

Strategic Behavior, Truthfulness and Optimality of Waiting Option in the Duopoly Forecasting Announcement Market ¹

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Abstract

In this paper, we discuss the possibility of strategic behavior and truthful reporting in a two players' announcement game when there is a cost for getting information. First we show that the best strategy of each player is to announce the observed signal truthfully if the announcement is made simultaneously. Second we show that if the order of announcement is given exogenously, the player who moves first reports her information truthfully always. But the optimal strategies of the player who moves later depends on the information cost, the belief in the information quality and payoffs. These can be explained by three factors, the incentive to be differentiated, negative effect of the information cost and the blame-sharing effect. Also we discuss what is a good scheme to induce the subsequent player to observe her signal and announce truthfully. Finally, we discuss about the welfare of using the waiting option in an endogenous ordering. We show that player can be better off in terms of ex-ante compared to the simultaneous announcement case even if she uses the waiting option for delaying her decision. This is a interesting result because the possibility of waiting option is usually understood as a main reason to make player worse-off compared to the simultaneous movement case. The conditions under which each player can be better off or worse off from using the option are explained. Finally the results of the experiments that support our model are denoted. Some assertions in this paper can be used as an alternative explanation for the phenomenon of the coincidence or discrepancy in the sovereign credit rating or corporate credit rating market by professional analysts and the reputation markets.

JEL Classification: C72, D83, L13. L84

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1 Introduction

In this paper, we discuss the phenomenon of player's strategic behavior under the framework of two players' announcement game. Usually, the strategic behavior with career concern or reputation is understood as the main reason that induces players' inefficient decision making when players are competing. Herding and anti-herding are good examples of inefficient results when players behave strategically with that motive. Shartstein and Stein (1990) first introduced a model that explains herding phenomenon with an example of investment decision. Also Effinger and Polborn (2001) introduced the reverse concept, anti-herding using an example of financial analysts' behavior.² The herding explains the situation that player makes the same decision with others and the anti-herding explains the situation that player makes the different decision from others. The important point of both concepts is that player behaves strategically with ignoring her own information. So herding is a meaningful concept when player chooses the same action with other players however she observed different signals from others. On the contrary, the anti-herding is meaningful especially when player deviates from the other players however she observed same signal with other players.

Each player has an opportunity to observe her signal that partially reveals the information about true state, but not perfectly. Each player's objective is to select the strategy to maximize her expected utility. Her payoff depends on her strategy and also on the other player's strategy. So the best case for each player is when she is the only one player who forecasted correctly and the worst case is when she is the only one player who announced wrong information. The payoff system assumed in this paper gives two conflicting incentives to each player. First, player has an incentive to make same announcement because the penalty when both are wrong is greater than the penalty when she is the only one player who was wrong. This is caused from the risk aversions to the situation that she is the unique player who announced wrong information. That is to say, so called "sharing blame effect" works.³ Second, player has the other contrary incentive to announce different information because the reward when she is the unique player who announces correct information is the greatest one in payoffs system. This is caused from the desire to be differentiated from the other player.

This paper follows similar framework with papers that treated the topic of herding and anti-herding, but introduces the model that explains the herding and anti-herding phenomenon together. Also the factors such as belief in the information quality, the information cost and the asymmetry in payoffs are introduced. In our model, each player's best strategies are described as a function of these parameters, so we show that these factors play a critical role in determining each player's strategy.

The most important new approaches of our model are the assumptions about players' type and the information cost. Many papers with a topic of herding or anti-herding usually assumed that players are heterogeneous in their type. Here, the meaning of being heterogeneous is that players have different ability in inferring information about true state from observed signal. Or it is used for denoting that the observed signals by each player have different information quality. In those settings, whether each player knows her type or not is an important factor in the analysis. So whether we can get the result

²Sharfstein & Stein (1990) also mentioned the possibility of anti-herding or deviation in their paper. But they just focused on the explanation of herding effect induced from reputational concern or career concern.

³The term, sharing blame effect was introduced in Sharfstein & Stein (1990).

of herding or anti-herding that strongly depends on the assumption about heterogeneous types.⁴ But in our paper, there is no assumption about player's types. Each player who participates in this game is a homogeneous type in the sense that there is no difference in player's information updating procedure from observed signal. Also the signal observed by each player has the same information quality and each player gives same weight for the observed information. However each player doesn't know which signal does the other player observe, she knows that the other player gives same weight to her signal if it is observed and she also gives same weight to the other player's information. If we can show the possibility of strategic deviation and imitation under this assumption of homogeneous types players, we may get similar results also under the assumption of heterogeneous types players.

The other important assumption that we use in our model is the positive information cost for observing signal. Till now, all papers with a topic of herding or anti-herding assumed that there is no cost for observing signal. So whether she will observe her signal or not is not considered as an important problem. But for the topic of strategic behavior of players, we think players' decision whether she will observe her signal or not should be considered as the important question because the degree of herding or anti-herding will be excessive especially if each player makes decision only based on the other player's announcement without her own signal. Also the free information cost assumption is not realistic because we can find many examples that people should pay cost for getting information. So we assert that the information cost for getting signal should be regarded as an important factor in the analysis of players' decision procedure. For example, if players should pay very high information cost for getting signal, sometimes it is rational for players to behave without signal with giving up the opportunity to observe signal. In this way, the information cost is strongly linked with player's decision and this is the reason why the information cost should be considered as an important factor. In this paper, we show that the analysis under the information cost enriches the results of our analysis compared to other works.⁵

Finally, the belief in the information quality plays an important role in each player's decision rule in our model. Each player's strategy can be expressed as the function of this, so we can get intuitive results and interpretation.

Under above basic frameworks, first, we analyze the simultaneous announcement and sequential announcement cases separately and compare the results of those cases. First, we show that there exists unique Bayesian Nash Equilibrium in a simultaneous announcement case. In this case, each player's best strategy is to report her signal truthfully. Second, under the assumption of sequential ordered announcement, player's best strategy depends on the information cost, payoffs and the belief in the information quality. Here, we can check the the three factors, the incentive to be differentiated, the negative effect of the information cost and the blame sharing effect. Let's denote that γ is the reward that the player gets if she is the unique player who made a correct announcement. Also let ϕ be the penalty that she gets if she is the unique player who made a false announcement.

If the cost for getting information is sufficiently greater than the critical level, the subsequent player

⁴For the brief summary about this type's assumption for many papers that treated the topic of behavior with career or reputation concern, please check Levy G. (2002), "Anti-herding and Strategic Consultation", p2-p5.

⁵According to our knowledge, this is the first paper that discussed the role of information cost for the explanation of strategic behavior with reputation concern.

gives up the opportunity to observe signal and behaves strategically without signal. That is to say, only blame sharing effect works. Whether she will imitate or deviate from the other player who moved first depends on the belief in the information quality.

If the given information cost is less than this critical level, we have some multiple results that depend on the parameters. First, suppose that $\gamma > \phi$. If the difference in the reward and penalty is not so high ($0 < \gamma - \phi < 2$), her strategy set consists of three elements, observing signal, imitation or deviation without observing signal. However the reward is relatively greater than the penalty ($\gamma - \phi > 2$), if the information cost is not sufficiently low, her strategy set consists of three strategies again. But if the information cost is not sufficiently low, her strategy set consists of only two elements, observing signal or deviation without signal. First, whether the strategy, deviation without signal, is included in the strategy set or not is determined by the incentive to be differentiated. If $\gamma > \phi$, the incentive to be differentiated exists always. Second, whether the incentive to be differentiated dominates the negative effect of information quality or not has an effect on whether the strategy, observing signal with information cost exists or not. If $0 < \gamma - \phi < 2$ or $\gamma - \phi > 2$ with relatively high information cost, we can find the belief interval in which the strategy of observing signal is the element of the strategy set. Finally, whether the strategy of imitation without signal is the element of the strategy set or not also depends on whether γ is sufficiently high or not. For a sufficiently high γ , the incentive to be differentiated dominates the blame sharing effect. So the strategy of imitation without signal is not included in the strategy set. But for γ which is not sufficiently high, it is included in the strategy set. That is to say, there exists belief interval in which the blame sharing effect dominates the incentive to be differentiated.

Second, suppose that the penalty is greater than the reward. Then the subsequent player always imitates the other player always. This comes from the reasoning that the greater penalty makes the so-called blame-sharing effect work and it induces her to imitate without signal always.

Finally, we discuss the welfare of using an waiting option with a view of ex-ante in an endogenous ordering under incomplete information. Each player has an incentive to use the waiting option for observing the other player's announcement because this makes it possible for her to have more information about true state. Moreover, because we assumed a positive information cost for getting signal, there may exist an excessive tendency of using an waiting option. Choi (1997) discussed this problem, so-called Penguin effect, with the example of technology adoption in which players hesitate to experiment a new but risky technology for the fear of being stranded. In his paper, it is asserted that, under some conditions, using the waiting option in endogenous ordering causes the decrease of player's welfare compared to the simultaneous movement case. In our model, we specify the conditions under which each player become better off or worse off from using the waiting option in an endogenous ordering compared to the simultaneous case with a view of ex-ante.

First, if the information cost sufficiently high and the reward is greater than penalty, whether waiting option makes player better off or worse off depends on the values of reward and penalty. If the reward is relatively greater than the penalty, the waiting option makes player worse off always. (Negative Penguin effect). If not, player become better off from using waiting option. (Positive Penguin effect) Second, if the penalty is greater than the reward, the waiting option makes player better off always. (Positive Penguin

effect) Third, suppose that the reward is greater than penalty and the information cost is relatively low. In this case, if the difference in the reward and penalty is not so high or if the information cost is relatively high however the difference in the reward and penalty is high, the optimality of using the waiting option depends on the probability of imitation and deviation without signal. Each player can be better off from using the waiting option if $Pr(Imitation) < Pr(Deviation)$. (Positive Penguin effect) Also she is worse off if $Pr(Imitation) > Pr(Deviation)$. (Negative Penguin effect) If the information cost is relatively low when the difference in the reward and penalty is high, using the waiting option makes player better off always. (Positive Penguin effect)

The last of our paper is organized as follows. Section 2 discusses the case that there is no information cost for getting signal. The simultaneous and exogenously ordered sequential cases are discussed and each player's best strategies are explained. In section 3, we assume that there is an information cost for observing signal. Under this, we discuss each player's best strategy when order of announcement is given exogenously. In section 4, the welfare of using the waiting option in endogenous ordering is discussed. Finally, in section 5, the applications of our model and some results of the experiment that support our model are introduced.

2 Basic Framework

In this section, we describe a basic framework of information structure and Bayesian game that we will use in our paper. Let $N = \{1, 2\}$ be a set of players, that is to say, there are two players, i and j in this game. They have to announce the forecasting about the state of next period. We assume that there are two true binary states in this game, $\Omega = \{H, L\}$.⁶ Those states are mutually exclusive in the sense that if one state happens, the other state can't happen. The prior probability of each state is given by $Pr(S = H) = Pr(S = L) = \frac{1}{2} = \rho$. Now each player can observe the signal that partially reveals information about true state. I assume that these signals are independently distributed given the state and each signal is binary, $\theta = h$ or $\theta = l$. So the set of signal can be expressed by $\Theta = \{\theta = h, \theta = l\}$ and there is a mapping $\tau : \Omega \rightarrow \Theta$. Each given signal partially reveals information in the sense that $Pr(\theta_i = h | S = H) = p$, $Pr(\theta_i = l | S = L) = p$, $Pr(\theta_i = h | S = L) = 1 - p$ and $Pr(\theta_i = l | S = H) = 1 - p$. Here, $p \in (\frac{1}{2}, 1)$. This means that each player is certain about precision for each signal that she observed, but not perfectly certain. Also this $Pr(\theta_i | S)$ can be interpreted as player's belief in the information quality. We assume the θ is a private signal, so each player doesn't know whether

⁶The binary discrete signal space that we assumed in this paper is from the strong belief that people categorize the announced information. Here, categorization means that however people observed announced information based on the continuous signal space by both players, people will set the categorization in interpreting announced information such that H or L . Especially, this can be understood as a reasonable assumption because we mainly focus on the strategic behavior of players who announced information. For example, suppose that player i announced information $a_i = 0.8$ based on the continuous signal space $\theta \in [-1, 1]$. Now when player j wants to deviate from that announcement regardless of her observed signal, $a_j = 0.7$ or 0.6 will not be understood by people as alternative new information. So if player j wants to be interpreted as the player who announced meaningful alternative information, that should be very extreme such as $a_j \in [-1, 0]$. From this reasoning, we can regard our given discrete signal space as one that catches above intuition and also makes our analysis simple. So we use discrete signal space instead of continuous signal space.

the other player observed signal $\theta = h$ or $\theta = l$, but the precision of each player's signal is a common knowledge, so each player believes that if the other player observes signal θ , the other player gives same weight to that signal as she does. But the belief in the information quality is the common knowledge in the sense that player knows that the other player has the same belief in the information quality with her. Each player has a set of payoff-relevant announcements and announcement set is denoted by $A_n(n = i, j)$. The utility function of each player is given as a risk neutral utility function, $U(\pi) = \pi$. So the preference ordering \succ_n^* of each player is defined by $a^* \succ_n^* b^*$ if and only if $U_n(a^*) > U_n(b^*)$ for $n = i, j$. So our Bayesian game is characterized by

$$\langle N, \Omega, (A_n), (\Theta_n), (\tau_n), (\rho_n), (\succ_n^*) \rangle \text{ for } n = i, j.$$

Because each player is identical except for the signal that she gets randomly, our equilibrium is symmetric one.

Now let's define the payoff of two players in this announcement game. The LHS matrix is for the case that $S = H$ and the RHS one is for the case that $S = L$.

$S = H$	$a_i = h$	$a_i = l$
$a_j = h$	1, 1	$\gamma, -\phi$
$a_j = l$	$-\phi, \gamma$	-1, -1

$S = L$	$a_i = h$	$a_i = l$
$a_j = h$	-1, -1	$-\phi, \gamma$
$a_j = l$	$\gamma, -\phi$	1, 1

Here, we assume that $\gamma > 1$ and $\phi > 1$. The payoffs of each player depend on her announcement and also the other player's announcement. The above payoffs can be explained as follows. If both players coincide in their announcement and the announcement correctly revealed the true information about oncoming period, the payoffs of both players are 1. However the announcements of both agencies coincide, both players' payoff is -1 if they announced false information. If the announcement of both players are different, the player who announced a correct information gets reward $\gamma > 1$ and the other player who made a mistake gets penalty $-\phi < -1$. Here, the payoffs when both players coincided in their announcement are normalized by 1 and -1.⁷

Finally, let's define the concepts that we will use in this paper.⁸ Here, $n = \{i, j\}$ and $-n = j$ for i and $-n = i$ for j .

Definition 1: Truthful announcement: We say that player makes a **truthful announcement** if $a_n = \theta_n$.

⁷Suppose that original given payoffs are $-a, -b, b, c$ ($a > 0, b > 0, c > 0$). If we divide the given payoffs by b , then the payoffs are $-\frac{a}{b}, -1, 1, \frac{c}{b}$. So we can write $-\frac{a}{b} = -\phi, -1, 1, \frac{c}{b} = \gamma$ ($\phi > 0, \gamma > 0$). Also we can set the different value for the case that both revealed false information together or revealed true information like $-a, -b, d, c$. But this assumption that $|\pi_i(S \neq a_i = a_j)| = |\pi_i(S = a_i = a_j)|$ makes our analysis easy and helps to get intuitive result in a simple way.

⁸For the terminologies, we use a term "imitation" instead of "herding" and use a term "deviation" instead of "anti-herding". Usually, the models that treated strategic behavior assumed two players and two period game as assumed in our model. But usually, herding and anti-herding are the definitions used for infinite players or finitely many players in long run periods. So using the term "herding" and "anti-herding" in models that treat two players game in two periods is not suitable. So we use the term "imitation" instead of "herding" to define the situation that player selects same decision with other players however the observed signals are different. Also we use the term "deviation" instead of "anti-herding" to define the situation that player selects different decision from other players however the observed signals are same.

Definition 2: Strategic Imitation: We say that player **imitates the other player strategically** if $a_n = a_{-n}$, $\theta_n \neq a_{-n}$.

Definition 3: Strategic Deviation: We say that player **deviates from the other player strategically** if $a_n \neq a_{-n}$, $\theta_n = a_{-n}$.

Definition 4: Strategic Imitation without signal: We say that player **imitates the other player without signal** if $a_n = a_{-n}$ without θ_n .

Definition 5. Strategic Deviation without signal: We say that player **deviates from the other player without signal** if $a_n \neq a_{-n}$ without θ_n .

In this paper, we want to tell by the concept of strategic behavior with signal and the one without signal. Finally, we assume that we exclude the possibility of announcement by guessing by each player without information.⁹

Assumption

There is no strategy of guessing in each player's strategy set.

3 When there is no cost in observing signal.

In section 2, we assume that there is no cost for observing signal. So players who participate in this game observe their signal always. In case of simultaneous announcement case, the only source of information that each player can use is her observed signal. But in case of sequential announcement, the subsequent player has more information because she can also observe the other player's announcement together with her own signal. But the player who announces first has exactly same information with the simultaneous announcement case. So there is no change in the best strategy of player who moves first whether announcement is made simultaneously or sequentially.

