{\sigma_r}\int_0^1 r^2(t)dz(t)}_{(b)} + \underbrace{B\sigma_r \int_0^1 dz(t)}_{(c)}. \tag{6}$$

The terms labeled (a), (b), and (c) are separate components of the true error from the regression of the future depreciation on the current interest differential. A further decomposition of the discrete-time change gives

$$s(1) - s(0) = r(0)[1 + \epsilon(1)] + v(1), \tag{7}$$

where

$$\epsilon(1) = \frac{1}{\sigma_r}[r(0) - 2\sigma_r z(0)] \underbrace{\int_0^1 dz(t)}_{(i)} + 2\underbrace{\int_0^1 z(t)dz(t)}_{(ii)}, \tag{8}$$

$$\begin{aligned} v(1) = {} & \sigma_r z^2(0) \underbrace{\int_0^1 dz(t)}_{(i)} + \sigma_r \underbrace{\int_0^1 z(t)dt}_{(iii)} - 2\sigma_r z(0) \underbrace{\int_0^1 z(t)dz(t)}_{(ii)} \\ & + \sigma_r \underbrace{\int_0^1 z^2(t)dz(t)}_{(iv)} - \sigma_r z(0) + B\sigma_r \underbrace{\int_0^1 dz(t)}_{(i)}. \end{aligned} \tag{9}$$

(7) is the 'big-news' representation suggested by Schotman et.al. (1997). They posit the parametric representation of UIP $s_{t+1} - s_t = r_t(1+\epsilon_{t+1}) + v_{t+1}$ where ϵ_{t+1} and v_{t+1} are conditionally zero-mean innovations. The multiplicative error $r_t\epsilon_{t+1}$ is called "big news" whereas the additive error v_{t+1} is regular news. They suggest that the forward premium anomaly may be a statistical artifact resulting from poor sampling properties of the OLS estimator when the observations are generated by this representation and where the interest differential is also drawn from a heavy-tailed distribution.[15]

From (7)-(9) it can be seen that the distribution of the big news is leptokurtotic and the ARCH effects are preserved under discretization through seen by the dependence of the conditional variance of the big news component $\mathrm{E}_0[r(0)^2\epsilon(1)^2]$ on $r(0)$. Several of the error components in (8) and (9) have more familiar representations. The term labeled (i) is $\int_0^1 dz(t) \sim N(0,1)$, the term labeled (ii) is $\int_0^1 z(t)dz(t) \sim \chi^2(1)-1$, which is skewed, and the term labeled (iii) is $\int_0^1 z(t)dt \sim N\left(0, \frac{1}{3}\right)$. The term labeled (iv) is $\int_0^1 z^2(t)dz(t)$ which is nonstandard. We investigate its properties by simulation and find it to be zero-meaned with a symmetrically leptokurtotic distribution (coefficient of excess kurtosis equal to 86.56).

When working with discrete time sampled (say weekly) data, we are interested in regressing the weekly depreciation on the 1-week interest differential, $R(0,1)$, and not on the instantaneous return differential $r(0)$. We appeal to the expectations hypothesis of the term structure of interest rates, $R(0,1) = \mathrm{E}_t(\int_0^1 r(u)du) = r(0)\int_0^1 du = r(0)$, the discretized representation (6) corresponds to the regression run on the data even though $r(0)$ is the instantaneous yield.

[15] Big news is not the only ingredient in Schotman et.al. story of poor small sample properties of OLS. They also assume that the interest differential is drawn from a leptokurtotic distribution. In work along similar lines, Baillie and Bollerslev (2000) demonstrate that the inter 90-percentile range of the OLS empirical distribution from regressing the exchange rate return on the interest differential is (-5.14,10.9) when the conditional volatility in the interest differential follows their calibrated fractionally integrated generalized ARCH process.

3 Simulated method of moments estimation

We first estimate the parameters $(\bar{r}, \sigma_r)$ of the interest rate processes using the simulated method of moments (SMM).[16] We begin by dividing each of the T weekly observations into N subintervals, of length $\delta_N = 1/N \simeq dt$, and use Euler's method to approximate the continuous-time model

$$r_j = r_{j-1} + \sigma_r \epsilon_j \sqrt{\delta_N}, \tag{10}$$

where $j = 1, \ldots, NT$, $\epsilon_j \overset{iid}{\sim} N(0,1)$, and σ_r is the weekly standard deviation of the instantaneous rate of return differential $r(t)$. The parameters $\bar{r}$ and σ_r are chosen such that the implied moments generated from simulations of (10) minimize a measure of quadratic distance between the set of simulated and sample moments. Using the 3 moments $\mathrm{E}(\Delta r_t, \Delta r_t^2, r_t r_{t-1})$, we obtain estimated values of $\hat{\sigma}_r = 0.576$ (s.e.=0.070), $\hat{\bar{r}} = 5.632$ (s.e.=0.643). The J-statistic for the chi-square test of the over identifying restrictions is 0.008 which with one degree of freedom gives a p-value of 0.930 and is not rejected by the data.

