Coalition Formation and Asymmetric Information in a Legislative Bargaining Game

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Abstract

To investigate players' incentives in coalition formation, we consider a legislative bargaining game with asymmetric information about time preferences. The force that does not exist in usual bargaining games with unanimity is that due to majority rule, if a player signals himself as the patient type, other players wish to exclude him from their coalitions because more is required to buy his vote. As a result, separating equilibria become harder to support, in the sense that type difference needs to be large enough. If there exists one, we show that the only separating equilibrium that can survive under the intuitive criterion is such that the patient player forms oversized coalitions, and the impatient player prefers minimal-winning coalitions.

Keywords: Asymmetric information, coalition formation, legislative bargaining.

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1 Introduction

Most models of legislative bargaining have minimal-winning coalitions as their key predictions for the equilibria. That is, proposers will never buy more than the smallest number of players needed to win. Empirical studies, however, find that oversized coalitions are very often observed as well. Researchers have been trying to rationalize the formation of oversized coalitions.[1] One popular explanation is uncertainty (Riker, 1962). With complete information, forming minimal-winning coalitions is usually the cheapest way to win.[2] However, if there is uncertainty about which minimal-winning coalition will actually form, then *ex ante*, proposers may wish to buy more votes than needed to reduce the uncertainty. Weingast (1979) and others offer a theory of "universalism," a norm built in order for everyone to receive a benefit due to the fear that he will be excluded from the minimal-winning coalitions.

The arguments above fail to address how these cooperative motivations to form oversized coalitions can sustain over time. More importantly, it remains to explain whether the incentives to attract more votes than needed are sequential rational, particularly when some information about their bargaining position is revealed during the process. There may be some other ways to incorporate uncertainty about what is to take to get a majority into a dynamic model, but it is the most natural to incorporate incomplete information in players' preferences into a legislative bargaining game. Thus, people have to take into account the signaling effects of how their proposals and votes to reject affect their future bargaining position. There is no other paper, as far as we know, which also analyzes a bargaining model with incomplete information where agreement requires less than unanimous approval.

We therefore consider a simple version of Baron and Ferejohn (1989) with three players and three periods. Their influential model has become a workhorse tool for analyzing many

[1] See Volden and Carrubba (2004) for the survey on several theories.

[2] This argument applies to almost all formal models. Norman (2000) argues an equilibrium with oversized coalitions can be supportable when players use history dependent strategies. Groseclose and Snyder (1996) also get supermajority coalitions in a stylized model of vote buying.

economic and political problems. In each period, nature selects a proposer who can divide a pie. Legislators then vote once the proposal in on the floor, and it passes if it obtains a majority of the votes, otherwise bargaining goes to the next period and the same process is repeated. In this paper, we assume that the discount rate of each player is private information.

There is a huge literature in bilateral bargaining with two-sided incomplete information, and much has been known about them.[3] For example, with continuous type space and infinite horizons, Cramton (1984) considers the case of one-sided offers, and Cramton (1992) considers the case of alternating offers. Chatterjee and Samuelson (1987) deal with alternating offers and binary type space. Watson (1998) also considers an alternating-offer game with two-sided incomplete information about player's discount rates. Fudenberg and Tirole (1983) provides a entire profile of equilibria in a two-period game with binary type space, in which offers can be made alternatingly. The literature in multilateral bargaining is relatively scarce. Baliga and Serrano (1995) consider a multilateral bargaining with imperfect information, in that players only knows his own share but not the ones offered to others. All of these papers concern about unanimity rule.

It is natural to conjecture that adding incomplete information to Barn-Ferejohn model will deliver a model sufficiently similar to the usual incomplete information bargaining games that these results will carry over in some form. However, it is not correct. In typical incomplete information bargaining games, a proposer wishes to signal that he is a "strong" bargainer (is patient or has a low valuation) since this is what will convince the other players to concede the most. In legislative bargaining games, however, a proposer's incentives are much more mixed. Unlike in bargaining with unanimity, being strong in the future is bad in the present: to form a winning coalition, only a subset of the players need to agree, so the current proposer will bribe a minimal-winning coalition with the players with the lowest continuation payoffs. If the proposer signals that he is a strong bargainer, it is possible that no later proposers will

[3]See Kennan and Wilson (1993) for a thorough survey.

include him in their coalitions because it is more expensive to buy his vote. As the result, separation is harder to get: for separation the proposal by the strong type must be rejected with positive probability; but if the strong type is revealed, he may not be included in the later majority if the proposal is rejected.

We show that if the types are not very far apart, there exists no separating equilibrium, because both types have the incentives to pass their proposal immediately. Only pooling equilibria with minimal-winning coalitions can be supported. However, oversized coalitions can be optimal if the types are different enough, and they must fail with some positive probability. In a separating equilibrium, the patient proposer tends to form oversized coalitions to increase the probability of acceptance, when it is not too expensive to include two relatively impatient opponents in the coalitions. The impatient proposer prefers minimal-winning coalitions in which his proposal passes for sure immediately, since he has less incentives to delay. Therefore, proposers separate not only in making different proposals, but also in choosing different forms of coalitions.

The remainder of the paper is organized as follows. Section 2 introduces the model. In section 3, we analyze separating equilibria and pooling equilibria which can survive under symmetric tie-breaking rules and the intuitive criterion. Section 4 makes some further discussions and section 5 concludes.

2 The Model

The model is a three-player, three-period version of Baron and Ferejohn (1989). In the beginning of each period if it is reached, nature selects a player as the proposer with equal probability to divide a dollar. A proposal $x^i = (x_1^i, x_2^i, x_3^i)$ satisfies $x_j^i \geq 0$ and $\sum_{j=1}^{3} x_j^i \leq 1$ for all i, $i \in \{1, 2, 3\}$, where x_j^i is the share that the proposer i offers to player j. Once the proposal is on the floor, the legislature votes simultaneously, and it passes if it wins a majority of the votes; if it fails, the game goes to the next round, and nature selects a proposer again and the same process is repeated. If there is still no agreement at period 3, the game ends and everyone gets 0.

Players are risk neutral: the utility function of player j is $u_j(x^i, t) = \delta_j^{t-1} x_j^i$, $t = 1, 2, 3$, where $\delta_j \in (0, 1]$ is the discount rate. We assume that δ_j is private information to player j, and it is common knowledge that for each player, the distribution function is binary: $\delta_j \in \Omega = \{\underline{\delta}, \bar{\delta}\}$, $\bar{\delta} > \underline{\delta}$, with probability $\frac{1}{2}$. Henceforth, we call player j of type $\bar{\delta}$ player jH and player 1 of type $\underline{\delta}$ player jL.

A history at the beginning of period t is $h_t = \{i(n), x^i(n), a^1(n), a^2(n), a^3(n)\}_{n=0}^{t-1}$, where $a^j \in \{\text{Yes}, \text{No}\}$ is j's voting decision upon the proposal x^i. A strategy of j is $\sigma^j : h_t \times \Omega \to [0, 1]^3$ when j is recognized to make a proposal at time t, and $\sigma^j : h_t \times [0, 1]^3 \times \Omega \to \{\text{Yes}, \text{No}\}$ when j votes on the proposal on the floor. Without loss of generality, we assume player 1 is the proposer in the first period. We look for sequential equilibria in this game.