3.1 The optimal strategy under simultaneous announcement

Now suppose that each player has to make an announcement simultaneously. Now we can described our game as follows. First, the player i, j observes the payoff conditional on the true state. Also each player observes her signal and behaves as an expected utility maximizer. Each one has two strategies and decides whether she will announces truthfully or distort the given signal after considering the expected utility. The forecasting announcement about the true state is made simultaneously. The game procedure can be summarized as follows.

T1) Player $i(j)$ observes the payoffs and her signal.

T2) Player $i(j)$ decides whether she will announce truthfully or distort the observed signal.

T3) a_i and a_j are announced.

⁹The subsequent player's decision based on the other player's announcement is not the one by guessing. Also from this assumption, the player who announces first should observe her signal always.

T4) State S is realized and each player gets payoff depending on her announcement, the other player's announcement and the true state.

We can model this simultaneous announcement problem as the Bayesian game with incomplete information. Let's define player i 's posterior belief Π about the state of the world given her own signal θ_i as follows.

$$\Pi_i \equiv \Pr(S, \theta_j | \theta_i)$$

In this situation, each player has to consider the true state and the other player's signal under the observed her signal because she can't observe the other player's signal. So the only source of information updating about true state is her observed signal. So her posterior belief is described as $\Pi_i \equiv \Pr(S, \theta_j | \theta_i)$. Also Player i 's strategy can be defined as

$$\sigma_i; \{\theta_i \longrightarrow a_i \text{ given } \theta_i \text{ and } \Pi_i \equiv \Pr(S, \theta_j | \theta_i)\}$$

In our setting, Bayesian Nash equilibrium consists of $\sigma = \{\sigma_n; n = i, j\}$ and a beliefs $\mu = \{\mu_n : n = i, j\}$ such that μ is consistent with μ and σ in terms of Bayesian updating. Now let's characterize the equilibrium of simultaneous announcement case. From following proposition, we find that there exists unique Bayesian Nash equilibrium if the announcement is made simultaneously. Each player's best strategy is to report her observed signal truthfully without distortion always.

Proposition 1

Suppose that player i and j announce simultaneously. Then there exists unique Bayesian Nash Equilibrium such that each player announces her signal truthfully without distortion always.

The proof of this proposition can be described simply as follows. First, let's think about the belief updating about the true state of player j . Now player j observes her own signal θ_j . Then player j updates her belief as follows by the Bayesian updating rule.

$$\Pr(S, \theta_i | \theta_j) = \frac{\Pr(S, \theta_i, \theta_j)}{\Pr(\theta_j)} = \frac{\Pr(S, \theta_i, \theta_j) \Pr(S)}{\Pr(S) \Pr(\theta_j)} = \frac{\Pr(\theta_i, \theta_j | S) \Pr(S)}{\Pr(\theta_j)}$$

So

$$\Pr(S, \theta_i | \theta_j) = \frac{\Pr(\theta_i, \theta_j | S) \Pr(S)}{\sum_{S, \theta_i} (\Pr(\theta_i, \theta_j | S) \Pr(S))} \quad (S = H, L \text{ and } \theta_i = h_i, l_i)$$

Then player j 's expected payoff from choosing action a_i when $\theta_j = h_j$ is as follows.

$$E(\pi_j(S, a_i, a_i | \theta_j = h_j)) = \sum_{S, \theta_i} \left[\frac{\Pr(\theta_i, \theta_j | S) \Pr(S)}{\sum_{S, \theta_i} (\Pr(\theta_i, \theta_j | S) \Pr(S))} \pi_j(S, a_i, a_j) \right]$$

Now suppose that player j observed that $\theta_j = h$. Then player can select the strategy to report truthfully or to deviate strategically from her observed signal. The payoffs depending on her strategy can be described as follows. Here, LHS table is for the strategy to report truthfully and RHS is for the

strategy to deviate from her observed signal.

π_j	1	γ	-1	$-\phi$	π_j	$-\phi$	-1	γ	1
S	H	H	L	L	S	H	H	L	L
$Pr(\cdot a_j)$	p^2	$p(1-p)$	$(1-p)^2$	$p(1-p)$	$Pr(\cdot a_j)$	p^2	$p(1-p)$	$(1-p)^2$	$p(1-p)$
a_i	h	l	h	l	a_i	h	l	h	l
a_j	h	h	h	h	a_j	l	l	l	l

From above, we can check that

$$E(\pi_j(S, a_i, a_j = h_j | \theta_j = h)) = p^2(\phi - \gamma) + p(\gamma - \phi + 2) - 1$$

$$E(\pi_j(S, a_i, a_j = h_j | \theta_j = h)) = p^2(\gamma - \phi) - 2p\gamma + \gamma$$

So $E^T \pi_j(\cdot) - E^D(\cdot) = (2p - 1)\{\gamma(1 - p) + p\phi + 1\} > 0$. So player's optimal strategy is to report her signal truthfully. Also same reasoning is possible when $\theta_j = l$. So we can say that the strategy to announce signal truthfully dominates the strategy to deviate from the observed signal always if the announcement is made simultaneously.

We can define the case of simultaneous announcement game as the benchmark. First, the sufficient information provision is possible by both players. Also there is no distortion in the information provided by both players. We will use this result of benchmark case again in the section 4 for the analysis of the optimality of waiting option in the endogenous ordering compared to the simultaneous announcement case.

Corollary 1

If the announcement is made simultaneously by both players, the changes in value of γ and ϕ have no effect on the best strategy of each player.

3.2 The optimal strategy under sequential announcement

Now let's discuss about sequential announcement case. Suppose that the order of announcement is given exogenously and there is a time lag in the announcement of both players. Let's assume that player i announces first and player j announces later. We also assume that player who moves later can observe the announcement of the other player who announced first. Then the procedure of this sequential announcement game can be summarized as follows.

T1) Player i enters to this game and observes payoff.

T2) Player i decides whether she will announces truthfully or distort the observed signal and a_i is announced.

T3) Player j enters to this game and observes the payoff and a_i .

T4) Player j decides whether she will announce truthfully or distort her observed signal and a_j is announced.

T5) State S is realized and each player gets payoff depending on both players' announcements and the realized true state.

We can describe this sequential announcement problem as a Bayesian game with incomplete information. First, let's define follower's posterior belief Π_j (player j 's posterior belief Π) about the true state given signal θ_j as follows.

$$\Pi_j \equiv \Pr(S \mid a_i, \theta_j)$$

We assume that there is one to one mapping from the action space to signal space. So player j can infer the signal of player i after observing player i 's announcement. We denote this mapping of player j as χ such that

$$\chi_j : a_i \longrightarrow \theta_i$$

Now the strategy of player j who is a follower can be defined as

$$\sigma_j; \{\theta_j \longrightarrow a_j \text{ given } \theta_j \text{ and } \Pi_j \equiv \Pr(S \mid a_i, \theta_j) \equiv \Pr(S \mid \chi(a_i), \theta_j)\}$$

Also the strategy of player i who moves first can be defined as follows.

$$\sigma_i; \{\theta_i \longrightarrow a_i \text{ given } \theta_i \text{ and } \Pi_i \equiv \Pr(S, a_i \mid \theta_j)\}$$

Then Bayesian Nash equilibrium consists of $\sigma = \{\sigma_n; n = i, j\}$ and a beliefs $\mu = \{\mu_n : n = i, j\}$ such that μ is consistent with μ and σ in terms of Bayesian updating. Here, we can check that, in case of player i , there is no change in her decision process compared to the simultaneous announcement case. So we can easily conjecture that there will be no change in her best strategy and this can be shown with simple procedure. Now let's characterize the equilibrium of this sequential announcement case. From the analysis, we can get following result.

Proposition 2

Suppose that player i announces first and player j announces later when there is no information cost. Also the signal observed by player j coincides with the announcement by player i , $\theta_j = a_i$. Here, $p^* = \frac{1}{\gamma - \phi} ((\gamma + 1) - \sqrt{\gamma + \phi + \gamma\phi + 1})$

1) If $\gamma > \phi$, there exists p^* such that player j deviates strategically if $p \in (\frac{1}{2}, p^*)$ and announces truthfully for $p \in (p^*, 1)$.

2) If $\gamma < \phi$, player always announces truthfully for $\forall p \in (\frac{1}{2}, 1)$.

Proof

Suppose player i announced $a_1 = h$ and the given signal of player j is $\theta_2 = h$. ($\theta_i = a_i = \theta_j$). In this case, player j has two strategies to take. The first is to report truthfully and the other is to deviate. Following are the two tables of each strategy. The left one is for the truthful announcement and the right one is for the deviation.

$\pi_j(S, a_i, a_j)$	1	-1
$Pr(S \mid a_i, \theta_j)$	$\frac{p^2}{2p^2 - 2p + 1}$	$\frac{(1-p)^2}{2p^2 - 2p + 1}$
S	H	L
a_i	h	h
a_j	h	h

$\pi_j(S, a_i, a_j)$	$-\phi$	γ
$Pr(S \mid a_i, \theta_j)$	$\frac{p^2}{2p^2 - 2p + 1}$	$\frac{(1-p)^2}{2p^2 - 2p + 1}$
S	H	L
a_i	h	h
a_j	l	l

After entering this game, player j observes her own signal and player i 's announcement. Now let's see the expected payoffs of each strategy. So player j 's posterior belief can be defined as follows $\Pr(S | a_i, \theta_j)$. First, let's think about the truthful announcement. Here, $\Pr(H | h_i, h_j)$ and $\Pr(L | h_i, h_j)$ can be attained by following Bayesian updating respectively.

$$\Pr(H | h_i, h_j) = \frac{\Pr(H, h, h)}{\Pr(h, h)} = \frac{\Pr(h, h | H) \Pr(H)}{\Pr(h, h | H) \Pr(H) + \Pr(h, h | L) \Pr(L)} = \frac{p^2}{2p^2 - 2p + 1}$$

$$\Pr(L | h_i, h_j) = \frac{\Pr(h, h | L) \Pr(L)}{\Pr(h, h | L) \Pr(L) + \Pr(h, h | H) \Pr(H)} = \frac{(1-p)^2}{2p^2 - 2p + 1}$$

Now let's denote that $E^T \pi_j(\cdot) = E^T \pi_j(S, a_i, a_j = h | a_i, \theta_j = h)$ and $E^D \pi_j(\cdot) = E^D \pi_j(S, a_i, a_j = l | a_i, \theta_j = h)$. Then the expected payoffs depending on strategies are as follows.

$$E^T \pi_j(\cdot) = \frac{2p-1}{2p^2-2p+1}, \quad E^D \pi_j(\cdot) = -\frac{p^2}{2p^2-2p+1} \phi + \frac{(1-p)^2}{2p^2-2p+1} \gamma$$

If we compare $E^T \pi_j(S | a_i, a_j)$ and $E^D \pi_j(S | a_i, a_j)$,

$$E^T \pi_j(\cdot) - E^D \pi_j(\cdot) = \frac{p^2}{2p^2-2p+1} (1+\phi) - \frac{(1-p)^2}{2p^2-2p+1} (1+\gamma)$$

Then whether $E^T \pi_j(S | a_i, a_j) \geq E^D \pi_j(S | a_i, a_j)$ depends on

$$(\phi - \gamma)p^2 + 2(1 + \gamma)p - (1 + \gamma) \geq 0$$

Now let's denote $f(p)$ as $(\phi - \gamma)p^2 + 2(1 + \gamma)p - (1 + \gamma)$. Then there are two cases depending on γ and ϕ .

Case 1) When $\phi > \gamma$

Let $f(p) = (\phi - \gamma)p^2 + 2(1 + \gamma)p - (1 + \gamma)$. Then we know the shape of function because $\phi > \gamma$. Also $f(\frac{1}{2}) = \frac{\phi - \gamma}{4} > 0$, $f(1) = 1 + \phi > 0$. So we can check that $f(p) > 0$ for $\forall p \in (\frac{1}{2}, 1)$. That is to say, if $\phi > \gamma$, $E^T \pi_j(S, a_i, a_j) > E^D \pi_j(S, a_i, a_j)$. So player always reports the observed signal truthfully.

Case 2)

Now let's suppose that $\gamma > \phi$. Then we can find that there exists p^* such that $f(p) < 0$ if $p \in (\frac{1}{2}, p^*)$ and $f(p) > 0$ if $p \in (p^*, 1)$ where $p^* = \frac{1}{\gamma - \phi} ((\gamma + 1) - \sqrt{\gamma + \phi + \gamma\phi + 1})$. That is to say,

$$\begin{cases} E^T \pi_j(\cdot) < E^D \pi_j(\cdot) & \text{if } p \in (\frac{1}{2}, p^*) \\ E^T \pi_j(\cdot) > E^D \pi_j(\cdot) & \text{if } p \in (p^*, 1) \end{cases}$$

Finally, we can say as follows. Suppose that $\theta_j = a_i$. Then if $\phi > \gamma$, player who moves later always reports her own signal truthfully. If $\gamma > \phi$, there exists p^* such that player deviates strategically from her signal if $p \in (\frac{1}{2}, p^*)$ and reports signal truthfully if $p \in (p^*, 1)$. So proved. **Q.E.D.**

Here, it is an interesting result that the subsequent player deviates from her observed signal however it coincides with the other player's announcement if the belief in the information quality for her and the other player is extremely low, $p \in (\frac{1}{2}, p^*)$.

Corollary 2

Suppose that player i announces first and player j announces later when there is no information cost. Also $\theta_j = a_i$, $\gamma > \phi$ and $p \in (\frac{1}{2}, p^*)$. $p^* = \frac{1}{\gamma - \phi} ((\gamma + 1) - \sqrt{\gamma + \phi + \gamma\phi + 1})$

- 1) As γ increases, the subsequent player has a more incentive to deviate strategically.
- 2) As ϕ increases, the subsequent player has a more incentive to announce truthfully.

Proof

From above, $Pr(\text{Strategic deviation}) = (\phi - \gamma)^{-1} (2\sqrt{\gamma + \phi + \gamma\phi + 1} - \phi - \gamma - 2)$.

So from this,

$$\frac{\partial Pr(sd)}{\partial \gamma} = \frac{(\gamma + \phi - 2\sqrt{\gamma + \phi + \gamma\phi + 1} + 2)(\phi + 1)}{(\phi - \gamma)^2 (\sqrt{\gamma + \phi + \gamma\phi + 1})} > 0$$

because $\gamma + \phi - 2\sqrt{\gamma + \phi + \gamma\phi + 1} + 2 > 0$ always. Also

$$\frac{\partial Pr(sd)}{\partial \phi} = -\frac{(\gamma + \phi - 2\sqrt{\gamma + \phi + \gamma\phi + 1} + 2)(\gamma + 1)}{(\phi - \gamma)^2 (\sqrt{\gamma + \phi + \gamma\phi + 1})} < 0$$

So proved.

Now our second case is when $\theta_2 \neq a_1$. In this case, we get following result.

Proposition 3

Suppose that player i announces first and player j announces later when there is no information cost. Also the signal observed by player j coincides with the announcement by player i , $\theta_j \neq a_i$.

- 1) If $\gamma > \phi$, player j announces her signal truthfully.
- 2) If $\gamma < \phi$, player j imitates player i 's announcement after deviating from her observed signal.

Proof

We assume that $a_i = h$ and $\theta_j = l$. Following are the payoff tables according to player j 's strategy. The left one is for the strategy to report truthfully and the right one is for the strategy to imitate the other player.

$\pi_j(S, a_i, a_j)$	$-\phi$	γ
$Pr(S a_i, a_j)$	1/2	1/2
S	H	L
a_i	h	h
a_j	l	l

$\pi_j(S, a_i, a_j)$	1	-1
$Pr(S a_i, a_j)$	1/2	1/2
S	H	L
a_i	h	h
a_j	h	h

In this case, the posterior belief player j about true state is updated as follows.

$$Pr(H | h^1, l^2) = \frac{Pr(H, h^1, l^2)}{Pr(h^1, l^2)} = \frac{Pr(h^1, l^2 | H) Pr(H)}{Pr(h^1, l^2 | H) Pr(H) + Pr(h^1, l^2 | L) Pr(L)} = \frac{1}{2}$$

also

$$Pr(L | h^1, l^2) = \frac{Pr(L, h^1, l^2)}{Pr(h^1, l^2)} = \frac{Pr(h^1, l^2 | L) Pr(L)}{Pr(h^1, l^2 | L) Pr(L) + Pr(h^1, l^2 | H) Pr(H)} = \frac{1}{2}$$

Now let's denote that $E^T \pi_j(\cdot) = E^T \pi_j(S, a_i, a_j = h | a_i, \theta_j = h)$ and $E^D \pi_j(\cdot) = E^D \pi_j(S, a_i, a_j = l | a_i, \theta_j = h)$. Then we can find that

$$E^T \pi_j(\cdot) = \frac{1}{2}[-\phi + \gamma] \text{ and } E^D \pi_j(\cdot) = 0$$

So

$$E^T \pi_j(\cdot) - E^D \pi(\cdot) = \frac{1}{2}[\gamma - \phi]$$

So we find following relation.

$$\begin{cases} E^T \pi_j(\cdot) > E^D \pi(\cdot) & \text{if } \gamma > \phi \\ E^T \pi_j(\cdot) < E^D \pi(\cdot) & \text{if } \gamma < \phi \end{cases}$$

So we can say that the strategy to announce truthfully dominates the strategy to deviate from observed signal and imitate if $\gamma > \phi$. But the strategy to imitate dominates the strategy to announce truthful information if $\phi > \gamma$. **Q.E.D.**

Now using above propositions, we shows that the best strategy of player who moves first is to report truthfully always.