Because the occasional violations version of the model does not give guidance for setting the parameter B, we use SMM estimates of B from the data. We obtain an estimate of B by applying the Euler method to the exchange rate (3) using the moments $(\mathrm{E}\Delta s_t, \mathrm{E}\Delta s_t^2, \mathrm{E}\Delta s_t r_{t-1})$. This gives $\hat{B} = 102$ (s.e.=6.11). The positive point estimate is evidence against the hypothesis that UIP holds at intervention points. Although carrying out classical hypothesis tests of the model are not our primary interest, we note that tests of the over identifying restrictions are somewhat unfavorable to the model (J=43, p-value=0.00).

4 Quantitative properties of the model

We now investigate the extent to which the model can quantitatively account for the features of the data described in Section 1. We conduct a series of 5000 model simula-

[16] Lee and Ingram (1991), Duffie and Singleton (1993).

Table 2: Statistical Properties of Calibrated Model (Percentiles of Monte Carlo Distributions from 5000 Replications of 1200 Weekly Observations).

	Occasional Violations			UIP		
	2.5%	50%	97.5%	2.5%	50%	97.5%
β	-4.020	-2.156	-1.565$^{a/}$	0.670$^{b/}$	1.105	2.104
asy-t	-5.238	-4.156	-3.793$^{a/}$	-1.732	0.399	2.511
LM: $\chi^2(1)$	321.607$^{b/}$	366.263	405.472	242.735	299.270	352.049
skewness	-0.160	0.002	0.164	-0.227	-0.002	0.230
excess kurtosis	-0.047	0.276	0.717$^{a/}$	0.526	1.211	2.313
VR(2)	0.895	0.956	1.017	0.927	0.992	1.060
VR(12)	0.602	0.767	0.957$^{a/}$	0.712	0.920	1.174
VR(24)	0.471	0.663	0.898$^{a/}$	0.586	0.846	1.179
$\sigma_{\Delta s}$	69.858	74.912	79.798	34.503	39.912	44.673$^{a/}$
$\rho_{\Delta s}(1)$	-0.070	-0.004	0.064	-0.072	-0.007	0.061
σ_r	2.270	3.088	3.628	2.270	3.088	3.628
$\rho_r(1)$	0.969	0.983	0.989	0.969	0.983	0.989
$\rho_r(12)$	0.682	0.828	0.896	0.682	0.828	0.896
$\rho_r(24)$	0.447	0.688	0.819	0.447	0.688	0.819

Note: $^{a/}$–less than estimate from data. $^{b/}$–greater than estimate from data.

tions with parameter values calibrated to our SMM estimates from US-German data. Each simulation begins with a realization of the Euler–approximated continuous-time interest differential and exchange rate with the weekly time interval divided into 84 subintervals. The initial value of the interest differential is drawn from the uniform distribution with support $[-\bar{r}, \bar{r}]$. We then draw 1200 observations at weekly intervals, to conform to the number of data points in the sample. These observations are then employed to calculate the statistics that were used to characterize the data. The results are contained in Tables 2 and 3.

Table 2 reveals the following. First, the model generates a pronounced forward premium anomaly in which the entire inter-95-percentile range of the OLS slope coefficient lies below the point estimate from the data. Second, as in the data, the simulated observations from the occasional violations model exhibit strong ARCH

Table 3: Volatility Properties. Median values and inter 90-percentile range

		Occasional Violations	UIP
LM Test for ARCH(1)	$\chi^2(1)$	366.26	299.27
GARCH(1,1)	δ	0.019	0.156
$h_{t+1} = \omega + \delta\epsilon_t^2 + \gamma h_t$	(5%:95%)	(0.008:0.034)	(0.074:0.288)
	γ	0.977	0.834
	(5%:95%)	(0.942:0.991)	(0.724:0.910)
$h_t = a_0 + a_1 \lvert r_{t-1} \rvert + v_t$	a_1	634.2211	-423.3474
		(445.3752: 838.0825)	(-400.2462: -349.3086)
	t-ratio	15.1162	-26.6304
		(10.1836 : 209.3852)	(-32.7553: -20.4963)
	R^2	0.8316	0.8029
		(0.7997:0.8567)	(0.7590:0.8357)

Note: t-ratio constructed to test hypothesis that $a_1 = 0$.

effects. The inter 95-percentile of the LM statistic distribution lies above the point estimate from the data while the simulated error distribution is symmetric and exhibits excess kurtosis. The implied log exchange is highly persistent although less so than in the data, as seen from the quantiles of the variance ratio statistics. Third, the model can match the volatility in the exchange rate return and the interest differential, which differ from each other by a factor of 20. Finally, the regulated Brownian motion matches the volatility and the persistence in the interest differential.

