Overconfidence in Economic Contests*

Munetomo Ando†
National Graduate Institute for Policy Studies

First Draft: September 10, 2003
This Version: March 20, 2004

Abstract

This paper studies an economic contest with two participants, who have overconfidence in their own relative abilities. We examine two different sources of overconfidence, overestimation of one's own ability and underestimation of the rival's ability, and compare the behavioral consequences of each situation with the correctly estimated case. The main result is that the former always induces the participants' aggressive behavior, while the latter does not.

JEL classification numbers: D44, D82.
Keywords: Overconfidence, economic contest, relative ability, all-pay auction, distribution of abilities.

*The author is grateful to Michihiro Kandori, Hajime Kobayashi, Akihiko Matsui, Toshihiro Matsumura, Hitoshi Matsushima, Daisuke Oyama, Noriyuki Yanagawa, and seminar participants at Contract Theory Workshop (CTW), Hosei University, Kyoto Game Theory Workshop, Kyoto University and the University of Tokyo for their comments.

†Address: 2-2 Wakamatsu-cho, Shinjuku-ku Tokyo, 162-8677 Japan, TEL. +81-3-3341-0748, FAX. +81-3-3341-0220, E-mail: m.ando@grips.ac.jp (M. Ando).

1 Introduction

Overconfidence is one of the famous stylized facts about human behavior. Several studies in psychology and experimental economics show that humans are overconfident in their own (relative) abilities. For example, Svenson (1981) reports that almost all drivers in Texas believe that their own driving skills are above average. In addition, the literature usually indicates that overconfidence induces people's aggressive behavior. For example, Camerer and Lovallo (1999) show that overconfidence brings about excess entry and business failure by using an experimental approach.[1]

The aim of this paper is to study the behavioral consequences of participants' overconfidence in economic contests with two participants. In the contests, each participant has overconfidence in his relative ability. There are some possible situations where a participant has overconfidence in his relative ability. Overconfidence in relative ability comes from overestimation of one's own ability and/or underestimation of the rival's ability.

To simplify the exposition, we examine two extreme situations where each participant has overestimation of his own ability *or* underestimation of the rival's ability, and compare the behavioral consequences of each situation with the correctly estimated case. By this comparison, we will show that the different sources of overconfidence have different behavioral consequences. More precisely, we will show that in the former case overconfidence always induces the participants' aggressive behavior, while in the latter case it does not.

In this paper, we study the following contest game. The principal hires two risk neutral agents for a specific period of time and assigns a task for each of them. Each agent outlays his effort for winning. Different agents have different types, which are equal to their monetary value of winning the contest. We read agents' types as their abilities since a higher ability agent can obtain a higher return when he wins the contest. Examples of such a situation are promotion contests in firms and political elections. The types are independently and identically distributed. Each agent has his prior belief about his own type and the distribution of types, and his beliefs may or may not be correct. That is, he may have incorrect information about his type or the distribution of types.[2] Each agent chooses his effort to maximize his expected profit. The effort level of each agent is observable by all players at the end when the agents already have chosen their effort levels. For the principal, larger efforts are profitable, so that she welcomes a larger expected

[1] See Camerer (1997) for further references.

[2] Under an auction setting called the all-pay auction under incomplete information, we formulate the situation where a participant underestimates the rival's ability by dominance relations between the true distribution of types and the subjective belief about type distribution. This means that the participant believes that there are many lower ability types compared to the true distribution of types.

effort per agent.

Our results are as follows: First, we derive the symmetric Bayesian Nash equilibrium strategy in the contest without overconfidence as the benchmark. Because of the fact that the contest game considered here is the standard all-pay auction, we can identify the equilibrium effort strategies and derive the expected effort level per agent by using the standard method in auction literature. The equilibrium effort levels are strictly increasing with respect to types, so that the probability of winning in the contest is equivalent to the probability that his type is no lower than the rival's type.

