Constructing a Coincident Index of Business Cycles Without Assuming a One-Factor Model

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Abstract

The Stock–Watson coincident index and its subsequent extensions assume a static linear one-factor model for the component indicators. Such assumption is restrictive in practice, however, with as few as four indicators. In fact, such assumption is unnecessary if one poses the index construction problem as optimal prediction of latent monthly real GDP. This paper estimates a VAR model for latent monthly real GDP and other indicators using the observable mixed-frequency series. The EM algorithm is useful for overcoming the computational difficulty, especially in model selection. The smoothed estimate of latent monthly real GDP is the proposed index.

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1 Introduction

Since the seminal work by Stock and Watson (1989, 1991), it has been standard in the literature on business cycle indices to assume a static linear one-factor model for coincident indicators, and use the estimated “common factor” as a coincident index; e.g., Kim and Yoo (1995), Diebold and Rudebusch (1996), Chauvet (1998), Kim and Nelson (1998), and Mariano and Murasawa (2003). This one-factor structure assumption is restrictive in practice, however. Indeed, Murasawa (2003) tests the covariance structure of the four US coincident indicators used in these works, and finds a strong evidence against the one-factor structure assumption.

This paper proposes a method for constructing a coincident index without assuming a one-factor model. The idea is simple. Many, if not all, will agree that if we observe real GDP promptly on monthly basis, then we do not need a coincident index. If so, then it suffices to predict the current monthly real GDP, which does not require a one-factor model. Moreover, as Mariano and Murasawa (2003) point out, an index must have an economic interpretation, because the heights of the peaks and the depths of the troughs depend on the choice of an index. While that paper includes real GDP in the one-factor model to relate the common factor to monthly real GDP, this paper estimates monthly real GDP directly.

We use a VAR model for prediction; thus we estimate a VAR model for monthly real GDP and other indicators using the observable mixed-frequency series. As in Mariano and Murasawa (2003), we derive a state-space model for the observable mixed-frequency series, and treat the mixed-frequency series as monthly series with missing observations. ML estimation of a linear Gaussian state-space model with missing observations is standard. The smoothed estimate of monthly real GDP is the proposed index. Note that this is a “composite” index, because the smoothing algorithm combines the component indicators.

In practice, quasi-Newton methods may fail because VAR models often involve too many parameters. An alternative method is the EM algorithm. Shumway and Stoffer (1982) derive the EM algorithm for estimating a linear Gaussian state-space model with missing observations. Since the EM algorithm slows down significantly near the optimum, Watson and Engle (1983) suggest using the EM algorithm to obtain a good initial value for a quasi-Newton method; see also Demos and Sentana (1998).

Our approach relates index construction to interpolation of quarterly real GDP, for which one also uses a state-space model; e.g., Bernanke, Gertler, and Watson (1997), Cuche and Hess (1999, 2000), and Liu and Hall (2001). One usually estimates a single equation for interpolation, however, which may be less efficient but more tractable than estimating the whole VAR model. Also, one often interpolates the components of real GDP first, and then add them up.

One can extend our framework in several ways. First, one can predict monthly real GDP, say, six months ahead and behind, and use them as leading and lagging indices. Second, one can use a factor model instead of a VAR model, not to extract the common factor but to reduce the number of the parameters (hence it can be a multi-factor model). Third, one can set up a VAR model for monthly real GDP and principal components extracted from many indicators, or “diffusion indices”; see Stock and Watson (2002). Fourth, one can introduce Markov-switching into a VAR model; see Krolzig (1997). Fifth, one can predict the components of monthly real GDP first, and then add them up.

The plan of the paper is as follows. Section 2 sets up a VAR model for monthly series, some of which are latent, and derives a state-space model for the observable mixed-frequency series. Section 3 summarizes the Kalman filtering and smoothing algorithm, and Section 4 explains the EM algorithm for estimating the state-space model. Section 5 applies the method to the US quarterly real GDP and monthly coincident indicators to obtain a new coincident index, and compares it with other indices. Section 6 discusses remaining issues.

2 Mixed-Frequency VAR Model

2.1 VAR Model

Let $\{Y_t\}$ be an N-variate random sequence. Assume that $\{\ln Y_t\}$ is integrated of order 1. Write $Y_t := (Y_{t,1}', Y_{t,2}')'$, where $\{Y_{t,1}\}$ is an N_1-variate quarterly sequence (observable every third period) and $\{Y_{t,2}\}$ is an N_2-variate monthly sequence.