Proposition 4

The player who has to announce first makes the truthful announcement always.

Proof

We use the backward induction for analyzing the strategy of player who moves first.

First, let's think about the case that $\theta_j = a_i$. In this case, player i knows that player j will announces her own observed signal truthfully if $\phi > \gamma$ or if $\phi < \gamma$ and $p \in (p^*, 1)$. Then this is the same case with the simultaneous announcement for player i . So player i will announce truthfully. Now let's think about the case of $\phi < \gamma$ and $p \in [\frac{1}{2}, p^*]$. Then player i knows that player j deviates strategically, so always there is a difference between the announcements of two players. Suppose that $\theta_i = h$. Then following are the payoff matrix of $a_j = h$ and $a_j = l$.

$\pi_i(S, a_i, a_j)$	γ	$-\phi$
$Pr(S \theta_i)$	p	$1 - p$
S	H	L
a_i	h	h
a_j	l	l

$\pi_i(S, a_i, a_j)$	$-\phi$	γ
$Pr(S \theta_i)$	p	$1 - p$
S	H	L
a_i	l	l
a_j	h	h

Then $E^T(\pi) = p\gamma - (1-p)\phi$ and $E^D(\pi) = -p\phi + \gamma(1-p)$. So always $E^T(\pi) - E^D(\pi) = (\gamma + \phi)(2p - 1) > 0$. So the player who announces first reports truthfully if $\theta_j = a_i$.

Second, suppose that $\theta_j \neq \theta_i$ and $\gamma > \phi$, then player i knows that $a_j = \theta_j \neq a_i = \theta_i$, that is to say, player i knows that player j will make the truthful announcement. Then from the same reasoning with former case, the best strategy for player i is to make the truthful announcement. Now suppose that $\theta_j \neq \theta_i$ and $\gamma < \phi$. Then player i knows that $a_j = a_i \neq \theta_j$, that is to say, player i knows that player j imitates the announcement of player i . Then there are two strategies that player i can take, truthful announcement or the deviation from her observed information. Now suppose that player i observed

$\theta_i = h$. Then the payoffs of each strategy are as follows,

$\pi_i(S, a_i, a_j)$	1	-1
$Pr(S \theta_i)$	p	$1 - p$
S	H	L
a_i	h	h
a_j	h	h

$\pi_i(S, a_i, a_j)$	-1	1
$Pr(S \theta_i)$	p	$1 - p$
S	H	L
a_i	l	l
a_j	l	l

Here, the LHS matrix is for the case that player i makes the truthful announcement and RHS one is for the case that player i deviates after observing signal $\theta_i = h$. In this case, Player i knows that player j will imitate player i 's announcement. Then the expected payoff for the truthful announcement is $2p - 1$ and that for the deviation is $1 - 2p$. Now let's denote that $E^T(\cdot)$ is the expected payoff of the strategy to report truthfully and $E^D(\pi)$ is the expected payoff of the strategy to deviate from observed signal. Then it is obvious that player i will make the truthful announcement because $E^T(\pi) - E^D(\pi) = 4p - 2 > 0$ from the assumption $p > 1/2$. Finally, we can say that player i 's best strategy is to make the truthful announcement for all cases. **Q.E.D.**

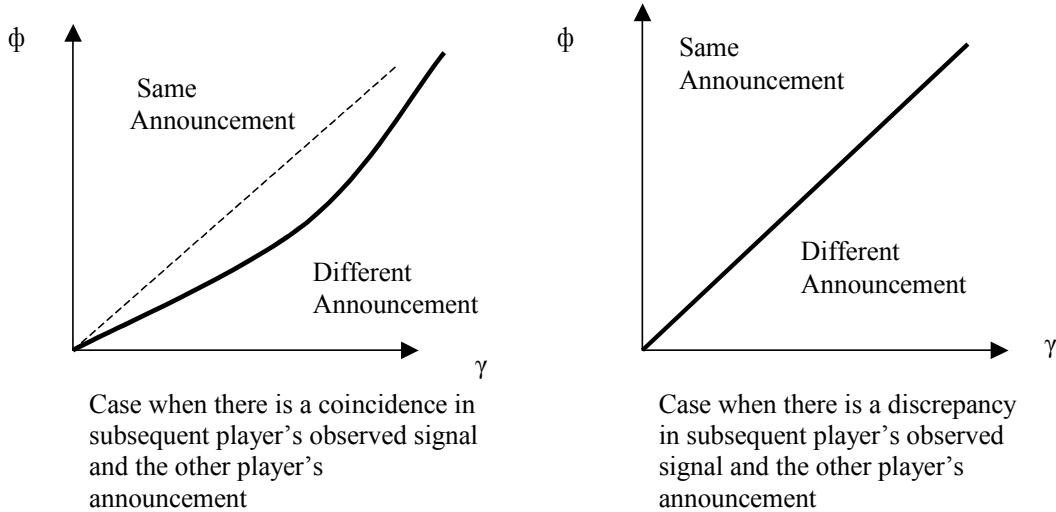
Above proposition explains the condition of the optimal strategy of player who moves later. Especially, we can find interesting result when there is a coincidence in her observed signal and the other player's announcement. Surprisingly, there is a positive possibility of strategic deviation however the observed signal is same with the other player's announcement. It says that the condition for this strategic deviation is $\gamma > \phi$ and $p \in (\frac{1}{2}, p^*)$. That is to say, if the reward when only one player was right is greater than the penalty when only one player was wrong and the belief in the information's quality is very low, player who moves later deviates strategically. This possibility comes from two effects. First, if player believes that her observed information has very low level of quality, this player doesn't give much weight for this information. Also she will give small weight for the other player's information quality. Also, if $\gamma > \phi$, she has a strong incentive to deviate strategically because she can get greater payoff with the strategy of deviation for being differentiated. If $p \in (p^*, 1)$, this may not happen however $\gamma > \phi$. In case of $\gamma < \phi$, the fear of being penalized with a big penalty when she is the only one who announced wrong information dominates the incentive to be differentiated with different information.

Now let's just discuss the case when there is a difference between the observed signal and the other player's announced information. Through the Bayesian updating, player j knows that the probabilities of good state and bad state are equal. Then there is no incentive for player j to ignore her own given signal because she gives equal weight for her information and the other's one. So information quality doesn't matter in her decision in this case. Then her next criteria for selecting optimal strategy depends on the relation of payoffs, γ and ϕ . Then if $\gamma > \phi$, the incentive to be differentiated makes her report her signal truthfully. But if $\gamma < \phi$, however she gives same weight to the signals of her and the other player, the fear of being penalized makes her ignore her signal and imitate player i 's signal. That is to say, the risk that she can be the only one player who announced wrong information makes her hesitate to announce the different information truthfully and this induces the imitation.

Finally, the information gain that player j got from sequential announcement is the perfect inference of player i 's signal and the equal probability of two different states induces her to select her strategy

just depending on the given relation of reward and payoff. Each player's belief in the precision of the other player's information doesn't matter in this case.

From above, the result of both players' announcement can be described with a diagram as follows. From this, we can check that there is a bias to the same announcements results and this can be interpreted as the bias to the imitation.



The result of the announcement by both players under the simultaneous announcement case.

4 When there is a positive cost in observing signal

In above section, we assumed that there is no cost in observing given signal. But this assumption can be regarded as a too strong one when we think about reality. We can find many examples that do not compatible with this no information cost assumption. So in this section, we assume that players should pay cost for observing their signal. This information cost can be understood as the time and money the players should spend for getting information. If player has to pay positive information fee, we can easily conjecture that player who moves later has new additional strategy, the announcement without signal. This strategy should be regarded as a meaningful one under the existence of positive information cost. If player doesn't observe her own signal, there is a negative effect in the sense that she misses the opportunity to use meaningful information, her own signal, with which she infer the true state. Then the only source of belief updating about true state is the announcement made by the other player who moved first. But there is also positive effect because she can save the information cost and this positive effect may dominates the negative effect more as the probability of correct announcement by the other player increases. So in this case, the best strategy of player who moves later is strongly related with the belief in the precision of the other player's information. Here, the higher belief in the other player's information quality means the greater probability that the other player has a correct information.

Now let's analyze the optimal strategy of the player who moves later. We already have mentioned there is a possibility that player may have more incentive to behave without signal as the probability of

correct announcement by the other player increases. We show that this conjecture is correct and asserts that the value of the information cost also plays an important role in determining the optimal strategy of the subsequent player.

In this section, we assume again that player i moves first and player j moves later.

4.1 When the subsequent player observes her signal with paying information cost

This case can be easily analyzed from the result of section 2. In section 2, we already found the optimal strategy of player who moves later. The only changed assumption is that player should pay cost for getting information. But there is no change in her decision rule however there is a information cost because the information fee can be considered as a sunk cost. The only thing that is changed is the expected payoff because c should be taken off. So the following decision rule of the subsequent player is again the repetition of the former case when there is no information cost.

Decision rule of player j

1) Suppose that the signal observed by player j coincides with the announcement of player i . ($\theta_j = a_i$) Then the subsequent player always announces truthfully for $\forall p \in (\frac{1}{2}, 1)$.

2) Suppose that the signal observed by player j doesn't coincide with that of player i . ($\theta_j \neq a_i$). Then
1) If $\gamma > \phi$, player j announces her signal truthfully.

2) If $\gamma < \phi$, player j imitates player i 's announcement after deviating strategically from her signal. If the reward when only one

Then player's expected payoffs when the subsequent player follows above decision rules are as follows.

Case 1) When $\theta_j = a_i$

If $\gamma > \phi$, $p \in [p^*, 1]$ or $\gamma < \phi$, player reports signal truthfully. So $E^T \pi_j(S, a_i \neq a_j \mid \theta_j = a_i) = \frac{2p-1}{2p^2-2p+1} - c$.

Case 2) When $\theta_j \neq a_i$.

If $\gamma > \phi$, player reports the observed signal truthfully. So $E^T \pi_j(S, a_i \neq a_j \mid \theta_j \neq a_i) = \frac{1}{2}(\gamma - \phi) - c$.

If $\gamma < \phi$, player imitates the other player strategically. So $E^T \pi_j(S, a_i \neq a_j \mid \theta_j \neq a_i) = -c$.

4.2 Optimal strategy of the subsequent player when she makes a decision without signal

Now suppose that player j decided not to observe a signal. Then the only source of information that she can use is the announcement by player i . Now suppose that player i announced $a_i = h$. Then player j has two strategies. The first is the imitation without signal and the second is the deviation without signal. Following are the matrix tables that show player j 's expected payoff depending on her strategy. LHS is for the strategy to imitate the other player's announcement and RHS is for the strategy

to deviate from the other player's one.

$\pi_j(S, a_i, a_j)$	1	-1
$Pr(S a_i)$	p	$1 - p$
S	H	L
a_i	h	h
a_j	h	h

$\pi_j(S, a_i, a_j)$	$-\phi$	γ
$Pr(S a_i)$	p	$1 - p$
S	H	L
a_i	h	h
a_j	l	l

In this case, the posterior belief of player j about true state is updated as follows.

$$Pr(H | a_i = h) = \frac{Pr(H, h)}{Pr(h)} = \frac{Pr(h | H) Pr(H)}{Pr(h | H) Pr(H) + Pr(h | L) Pr(L)} = p$$

also

$$Pr(L | a_i = h) = \frac{Pr(L, h)}{Pr(h)} = \frac{Pr(h | L) Pr(L)}{Pr(h | L) Pr(L) + Pr(h | H) Pr(H)} = 1 - p$$

Now suppose that player j imitates player i 's announcement without signal. Let's denote that $E^M(\cdot)$ is the expected payoff when she selects the strategy of imitation without signal. Then

$$E_j^M(\pi(S | a_i)) = 2p - 1.$$

Now suppose that player j deviates from player i 's announcement without signal. Then the expected payoff when player j selected the strategy of deviation from player i 's announcement is

$$E_j^D(\pi(S | a_i)) = -p\phi + (1 - p)\gamma$$

Here, there is no change in posterior belief, $Pr(S | a_i)$, whether player j imitates or deviates. Now if we compare the expected payoffs, we can get following proposition.

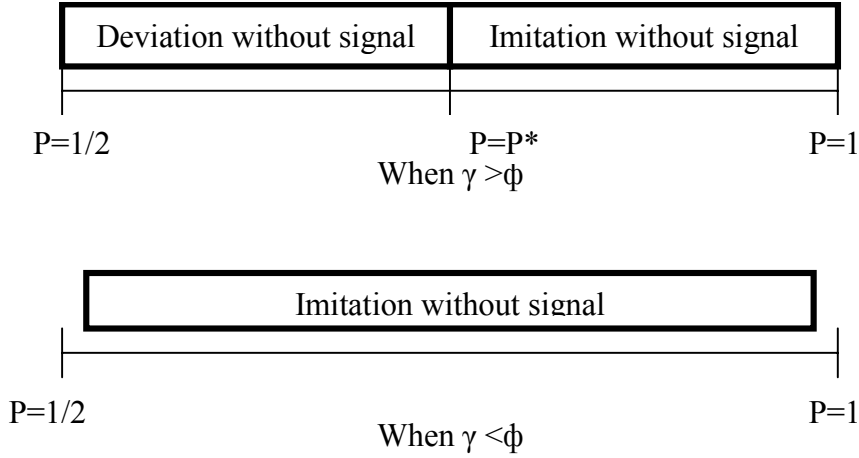
Lemma 1

Suppose that the subsequent player doesn't observe signal.

- 1) *If $\gamma > \phi$ and $p > \frac{1+\gamma}{2+\gamma+\phi} > \frac{1}{2}$, the subsequent player imitates the other player without signal.*
- 2) *If $\gamma > \phi$ and $\frac{1+\gamma}{2+\gamma+\phi} > p > \frac{1}{2}$, the subsequent player deviates from the other player without signal.*
- 3) *If $\gamma < \phi$, player who announces later imitates the other player without signal for $\forall p$ such that $p \in [1/2, 1]$.*

Above proposition can be proved in a easy way. First, $E^M(\pi(\cdot)) - E^V(\pi(\cdot)) = p(2 + \gamma + \phi) - (1 + \gamma)$. So if $p > \frac{1+\gamma}{2+\gamma+\phi}$, $E^M(\pi(\cdot)) > E^V(\pi(\cdot))$. So player j imitate. Also if $\frac{1+\gamma}{2+\gamma+\phi} > p$, $E^M(\pi(\cdot)) < E^V(\pi(\cdot))$. So player j will deviate. But we have a constraint for p such that $p > \frac{1}{2}$. So if we compare $\frac{1+\gamma}{2+\gamma+\phi}$ and $\frac{1}{2}$, we can find that $\frac{1+\gamma}{2+\gamma+\phi} > \frac{1}{2}$ if $\gamma > \phi$ and $\frac{1+\gamma}{2+\gamma+\phi} < \frac{1}{2}$ if $\gamma < \phi$. Finally, we can say that if $\gamma > \phi$ and $\frac{1+\gamma}{2+\gamma+\phi} < p < 1$, $E^M(\pi(\cdot)) > E^V(\pi(\cdot))$. Also if $\gamma > \phi$ and $\frac{1}{2} < p < \frac{1+\gamma}{2+\gamma+\phi}$, $E^M(\pi(\cdot)) < E^V(\pi(\cdot))$. Finally, if $\gamma < \phi$ and $p > \frac{1}{2}$, $E^M(\pi(\cdot)) > E^V(\pi(\cdot))$ for all $p > \frac{1}{2}$, because if $\gamma < \phi$, $\frac{1+\gamma}{2+\gamma+\phi} > \frac{1}{2}$ is automatically satisfied. So we proved.

We can explain above result intuitively with following diagram. In this diagram, $p^* = \frac{1+\gamma}{2+\gamma+\phi}$.



The best strategy of the subsequent player when she decides not to observe her signal.

According to our proposition, if $\gamma > \phi$ and $\frac{1}{2} < p < \frac{1+\gamma}{2+\gamma+\phi}$, player j deviates from player i 's announcement. That is to say, $a_i \neq a_j$. This explains following situation. Suppose that the reward when only I was right is greater than the penalty when only I was wrong, $\gamma > \phi$. Now if the belief in the precision of other player's information is bounded above by $\frac{1+\gamma}{2+\gamma+\phi}$, player strategically deviates from the other player's announcement. Here, the fact that player j 's belief in the player i 's information quality is bounded above means that she doesn't have a strong belief in the information quality. Here, p is only the player i 's information quality because player j doesn't observe her signal. Then player j has much incentive to deviate from player i 's announcement from this weak belief in the other player's information. Especially, the incentive to be differentiated from the other player under $\gamma > \phi$ makes her select the strategy of deviation.