As a point of comparison, the table also displays the simulation results for the no UIP violations specification. Here, the median of the OLS slope coefficient distribution lies slightly above 1, which goes in the opposite direction of the forward premium anomaly. Another feature worth pointing out is that this model does not generate sufficient volatility in the exchange rate return. The remaining aspects of the simulated observations shown in the table are quite similar to the occasional violations model.

To examine the volatility properties of the model in more detail, Table 3 reports selected percentiles of the GARCH(1,1) parameter estimates and coefficient estimates of the regression of the conditional variance on the absolute interest differential. In regard to the GARCH(1,1) model, it can be seen that the estimated values of δ and γ from the data lie within the inter 95-percentile range of the statistical distributions. In regard to the implied positive relationship between the exchange rate conditional volatility and the absolute magnitude of the interest differential also conforms to that found in the data.[17]

Comparing the volatility properties to the no violations in UIP specification, the GARCH(1,1) patterns are also seen to conform to those found in the data but it has the counterfactual property that the conditional variance of the exchange rate is negatively related to the absolute size of the interest differential. This is because that at the instant of intervention say at the upper band, UIP holds exactly with

[17] The heightened volatility associated with large interest differentials and intervention points predicted by the occasional violations model is consistent with empirical findings of Dominguez (1989), Anderson et.al. (2002) and others who find that heightened exchange rate volatility shortly following central bank interventions.

$ds(t) = \bar{r}dt$ so that the conditional variance of the exchange rate collapses to zero at this point. We note also that the ARCH effects generated by the no violations UIP model are less systematic in the sense that this specification generates much wider inter 95-percentile ranges for the GARCH parameter estimators than does the occasional violations model

How frequently do these violations occur in producing these results? Based upon the estimated model and a weekly sampling interval of the observations, the unconditional probability of touching either of the bands is 0.081. Over the course of a sample of 23 years, this amounts to interventions in approximately 98 out of the total 1200 weekly observations. This is quite similar to the actual intervention record for the Fed. Dominguez (2003) reports the record of Fed interventions from 1987 to 1995, from which it can be inferred that the unconditional probability of Fed interventions over that time period in the dollar–DM rate was 0.084.

5 Interventions and the forward premium anomaly

In this section, we present empirical results that focus on two predictions of the model. The first of these is that UIP is violated in the direction of the forward premium anomaly only during times of intervention. The second is that foreign exchange interventions are more likely to occur the larger is the magnitude of the interest differential. We address these issues with data on Fed and Bundesbank interventions in the dollar–DM market.[18] These are daily indicators that show whether the Fed or the Bundesbank had either bought or sold dollars.

We begin by examining whether the forward premium anomaly intensifies during interventions. First, we note that interventions occurred relatively more frequently from the period 1977 to 1991.[19] Simply splitting the sample at January 1992 and

[18]These are Reuters reports of central bank intervention. We thank Kathryn Dominguez for these data.

[19]The number of days in which either the Fed or the Bundesbank was reported to have intervened in the dollar-DM market was 173 (1977), 217 (1978), 217 (1979), 216 (1980), 139 (1981), 84 (1982), 130 (1983), 162 (1984), 55 (1985), 9 (1986),73 (1987),96 (1988), 110 (1989),34 (1990), 25 (1991), 8 (1992),5 (1993), 5 (1994), 8 (1995),0 (1996),0 (1997), 1 (1998).

running the regression (1) gives an estimated slope coefficient of -2.268 (s.e.=1.22) for the early portion of the sample and an estimate of 0.249 (s.e.=1.41) for the latter portion. This very coarse look at the data suggests that the forward premium anomaly is found in the dollar-DM market during that portion of the sample when interventions occurred most frequently.

To take a finer look at the data, we define an intervention period as one in which a central bank (either the Fed or the Bundesbank) has intervened within a window of time around Friday, which is the day that the exchange rate and interest rate are sampled. We then classify the observations as to whether or not they were drawn from an intervention period and then estimate the regression (1) for each classification. We considered three intervention windows: i) a 5 day lead and lag window, ii) a 5 day lead window, and iii) a 5 day lag window.