Let $\{Y_{t,1}^*\}$ be a latent sequence underlying $\{Y_{t,1}\}$ such that for all t,

$$\ln Y_{t,1} = \frac{1}{3}\left(\ln Y_{t,1}^* + \ln Y_{t-1,1}^* + \ln Y_{t-2,1}^*\right), \tag{1}$$

i.e., $Y_{t,1}$ is the geometric mean of $Y_{t,1}^*$, $Y_{t-1,1}^*$, and $Y_{t-2,1}^*$. Taking the three-period differences, for all t,

$$\begin{aligned}\ln Y_{t,1} - \ln Y_{t-3,1} &= \frac{1}{3}\left(\ln Y_{t,1}^* - \ln Y_{t-3,1}^*\right) + \frac{1}{3}\left(\ln Y_{t-1,1}^* - \ln Y_{t-4,1}^*\right) \\ &\quad + \frac{1}{3}\left(\ln Y_{t-2,1}^* - \ln Y_{t-5,1}^*\right),\end{aligned}$$

or

$$\begin{aligned}y_{t,1} &= \frac{1}{3}\left(y_{t,1}^* + y_{t-1,1}^* + y_{t-2,1}^*\right) + \frac{1}{3}\left(y_{t-1,1}^* + y_{t-2,1}^* + y_{t-3,1}^*\right) \\ &\quad + \frac{1}{3}\left(y_{t-2,1}^* + y_{t-3,1}^* + y_{t-4,1}^*\right) \\ &= \frac{1}{3}y_{t,1}^* + \frac{2}{3}y_{t-1,1}^* + y_{t-2,1}^* + \frac{2}{3}y_{t-3,1}^* + \frac{1}{3}y_{t-4,1}^*,\end{aligned}$$

where $y_{t,1} := \Delta_3 \ln Y_{t,1}$ and $y_{t,1}^* := \Delta \ln Y_{t,1}^*$. We observe $y_{t,1}$ every third period, and never observe $y_{t,1}^*$.

Let for all t,

$$y_t := \begin{pmatrix} y_{t,1} \\ y_{t,2} \end{pmatrix}, \quad y_t^* := \begin{pmatrix} y_{t,1}^* \\ y_{t,2} \end{pmatrix},$$

where $y_{t,2} := \Delta \ln Y_{t,2}$. Let

$$\begin{aligned}H(L) &:= \begin{bmatrix} (1/3)I_{N_1} & 0 \\ 0 & I_{N_2} \end{bmatrix} + \begin{bmatrix} (2/3)I_{N_1} & 0 \\ 0 & 0 \end{bmatrix} L + \begin{bmatrix} I_{N_1} & 0 \\ 0 & 0 \end{bmatrix} L^2 \\ &\quad + \begin{bmatrix} (2/3)I_{N_1} & 0 \\ 0 & 0 \end{bmatrix} L^3 + \begin{bmatrix} (1/3)I_{N_1} & 0 \\ 0 & I_{N_2} \end{bmatrix} L^4.\end{aligned}$$

Then for all t,

$$y_t = H(L) y_t^*. \tag{2}$$

Assume a VAR(p) model for $\{y_t^*\}$ such that for all t,

$$\Phi(L)(y_t^* - \mu^*) = w_t, \tag{3}$$

$$w_t \sim \mathrm{NID}(0, \Sigma). \tag{4}$$

2.2 A State-Space Representation

If $p \leq 5$, then define the state vector as for all t,

$$s_t := \begin{pmatrix} y_t^* - \mu^* \\ \vdots \\ y_{t-4}^* - \mu^* \end{pmatrix}.$$