Now think about second case. It says that If $\gamma > \phi$ and $\frac{1+\gamma}{2+\gamma+\phi} < p < 1$, player j imitates player i 's announcement. Here, the condition $\frac{1+\gamma}{2+\gamma+\phi} < p < 1$ explains that player j 's belief in the player i 's information quality is bounded below by $\frac{1+\gamma}{2+\gamma+\phi}$ and this means that she has a strong belief in player i 's information. Then this will induce player j to imitate player i 's announcement. So we can say that, in this case, the strong belief in other player's information dominates the incentive to be differentiated however $\gamma > \phi$. Finally, if $\gamma < \phi$, player j imitates player i 's announcement always for any level of $p \in (\frac{1}{2}, 1)$. Here, we can check the risk averse attitude to the situation that she can be the only one player who made a mistake. That is to say, the blame sharing effect works for any level of belief in the information quality.

If we compare the behavior of player j when $\gamma > \phi$ and $\gamma < \phi$, we can find asymmetry in her behavior to the risky situation. However there is no change in her decision procedure whether $\gamma > \phi$ or $\gamma < \phi$, player j shows a strong risky averse attitude to the situation that she is the only one who made a false announcement and it is natural that this will be exaggerated when $\gamma < \phi$.

From above, we also can get the following comparative statics that explains the effect of change in the payoff on the best strategy of the subsequent player.

Corollary 3

Suppose $\gamma > \phi$ and the player who announces later decided not to observe her signal.

1) As γ increases, the possibility of imitation without signal decreases and the possibility of deviation without signal increases.

2) As ϕ increases, the possibility of imitation without signal increases and the possibility of deviation without signal decreases.

This can be proved easily. First, suppose $\gamma > \phi$. Then we know that player will imitate the other player when $\frac{1+\gamma}{2+\gamma+\phi} < p < 1$ and will deviate from the other player when $\frac{1}{2} < p < \frac{1+\gamma}{2+\gamma+\phi}$. So we can say that

$$\begin{aligned} \text{Prob}(\text{Imitation}) &= \frac{2(\phi+1)}{\gamma+\phi+2} \\ \text{Prob}(\text{Deviation}) &= \frac{\gamma-\phi}{\gamma+\phi+2} \end{aligned}$$

So from these, we can get that

$$\begin{aligned} \frac{\partial \text{Prob}(\text{Imitation})}{\partial \gamma} &= (-2)(\gamma+\phi+2)^{-2}(\phi+1) < 0 \\ \frac{\partial \text{Prob}(\text{Deviation})}{\partial \gamma} &= 2(\gamma+\phi+2)^{-2}(\gamma+1) > 0 \\ \frac{\partial \text{Prob}(\text{Imitation})}{\partial \phi} &= (-2)(\gamma+\phi+2)^{-2}(\gamma+1) < 0 \\ \frac{\partial \text{Prob}(\text{Deviation})}{\partial \phi} &= 2(\gamma+\phi+2)^{-2}(\phi+1) > 0 \end{aligned}$$

Intuitively, we can check again that γ works for the incentive to be differentiated and ϕ works for the blame sharing effect.

4.3 The optimal strategy of the subsequent player when there is a positive information cost

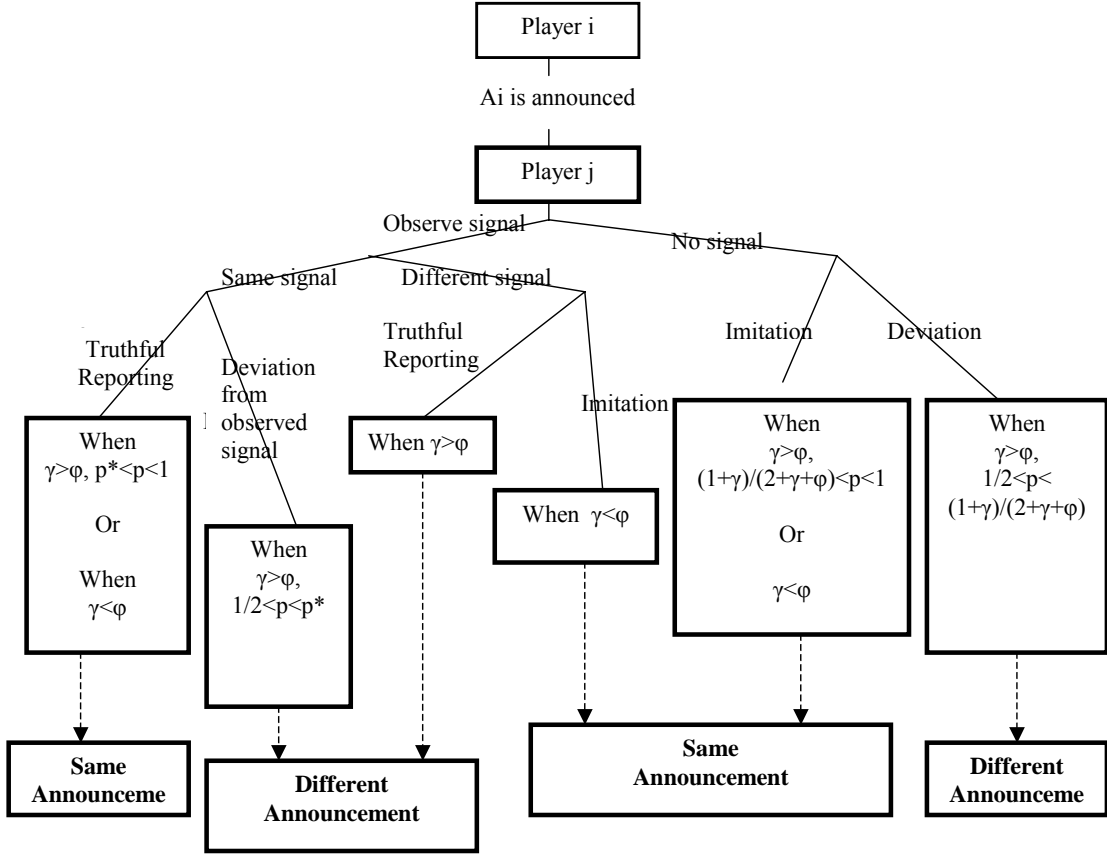
In this section, we discuss about the best strategy of player j who moves later when she has to pay information cost for observing signal. In above, we checked the best strategy of the subsequent player that depends on her decision whether she observe signal and or not. We already mentioned that the value of of information cost will play an important role in her decision. So first, we check the existence of critical value of information cost on which her decision depends whether she will observe her signal or not. Then we get player j 's best strategy that depends on payoffs and the belief in the information quality.

Now we consider following four cases. Case 1) $\gamma > \phi$ and $\frac{1+\gamma}{2+\gamma+\phi} < E(p) < 1$, Case 2) $\gamma > \phi$ and $p^* < E(p) < \frac{1+\gamma}{2+\gamma+\phi}$, Case 3) $\gamma > \phi$ and $\frac{1}{2} < E(p) < p^*$, Case 4) $\gamma < \phi$. After entering the game, the subsequent player observes the information cost, c and the payoffs, γ and ϕ . Also she has the expectation about the other player's information quality. If she decides to observe her signal, this p measures both her information quality and the other player's information quality. If not, p measures only this player's belief in the other player's information quality.

Also After entering game, she has to decide whether she will observe her signal or not. In case of the decision to observe signal, she has no knowledge whether she will observe same signal with a_i or not.

So the expected payoff of the decision to observe her signal is the results after considering both cases $a_i = \theta_j$ and $a_i \neq \theta_j$. But in case of the decision not to observe her signal, she knows whether she will select the strategy of imitation or deviation without signal because she already knows γ, ϕ, c and $E(p)$.

Following is the procedure of our game. Here, player i is the player who announces first and player j is the subsequent player in our announcement game. Here, $p^* = \frac{1}{\gamma - \phi} ((\gamma + 1) - \sqrt{\gamma + \phi + \gamma\phi + 1})$.



4.3.1 Case 1) When $\gamma > \phi$ and $\frac{1+\gamma}{2+\gamma+\phi} < E(p) < 1$

Suppose that $\gamma > \phi$ and $\frac{1+\gamma}{2+\gamma+\phi} < E(p) < 1$. Now the subsequent player can decide whether she will observe her signal or imitate. Here, the strategy of deviation is excluded because $\frac{1+\gamma}{2+\gamma+\phi} < p < 1$. We can conjecture that there may exist the critical level of cost such that if the given information cost is less than critical level, she will take into account the strategy to observe her signal. If not, she will give up the opportunity to observe her signal.

Suppose that $p = \frac{1+\gamma}{2+\gamma+\phi}$. Then this given p is this lowest level for given level $p \in \left(\frac{1+\gamma}{2+\gamma+\phi}, 1\right)$. So if there is no cost for observing signal, player will always select the strategy of observing signal. But because of the existence of positive cost for signal, $E^S \pi(p = \frac{1+\gamma}{2+\gamma+\phi}) < E^I \pi(p = \frac{1+\gamma}{2+\gamma+\phi})$ is possible. From now, we denote $E^S \pi(p = \frac{1+\gamma}{2+\gamma+\phi})$ as $E^S \pi(\cdot)$ and $E^I \pi(p = \frac{1+\gamma}{2+\gamma+\phi})$ as $E^I \pi(\cdot)$. Then $E^S \pi(\cdot) - E^I \pi(\cdot) < 0$

can be written as

$$E^I \pi(\cdot) - E^S \pi(\cdot) = \frac{2 \left(\frac{1+\gamma}{2+\gamma+\phi} \right) - 1}{4 \left(\frac{1+\gamma}{2+\gamma+\phi} \right)^2 - 4 \left(\frac{1+\gamma}{2+\gamma+\phi} \right) + 2} + \frac{\gamma - \phi}{4} - c - \left(2 \left(\frac{1+\gamma}{2+\gamma+\phi} \right) - 1 \right) < 0$$

So from above, we can get the critical value of c , $\hat{c} = \frac{2 \left(\frac{1+\gamma}{2+\gamma+\phi} \right) - 1}{4 \left(\frac{1+\gamma}{2+\gamma+\phi} \right)^2 - 4 \left(\frac{1+\gamma}{2+\gamma+\phi} \right) + 2} + \frac{\gamma - \phi}{4} - \left(2 \left(\frac{1+\gamma}{2+\gamma+\phi} \right) - 1 \right)$.

Then if $c > \hat{c}$, $E^I \pi(\cdot) > E^S \pi(\cdot)$, so player j will select the strategy of imitation without signal and if $0 < c < \hat{c}$, she takes into account the strategy of observing signal as possible one.

Lemma 2

Suppose $\gamma > \phi$, $p > \frac{1+\gamma}{2+\gamma+\phi}$ and $c > \hat{c}$. Then player who moves later always imitates the other player's announcement without signal.

Now let's think about the case $0 < c < \hat{c}$. If our given c is $c \in (0, \hat{c})$, player's strategy set consists of two elements. The first is to observe her signal and the second is to imitate without signal. Let's denote that the expected payoff of the strategy to observe signal as $E^S \pi$ and the one of the strategy to imitate without signal as $E^I \pi$. Then we know that

$$E^S \pi(\cdot) - E^I \pi(\cdot) = -\frac{(2p-1)^3}{4p-4p^2-2} - \frac{4c-\gamma+\phi}{4}$$

In above formula, we can check $\frac{4c-\gamma+\phi}{4} < 0$ if should be satisfied because $\frac{-(2p-1)^3}{4p^2-4p+2} < 0$ and this means that $c < \frac{\gamma-\phi}{4}$ is a condition.

First, let's denote $f(p) = \frac{-(2p-1)^3}{4p^2-4p+2}$. Then we can check that this function is a decreasing function for $\forall p$ from $\frac{\partial f(p)}{\partial p} < 0$. Also $f(\frac{1}{2}) = 0$ and $f(1) = -\frac{1}{2}$. Here, our condition says $p \in (\frac{1+\gamma}{2+\gamma+\phi}, 1)$. So $f(p) \in (-\frac{1}{2}, f(\frac{1+\gamma}{2+\gamma+\phi}))$. Then if we compare $f(p)$ and $\frac{4c-\gamma+\phi}{4}$, following three cases are possible, Case a) $\frac{4c-\gamma+\phi}{4} > f(\frac{1+\gamma}{2+\gamma+\phi})$, Case b) $-\frac{1}{2} < \frac{4c-\gamma+\phi}{4} < f(\frac{1+\gamma}{2+\gamma+\phi})$ and Case c) $\frac{4c-\gamma+\phi}{4} < -\frac{1}{2}$. In following, $\frac{(6\gamma+6\phi+8\gamma\phi+2\gamma^2+\gamma^3+2\phi^2+\phi^3+\gamma\phi^2+\gamma^2\phi+4)(\gamma-\phi)}{4(2\gamma+2\phi+\gamma^2+\phi^2+2)(\gamma+\phi+2)}$ is denoted by \hat{c} .

Case a) When $\frac{4c-\gamma+\phi}{4} > f(\frac{1+\gamma}{2+\gamma+\phi})$.

Now $f(\frac{1+\gamma}{2+\gamma+\phi}) - \frac{4c-\gamma+\phi}{4} = \frac{(6\gamma+6\phi+8\gamma\phi+2\gamma^2+\gamma^3+2\phi^2+\phi^3+\gamma\phi^2+\gamma^2\phi+4)(\gamma-\phi)}{4(2\gamma+2\phi+\gamma^2+\phi^2+2)(\gamma+\phi+2)} - c$. But we can check that $f(\frac{1+\gamma}{2+\gamma+\phi}) - \frac{4c-\gamma+\phi}{4} = \hat{c}$. So $f(\frac{1+\gamma}{2+\gamma+\phi}) - \frac{4c-\gamma+\phi}{4} = \hat{c} - c > 0$ always from the assumption $0 < c < \hat{c}$. So always $f(\frac{1+\gamma}{2+\gamma+\phi}) > \frac{4c-\gamma+\phi}{4}$ should be satisfied. So case a) is excluded in our analysis.

Case b) When $-\frac{1}{2} < \frac{4c-\gamma+\phi}{4} < f(\frac{1+\gamma}{2+\gamma+\phi})$.

This case means that there exists \hat{p} such that if $p \in (\frac{1+\gamma}{2+\gamma+\phi}, \hat{p})$, $E^S \pi > E^I \pi$ and if $p \in (\hat{p}, 1)$, $E^S \pi < E^I \pi$. Also we can get $c > \frac{\gamma-\phi-2}{4}$ from $-\frac{1}{2} < \frac{4c-\gamma+\phi}{4}$ and $c < \hat{c} = \frac{(6\gamma+6\phi+8\gamma\phi+2\gamma^2+\gamma^3+2\phi^2+\phi^3+\gamma\phi^2+\gamma^2\phi+4)(\gamma-\phi)}{4(2\gamma+2\phi+\gamma^2+\phi^2+2)(\gamma+\phi+2)}$ from $\frac{4c-\gamma+\phi}{4} < f(\frac{1+\gamma}{2+\gamma+\phi})$. If we think the conditions $c > \frac{\gamma-\phi-2}{4}$, $c < \hat{c}$ and $c < \frac{\gamma-\phi}{4}$ together, we can get $\frac{\gamma-\phi-2}{4} < c < \hat{c}$ because $\frac{\gamma-\phi}{4} > \hat{c}$. So we can say that if $\frac{\gamma-\phi-2}{4} < c < \hat{c}$, there exists p_3^* such that $E^S \pi > E^I \pi$ if $p \in (\frac{1+\gamma}{2+\gamma+\phi}, p_3^*)$ and $E^S \pi < E^I \pi$ if $p \in (p_3^*, 1)$.¹⁰

¹⁰ Here, \hat{p} can be attained from $f(p) = \frac{-(2p-1)^3}{4p^2-4p+2} = \frac{4c-\gamma+\phi}{4}$ when $p \in [\frac{1+\gamma}{2+\gamma+\phi}, 1]$. So from $\frac{-(2p-1)^3}{4p^2-4p+2} = \frac{4c-\gamma+\phi}{4}$,

Case c) When $\frac{4c-\gamma+\phi}{4} < -\frac{1}{2}$

This case means that $E^S\pi < E^I\pi$ always. So player will imitates the other player without observing signal always. The condition $\frac{4c-\gamma+\phi}{4} < -\frac{1}{2}$ means $c < \frac{\gamma-\phi-2}{4}$. So if this is satisfied, player will imitate the other player for $\forall p \in (\frac{1+\gamma}{2+\gamma+\phi}, 1)$. But this also depends on γ and ϕ because if $\gamma - \phi > 2$, $\frac{\gamma-\phi-2}{4} > 0$, but if $\gamma - \phi < 2$, $\frac{\gamma-\phi-2}{4} < 0$.

Case c-1) $0 < \gamma - \phi < 2$

Then case c) says that $E^S\pi > E^I\pi$ for $\forall p \in [\frac{1+\gamma}{2+\gamma+\phi}, 1]$ if $0 < c < \frac{\gamma-\phi-2}{4}$.