3 Equilibria with Symmetric Tie-breaking Rules

In the third period if it is reached on the equilibrium path, no matter what beliefs players hold, the proposer will demand the whole dollar and everyone will vote for it. The continuation value for each player at the beginning of the third period is therefore either $\frac{1}{3}\bar{\delta}$ or $\frac{1}{3}\underline{\delta}$. Thus, if the second period is reached on the equilibrium path, player jH (jL) will vote for i's proposal, $y^i = (y_1^i, y_2^i, y_3^i)$, if and only if $y_j^i \geq \frac{1}{3}\bar{\delta}$ ($y_j^i \geq \frac{1}{3}\underline{\delta}$). Based on this fact, in the following proposition we show that a minimal-winning coalition always forms in the second period.

Proposition 1. *Following equilibrium path proposals in the second period if it is reached:*

1. *A minimal-winning coalition always forms.*

2. *If player 1 is perceived as a L type after a rejection in the first period, he is always included in the majority in the second period.*

Proof. See the Appendix.

If player 1's proposal does not pass in the first period, players update their beliefs as $P(h) = (p_1, p_2, p_3)$, given the history h. Due to the skimming property, if the second period is reached on the equilibrium path, it is more likely that players 2 and 3 are of H type, that is, $p_2 \geq \frac{1}{2}$ and $p_3 \geq \frac{1}{2}$. However, it is not optimal for player 1 to make the first-period proposal accepted *neither* by players 2 *nor* by 3, given the fact that the pie shrinks and player 1 may lose control in the agenda setting when the bargaining is delayed to the next period, so that at least one player between players 2 and 3 is of H type if the second period is reached on the equilibrium path. Thus, when player 1 is the proposer again, he will satisfy one player of H type and make the proposal pass (by offering $\frac{1}{3}\bar{\delta}$), instead of forming an oversized coalition so that with a chance, the agreement is postponed to the third period.

When player 2 (3) is the proposer in the second period, he can choose either player 1 or player 3 (2) in the coalition. Given the second-period strategies played, offering $\frac{1}{3}\bar{\delta}$ to another player j will ensure it to pass. Again, instead of buying a supermajority, it is optimal to form a minimal-winning coalition and bargaining concludes in the second period. Moreover, if player 1 is perceived as a L type, he is in the majority for sure since it is cheaper to buy his vote so that both players 2 and 3 will choose him in their coalitions. On the other hand, if he is perceived as a H type, then there is a probability that he is not in player 2 or 3's coalition. In any case, a minimal-wining coalition forms in the second period if it is reached on the equilibrium path.

3.1 Separating Equilibria

In a separating equilibrium, the first-period proposal $x^1 \in \{x^H, x^L\}$ made by players $1H$ and $1L$ are different; that is, $x^H \neq x^L$, so that player 1's type is revealed in the first period. Given a history h, supposing that $V_j(h)$ is the continuation value at the beginning of the second period for player j, then the equilibrium strategies call for player $2H$ to vote for x^1 if and only if $x_2^1 \geq \bar{\delta} V_2(h)$, and for player $2L$ to vote for it if and only if $x_2^1 \geq \underline{\delta} V_2(h)$. Player 3 is analogous.

Many separating equilibria can be supportable, but most of them are based on unreasonable off-equilibrium beliefs. We then use the *intuitive criterion* as the refinement. However, since a separating equilibrium puts no restriction on the tie-breaking rule, there are still many equilibria that can be supportable after any refinement. We therefore focus on those equilibria with *symmetric* tie-breaking rules; that is, the proposer selects one of the other players in his coalition with *equal* probability when there is a tie. Restricting our attention to this rule yields clear predictions for the equilibria. In the next section, we will discuss asymmetric tie-breaking rules, in which the problem of multiple equilibria becomes more serious.

Proposition 2. *Under symmetric tie-breaking rules and the intuitive criterion:*

1. *If* $\bar{\delta} < 2\underline{\delta}$*, there exists no separating equilibrium.*
2. *If* $\bar{\delta} \geq 2\underline{\delta}$*, the only separating equilibrium that can be supportable is such that* $1H$ *forms an oversized coalition, and* $1L$ *forms a minimal-winning coalition. Namely,* $x^H = (1 - \frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$*, and* $x^L = (1 - \frac{1}{3}\bar{\delta} + \frac{1}{9}\bar{\delta}\underline{\delta} - \frac{1}{18}\bar{\delta}^2, \frac{1}{3}\bar{\delta} - \frac{1}{9}\bar{\delta}\underline{\delta} + \frac{1}{18}\bar{\delta}^2, 0)$ *or* $(1 - \frac{1}{3}\bar{\delta} + \frac{1}{9}\bar{\delta}\underline{\delta} - \frac{1}{18}\bar{\delta}^2, 0, \frac{1}{3}\bar{\delta} - \frac{1}{9}\bar{\delta}\underline{\delta} + \frac{1}{18}\bar{\delta}^2)$.

Proof. See the Appendix.

[Insert Figure 1]

In a separating equilibrium, as shown in the Appendix, the incentive constraints imply that x^H must be rejected with some probability on the equilibrium path. Thus, there are only two candidates of x^H for a separating equilibrium: (i) x^H is made such that players $2L$ and $3L$ vote for it, but players $2H$ and $3H$ vote against it; (ii) x^H is made such that only either player $2L$ *or* $3L$ votes for it. Proposals other than those ones cannot increase the probability of passing it but will give player $1H$ a lower share, and thus they can be eliminated by the intuitive criterion.

We also argue that it is not optimal that x^L is made such that only one player of L type votes for it, because although it saves some costs in the first period, the reduction in the probability of acceptance cannot compensate the proposer. Hence, there are only two candidates of x^L for a separating equilibrium: (i) x^L is made such that players $2L$ and $3L$ vote for it, but players $2H$ and $3H$ reject it; (ii) x^L is made such that only either player $2H$ or $3H$ votes for it.

The results show that, in order to support a separating equilibrium, $1H$ has to be patient enough, otherwise, since he has some probability not being included in others' coalitions when his first-period proposal is rejected, he has less incentives to reveal his information. Therefore, since x^H must fail with some positive probability, if $\bar{\delta}$ and $\underline{\delta}$ are too close, no separating equilibrium can be supported because $1H$ wishes to pass his proposal immediately.

On the other hand, if $\bar{\delta}$ and $\underline{\delta}$ are distant enough, a enough patient $1H$ has the incentives to form an oversized coalition, since when $\underline{\delta}$ is small, it is not too expensive to buy one more vote than needed to increase the probability of acceptance, although it means a worse bargaining position when he reveals as a H type. For $1L$, when $\underline{\delta}$ is small, he has less incentives to delay, so that a minimal-winning coalition is the optimal choice because it passes in the first period for sure. Therefore, "separation" not only represents that the two types offer different proposals in equilibrium, but also means that they choose different forms of coalitions.

Hence, majority rule creates a tradeoff that does not exist in traditional bargaining games. In unanimity games, the patient player has more incentives to wait for better offers, so that one typical equilibrium is that the impatient player wishes to pass his proposal immediately, but the patient player wishes to delay with some probability. In the legislative bargaining problems, there is another force such that the H type may sometimes have no incentives to wait. According to Proposition 1, a L type player can obtain a better position in later stages of bargaining because other players will include him in their coalitions. The tradeoff thus is between being included in the future majority for sure and demanding a larger share of the pie today.