Second, we show that overestimation of one's own type always increases agents' efforts, and therefore, it is profitable for the principal. The reason why overestimation of one's own type increases agents' efforts is as follows. In the situation of overestimation of one's own type, an agent behaves as if he has a higher type. Since the equilibrium strategy is increasing in types, he chooses a higher effort compared to the benchmark case.

Third, we show that underestimation of the type distribution increases efforts of agents with low abilities, while it decreases efforts of agents with high abilities. Therefore, it may or may not be profitable for the principal. Additionally, we show that underestimation increases an agent's expected efforts in some cases, while it decreases in other cases.

The fundamental reason why underestimation of the type distribution changes agents' efforts is that underestimation changes each type's subjective probability of winning. In underestimated situations, since the probability of winning at the left tail increases faster compared to the benchmark case (i.e., the correctly estimated case), underestimation induces more aggressive efforts at the left tail. For the higher types, underestimation decreases the gradient of the probability of winning function. This reflects decreases in the gradient of the effort strategy function for the higher types.

There are two strands of theoretical works related to the present paper. The first class is analyses of economic contests as a variation of the all-pay auction.[3] Recently, the contest design problems from the viewpoint of the contest designer have attracted much attention. Examples include Singh and Wittman (1998, 2001), Moldovanu and Sela (2001), and Ando (2003). However, these studies usually assume that the prior beliefs about one's own type and the type distribution are correct. Hence the present paper is different from these studies.

The second class is studies of overconfidence in the field of behavioral economics.[4] Most studies of overconfidence focus on the situations of one person's decision making. For example, Dubra (2003) studies a search model

[3]Amann and Leininger (1996) and Krishna and Morgan (1997) are examples of theoretical studies on all-pay auctions. Noussair and Silver (2003) is an experimental study of all-pay auctions under incomplete information.

[4]Camerer (2003) is a useful reference of the behavioral game theory. Itoh (2003) is a study toward the behavioral contract theory.

with an optimistic individual. In contrast to these studies, the present paper studies an auction setting incorporated with overconfidence.

The rest of this paper proceeds as follows. Section 2 describes the environment of our model. Section 3 examines the benchmark case, the behavioral consequences of overestimation of one's own type, and the behavioral consequences of underestimation of the type distribution in turn. Section 4 concludes.

2 Model

We consider an economic contest with two risk neutral agents, agents 1 and 2. They compete to win the contest. Each agent i decides his effort e_i. Efforts are outlaid simultaneously and independently. The agents' effort levels are observable by the principal and the agents at the end when the agents have already chosen their effort levels.

Each agent has a different type, which represents his monetary value of winning. The true type of agent i is denoted by θ_i and the exact value may be perceptible when he wins. We assume that each agent has his prior (or subjective) belief about his type, a_i, and this is his private information. The true type of each agent is drawn from an interval $[0, 1]$ according to the distribution function F that has a continuous and everywhere strictly positive density function f.[5]

Note that, in the standard model of economic contests based on all-pay auctions, each agent's prior belief about his own type is exactly the same as the true value and the distribution function is assumed to be common knowledge for the principal and both agents. However, in this paper we relax these assumptions. Agent i believes that his type is a_i but it may not be equal to the true type θ_i. Moreover, he believes that his type is drawn from an interval $[0, 1]$ according to the distribution function G_i and the rival's type is also drawn independently according to the distribution function G_i but G_i may not be equal to the true distribution F. Additionally, he believes that the rival j's prior belief about the type distribution is also G_i, the rival knows his own true type (i.e., $a_j = \theta_j$), and these belief structures are common knowledge.

In this contest, the real payoff of agent i is, $\theta_i - e_i$ if he wins, and $-e_i$ if he does not. However each agent chooses his effort in order to maximize his expected payoff based on his prior belief about his own type a_i and the knowledge about the type distribution G_i. For the principal, larger efforts are profitable, so that she welcomes a larger expected effort per agent.

[5] We assume that the type space is $[0, 1]$. This restriction is only for an analytical convenience. We can preserve all our results in any non-negative types with bounded support cases.

