Agency costs and asymmetric information in a small open economy*

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Abstract

This paper develops a theoretical model of a small open economy to assess the effects of agency costs and asymmetric information in credit markets. The framework of the analysis is a dynamic general equilibrium model with microeconomic foundations, where agents' decisions are derived from optimising behaviour and prices are sticky. Agency costs arise from an ex post information asymmetry between borrowers and lenders and raise the cost of external financing. To assess the long-run effects of agency costs and their impact on business cycle fluctuations the model, which is calibrated for New Zealand, is solved with and without agency costs. A decline in the degree of information asymmetry and hence agency costs increases the steady state level of capital, investment and output. Agency costs also have important effects on the business cycle and the adjustment paths of interest and exchange rates.

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1 Introduction and motivation

This paper develops a computable general equilibrium model of a small open economy in which asymmetric information in credit markets can affect business cycle fluctuations. Modern macroeconomic models used by policy makers generally do not account explicitly for asymmetric information between borrowers and lenders or credit market frictions more generally.[1] The assumption in these models is that the Modigliani and Miller (1958) theorem holds. Under the assumption that financial markets are complete and information and transaction costs are non-existent, the Modigliani and Miller (1958) theorem states that the mix of debt and equity used to finance firms' expenditures does not affect the expected profitability of a project – the same investment decisions would be made, irrespective of the mix of debt and equity finance.

The assumption of zero information and transaction costs (or perfect information) has come under increasing criticism since Akerlof's (1970) seminal paper. It illustrates how imperfect information between buyers and sellers can cause market malfunctioning and how efficient markets need some mechanism for overcoming the imperfect information problem.

The credit channel literature examines the impact of asymmetric information and other credit market frictions on real spending and economic activity and the implications for monetary policy.[2] Asymmetric information between borrowers (investors) and lenders (savers) is thought to affect the transmission of shocks to the economy in two ways. First, a shock to the economy can influence financial intermediaries' willingness to provide loans. This channel is referred to as the bank lending channel. The second channel is the balance sheet channel. It focuses on the impact of shocks on firms' financial positions and their ability to borrow. The focus in this paper is on the balance sheet channel. The bank lending channel is assessed in Claus (2004). Using a computable dynamic general equilibrium model Claus (2004) finds that the effects of bank lending are likely to be small.

The balance sheet channel is caused by the presence of agency costs. Agency costs arise in a principal-agent relation when agents have an incentive not to perform in the best interest of the principal. In credit markets, agency costs occur whenever lenders delegate control over resources to borrowers, leading to adverse selection, moral hazard and monitoring costs because of the inability of

[1] See, for example, the Reserve Bank of New Zealand's *Forecasting and Policy System* (Black, Cassino, Drew, Hansen, Hunt, Rose and Scott 1997), the Federal Reserve Board of Governors' *FRB/US* model (Brayton and Tinsley 1996), the International Monetary Fund's *MULTIMOD* model for industrial countries (Laxton, Isard, Faruqee, Prasad and Turtelboom 1998), the Bank of Canada's *Quarterly Projection Model* (Black, Laxton, Rose and Tetlow 1994) or the Australian Treasury's *TRYM* model (Commonwealth Treasury 1996a, 1996b). Credit market effects are not completely ignored by these models, but tend to be incorporated in an ad hoc manner as borrowing or cash flow constraints.

[2] See Bernanke and Gertler (1995), Mishkin (1995) and Walsh (1998) among others.

lenders to monitor borrowers or share in borrowers' information costlessly.

In this model, agency costs arise because of an ex post information asymmetry, i.e. only borrowers can costlessly observe actual returns after project completion. The imperfect information leads to a moral hazard problem and lowers the probability that a loan will be repaid. Financial intermediaries help overcome the information asymmetry by lending to entrepreneurs via a debt contract and monitoring entrepreneurs who default on their loans.

The credit channel literature has tended to use calibrated general equilibrium models. This is because assessing the quantitative effects empirically is difficult due to limited data, for example, on balance sheets or lending criteria. Important advances have been made, such as incorporating heterogeneous agents and asymmetric information into a representative agent framework. However, much of the literature to date has focused on the United States, a large semi-closed economy.[3] The credit channel has not yet been incorporated in a model of an open economy with a floating exchange rate.[4] This paper is a step toward filling this gap. It also contributes to the new open economy macroeconomics literature that began to emerge with Obstfeld and Rogoff's (1995) *Redux* model.[5] This literature focuses on developing open economy dynamic general equilibrium models that incorporate imperfect competition and nominal rigidities.

The paper proceeds as follows. Section 2 describes the theoretical model, which is calibrated for New Zealand. Section 3 discusses the steady state model with and without agency costs. Next, the effects of agency costs on business cycle fluctuations are evaluated. The dynamic model is solved in section 4 and the dynamic properties with and without agency costs are assessed in section 5. The last section summarises and concludes.

2 The general equilibrium model

The theoretical model builds on Carlstrom and Fuerst's (1997) closed economy agency cost model. It is extended to include a foreign sector, a floating exchange rate, sticky prices, an inflation targeting monetary authority and a government. The foreign sector is incorporated following McCallum and Nelson (1999). Firms use commodity inputs, which they import at the beginning of each period, to produce consumption goods. They sell the output to domestic and foreign consumers. Exports are a function of the real exchange rate and foreign demand.

The domestic economy operates under a flexible exchange rate and uncovered interest rate parity holds. Domestic prices are assumed to adjust only sluggishly.

[3] See Bernanke, Gertler and Gilchrist (1998), Fuerst (1995), Carlstrom and Fuerst (1997) and Fisher (1999) among others.

[4] Edwards and Végh (1997) develop a theoretical model of a small open economy with a predetermined exchange rate. In their model, the policy maker sets the exchange rate and stands ready to exchange domestic money for international reserves (or vice versa) at the prevailing exchange rate.

[5] For a review of the new open economy macroeconomics literature see Lane (2001).

2.1 Overview of the model

There are six agents in the economy: households, entrepreneurs, firms, financial intermediaries, a government and a monetary authority. Households and entrepreneurs form a continuum of agents with unit mass. The proportion of households is given by $1 - \eta$ and of entrepreneurs by η.

Financial intermediaries are modelled as in Carlstrom and Fuerst (1997). The set-up for households and entrepreneurs is slightly modified in that households and entrepreneurs purchase an index of consumption goods rather than a single good. The specification for firms also differs to allow for nominal rigidities and a foreign sector. Moreover, a monetary authority and a government are included, which Carlstrom and Fuerst (1997) do not incorporate.

Households provide labour and capital to firms and consume. They face a deposit-in-advance constraint and must hold demand deposits to purchase consumption goods. Households also hold bonds in the form of domestic and foreign securities.

Firms are monopolistic competitors and produce consumption goods by hiring labour and renting capital from households and entrepreneurs. They also use imported commodity inputs. Firms sell their output to domestic households, entrepreneurs, the government and foreign consumers, who purchase quantities of an index of goods.

Entrepreneurs produce the capital good that firms use in the production of consumption goods. The capital good may be thought as a durable asset like trees and consumption goods can be thought of as a non-durable commodity like fruit. To produce the capital good entrepreneurs use external financing and their own net worth consisting of wage earnings and their own capital. Entrepreneurs obtain external financing from households through financial intermediaries (capital mutual funds). For each unit of capital good that households wish to obtain they give $\hat{\Psi}_t$ units of their consumption good index to financial intermediaries.[6] $\hat{\Psi}_t$ thus denotes the real price of capital in consumption goods. Households provide external finance to entrepreneurs via financial intermediaries because the production of capital is subject to idiosyncratic technology shocks. This leads to agency costs because the technology shocks are freely observable only by entrepreneurs. Financial intermediaries help overcome the information asymmetry problem by lending to entrepreneurs via a debt contract and monitoring entrepreneurs who default on their debt.

The government collects taxes from households and entrepreneurs to purchase consumption goods from firms. The monetary authority has an explicit consumer

[6] A change in variables is introduced as inflation is positive in steady state (discussed further below) and nominal variables are trending. Let $\frac{\Psi_{t+i}}{P_{t+i}} = \hat{\Psi}_{t+i}$, $\frac{W_t^h}{P_t} = \hat{W}_t^h$, $\frac{W_t^e}{P_t} = \hat{W}_t^e$, $\frac{D_{t+i}}{P_{t-1+i}} = \hat{D}_{t+i}$, $\frac{B_{t+i}^h}{P_{t-1+i}} = \hat{B}_{t+i}^h$ and $\frac{B_{t+i}^{h*}}{P_{t-1+i}^*} = \hat{B}_{t+i}^{h*}$ for $i = 0,\ 1$. Capital letters with a " ˆ " thus denote real values of nominal variables. The variables are defined further in the text.