We can write for all t,

$$\begin{aligned}
y_t^* - \mu^* &= [\,\Phi_1 \quad \ldots \quad \Phi_p \quad O_{N\times(5-p)N}\,] \begin{pmatrix} y_{t-1}^* - \mu^* \\ \vdots \\ y_{t-4}^* - \mu^* \end{pmatrix} + w_t \\
&= [\,\Phi \quad O_{N\times(5-p)N}\,]\, s_{t-1} + w_t \\
&= \begin{bmatrix} \phi_1' & o_{(5-p)N}' \\ \vdots & \vdots \\ \phi_N' & o_{(5-p)N}' \end{bmatrix} s_{t-1} + w_t \\
&= \begin{bmatrix} s_{t-1}' \begin{pmatrix} \phi_1 \\ o_{(5-p)N} \end{pmatrix} \\ \vdots \\ s_{t-1}' \begin{pmatrix} \phi_N \\ o_{(5-p)N} \end{pmatrix} \end{bmatrix} + w_t \\
&= \begin{bmatrix} s_{t-1}' & & 0 \\ & \ddots & \\ 0 & & s_{t-1}' \end{bmatrix} \begin{pmatrix} \begin{pmatrix} \phi_1 \\ o_{(5-p)N} \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \phi_N \\ o_{(5-p)N} \end{pmatrix} \end{pmatrix} + w_t \\
&= \left(I_N \otimes s_{t-1}'\right) F\phi + w_t,
\end{aligned}$$

where

$$F := \begin{bmatrix} I_{pN} & & 0 \\ O_{(5-p)N\times N} & & \\ & \ddots & \\ & & I_{pN} \\ 0 & & O_{(5-p)N\times N} \end{bmatrix}.$$

A state-space representation is for all t,

$$\begin{aligned}
s_{t+1} &= As_t + Bz_t, && (5) \\
y_t &= \mu + Cs_t + Dz_t, && (6) \\
z_t &\sim \mathrm{NID}(0, I_N), && (7)
\end{aligned}$$

where

$$\begin{aligned}
A &:= \begin{bmatrix} \Phi_1 & \ldots & \Phi_p & O_{N\times(5-p)N} \\ & I_{4N} & & O_{4N\times N} \end{bmatrix}, \\
B &:= \begin{bmatrix} \Sigma^{1/2} \\ O_{4N\times N} \end{bmatrix}, \\
C &:= [\,H_0 \quad \ldots \quad H_4\,], \\
D &:= O_{N\times N}.
\end{aligned}$$

If $p \geq 5$, then define the state vector as for all t,

$$s_t := \begin{pmatrix} y_t^* - \mu^* \\ \vdots \\ y_{t-p+1}^* - \mu^* \end{pmatrix}.$$

We can write for all t,

$$y_t^* - \mu^* = \left(I_N \otimes s_{t-1}'\right) \phi + w_t.$$

We have the same state-space representation except that

$$\begin{aligned} A &:= \begin{bmatrix} \Phi_1 & \dots & \Phi_{p-1} & \Phi_p \\ & I_{(p-1)N} & & O_{(p-1)N \times N} \end{bmatrix}, \\ B &:= \begin{bmatrix} \Sigma^{1/2} \\ O_{(p-1)N \times N} \end{bmatrix}, \\ C &:= \begin{bmatrix} H_0 & \dots & H_4 & O_{N \times (p-5)N} \end{bmatrix}. \end{aligned}$$

2.3 Missing Observations

Let for all t,

$$y_{t,1}^+ := \begin{cases} y_{t,1} & \text{if } y_{t,1} \text{ is observable} \\ v_t & \text{otherwise} \end{cases}.$$

Write for all t,

$$\begin{pmatrix} y_{t,1} \\ y_{t,2} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} s_t.$$

Then for all t,

$$\begin{pmatrix} y_{t,1}^+ \\ y_{t,2} \end{pmatrix} = \begin{pmatrix} \mu_{t,1} \\ \mu_2 \end{pmatrix} + \begin{bmatrix} C_{t,1} \\ C_2 \end{bmatrix} s_t + \begin{pmatrix} D_{t,1} \\ 0 \end{pmatrix} v_t,$$

where

$$\begin{aligned} \mu_{t,1} &= \begin{cases} \mu_1 & \text{if } y_{t,1} \text{ is observable} \\ 0 & \text{otherwise} \end{cases}, \\ C_{t,1} &= \begin{cases} C_1 & \text{if } y_{t,1} \text{ is observable} \\ 0 & \text{otherwise} \end{cases}, \\ D_{t,1} &= \begin{cases} 0 & \text{if } y_{t,1} \text{ is observable} \\ I_{N_1} & \text{otherwise} \end{cases}. \end{aligned}$$