3 Analysis

3.1 The benchmark case

In this subsection, we consider the standard all-pay auction as the benchmark case. That is, we consider the situation where $a_i = \theta_i$ and $G_i = F$, $i = 1, 2$.

The symmetric equilibrium effort strategy in this contest game is as follows.

Proposition 1. *In a symmetric equilibrium, the effort strategy is*

$$\beta(\theta, F) = \int_0^\theta y f(y) dy, \tag{1}$$

and the symmetric equilibrium is unique.

Proof. The equilibrium strategy can be easily derived with the standard method in auction litarature. The uniquness can be shown in the process of derivation. Hence the proof is omitted. ∎

In the symmetric equilibrium, since $\beta(\cdot)$ is strictly increasing, agent i's probability of winning (hereafter $p(\theta_i, F)$) is equivalent to the probability that his type is no lower than the rival's type, that is, $p(\theta_i, F) = F(\theta_i)$.

In this contest, the expected effort per agent is

$$E(e, F) = \int_0^1 \beta(\theta) f(\theta) d\theta. \tag{2}$$

By using integration by parts with expressions (1) and (2), we obtain the following.

Proposition 2. *The expected effort per agent is*

$$E(e, F) = \int_0^1 (1 - F(\theta))\theta f(\theta) d\theta. \tag{3}$$

We provide an example.

Example 3.1. *If* $F(\theta) = \theta$, $\beta(\theta, F) = \theta^2/2$ *and* $E(e, F) = 1/6$.

3.2 Overestimation of one's own type

In this subsection, we examine the consequences of overestimation of one's own type in economic contests. We assume that the prior knowledge about the type distribution is correct (i.e., $G_1 = G_2 = F$). However, they have incorrect information about their own types. We assume that they overestimate their types in a systematic way, for given $\theta_i \in [0, 1)$, $a_i > \theta_i$, $i = 1, 2$.

In this situation, each agent derives the equilibrium strategy $\beta(\cdot, F)$ by F. Then he chooses his action by calculation with $\beta(\cdot, F)$ and a_i. From the above facts, we obtain the following.

Proposition 3. *For an agent i with true type $\theta_i \in [0,1)$, overestimation of his own type increases his effort (i.e., $\beta(a_i, F) > \beta(\theta_i, F)$ for all $\theta_i \in [0,1)$ and for all $a_i > \theta_i$).*

Proof. The effort strategy $\beta(\cdot, F)$ is constructed by F, and thereofore, $\beta(a, F) = \int_0^a yf(y)dy$. Since $yf(y)$ is positive for all $y \in (0,1]$, $\int_0^a yf(y)dy > \int_0^\theta yf(y)dy$ for all $a > \theta$. Thus, we obtain $\beta(a_i, F) > \beta(\theta_i, F)$. ∎

This proposition implies that, in the situations of overestimation of one's own type, an agent who has a true type θ_i behaves as if he has a higher type, a_i. Since the equilibrium strategy is increasing in types, he chooses a higher effort compared to the benchmark case. Thus, overconfidence from overestimation of one's own type is always profitable for the principal.

We provide an example. In this example, an agent with a true type $\theta \in (0,1)$ has an overestimation of his type that is constructed by the following simple rule, $a = \sqrt{\theta}$. This formulation permits us to directly calculate the expected effort level.

Example 3.2. *Suppose that $F(\theta) = \theta$ and $a_i = \sqrt{\theta_i}$. In this situation, the effort strategy is $\beta(a_i, F) = \theta_i/2$ and the expected effort level per agent is $E(\beta(a_i, F)) = 1/4$. If the agent knows his true type, he follows the following strategy $\beta(\theta_i, F) = {\theta_i}^2/2$ and the expected effort level per agent is $E(\beta(\theta_i)) = 1/6$.*

The strategies in the above example are depicted in Figure 1. The horizontal axis is the true types and the vertical axis is the effort levels.