Table 1: Chronology of events in the agency cost model

Firms import commodity inputs, hire labour and rent capital from households and entrepreneurs to produce consumption goods.
The government and foreign consumers purchase consumption goods.
Households decide how much of their income to hold in financial assets. They purchase an index of consumption goods and decide how much to consume immediately and how much to use to obtain the capital good. For each unit of capital that households wish to acquire, they give $\hat{\Psi}_t$ units of the consumption good index to financial intermediaries.
Financial intermediaries use households' funds to provide loans to entrepreneurs via a debt contract to produce the capital good.
Entrepreneurs borrow from financial intermediaries and place all funds, both loans and their own net worth, into their capital-creating technology.
The idiosyncratic technology shock of each entrepreneur is realised. Entrepreneurs either produce the capital good and repay their loan in capital goods or declare bankruptcy. In the case of bankruptcy financial intermediaries monitor entrepreneurs and absorb any losses.
Entrepreneurs, who are still solvent, make their consumption decision.
Financial intermediaries deliver households' capital goods.

price inflation target and acts to achieve this target by adjusting the rate of interest paid on domestic bonds.

Table 1 summarises the chronology of events.

2.2 Financial intermediaries

The primary role of financial intermediaries is to channel funds from households to entrepreneurs.[7] Households have savings, but no productive uses for them, while entrepreneurs have productive investments, but insufficient funds to carry them out. Financial intermediaries reduce the costs of moving funds between borrowers (investors) and lenders (savers) because they can enhance risk diversification and help overcome an information asymmetry problem, leading to a more efficient allocation of resources. By taking advantage of economies of scale

[7] Financial intermediaries also hold households' demand deposits and issue domestic bonds.

financial intermediaries can promise households a higher payoff relative to the non-intermediated case. To resolve the imperfect information problem between borrowers and lenders financial intermediaries lend to entrepreneurs via a debt contract, an agreement by the borrower to pay the lender a fixed amount. Debt contracts are negotiated at the beginning of each period and resolved by the end of the period.

The information asymmetry arises because entrepreneurs must use external financing to produce the capital good and because their production technology is subject to idiosyncratic shocks. The set-up for financial intermediaries follows Carlstrom and Fuerst (1997). Each entrepreneur i borrows $(IN_t(i) - NW_t(i))$ consumption goods, where $IN_t(i)$ is the size of entrepreneur i's investment project and $NW_t(i)$ is entrepreneur i's net worth or internal funds. After capital is produced loans are repaid in capital goods. Each entrepreneur i has access to a stochastic technology, $\omega_t(i)$, that transforms an input of IN_t consumption goods into $\omega_t(i)\, IN_t$ units of new capital. The random variable $\omega_t(i)$ is assumed to be lognormally distributed across time and entrepreneurs, i.e. $\ln(\omega_t(i)) \sim N\left(\tilde{\mu},\ \tilde{\sigma}^2\right)$, with a mean of unity and a standard deviation of σ. The distribution function and density of $\omega_t(i)$ are given by $\Phi(\omega_t(i))$ and $\phi(\omega_t(i))$.

Agency costs arise because $\omega_t(i)$ can only be observed costlessly by the entrepreneur. Others (lenders) can only observe $\omega_t(i)$ at a monitoring cost of $\alpha IN_t(i)$ capital inputs, i.e. there is costly state verification (Townsend 1979). The information asymmetry creates a moral hazard problem because entrepreneurs have an incentive to underreport their true value of $\omega_t(i)$. The optimal debt contract is structured so that entrepreneurs always truthfully report the value of $\omega_t(i)$.

The optimal contract is risky debt and is characterised by the size of entrepreneur i's project, $IN_t(i)$, and a critical $\omega_t(i)$ that triggers bankruptcy, denoted by $\varpi_t(i)$. If the realisation of the technology shock $\omega_t(i)$ is below the critical $\varpi_t(i)$, the entrepreneur becomes bankrupt and defaults on the debt contract. In the event of default, the financial intermediary monitors the entrepreneur, as in Williamson (1986), occurs the monitoring cost, confiscates all returns from the project and absorbs any losses.

To derive the optimal project size $IN_t(i)$ and the critical $\varpi_t(i)$ that triggers bankruptcy it is convenient to define the functions $f(\varpi)$ and $g(\varpi)$. They are the fractions of the expected net capital output received by the entrepreneur and by the lender. Time and entrepreneur subscripts have been dropped for simplicity. The functions $f(\varpi)$ and $g(\varpi)$ are given by

$$\begin{aligned} f(\varpi) &= \int_{\varpi}^{\infty} (\omega - \varpi)\, d\Phi(\omega) \\ &= \int_{\varpi}^{\infty} \omega d\Phi(\omega) - [1 - \Phi(\varpi)]\, \varpi \end{aligned} \tag{1}$$

and

$$g(\varpi) = \int_{0}^{\varpi} \omega d\Phi(\omega) - \alpha\Phi(\varpi) + [1 - \Phi(\varpi)]\varpi \tag{2}$$

Note that $f(\varpi)$ integrates only over values of ω in excess of ϖ and $g(\varpi)$ integrates over 0 to ϖ. Moreover, the two functions do not sum to one because of expected bankruptcy and monitoring costs, i.e.

$$f(\varpi) + g(\varpi) = 1 - \alpha\Phi(\varpi) \tag{3}$$

The expected net capital output received by the entrepreneur and lender from entrepreneur i's project is given by $f(\varpi_t(i))\hat{\Psi}_t IN_t(i)$ and $g(\varpi_t(i))\hat{\Psi}_t IN_t(i)$, where $\hat{\Psi}_t$ denotes the aggregate price of capital in terms of consumption goods. The optimal contract between the lender and entrepreneur is given by the pair $(IN_t(i),\ \varpi_t(i))$ that maximises the entrepreneur's net capital output subject to the lender being indifferent between loaning the funds to the entrepreneur and retaining them, i.e.

$$\max_{IN_t(i),\ \varpi_t(i)} f(\varpi_t(i))\hat{\Psi}_t IN_t(i) \tag{4}$$

subject to[8]

$$g(\varpi_t(i))\hat{\Psi}_t IN_t(i) \geq IN_t(i) - NW_t(i) \tag{5}$$

The first-order conditions of the optimisation problem are given by

$$\frac{f(\varpi_t(i))}{f'(\varpi_t(i))} = \frac{g(\varpi_t(i))\hat{\Psi}_t - 1}{g'(\varpi_t(i))\hat{\Psi}_t} \tag{6}$$

and

$$IN_t(i) = \frac{NW_t(i)}{1 - g(\varpi_t(i))\hat{\Psi}_t} \tag{7}$$

Using (3) equation (6) can be written as

$$\hat{\Psi}_t \left(1 - \alpha\Phi(\varpi_t(i)) + \frac{\alpha\phi(\varpi_t(i))f(\varpi_t(i))}{f'(\varpi_t(i))}\right) = 1 \tag{8}$$

Equation (8) defines the critical $\varpi_t(i)$ as a function of the aggregate price of capital, $\hat{\Psi}_t$, the distribution of the stochastic technology shock, $\omega_t(i)$, and the monitoring cost, α. The critical $\varpi_t(i)$ is independent of i; that is, all entrepreneurs receive the same basic terms on their debt contract. Contracts only differ in terms of size – entrepreneurs with larger net worth receive a proportionately larger loan (equation 7). This result is important as it overcomes the heterogeneity problem with entrepreneurs that arises from the idiosyncratic technology shock. The result follows from the assumption of linear monitoring costs and investment technology. Variables specific to i can henceforth be interpreted as averages.

Using equation (7), the expected return to internal funds, IR_t, is given by

$$IR_t = \frac{f(\varpi_t)\hat{\Psi}_t IN_t}{NW_t} = \frac{f(\varpi_t)\hat{\Psi}_t}{1 - g(\varpi_t)\hat{\Psi}_t} \tag{9}$$

where $\frac{f(\varpi_t)\hat{\Psi}_t IN_t}{NW_t}$ denotes the expected net capital output received by entrepreneurs per unit of leveraged net worth.

[8] At an optimum, equation (5) holds as an equality.

2.3 Firms

Firms are monopolistic competitors and specialise in production. They produce aggregate output of consumption goods, Y_t, under a constant elasticity of substitution (CES) technology by hiring household and entrepreneurial labour, L_t^h and L_t^e, using capital, K_t, and commodity inputs, IM_t. Commodity inputs, IM_t, are imported at the beginning of each period. Returning to the earlier analogy, firms can be thought of as producing consumption goods, fruit, using trees, labour and commodity imports as production inputs, where commodity imports may be fertilisers.

The aggregate production function is given by

$$\begin{aligned} Y_t = &\left(\eta_l \left(Z_t L_t^h\right)^\nu + \eta_k \left(K_t\right)^\nu \right. \\ &\left. + \eta_{im} \left(IM_t\right)^\nu + \left(1 - \eta_l - \eta_k - \eta_{im}\right) \left(L_t^e\right)^\nu\right)^{\frac{1}{\nu}} \end{aligned} \tag{10}$$

where η_l, η_k, $\eta_{im} \in (0, 1]$ are parameters and $\nu < 1$; that is, the marginal return to each input is diminishing. Z_t denotes aggregate productivity and the elasticity of substitution in production is given by $\frac{1}{1-\nu}$.[9]

The assumption of monopolistic competition in the consumption goods market allows pricing decisions to be determined explicitly and provides a channel for introducing nominal rigidities. A firm treats the price in domestic currency, $P_t(j)$, of the consumption good j it produces as a choice variable, while taking the domestic aggregate price level, P_t, the nominal exchange rate, S_t, and the foreign price level, P_t^*, as given.[10] Having chosen $P_t(j)$, the firm then produces the quantity of output demanded at that price. Firms may not price discriminate and the price of good j sold to foreign consumers (denominated in foreign currency) is given by $\frac{P_t(j)}{S_t}$.