The results are shown in Table 4. Using the full sample, we see that in the absence of intervention, there is no forward premium anomaly. During non-intervention periods using the lead and lag window definition, the slope coefficient point estimate slightly exceeds but is insignificantly different from 1, whereas intervention periods, the forward premium anomaly is present with $\hat{\beta} < 0$ and significantly less than 1. The results are robust to the other two intervention window definitions. The sub sample analysis tells a similar story. The forward premium anomaly is present when central banks intervene. In the absence of intervention, a forward premium bias remains, but the anomaly is absent.

Lastly, we examine the relationship between the likelihood of an intervention and size of the interest differential. Consider a model where the central bank's propensity to intervene y_t^*, depends on the magnitude of the interest differential,

$$y_t^* = \theta_0 + \theta_1 |r_t| + u_t.$$

A central bank then intervenes $y_t = 1$ (or does not $y_t = 0$) if y_t^* exceeds a threshold of 0,

$$y_t = \begin{cases} 1 & \text{if} \quad y_t^* > 0 \\ 0 & \text{o.w.} \end{cases}$$

Table 4: The Forward Premium Anomaly During Normal and Intervention Periods

	Interventions			No Interventions		
	Lead&Lag	Lead	Lag	Lead&Lag	Lead	Lag
Sample: 1977-1998						
Observations	632	553	552	520	599	600
Slope	-1.591	-2.183	-2.346	1.195	1.130	1.601
t-ratio	-2.110	-2.255	-2.380	0.147	0.111	0.506
Sample: 1977-1991						
Observations	592	531	530	194	255	256
Slope	-2.179	-2.788	-2.340	0.242	0.363	0.894
t-ratio	-2.112	-2.252	-2.042	-0.317	-0.329	-0.053
Sample: 1992-1998						
Observations	40	22	22	326	344	344
Slope	-4.198	-7.630	-3.896	0.711	0.652	0.671
t-ratio	-1.377	-2.011	-0.879	-0.184	-0.231	-0.220

Note: t-ratio constructed to test the hypothesis that slope equals 1.

Fitting a probit to this model gives $\hat{\theta}_0 = -0.726, (t = -9.056)$ and $\hat{\theta}_1 = 0.208, (t = 9.500)$. That the likelihood of central bank interventions is found to increase with the size of the interest differential is consistent with the idea that the exchange rate lies relatively far from the central banks target rate during periods of large absolute interest differentials and that central banks are more likely to intervene during these times with the dual objective of guiding the exchange rate towards its target value and to reduce market volatility.[20]

6 Conclusion

This paper was motivated in part by the long-standing difficulty encountered by research in the area to establish that measured deviations from UIP vary systematically with economic fundamentals in ways predicted by standard asset pricing theory.

[20]Baillie and Osterberg (2000) report that the magnitude of deviations from UIP increase during intervention periods, but they do not directly address the effect on the forward premium anomaly. Baillie and Osterberg (1997a,b) and Dominguez (1989) find evidence that interventions are in part motivated to reduce market volatility.

As an alternative, we have put forth a simple model in which the forward premium anomaly emerges as a result of unanticipated central bank interventions in the foreign exchange market. In this model, the violations to UIP do not reflect unexploited profit opportunities or systematic risk. The model is able to generate a foreign premium anomaly that is quantitatively reasonable in size. In addition, it is able to match or reasonably approximate many other notable features of the data. Further support for the model is obtained from an analysis of Fed and Bundesbank interventions which finds that the forward premium anomaly intensifies during periods in which central banks are intervening.

It might be argued that the model we present is a poor candidate for providing a literal description of the foreign exchange market due to its highly stylized nature. There are no good reasons other than tractability for the many assumptions that we made, such as the purely unanticipated nature of the interventions their unlearnability, that interventions only occur at the margins of this fixed but informal exchange rate band. The model might best be viewed as a parable for a more realistic environment in which the interest differential may be subject to occasional shifts into an evolving menu of regimes. Market participants in this environment, although attempting to learn about the underlying rules, are only able to do so exceptionally slowly.