Thus we have a state-space model for $\left\{y_t^+\right\}$ s.th. for all t,

$$s_{t+1} = As_t + Bz_t, \tag{8}$$
$$y_t^+ = \mu_t + C_t s_t + D_t v_t, \tag{9}$$
$$z_t \sim \text{NID}(0, I_N). \tag{10}$$

3 Kalman Filtering and Smoothing

3.1 Updating

Let for all t,

$$\begin{aligned} S_t &:= (s_1, \dots, s_t), \\ Y_t^+ &:= \left(y_1^+, \dots, y_t^+\right). \end{aligned}$$

Let for all t, s,

$$\begin{aligned} s_{t|s} &:= \mathrm{E}\left(s_t | Y_s^+\right), \\ P_{t|s} &:= \mathrm{V}\left(s_t | Y_s^+\right). \end{aligned}$$

We have for all t,

$$\begin{pmatrix} s_t \\ y_t^+ \end{pmatrix} | Y_{t-1}^+ \sim \mathrm{N}\left(\begin{pmatrix} s_{t|t-1} \\ \mu_t + C_t s_{t|t-1} \end{pmatrix}, \begin{bmatrix} P_{t|t-1} & P_{t|t-1} C_t' \\ C_t P_{t|t-1} & C_t P_{t|t-1} C_t' + D_t D_t' \end{bmatrix} \right).$$

Hence the updating equations are for all t,

$$\begin{aligned} s_{t|t} &= s_{t|t-1} + P_{t|t-1} C_t' \left(C_t P_{t|t-1} C_t' + D_t D_t' \right)^{-1} \left(y_t - \mu_t - C_t s_{t|t-1} \right) \\ &= s_{t|t-1} + K_t e_t, \\ P_{t|t} &= P_{t|t-1} - P_{t|t-1} C_t' \left(C_t P_{t|t-1} C_t' + D_t D_t' \right)^{-1} C_t P_{t|t-1} \\ &= (I_M - K_t C_t) P_{t|t-1}, \end{aligned}$$

where

$$\begin{aligned} K_t &:= P_{t|t-1} C_t' \left(C_t P_{t|t-1} C_t' + D_t D_t' \right)^{-1}, \\ e_t &:= y_t^+ - \mu_t - C_t s_{t|t-1}. \end{aligned}$$

3.2 Prediction

The prediction equations are for all t,

$$\begin{aligned} s_{t+1|t} &= A s_{t|t}, \\ P_{t+1|t} &= A P_{t|t} A' + B B'. \end{aligned}$$

3.3 Fixed-Interval Smoothing

The following algorithm proposed by de Jong (1989) avoids inversion of large matrices, and hence is more efficient than the standard one; see Durbin and Koopman (2001, sec. 4.3). Let $r_{T+1} := 0$, $R_{T+1} := 0$, and for $t = T, \ldots, 1$,

$$r_t = C_t' \left(C_t P_{t|t-1} C_t' + D_t D_t' \right)^{-1} e_t + L_t' r_{t+1}, \tag{11}$$

$$R_t = C_t' \left(C_t P_{t|t-1} C_t' + D_t D_t' \right)^{-1} C_t + L_t' R_{t+1} L_t, \tag{12}$$

where

$$L_t := A(I_M - K_t C_t).$$

The smoothing equations are for all t,

$$s_{t|T} = s_{t|t-1} + P_{t|t-1} r_t, \tag{13}$$

$$P_{t|T} = P_{t|t-1} - P_{t|t-1} R_t P_{t|t-1}. \tag{14}$$

The EM algorithm for estimating the state-space model requires the smoothed autocovariance matrices as well. Let for all t, s, r,

$$P_{t,s|r} := \mathrm{Cov}\left(s_t, s_s | Y_r^+\right).$$

De Jong and MacKinnon (1988) show that for all t, for $s \geq 1$,

$$P_{t+s,t|T} = \left(I_M - P_{t+s|t+s-1} R_{t+s} \right) L_{t+s-1} \cdots L_t P_{t|t-1}. \tag{15}$$