Firms sell their output of consumption goods, Y_t, to domestic households and entrepreneurs and the government. They also export to the rest of the world. Let $C_t^h(j)$, $C_t^e(j)$, $EX_t(j)$ and $G_t(j)$ be the quantity of consumption good j demanded by a typical household, entrepreneur and foreign consumer, and the government, i.e. $Y_t(j) = C_t^h(j) + C_t^e(j) + EX_t(j) + G_t(j)$. It can be shown (e.g. Obstfeld and Rogoff 1996) that the household's, entrepreneur's and government's demand functions for good j are given by

$$C_t^h(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} C_t^h \tag{11}$$

$$C_t^e(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} C_t^e \tag{12}$$

[9] The production function is similar to that in the New Zealand Treasury macroeconomic model. See Szeto (2002) for details.

[10] The nominal exchange rate, S_t, is measured as the price of foreign currency in units of domestic currency, i.e. an increase in S_t indicates a depreciation. S_t is given in period t.

and

$$G_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} G_t \tag{13}$$

where C_t^h, C_t^e and G_t denote total consumption by the typical household and entrepreneur and the government. The aggregate price level, P_t, is an index given by $P_t = \left[\int_0^1 P_t(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}$, where θ is the price elasticity of demand faced by each monopolistic competitive firm. Similarly, foreign demand for consumption good j is given by

$$EX_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} EX_t \tag{14}$$

where EX_t denotes aggregate exports.[11] Aggregate export demand is a function of the real exchange rate, $Q_t \equiv \frac{S_t P_t^*}{P_t}$, and foreign demand for the domestic country's output, Y_t^*.[12, 13] Thus, the economy's aggregate exports are assumed to be given by

$$EX_t = \left(\frac{S_t P_t^*}{P_t}\right)^{\kappa} (Y_t^*)^{\varsigma} \tag{15}$$

where κ, $\varsigma > 0$ are the price and foreign demand elasticities of exports.

Firms choose the optimal value of inputs $\left\{L_t^h,\ L_t^e,\ IM_t,\ K_t\right\}$. In a symmetric equilibrium, all firms charge the same relative price, employ the same labour and use the same capital and commodity inputs. This leads to the following first-order conditions for the typical firm

$$\hat{W}_t^h = \frac{\eta_l (Z_t)^{\nu} \left(\frac{Y_t}{L_t^h}\right)^{1-\nu}}{\xi_t} \tag{16}$$

$$\hat{W}_t^e = \frac{(1-\eta_l-\eta_k-\eta_{im})\left(\frac{Y_t}{L_t^e}\right)^{1-\nu}}{\xi_t} \tag{17}$$

$$R_t = \frac{\eta_k \left(\frac{Y_t}{K_t}\right)^{1-\nu}}{\xi_t} \tag{18}$$

and

$$Q_t = \frac{\eta_{im}\left(\frac{Y_t}{IM_t}\right)^{1-\nu}}{\xi_t} \tag{19}$$

where $\hat{W}_t^h$ and $\hat{W}_t^e$ denote households' and entrepreneurs' real wage rate, R_t is the rental rate of capital and $Y_t = \xi_t \left(\hat{W}_t^h L_t^h + \hat{W}_t^e L_t^e + R_t K_t + Q_t IM_t\right)$. The first-order conditions show that firms sell their output of consumption goods at a mark-up, ξ_t, over production costs and factor prices are below their marginal products. The mark-up, ξ_t, is the ratio of the price level to aggregate marginal cost. Under price flexibility, it is constant and equal to $\frac{\theta}{\theta-1}$.

[11] The exchange rate cancels out in the relative price term.

[12] The domestic economy's exports are assumed to form an insignificant proportion of foreigners' demand and have a negligible weight in the rest of the world's price index.

[13] The real exchange rate, Q_t, is given in period t.

2.4 Households

Households are infinitely lived. A typical household values streams of consumption and leisure according to

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln\left(C_{t+j}^h\right) + \gamma\left(1 - N_{t+j}\right)\right\} \tag{20}$$

where $\gamma > 0$ is a parameter, $\beta \in (0, 1)$ denotes the household's discount factor and C_t^h is an index of household consumption in period t. Time is normalised to one – the household's labour supply is given by N_t and $(1 - N_t)$ is leisure. E_t is the conditional expectations operator with respect to information available at time t.

Each household consumes many goods, all of which are domestically produced. C_t^h is the quantity consumed in period t of an index of these goods with $C_t^h = \left[\int_0^1 C_t^h(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$, where $C_t^h(j)$ denotes the household's period t consumption of good j and $\theta > 0$ (Dixit and Stiglitz 1977).

Households hold financial assets consisting of demand deposits with financial intermediaries, domestic bonds issued by financial intermediaries and foreign bonds.[14] The typical household's financial wealth in real terms, $\hat{A}_t^h$, is given by

$$\hat{A}_t^h = \hat{D}_t + \hat{B}_t^h + Q_t \hat{B}_t^{h*} \tag{21}$$

where $\hat{D}_t$, $\hat{B}_t^h$ and $\hat{B}_t^{h*}$ denote the real stock of demand deposits and domestic and foreign bonds and Q_t is the real exchange rate. In addition, households own capital, K_t^h, that they rent to firms.

Households derive income from three sources. First, the typical household earns wage income, $\hat{W}_t^h N_t$, from supplying labour, N_t, to firms, where $\hat{W}_t^h$ denotes households' real wage rate. Second, households receive interest from holding financial assets. Domestic bonds, $\hat{B}_t^h$, earn a nominal return (in terms of domestic currency) of I_t and the rate of interest paid on foreign bonds, $\hat{B}_t^{h*}$, is given by I_t^*. Demand deposits, $\hat{D}_t$, do not earn any interest – they are held to purchase consumption goods. Third, households earn income from renting their accumulated capital holdings, K_t^h, at rate R_t to firms. Households pay taxes on their wage and rental income. For simplicity, it is assumed that households' interest income and capital gains from exchange rate and capital price movements are not taxed. The tax rate imposed by the government is given by τ.

The typical household's flow constraint in real terms is given by

$$\begin{aligned} &(1-\tau)\hat{W}_t^h N_t + \tfrac{(1+I_t)\hat{A}_t^h}{1+\Pi_t} + \Big((1-\delta)\hat{\Psi}_t + (1-\tau)R_t\Big) K_t^h \\ &-C_t^h - \tfrac{I_t \hat{D}_t}{1+\Pi_t} - \hat{A}_{t+1}^h - \hat{\Psi}_t K_{t+1}^h = 0 \end{aligned} \tag{22}$$

[14] In the analysis domestic bond holdings are assumed to be zero and demand deposits earn no interest. These assumptions are made for simplicity and allow to abstract from financial intermediaries' budget constraint.

where $\frac{I_t \hat{D}_t}{1+\Pi_t}$ denotes the opportunity cost of having to hold demand deposits and δ is the depreciation rate of capital.[15] The flow constraint can be interpreted as follows. Each period, households receive income from supplying labour. They also earn a real return on their financial assets and accumulated capital holdings. Households then sell all their financial assets and capital net of depreciation to purchase consumption goods, new financial assets and capital. The price of capital in terms of consumption goods is given by $\hat{\Psi}_t$. The budget constraint is binding and households' expenditure is equal to their income.

The typical household's deposit-in-advance constraint in real terms is given by

$$C_t^h \leq \hat{D}_t \tag{23}$$

and will hold as an equality, at an optimum, if $I_t > 0$. The deposit-in-advance constraint provides a channel for incorporating monetary policy into the model similar to that in Carlstrom and Fuerst (2000). The typical household's lifetime budget constraint can then be written as

$$\begin{aligned} &\hat{A}_0^h + E_t \sum_{j=0}^{\infty} \frac{1}{\left(\frac{1+I_t}{1+\Pi_t}\right)^j} \{(1-\tau)\, \hat{W}_{t+j}^h N_{t+j} \\ &+ \left((1-\delta)\, \hat{\Psi}_{t+j} + (1-\tau)\, R_{t+j}\right) K_{t+j}^h \\ &- \left(1 + \frac{I_{t+j}}{1+\Pi_{t+j}}\right) C_{t+j} - \hat{\Psi}_{t+j} K_{t+1+j}^h \} = 0 \end{aligned} \tag{24}$$

The household's optimisation problem consists of choosing $\left\{C_t^h,\ N_t,\ K_{t+1}^h\right\}$ for all $t \in [0, \infty)$ to maximise lifetime utility (equation 20) subject to equation (24). The first-order conditions are given by

$$\frac{1}{\gamma C_t^h} - \frac{\left(1+\frac{I_t}{1+\Pi_t}\right)}{(1-\tau)\hat{W}_t^h} = 0 \tag{25}$$

and

$$\frac{\hat{\Psi}_t}{C_t^h\left(1+\frac{I_t}{1+\Pi_t}\right)} - E_t\left[\frac{\beta\left((1-\delta)\hat{\Psi}_{t+1}+(1-\tau)R_{t+1}\right)}{C_{t+1}^h\left(1+\frac{I_{t+1}}{1+\Pi_{t+1}}\right)}\right] = 0 \tag{26}$$

Equation (25) indicates that, at an optimum, the marginal rate of substitution between consumption and leisure is equal to the relative price of consumption; that is, the ratio of the effective price of consumption and the after-tax real wage rate. The effective price of consumption is the sum of its market price (equal to unity) and the opportunity cost of having to hold demand deposits to purchase consumption goods $\left(\frac{I_t}{1+\Pi_t}\right)$. Equation (26) implies that the marginal rate of substitution between consumption today and next period is equal to a unit value of the capital stock net of depreciation plus the after-tax rate of return on capital.

