

A WAY TO SELL GOODS WITH
NETWORK EXTERNALITIES

by

Tatsuhiko SHICHIJO

Yuji NAKAYAMA

Osaka Prefecture University

2004

abstract

There are a lot of goods which have network externalities. While the number of players who have such a good is small, they may not get enough utility from the goods. That is, players have an incentive to delay their decision, when they purchase the goods with network externalities. Delay causes negative effects on players' utility, so equilibrium with delay is inefficient. We propose a way to settle this problem using a kind of call option. If we use the way and some conditions are satisfied, all players purchase the good and the delay decreases in equilibrium.

Keywords and phrases: Network externality, strategic delay, coordination game

JEL Code: C72, D43, L13

1. Introduction

People have an incentive to delay their decision in some economic environments. For example, when new durable products such as digital cameras or DVD recorders are introduced in the market, many consumers do not purchase them immediately; they wait and see whether the products sell well in the market. Similarly, firms with an investment opportunity (e.g. a plan to establish large plants in China) do not necessarily exercise it in advance of rival firms; they postpone their action until the rivals carry out their option and it ends in a success.

The above examples contain common features: uncertainty about results and irreversibility of actions. Consumers put off their decision to buy a new expensive durable, because the quality of the durable is uncertain and, once they buy it, they must use it during long periods even though not suitable for them. The same logic can apply to firms' behavior; firms hesitate to carry out a project with huge costs because they do not have confidence about the success of the project.

In addition, there are cases where it is beneficial for people to coordinate their action with others'. Some consumer goods have a feature of so called "network externalities". For example, the value of a cellular phone network that each subscriber obtains increases with the number of subscribers of the network, because each user can make a contact with many other users with low costs. Similarly, a firm has an incentive to make a plant in an area where many other firms have already established their plants because the productivity of the plant increases due to "agglomeration economies". Therefore, in facing with network externalities (or agglomeration economies), consumers (or firms) have a motivation to observe others' decision and to delay their decision for fear that they choose a different choice from others ("coordination failures").

In this paper, we consider such a coordination problem in a dynamic framework, and investigate a way to remedy the problem. Related issues have been examined in literatures on network economics and coordination games. In the following, we briefly mention papers closely related with ours in these fields.

In network economics¹, Farrell and Saloner (1985) analyze a technology adoption problem using a model of incomplete information about firms' preference for new technology. They show that there are cases that a socially beneficial technology is not adopted as an outcome of a perfect Bayesian Nash equilibrium ("excess inertia"). This can arise because each firm fears to adopt it solely, then waits and sees other firms' behavior, and as a result all firms remain status quo. They also show that nonbinding communication about preferences can remove some, but not all, types of the inefficiency.

In coordination game literature, Gale (1995) examines an investment decision of N players using a dynamic version of a direct payoff complementarity game (i.e. the payoff of each player is an increasing function of the number of investment activities). He shows that, given N , all players immediately invest after the game starts in any subgame perfect equilibrium, when the length of a unit period becomes sufficiently shorter. The reason for this result is that each

¹ Katz and Shapiro (1985,1986) are another early contributions of network economics in the framework of New IO. Economides (1996) offers a useful overview of this field. Choi (1997) is a recent research on a similar theme.

player recognizes that his investment precipitates a subgame with few players, and that the assumption of the shorter unit period makes the quick attainment of the critical mass (i.e. the number of investment which makes each investment profitable). Therefore, this result indicates that the coordination problem disappears when the game has multiple decision stages for each player, if each player reacts quickly against other players' actions (an interpretation of a short unit period). However, he also shows that, given the length of a unit period, there exists a, but not a unique, subgame perfect equilibrium where no player invests, when the number of players becomes sufficiently larger. This is why, because of the large number of players, the critical mass attains after longer period which makes an investment decision of each player unprofitable².

While researches in network economics search for resolving on the coordination problem through establishing institutions or procedures among parties concerned, ones in coordination games examine more fundamental subjects such as the relationship between the structure of the game and equilibria attained. On the other hand, This paper adds some rules, which are discussed below for detail, to a model of Gale (1995), and examines whether the coordination among players is easily accomplished in an equilibrium.