In particular, for all t,

$$P_{t+1,t|T} = \left(I_M - P_{t+1|t} R_{t+1} \right) L_t P_{t|t-1}. \tag{16}$$

4 Parameter Estimation

4.1 Likelihood Function

Assume that $p \leq 5$ (the derivation is easier when $p \geq 5$). Assume for simplicity that μ^* is known to be 0. Let $\theta := (\text{vec}(\Phi)', \text{vech}(\Sigma)')'$. We consider an approximate ML estimator of θ, taking s_0 as given. Let $\Omega \subset \{1, \ldots, T\}$ be the set of periods for which $y_{t,1}$ is missing. By the prediction error decomposition,

$$\begin{aligned} f\left(Y_T^+, S_T; \theta\right) &= \prod_{t=1}^{T} f\left(y_{t,1}^+, y_{t,2} | s_t, Y_{t-1}^+, S_{t-1}; \theta\right) f\left(s_t | Y_{t-1}^+, S_{t-1}; \theta\right) \\ &= \prod_{t=1}^{T} f\left(y_{t,1}^+ | s_t; \theta\right) f\left(s_t | s_{t-1}; \theta\right) \\ &= \prod_{t \in \Omega} f(v_t) \prod_{t=1}^{T} f(y_t^* | s_{t-1}; \theta). \end{aligned}$$

Let

$$\begin{aligned} G &:= \begin{bmatrix} I_N & O_{N \times 4N} \end{bmatrix}, \\ \Phi_0 &:= \begin{bmatrix} \Phi & O_{N \times (5-p)N} \end{bmatrix}, \end{aligned}$$

so that for all t, $Gs_t = y_t^* = \Phi_0 s_{t-1} + w_t$. Then the log-likelihood function of θ given $\left(Y_T^+, S_T\right)$ is

$$\begin{aligned} \ln L\left(\theta; Y_T^+, S_T\right) &= \sum_{t \in \Omega} \ln f(v_t) - \frac{NT}{2} \ln 2\pi - \frac{T}{2} \ln \det(\Sigma) \\ &\quad - \frac{1}{2} \sum_{t=1}^{T} (y_t^* - \Phi_0 s_{t-1})' \Sigma^{-1} (y_t^* - \Phi_0 s_{t-1}) \\ &= \sum_{t \in \Omega} \ln f(v_t) - \frac{NT}{2} \ln 2\pi - \frac{T}{2} \ln \det(\Sigma) \\ &\quad - \frac{1}{2} \sum_{t=1}^{T} \left[Gs_t - \left(I_N \otimes s_{t-1}'\right) F\phi\right]' \Sigma^{-1} \end{aligned}$$

4.2 Score Function

The score functions are

$$\begin{aligned} &\frac{\partial \ln L\left(\theta; Y_T^+, S_T\right)}{\partial \phi} \\ &= \sum_{t=1}^{T} F' \left(I_N \otimes s_{t-1}'\right)' \Sigma^{-1} \left[Gs_t - \left(I_N \otimes s_{t-1}'\right) F\phi\right] \\ &= F' \left(\Sigma^{-1} \otimes I_M\right) \text{vec} \left(\sum_{t=1}^{T} s_{t-1} s_t' G'\right) - F' \left(\Sigma^{-1} \otimes \sum_{t=1}^{T} s_{t-1} s_{t-1}'\right) F\phi, \end{aligned}$$

$$\begin{aligned} &\frac{\partial \ln L(\theta; Y_T, S_T)}{\partial \Sigma^{-1}} \\ &= \frac{T}{2} \Sigma - \frac{1}{2} \sum_{t=1}^{T} (Gs_t - \Phi_0 s_{t-1})(Gs_t - \Phi_0 s_{t-1})' \\ &= \frac{T}{2} \Sigma - \frac{1}{2} \sum_{t=1}^{T} \left(Gs_t s_t' G' - Gs_t s_{t-1}' \Phi_0' - \Phi_0 s_{t-1} s_t' G' + \Phi_0 s_{t-1} s_{t-1}' \Phi_0'\right). \end{aligned}$$

4.3 EM Algorithm

Let for all t, s,

$$\begin{aligned} M_{t|s} &:= \mathrm{E}\left(s_t s_t' | Y_s^+\right) \\ &= P_{t|s} + s_{t|s} s_{t|s}'. \end{aligned}$$

Let for all t, s, r,

$$\begin{aligned} M_{t,s|r} &:= \mathrm{E}\left(s_t s_s' | Y_r^+\right) \\ &= P_{t,s|r} + s_{t|r} s_{s|r}'. \end{aligned}$$