[15] Uncovered interest rate parity is assumed to hold.

2.5 Entrepreneurs

The specification for entrepreneurs follows Carlstrom and Fuerst (1997) except for two differences. First, firms and entrepreneurs are modelled separately because of the assumption of monopolistic competition in the consumption goods market. In Carlstrom and Fuerst (1997) there are no nominal rigidities. Firms are perfectly competitive and owned by entrepreneurs. Second, entrepreneurs consume an index of consumption goods rather than a single good.

Entrepreneurs are infinitely lived and produce the capital (investment) good, which firms use as an input in the production of consumption goods. Or using the earlier analogy, entrepreneurs grow trees, which are a production input to produce fruit. The production of the capital good uses consumption goods; that is, entrepreneurs grow trees from fruit.

Each entrepreneur has access to a stochastic technology, ω_t, that transforms an input of IN_t consumption goods (fruit) into $\omega_t IN_t$ units of new capital (trees). To finance the production of the capital good (trees), entrepreneurs use external financing and their own net worth. External financing is obtained from financial intermediaries and is the proportion of the consumption goods (fruit) that households handed over to financial intermediaries to obtain capital goods (trees), which they rent to firms to produce consumption goods (fruit). Entrepreneurs' net worth consists of their after-tax wage earnings and the market value of their capital stock (trees). A typical entrepreneur's net worth, NW_t, in real terms is given by

$$NW_t = (1-\tau)\,\hat{W}_t^e + \Big((1-\delta)\,\hat{\Psi}_t + (1-\tau)\,R_t\Big)\,K_t^e \tag{27}$$

where $\hat{W}_t^e$ is the real wage rate for entrepreneurial labour, with entrepreneurial labour supply equal to unity, and K_t^e is the entrepreneur's capital stock (trees).[16] The assumption of entrepreneurial labour income ensures that entrepreneurs always have a nonzero level of net worth.

After production of the capital good commences the idiosyncratic technology shock, ω_t, is realised. Entrepreneurs who are still solvent after the shock occurs repay their loans and make their consumption decision. The typical entrepreneur's utility function is given by

$$E_t \sum_{j=0}^{\infty} (\zeta\beta)^j\, C_{t+j}^e \tag{28}$$

where C_t^e is an index of entrepreneurial consumption (of fruit) in period t and $\zeta \in (0,1)$ is an additional discount factor.[17] Entrepreneurs are assumed to discount the future more heavily than households to ensure that they use external

[16] The production of capital goods (trees) uses consumption goods (fruit) and entrepreneurs must first sell their accumulated capital stock (trees) to financial intermediaries for consumption goods (fruit).

[17] As households, each (solvent) entrepreneur consumes many goods and C_t^e is the quantity

financing. Agency costs imply that the return to internal funds is greater than the return to external funds and entrepreneurs have an incentive to postpone all consumption and accumulate internal funds to self-finance.[18]

The typical entrepreneur's budget constraint, after loan repayment in the form of newly created capital (trees), is given by

$$\begin{aligned}&\left((1-\tau)\,\hat{W}_t^e+\left((1-\delta)\,\hat{\Psi}_t+(1-\tau)\,R_t\right)K_t^e\right)\tfrac{f(\varpi_t)\hat{\Psi}_t}{1-g(\varpi_t)\hat{\Psi}_t}\\&-C_t^e-\hat{\Psi}_tK_{t+1}^e=0\end{aligned}\tag{29}$$

Note that, as in the case of households, entrepreneurs' wage earnings and return to capital are taxed but capital gains from capital price movements are not. Equation (29) states that the entrepreneur's net worth, $(1-\tau)\,\hat{W}_t^e+((1-\delta)\,\hat{\Psi}_t+(1-\tau)\,R_t)K_t^e$, earns an expected return to internal funds of $\frac{f(\varpi_t)\hat{\Psi}_t}{1-g(\varpi_t)\hat{\Psi}_t}$. The entrepreneur then sells a proportion of this newly created capital (trees) to financial intermediaries to purchase consumption goods (fruit), C_t^e, and K_{t+1}^e denotes the entrepreneurial capital (trees) left after consumption.

The entrepreneur's optimisation problem consists of choosing $\left\{C_t^e,\ K_{t+1}^e\right\}$ for all $t\in[0,\infty)$ to maximise lifetime utility (equation 28) subject to equation (29). The entrepreneur's first-order condition is given by

$$\hat{\Psi}_t=E_t\left[\frac{\zeta\beta\left((1-\delta)\hat{\Psi}_{t+1}+(1-\tau)R_{t+1}\right)f(\varpi_{t+1})\hat{\Psi}_{t+1}}{1-g(\varpi_{t+1})\hat{\Psi}_{t+1}}\right]\tag{30}$$

The term $\frac{f(\varpi_{t+1})\hat{\Psi}_{t+1}}{1-g(\varpi_{t+1})\hat{\Psi}_{t+1}}$ in equation (30) is the gross expected return on internal funds and is greater than one. It is this additional return that encourages entrepreneurs to accumulate capital (trees) even though they discount the future more than households. To avoid self-financing, in the calibration ζ is set to offset the steady state internal return, i.e. $\frac{\zeta f(\bar{\varpi})\bar{\Psi}}{1-g(\bar{\varpi})\bar{\Psi}}=1$.[19]

2.6 Government

The government's budget constraint is given by

$$\tau\left(\hat{W}_t^hL_t^h+\hat{W}_t^eL_t^e+R_tK_t\right)-G_t=0\tag{31}$$

The government collects taxes on households' and entrepreneurs' wage and rental income, $\tau\left(\hat{W}_t^hL_t^h+\hat{W}_t^eL_t^e+R_tK_t\right)$. It uses this revenue to purchase an index of

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consumed in period t of an index of these goods with $C_t^e=\left[\int\limits_0^1 C_t^e\,(j)^{\frac{\theta-1}{\theta}}\,dj\right]^{\frac{\theta}{\theta-1}}$, where $C_t^e\,(j)$ denotes the typical (solvent) entrepreneur's period t consumption of good j.

[18]With no external financing, agency costs disappear. The additional discount factor avoids this outcome.

[19]Letters with a " $-$ " indicate (average) steady state levels.

consumption goods, G_t, from firms.[20] For simplicity, the government's budget constraint is assumed to balance in each period, i.e. there is no debt financing.

2.7 Monetary authority

The monetary authority has an explicit consumer price inflation target, Π^T. To maintain this target following a shock to the economy the central bank adjusts the nominal rate of interest paid on domestic bonds, I_{t+1}. Its reaction function is based on a variant of the Taylor rule (Taylor 1993) and depends on deviations of inflation from target and deviations of output from full capacity flexible price output as in a Taylor rule, and the previous interest rate, I_t. Full capacity flexible price output and the central bank's reaction function are discussed further in section 4.

2.8 Market clearing and equilibrium conditions

There are four domestic markets in the economy: two labour markets, a consumption goods market and a capital goods market. The clearing conditions are given by

$$L_t^h = (1-\eta) N_t \tag{32}$$

$$L_t^e = \eta \tag{33}$$

$$Y_t = (1-\eta) C_t^h + \eta C_t^e + G_t + EX_t + \eta IN_t \tag{34}$$

$$K_{t+1} = (1-\delta) K_t + \eta IN_t (1 - \alpha\Phi(\varpi_t)) \tag{35}$$

Moreover, uncovered interest rate parity holds

$$1 + I_{t+1} = E_t \left[\left(1 + I_{t+1}^*\right) \tfrac{S_{t+2}}{S_{t+1}} \right] \tag{36}$$

and the sequences of foreign interest rates, prices, inflation and foreign demand $\{I_t^*,\ P_t^*,\ \Pi_t^*,\ Y_t^*\}$ are given to the small open economy. For simplicity, it is assumed that all households' bond holdings are in the form of foreign securities, i.e. $\hat{B}_t^h = 0$ for all t.

3 The steady state model

This section derives the steady state model with and without agency costs. Parameter values are chosen so that the steady state of the agency cost model is broadly consistent with New Zealand data and/or assumptions made in the literature. A period in the model is assumed to correspond to one quarter and the following parameters are chosen.