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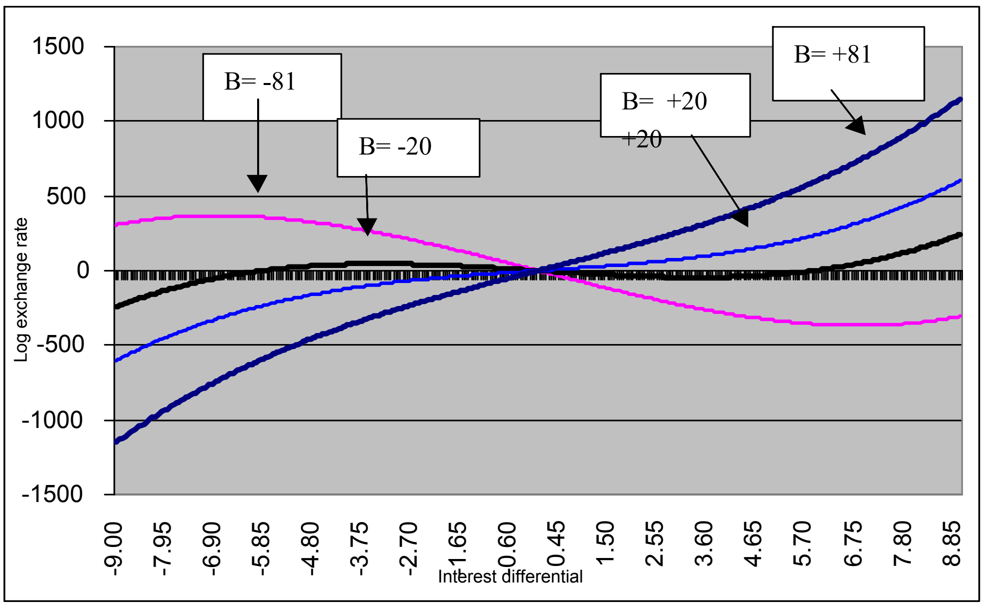

Figure 1: Nonlinear relation between log exchange rate and interest differential: Plots of equation (3) with alternative values of B.

Appendix

A.1 Exchange rate solution

We first derive the solution in the text following Krugman's (1992) use of the method of undetermined coefficients. We guess that the solution takes the form,

$$s(t) = G[r(t)], \tag{A.1}$$

where $G(\cdot)$ is a time-invariant continuous and twice differentiable function of r. Using Ito's lemma to take the total differential of (A.1) gives

$$ds(t) = G'[r(t)]dr(t) + \frac{1}{2}G''[r(t)][dr(t)]^2 \tag{A.2}$$

where $G' = dG(r)/dr$ and $G'' = d^2G(r)/dr^2$.

Derivation of eq.(3). If the interest differential evolves according to (2), then $dr(t) = \sigma_r dz(t)$ and $dr(t)^2 = \sigma_r^2 dt$. Upon substitution into (A.2), we get

$$ds(t) = G'[r(t)]\sigma_r dz(t) + \frac{\sigma_r^2}{2}G''[r(t)]dt \tag{A.3}$$

Now take expectations of both sides of (A.3) conditional on information known at instant t,

$$\mathrm{E}_t[ds(t)] = \frac{\sigma_r^2}{2}G''[r(t)]dt = r(t)dt \tag{A.4}$$

where the second equality is obtained by UIP. Now we seek to solve the differential equation,

$$\frac{\sigma_r^2}{2}G''[r(t)] = r(t) \tag{A.5}$$

Let the solution to the homogeneous part of (A.5) be G_h. This solution must satisfy $G_h'' = 0$ and is satisfied by setting $G_h = A + Br$. Next, we guess that the solution to the nonhomogeneous part be $G_n = kr^3$. Then $G_n' = 3kr^2, G_n'' = 6kr$. Upon substitution into (A.5), we obtain $k = 1/3\sigma_r^2$. The general solution is therefore $s(t) = G_h + G_n = A + Br + r^3/(3\sigma_r^2)$, which is eq.(3). $\|$

Derivation of restriction $B = -\bar{r}^2/\sigma_r^2$ under the no UIP violations specification. Here, we exploit knowledge of behavior at the bands to determine B. Due to the symmetric nature of the bands, we need only examine behavior at one of the bands. Suppose that $r(t)$ attains the upper band $\bar{r}$. At that instant, $G'[\bar{r}] = 0 = B + \bar{r}^2/\sigma_r^2$ and solving yields $B = -(\bar{r}^2/\sigma_r^2)$. $\|$

Derivation of B for Flood-Garber interventions in footnote 9. As in the abofe derivation for B under the marginal intervention rule, the solution to the nonhomogenous part of the differential equation is given by $G_n = r^3/(3\sigma_r^2)$. A general guess solution can be written explicitly in terms of the bands as,

$$G(r|\bar{r}) = A + \frac{r^3}{3\sigma_r^2} + B(r - \bar{r}) + C(r + \bar{r}) \tag{A.6}$$