Rauch (1993) has the similar spirit with our paper. He considers a relocation problem of firms from an old high-cost industrial park to a new low-cost park from a view point of a developer of the new park. When an industry is subject to agglomeration economies, the above-mentioned coordination problem occurs; no firm wants to move first to fear for being stranded in the new site, as a result, all firms do not move. He demonstrates that the developer can beat this passivity through discriminatory pricing of land over time. He also shows that empirical evidence in the United States supports his result³.

² Another strand of the field, which is called "herding" or "informational cascades", deals with games without direct payoff complementarity to analyze delay for learning some information from the behavior of others. Early contributions of the area are Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992). Bikhchandani, Hirshleifer and Welch (1998) provide a compact review of this field, while Chamely (2003) is a textbook of comprehensive coverage of this area. Dasgupta (2000) is a recent research combining the element of herding with that of direct payoff complementarity.

³ Matsumura and Ueda (1996) also show a similar result in a model of a technology adoption under incomplete information about the number of firms that can adopt the new technology

A model developed in our paper differs from Rauch (1993) since we are concerned with some "neutral" rules from a view point of the government rather than profit-oriented rules by a developer.

2. The basic model

We assume the same model as Gale (1995) and Gale (2000). We use notation of Gale (2000), because the notation is more sophisticated than Gale (1995).

Consider a game with N players. Time is represented by a sequence of dates $t = 1, 2, \dots, \infty$. For simplicity, we assume that each player buys only one unit of the goods and the price of the goods is $C > 0$. A player chooses a date t at which she buys the goods. Let x_{it} describe state of player i at date t . If player i has not bought by the end of the date t , let $x_{it} = 0$. If player i has bought by the end of the date t , let $x_{it} = 1$. The state of the game at date t is denoted by the vector of x_{it} , $x_t = (x_{1t}, x_{2t}, \dots, x_{Nt})$. Let X denote set of all states. The history of the game at date t is denoted by $h_t = (x_1, \dots, x_{t-1})$. Let H_t denote the set of histories at date t and Let $H = \cup H_t$ denote the set of all histories. A pure strategy for player i is a function $\sigma_i : H \rightarrow \{0, 1\}$, where $\sigma_i(h_t)$ for $h_t \in H_t$ is the state of the player i at date t , that is $x_{it} = \sigma_i(x_{1t}, \dots, x_{t-1})$. Let $\sigma \equiv \sigma_1 \times \sigma_2 \times \dots \times \sigma_N$. We assume strategies satisfy the following condition.

Assumption 1

$$\sigma_i(h_t) = 1 \text{ if } x_{it-1} = 1 \text{ in history } h_t$$

The above assumption means players do not throw away the goods. From the assumption 1, states satisfy $x_s \geq x_{s-1}$.

The proportion of the players who have bought the goods by the end of date t is $\alpha_t = \frac{1}{N} \sum_{i=1}^N x_{it}$. The more players buy the goods, the larger the utility that a

given user derives from the goods at date t , $R(\alpha_t)$ is. That is, the utility from the goods at date t is an increasing function of α_t . Since the price of the goods is C , the payoff of purchaser at date t is $\sum_{s=t}^{\infty} R(\alpha_s) \delta^{s-1} - \delta^{t-1} C$, where $0 < \delta < 1$

is the common discount factor. The payoff to players who do not buy is zero. According to Gale(2000), let $v(\alpha_t) = R(\alpha_t) - (1 - \delta)C$. Using the function v ,

the payoff of purchaser at date t is $\sum_{s=t}^{\infty} v(\alpha_s) \delta^{s-1}$. We assume the payoff

function v satisfies the following conditions.

Assumption 2.

- (a) $v(1) > 0$
- (b) $v\left(\frac{1}{N}\right) < 0$
- (c) v is continuous and strictly increasing.

Assumption 2(a) guarantees that players can get positive payoff if all players buy the goods. That is, the state where all players buy the goods is better than the state where no player buy ones. However, from assumption 2(b), if only one player buys, the player gets negative payoff. Thus each does not wish to buy if all other players do not buy.

From the assumption 2, there is a condition such that buying the goods becomes a dominant strategy.