Case c-2) $\gamma - \phi > 2$

Then case c) is excluded because the condition $\frac{4c-\gamma+\phi}{4} < -\frac{1}{2} \implies c < \frac{\gamma-\phi-2}{4} < 0$ is meaningless.

Finally we can get following proposition from above analysis.

Lemma 3

Suppose $\gamma > \phi$, $\frac{1+\gamma}{2+\gamma+\phi} < p < 1$. Here, $\hat{c} = \frac{(6\gamma+6\phi+8\gamma\phi+2\gamma^2+\gamma^3+2\phi^2+\phi^3+\gamma\phi^2+\gamma^2\phi+4)(\gamma-\phi)}{4(2\gamma+2\phi+\gamma^2+\phi^2+2)(\gamma+\phi+2)}$.

1) Suppose $0 < \gamma - \phi < 2$. Then, for $0 < c < \hat{c}$, $\exists \hat{p}$ such that player who moves later reports truthfully after observing signal if $p \in (\frac{1+\gamma}{2+\gamma+\phi}, p_3^*)$ and imitates the other player without signal if $p \in (p_3^*, 1)$.

2) Suppose $\gamma - \phi > 2$. Then, for $\frac{\gamma-\phi-2}{4} < c < \hat{c}$, $\exists \hat{p}$ such that player who moves later reports truthfully after observing signal if $p \in (\frac{1+\gamma}{2+\gamma+\phi}, p_3^*)$ and player imitates the other player without signal if $p \in (p_3^*, 1)$. Also, for $0 < c < \frac{\gamma-\phi-2}{4}$, player who moves later reports truthfully after observing signal for $\forall p \in (\frac{1}{2}, 1)$.

This proposition says that if $0 < c < \hat{c}$, $\gamma > \phi$, player's optimal decision rule depends on the belief in the other player's information quality and payoffs. How can we interpret this intuitively? From the formula, $E^S\pi - E^I\pi$, we can check that $\frac{\partial(E^S\pi - E^I\pi)}{\partial p} < 0$. Then $\frac{\partial(E^S\pi - E^I\pi)}{\partial p} < 0$ means that

$$p \uparrow \implies (E^S\pi - E^I\pi) \downarrow \text{ and } p \downarrow \implies (E^S\pi - E^I\pi) \uparrow.$$

Also we can check $\frac{\partial E^S\pi}{\partial p} > 0$ and $\frac{\partial E^I\pi}{\partial p} > 0$. So the fact " $p \uparrow$, $(E^S\pi - E^I\pi) \downarrow$ " means that the effect of $\frac{\partial E^I\pi}{\partial p}$ dominates $\frac{\partial E^S\pi}{\partial p}$. Here, for $\frac{\partial E^S\pi}{\partial p}$, the increase in p means that the quality of information increases. So the expected payoff when she decides to observe her signal will increase. So this player has more incentive to observe her signal as p increases. But also if we think about $\frac{\partial E^I\pi}{\partial p}$, the increase in p means that the quality of information that **the other player** has is getting better. So $\frac{\partial E^I\pi}{\partial p} > 0$ also means that the expected payoff when she decides not to observe her own signal and to imitate increases. So the fact " $p \uparrow$, $(E^S\pi - E^I\pi) \downarrow$ " means however there is a positive effect from the increase in p for observing signal and also for imitation, the effect of $\frac{\partial E^I\pi}{\partial p}$ dominates $\frac{\partial E^S\pi}{\partial p}$. That is to say, if the given quality of information is very high, the strategy of imitation is a better strategy compared to that of observing signal, so player j will imitate. We also can apply same reasoning for $p \downarrow$, $(E^S\pi - E^I\pi) \uparrow$. This means that as p decreases, the effect of $E^S\pi$ dominates the effect of $E^I\pi$. So we can infer that if the given p is relatively low in $p \in (\frac{1+\gamma}{2+\gamma+\phi}, 1)$, player j has more incentive to observe her signal however she has to pay information cost. Then we can check again the existence of \hat{p} such that $E^S\pi(\hat{p}) = E^I\pi(\hat{p})$ intuitively.

4.3.2 Case 2) When $\gamma > \phi$ and $\frac{1}{2} < p < p^*$

Now our second case is when $\gamma > \phi$ and $\frac{1}{2} < p < p^*$. From our game tree, we can check there is a difference in the announcements by both players always. First if the subsequent player j observes her signal and $\theta_j = a_i$, $a_i \neq a_j$ because she deviates from her observed signal. Also if $\theta_j \neq a_i$, $a_i \neq a_j$ again because she reports truthfully. However she doesn't observe her signal, she just deviates from the other player's announcement for the extremely low level of belief in the other player's information quality. So $a_i \neq a_j$. That is to say, always $a_i \neq a_j$ and there is no reason for her to observe her signal with positive information cost. So we can say follows.

Proposition

Suppose that $\gamma > \phi$ and $\frac{1}{2} < p < p^*$. Then the subsequent player doesn't select the strategy of observing signal.

Now however we know the best strategy of the subsequent player when $\gamma > \phi$ and $\frac{1}{2} < p < p^*$, let's do a following procedure. Later in case 3), we will use the condition that we get from this procedure for solving the function that is not the closed form. Now let's think about the case that $\gamma > \phi$ and $\frac{1}{2} < p < p^*$. In this case, player announces different information from the other player whatever her observed signal it is. Especially, she deviates from her signal however she observes same signal with the other player if the belief in the information quality is extremely low, $\frac{1}{2} < p < p^*$.

First, the expected payoff when she observes her signal is

$$E^S(\pi) = \frac{1}{2} \left(-\frac{p^2}{2p^2 - 2p + 1} \phi + \frac{(1-p)^2}{2p^2 - 2p + 1} \gamma + \frac{\gamma - \phi}{2} \right) - c$$

This comes from the fact that $E_{same}^S = -\frac{p^2}{2p^2 - 2p + 1} \phi + \frac{(1-p)^2}{2p^2 - 2p + 1} \gamma - c$ and $E_{different}^S = \frac{\gamma - \phi}{2} - c$ with $Prob(same\ S) = Prob(different\ S) = \frac{1}{2}$. Here, E_{same}^S is the expected payoff when she observes same signal and $E_{different}^S$ is the one when she observes different signal.

Second, the expected payoff when she doesn't observe her signal is

$$E^{NS}(\pi) = -p\phi + (1-p)\gamma.$$

Then

$$E^S(\pi) - E^{NS}(\pi) = \frac{1}{2} \left(-\frac{p^2}{2p^2 - 2p + 1} \phi + \frac{(1-p)^2}{2p^2 - 2p + 1} \gamma + \frac{\gamma - \phi}{2} \right) - c - (-p\phi + (1-p)\gamma)$$

So whether $E^S(\pi) \geq E^{NS}(\pi)$ or not depends on following formula,

$$\frac{p^3(4\gamma + 4\phi) + p^2(-3\gamma - 5\phi) + 2p\phi + \gamma}{4p^2 - 4p + 2} \geq \frac{1}{4}(4c + 3\gamma + \phi)$$

Let

$$h(p) = \frac{p^3(4\gamma + 4\phi) + p^2(-3\gamma - 5\phi) + 2p\phi + \gamma}{4p^2 - 4p + 2}$$

Here, $h(p)$ is an increasing function for $\forall p$ from $\frac{\partial(h(p))}{\partial p} = \frac{(\gamma + \phi)(2p - 1)^2(p^2 - p + 1)}{(2p^2 - 2p + 1)^2} > 0$. Also because $p \in (\frac{1}{2}, p^*)$, $h(p) \in (\frac{1}{4}(3\gamma + \phi), h(p^*))$. Then following three cases are possible. Case 1) $h(p = \frac{1}{2}) > h(p^*)$, Case 2) $h(p = \frac{1}{2}) < \frac{1}{4}(4c + 3\gamma + \phi) < h(p^*)$, Case 3) $\frac{1}{4}(4c + 3\gamma + \phi) < h(p = \frac{1}{2})$.

Case 1) $\frac{1}{4}(4c + 3\gamma + \phi) > h(p^*)$.

From this, we can find that if $c > h(p^*) - \frac{(3\gamma + \phi)}{4}$, $E^S < E^{NS}$ for $\forall p \in (\frac{1}{2}, p^*)$

Case 2) $\frac{1}{4}(3\gamma + \phi) < \frac{1}{4}(4c + 3\gamma + \phi) < h(p^*)$

From this, we can find that if $0 < c < h(p^*) - \frac{(3\gamma + \phi)}{4}$, there exists p_4^* such that $E^S < E^{NS}$ for $p \in (\frac{1}{2}, p_4^*)$ and $E^S > E^{NS}$ for $p \in (p_4^*, p^*)$

Case 3) $\frac{1}{4}(4c + 3\gamma + \phi) < \frac{1}{4}(3\gamma + \phi)$

This case is excluded because this is impossible.

So we can summarize above analysis as follows.

Result 1) Suppose $h(p^) > \frac{1}{4}(3\gamma + \phi)$.*

If $c > h(p^) - \frac{(3\gamma + \phi)}{4}$, $E^S < E^{NS}$ for $\forall p \in (\frac{1}{2}, p^*)$.*

If $0 < c < h(p^) - \frac{(3\gamma + \phi)}{4}$, there exists \tilde{p} such that $E^S < E^{NS}$ for $p \in (\frac{1}{2}, p_4^*)$ and $E^S > E^{NS}$ for $p \in (p_4^*, p^*)$.*

Result 2) Suppose $h(p^) < \frac{1}{4}(3\gamma + \phi)$.*

$E^S < E^{NS}$ for $\forall p \in (\frac{1}{2}, p^)$ and $c > 0$.*

But here, result 1) is excluded because we know that the subsequent player selects the strategy of deviation without signal for $\forall p \in (\frac{1}{2}, p^*)$ and for $\forall c > 0$ from the reasoning described above. So the unique condition that supports our reasoning is

$$h(p^*) < \frac{1}{4}(3\gamma + \phi)$$

This condition will be used again in the analysis when $\gamma > \phi$, and $p^* < p < \frac{1+\gamma}{2+\gamma+\phi}$.

4.3.3 Case 3) When $\gamma > \phi$, and $p^* < p < \frac{1+\gamma}{2+\gamma+\phi}$

Suppose that $\gamma > \phi$ and $p^* < p < \frac{1+\gamma}{2+\gamma+\phi}$. Now the subsequent player can decide whether she will observe her signal or deviate without signal. Now we can conjecture that there may exist the critical level of information cost under which she takes into account the strategy to observe her signal as a possible option. If the information cost is greater than the critical level, she may give up the opportunity to observe signal and she will deviate.

Let's think about following case. If the information quality p is given as $p = \frac{1+\gamma}{2+\gamma+\phi}$, this is the highest level of p under our assumption $p \in (\frac{1}{2}, \frac{1+\gamma}{2+\gamma+\phi})$. So in this case, player may select the strategy of observing signal if there is no information cost because the deviation without signal under the high belief in the other player's information quality is not optimal for her. But if we think about the existence of positive cost for signal, sometimes $E^S \pi(p = \frac{1+\gamma}{2+\gamma+\phi}) - E^D \pi(p = \frac{1+\gamma}{2+\gamma+\phi}) < 0$ is possible if the information cost is very high. From now, we denote $E^S \pi(p = \frac{1+\gamma}{2+\gamma+\phi})$ as $E^S \pi(\cdot)$ and $E^D \pi(p = \frac{1+\gamma}{2+\gamma+\phi})$ as $E^D \pi(\cdot)$. Then from we can get the critical level of c , $\check{c} = \frac{2(\frac{1+\gamma}{2+\gamma+\phi}) - 1}{4(\frac{1+\gamma}{2+\gamma+\phi})^2 - 4(\frac{1+\gamma}{2+\gamma+\phi}) + 2} + \frac{\gamma - \phi}{4} - \gamma + \frac{1+\gamma}{2+\gamma+\phi}(\gamma + \phi)$ that makes $E^S \pi(\cdot) - E^D \pi(\cdot) < 0$. So we can find that if the given information cost is greater than the critical level, \check{c} , player always selects the strategy of deviation without signal. If not player takes into account the strategy of observing signal as possible one. Then we can get following proposition.

Lemma 4

Suppose $\gamma > \phi$ and $p^* < p < \frac{1+\gamma}{2+\gamma+\phi}$. If $c > \check{c} = \frac{2(\frac{1+\gamma}{2+\gamma+\phi})-1}{4(\frac{1+\gamma}{2+\gamma+\phi})^2-4(\frac{1+\gamma}{2+\gamma+\phi})+2} + \frac{\gamma-\phi}{4} - \gamma + \frac{1+\gamma}{2+\gamma+\phi}(\gamma + \phi)$, player who moves later always deviates from the other player without signal.

Now let's think the case $c < \hat{c}$. If $c < \hat{c}$, the subsequent player's strategy set consists of two elements. The first is to observe signal and the second is to deviate without signal. Let's denote that the expected payoff of the strategy to observe signal as $E^S\pi(\cdot)$ and the one of the strategy to deviate without signal as $E^D\pi(\cdot)$. Then

$$\begin{aligned} E^S\pi - E^D\pi &= \frac{2p-1}{4p^2-4p+2} + \frac{\gamma-\phi}{4} - c - \gamma + p(\gamma + \phi) \\ E^S\pi &= E^D\pi \iff \frac{2p-1}{4p^2-4p+2} + p(\gamma + \phi) = \frac{1}{4}(4c + 3\gamma + \phi) \end{aligned}$$

Now let's denote that $g(p) = \frac{2p-1}{4p^2-4p+2} + p(\gamma + \phi)$. Then $g(p)$ is a increasing function of $p \in (p^*, \frac{1+\gamma}{2+\gamma+\phi})$. Also we can get $g(\frac{1}{2}) = \frac{\gamma+\phi}{2}, g(0) = -\frac{1}{2}$ and $g(1) = \gamma + \phi + \frac{1}{2}$. Here, $p \in [p^*, \frac{1+\gamma}{2+\gamma+\phi}]$, so $g(p) \in [g(p^*), g(\frac{1+\gamma}{2+\gamma+\phi})]$. Now if we compare $g(p)$ and $\frac{3\gamma+\phi}{4} + c$, following three cases are possible. Case a) $\frac{3\gamma+\phi}{4} + c > g(\frac{1+\gamma}{2+\gamma+\phi})$, Case b) $g(p^*) < \frac{3\gamma+\phi}{4} + c < g(\frac{1+\gamma}{2+\gamma+\phi})$ and Case c) $\frac{3\gamma+\phi}{4} + c < g(p^*)$.

Case a) When $\frac{3\gamma+\phi}{4} + c > g(\frac{1+\gamma}{2+\gamma+\phi})$

Now if we compare $g(\frac{1+\gamma}{2+\gamma+\phi})$ and $\frac{3\gamma+\phi}{4} + c$, $g(\frac{1+\gamma}{2+\gamma+\phi}) - \frac{1}{4}(4c + 3\gamma + \phi) = \frac{(6\gamma+6\phi+8\gamma\phi+2\gamma^2+\gamma^3+2\phi^2+\phi^3+\gamma\phi^2+\gamma^2\phi+4)(\gamma-\phi)}{4(2\gamma+2\phi+\gamma^2+\phi^2+2)(\gamma+\phi+2)} - c$. Now let's denote $\frac{(6\gamma+6\phi+8\gamma\phi+2\gamma^2+\gamma^3+2\phi^2+\phi^3+\gamma\phi^2+\gamma^2\phi+4)(\gamma-\phi)}{4(2\gamma+2\phi+\gamma^2+\phi^2+2)(\gamma+\phi+2)} = \check{c}$. Then $g(\frac{1+\gamma}{2+\gamma+\phi}) - \frac{3\gamma+\phi}{4} - c = \check{c} - c$. But $g(\frac{1+\gamma}{2+\gamma+\phi}) > \frac{3\gamma+\phi}{4} + c$ always because our assumption says $0 < c < \check{c}$. So Case a) is excluded because always $g(\frac{1+\gamma}{2+\gamma+\phi}) > \frac{3\gamma+\phi}{4} + c$ should be satisfied.

Case b) When $g(p^*) < \frac{3\gamma+\phi}{4} + c < g(\frac{1+\gamma}{2+\gamma+\phi})$

This case means that there exists p_2^* such that if $p \in (p^*, p_2^*)$, $E^S\pi < E^D\pi$ and if $p \in (p_2^*, \frac{1+\gamma}{2+\gamma+\phi})$, $E^S\pi > E^D\pi$. From $g(p^*) < \frac{3\gamma+\phi}{4} + c$, we can get $c > g(p^*) - \frac{3\gamma+\phi}{4}$ and $c < \check{c} = \frac{(6\gamma+6\phi+8\gamma\phi+2\gamma^2+\gamma^3+2\phi^2+\phi^3+\gamma\phi^2+\gamma^2\phi+4)(\gamma-\phi)}{4(2\gamma+2\phi+\gamma^2+\phi^2+2)(\gamma+\phi+2)}$ from $\frac{3\gamma+\phi}{4} + c < g(\frac{1+\gamma}{2+\gamma+\phi})$. Finally, if we think the range of c , $g(p^*) - \frac{3\gamma+\phi}{4} < c < \check{c}$ is derived.