Now suppose the upper band is hit at the instant t_0, $r(t_0) = \bar{r}$, it follows that

$$s(t_0) = A + \frac{\bar{r}^3}{3\sigma_r^2} + 2C\bar{r}. \tag{A.7}$$

At the next instant, the interest differential is set to 0. Since these actions are known with certainty, we have,

$$s(t_0 + dt) = \mathrm{E}_t s(t_0 + dt) = G(0|\bar{r}) = A + (C - B)\bar{r}. \tag{A.8}$$

Ruling out arbitrage profits requires that $s(t_0 + dt) = s(t_0)$. Thus equating (A.7) and (A.8) gives $(B + C) = -\bar{r}^2/(3\sigma_r^2)$. Due to the symmetry of the bands, we have $B = C = -\bar{r}^2/(6\sigma_r^2)$. Substituting back into (A.6) gives,

$$G(r|\bar{r}) = A - \frac{\bar{r}^2}{3\sigma_r^2} r + \frac{r^3}{3\sigma_r^2} \tag{A.9}$$

||

Derivation of B for Bertola-Caballero interventions in footnote 9. Again, begin by writing the guess solution explicitly in terms of the bands,

$$G(r|\bar{r}) = A + \frac{r^3(t)}{3\sigma_r^2} + B(r - \bar{r}) + C(r - \underline{r}) \tag{A.10}$$

where $\underline{r} = -\bar{r}$, and where we have already made use of the solution to the nonhomogeneous part of the differential equation. Under symmetric intervention points, we know that $B = C$. Let the bandwidth be $b = \bar{r} - \underline{r} = 2\bar{r}$. It follows that,

$$\begin{aligned} B(r - \bar{r}) + C(r - \underline{r}) &= B(r - \bar{r} + r - \underline{r}) \\ &= B[2(r - \bar{r}) + b] \end{aligned}$$

which we can use to rewrite (A.10) as,

$$G(r|\underline{r}, \bar{r}) = A + \frac{r^3(t)}{3\sigma_r^2} + 2Br \tag{A.11}$$

Now suppose that the upper band $\bar{r}$ is attained at instant t_0. Then

$$s(t_0) = G(\bar{r}|\underline{r}, \bar{r}) = A + \frac{\bar{r}^3}{3\sigma_r^2} + Bb \tag{A.12}$$

At the next instant, the authorities revalue with probability p to $G(\bar{r}+(b/2)|\bar{r}, \bar{r}+b) = A + (\bar{r} + (b/2))^3/(3\sigma_r^2)$ or defend with probability $1-p$ by setting the exchange rate to $G(\bar{r} - (b/2)|\underline{r}, \bar{r}) = A + (\bar{r} - (b/2))^3/(3\sigma_r^2)$. That is,

$$s(t_0 + dt) = \begin{cases} A + (\bar{r} + (b/2))^3/3\sigma_r^2 & \text{w.p. } p \\ A + (\bar{r} - (b/2))^3/(3\sigma_r^2) & \text{w.p. } (1-p) \end{cases} \tag{A.13}$$

To rule out expected arbitrage profits, we require $s(t_0) = \mathrm{E}_t[s(t_0 + dt)]$ from which it follows that,

$$A + \frac{\bar{r}^3}{3\sigma_r^2} + Bb = A + p\frac{(\bar{r} + (b/2))^3}{3\sigma_r^2} + (1-p)\frac{(\bar{r} - (b/2))^3}{3\sigma_r^2}. \tag{A.14}$$

Solving (A.14) for B gives,

$$B = \frac{p(\bar{r} + (b/2))^3 + (1-p)(\bar{r} - (b/2))^3 - \bar{r}^3}{3b\sigma_r^2} = (8p-1)\frac{\bar{r}^2}{6\sigma_r^2} \tag{A.15}$$

where the second equality follows from the symmetry conditions. $\|$

Derivation of (6). We begin with (4) which, for convenience, we reproduce here as

$$ds(t) = r(t)dt + \left(B + \frac{r^2(t)}{\sigma_r^2}\right)\sigma_r dz(t).$$

Integration gives,

$$\begin{aligned} s(1) - s(0) &= \int_0^1 ds(t) \\ &= \underbrace{\int_0^1 r(t)dt}_{(a)} + \underbrace{\frac{1}{\sigma_r}\int_0^1 r^2(t)dz(t)}_{(b)} + \underbrace{B\sigma_r \int_0^1 dz(t)}_{(c)} \end{aligned} \tag{A.16}$$

Since

$$r(t) - r(0) = \int_0^t dr(u) = \sigma_r \int_0^t dz(u) = \sigma_r[z(t) - z(0)], \tag{A.17}$$

it follows that

$$(a) = \int_0^1 r(t)dt = r(0) + \sigma_r \int_0^1 z(t)dt - \sigma_r z(0).$$

Next, squaring the interest differential using (A.17) gives

$$r^2(t) = r^2(0) + \sigma_r^2[z^2(t) + z^2(0) - 2z(0)z(t)] + 2r(0)\sigma_r[z(t) - z(0)] \tag{A.18}$$