Definition 1 (critical number of purchasers)

Define i^* as a natural number that satisfies $v\left(\frac{i^*}{n}\right) \leq 0 < v\left(\frac{i^*+1}{n}\right)$

If i^* players have bought the goods, a player who has not bought yet obtains positive payoff with probability 1 by buying the goods. The above structure of the game is the same as the Gale(1995).

3. The result of Basic model

We will refer some results of Gale(1995).

Theorem 1 (Gale 1995)

There exists a SPE in which all players buy the goods at date i^* .

The proof is in theorem 2 in Gale (1995). From the above theorem, strategic delay possibly occurs.

Theorem 2 (Gale1995)

If $v\left(\frac{1}{n}\right)(1 + \delta + \delta^2 + \dots + \delta^{i^*-2}) + v(1)\frac{\delta^{i^*-1}}{1-\delta} < 0$, then there exists a SPE in which no player buy the goods.

From theorem 2, there is possibility such that no player buys the goods. This is Pareto inefficient equilibrium.

We define partial order in history space as follows.

Definition 2 (Partial order in finite history space)

A partial order in histories is defined by $h_t \preceq \hat{h}_s$ for $h_t = (x_1, \dots, x_{t-1}) \in H_t$

$$\hat{h}_s = (\hat{x}_1, \dots, \hat{x}_{s-1}) \in H_s, t \geq s \Leftrightarrow x_{ir} \leq \hat{x}_{ir} \text{ for } r \leq s-1 \text{ and } x_{ir} \leq \hat{x}_{is} \text{ for}$$

$$t-1 \geq r > s-1.$$

Definition 3 (Partial order for infinite histories)

Let $h, \hat{h} \in H$ be infinite histories $h = (x_1, x_2, \dots)$, $\hat{h} = (\hat{x}_1, \hat{x}_2, \dots)$. Let

$\rho_t(h) = (x_1, x_2, \dots, x_{t-1})$, $\rho_t(\hat{h}) = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_{t-1})$ be the first $t-1$ history of h, \hat{h} .

We say that $h \preceq \hat{h}$ if $\rho_t(h) \preceq \rho_t(\hat{h})$ for any t .

Example 1

Consider the following two histories:

- (a) Player 1 bought the good at date 1. Player 2 bought the good at date 2.
- (b) Player 1 bought the good at date 1. Player 2 and Player 3 bought the good at date 2.

More players bought goods in history (b) than in history (a). So we can guess more goods will be sold in history (b) than in history (a). By definition 3, history (b) is larger than history (a).

Example 2

Consider the following two histories:

- (a) Player 1 bought the good at date 1. Player 2 bought the good at date 2.
- (b) Player 1 and player 2 bought the good at date 1. Date 1 is the end if this history.

2 goods are sold earlier in history (b) than in history (a). So we can guess goods will be sold out earlier in history (b) than in history (a). By definition 3, history (b) is larger than history (a).

The order implies the degree how early and many players buy the goods. It is natural for players to have the conjecture that the greater the present history is, the earlier other players will buy the goods and the more riskless her purchase becomes. The following definition of monotonicity of strategy describes strategy under such conjecture.

Definition 4 (monotonicity of strategy)

A strategy satisfies monotonicity if $\sigma_i(\hat{h}_s) \geq \sigma_i(h_t)$ for all i and all histories $\hat{h}_s \in H_s, h_t \in H_t$ which satisfies $\hat{h}_s \succeq h_t$.

It is natural for strategies of all players to have above monotonicity. So we assume the following assumption.

Assumption 3

Strategies of all players satisfy the monotonicity.

Let $\Gamma(h)$ denote subgame which begins after finite history $h \in H$. Given a pure strategy profile σ , there is the only one history which will be realized in $\Gamma(h)$. Let $\eta(h, \sigma)$ denote a history on $\Gamma(h)$ which is determined by σ . Note that state in the first date of $\eta(h, \sigma)$ is $\sigma(h)$ not the first state in h . From the Assumption 1, we can get the following lemma immediately.

Lemma 1(monotonicity of histories)

If $h \prec \hat{h}$, where h and \hat{h} are finite histories, then $\eta(h, \sigma) \preceq \eta(\hat{h}, \sigma)$ for any σ .