3.2 Pooling Equilibria

In a pooling equilibrium, both types offer the same proposal in the first period, that is, $x^L = x^H = x$. Thus, if the second period is on the equilibrium path, then $p_1 = \frac{1}{2}$. In this case, after a rejection to x, no one can always be included in everyone's coalition in the second period. The following proposition claims that there are two kinds of proposals supportable in a pooling equilibrium, depending on the difference of $\bar{\delta}$ and $\underline{\delta}$.

Proposition 3. *Under symmetric tie-breaking rules and the intuitive criterion, the first-period proposals in a pooling equilibrium are such that:*

1. *If $\bar{\delta}$ and $\underline{\delta}$ are close, a minimal-winning coalition forms, i.e.* $x = (1 - \frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ *or* $x = (1 - \frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$.

2. *If $\bar{\delta}$ and $\underline{\delta}$ are far apart, an oversized coalition forms, i.e.* $x = (1 - \frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$.

Proof. See the Appendix.

[Insert Figure 2]

When $\bar{\delta}$ and $\underline{\delta}$ are close, both $1H$ and $1L$ have incentives to pass their proposals immediately because it is not too expensive to satisfy a H type, so that a minimal-winning coalition forms. On the other hand, if $\bar{\delta}$ and $\underline{\delta}$ are very different, the costs of buying two L type players are relatively small although with some probability the proposal fails. Thus, an oversized proposal is optimal for both types.

It is still possible to consider uncertainty as a good reason which indeed can lead to the formation of oversized coalitions; however, in contrast to the previous literature, the winning coalitions are oversized in an *ex post* sense: no one is quite sure what it takes to get a majority, and oversized proposals are made to be rejected with some probabilities. It is shown that there must exist this kind of pooling equilibrium sometimes, where the proposer prefers to make a proposal that is sure to attract too many votes, rather than take a bigger chance that a more aggressive proposal will fail.

4 Discussions

4.1 An Asymmetric Equilibrium

The reason to focus on equilibria with symmetric tie-breaking rules is that it can produce clear predictions for the equilibrium outcomes. When the tie-breaking rules are asymmetric, much more equilibria can be supportable after any refinement, because players can punish and reward their opponents by manipulating the way they break the tie. Consider the following example, in which H type proposer is always included in the majority for sure in the second period when there is a tie such that he has enough incentives to separate, even when $\bar{\delta}$ and $\underline{\delta}$ are close.

When there is a tie, we denote by $\bar{\rho}^i_j$ the probability that proposer iH chooses j in his coalition in the second period, and by $\underline{\rho}^i_j$ the probability that proposer iL chooses j. Then:

Proposition 4. *There exists a tie-breaking rule* $(\bar{\rho}^i_j, \underline{\rho}^i_j)$ *such that the following separating equilibrium can be supportable when* $\bar{\delta} < 2\underline{\delta}$*:*

1. $x^H = (1 - \frac{2}{3}\underline{\delta} + \frac{1}{9}\bar{\delta}\underline{\delta}, \frac{1}{3}\underline{\delta} - \frac{1}{9}\bar{\delta}\underline{\delta}, \frac{1}{3}\underline{\delta})$ *which is accepted by player* $2L$ *and* $3L$*;*

2. $x^L = (1 - \frac{1}{3}\bar{\delta} + \frac{1}{9}\bar{\delta}\underline{\delta} - \frac{1}{9}\bar{\delta}^2\underline{\rho}^1_2, \frac{1}{3}\bar{\delta} - \frac{1}{9}\bar{\delta}\underline{\delta} + \frac{1}{9}\bar{\delta}^2\underline{\rho}^1_2, 0)$ *which is accepted only by* $2H$*.*

3. *Tie-breaking rule when applicable:*
$$\bar{\rho}^j_1(x^1) = \begin{cases} 1 \quad if \quad x^1 = x^H \\ 0 \quad\quad otherwise \end{cases} \quad for \quad j \neq 1; \;\; \bar{\rho}^1_2 = 0; \;\; \underline{\rho}^1_2 \in [0,1].$$

Proof. See the Appendix.

This example is cooked such that players always choose $1H$ when there is a tie in the second period. This can change his incentive in separation, since when $\bar{\delta}$ and $\underline{\delta}$ are close under the symmetric case, $1H$ has no incentive to wait because he has some significant probabilities of not being in other's coalitions once his proposal is rejected. Here, on the contrary, since there is no restrictions put on the tie-breaking rules used off the equilibrium path, by carefully choosing the tie-breaking rules on and off the equilibrium path, this separating equilibrium can be supportable even if the types difference is small, and the range of non-separation is in fact smaller than the previous case.

4.2 Unanimity Rule

In the traditional bargaining game with unanimity, the proposer has to offer everyone a positive share in order to get enough votes, otherwise the proposal will fail. Thus, a patient player can demand more because in case where the proposal is rejected, no one will be excluded. He then has more incentives to separate herself by delaying the agreement.

Under the same game except the unanimity, there does not always exist a separating equilibrium, especially when both types are less patient. However, contrary to the previous case, we are able to construct a separating equilibrium in some range of the parameters, *even when the two types are very close.* In this separating equilibrium, player $1H$ buys votes from $2H$ and $3L$ (or $2L$ and $3H$), in which case the proposals will be rejected with some probability; and $1L$ buys votes from $2H$ and $3H$, which passes for sure immediately.[4] Thus, in the sense that the types difference has to be larger, majority rule makes separation harder.

5 Conclusion

This paper studies a legislative bargaining model with asymmetric information. Equilibria under majority rule can be quite different from the ones in typical bargaining games. In unanimity games, patient players can demand more since it is less costly to delay. In legislative bargaining, since only a majority is needed to pass a proposal, a player who is strong in

[4]This equilibrium is discussed in Appendix B.

the future is now at a disadvantage because it is more expensive to buy his vote, so that he is excluded from others' coalitions. This new force makes separating equilibria harder to support, in the sense that the type differences have to be large enough. We show that oversized coalitions can form in a pooling equilibrium sometimes, in which the first-period proposal will be rejected with some positive probability. Moreover, the only possible separating equilibrium that can survive under the intuitive criterion is such that the patient type forms an oversized coalition, and the impatient one chooses a minimal-winning coalition. Therefore, "separation" here also represents that they choose different forms of coalitions.

Future research should address whether the results hold for more general environments. The answer is more likely to be positive when the time horizons are extended to finite periods more than three, because the effect of exclusion after separation may still make the "strong" player (who may not always be the patient player now) hesitate in revealing his private information. Another direction is to look at what happens as the number of legislators increases, although its complexity will make it much difficult to predict the results. Our haunch is that, it becomes very hard to support a "universal" coalition if it fails with some probability as the level of uncertainty increases. Proposers sometimes may still have the incentives to buy more votes than needed, because the advantage to be perceived as a "weak" type at present should become larger when there are more players in the game, who wish to include him in their coalitions. Therefore, we expect that some interesting implications can be drawn from the pursuit of the endogenous relationship between the "optimal supermajority" and the level of uncertainty.

Appendix A

After history h, we define $p_j(h)$ the posterior beliefs in the second period that player j is of H type, and $P(h) = (p_1, p_2, p_3)$. Also, $x^i = (x_1^i, x_2^i, x_3^i)$ is the proposal offered by proposer i in the first period, and if the second period is reached on the equilibrium path, $y^i = (y_1^i, y_2^i, y_3^i)$ is the proposal by i. When there is a tie, $\bar{\rho}_j^i$ is the probability that proposer iH chooses j in his coalition in the second period, and $\underline{\rho}_j^i$ is the probability that proposer iL chooses j.