Integrating (A.18) with respect to $dz(t)$ gives,

$$\begin{aligned}
(b) = \frac{1}{\sigma_r}\int_0^1 r^2(t)dz(t) &= \frac{1}{\sigma_r}\{r^2(0)\int_0^1 dz(t) + \sigma_r^2\int_0^1 z^2(t)dz(t) \\
&\quad +\sigma_r^2 z^2(0)\int_0^1 dz(t) - 2\sigma_r^2 z(0)\int_0^1 z(t)dz(t) \\
&\quad +2r(0)\sigma_r\int_0^1 z(t)dz(t) - 2r(0)z(0)\sigma_r\int_0^1 dz(t)\} \\
&= \frac{1}{\sigma_r}[r(0) - \sigma_r z(0)]^2\int_0^1 dz(t) + 2[r(0) - \sigma_r z(0)]\int_0^1 z(t)dz(t) \\
&\quad +\sigma_r\int_0^1 z^2(t)dz(t)
\end{aligned}$$

Now for part (c), we simply note that $B\sigma_r\int_0^1 dz(t) = B\sigma_r[z(1) - z(0)]$. Substitute these expressions back into (A.16) to get

$$\begin{aligned}
s(1) - s(0) &= r(0) + \sigma_r\int_0^1 z(t)dt - \sigma_r z(0) + \frac{1}{\sigma_r}[r(0) - \sigma_r z(0)]^2\int_0^1 dz(t) \\
&\quad +2[r(0) - \sigma_r z(0)]\int_0^1 z(t)dz(t) + \sigma_r\int_0^1 z^2(t)dz(t) + B\sigma_r\int_0^1 dz(t) \\
&\equiv r(0) + \eta(1)
\end{aligned}$$

Decomposing $\eta(1)$ into terms that depend on $r(0)$ and those that do not gives $\eta(1) = r(0)\epsilon(1) + v(1)$ where $\epsilon(1)$ is given by (8) and $v(1)$ is given by (9). $\|$

Asymmetric Bands. The symmetric band assumption is not key and can be relaxed. Here, we derive the exchange rate solution when $\underline{r} = -\alpha\bar{r}$. As above, the solution to the nonhomogeneous part of the differential equation is $G_n = r^3/(3\sigma_r^2)$. We write the general guess solution explicitly in terms as

$$s(t) = G(r|\underline{r},\bar{r}) = A + \frac{r^3(t)}{3\sigma_r^2} + B[\bar{r} + r(t)/\alpha] + C[\underline{r} + \alpha r(t)]. \tag{A.19}$$

At $r(t) = \bar{r}$,

$$G(\bar{r}|\underline{r},\bar{r}) = A + \frac{\bar{r}^3(t)}{3\sigma_r^2} + B[\bar{r} + \bar{r}/\alpha], \tag{A.20}$$

and

$$G'(\bar{r}|\underline{r},\bar{r}) = 0 = B\left[\frac{1+\alpha}{\alpha}\right] + \frac{\bar{r}}{\sigma_r^2}, \tag{A.21}$$

which gives

$$B = -\left[\frac{\alpha}{1+\alpha}\right]\frac{\bar{r}^2}{\sigma_r^2}. \tag{A.22}$$

Similarly, at $r(t) = \underline{r}$,

$$G(\underline{r}|\underline{r}, \bar{r}) = A + \frac{\underline{r}^3}{3\sigma_r^2} + C[\underline{r} + \alpha\underline{r}], \tag{A.23}$$

and

$$G'(\underline{r}|\underline{r}, \bar{r}) = 0 = C(1+\alpha) + \frac{\underline{r}^2}{\sigma_r^2} \tag{A.24}$$

which gives,

$$C = -\left[\frac{1}{1+\alpha}\right]\frac{\underline{r}^2}{\sigma_r^2} = -\left[\frac{\alpha^2}{1+\alpha}\right]\frac{\bar{r}^2}{\sigma_r^2} = -\alpha B \tag{A.25}$$