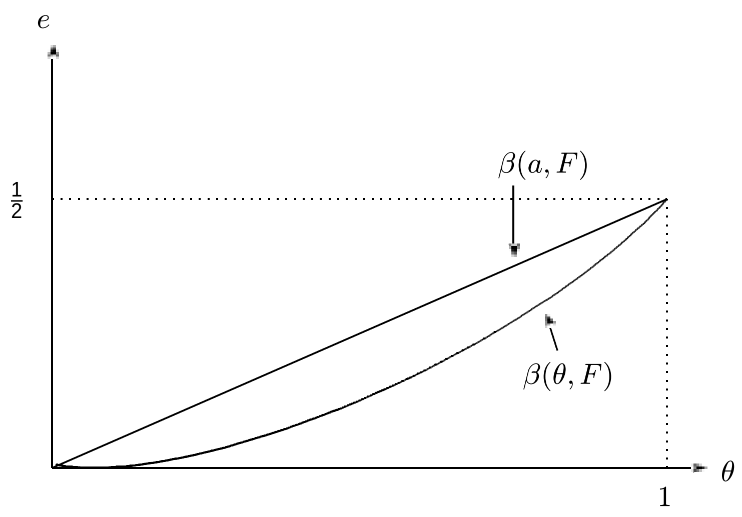

Figure 1:

3.3 Underestimation of the type distribution

In this subsection, we examine the consequences of underestimation of the type distribution. We assume that each agent i knows his true type, that is, $a_i = \theta_i$, $i = 1, 2$. However, both agents have incorrect information about the type distribution. We assume that, $G_1 = G_2 = G \neq F$.

The next proposition shows the effort strategy under incorrect information about the type distribution.

Proposition 4. *If both agents have incorrect information about the type distribution, they choose their effort as follows:*

$$\beta(\theta, G) = \int_0^\theta yg(y)dy. \tag{4}$$

Proof. The proof is omitted. ∎

In this situation, the expected effort per agent is

$$E(e, G) = \int_0^1 \beta(\theta, G)f(\theta)d\theta. \tag{5}$$

Note that this expected effort is derived from the following facts. Each type's effort level is calculated by $\beta(\cdot, G)$ and θ_i. However, the expectation is based on the true distribution of types, F.

By using integration by parts with expressions (4) and (5), we obtain the following.

Proposition 5. *The expected effort per agent is*

$$E(e, G) = \int_0^1 (1 - F(\theta))\theta g(\theta)d\theta. \tag{6}$$

Now, we turn our attention to the characteristics of incorrect information about the type distribution. We define the following.

Definition 1. *For $G \neq F$, both agents underestimate the type distribution if F first order stochastically dominates G, that is,*

$$\forall\theta \in [0, 1], G(\theta) \geq F(\theta).$$

An intuitive explanation of underestimation is that both agents believe that there are many lower types compared to the true distribution of types.

Next, we describe how underestimation changes agents' behavior.

Proposition 6. *Compared with the benchmark case, underestimation of type distribution changes agents' behavior as follows. For $G \neq F$,*

1. *there exists $\hat{\theta} \in (0, 1)$ such that, for all $\theta \in (\hat{\theta}, 1]$, $\beta(\theta, G) < \beta(\theta, F)$, and*

2. *there exists an interval of types* (θ_a, θ_b) *such that for all* $\theta \in (\theta_a, \theta_b)$, $\beta(\theta, G) > \beta(\theta, F)$.

Proof. The proof of the former statement is as follows. $\beta(\theta, G) - \beta(\theta, F) = \int_0^\theta yg(y)dy - \int_0^\theta yf(y)dy$. By using integration by parts we obtain $\beta(\theta, G) - \beta(\theta, F) = \theta(G(\theta) - F(\theta)) - \int_0^\theta (G(y) - F(y))dy$. This is strictly negative at $\theta = 1$, since $\theta(G(\theta) - F(\theta))$ is zero and $\int_0^\theta (G(y) - F(y))dy$ is strictly positive at $\theta = 1$ by definition. By the facts that β is increasing and continuous, we can conclude that there exists $\hat{\theta} \in (0, 1)$ such that, for all $\theta \in (\hat{\theta}, 1]$, $\beta(\theta, G) < \beta(\theta, F)$.