Proof

Suppose that $h_t = (x_1, x_2, \dots, x_{t-1}) \in H_t$ $\hat{h}_s = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_{s-1}) \in H_s$ are two finite histories such that $h_t \prec \hat{h}_s$ $t \geq s$. Suppose that $\eta = (y_1, y_2, \dots)$,

$\hat{\eta} = (\hat{y}_1, \hat{y}_2, \dots) \in H$ are two histories such that $\eta \preceq \hat{\eta}$ and $x_{t-1} \leq y_1$ $\hat{x}_{s-1} \leq \hat{y}_1$.

Since $\hat{y}_i \geq x_{s-1+i}$ for $i \leq t-s$ and $\hat{y}_i \geq y_{i+s-t}$, $(h_t, \eta) \preceq (\hat{h}_s, \hat{\eta})$ where

$$(h_t, \eta) = (x_1, x_2, \dots, x_{t-1}, y_1, y_2, \dots) \text{ and } (\hat{h}_t, \hat{\eta}) = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_{s-1}, \hat{y}_1, \hat{y}_2, \dots).$$

Suppose that $\eta(h, \sigma) = (y_1, y_2, \dots)$ and that $\eta(\hat{h}, \sigma) = (\hat{y}_1, \hat{y}_2, \dots)$. If $h \preceq \hat{h}$,

then $\sigma(h) \leq \sigma(\hat{h})$ from monotonicity of strategies. Since $\sigma(h) = y_1$ and

$$\sigma(\hat{h}) = \hat{y}_1, y_1 \leq \hat{y}_1. \text{ Since, by the first consideration, } (h, y_1) \preceq (\hat{h}, \hat{y}_1),$$

$$\sigma(h, y_1) \leq \sigma(\hat{h}, \hat{y}_1). \text{ Hence } (y_1, y_2) \preceq (\hat{y}_1, \hat{y}_2). \text{ By induction } \eta(h, \sigma) \preceq \eta(\hat{h}, \sigma).$$

End of Proof

Because state at each date in $\Gamma(h)$ is determined by a strategy σ . Payoff is also determined by a strategy σ .

Lemma 2(monotonicity of payoff)

Let $B(h, \sigma)$ be payoff for a player who has bought in $\Gamma(h)$ when players obey strategy profile σ . If $h \preceq \hat{h}$, then $B(h, \sigma) \leq B(\hat{h}, \sigma)$

Proof

Let x and \hat{x} be states. If $x \leq \hat{x}$, then the payoff for players who have bought in state \hat{x} is equal or larger than that in state x . Moreover the payoff for players who do not buy is fixed value zero. Hence the payoff for any players in state \hat{x} is equal to or larger than that in x . From lemma 1, $B(h, \sigma) \leq B(\hat{h}, \sigma)$.

End of proof.

Theorem 1

There is no subgame perfect equilibrium of any subgame $\Gamma(h)$ such that only one player purchase a good at date 2 and she gets positive payoff.

Proof.

Let x_1, x_2 be the state at date 1 and 2 in $\Gamma(h)$, respectively. Suppose that player i buys the goods at date 2 in $\Gamma(h)$ and that the other players do not buy at date 2 in $\Gamma(h)$. Let $\#x$ is the number of players who have bought in state x . Payoff

for player i is $v\left(\frac{\#x_2}{n}\right)\delta + \delta^2 B((h, x_1, x_2), \sigma)$. When player i bought at date 1 in

$\Gamma(h)$, player i gets. Since $(h, x_2) \succeq (h, x_1, x_2)$, $B((h, x_2), \sigma) \geq B((h, x_1, x_2), \sigma)$ by

lemma 2. Thus if $v\left(\frac{\#x_2}{n}\right)\delta + \delta^2 B((h, x_1, x_2), \sigma) > 0$, then

$v\left(\frac{\#x_2}{n}\right)\delta + \delta^2 B((h, x_1, x_2), \sigma) < v\left(\frac{\#x_2}{n}\right) + \delta B((h, x_2), \sigma)$. This contradicts with

the equilibrium condition.

End of Proof

Note that monotonicity does not solve the problem of strategic delay. However monotonicity, especially Theorem 1, plays an important role in the next section.