[20] G_t is the quantity consumed by the government in period t of an index of consumption goods with $G_t = \left[\int_0^1 G_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$, where $G_t(j)$ denotes the government's period t consumption of good j.

Households' discount rate, β, equals 0.9902 and leads to an annual steady state, pre-tax real domestic interest rate of 4 percent. The coefficient on leisure, γ, in households' utility function is chosen so that work effort accounts for a third of the time endowment in steady state. The ratio of entrepreneurs to households, η, is arbitrarily set to 0.1.

Labour-augmenting productivity, $\bar{Z}$, is normalised to 1 in steady state. The elasticity of substitution between labour, capital and commodity inputs, $\frac{1}{1-\nu}$, is set to $\frac{1}{1.1}$ to approximate the consumption production technology assumed in Carlstrom and Fuerst (1997), which is Cobb-Douglas. The coefficients on household labour, η_l, capital, η_k, and commodity inputs, η_{im}, in firms' production function are 0.5631, 0.3168 and 0.12 respectively. These assumptions are broadly in line with the New Zealand input-output data for 1995-96 and yield a steady state ratio of imports to output of about 0.12, the same as in McCallum and Nelson (1999). The capital depreciation rate is set to 8.5 percent per annum, the same as in the Reserve Bank of New Zealand' macroeconomic model.[21] Firms' mark-up in steady state is 20 percent ($\frac{\theta}{\theta-1} = 1.2$), i.e. $\theta = 6$, the same as in McCallum and Nelson (1999).

The assumptions for entrepreneurs' discount factor and for financial intermediaries are the same as in Carlstrom and Fuerst (1997). Entrepreneurs' extra discount factor, ζ, is 0.947. The monitoring cost, α, is set to 0.25. The bankruptcy rate, $\Phi(\bar{\varpi})$, is 0.974 percent per quarter and the standard deviation of the idiosyncratic technology shocks, σ, is 0.207.[22]

The annual domestic steady state inflation rate is equal to the Reserve Bank of New Zealand's inflation target rate, Π^T, of 2 percent, which is the mid-point of the 1 to 3 percent target band for inflation. The tax rate, τ, equals 17 percent in line with the income tax assumption in the Reserve Bank's model.

For simplicity, the steady state foreign inflation rate, $\bar{\Pi}^*$, and nominal bond rate, $\bar{I}^*$, are assumed to be the same as for the domestic economy and the steady state real exchange rate, $\bar{Q}$, is normalised to 1. The price and foreign demand elasticities of exports, κ and ς, are equal to unity, as in McCallum and Nelson (2001) and foreign demand is chosen to yield a steady state ratio of exports to output of 0.11, the same as in McCallum and Nelson (1999).

With these assumptions the steady state agency cost model can be solved as follows:

$$\bar{L}^h = \frac{0.3(1-\eta)}{\eta} \tag{37}$$

$$\Phi(\bar{\varpi}) = 0.00974 \tag{38}$$

$$\bar{\Psi} = \frac{1}{1-\alpha\Phi(\bar{\varpi}) + \frac{\alpha\phi(\bar{\varpi})f(\bar{\varpi})}{f'(\bar{\varpi})}} \tag{39}$$

[21] See Black et al. (1997) for details.

[22] The standard deviation and entrepreneurs' discount factor imply an annual risk premium of 187 basis points. The steady state quarterly risk premium is given by $\frac{\bar{\Psi}\bar{\varpi}\bar{IN}}{\bar{IN}-\bar{NW}} - 1$.

$$\bar{IR} = \frac{f(\bar{\varpi})\bar{\Psi}}{1-g(\bar{\varpi})\bar{\Psi}} - 1 \tag{40}$$

$$\bar{R} = \frac{\frac{\bar{\Psi}}{\beta}-1}{1-\tau} \tag{41}$$

Using

$$\bar{K} = \bar{Y}\left(\frac{\left(\left(1+(1-\tau)\bar{R}\right)-(1-\delta)\bar{\Psi}\right)\frac{\theta}{\theta-1}}{(1-\tau)\eta_k}\right)^{\frac{1}{\nu-1}} \tag{42}$$

$$\bar{IM} = \bar{Y}\left(\frac{\frac{\theta}{\theta-1}\bar{Q}}{\eta_{im}}\right)^{\frac{1}{\nu-1}} \tag{43}$$

and

$$\bar{Y}^{\nu} = \eta_l\left(\bar{Z}\bar{L}^h\right)^{\nu} + \eta_k\bar{K}^{\nu} + \eta_{im}\bar{IM}^{\nu} + (1-\eta_l-\eta_k-\eta_{im}) \tag{44}$$

$\bar{Y}$ can be derived as

$$\bar{Y} = \left(\frac{\eta_l\left(\bar{Z}\bar{L}^h\right)^{\nu}+(1-\eta_l-\eta_k-\eta_{im})}{1-\eta_k\left(\frac{\left((1+(1-\tau)\bar{R})-(1-\delta)\bar{\Psi}\right)\frac{\theta}{\theta-1}}{(1-\tau)\eta_k}\right)^{\frac{\nu}{\nu-1}}-\eta_{im}\left(\frac{\frac{\theta}{\theta-1}\bar{Q}}{\eta_{im}}\right)^{\frac{\nu}{\nu-1}}}\right)^{\frac{1}{\nu}} \tag{45}$$

$$\bar{IN} = \frac{\delta\bar{K}}{1-\alpha\Phi(\bar{\varpi})} \tag{46}$$

$$\bar{NW} = \bar{IN}\left(1-g\left(\bar{\varpi}\right)\bar{\Psi}\right) \tag{47}$$

$$\bar{K}^e = \frac{\bar{NW}-\frac{(1-\tau)(1-\eta_l-\eta_k-\eta_{im})\bar{Y}^{1-\nu}}{\frac{\theta}{\theta-1}}}{\left(1+(1-\tau)\bar{R}\right)} \tag{48}$$

$$\bar{C}^e = \bar{NW}\left(1+\bar{IR}\right) - \bar{\Psi}\bar{K}^e \tag{49}$$

$$\bar{EX} = 0.11\cdot\bar{Y} \tag{50}$$

$$\bar{G} = \tau\left(\frac{\bar{Y}-\eta_{im}\bar{Y}^{1-\nu}\bar{IM}^{\nu}}{\frac{\theta}{\theta-1}}\right) \tag{51}$$

$$\bar{C}^h = \frac{\eta\left(\bar{Y}-\bar{C}^e-\bar{G}-\bar{EX}-\bar{IN}\right)}{(1-\eta)} \tag{52}$$

$$\bar{C} = \frac{(1-\eta)\bar{C}^h}{\eta} + \bar{C}^e \tag{53}$$

The steady state values for [K_{t+1}, K^e_{t+1}, NW_t, IN_t, C^h_t, C^e_t, C_t, G_t, EX_t, Y_t, L^h_t, IM_t, $\hat{\Psi}_t$, R_t, IR_t] are summarised in Table 2. Column (1) reports the results for the agency cost model.

The steady state ratios of aggregate (household and entrepreneurial) consumption, government consumption, investment and exports to output are 0.6477, 0.1207, 0.1216 and 0.11 respectively. The ratio of imports to steady state output is 0.1233. These ratios are lower than in the Reserve Bank of New Zealand's

Table 2: Numerical steady state of the agency cost model

		agency cost model (1)	no agency costs (2)	difference (3)
$\bar{K}$	capital	23.9179	24.6722	3.1536
$\bar{K}^e$	entrepreneurial capital	0.1820	0.0000	-100.0000
$\bar{NW}$	entrepreneurial net worth	0.1885	0.0000	-100.0000
$\bar{IN}$	investment	0.4940	0.5084	2.9024
$\bar{C}^h$	household consumption	0.2911	0.2943	1.0954
$\bar{C}^e$	entrepreneurial consumption	0.0127	0.0000	-100.0000
$\bar{C}$	aggregate consumption	2.6323	2.6483	0.6080
$\bar{G}$	government consumption	0.4905	0.4953	0.9706
$\bar{EX}$	exports	0.4470	0.4514	0.9706
$\bar{Y}$	output	4.0638	4.1033	0.9706
$\bar{L}^h$	household labour	2.7000	2.7000	0.0000
$\bar{IM}$	imports	0.5010	0.5059	0.9706
$\bar{\Psi}$	price of capital	1.0238	1.0000	-2.3255
$\bar{R}$	rental rate of capital	0.0408	0.0119	-2.8968
$\bar{IR}$	return to internal funds	0.0559	*n/a*	*n/a*
$\bar{C}/\bar{Y}$		0.6477	0.6454	-0.2326
$\bar{G}/\bar{Y}$		0.1207	0.1207	0.0000
$\bar{IN}/\bar{Y}$		0.1216	0.1239	0.2326
$\bar{EX}/\bar{Y}$		0.1100	0.1100	0.0000
$\bar{IM}/\bar{Y}$		0.1233	0.1233	0.0000
$\bar{NW}/\bar{IN}$		0.3816	*n/a*	*n/a*
$\bar{C}^e/\bar{NW}$		0.0673	*n/a*	*n/a*

Note: $\bar{K}$, $\bar{K}^e$, $\bar{NW}$, $\bar{IN}$, $\bar{C}^h$, $\bar{C}^e$, $\bar{C}$, $\bar{G}$, $\bar{EX}$, $\bar{Y}$, $\bar{L}^h$ and $\bar{IM}$ denote steady state averages. All variables are reported at quarterly rates. All differences are in percent except for $\bar{R}$, $\bar{C}/\bar{Y}$, $\bar{G}/\bar{Y}$, $\bar{IN}/\bar{Y}$, $\bar{EX}/\bar{Y}$ and $\bar{IM}/\bar{Y}$, which are in percentage points.

macroeconomic model, in part, because in this model all imports are intermediate goods whereas in the Reserve Bank's model a proportion of imports is for final demand.