Let

$$\begin{aligned} \bar{M} &:= \frac{1}{T} \sum_{t=1}^{T} M_{t|T}, \\ \bar{M}_1 &:= \frac{1}{T} \sum_{t=1}^{T} M_{t,t-1|T}, \\ L\bar{M} &:= \frac{1}{T} \sum_{t=1}^{T} M_{t-1|T}. \end{aligned}$$

Taking the conditional expectations of the first-order conditions given Y_T^+,

$$\begin{aligned} F'\left(\Sigma^{-1} \otimes I_M\right) \operatorname{vec}\left(\bar{M}_1' G'\right) - F'\left(\Sigma^{-1} \otimes L\bar{M}\right) F\phi &= 0, \\ \Sigma - \left(G\bar{M}G' - G\bar{M}_1\Phi_0' - \Phi_0 \bar{M}_1' G' + \Phi_0 L\bar{M}\Phi_0'\right) &= 0, \end{aligned}$$

or

$$\begin{aligned} \phi &= \left[F'\left(\Sigma^{-1} \otimes L\bar{M}\right) F\right]^{-1} F'\left(\Sigma^{-1} \otimes I_M\right) \operatorname{vec}\left(\bar{M}_1' G'\right), & (17) \\ \Sigma &= G\bar{M}G' - G\bar{M}_1\Phi_0' - \Phi_0 \bar{M}_1' G' + \Phi_0 L\bar{M}\Phi_0'. & (18) \end{aligned}$$

One can solve this system of equations by the Gauss–Seidel method. When $p \geq 5$, the equations simplifies to

$$\begin{aligned} \phi &= \left(I_N \otimes L\bar{M}^{-1}\right) \operatorname{vec}\left(\bar{M}_1' G'\right), & (19) \\ \Sigma &= G\bar{M}G' - G\bar{M}_1\Phi' - \Phi\bar{M}_1' G' + \Phi L\bar{M}\Phi'. & (20) \end{aligned}$$

The EM algorithm proceeds as follows:

1. Pick an initial value $\theta^{(0)}$.
2. (E step) Compute $\left\{s_{t|T}\right\}$, $\left\{P_{t|T}\right\}$, and $\left\{P_{t,t-1|T}\right\}$.
3. (M step) Compute Φ and Σ, and use it as $\theta^{(1)}$.
4. Repeat until convergence.

5 Application

5.1 Data

We apply the method to US business cycle indicators to construct a new coincident index, i.e., we predict latent monthly real GDP using quarterly real GDP and the four monthly coincident indicators that currently make up the composite index (CI) released by The Conference Board. Table 1 describes the indicators. The sample period is from January 1959 to December 2002. We take the first difference of the log of the series and multiply it by 100, which is essentially the percentage growth rate series (quarterly or monthly). Table 2 summarizes descriptive statistics of this growth rate series.

Table 1: US Coincident Indicators

Indicator	Description
	Quarterly
GDP	Real GDP (billions of chained 2000 dollars, SA, AR)
	Monthly
EMP	Employees on nonagricultural payrolls (thousands, SA)
INC	Personal income less transfer payments (billions of chained 1996 dollars, SA, AR)
IIP	Index of industrial production (1997 = 100, SA)
SLS	Manufacturing and trade sales (millions of chained 1996 dollars, SA)

Note: SA means "seasonally-adjusted" and AR means "annual rate."

Table 2: Descriptive Statistics of the Indicators

Indicator	Mean	S.D.	Min.	Max.
		Quarterly		
GDP	0.84	0.88	−2.04	3.86
		Monthly		
EMP	0.17	0.23	−0.88	1.23
INC	0.27	0.56	−4.95	3.70
IIP	0.26	0.83	−3.66	6.00
SLS	0.27	1.05	−3.21	3.54

Note: Statistics are for the first difference of the log times 100.

Table 3: Model Selection

p	Log-likelihood	AIC	SBIC
1	-1825.3	-3.5111	-3.6123
2	-1766.7	-3.4472	-3.6496
3	-1723.9	-3.4134	-3.7171
4	-1697.2	-3.4102	-3.8150
5	-1673.4	-3.4126	-3.9187
6	-1639.0	-3.3946	-4.0019
7	-1607.6	-3.3825	-4.0910
8	-1570.1	-3.3589	-4.1686
9	-1553.5	-3.3748	-4.2857
10	-1516.8	-3.3526	-4.3648
11	-1513.8	-3.3943	-4.5077
12	-1476.4	-3.3709	-4.5854

5.2 Model Selection

We take two shortcuts in estimation. First, to reduce the number of the parameters, we demean the series, and delete the constant term from the model. Second, we use the approximate ML estimator instead of the exact one regarding the initial state for the Kalman filter. Recall that we often estimate a VAR model without missing observations by applying OLS to the demeaned series. We take the same shortcuts here.