A.2 Simulated method of moments estimation

Let the simulated observations be denoted with a 'tilde.' For the discretized regulated Brownian motion we divided each of the $T = 1200$ weekly time periods into $N = 14$ subintervals. Experimentation using $N = 7$ and $N = 21$ subintervals produced little differences in the results. Setting $\delta_N = (1/N) \simeq dt$, we simulate sequences of (10) by

$$\tilde{r}_j = \tilde{r}_{j-1} + \sigma_r \epsilon_j \sqrt{\delta_N} \tag{A.26}$$

where $\epsilon_j \overset{iid}{\sim} N(0,1)$ and

$$\tilde{r}_j = \begin{cases} \underline{r} & \text{if} \quad \tilde{r}_{t-1} < \underline{r} \\ \tilde{r}_j & \text{if} \quad \underline{r} \le \tilde{r}_j \le \bar{r} \\ \bar{r} & \text{if} \quad \tilde{r}_{t-1} > \bar{r} \end{cases}$$

for $j = 1, \ldots, NMT$. The observations were then re-sampled at weekly intervals giving us a sequence of MT weekly observations (we use $M = 30$).

SMM estimation of this model proceeds as follows. Let $\underline{\beta}$ be the vector of parameters to be estimated, $\underline{r}' = (r_1, r_2, \ldots, r_T)$ denote the collection of the actual time-series observations, and $\{\tilde{r}_i(\underline{\beta})\}_{i=1}^M$ be the computer simulated time-series of length M which we generate according to (A.26). $\underline{\tilde{r}}'(\underline{\beta}) = (\tilde{r}_1(\underline{\beta}), \tilde{r}_2(\underline{\beta}), \ldots, \tilde{r}_M(\underline{\beta}))$ denotes the collection of these M observations. To estimate σ_r and $\bar{r}$ by matching $\mathrm{E}(\Delta r_t), \mathrm{E}(\Delta r_t)^2$, and $\mathrm{E}(r_t r_{t-1})$, we let the vector function of the data from which to simulate the moments be $\underline{h}(r_t) = (r_t, r_t^2, r_t r_{t-1})'$ and the vector of sample moments be $\underline{H}_T(\underline{r}) = \frac{1}{T}\sum_{t=1}^T \underline{h}(r_t)$. The corresponding vector of simulated moments is $\underline{H}_M(\underline{\tilde{r}}(\underline{\beta})) = \frac{1}{M}\sum_{i=1}^M \underline{h}(\tilde{r}_i(\underline{\beta}))$, where the length of the simulated series is M. Now let $\underline{u}_t = \underline{h}(r_t) - \underline{H}_T(\underline{r})$ be the deviation of $\underline{h}$ from its mean, $\hat{\mathbf{\Omega}}_0 = \frac{1}{T}\sum_{t=1}^T \underline{u}_t \underline{u}_t'$ be the sample short-run variance of $\underline{u}_t$, and $\hat{\mathbf{\Omega}}_j = \frac{1}{T}\sum_{t=1}^T \underline{u}_t \underline{u}_{t-j}'$ be the sample cross-covariance matrix of $\underline{u}_t$, $\hat{\mathbf{W}}_T = \hat{\boldsymbol{\Omega}}_0 + \frac{1}{T}\sum_{j=1}^{m}(1-\frac{j+1}{T})(\hat{\boldsymbol{\Omega}}_j + \hat{\boldsymbol{\Omega}}'_j)$ is the Newey and West (1987) estimate of the long-run covariance matrix of $\underline{u}_t$.

If we let $\underline{g}_{T,M}(\underline{\beta}) = \underline{H}_T(\underline{r}) - \underline{H}_M(\underline{\tilde{r}}(\underline{\beta}))$ be the deviation of the sample moments from the simulated moments, then the SMM estimator, $\underline{\hat{\beta}}_S$, is that value of $\underline{\beta}$ that minimizes the quadratic distance between the simulated moments and the sample moments

$$g_{T,M}(\underline{\beta})' \left[\mathbf{W}_{T,M}^{-1}\right] g_{T,M}(\underline{\beta}), \tag{A.27}$$

where $\mathbf{W}_{T,M} = \left[\left(1+\frac{T}{M}\right)\mathbf{W}_T\right]$ and is asymptotically normally distributed with

$$\sqrt{T}(\underline{\hat{\beta}}_S - \underline{\beta}) \xrightarrow{D} N(0, \mathbf{V}_S),$$

as T and $M \rightarrow \infty$ where $\mathbf{V}_S = \left[\mathbf{B}'\left[\left(1+\frac{T}{M}\right)\mathbf{W}\right]\mathbf{B}\right]^{-1}$ and $\mathbf{B} = \frac{\mathrm{E}\partial \underline{h}[\tilde{q}_j(\underline{\beta})]}{\partial \underline{\beta}}$.

We estimated $\bar{r}$ and σ_r by doing a grid search over $\bar{r} \in [2.0, 15.0]$ and minimizing with respect to σ_r for each candidate value of $\bar{r}$.