The internal financing percentage, i.e. the ratio of net worth to investment, at 0.3816 and the ratio of entrepreneurial consumption to net worth at 0.0673 are in line with the results in Carlstrom and Fuerst (1997). The price of capital, the rental rate of capital and the rate of return to internal funds are 1.0238, 0.0408 and 0.0559, the same as in Carlstrom and Fuerst (1997).

Next, the steady state model is solved without agency costs to assess the long-run real effects of asymmetric information between borrowers and lenders. Agency costs arose because entrepreneurs' production technology is stochastic and because they must use external financing. Agency costs disappear when entrepreneurs' production process becomes certain. With no idiosyncratic technology shocks entrepreneurs no longer become bankrupt and default on their debt. Financial intermediaries' monitoring costs become zero and entrepreneurs can borrow directly from households. In fact, entrepreneurs obtain all external financing from households and do not need to accumulate net worth. As a result, entrepreneurs' capital is zero. Wage earnings and consumption are also set to zero for simplicity. Hence, the steady state model (equations 37 to 53) with no agency costs can be solved by setting $\alpha = \bar{NW} = \bar{C}^e = \bar{W}^e = \bar{K}^e = 0$. Moreover, the price of capital is the same as for consumption, i.e. $\bar{\Psi} = 1$. The results are reported in Table 2, column (2). Column (3) reports the differences between the model without agency costs and the agency cost model.

Without agency costs and net worth, the price and rental rate of capital are lower, leading to a rise in the steady state level of investment and capital. Steady state investment is about 3 percent and capital about 3.2 percent higher than in the agency cost model. Output rises by about 1 percent. The increase in output raises households' wages – households' labour is assumed constant. Households' rental income also increases despite the fall in the return to capital because of a larger capital stock. The increase in households' income raises aggregate consumption. Government consumption is also higher due to an increase in tax revenue. Commodity imports, which are an input into production, increase with the rise in output and steady state exports, which are a fixed proportion of output, are also higher.

The steady state comparisons of the model with and without agency costs lead to the following conclusions. Agency costs arise from an information asymmetry between borrowers and lenders and increase the cost of external financing. A reduction in agency costs lowers the price and rental rate of capital, leading to a rise in the long-run level of steady state capital, investment, consumption and output.

4 The dynamic model

Next, the impact of agency costs on business cycle fluctuations is assessed. To evaluate the effects, the economy is subjected to aggregate shocks and the adjustment paths of the agency cost model back to steady state are compared to those of the model without agency costs. The dynamic responses of the two models are derived in terms of logarithmic deviations from steady state (denoted by lower case letters).

Following an aggregate shock firms adjust prices and the monetary authority adjusts interest rates as inflation deviates from target and the economy operates above or below full capacity output. The price adjustment follows Calvo (1983) and is assumed to be sluggish. Each period there is a constant probability, φ, that firms can adjust their prices. This leads to the following price adjustment equation

$$E_t\left[\pi_{t+1}\right] = \beta E_t\left[\pi_{t+2}\right] + \varrho\left(y_t - \bar{y}_t\right) \tag{54}$$

where $\varrho = \frac{\varphi(1-(1-\varphi)\beta)}{(1-\varphi)\theta}$ and $\bar{y}_t$ is the log level of aggregate flexible price output of consumption goods. π_t is given in period t and $E_t\left[\pi_{t+1}\right]$ is the inflation rate following an unanticipated shock to the economy. Equation (54) thus states that inflation is a function of expected future inflation and the output gap, defined as deviations of output from full capacity, $y_t - \bar{y}_t$. In the dynamic analysis below the probability that firms can adjust prices is set to 0.33, i.e. prices remain unchanged on average for three quarters.

Full capacity, flexible price output is derived next.[23] It is the total domestic output of consumption goods that would be produced under price flexibility, i.e. in the absence of any restrictions of adjusting prices. Under price flexibility, aggregate output of consumption goods, $\bar{Y}_t$, is given by

$$\begin{aligned}\bar{Y}_t = &\left(\eta_l\left(Z_t\bar{L}_t^h\right)^\nu + \eta_k\left(\bar{K}_t\right)^\nu + \eta_{im}\left(I\bar{M}_t\right)^\nu\right.\\ &\left.+\left(1-\eta_l-\eta_k-\eta_{im}\right)\eta^\nu\right)^{\frac{1}{\nu}}\end{aligned} \tag{55}$$

where $\bar{L}_t^h$, $\bar{K}_t$ and $I\bar{M}_t$ denote the flexible price level of household labour, capital and commodity imports and $\bar{L}_t^e = \eta$ for all t. Log-linearising equation (55) yields

$$\begin{aligned}\bar{y}_t = &\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^\nu z_t + \eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^\nu \bar{l}_t^h + \eta_k\left(\frac{\bar{K}}{\bar{Y}}\right)^\nu \bar{k}_t\\ &+\eta_{im}\left(\frac{I\bar{M}}{\bar{Y}}\right)^\nu i\bar{m}_t\end{aligned} \tag{56}$$

The log level of flexible price household labour, $\bar{l}_t^h$, can be derived from households' first-order condition that the marginal utility of leisure is equal to the after-tax real wage rate and firms' first-order condition determining labour demand (equation 16). It is given by

$$\bar{l}_t^h = \bar{y}_t + \frac{\nu}{1-\nu} z_t \tag{57}$$

[23] The derivation partly follows McCallum and Nelson (1999).

where the mark-up ξ_t is constant and equal to $\frac{\theta}{\theta-1}$. The log levels of flexible price capital, $\bar{k}_t$, and commodity imports, $\bar{im}_t$, can be derived from firms' first-order conditions (18) and (19) with ξ_t equal to the constant mark-up $\frac{\theta}{\theta-1}$. They are given by

$$\bar{k}_t = \bar{y}_t - \tfrac{1}{1-\nu} r_t \tag{58}$$

and

$$\bar{im}_t = \bar{y}_t - \tfrac{1}{1-\nu} q_t \tag{59}$$

Substituting (57), (58) and (59) into (56) yields the flexible price log output of consumption goods

$$\bar{y}_t - \tfrac{1}{1-\nu} z_t + \frac{\eta_k \left(\frac{\bar{K}}{\bar{Y}}\right)^{\nu}}{\eta_l (1-\nu) \left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}} r_t + \frac{\eta_{im} \left(\frac{\bar{IM}}{\bar{Y}}\right)^{\nu}}{\eta_l (1-\nu) \left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}} q_t = 0 \tag{60}$$

Equation (60) states that the flexible price level of log output is a function of labour-augmenting productivity, the rental rate of capital and the real exchange rate.

Following a shock to the economy the monetary authority adjusts nominal interest rates according to the following reaction function

$$i_{t+1} - \mu_1 \pi_t - \mu_2 \left(y_t - \bar{y}_t\right) - \mu_3 i_t + \epsilon_{i,t} = 0 \tag{61}$$

where $\epsilon_{i,t} \sim i.i.d.\ N\left(0; \sigma_i^2\right)$ is an exogenous shock that can be interpreted as a policy error. The coefficients on inflation, the output gap and past interest rate are given by $\mu_1 = 1.5$, $\mu_2 = 0.5$ and $\mu_3 = 0.8$. The choice for μ_1 and μ_2 is based on the parameter values in a Taylor rule (Taylor 1993).[24] The coefficient on the lagged interest rate is the same as in McCallum and Nelson (1999) and in line with estimates for New Zealand by Huang, Margaritis and Mayes (2001), who find strong evidence of interest rate smoothing.