The proof of the latter statement is as follows. We define $\theta_c = \inf\{\theta \mid G(\theta) > F(\theta)\}$. For any $\theta \in [0, \theta_c)$, $\beta(\theta, G) = \beta(\theta, F)$, since for any $\theta \in [0, \theta_c)$, $g(\theta) = f(\theta)$, and $\beta(\theta, G) = \int_0^\theta yg(y)dy$ and $\beta(\theta, F) = \int_0^\theta yf(y)dy$. For sufficiently small $\varepsilon > 0$, $\beta(\theta_c + \varepsilon, G) - \beta(\theta_c + \varepsilon, F) = \int_{\theta_c}^{\theta_c+\varepsilon} y(g(y) - f(y))dy$ and this is strictly positive, since $g(\theta_c + \varepsilon) > f(\theta_c + \varepsilon)$ by definition of θ_c. By the facts that β is increasing and continuous, we can conclude that there exists an interval of types (θ_a, θ_b) such that for all $\theta \in (\theta_a, \theta_b)$, $\beta(\theta, G) > \beta(\theta, F)$. ∎

The implication of the above proposition is that, if both agents underestimate the type distribution, some types over work and other types under work compared with the benchmark case. There is a possibility that the sign of $\beta(\theta, G) - \beta(\theta, F)$ changes more than once (that is, $\theta_b \neq \hat{\theta}$). However, to simplify the exposition, we restrict our attention to the situations where $\beta(\theta, G)$ and $\beta(\theta, F)$ are single-crossing in the interval $(0, 1)$ (that is, $\theta_a = 0$ and $\theta_b = \hat{\theta}$) in the rest of this paper.

We can describe a sufficient condition for single-crossing of $\beta(\theta, G)$ and $\beta(\theta, F)$.

Proposition 7. $\beta(\theta, G)$ *and* $\beta(\theta, F)$ *are single-crossing in the interval* $(0, 1]$, *if there exists* $\bar{\theta} \in (0, 1)$, *such that* $\forall\theta < \bar{\theta}$, $f(\theta) < g(\theta)$, *and* $\forall\theta > \bar{\theta}$, $f(\theta) > g(\theta)$.

Proof. $\beta(\theta, G) - \beta(\theta, F) = \int_0^\theta y(g(y) - f(y))dy$. This is positive at the type just by zero, since $g(y) > f(y)$ at $y = 0$. Moreover, $g(y)$ and $f(y)$ are crossing only once at a certain $y \in (0, 1)$ and at the left hand side of there, $g(y) > f(y)$, and at the right hand side, $g(y) < f(y)$. Since $\int_0^1 (g(y) - f(y))dy = 0$ and y is strictly increasing function, we obtain $\int_0^1 y(g(y) - f(y))dy < 0$.

Next, we can show that $\exists\acute{\theta}$, $\forall\theta < \acute{\theta}, \partial\beta(\theta, G)/\partial\theta > \partial\beta(\theta, F)/\partial\theta$ and $\forall\theta > \acute{\theta}, \partial\beta(\theta, G)/\partial\theta < \partial\beta(\theta, F)/\partial\theta$, since $\partial\beta(\theta, G)/\partial\theta = \theta g(\theta)$ and $\partial\beta(\theta, F)/\partial\theta = \theta f(\theta)$.

From the above facts and the facts that β is increasing and continuous, we can conclude that $\beta(\theta, G)$ and $\beta(\theta, F)$ are single-crossing in the interval $(0, 1)$. ∎

This proposition tells us that, if the density functions are single-crossing, $\beta(\theta, G)$ and $\beta(\theta, F)$ are also single-crossing in the interval $(0, 1)$.