Lemma 1. *Given history h, following the equilibrium path proposal in the second period if it is reached:*

1. *if $\min\{p_j, p_k\} > \frac{\bar{\delta}-\underline{\delta}}{3-\bar{\delta}-\underline{\delta}}$ and $p_j p_k > \frac{\bar{\delta}-2\underline{\delta}}{3-\bar{\delta}-2\underline{\delta}}$, then iH randomizes between $y^i = (1 - \frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ and $(1 - \frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$ with probability $\bar{\rho}_j^i \in [0, 1]$;*

2. *if $\min\{p_j, p_k\} > \frac{\underline{\delta}}{3-\bar{\delta}-\underline{\delta}}$, and $p_j p_k < \frac{\bar{\delta}-2\underline{\delta}}{3-\bar{\delta}-2\underline{\delta}}$, then iH proposes $y^i = (1 - \frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$ with probability 1;*

3. *if $\min\{p_j, p_k\} < \frac{\bar{\delta}-\underline{\delta}}{3-\bar{\delta}-\underline{\delta}}$ and $p_j p_k > \frac{\bar{\delta}-2\underline{\delta}}{3-\bar{\delta}-2\underline{\delta}}$, or if $\min\{p_j, p_k\} < \min\{\frac{\bar{\delta}-\underline{\delta}}{3-\bar{\delta}-\underline{\delta}}, \frac{\underline{\delta}}{3-\bar{\delta}-\underline{\delta}}\}$ and $p_j p_k < \frac{\bar{\delta}-2\underline{\delta}}{3-\bar{\delta}-2\underline{\delta}}$, then iH proposes $y^i = (1 - \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, 0)$ with probability 1 if $p_j < p_k$, and randomizes between $(1 - \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, 0)$ and $(1 - \frac{1}{3}\underline{\delta}, 0, \frac{1}{3}\underline{\delta})$ with a probability $\bar{\rho}_j^i \in [0, 1]$ if $p_j = p_k$;*

where $i, j, k \in \{1, 2, 3\}, i \neq j \neq k$. The cases for proposer iL are analogous.

Proof. First of all, the continuation value for each player in the beginning of the third period is $\frac{1}{3}$. Thus, in the second period if it is reached, player jH votes for any proposal where $y_j^i \geq \frac{1}{3}\bar{\delta}$ and jL votes for any proposal where $y_j^i \geq \frac{1}{3}\underline{\delta}$.

Suppose player $1H$ is the proposer (again) in the second period. Without loss, suppose $p_2 \leq p_3$. Proposal $(1 - \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, 0)$ yields an expected payoff $(1 - \frac{1}{3}\underline{\delta})(1 - p_2) + \frac{1}{3}\bar{\delta} \cdot p_2$; proposal $(1 - \frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$ gives him $(1 - \frac{2}{3}\underline{\delta})(1 - p_2 p_3) + \frac{1}{3}\bar{\delta} \cdot p_2 p_3$, and proposal $(1 - \frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ or $(1 - \frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$ gives him $1 - \frac{1}{3}\bar{\delta}$.

If $p_2p_3 > \frac{\bar{\delta}-2\delta}{3-\bar{\delta}-2\underline{\delta}}$, then for player $1H$, proposal $(1-\frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ or $(1-\frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$ is better than $(1-\frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$. And if $p_2 > \frac{\bar{\delta}-\delta}{3-\bar{\delta}-\underline{\delta}}$, then proposal $(1-\frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ or $(1-\frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$ is better than $(1-\frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, 0)$. Thus, $(1-\frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ or $(1-\frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$ is the best proposal under the conditions in part 1.

Suppose that $p_2p_3 < \frac{\bar{\delta}-2\delta}{3-\bar{\delta}-2\underline{\delta}}$. For player $1H$, proposal $(1-\frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$ is better than $(1-\frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ or $(1-\frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$. If $p_2 > \frac{\bar{\delta}-\delta}{3-\bar{\delta}-\underline{\delta}}$, then proposal $(1-\frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ or $(1-\frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$ is better than $(1-\frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, 0)$, which means that $(1-\frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$ is the best proposal.

For the rest of the cases, $(1-\frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, 0)$ is the best proposal. If $p_2 = p_3$, then player $1H$ is indifferent between $(1-\frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, 0)$ and $(1-\frac{1}{3}\underline{\delta}, 0, \frac{1}{3}\underline{\delta})$.

The cases where player 2 or 3 is the proposer can be similarly computed. Also, the cases for $1L$ are analogous. $\square$

Proof of Proposition 1.

If the second period is reached on the equilibrium path, that is, x^1 is rejected with some positive probability, then by the skimming property, $p_2 \geq \frac{1}{2}$ and $p_3 \geq \frac{1}{2}$. In a separating equilibrium, if x^L is rejected, $p_1 = 0$. By Lemma 1, when player $2H$ ($3H$) is the proposer in the second period, he will *always* choose player $1L$ and offer $y^2 = (\frac{1}{3}\underline{\delta}, 1-\frac{1}{3}\underline{\delta}, 0)$, and $1L$ will accept it. On the other hand, if x^H is rejected, $p_1 = 1$. Then player 2 will randomize between $(\frac{1}{3}\bar{\delta}, 1-\frac{1}{3}\bar{\delta}, 0)$ and $(0, 1-\frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta})$, because $\frac{1}{2} > \frac{\bar{\delta}-\delta}{3-\bar{\delta}-\underline{\delta}} > \frac{\bar{\delta}-2\delta}{3-\bar{\delta}-2\underline{\delta}}$. That is, player $1H$ is not always in the majority. The case for player $2L$ ($3L$) being the proposer is similar.

When player 1 is the proposer in the second period again, according to Lemma 1, he would propose $y^1 = (1-\frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$ only if *both* types of both players 2 and 3 vote against x^1 with some positive probabilities (i.e. when $p_2p_3 < \frac{\bar{\delta}-2\delta}{3-\bar{\delta}-2\underline{\delta}} < \frac{1}{2}$). Give that the pie shrinks in a discount rate and player 1 may lose the control in agenda setting in the next period, this case cannot happen *on the equilibrium path*, that is, at least one of p_2 and p_3 is equal to 1. Therefore, player 1 will randomize between $(1-\frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ and $(1-\frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$.

In a pooling equilibrium, if x^1 is rejected on the equilibrium path, $p_1 = \frac{1}{2}$. According to Lemma 1, each proposer will offer $1 - \frac{1}{3}\bar{\delta}$ to himself, and $\frac{1}{3}\bar{\delta}$ to one of the other players randomly in the second period. Therefore, we can see that a minimal-winning coalition always forms on the equilibrium path if the second period is reached. □

Proof of Proposition 2.

There are several steps to prove this proposition.