4. Heterogeneous options with refund contract

In order to solve the problem of strategic delay, we propose heterogeneous options with refund contract (hereafter HORC). This is a right to buy the good at fixed price which is similar to a call option in financial market. However, if

some condition is satisfied, purchasers of the option can get refund. And the condition is different among options.

Definition 5 (Heterogeneous options with refund contract)

Let O_i be one of heterogeneous options with refund contract (HORC). There are n kinds of HORC, O_i ($i=0,1, \dots, n-1$). Purchasers of HORC have a right to buy the good at a fixed price. However if i players buy the good at a date and if purchaser of O_i have not bought the goods by the end of the next date, then she loses her right to buy the good. Moreover purchaser of O_i can get refund if one of the following conditions are satisfied:

- a) She buys the good before she loses her right to buy.
- b) Some of O_j $j=0, \dots, n-1$, remain unsold.
- c) The number of the players who have bought the goods is smaller than i by the end of a period.

In this section we impose the next assumption.

Assumption 4

If a purchaser of HORC loses her right, then she can not buy the good forever.

This assumption is probably too strong, because we can not implement the assumption perfectly. Other players can buy the good for her and we can not detect every such cheating. However, the assumption helps to simplify the result of our model, so it is useful for understanding of the essence of HORC. We will loose this assumption in the next section.

From now, we will consider the case all players have bought HORC in this section. That is we will consider what would happen after HORC are sold out.

Let $S_i \subset H_i$ be set of histories such that every player does not lose their right.

Theorem 2

Let $[z]$ be the maximum integer less than z . Suppose that $l^* = \left\lfloor \frac{i^* - 1}{2} \right\rfloor$. If $v\left(\frac{1}{n}\right) + \sum_{i=1}^{l^*} v\left(\frac{2i}{n}\right)\delta^i + \frac{v(1)\delta^{l^*+1}}{1-\delta} > 0$, then all players buy the good within $\left\lfloor \frac{i^*}{2} \right\rfloor$ in every SPE after they bought HORC.

Proof

Suppose that $S^k \subset H$ is a subset of histories such that no player lose their right and just i players have bought at the last date. Let σ_h^* be SPE strategy in. we will argue by induction. Let $A(k)$ for $k \leq i^*$ denote an assertion such that $B(\sigma_h^*, h) \geq \sum_{j=0}^{k^*(k)} v\left(\frac{2j+k+1}{n}\right)\delta^j + \frac{v(1)\delta^{k^*(k)+1}}{1-\delta}$ for $\forall h \in S^k \quad \forall \sigma_h^*$, where $k^*(k) = \left\lfloor \frac{i^* - k - 2}{2} \right\rfloor$. $A(i^*)$ is true, because every player gets positive payoff when buying goods. We will show $A(k-1)$ is true if $A(k)$ is true. Consider $\Gamma(h)$ for $h \in S^{k-1}$. There is at least one player at the beginning of the subgame who has to decide whether she buys the good or loses her right to buy. She gets zero payoff if she loses their right, so she will buy the goods if they can get positive payoff by doing so.

$$\text{If } A(k) \text{ is true, for } h \in S^{k-1}, B(\sigma_h^*, h) \geq v\left(\frac{k}{n}\right) + \sum_{j=0}^{k^*(k)} v\left(\frac{2j+k+1}{n}\right)\delta^{j+1} + \frac{v(1)\delta^{k^*(k)+2}}{1-\delta} \geq v\left(\frac{1}{n}\right) + \sum_{i=1}^{l^*} v\left(\frac{2i}{n}\right)\delta^i + \frac{v(1)\delta^{l^*+1}}{1-\delta} > 0 .$$

Hence players who have to buy or lose their right will buy the goods. From theorem 1, they will get at least $\sum_{j=0}^{k^*(k-1)} v\left(\frac{2j+k}{n}\right)\delta^j + \frac{v(1)\delta^{k^*(k-1)+1}}{1-\delta}$. Thus $A(k-1)$ is true. By induction, $A(k)$ is true for $k=1,2,\dots,i^*$.

End of proof

From Theorem 2, if every player buys HORB, then every player gets positive payoff. If some players do not buy HORB, purchasers of HORB can get their money back due to refund contract. Buying HORB is weakly dominant strategy.