We briefly describe the reason why underestimation of the type distribution changes agents' efforts under single-crossing situations. When an agent with true type θ underestimates the type distribution, his subjective probability of winning in the contest is changed from $F(\theta)$ to $G(\theta)$. In this underestimated situation, since the probability of winning at the left tail increases faster compared to the benchmark case, underestimation induces more aggressive effort at the left tail. For the higher types, underestimation decreases the gradient of the probability of winning function. This reflects decreases in the gradient of the effort strategy function for the higher types.

We provide an example.

Example 3.3. *Suppose that* $a_i = \theta_i, i = 1, 2$, *and* $F(\theta) = \theta$ *and* $G(\theta) = 2\theta - {\theta_i}^2$. *In this situation,* $\beta(\theta, F) = \theta^2/2$ *and* $\beta(\theta, G) = \theta^2 - 2\theta^3/3$.

The above example is depicted in Figure 2. The horizontal axis is the types and the vertical axis is the effort levels.

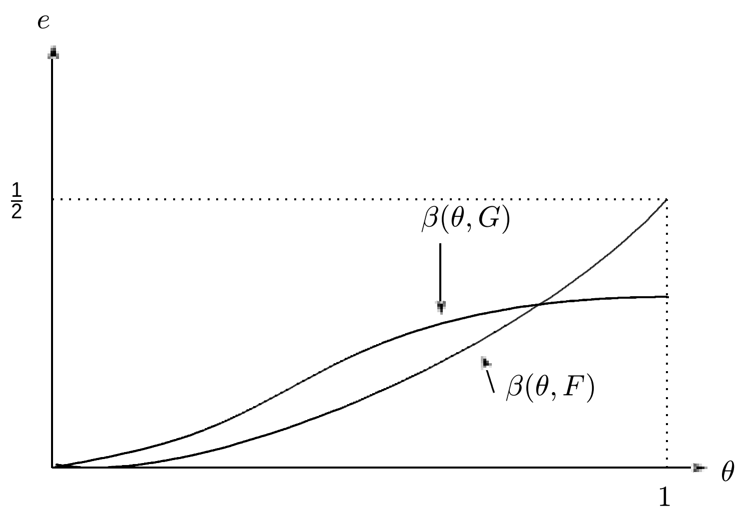

Figure 2:

Finally, we describe changes in the expected effort brought by underestimation of type distribution. In the following proposition, we show that the necessary and sufficient condition for $E(e, G) > E(e, F)$. If the condition holds, the expected value of increases in the lower types' efforts is larger than that of decreases in the higher types' efforts, so that underestimation increases the agents' expected efforts. Consequently, the underestimation is profitable for the principal. Otherwise, underestimation decreases the agents' expected efforts and is not profitable for the principal.

Proposition 8. *$E(e, G) > E(e, F)$ if and only if* $\int_0^1 (1 - F(\theta))\theta(g(\theta) - f(\theta))d\theta$.

Proof. This statement is straightforward from Propositions 2 and 5. ∎

We provide two examples. The former shows that underestimation may increase the agents' expected efforts and the latter shows that underestimation may decreases the agents' expected efforts.

Example 3.4. *Suppose that $a_i = \theta_i, i = 1, 2$, and $F(\theta) = \theta$. In the correctly estimated case, $E(e, F) = 1/6$.*

- *If $G(\theta) = (3\theta - \theta^3)/2$, $E(e, G) = 7/40$.*
- *If $G(\theta) = 3\theta - 2\theta^{3/2}$, $E(e, G) = 11/70$.*

4 Concluding Remarks

In this paper, we have examined two different sources of overconfidence and have compared the behavioral consequences of each situation with the benchmark case. The main result is that overestimation of one's own ability always induces participants' aggressive behavior, while underestimation of the rival's ability does not. To conclude the paper, three remarks are in order.