The rest of the dynamic model is described by the following equations:

$$(1-\nu) y_t - (1-\nu) l_t^h - c_t^h + \nu z_t + \tfrac{1}{\theta} \left(y_t - \bar{y}_t\right) - i_t + \pi_t = 0 \tag{62}$$

$$\bar{K} k_{t+1} - (1-\delta) \bar{K} k_t - \bar{IN} \left(1 - \alpha \Phi\left(\bar{\varpi}\right)\right) in_t + \bar{IN} \alpha \phi\left(\bar{\varpi}\right) \bar{\varpi} \boldsymbol{\varpi}_t = 0 \tag{63}$$

$$\begin{aligned} &\frac{(1-\tau)(1-\nu)\eta_k \left(\frac{\bar{Y}}{\bar{K}}\right)^{1-\nu}}{\frac{\theta}{\theta-1}} y_t + (1-\delta) \bar{\Psi} \psi_t - \frac{(1-\tau)(1-\nu)\eta_k \left(\frac{\bar{Y}}{\bar{K}}\right)^{1-\nu}}{\frac{\theta}{\theta-1}} k_t \\ &+ \frac{(1-\tau)\eta_k \left(\frac{\bar{Y}}{\bar{K}}\right)^{1-\nu}}{\frac{\theta^2 \bar{\Psi}}{\theta-1}} \left(y_t - \bar{y}_t\right) - \left(1 + (1-\tau) \bar{R}\right) r_t = 0 \end{aligned} \tag{64}$$

$$\frac{(1-\eta)\bar{C}^h}{\eta} c_t^h + \bar{C}^e c_t^e + \bar{G} g_t + \bar{EX} ex_t + \bar{IN} in_t - \bar{Y} y_t = 0 \tag{65}$$

$$\tfrac{1}{\bar{\Psi}} \psi_t + \tfrac{f(\bar{\varpi})}{f'(\bar{\varpi})} \left(\tfrac{\phi'(\bar{\varpi})}{\phi(\bar{\varpi})} - \tfrac{f''(\bar{\varpi})}{f'(\bar{\varpi})}\right) \alpha \phi\left(\bar{\varpi}\right) \bar{\varpi} \boldsymbol{\varpi}_t = 0 \tag{66}$$

[24] The original Taylor rule does not include the lagged interest rate.

$$ir_t + nw_t - \psi_t - \frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}\boldsymbol{\varpi}_t - in_t = 0 \tag{67}$$

$$\begin{aligned}&\frac{(1-\tau)(1-\nu)(1-\eta_l-\eta_k-\eta_{im})\bar{Y}^{1-\nu}}{\frac{\theta}{\theta-1}}y_t + \frac{(1-\tau)(1-\eta_l-\eta_k-\eta_{im})\bar{Y}^{1-\nu}}{\frac{\theta^2}{\theta-1}}\left(y_t - \bar{y}_t\right) \\ &+ \left(1 + (1-\tau)\bar{R}\right)\bar{K}^e k_t^e + \left(1 + (1-\tau)\bar{R}\right)\bar{K}^e r_t - \bar{NW} nw_t = 0\end{aligned} \tag{68}$$

$$\bar{NW}\left(1 + \bar{IR}\right)nw_t + \bar{NW}\left(1 + \bar{IR}\right)ir_t - \bar{C}^e c_t^e - \bar{\Psi}\bar{K}^e\psi_t - \bar{\Psi}\bar{K}^e k_{t+1}^e = 0 \tag{69}$$

$$\frac{1}{1-g(\bar{\varpi})\bar{\Psi}}\psi_t + \left(\frac{g'(\bar{\varpi})\bar{\Psi}}{1-g(\bar{\varpi})\bar{\Psi}} + \frac{f'(\bar{\varpi})}{f(\bar{\varpi})}\right)\bar{\varpi}\boldsymbol{\varpi}_t - ir_t = 0 \tag{70}$$

$$\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^\nu z_t + \eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^\nu l_t^h + \eta_k\left(\frac{\bar{K}}{\bar{Y}}\right)^\nu k_t + \eta_{im}\left(\frac{\bar{IM}}{\bar{Y}}\right)^\nu im_t - y_t = 0 \tag{71}$$

$$\begin{aligned}&\frac{\tau\left(\bar{Y}-\eta_{im}(1-\nu)\bar{Y}^{1-\nu}\bar{IM}^\nu\right)}{\frac{\theta}{\theta-1}}y_t + \frac{\tau\left(\bar{Y}-\eta_{im}\bar{Y}^{1-\nu}\bar{IM}^\nu\right)}{\frac{\theta^2}{\theta-1}}\left(y_t - \bar{y}_t\right) \\ &-\frac{\tau\eta_{im}\nu\bar{Y}^{1-\nu}\bar{IM}^\nu}{\frac{\theta}{\theta-1}}im_t - \bar{G}g_t = 0\end{aligned} \tag{72}$$

$$ex_t - \kappa q_t - \varsigma y_t^* = 0 \tag{73}$$

$$q_t - (1-\nu)\,y_t - \frac{1}{\theta}\left(y_t - \bar{y}_t\right) + (1-\nu)\,im_t = 0 \tag{74}$$

$$c_t^h + i_t - \pi_t - \psi_t + E_t\left[r_{t+1}\right] - E_t\left[c_{t+1}^h\right] - i_{t+1} + E_t\left[\pi_{t+1}\right] = 0 \tag{75}$$

$$E_t\left[r_{t+1}\right] + E_t\left[ir_{t+1}\right] - \psi_t = 0 \tag{76}$$

$$i_{t+1} - E_t\left[i_{t+1}^*\right] - E_t\left[s_{t+2}\right] + E_t\left[s_{t+1}\right] = 0 \tag{77}$$

$$E_t\left[q_{t+2}\right] - E_t\left[q_{t+1}\right] - E_t\left[s_{t+2}\right] + E_t\left[s_{t+1}\right] + E_t\left[\pi_{t+2}\right] - E_t\left[\pi_{t+2}^*\right] = 0 \tag{78}$$

The system of log-linearised equations (54, 60, 61 and 62 to 78) can be solved with the method of undetermined coefficients, using Uhlig's (1997) procedures for MATLAB. Using Uhlig's (1997) notation, the dynamic stochastic model can be described as follows. The vector of endogenous state variables is given by $\mathbf{x}_{t+1}$ containing capital, k_{t+1}, entrepreneurial capital, k_{t+1}^e, the domestic bond rate, i_{t+1}, inflation, π_{t+1}, and the nominal and real exchange rates, s_{t+1} and q_{t+1}. The vector of endogenous variables is given by $\mathbf{y}_t$ and contains entrepreneurial net worth, nw_t, investment, in_t, household and entrepreneurial consumption, c_t^h and c_t^e, government consumption, g_t, exports, ex_t, output, y_t, household labour, l_t^h, imports, im_t, the price of capital, ψ_t, the rental rate of capital, r_t, the rate of return to internal funds, ir_t, the bankruptcy rate, $\boldsymbol{\varpi}_t$, and flexible price output, $\bar{y}_t$. The vector of exogenous state variables is given by $\mathbf{z}_t$ and discussed further in the next section.

The following equilibrium relationships between $\mathbf{x}_{t+1}$, $\mathbf{y}_t$ and $\mathbf{z}_t$ are assumed

$$0 = A\mathbf{x}_{t+1} + B\mathbf{x}_t + C\mathbf{y}_t + D\mathbf{z}_t \tag{79}$$

$$0 = E_t\left[F\mathbf{x}_{t+2} + G\mathbf{x}_{t+1} + H\mathbf{x}_t + J\mathbf{y}_{t+1} + K\mathbf{y}_t + L\mathbf{z}_{t+1} + M\mathbf{z}_t\right] \tag{80}$$

$$\mathbf{z}_{t+1} = N\mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1};\ E_t\left[\boldsymbol{\varepsilon}_{t+1}\right] = 0 \tag{81}$$

where C is of size 15×14 and has full rank. F is of size 5×6 and the matrix N is assumed to only have stable eigenvalues. The next step is to solve for the recursive equilibrium law of motion given by

$$\mathbf{x}_{t+1} = P\mathbf{x}_t + Q\mathbf{z}_t \tag{82}$$

$$\mathbf{y}_t = R\mathbf{x}_{t+1} + S\mathbf{z}_t \tag{83}$$

The coefficient matrices P, Q, R and S can be calculated using the method of undermined coefficients. If there exists a recursive equilibrium law of motion that solves equations (79) to (81), then the coefficient matrices, P, Q, R and S, can be found as follows:

1. P satisfies the matrix quadratic equations

$$C^0 AP + C^0 B = 0 \tag{84}$$

$$\begin{aligned} &\left(F - JC^{+}A\right)P^2 - \left(JC^{+}B - G + KC^{+}A\right)P \\ &-KC^{+}B + H = 0 \end{aligned} \tag{85}$$

   where C^+ denotes the pseudo-inverse of C and C^0 is a 1×15 matrix, whose row forms a basis of the null space of C'. The equilibrium described by the recursive equilibrium law of motion (equations 82 and 83) and equation (81) is stable if and only if all eigenvalues of P are smaller than unity in absolute value.

2. R is given by

$$R = -C^{+}\left(AP + B\right) \tag{86}$$

3. Given P and R, let V

$$V = \begin{bmatrix} I_k \otimes A, & I_k \otimes C \\ N' \otimes F + I_k \otimes \left(FP + JR + G\right), & N' \otimes J + I_k \otimes K \end{bmatrix} \tag{87}$$

   where I_k is the identity matrix of size $k \times k$ and k is the number of exogenous state variables. Then

$$V = \begin{bmatrix} vec\left(Q\right) \\ vec\left(S\right) \end{bmatrix} = - \begin{bmatrix} vec\left(D\right) \\ vec\left(LN + M\right) \end{bmatrix} \tag{88}$$

   where $vec\left(.\right)$ denotes columnwise vectorisation. The solution for Q and S is obtained by multiplying equation (88) with V^{-1} if V is invertible.