Lemma 2. *In a separating equilibrium, x^H must be rejected with a positive probability.*

Proof. Suppose that on the equilibrium path, x^H passes for sure in the first period. It cannot be a separating equilibrium when x^L also passes for sure on the equilibrium path in the first period, because one of the types will deviate to the proposal with a higher payoff. Suppose that x^L fails with some probability $\gamma^L > 0$, then we have the incentive constraints:

$$(1-\gamma^L)x_1^L + \gamma^L \underline{\delta} V_1(x^L) \geq x_1^H \tag{1}$$

$$x_1^H \geq (1-\gamma^L)x_1^L + \gamma^L \bar{\delta} V_1(x^L) \tag{2}$$

(1) and (2) cannot be both satisfied. Thus, x^H must be rejected with a positive probability on the equilibrium path. □

Proposition 2, part 1. If $\bar{\delta} < 2\underline{\delta}$, there exists no separating equilibrium under symmetric tie-breaking rules and the intuitive criterion.

Proof. Since in a separating equilibrium, x^H is rejected with a positive probability in the first period on the equilibrium path, then based on the skimming property, $P(x^H) = (1, p_2 \geq \frac{1}{2}, p_3 \geq \frac{1}{2})$, and $p_2 p_3 \geq \frac{1}{2}$, and we can compute the continuation values in the second period, given the strategies played in the second period as in Lemma 1:

$$V_1(x^H) = V_2(x^H) = V_3(x^H) = \frac{1}{3}$$

Thus, player j votes for x^H if $x_j^H \geq \frac{1}{3}\delta$, where $\delta \in \{\bar{\delta}, \underline{\delta}\}$. In the first period, player 1 can consider one of the following proposal: $x^H = (1 - \frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$, $(1 - \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, 0)$, $(1 - \frac{1}{3}\underline{\delta}, 0, \frac{1}{3}\underline{\delta})$,

$(1-\frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$, or $(1-\frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$. Proposals other than those ones will give player 1 a smaller share in the first period but will not increase the probability of passing, and thus they can be ignored.

If $x^H = (1-\frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$, then in equilibrium player $2L$ and $3L$ will vote for it, but player $2H$ and $3H$ will vote against it. Therefore, the expected payoff for player $1H$ from this proposal is

$$\frac{3}{4}(1-\frac{2}{3}\underline{\delta}) + \frac{1}{4}\cdot\frac{1}{3}\bar{\delta} = \frac{3}{4} - \frac{1}{2}\underline{\delta} + \frac{1}{12}\bar{\delta}.$$

If $x^H = (1-\frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, 0)$ ($x^H = (1-\frac{1}{3}\underline{\delta}, 0, \frac{1}{3}\underline{\delta})$ is analogous), then only player $2L$ will vote for it, but players $2H$ and 3 ($3H$ and $3L$) will vote against it. Therefore, the expected payoff for player $1H$ from this proposal is

$$\frac{1}{2}(1-\frac{1}{3}\underline{\delta}) + \frac{1}{2}\cdot\frac{1}{3}\bar{\delta} = \frac{1}{2} - \frac{1}{6}\underline{\delta} + \frac{1}{6}\bar{\delta}.$$

If $x^H = (1-\frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ (and $x^H = (1-\frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$ is analogous), both players $2L$ and $2H$ vote for it so that it passes in the first period. Thus, the payoff is $1-\frac{1}{3}\bar{\delta}$.

It is easily to verify that $1-\frac{1}{3}\bar{\delta} > \frac{1}{2} - \frac{1}{6}\underline{\delta} + \frac{1}{6}\bar{\delta}$ for any possible parameters $\bar{\delta}$ and $\underline{\delta}$, and $1-\frac{1}{3}\bar{\delta} > \frac{3}{4} - \frac{1}{2}\underline{\delta} + \frac{1}{12}\bar{\delta}$ if $\bar{\delta} < 2\underline{\delta}$.[5] That is, $x^H = (1-\frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ is the best proposal for player $1H$ *if he is perceived as a H type.* Any other proposals which may be able to support a separating equilibrium must be based on the belief that there is some probability that this deviating proposal, $x' = (1-\frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$, is offered by $1L$, so that player 2 might be better off to reject it. It is not the case when $\bar{\delta} < 2\underline{\delta}$, because if $P(x') = (0, p_2 \geq \frac{1}{2}, p_3 \geq \frac{1}{2})$ is the off-equilibrium belief,[6] the continuation value in the second period, following the strategies played in that period, is

$$V_2(x') = \frac{1}{3} - \frac{1}{9}\underline{\delta} + \frac{1}{18}\bar{\delta} < \frac{1}{3},$$

so that both types of player 2 are still better off to accept it, instead of rejecting it, and thus

[5] It is required that $\bar{\delta} < \frac{3+6\underline{\delta}}{5}$, which is impled by $\bar{\delta} < 2\underline{\delta}$ for any $\bar{\delta}, \underline{\delta} \in (0, 1]$.

[6] $p_2 = 0$ or $p_3 = 0$ cannot be reasonable beliefs, because player $2L$ or $3L$ cannot be better off to reject the proposal no matter what off-equilibrium beliefs players hold.

player $1H$ is better off to deviate. Thus, no matter what off-equilibrium beliefs that players may hold, it is optimal for $1H$ to offer $x^H = (1-\frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ or $(1-\frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$.

However, this contradicts to Lemma 2, in which we claim that the proposal must fail with some probability in the first period in a separating equilibrium. This shows that there does not exist any separating equilibrium if $\bar{\delta} < 2\underline{\delta}$ under the intuitive criterion. □

Lemma 3. *In a separating equilibrium, it is not optimal for player $1L$ that x^L is made such that only one player of L type accepts it in the first period.*

Proof. Suppose that x^L is accepted only by $2L$ in the first period, then in the second period if it is reached, $P(x^L) = (0, 1, \frac{1}{2})$, and $V_1(x^L) = \frac{1}{3}(1-\frac{1}{3}\bar{\delta}) + \frac{2}{3}\cdot\frac{1}{3}\underline{\delta}$. Since $2L$ accepts x^L iff $x_2 \geq \underline{\delta}[\frac{1}{3}(1-\frac{1}{3}\underline{\delta})]$, in which $1L$ can set $\underline{\rho}_3^1 = 1$ to break the tie (that is, player $1L$ can always choose player 3 if the second period is reached), the highest expected payoff in the first period player $1L$ can obtain from this proposal is $\frac{1}{2}(1+\frac{1}{3}\underline{\delta}^2 - \frac{1}{9}\bar{\delta}\underline{\delta})$ (which is certainly higher than the one under symmetric tie-breaking rules).

However, if $1L$ instead offers a proposal such that $2H$ accepts it, then no matter what off-equilibrium beliefs players could hold, player $2H$ accepts it iff $x_2 \geq \bar{\delta}[\frac{1}{3}(1-\frac{1}{3}\underline{\delta}) + \frac{1}{3}(\frac{1}{3}\bar{\delta})]$. In this case, the least that player $1L$ can get from this proposal is $(1-\frac{1}{3}\bar{\delta}+\frac{1}{9}\bar{\delta}\underline{\delta}-\frac{1}{9}\bar{\delta}^2)$, which is always higher than $\frac{1}{2}(1+\frac{1}{3}\underline{\delta}^2-\frac{1}{9}\bar{\delta}\underline{\delta})$. Hence, it is never optimal that x^L is made such that only one player of L type accepts it. □

Lemma 4. *In a separating equilibrium, if $\bar{\delta} > 2\underline{\delta}$, then under the intuitive criterion, the only possible x^H is $x^H = (1-\frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$.*