5. Price of HORB

In the last section, assumption 4 plays an important role. Due to assumption 4, the price of HORB can be zero. Theorem 2 does not need for HORB to have positive price. In this section, instead of assumption 4, we assume HORB has some positive price. If k goods are sold, players who bought O_i $i \geq k$ have to decide whether to buy the good or to lose their right to get refund. We can guess players buy the goods without Assumption 4 if the price of HORB is high. Next theorem determines how much HORB should be.

Theorem 3

Let ξ be the price of HORB. If $\xi > -v\left(\frac{1}{n}\right) - \sum_{i=1}^{l^*} v\left(\frac{2i}{n}\right)\delta^i$ and $v\left(\frac{1}{n}\right) + \sum_{i=1}^{l^*} v\left(\frac{2i}{n}\right)\delta^i + \frac{v(1)\delta^{l^*+1}}{1-\delta} > 0$, then all players buy the good within $\left[\frac{i^*}{2}\right]$ in every SPE.

Proof.

We will use the same notation as theorem 2. $A(i^*)$ is true in this game. We will show $A(k-1)$ is true if $A(k)$ is true. If $A(k)$ is true, then, for $h \in S^{k-1}$, $B(\sigma_h^*, h) \geq v\left(\frac{k}{n}\right) + \sum_{j=0}^{k^*(k)} v\left(\frac{2j+k+1}{n}\right)\delta^{j+1} + \frac{v(1)\delta^{k^*(k)+2}}{1-\delta}$. Because of monotonicity of strategy, the difference between payoff of losing right to get refund and of buying the goods is at most $-\xi - v\left(\frac{k}{n}\right) - \sum_{j=0}^{k^*(k)} v\left(\frac{2j+k+1}{n}\right)\delta^{j+1} \leq -\xi - v\left(\frac{1}{n}\right) - \sum_{i=1}^{l^*} v\left(\frac{2i}{n}\right)\delta^i < 0$. Thus, buying the

goods is a dominant strategy. From theorem 2,

$$B(\sigma_h^*, h) \geq \sum_{j=0}^{k^*(k-1)} v \left(\frac{2j+k}{n} \right) \delta^j + \frac{v(1)\delta^{k^*(k-1)+1}}{1-\delta}$$

End of proof

References

- Banerjee, Abhijit V. "A Simple Model of Herd Behavior" Quarterly Journal of Economics, 1992 (107) pp 797-817.
- Bikhchandani, Sushil. Hirshleifer, David. Welch, Ivo. "A Theory of Fads, Fashion, Custom, and Cultural Change in Informational Cascades" Journal of Political Economy, 1992 (100) pp 992-1026.
- Chamley, Christophe. Rational Herds: Economic Models of Social Learning, Cambridge University Press, 2004.
- Choi, Jay Pil. "Herd Behavior, the "Penguin Effect," and the Suppression of Informational Diffusion: An Analysis of Informational Externalities and Payoff Interdependency" RAND Journal of Economics, 1997 (28) pp 407-425.
- Dasgupta, Amil. "Social Learning with Payoff Complementarities" mimeo (available at <http://fmq2.lse.ac.uk/~amil/research.html>) 2000.
- Economides, Nicholas. "The Economics of Networks" International Journal of Industrial Organization, 1996 (14) pp 673-699.
- Farrell, Joseph. Saloner, Garth. "Standardization, Compatibility, and Innovation" RAND Journal of Economics, 1985 (16) pp 70-83.
- Gale, Douglas. "Dynamic Coordination Games" Economic Theory, 1995 (5) pp 1-18.
- Gale, Douglas. "Dynamic Coordination Games" mimeo 2000.
- Katz, Michael L.. Shapiro, Carl. "Network Externalities, Competition, and Compatibility" American Economic Review, 1985 (75) pp 424-440.
- Katz, Michael L.. Shapiro, Carl. "Technology Adoption in the Presence of Network Externalities" Journal of Political Economy, 1986 (94) pp 822-841.

Matsumura, Toshihiro. Ueda, Masako. "Endogenous Timing in the Switching of Technology with the Marshallian Externalities" *Journal of Economics*, 1996 (63) pp 41-56.

Rauch, James E. "Does History Matter Only When It Matters Little? The Case of City-Industry Location" *Quarterly Journal of Economics*, 1993 (108) pp 843-867.