First, for an agent, both overestimation of his own type and underestimation of the type distribution increase his subjective probability of winning in the contest. For example, consider an agent with true type $1/2$ in the situation where $F(\theta) = \theta$. In this situation, his true probability of winning is $1/2$. However, in the situation of overestimation of his own type as $a_i = \sqrt{\theta_i}$, his subjective probability of winning is $\sqrt{1/2}$ that is strictly larger than the true value, $1/2$. In the situation where he underestimates the type distribution as $G(\theta) = 2\theta - \theta^2$, his subjective probability of winning is $3/4$ that is also strictly larger than the true value, $1/2$. In both situations, he has an overestimation about the probability of winning in the contest. However, these two situations may yield different consequences. The former is always profitable for the principal but the latter may not be.

Second, experiments of economic contests under the situation where participants have overconfidence may be interesting topics. However, they involve some difficulties. In the all-pay auction experiments without overconfidence, Noussair and Silver (2003) observed over-bidding behavior of the subjects.[6] So, if we observe over-bidding (or under-bidding) in experiments

[6] Dorsey and Razzolini (2002) attempted to describe the source of over-bidding behavior of bidders in first-price auctions by auction experiments and compared the choices under a first-price auction and incentive-wise identical lottery. They concluded that the subjects cannot calculate the probability of winning in auctions appropriately, so that over-bidding occurs.

with overconfidence, we cannot conclude immediately that overconfidence yields over-bidding (or under-bidding). Consequently, we should design experiments carefully to reach a conclusion.

Third, we have showed that the different sources of overconfidence have different behavioral consequences. This type of conclusion is not only in situations of contests or auctions. Consider Bertrand competition in differentiated duopoly. Since strategic complementarities exist, lower marginal costs of one's own firm (i.e., higher one's own relative abilities) yield his aggressive behavior and higher marginal costs of the rival firm (i.e., lower the rival's relative abilities) yield his less aggressive behavior.

References

[1] Amann, E., and Leininger, W., 1996. Asymmetric All-Pay Auctions with Incomplete Information: The Two-Player Case, *Games Econ. Behav.* **14**, 1-18.

[2] Ando, M., 2003. Division of a Contest with Identical Prizes, forthcoming in *J. Japanese Int. Economies.*

[3] Camerer, C., 1997. Progress in Behavioral Game Theory, *J. Econ. Perspect.* 167-188.

[4] Camerer, C., 2003, *Behavioral Game Theory: Experiments in Strategic Interaction.* Princeton University Press.

[5] Camerer, C., and Lovallo, D., 1999. Overconfidence and Excess Entry: An Experimental Approach, *Amer. Econ. Rev.* **89**, 306-318.

[6] Dorsey, R., and Razzolini, L., 2002. Explaining Overbidding in First Price Auctions Using Controlled Lotteries, *Exper. Econ.* **6**, 123-140.

[7] Dubra, J., 2004. Optimism and Overconfidence in Search, *Rev. Econ. Dynam.* **7**, 198-218.

[8] Itoh, H., 2003. Moral Hazard and Other-Regarding Preferences, mimeograph, Hitotsubashi University.

[9] Krishna, V., and Morgan, J., 1997. An Analysis of the War of Attrition and the All-Pay Auction, *J. Econ. Theory* **72**, 343-362.

[10] Moldovanu, B., and Sela, A., 2001. The Optimal Allocation of Prizes in Contests, *Amer. Econ. Rev.* **91**, 542-558.

[11] Noussair, C., and Silver, J., 2003. Behavior in All-Pay Auctions with Incomplete Information, mimeograph, Emory University.

[12] Singh, N. and Wittman, D., 1998. Contest Design and the Objective of the Contest Designer: Sales Promotion, Sporting Events, and Patent Races, *in* "Advances in Applied Microeconomics," Volume 7, pp. 139-167, JAI Press.

[13] Singh, N. and Wittman, D., 2001. Contests Where There is Variation in the Marginal Productivity of Effort, *Econ. Theory* **18**, 711-744.

[14] Svenson, O., 1981. Are We All Less Risky and More Skillful Than Our Fellow Drivers? *Acta Psychologica* **47**, 143-148.