Proof. By Lemma 2, $x^H = (1-\frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ or $(1-\frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$ cannot be a candidate in a separating equilibrium. Also, $\frac{3}{4}-\frac{1}{2}\underline{\delta}+\frac{1}{12}\bar{\delta} > \frac{1}{2}-\frac{1}{6}\underline{\delta}+\frac{1}{6}\bar{\delta}$ is always true when $\bar{\delta} > 2\underline{\delta}$. On the other hand, there exist a range in $\bar{\delta} > 2\underline{\delta}$ such that $\frac{3}{4}-\frac{1}{2}\underline{\delta}+\frac{1}{12}\bar{\delta} > 1-\frac{1}{3}\bar{\delta}$. Therefore, if $\bar{\delta} > 2\underline{\delta}$ and a separating equilibrium exists, the only possible candidate of x^H under the intuitive criterion is $x^H = (1-\frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$, because this is the best proposal that player $1H$

can obtain if he is perceived as a H type. $\square$

Proposition 2, part 2. If $\bar{\delta} \geq 2\underline{\delta}$, the only separating equilibrium that can be supportable under the intuitive criterion is such that $x^H = (1 - \frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$, and $x^L = (1 - \frac{1}{3}\bar{\delta} + \frac{1}{9}\bar{\delta}\underline{\delta} - \frac{1}{18}\bar{\delta}^2, \frac{1}{3}\bar{\delta} - \frac{1}{9}\bar{\delta}\underline{\delta} + \frac{1}{18}\bar{\delta}^2, 0)$ or $(1 - \frac{1}{3}\bar{\delta} + \frac{1}{9}\bar{\delta}\underline{\delta} - \frac{1}{18}\bar{\delta}^2, 0, \frac{1}{3}\bar{\delta} - \frac{1}{9}\bar{\delta}\underline{\delta} + \frac{1}{18}\bar{\delta}^2)$.

Proof. By Lemma 2, 3 and 4, the only possible candidates for a separating equilibrium are: (i) $1H$ buys two votes from $2L$ and $3L$, and $1L$ buys two votes from $2L$ and $3L$; and (ii) $1H$ buys two votes from $2L$ and $3L$, and $1L$ buys one vote from $2H$ or $3H$.

Case (i):

Since both x^H and x^L are rejected with positive probabilities in the first period on the equilibrium path, then $P(x^H) = (1,1,1)$ and $P(x^L) = (0,1,1)$. Again, the continuation values in the second period can be obtained, and so $x^H = (1 - \frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$, and $x^L = (1 - \frac{2}{3}\underline{\delta} + \frac{2}{9}\underline{\delta}^2 - \frac{1}{9}\bar{\delta}\underline{\delta}, \frac{1}{3}\underline{\delta} - \frac{1}{9}\underline{\delta}^2 + \frac{1}{18}\bar{\delta}\underline{\delta}, \frac{1}{3}\underline{\delta} - \frac{1}{9}\underline{\delta}^2 + \frac{1}{18}\bar{\delta}\underline{\delta})$. The incentive constraints require:

$$(1-\gamma^L)x_1^L + \gamma^L \underline{\delta} V_1(x^L) \geq (1-\gamma^H)x_1^H + \gamma^H \underline{\delta} V_1(x^H) \tag{3}$$

$$(1-\gamma^H)x_1^H + \gamma^H \bar{\delta} V_1(x^H) \geq (1-\gamma^L)x_1^L + \gamma^L \bar{\delta} V_1(x^L) \tag{4}$$

Since in this case, $\gamma^L = \gamma^H = \frac{1}{2}$ in equilibrium, and $V_1(x^H) = \frac{1}{3} > V_1(x^L) = \frac{1}{3} - \frac{1}{9}\bar{\delta} + \frac{2}{9}\underline{\delta}$ if $\bar{\delta} > 2\underline{\delta}$, it is equivalent to:

$$x_1^L \geq x_1^H, \tag{5}$$

or $1 - \frac{2}{3}\underline{\delta} + \frac{2}{9}\underline{\delta}^2 - \frac{1}{9}\bar{\delta}\underline{\delta} \geq 1 - \frac{2}{3}\underline{\delta}$, which is not true if $\bar{\delta} > 2\underline{\delta}$. Therefore, this case cannot be supported as a separating equilibrium.

Case (ii):

The only case left is where $1H$ buys two votes from $2L$ and $3L$, and $1L$ buys one vote from $2H$ or $3H$. We can show there exists a range of parameters in $\bar{\delta} > 2\underline{\delta}$ that supports this equilibrium. First of all, supposing that $1L$ buys one vote from $2H$, the reasonable beliefs must put $p_2 \geq \frac{1}{2}$, because it is not possible that $2L$ is better off by rejecting the proposal, but $2H$ is better off to reject it. Therefore, we have $x^L = (1 - \frac{1}{3}\bar{\delta} + \frac{1}{9}\bar{\delta}\underline{\delta} - \frac{1}{18}\bar{\delta}^2, \frac{1}{3}\bar{\delta} - \frac{1}{9}\bar{\delta}\underline{\delta} + \frac{1}{18}\bar{\delta}^2, 0)$, based on $P(x^L) = (0, p_2 \geq \frac{1}{2}, \frac{1}{2})$. The incentive constraints require

$$\frac{3}{4}(1 - \frac{2}{3}\underline{\delta}) + \frac{1}{4} \cdot \frac{1}{3}\bar{\delta} \geq 1 - \frac{1}{3}\bar{\delta} + \frac{1}{9}\bar{\delta}\underline{\delta} - \frac{1}{18}\bar{\delta}^2 \geq \frac{3}{4}(1 - \frac{2}{3}\underline{\delta}) + \frac{1}{4} \cdot \frac{1}{3}\underline{\delta}, \tag{6}$$

which gives us the range of parameters $(\bar{\delta}, \underline{\delta})$ that satisfies incentive compatibility.[7]

Under the intuitive criterion, it also requires that these proposals are the best one in a separating equilibrium for players $1H$ and $1L$, respectively, so that there are two more restrictions on parameters $(\bar{\delta}, \underline{\delta})$.[8] That is, they assure that the proposal is the best one for $1H$ ($1L$) when he is perceived as a H (L) type. Combining all these constraints, we can find a range of parameters $(\bar{\delta}, \underline{\delta})$ where this separating equilibrium can be supportable. For example, $\bar{\delta} = 1$ and $\underline{\delta} = 0.3$ and their neighborhood can satisfy all the constraints. □

Proof of Proposition 3.

In a pooling equilibrium, both types of player 1 offer the same proposal in the first period. Thus, if the second period is on the equilibrium path, $p_1 = \frac{1}{2}$. Similar to Lemma 3, it can still be shown that it is never optimal that x is made such that only one player of L type accepts it. There remain two other cases.

Case (i): Proposal x is made such that either player $2H$ or $3H$ accepts it:

Suppose in equilibrium, player $2H$ accepts x, and both player $3L$ and $3H$ reject it. In the second period if reached, reasonable beliefs are put such that $p_2 \geq \frac{1}{2}$, and thus $P(x) = (\frac{1}{2}, p_2 \geq \frac{1}{2}, \frac{1}{2})$. According to Lemma 1, every player, regardless of his type, offers $1 - \frac{1}{3}\bar{\delta}$ to himself and $\frac{1}{3}\bar{\delta}$ to one of the other players. Therefore, the continuation value for each player is $\frac{1}{3}$ under symmetric tie-breaking rules. Player 1 thus offers $x = (1 - \frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, 0)$ or $x = (1 - \frac{1}{3}\bar{\delta}, 0, \frac{1}{3}\bar{\delta})$ and player 2 (or player 3) will accept it on the equilibrium path.