1) Suppose $g(p^*) > \frac{3\gamma+\phi}{4}$. Then for $g(p^*) - \frac{3\gamma+\phi}{4} < c < \check{c}$, there exists \check{p} such that if $p \in (p^*, p_2^*)$, $E^S\pi < E^D\pi$ and if $p \in (p_2^*, \frac{1+\gamma}{2+\gamma+\phi})$, $E^S\pi > E^D\pi$.

2) Suppose $g(p^*) < \frac{3\gamma+\phi}{4}$. Then for $0 < c < \check{c}$, there exists \check{p} such that if $p \in (p^*, p_2^*)$, $E^S\pi < E^D\pi$ and if $p \in (p_2^*, \frac{1+\gamma}{2+\gamma+\phi})$, $E^S\pi > E^D\pi$.

Case c) When $\frac{3\gamma+\phi}{4} + c < g(p^*)$

Case c) is excluded.

Finally we can get following results from above analysis when $\gamma > \phi$, $p^* < p < \frac{1+\gamma}{2+\gamma+\phi}$

Result 1) Suppose $g(p^*) > \frac{3\gamma+\phi}{4}$. Then for $g(p^*) - \frac{3\gamma+\phi}{4} < c < \check{c}$, there exists \check{p} such that if $p \in (p^*, p_2^*)$, $E^S\pi < E^D\pi$ and if $p \in (p_2^*, \frac{1+\gamma}{2+\gamma+\phi})$, $E^S\pi > E^D\pi$.

Result 2) Suppose $g(p^) < \frac{3\gamma+\phi}{4}$. Then for $0 < c < \check{c}$, there exists p_2^* such that if $p \in (p^*, p_2^*)$, $E^S \pi < E^D \pi$ and if $p \in (p_2^*, \frac{1+\gamma}{2+\gamma+\phi})$, $E^S \pi > E^D \pi$.*

Now let's check $g(p^*) = h(p^*)$. We know that

$$g(p) = \frac{2p-1}{4p^2-4p+2} + p(\gamma+\phi)$$

$$h(p) = \frac{p^3(4\gamma+4\phi) + p^2(-3\gamma-5\phi) + 2p\phi + \gamma}{4p^2-4p+2}$$

So, if we let $\chi(p) = g(p) - h(p)$,

$$\chi(p) = \frac{(2p-\gamma+2p\gamma-p^2\gamma+p^2\phi-1)}{2(2p^2-2p+1)}$$

and $\chi(p)$ has root at $p = p^* = \frac{1}{\gamma-\phi} ((\gamma+1) - \sqrt{\gamma+\phi+\gamma\phi+1})$. So we can find that $g(p = p^*) = h(p = p^*)$.

Claim

$$g(p^*) = h(p^*)$$

Now in case 2), when $\gamma > \phi$ and $\frac{1}{2} < p < p^*$, we showed that the unique condition that is satisfied is $h(p^*) < \frac{1}{4}(3\gamma+\phi)$. Then from $g(p = p^*) = h(p = p^*)$, the unique condition is $g(p^*) < \frac{3\gamma+\phi}{4}$. So from this, result 1) is excluded and the only possible one is result 2). So we can simplify the best strategy of the subsequent player when $\gamma > \phi$, $p^* < p < \frac{1+\gamma}{2+\gamma+\phi}$ as follows.

Lemma 5

Suppose $\gamma > \phi$, $p^ < p < \frac{1+\gamma}{2+\gamma+\phi}$. Then for $0 < c < \check{c}$, there exists p such that if $p_2^* \in (p^*, p_2^*)$, $E^S \pi < E^D \pi$ and if $p \in (p_2^*, \frac{1+\gamma}{2+\gamma+\phi})$, $E^S \pi > E^D \pi$. So the subsequent player always observes signal in this case.*

4.3.4 Optimal strategy of the subsequent player when $\gamma > \phi$

First, we can get following lemma that explains the best strategy of the subsequent player when $\gamma > \phi$ and $p \in (\frac{1}{2}, \frac{1+\gamma}{2+\gamma+\phi})$ as follows.

Lemma 7

Suppose $\gamma > \phi$

1) $p^* < p < \frac{1+\gamma}{2+\gamma+\phi}$. Then for $0 < c < \check{c}$, there exists p_2^* such that if $p \in (p^*, p_2^*)$, $E^S \pi < E^D \pi$ and if $p \in (p_2^*, \frac{1+\gamma}{2+\gamma+\phi})$, $E^S \pi > E^D \pi$.

2) $\frac{1}{2} < p < p^*$. Then for $0 < c < \check{c}$, $E^S < E^{NS}$ for $\forall p \in (\frac{1}{2}, p^*)$

From this, we can get following results that explains the equilibrium of the subsequent players' strategy when $\gamma > \phi$ and $p \in (\frac{1}{2}, 1)$.

Proposition 5

Suppose that $\gamma > \phi$ and $c > c^*$. Then the best strategy of the subsequent player can be described as follows.

$$\text{The subsequent player} \begin{cases} \text{Imitates without signal if } p \in (\frac{1+\gamma}{2+\gamma+\phi}, 1) \\ \text{Deviates without signal if } p \in (\frac{1}{2}, \frac{1+\gamma}{2+\gamma+\phi}) \end{cases}$$

Proposition 6

Suppose that $\gamma > \phi$ and $0 < c < c^*$. Then the best strategy of the subsequent player can be described as follows.

Case 1) When $0 < \gamma - \phi < 2$

$$\text{The subsequent player} \begin{cases} \text{Imitates without signal if } p \in (p_3^*, 1) \\ \text{Observes signal if } p \in (p_2^*, p_3^*) \\ \text{Deviates without signal if } p \in (\frac{1}{2}, p_2^*) \end{cases}$$

Case 2) When $\gamma - \phi > 2$

1) If $\frac{\gamma-\phi-2}{4} < c < c^*$.

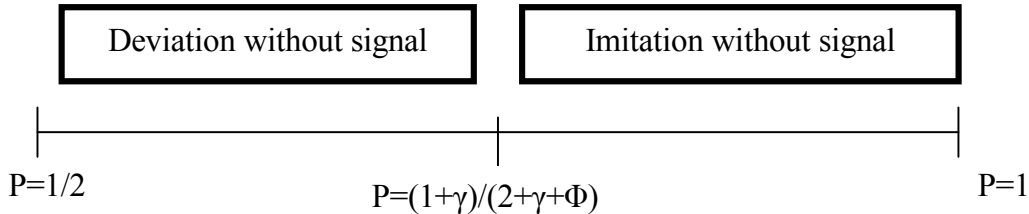
$$\text{The subsequent player} \begin{cases} \text{Imitates without signal if } p \in (p_3^*, 1) \\ \text{Observes signal if } p \in (p_2^*, p_3^*) \\ \text{Deviates without signal if } p \in (\frac{1}{2}, p_2^*) \end{cases}$$

2) If $0 < c < \frac{\gamma-\phi-2}{4}$

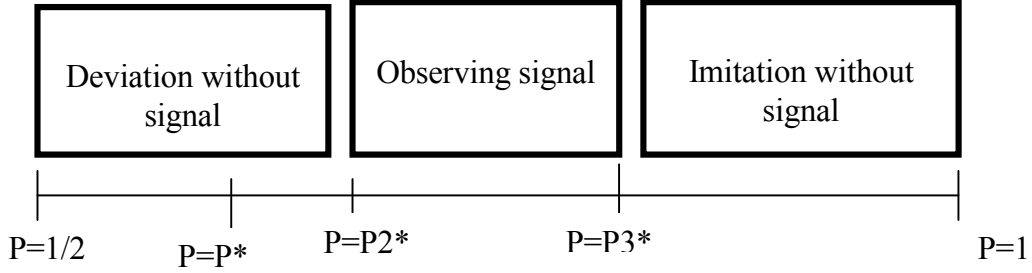
$$\text{The subsequent player} \begin{cases} \text{Observes signal if } p \in (p_2^*, 1) \\ \text{Deviates without signal if } p \in (\frac{1}{2}, p_2^*) \end{cases}$$

Now above decision rule can be described as follows as a function of belief in the information quality.

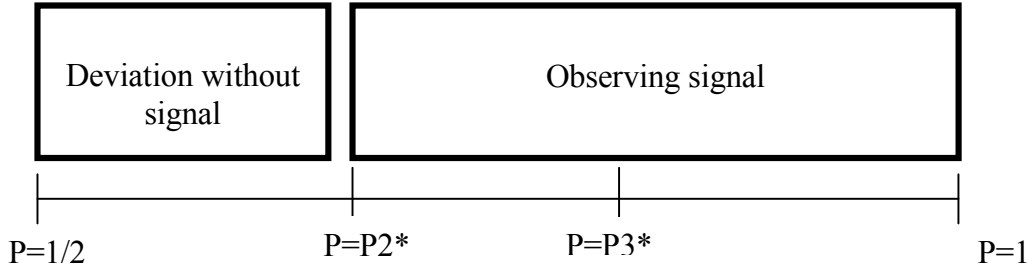
1) When $c > c^*$



2) When $0 < c < c^*$.



When 1) $0 < \gamma - \phi < 2$ or 2) $\gamma - \phi > 2, \frac{\gamma - \phi - 2}{4} < c < c^*$



When $\gamma - \phi > 2, 0 < c < \frac{\gamma - \phi - 2}{4}$

The first diagram is about the best strategy of the subsequent player for $c > c^*$ and other are for the case $0 < c < c^*$. So in case of $c > c^*$, the subsequent player's best strategy is the imitation or deviation without signal because she gives up the opportunity to observe her signal because of the high information cost. In this case, the only source of information that this player can use is the other player's announcement. If player behaves without her signal, her best strategy may depend on the belief in the other player's information quality, p . So if p is greater than the critical, $p^* = \frac{1+\gamma}{2+\gamma+\phi}$, this relatively high level of belief in the other player's information will make her follow the other player's announcement. Also if p is less than the critical level, $p^* = \frac{1+\gamma}{2+\gamma+\phi}$, this relatively low level of belief will make her deviate from the other player's announcement. Here, the negative effect of the information cost works.

Now other diagrams explain the best strategy of the subsequent player when $c < c^*$. In this case, player has an incentive to observe the signal because the information cost is not so high. Also the decision whether she will observe signal or not depends on the belief in the other player's information quality and the payoffs γ and ϕ . Now there are some points that we should mention for the diagrams under $c < c^*$.

First, if $0 < \gamma - \phi < 2$, her strategy set S is $S = \{DNS, SG, INS\}$ ¹¹. That is to say, however she can make use of the opportunity to get reward by being differentiated from $\gamma > \phi$, there exists the belief interval in which player imitates without signal.

Second, if $\gamma - \phi > 2$ and $\frac{\gamma - \phi - 2}{4} < c < c^*$, her strategy set S is $S = \{DNS, SG, INS\}$ again. But if $0 < c < \frac{\gamma - \phi - 2}{4} < c^*$, there is no interval of the belief in which the subsequent player imitates without signal. So in this case, $S = \{DNS, S\}$. This means that the incentive of being differentiated dominates the the negative effect of information cost. Here, the critical level of information cost is $\bar{c} = \frac{1}{4}(\gamma - \phi - 2)$ where $(0 < \bar{c} < c^*)$.

¹¹Here, *DNS* denotes deviation without signal, *SG* does observing signal and *INS* does imitation without signal respectively.

Now we can infer that three factors, the incentive to be differentiated, the negative effect of the information cost and the blame sharing effect works in determining the decision rule of the subsequent player. First, whether the strategy $s = \{DNS\} \in S$ or $s = \{DNS\} \notin S$ is determined by the incentive to be differentiated. In above, if $\gamma > \phi$, the incentive to be differentiated exists always because the strategy $s = \{DNS\} \in S$ whether $0 < \gamma - \phi < 2$ or $\gamma - \phi > 2$. Second, whether the incentive to be differentiated dominates the negative effect of information quality or not has an effect on whether $s = \{SG\} \in S$ or $s = \{SG\} \notin S$. Above result says that there exists the interval of the belief in which the incentive dominates the negative effect of the information quality always. For example, if $0 < \gamma - \phi < 2$ or $\gamma - \phi > 2$ and $\frac{\gamma - \phi - 2}{4} < c < c^*$, $s = \{SG\} \in S$ for $p \in (p_2^*, p_3^*)$. Also if $\gamma - \phi > 2$, $0 < c < \frac{\gamma - \phi - 2}{4} < c^*$, $s = \{SG\} \in S$ for $p \in (p_2^*, 1)$. Whether $s = \{SG\} \in S$ for $p \in (p_2^*, p_3^*)$ or $p \in (p_2^*, 1)$ depends on whether γ is sufficiently high or not. Finally, whether $s = \{INS\} \in S$ or $s = \{INS\} \notin S$ also depends on whether γ is sufficiently high or not. For a sufficiently high γ , $s = \{INS\} \notin S$. That is to say, the incentive to be differentiated dominates the blame sharing effect. But for not a sufficiently high γ , $s = \{INS\} \in S$. That is to say, there exists belief in terval in which the blame sharing effect dominates the incentive to be differentiated.

4.3.5 Optimal strategy of the subsequent player when $\gamma < \phi$

Now suppose that $\gamma < \phi$. If player who moves later decides not to observe signal, she always imitates the other player. And the expected payoff of selecting strategy to imitate without signal is

$$E^I \pi = 2p - 1$$

If player decides to observe her own signal, the expected payoff is

$$\begin{aligned} E^S \pi &= \frac{1}{2} E^S(\pi | \theta_j = a_i) + \frac{1}{2} E^S(\pi | \theta_j \neq a_i) \\ &= \frac{1}{2} \left(\frac{2p-1}{2p^2-2p+1} \right) + \frac{1}{2} \left(\frac{2p-1}{4p^2-4p+2} - c \right) = \frac{2p-1}{4p^2-4p+2} - c \end{aligned}$$

Then we can find that player who moves later imitates the other player without signal always.

Proposition 7

Suppose $\gamma < \phi$. Then the subsequent player always imitates the other player's announcement without observing her own signal.

Proof

We know that $E^S \pi = \frac{2p-1}{4p^2-4p+2} - c$ and $E^I \pi = 2p - 1$. So

$$E^S \pi - E^I \pi = \frac{-(2p-1)^3}{(2p-1)^2+1} - c < 0$$

So player who moves later imitates the other player without observing her signal always.

Above result says that if $\gamma < \phi$, the strategy of player who moves later does not depend on the belief in the other player's information. Our result says that if the penalty is greater than the reward, player

has a risk-averse attitude for the case that she is the unique player who made a mistake in forecasting. So her best strategy is to imitate the other player always. Only blame sharing effect works.

4.3.6 Optimal strategy of the player who moves first

Now let's discuss about the best strategy of player who moves first. In our example, we assumed that player i moves first. From backward induction, we can check that the best strategy of player who moves first is to reports truthfully after observing signal..

Proposition 8

The player who announces first reports truthfully her observed signal always.

Proof

Now let's assume that player i announces first and the player j announces later.

Case 1) When the subsequent player imitates without signal

Now suppose that player i observed the signal h_i . Then we can set up following payoff matrix for each strategy, to report truthfully and to announce distorted information, that is to say, deviates strategically. (The LHS table is for the strategy to announces truthfully and RHS one is for the case that player i announces distorted information.)

π_i	1	-1	π_i	-1	1
$Pr(s \theta_i)$	p	$1 - p$	$Pr(s \theta_i)$	p	$1 - p$
S	H	L	S	H	L
a_i	h	h	a_i	l	l
a_j	h	h	a_j	l	l

Then the expected payoff when player i announces truthfully is $E^T(\pi_i) = 2p - 1$ and the expected payoff when player i announces distorted information is $E^F(\pi_i) = 1 - 2p$. So we can check that it is better always for player i to announce truthfully.

Case 2) When the subsequent player deviates without signal

Now suppose that player i observed the signal h_i again. Then we can set up following payoff matrix for the each strategy to report truthfully or to announce distorted information. (The LHS one is for the strategy to announce truthfully and RHS one is for the strategy to announce distorted information.)

π_i	γ	$-\phi$	π_i	$-\phi$	γ
$Pr(s \theta_i)$	p	$1 - p$	$Pr(s \theta_i)$	p	$1 - p$
S	H	L	S	H	L
a_i	h	h	a_i	l	l
a_j	l	l	a_j	h	h

Then the expected payoff when player i announces truthfully is $E^T(\pi_i) = p\gamma - (1-p)\phi$ and the expected payoff when player i announces distorted information is $E^F(\pi_i) = -p\phi + (1-p)\gamma$. So it is better for player i to announce truthfully because $E^T(\pi_i) - E^F(\pi_i) > 0$.¹²

Case 3) When the subsequent player observes her signal

Then this is the same case with the simultaneous announcement case. So again player i announces truthfully.

Finally, we can say that player i announces truthfully always. **Q.E.D.**

So from above, we can check that the best strategy of player who moves first is to report truthfully her observed signal always.