We must determine p, the order of the VAR model used for prediction. One usually checks Akaike's information criterion (AIC) and Schwartz's Bayesian information criterion (SBIC) for that purpose. For our model,

$$\begin{aligned}
\mathrm{AIC} &:= -\frac{1}{T}\left\{\ln L\left(\hat{\theta}\right) - \left[pN^2 + \frac{N(N+1)}{2}\right]\right\}, \\
\mathrm{SBIC} &:= -\frac{1}{T}\left\{\ln L\left(\hat{\theta}\right) - \frac{\ln T}{2}\left[pN^2 + \frac{N(N+1)}{2}\right]\right\},
\end{aligned}$$

where $\hat{\theta}$ is the (approximate) ML estimator of θ.

To compute the AIC and SBIC for various p, we estimate each VAR model. We use Ox 3.3 by Doornik (2001) for computation. Since the quasi-Newton method in Ox fails when p is large, we use the EM algorithm. Our criterion for convergence of the log-likelihood divided by T is 10^{-8}.

We try up to $p = 12$, and find that AIC selects $p = 10$ while SBIC selects $p = 1$. One usually follows AIC for optimal prediction and SBIC for consistent model selection. It is not clear which criterion one should follow when one is interested in smoothing the state vector. We follow SBIC here, preferring the simpler model for illustration of our method.

5.3 VAR Coincident Index

A by-product of the EM algorithm is the smoothed estimate of monthly real GDP. If the selected model is small, then it is also possible to reestimate the model by a quasi-Newton method. We use SsfPack 2.2 on Ox for this as well as smoothing; see Koopman, Shephard, and Doornik (1999). We call a coincident index based on a VAR(p) model as a VAR(p) coincident index. Figure 1 plots the VAR(1) coincident index. Although it captures the NBER business cycle reference dates, it is rather

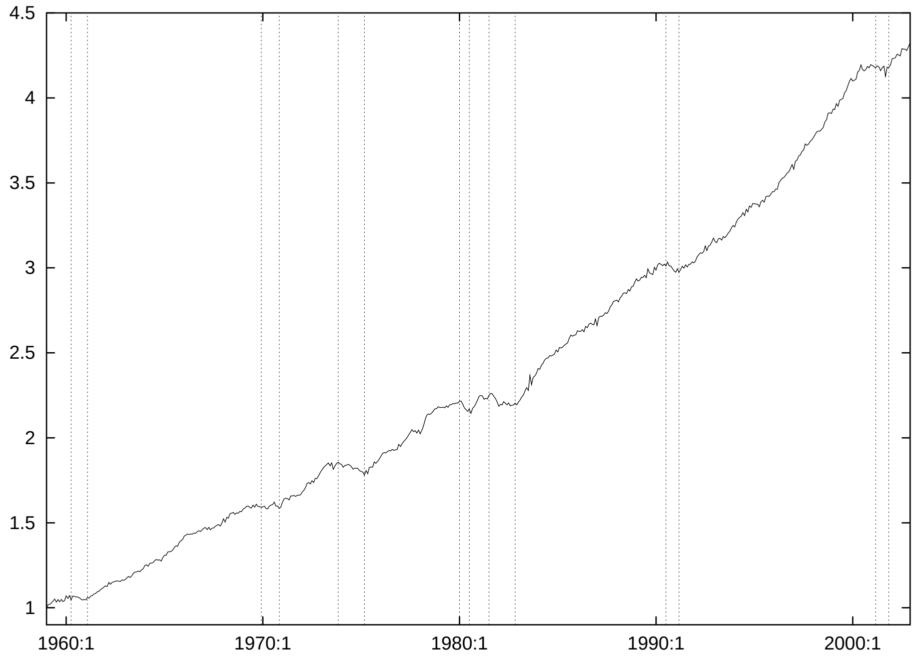

Figure 1: Historical Plot of the New Coincident Index (1959:1=1). The vertical lines are the NBER business cycle reference dates.

volatile. Perhaps we need further smoothing, e.g., by taking moving averages, to determine the turning points by a VAR coincident index.