[7] Namely, $\frac{4\underline{\delta} - 15 + \sqrt{16\underline{\delta}^2 + 24\underline{\delta} + 297}}{4} \leq \bar{\delta} \leq \frac{4\underline{\delta} - 12 + \sqrt{16\underline{\delta}^2 + 24\underline{\delta} + 216}}{4}$.

[8] Namely, $\frac{3 + 6\underline{\delta}}{5} \leq \bar{\delta} \leq \frac{-6 + 4\underline{\delta} + \sqrt{54 - 18\underline{\delta}}}{2}$.

This proposal can be supportable when $\bar{\delta}$ and $\underline{\delta}$ are close. One proposal that player 1 may consider to deviate to is such that only $2L$ and $3L$ vote for it. However, as the calculations in the separating equilibrium, when $\bar{\delta}$ and $\underline{\delta}$ are close (for example, $\bar{\delta} < 2\underline{\delta}$), the expected payoff for $1H$ from this deviation is always less than the pool one, no matter what off-equilibrium beliefs players hold.

Case (ii): Proposal x is made such that only players $2L$ and $3L$ accept it:

When the second period is reached on the equilibrium path, $P(x) = (\frac{1}{2}, 1, 1)$, and so in this case, $x = (1 - \frac{2}{3}\underline{\delta}, \frac{1}{3}\underline{\delta}, \frac{1}{3}\underline{\delta})$. The expected payoff for $1H$ is $\frac{3}{4}(1 - \frac{2}{3}\underline{\delta}) + \frac{1}{4} \cdot \frac{1}{3}\bar{\delta} = \frac{3}{4} - \frac{1}{2}\underline{\delta} + \frac{1}{12}\bar{\delta}$, and that for $1L$ is $\frac{3}{4} - \frac{5}{12}\underline{\delta}$.

Suppose that $1L$ deviates to buying one vote from $2H$ and offering him $\frac{1}{3}\bar{\delta}$. If this deviation is considered from a H type, then the expected payoff is $1 - \frac{1}{3}\bar{\delta}$ under the reasonable beliefs. Thus, as long as $\bar{\delta}$ and $\underline{\delta}$ are distant enough such that $\frac{3}{4} - \frac{5}{12}\underline{\delta} > 1 - \frac{1}{3}\bar{\delta}$, this is no a profitable deviation. If this deviation is considered from a L type, then the continuation value for player 2 is $\frac{1}{3} - \frac{1}{9}\underline{\delta} + \frac{1}{18}\bar{\delta}$, which is larger than $\frac{1}{3}$ if $\bar{\delta} > 2\underline{\delta}$. This means that player 2 is better off to reject this deviating proposal. Therefore, when $\bar{\delta}$ and $\underline{\delta}$ are distant enough, there is no profitable deviation so that this proposal can be supportable as a pooling equilibrium. The case for $1H$ is similar. □

Proof of Proposition 4.

Suppose that on the equilibrium path, x^H is accepted by $2L$ and $3L$. Hence, $P(x^H) = (1, 1, 1)$ when x^H is rejected in the first period. Based on the tie-breaking rules proposed, player $2L$ accepts x^H iff

$$x_2^H \geq \frac{1}{3}\underline{\delta}(1 - \frac{1}{3}\bar{\delta}) \tag{7}$$

and player $3L$ accepts it iff

$$x_3^H \geq \frac{1}{3}\underline{\delta}[(1 - \frac{1}{3}\bar{\delta}) + \frac{1}{3}\bar{\delta}] \tag{8}$$

Player $1H$ thus proposes x^H as stated. On the other hands, suppose that x^L is accepted only by $2H$. Since $P(x^L) = (0, p_2 \geq \frac{1}{2}, \frac{1}{2})$ is based on reasonable beliefs, player $2H$ accepts it iff

$$x_2^L \geq \frac{1}{3}\bar{\delta}[(1 - \frac{1}{3}\underline{\delta}) + \frac{1}{3}\bar{\delta} \cdot \underline{\rho}_2^1] \tag{9}$$

The incentive constraints implies

$$1 - \frac{1}{3}\bar{\delta} + \frac{1}{9}\bar{\delta}\underline{\delta} - \frac{1}{9}\bar{\delta}^2\underline{\rho}_2^1 \geq \frac{3}{4}(1 - \frac{2}{3}\underline{\delta} + \frac{1}{9}\bar{\delta}\underline{\delta}) + \frac{1}{4}\underline{\delta}(\frac{1}{3} + \frac{1}{9}\bar{\delta}) \tag{10}$$

$$\frac{3}{4}(1 - \frac{2}{3}\underline{\delta} + \frac{1}{9}\bar{\delta}\underline{\delta}) + \frac{1}{4}\bar{\delta}(\frac{1}{3} + \frac{1}{9}\bar{\delta}) \geq 1 - \frac{1}{3}\bar{\delta} + \frac{1}{9}\bar{\delta}\underline{\delta} - \frac{1}{9}\bar{\delta}^2\underline{\rho}_2^1 \tag{11}$$

Both (10) and (11) can be satisfied if $\underline{\rho}_2^1$ is chosen wisely even when $\bar{\delta} < 2\underline{\delta}$. For example, if $\bar{\delta}$ is close to but strictly less than 1 and $\underline{\delta} = \frac{1}{2}$, $\underline{\rho}_2^1 > \frac{5}{8}$ works.

While looking for any profitable deviation, the best deviating proposal x' for player $1H$ is to make one player of H type, say $2H$, accept it. In this case, $P(x') = (1, 1, \frac{1}{2})$. He can set $\underline{\rho}_2^1 = 0$, so that the best one is $x_2' = \frac{1}{3}\bar{\delta}[(1 - \frac{1}{3}\bar{\delta}) + \frac{1}{3}\bar{\delta} \cdot (1 - \bar{\rho}_1^3)]$ and $x_1' = 1 - x_2'$, given the tie-breaking rule which player 3 is applying. Since there is no restriction on the tie-breaking rules which people are using off the equilibrium path, there is an equilibrium supportable at $\bar{\rho}_1^3 = 0$, so that in some range of $\bar{\delta} < 2\underline{\delta}$, there is no profitable deviation given any off-equilibrium beliefs. That is, by carefully choosing the tie-breaking rules on and off the equilibrium path, this equilibrium can be supportable even if the type difference is small. $\square$

Appendix B

A Separating Equilibrium under Unanimity.

Again, every proposer demands 1 in the third period and the continuation value for each player in the third period is $\frac{1}{3}$, so that everyone votes for the second-period proposal y^i if and only if $y_j^i \geq \frac{1}{3}\delta$. In the second period, under the skimming property and the intuitive criterion, the reasonable beliefs after a rejection to the first-period proposal must be that both player 2 and 3 are of H types. Therefore, the optimal second-period proposal for player 1 is $(1 - \frac{2}{3}\bar{\delta}, \frac{1}{3}\bar{\delta}, \frac{1}{3}\bar{\delta})$. Likewise, in a separating equilibrium, player 2 offers $(\frac{1}{3}\bar{\delta}, 1 - \frac{2}{3}\bar{\delta}, \frac{1}{3}\bar{\delta})$ to

$1H$, and $(\frac{1}{3}\underline{\delta}, 1-\frac{1}{3}\bar{\delta}-\frac{1}{3}\underline{\delta}, \frac{1}{3}\bar{\delta})$ to $1L$. It is analogous for player 3. Therefore, the continuation values are $V_1(x^H) = V_2(x^H) = V_3(x^H) = \frac{1}{3}$, and $V_1(x^L) = \frac{1}{3} - \frac{2}{9}\bar{\delta} + \frac{2}{9}\underline{\delta}$, $V_2(x^H) = V_3(x^H) = \frac{1}{3} + \frac{1}{9}\bar{\delta} - \frac{1}{9}\underline{\delta}$.