5 Optimality of waiting option in the endogenous ordering with a view of ex-ante under incomplete information

According to the classical literature about endogenous ordering, the endogeneity of players' movement can cause the inefficiency compared to the simultaneous movement case. This inefficiency is caused from the fact that the subsequent player can free-ride on the other player who moved first because she can infer information from observing the other player's action. So every player has a tendency to delay her decision. Same reasoning can be applied to our model. Each player has an incentive to announce later because she can infer the signal of the other player from the announcement. Especially, if she is the first player who announces, she has to pay information cost for getting signal and this information cost may aggravate player's tendency to delay her decision.

Till now, many interesting topics were introduced for endogenous ordering problem in a strategic game. The most prominent topic is about the players timing of movement when players have heterogeneous belief. Chamley and Gale(1994) and Zhang(1997) showed that the player who has the highest belief in her information moves first in the endogenous ordering game. Especially, Zhang(1997) discussed this endogenous ordering problem with social learning literature together and showed that there is an onset of information cascade just after first mover's decision. In his model, the player who moves first is the one who has highest belief in her information. Choi(1997) also explained the endogenous ordering with a topic of the technology adoption under the network externality. In that paper, he explained that if the endogenous ordering is given for the adoption of new but risky technology, this endogenous ordering can make player be worse off and asserted that this loss of welfare is caused from the waiting option for learning from the other player's adoption.

In this section, we discuss the similar topic with Choi(1997), the welfare of waiting option in an endogenous ordering game with a view of ex-ante and shows the complementary results with Choi(1997). According to our result, the waiting option in an endogenous ordering can make player better off and

¹²Here, $E^T(\pi_i) - E^F(\pi_i) > 0$ because $E^T(\pi_i) - E^F(\pi_i) = p\gamma - (1-p)\phi - (-p\phi + (1-p)\gamma) > 0$ and this comes from the assumption $p > 1/2$ because $p > \frac{\phi + \gamma}{2(\phi + \gamma)} = \frac{1}{2}$.

also worse off compared to the simultaneous announcement. We will check the conditions that the welfare of player with a view of ex-ante can be better off and worse off.

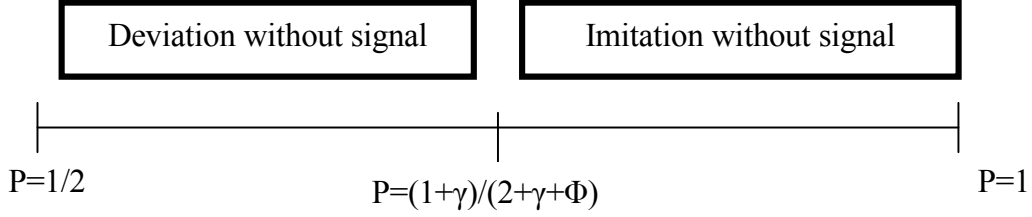
The frameworks of our analysis in this section are as follows. Suppose that there are two rounds in this announcement game. After entering into this game, both players observe the value of information cost and the payoffs, γ and ϕ . Our ordering of announcement is given endogenously. Because each player can observe the payoffs and the value of cost, players behavior can be analyzed with the result of exogenously ordering case. Each player can select the round in which she announces. But she doesn't know whether the other player will move in the first round or not. Player should decide when she will announce before the start of this game and the announcement is irreversible. So player has only one time chance to announce. Suppose that player decides to announce in the first round. Then after the announcement in the first round, she gets payoff depending on the other player's timing of announcement. If the other player moves in the first round together, she gets the payoff just after the first round. If not, she waits in the second round and gets payoff after the end of second round. Now suppose that player decides to move in the second round. After the end of the first round, we can check whether there was an announcement by the other player in the first round or not. If there was no announcement, there is nothing she can observe, so the second period problem is same with the initial period. If there was an announcement by the other player, she can observe the other player's announcement and selects her strategy. Here, each player has a trade off in her decision. If she announces in the first round, she can avoid the discount in her payoff. But she can't observe her the other player's announcement and also should pay information cost for observing signal. If she decides to announce in the second round, her payoff is discounted by δ . But there is a possibility that she can observe the other player's announcement, so her information set can be enriched. Also she can avoid the information cost if she decides to behave strategically without signal. But that happens only when the other player has already moved in the first round. So there is an uncertainty.

From now let's assume that we analyze the problem of player j . In following, $U(\cdot)$ represents uniform distribution. Also $\pi_{s_j}^{s_i}$ denotes the expected payoff when player j selects the strategy s_j and the player i selects the strategy s_i .

5.1 When $\gamma > \phi$ and the cost for observing signal is high, $c > c^*$.

Our first case is when $\gamma > \phi$ and $c > c^*$. After entering in this game, both players observe payoffs and the value of information cost. Now each player has to decide whether she will announce in the first round or in the second round. If she announces in the first round, we already checked that her best strategy is to observe her signal and report truthfully. If she decides to move in the second round, there are two strategies that she can select. She imitates the other player who moved in the first round without signal if $\frac{1+\gamma}{2+\gamma+\phi} < p < 1$ and deviates from the other player without signal if $\frac{1}{2} < p < \frac{1+\gamma}{2+\gamma+\phi}$. But the strategic behavior without signal is possible only when the other player has already moved in the first round. If there were no announcements by both player, the second round is exactly same with the initial first round. Now we suppose that q is the probability each player believes that the other player announces in the first round whether she announces in the first round or not. Also we assume that there is a discount

in the payoff if player makes an announcement in the second round by δ . Then we can set up the value of each player depending on the strategy to select when she announces as follows.



Strategy of announcement in the first round.

Let's think about the strategy that she moves in the first round. Then the value function of this strategy can be described as

$$\begin{aligned}
 V_1^j &= q\pi_S^S + (1-q) \left[\left(U(p^*) - U\left(\frac{1}{2}\right) \right) \pi_S^D + [U(1) - U(p^*)] \pi_S^I \right] - c \\
 &= q \left(\frac{\gamma - \phi}{4} \right) + (1-q) \left[\left(\frac{2+2\phi}{2+\gamma+\phi} \right) \left(\frac{\gamma - \phi}{2} \right) \right] - c
 \end{aligned}$$

In above formula, the first term, $\frac{q}{4}(\gamma - \phi)$, is the expected payoff when both player i and j announce in the first round with information cost. The second term, $(1-q) \left[\left(\frac{2+2\phi}{2+\gamma+\phi} \right) \left(\frac{\gamma - \phi}{2} \right) + \left(\frac{\gamma - \phi}{2+\gamma+\phi} \right) (0) \right]$, is the expected payoff when player j moves in the first round but player i delays her decision in the first round. Then player j 's expected payoff depends on the other player's strategy. If player i announces in the second round, her strategies consists of two elements, imitation or deviation because $c > c^*$.

First, let's get player i 's probability of the deviation in the second round. Player i deviates in the round two if $\frac{1}{2} < p < \frac{1+\gamma}{2+\gamma+\phi}$. So the probability of deviation is $\frac{2+2\phi}{2+\gamma+\phi}$ from $\Pr(\text{Deviation}) = \frac{1-p_1^*}{\frac{1}{2}} = \frac{1 - \frac{1+\gamma}{2+\gamma+\phi}}{\frac{1}{2}}$. Through same reasoning, we can get the probability of imitation, $\Pr(\text{Imitation}) = \frac{\gamma - \phi}{2+\gamma+\phi}$. If player i deviates from player j 's announcement in the round two, player j 's expected payoff is $\frac{\gamma - \phi}{2}$. If player i imitates, the expected payoff is 0. Finally, because she decides to announce in the first round, she has to observe her signal with paying information cost c .

Strategy of announcement in the second round.

Now, let's think about the strategy that player j moves in the second round. In this case, the value function is given as follows.

$$\begin{aligned}
 V_2^j &= \delta \left[q \left(U(p^*) - U\left(\frac{1}{2}\right) \right) \pi_D^S + [U(1) - U(p^*)] \pi_I^S \right] + (1-q)V \\
 &= \delta \left[q \left(\frac{2+2\phi}{2+\gamma+\phi} \right) \left(\frac{\gamma - \phi}{2} \right) + (1-q)V \right]
 \end{aligned}$$

Here, δ is the discount factor for the announcement in the second round and q is the probability that the other player announces in the first round again. If player j decides to announce in the second round, she can observe the other player's announcement with probability q . But with probability, $1 - q$, player

i doesn't announce in the first round and player j has same problem again with the initial period in this case. Here, V denotes the value of this announcement game in the initial period. Again, $\left(\frac{2+2\phi}{2+\gamma+\phi}\right)$ is player j 's probability of deviation.

First, we can check that there exists a unique $q^* \in (0, 1)$ such that $V_1(q^*) = V_2(q^*)$ from following proposition.

Lemma 8

In above formula, $\exists q^*$, equilibrium probability, such that $V_1(q^*) = V_2(q^*)$.

Proof

Let's denote that $\frac{\gamma-\phi}{4} = A$, $\left(\frac{2+2\phi}{2+\gamma+\phi}\right)\left(\frac{\gamma-\phi}{2}\right) = B$. Then we can write above value functions as follows.

$$\begin{aligned} V_1 &= q[A] + (1-q)[B] - c \\ V_2 &= \delta q[B] + \delta(1-q)V \end{aligned}$$

Then if we set, $V_1 = V_2$,

$$\begin{aligned} q[A] + (1-q)[B] - c &= \delta q[B] + \delta(1-q)V \\ \frac{q}{1-q} &= \frac{[\delta V + B + c]}{\delta B + c - A} \end{aligned}$$

Here, RHS formula, $\frac{[\delta V + B + c]}{\delta B + c - A}$ is independent of q and only LHS formula is a function of q . Also the numerator, q , is a increasing function of q and denominator, $1 - q$, is a decreasing function of q . So we can find that there exists q^* such that $V_1(q^*) = V_2(q^*)$.

Now let's discuss about the optimality of using waiting option in an endogenous ordering. We already mentioned that the simultaneous announcement case can be regarded as a bench mark case because both players' information can be aggregated and there is no distortion in reported information by both players. So we will compare the value of endogenous ordering when she can use the waiting option and the value when she has to announce simultaneously. From this, we can check the welfare of player when she can use the waiting option with a view of ex-ante.

Let's compare the value of above mixed equilibrium result with simultaneous announcement case. We know that there exists q such that $V = V_1(q^*) = V_2(q^*)$. Now let's denote the value of simultaneous announcement case as V_{11} . Then the player's problem can be represented as follows.

$$\begin{aligned} V^j(\gamma, \phi, c) &= \max\{V_{11}, V_1^j(q^*)\} \\ &= \max\left\{\frac{\gamma-\phi}{4} - c, q\left[\frac{\gamma-\phi}{4}\right] + (1-q)\left[\left(\frac{2+2\phi}{2+\gamma+\phi}\right)\left(\frac{\gamma-\phi}{2}\right)\right] - c\right\} \\ &= \max\left\{\frac{\gamma-\phi}{4}, q\left[\frac{\gamma-\phi}{4}\right] + (1-q)\left[\left(\frac{2+2\phi}{2+\gamma+\phi}\right)\left(\frac{\gamma-\phi}{2}\right)\right]\right\} \end{aligned}$$

Then we can get the following results from above problem.

Proposition 9

Suppose that $\gamma > \phi$, $c > c^* = \frac{(6\gamma+6\phi+8\gamma\phi+2\gamma^2+\gamma^3+2\phi^2+\phi^3+\gamma\phi^2+\gamma^2\phi+4)(\gamma-\phi)}{4(2\gamma+2\phi+\gamma^2+\phi^2+2)(\gamma+\phi+2)}$.

1) If $\gamma < 3\phi + 2$, the waiting option makes each player better off compared to the simultaneous announcement case.

2) If $\gamma > 3\phi + 2$, the waiting option makes each player worse off compared to the simultaneous announcement case.

Proof

From above,

$$V_1^j(q^*) = q \left[\frac{\gamma - \phi}{4} \right] + (1 - q) \left[\left(\frac{2 + 2\phi}{2 + \gamma + \phi} \right) \left(\frac{\gamma - \phi}{2} \right) \right] - c$$

Now let's denote that V_{11} is the expected value when both players make the announcements simultaneously. Then

$$V_{11} = \frac{\gamma - \phi}{4} - c$$

Then

$$V_{11} - V_1^j(q^*) = (1 - q) \left[\frac{\gamma - \phi}{4} - \left(\frac{2 + 2\phi}{2 + \gamma + \phi} \right) \left(\frac{\gamma - \phi}{2} \right) \right]$$

So whether $V_{11} > V_1$ or $V_{11} < V_1$ depends on

$$\frac{\gamma - \phi}{4} - \left(\frac{2 + 2\phi}{2 + \gamma + \phi} \right) \left(\frac{\gamma - \phi}{2} \right) \geq 0$$

So

$$V_{11} < V_1 \text{ if } 0 < \gamma - 3\phi < 2$$

$$V_{11} > V_1 \text{ if } \gamma - 3\phi > 2$$

So we proved. **Q.E.D.**

Above proposition says that if the reward is not so high compared to the penalty, $\gamma < 3\phi + 2$, the waiting option in her decision makes this player better off. On the contrary if not, that waiting option makes player worse off. Here, If the reward is not so high, player doesn't have much incentive to be the unique player who announces the correct information compared to the case when the reward is relatively high. Then it will be better for her to delay her decision for the observation of the other player's announcement. In that way, there is a positive Penguin effect. But if $\gamma > 3\phi + 2$, the negative Penguin effect exists.

5.2 When $\gamma < \phi$

Now suppose that each player observes $\gamma < \phi$. If $\gamma < \phi$, player who moves later always imitates the other player who moved first. Let's suppose again that q is the probability that the other player announces in the first round. Then we can set up the value function of player j as follows. Here, V_1^j is the value

function of strategy that moves in the first round and V_2^j is the value function of the strategy that moves in the second round.

$$\begin{aligned} V_1^j &= q\pi_S^S + (1-q)\pi_S^I - c = q\left(\frac{\gamma-\phi}{4}\right) - c \\ V_2^j &= q\pi_I^S + (1-q)V = (1-q)V \end{aligned}$$

Let's think about V_1^j first. The term, $q\left[\frac{\gamma-\phi}{4}\right]$ is the player j 's expected payoff when player i also moves in the first round with probability q . The second term, $(1-q)V$, is the expected payoff when the other player doesn't announce in the first round with probability $1-q$. Now we can check again that there exists equilibrium q^* such that $V_1(q^*) = V_2(q^*)$ is satisfied. So we will skip that proof.

Now the player's problem is

$$\begin{aligned} V^j(\gamma, \phi, c) &= \max\{V_{11}, V_1^j(q^*)\} \\ &= \max\left\{\frac{\gamma-\phi}{4} - c, q\left(\frac{\gamma-\phi}{4}\right) - c\right\} \\ &= \max\left\{\frac{\gamma-\phi}{4}, q\left(\frac{\gamma-\phi}{4}\right)\right\} \end{aligned}$$

Then we can get following results.

Lemma 9

In above formula, $\exists q^*$ such that $V_1(q^*) = V_2(q^*)$.

Now from above proposition, we can compare the value function of simultaneous case, V_{11}^j and V_1^j . Let's denote V_{11} as the value when both player announce simultaneously. Then

$$V_{11} = \frac{\gamma-\phi}{4} - c$$

So

$$V_{11} - V_1(q^*) = (1-q)\left(\frac{\gamma-\phi}{4}\right) < 0$$

because $\gamma < \phi$. So $V_{11} < V_1$ always. This means that the waiting option makes players better off always. So we can get following proposition.

Proposition 10

Suppose $\gamma < \phi$. Then the waiting option caused from endogenous ordering makes players better off compared to the simultaneous announcement case.

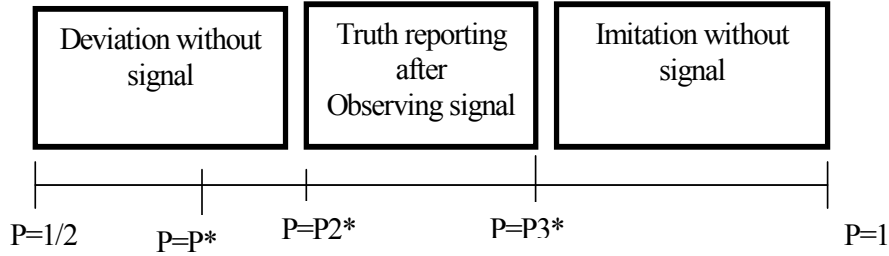
This proposition says that using the waiting option in an endogenous ordering makes player better off compared to the simultaneous announcement case if $\gamma < \phi$. This is a very interesting result. Usually, using waiting option for delaying her decision in an endogenous ordering is understood as the main reason to make players worse off compared to the simultaneous case. We already know that player imitates the other player if $\gamma < \phi$ when she is a player who moves later. That is caused from the fact that player has a strong risk aversion to the situation that she is the only one player who announced the false information.