Table 4 compares turning points determined by alternative indices with the NBER reference dates. The CI captures the NBER reference dates better than the VAR(1) index. It is possible to say, however, that the NBER reference dates do not coincides with the turning points determined by monthly real GDP. Harding and Pagan (2002) discuss how to determine the turning points.

Figure 2 plots the CI and the VAR(1) index from 1979 to 1983, during which there are two peaks and two troughs. The VAR(1) index is clearly more volatile than the CI. In particular, it picks a small "dip" in January 1982 as the trough instead of the official trough in November 1982, and gives a "false" signal. Again, whether it is false or not depends on how to define the turning points. The result simply implies that the trough of monthly real GDP was in fact January 1982 instead of November 1982.

6 Discussion

This paper is still preliminary. We will address the following issues in future revisions. First, we will consider VARMA models instead of restricting to VAR models, although there are some identification issues with VARMA models. Second, we will compare the VAR index with the index proposed by Mariano and Murasawa (2003). Their index relies on a misspecified model, but it is related to monthly real GDP, and it is less volatile than the VAR index. Thus imposing one-factor structure assumption has the effect of smoothing monthly real GDP. Third, we will discuss more about determining turning points.

Table 4: Business Cycle Turning Points Determined by Alternative Indices

NBER	CI	VAR(1) no MA	VAR(1) MA
Peaks			
1960/4	0	−1	+2
1969/12	−2	−3	−4
1973/11	0	0	0
1980/1	0	+1	0
1981/7	+1	+1	+1
1990/7	−1	+1	0
2001/3	−6	−3	−2
Troughs			
1961/2	0	−3	−2
1970/11	0	0	0
1975/3	+1	0	+1
1980/7	0	+1	0
1982/11	+1	−10	−2
1991/3	0	0	−1
2001/11	0	0	−1

Note: MA means "moving average" (3 months). The numbers are lags from the NBER business cycle reference dates.

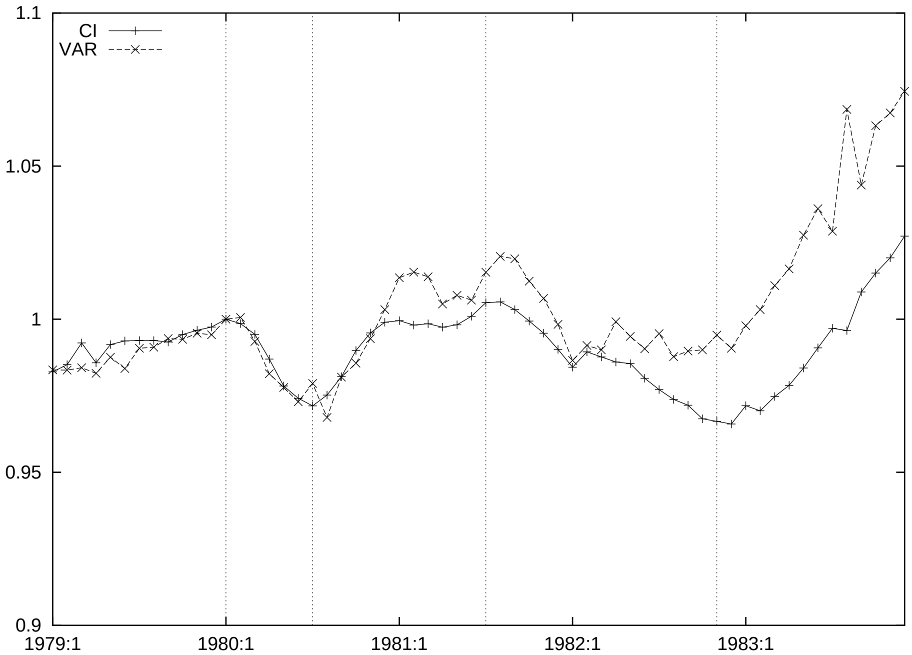

Figure 2: Comparison of Alternative Indices from 1979 to 1983 (1980:1=1). The vertical lines are the NBER business cycle reference dates.

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