It can be shown that it is never optimal for player 1 of both types buy votes from only $2L$ and $3L$ in the first period. Consider the case that $1H$ buys votes from $2H$ and $3L$, and $1L$ buys votes from $2H$ and $3H$. Then $x^H = (1-\frac{1}{3}\bar{\delta}-\frac{1}{3}\underline{\delta}, \frac{1}{3}\bar{\delta}, \frac{1}{3}\underline{\delta})$ and $x^L = (1-\frac{2}{3}\bar{\delta}-\frac{2}{9}\bar{\delta}^2+\frac{2}{9}\bar{\delta}\underline{\delta}, \frac{1}{3}\bar{\delta}+\frac{1}{9}\bar{\delta}^2-\frac{1}{9}\bar{\delta}\underline{\delta}, \frac{1}{3}\bar{\delta}+\frac{1}{9}\bar{\delta}^2-\frac{1}{9}\bar{\delta}\underline{\delta})$. The incentive constraints are

$$1-\frac{2}{3}\bar{\delta}-\frac{2}{9}\bar{\delta}^2+\frac{2}{9}\bar{\delta}\underline{\delta} \geq \frac{1}{2}(1-\frac{1}{3}\bar{\delta}-\frac{1}{3}\underline{\delta})+\frac{1}{2}\cdot\frac{1}{3}\underline{\delta} \tag{12}$$

$$\frac{1}{2}(1-\frac{1}{3}\bar{\delta}-\frac{1}{3}\underline{\delta})+\frac{1}{2}\cdot\frac{1}{3}\bar{\delta} \geq 1-\frac{2}{3}\bar{\delta}-\frac{2}{9}\bar{\delta}^2+\frac{2}{9}\bar{\delta}\underline{\delta} \tag{13}$$

The is a range of the parameters such that even when $\bar{\delta}$ and $\underline{\delta}$ are very close, (12) and (13) are both satisfied. Namely, $\frac{\underline{\delta}-3+\sqrt{\underline{\delta}^2-3\underline{\delta}+18}}{2} \leq \bar{\delta} \leq \frac{4\underline{\delta}-9+\sqrt{16\underline{\delta}^2-72\underline{\delta}+225}}{8}$. □

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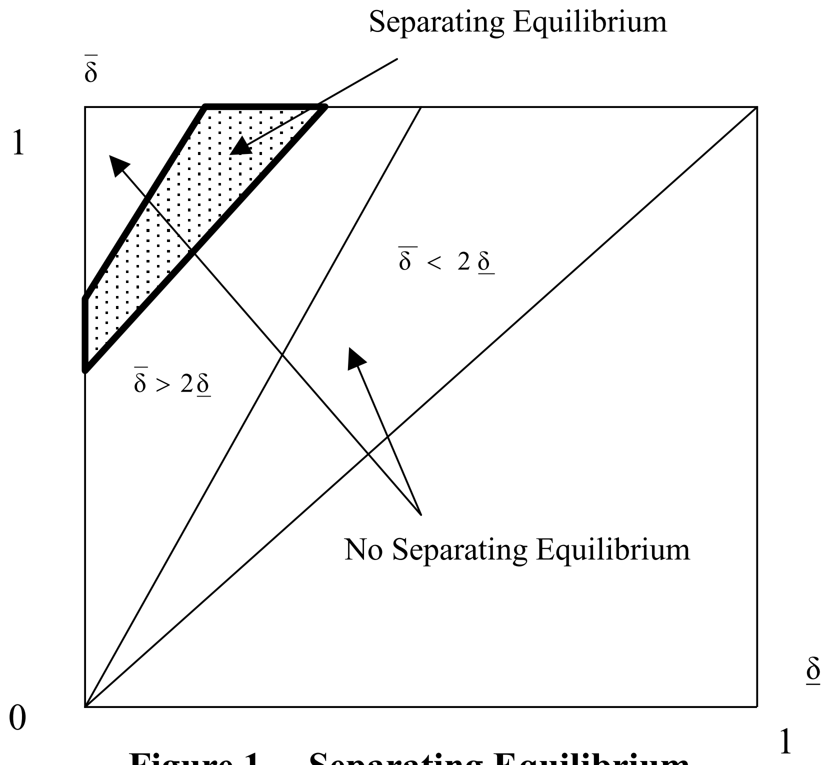

Figure 1. Separating Equilibrium

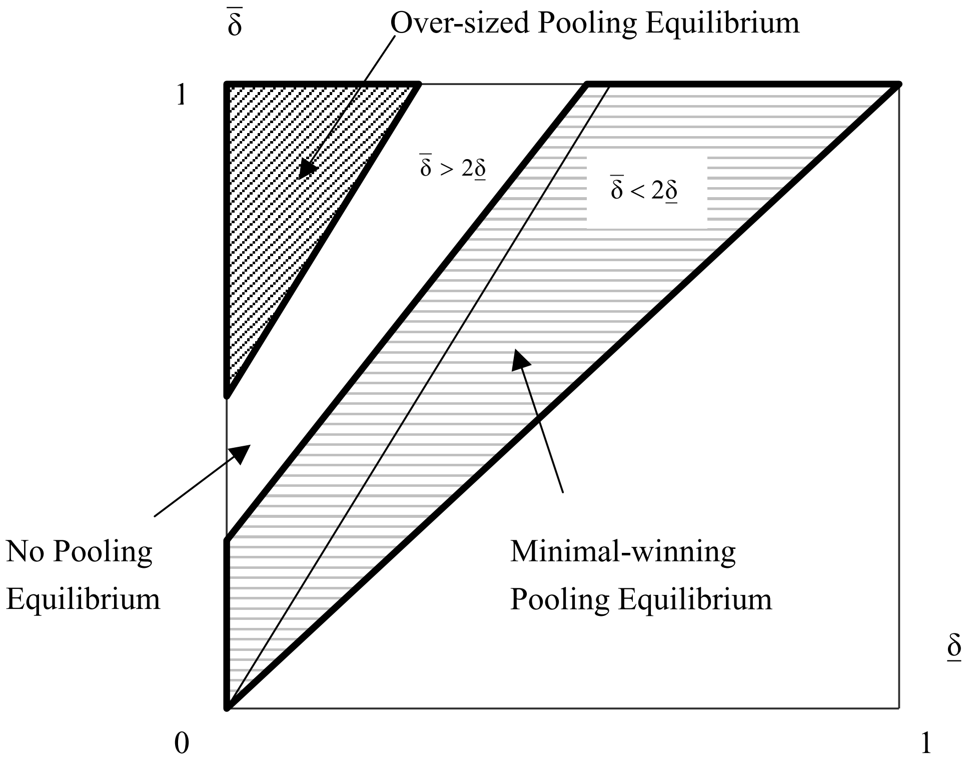

Figure 2. Pooling Equilibrium