In this case, if she can delay her decision and can observe the other player's announcement, she can avoid the case that she is the only one player who announced false announcement. In that sense, the possibility of waiting option makes player better off compared to the simultaneous announcement case. We have to be careful in interpreting this result that, here, the meaning of being better off is about the individual welfare with a view of ex-ante. So the possibility of avoiding severe penalty by herself makes player being better off compared to the simultaneous announcement case. The information cost makes the expected value of using waiting option greater because she has to pay information cost for the announcement in the first round. So in this case, the information cost increases the possibility of the positive penguin effect.

5.3 When $\gamma > \phi$ and the cost for observing signal is not so high, $c < c^*$

In last section, we checked that there are four possibilities in the subsequent player's best strategy when $\gamma > \phi$ and $c < c^*$. The point that we have to be careful is that however the subsequent player observes her signal with a positive information cost, whether she reports truthfully or not depends on her observed signal. That is to say, when player j is a subsequent player, she announces her signal truthfully if $\theta_j \neq a_i$. But if $\theta_j = a_i$, she reports her signal truthfully if $p > p^*$ and deviates from her observed signal if $\frac{1}{2} < p < p^*$.

Case 1)



In this case, the critical value above which player j observes her signal $p_2^* > p^*$. So whether $\theta_j \neq a_i$ or $\theta_j = a_i$, she always reports truthfully if she decides to observe her signal.

Now if player j decides to announce in the first round, the value function V_1 is as follows.

$$\begin{aligned} V_1^j &= q\pi_S^S + (1-q) \left[U \left(p_2^* - \frac{1}{2} \right) \pi_S^D + U(p_3^* - p_2^*) \pi_S^S + U(1 - p_3^*) \pi_S^I \right] - c \\ &= q \left[\frac{\gamma - \phi}{4} \right] + 2(1-q) \left[\left(p_2^* - \frac{1}{2} \right) \left(\frac{\gamma - \phi}{2} \right) + (p_3^* - p_2^*) \left(\frac{\gamma - \phi}{4} \right) \right] - c \end{aligned}$$

Also if player j decides to announce in the second round, the value function V_2 is

$$\begin{aligned} V_2^j &= q \left[U \left(p_2^* - \frac{1}{2} \right) \pi_S^D + U(p_3^* - p_2^*) \pi_S^S + U(1 - p_3^*) \pi_S^I \right] + (1-q)V \\ &\quad 2q \left[\left(p_2^* - \frac{1}{2} \right) \left(\frac{\gamma - \phi}{2} \right) + (p_3^* - p_2^*) \left(\frac{\gamma - \phi}{4} \right) \right] + (1-q)V \end{aligned}$$

Then we can check that there exists q^* that makes $V_1^j(q^*) = V_2^j(q^*)$ and the player's problem can be defined as follows.

$$\begin{aligned}
V^j(\gamma, \phi, c) &= \max\{V_{11}, V_1^j(q^*)\} \\
&= \max \left\{ \begin{array}{l} \frac{\gamma - \phi}{4} - c, \\ q \left[\frac{\gamma - \phi}{4} \right] + 2(1 - q) \left[\left(p_2^* - \frac{1}{2} \right) \left(\frac{\gamma - \phi}{2} \right) + (p_3^* - p_2^*) \left(\frac{\gamma - \phi}{4} \right) \right] - c \end{array} \right\} \\
&= \max \left\{ \begin{array}{l} \frac{\gamma - \phi}{4}, \\ q \left[\frac{\gamma - \phi}{4} \right] + 2(1 - q) \left[\left(p_2^* - \frac{1}{2} \right) \left(\frac{\gamma - \phi}{2} \right) + (p_3^* - p_2^*) \left(\frac{\gamma - \phi}{4} \right) \right] \end{array} \right\}
\end{aligned}$$

From this

$$V_{11} - V_1^j(q^*) = (1 - q) \left\{ \left[\frac{\gamma - \phi}{4} \right] - 2 \left[\left(p_2^* - \frac{1}{2} \right) \left(\frac{\gamma - \phi}{2} \right) + (p_3^* - p_2^*) \left(\frac{\gamma - \phi}{4} \right) \right] \right\}$$

So whether $V_{11} \geq V_1^j(q^*)$ depends on following relation,

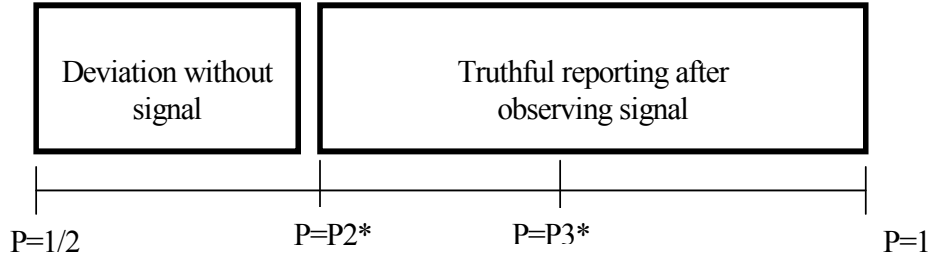
$$\begin{aligned}
f(\gamma, \phi) &= \left\{ \left[\frac{\gamma - \phi}{4} \right] - 2 \left[\left(p_2^* - \frac{1}{2} \right) \left(\frac{\gamma - \phi}{2} \right) + (p_3^* - p_2^*) \left(\frac{\gamma - \phi}{4} \right) \right] \right\} \\
&= \frac{1}{4} (\phi - \gamma) (2p_2 + 2p_3 - 3) \geq 0
\end{aligned}$$

So we can check whether $f(\gamma, \phi) \geq 0$ depends on $2p_2 + 2p_3 - 3 \leq 0$. Here, $2 < 2p_2 + 2p_3 < 4$ from $\frac{1}{2} < p_2, p_3 < 1$.

So if $p_2 + p_3 > \frac{3}{2}$, $f < 0 \implies V_{11} < V_1^j(q^*)$ and if $1 < p_2 + p_3 < \frac{3}{2}$, $f > 0 \implies V_{11} > V_1^j(q^*)$.

So in case 1), the waiting option makes player better off if $p_2 + p_3 > \frac{3}{2}$ and worse off if $1 < p_2 + p_3 < \frac{3}{2}$.

Case 2)



Again, the critical value above which player j observes her signal $p_2^* > p^*$. So whether $\theta_j \neq a_i$ or $\theta_j = a_i$, she always reports truthfully if she decides to observe her signal.

Now if player j decides to announce in the first round, the value function V_1 is as follows.

$$\begin{aligned}
V_1^j &= q\pi_S^S + (1 - q) \left[U \left(p_2^* - \frac{1}{2} \right) \pi_S^D + (1 - p_2^*) \pi_S^S \right] - c \\
&= q \left[\frac{\gamma - \phi}{4} \right] + 2(1 - q) \left[\left(p_2^* - \frac{1}{2} \right) \left(\frac{\gamma - \phi}{2} \right) + (1 - p_2^*) \left(\frac{\gamma - \phi}{4} \right) \right] - c
\end{aligned}$$

Also if player j decides to announce in the second round, the value function V_2 is

$$\begin{aligned}
V_2^j &= q \left[U \left(p_2^* - \frac{1}{2} \right) \pi_S^D + (1 - p_2^*) \pi_S^S \right] + (1 - q)V \\
&= 2q \left[\left(p_2^* - \frac{1}{2} \right) \left(\frac{\gamma - \phi}{2} \right) + (1 - p_2^*) \left(\frac{\gamma - \phi}{4} \right) \right] + (1 - q)V
\end{aligned}$$

Then we can check that there exists q^* that makes $V_1^j(q^*) = V_2^j(q^*)$ and from this

$$\begin{aligned} V^j(\gamma, \phi, c) &= \max\{V_{11}, V_1^j(q^*)\} \\ &= \max \left\{ \begin{array}{l} \frac{\gamma - \phi}{4} - c, \\ q \left[\frac{\gamma - \phi}{4} \right] + 2(1 - q) \left[\left(p_2^* - \frac{1}{2} \right) \left(\frac{\gamma - \phi}{2} \right) + (1 - p_2^*) \left(\frac{\gamma - \phi}{4} \right) \right] - c \end{array} \right\} \\ &= \max \left\{ \begin{array}{l} \frac{\gamma - \phi}{4}, \\ q \left[\frac{\gamma - \phi}{4} \right] + 2(1 - q) \left[\left(p_2^* - \frac{1}{2} \right) \left(\frac{\gamma - \phi}{2} \right) + (1 - p_2^*) \left(\frac{\gamma - \phi}{4} \right) \right] \end{array} \right\} \end{aligned}$$

Then

$$V_{11} - V_1^j(q^*) = (1 - q) \left\{ \left[\frac{\gamma - \phi}{4} \right] - 2 \left[\left(p_2^* - \frac{1}{2} \right) \left(\frac{\gamma - \phi}{2} \right) + (1 - p_2^*) \left(\frac{\gamma - \phi}{4} \right) \right] \right\}$$

So whether $V_{11} \geq V_1^j(q^*)$ depends on following relation,

$$g(\gamma, \phi) = \left[\frac{\gamma - \phi}{4} \right] - 2 \left[\left(p_2^* - \frac{1}{2} \right) \left(\frac{\gamma - \phi}{2} \right) + (1 - p_2^*) \left(\frac{\gamma - \phi}{4} \right) \right] \geq 0$$

Here, we can check $g(\gamma, \phi) < 0$ always because

$$g(\gamma, \phi) = \frac{1}{4} (\phi - \gamma) (2p_2^* - 1) < 0$$

So in case 2), always using the waiting option makes player better off compared to the simultaneous announcement case.

So from the results of case 1) to case 4), we can get following results.

Result

Suppose that $\gamma > \phi$ and $c < c^*$. In case 1), the waiting option makes player better off if $p_2 + p_3 > \frac{3}{2}$ and worse off if $1 < p_2 + p_3 < \frac{3}{2}$. In case 2), always using the waiting option makes player better off compared to the simultaneous announcement case.

Here, case 2) and case 4) are the ones that there is no probability of strategic imitation without signal. So our results say that using the waiting option makes player better off always compared to the simultaneous announcement case in that case. In case 1) whether the waiting option makes player better off or worse off depends on the value of $p_2 + p_3$. What is the intuition of this condition? First we prove following claim.

Claim

1) $p_2 + p_3 > \frac{3}{2} \iff \Pr(\text{Imitation}) < \Pr(\text{Deviation})$ and $p_2 + p_3 < \frac{3}{2} \iff \Pr(\text{Imitation}) > \Pr(\text{Deviation})$.

Proof

In following let's denote that $\Pr(I) = \Pr(\text{Imitation})$, $\Pr(D) = \Pr(\text{Deviation})$ and $\Pr(S) = \Pr(\text{Observing Signal})$.

1) First, let's think about the condition $p_2 + p_3 > \frac{3}{2}$ of case 1).

Then $p_2 + p_3 > \frac{3}{2} \iff p_2 - p_3 > \frac{3}{2} - 2p_3 \iff 2(p_3 - p_2) < 4p_3 - 3$
 $\iff 2(p_3 - p_2) < -[2(1 - p_3) - 2(p_3 - \frac{1}{2})]$. So

$$p_2 + p_3 > \frac{3}{2} \iff \Pr(S) < -[\Pr(I) - (1 - \Pr(I))] = 1 - 2\Pr(I)$$

So

$$\begin{aligned} 2\Pr(I) &< 1 - \Pr(S) = \Pr(I) + \Pr(D) \\ \iff \Pr(I) &< \Pr(D) \end{aligned}$$

So

$$\begin{aligned} p_2 + p_3 > \frac{3}{2} &\iff \Pr(Imitation) < \Pr(Deviation) \\ p_2 + p_3 < \frac{3}{2} &\iff \Pr(Imitation) > \Pr(Deviation) \end{aligned}$$

From above claim, we can check that the conditions that using the waiting option makes player better off or worse off are the relations between the probability of imitation and the probability of deviation. So we can rewrite our results as follows and we can get the interesting result that the waiting option in an endogenous ordering makes player better off compared to the simultaneous announcement case.

Proposition 11

Suppose that $\gamma > \phi$.

1) If $\gamma < \phi + 2, \forall c \in (0, c^*)$ or $\gamma > \phi + 2, \frac{\gamma - \phi - 2}{4} < c < c^*$, the waiting option makes player better off compared to the simultaneous announcement case if $\Pr(Imitation) < \Pr(Deviation)$ and worse off if $\Pr(Imitation) > \Pr(Deviation)$.

2) If $\gamma > \phi + 2, 0 < c < \frac{\gamma - \phi - 2}{4}$, the waiting option makes player better off compared to the simultaneous announcement case always.

6 Related literature with experiment

In this section, we introduce three papers that treated related topic. From these, we can find the results of experiments that support our model and also we can give an alternative explanation to the results of those experiments with a different view.

Derothea & Georg (2002) (from now, DG) introduced the experiment that discusses the failure of information cascade with limited depth of reasoning. Anderson & Holt (1997) (from now, AH) is an initiative paper that treated the topic of the formation of the information cascade with experiment. The most big difference in those two papers are the existence of information cost that is also an important assumption in our paper. In AH paper, players pay no information cost for observing signal and it says that the formation of information cascade can be checked with experiment. According to this

paper, in Bayesian Nash equilibrium, the first player buys a signal and makes a decision based on this signal. Then all subsequent players follow the first player's decision. So no further signals are revealed and cascade happens with certainty. DG changes the no information cost assumption and asserts that there is a difference in a formation of information cascade when players should pay information cost for observing signal. In this paper, the meaning of information cost is interpreted with a different view from our paper. Here, the information cost is interpreted as a main source that subsequent players can't believe the truthfulness of predecessor's decision.¹³ So this makes player who moves later observes her signals and makes a decision based on this. But the observing signal happens only for players who moves relatively early. If the order is relatively late, players are confident that previous decisions were made based on private signal and herd occurs.

Now how can we apply the results of this paper into our model? First, this paper introduced an error rate in the player's belief on the decision making procedure of other players' reasoning. Then this can be interpreted as the partial precision of given signal in our model. Because player in our model doesn't give 100% certainty to the other player's information, this plays a same role with the error rate of DG paper. In DG paper, the result that players who moves early observe their signal under positive information cost is explained by the reasoning that players doesn't give much weight to other player's decision for her giving high error rate to other players' decision. Then this is exactly coincides with the result of our model that player who moves later doesn't follow the announcement of player who moved first if the belief in the information quality is not so high. Of course, in DG model, the possibility of deviation without signal is not considered in experiment. Also in our model, players give same weight to the other player's information. The factor that affects her decision is the belief in the other player's information quality. But the assertion that player doesn't believe the other player's decision under some condition is same. Also the other result of experiment in DG paper that players who moves relatively later follow the decision of players who moved earlier when the majority of decision is formed can be explained with the reasoning of our model. Our model says that player who moves later imitates the other player without signal if the belief in the information quality is very high. The condition that the majority of decision is formed by player who moved earlier in DG model can be interpreted as the same condition with the high belief in the information quality in our model. The differences of our model and DG paper are as follows. First, DG paper assumed that there is a limit in the Bayesian information updating of players. So they introduced the error rate in interpreting the other players' decision and this assumption is used for the explanation of experiment results. But in our model, we assume that players are rational in the sense that they are perfect in Bayesian updating and use the information available to her, but we show that the there is a possibility of strategic behavior of imitation and deviation without signal. That is to say, the phenomena of imitation and deviation of our model are the results of fully rational behavior, but the results of DG paper are the results of irrational behavior with limited information updating. Finally, we strongly believe that to regard the option of deviation without signal as one element of strategy set will deepen the results of experiment.

The second paper that is related with our model is Andreoni & Harbaugh & Vesterlund (2003). This paper is about one shot proposer-responder game with a topic of systematic look at both reward

¹³In DG paper, truthfulness denotes the situation that player makes a decision based on observed signal.

and punishment together. The main set up of this paper's model is different with that of ours. In this game, proposers choose how much to share of a fixed pie with others, so it talks about the relation between selfishness and reward & punishment. They assert that the cooperation, that is to say the least selfishness, is strongly guaranteed when reward and punishment exist together. The punishment can help by getting people to move away from perfect selfishness. Also the reward is essential in the sense that it encourages further cooperation. So it says that when devising incentive systems, it is important to recognize that both tools, reward and punishment, should be present. This point gives reasoning why we have to give an asymmetry to payoffs for the case that she is the only one player who made a correct announcement and when she is the only one player who made a false announcement. In other papers, usually it is assumed that the reward when only one player is correct is symmetric with the punishment when only one player was wrong. So $\gamma = \phi$. But our model assume that there is an asymmetry in the reward and penalty and analyze the strategic behavior of players under the conditions of $\gamma > \phi$ and $\gamma < \phi$. As already mentioned, the best strategy of player who moves later in our announcement game depends on the value of γ and ϕ and this deepen our analysis for the players' strategic behavior. Also we can check that the increase in γ and the decrease in ϕ increase the probability of truthful announcement after observing signal when the information cost is not so high. Of course, the increase of γ and ϕ both together definitely increases the probability of truthful announcement compared to the case that only γ or ϕ increases separately. The meaning of "cooperation" in Andreoni & Harbaugh & Vesterlund (2003) can be understood as the truthfulness in our model.

7 Reference

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