5 Impulse response analysis

The solution to the recursive equilibrium law of motion can be used to evaluate the dynamic properties of the model with and without agency costs via impulse response analysis. Four types of aggregate shock are considered: a shock to domestic productivity, foreign demand, the foreign interest rate and monetary policy. The response of the endogenous variables to an unanticipated shock in these variables can be traced out using equations (82) and (83), under the assumption that there are no other shocks. If the economy starts out in equilibrium and there are no other shocks, $\mathbf{x}_0 = 0$ if the shock occurs in period 1, where $\mathbf{x}_0$ is the vector of deviations from steady state of the endogenous state variables in period 0.

All shocks are assumed normally distributed. Productivity, z_t, foreign demand, y_t^*, and the foreign interest rate, i_t^*, are univariate exogenous processes and evolve according to

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t}, \quad \text{where} \quad \epsilon_{z,t} \sim i.i.d.\ N\left(0; \sigma_z^2\right) \tag{89}$$

$$y_t^* = \rho_{y^*} y_{t-1}^* + \epsilon_{y^*,t}, \quad \text{where} \quad \epsilon_{y^*,t} \sim i.i.d.\ N\left(0; \sigma_{y^*}^2\right) \tag{90}$$

$$i_t^* = \rho_{i^*} i_{t-1}^* + \epsilon_{i^*,t}, \quad \text{where} \quad \epsilon_{i^*,t} \sim i.i.d.\ N\left(0; \sigma_{i^*}^2\right) \tag{91}$$

The choice of shock parameters follows McCallum and Nelson (2001), except for the foreign demand shock. McCallum and Nelson (2001) assume that the foreign demand shock is a random walk. Here, the shock is temporary and the autocorrelation coefficient is the same as for McCallum and Nelson's (2001) risk premium shock, i.e. $\rho_{y^*} = 0.5$. The parameters for the foreign interest rate shock are the same as for McCallum and Nelson's (2001) risk premium shock. The autocorrelation coefficient of the productivity shock, ρ_z, is 0.95 and the innovation variances are assumed to be given by $\sigma_z^2 = (0.007)^2$, $\sigma_{y^*}^2 = (0.02)^2$ and $\sigma_{i^*}^2 = (0.04)^2$. The standard deviation of the monetary policy shock is 0.8 percent per annum.

The impulse responses of the endogenous variables to a shock in productivity, foreign demand, the foreign interest rate and the domestic interest rate are plotted in Figures 1 to 4 as percent deviations from steady state together with the respective shock. The solid line shows the responses of the variables in the agency cost model and the dotted line is the economy without agency costs. All variables eventually return to steady state.

5.1 Productivity shock

Following a positive shock to aggregate productivity, investment, imports and output rise instantaneously in the model without agency costs and then slowly return to steady state as productivity starts declining (Figure 1). Household labour also increases sharply following the labour-augmenting productivity shock but with a lag. In the agency cost model, the dynamics of these variables are different because of the behaviour of net worth. The positive shock to productivity raises entrepreneurs' wage and rental income. However, net worth only increases with a lag because entrepreneurs' capital is initially fixed (see equation 69). Subsequently, entrepreneurial capital rises as increased demand for capital pushes up the price of capital and thus the return to internal funds.[25] The increase in the return to internal funds also leads entrepreneurs to reduce their consumption, accelerating further the accumulation of capital and net worth. The delayed increase in net worth causes a hump-shaped response in investment, imports and output. The response in household employment is also hump-shaped following an initial decline following the labour-augmenting productivity shock.[26]

Another difference between the agency cost model and the model without agency costs is in the adjustment path of the interest rate. In the agency cost model, the central bank lowers the nominal interest rate by more than without agency costs. This is because the output gap is more negative, putting additional downward pressure on inflation. The larger negative output gap arises because in the agency cost model full capacity, flexible price output increases faster than actual output – actual output adjusts more slowly due to the hump-shaped response.

Finally, the adjustment paths of the exchange rate and exports differ in the model with and without agency costs. In both models, the (real) exchange rate appreciates following the positive productivity shock, leading to an increase in the price of exports and a decline in foreign demand. However, in the agency cost model, the real exchange rate appreciates by less and the fall in exports is smaller because of a larger decline in the domestic interest rate.

5.2 Foreign demand shock

The responses in the agency cost model and the model without agency costs to a foreign demand shock are plotted in Figure 2. In both models, a positive shock to foreign demand increases exports, leading to an increase in output. But the effects on output are magnified in the agency cost model. Another difference is that household consumption initially falls in the agency cost model because of a decline in the price of capital and a smaller drop in investment. The decline in household consumption in the agency cost model leads to a larger increase in labour supply, which together with higher labour demand, raises employment by more than in the model without agency costs.

In the agency cost model, the increase in output raises entrepreneurs' wage rate. However, the increase in the wage rate is insufficient to offset the decline in the price of capital and a fall in the rental rate, leading to lower entrepreneurial

[25] The increase in the price of capital increases the bankruptcy rate, i.e. $\frac{f(\bar{\varpi})}{f'(\bar{\varpi})}\left(\frac{\phi'(\bar{\varpi})}{\phi(\bar{\varpi})} - \frac{f''(\bar{\varpi})}{f'(\bar{\varpi})}\right)\alpha\phi\left(\bar{\varpi}\right)\bar{\varpi}$ in equation 66 is negative.

[26] Government consumption in both models mainly follows the response of output.

Figure 1: Impulse responses to a productivity shock

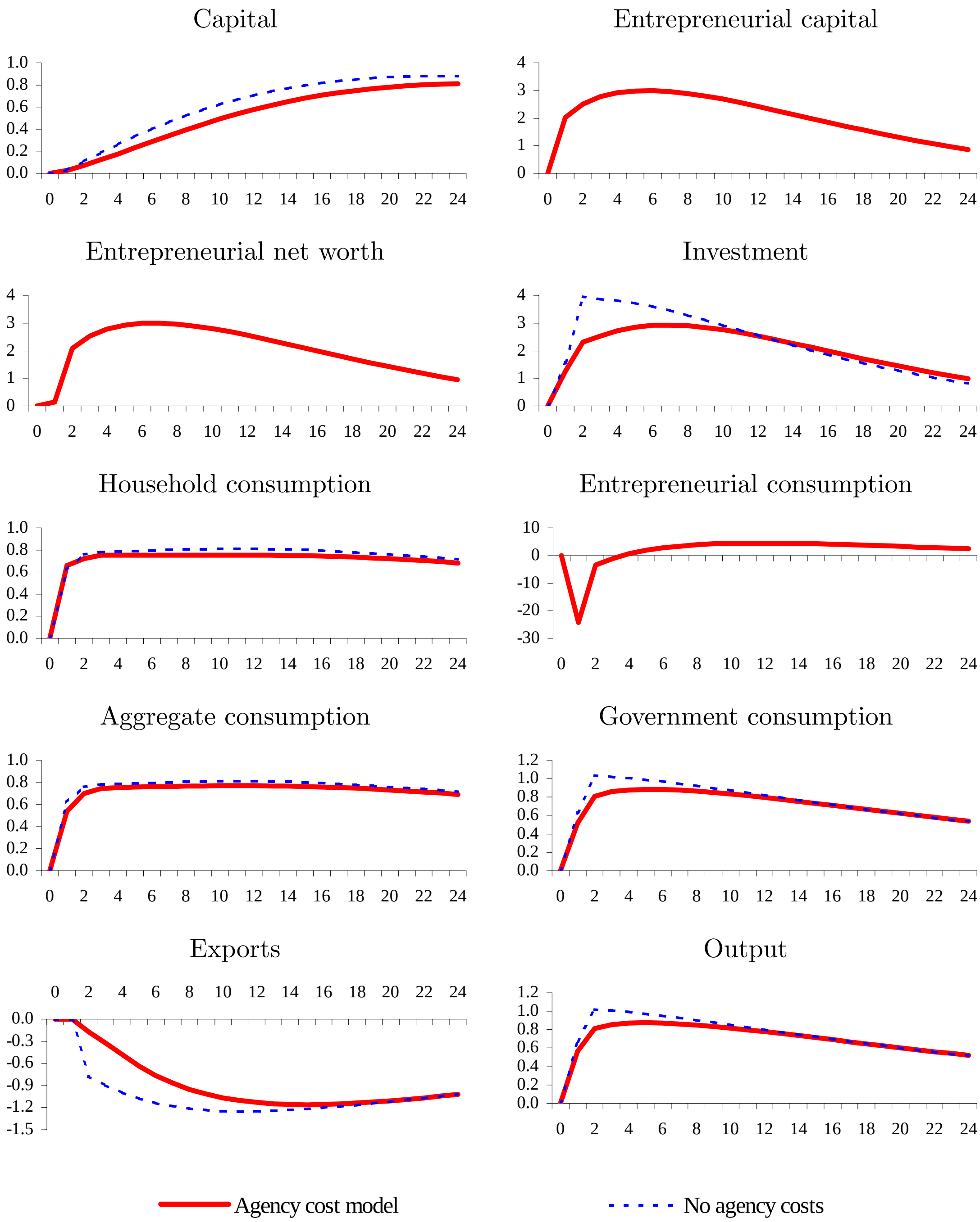


Figure 1 continued

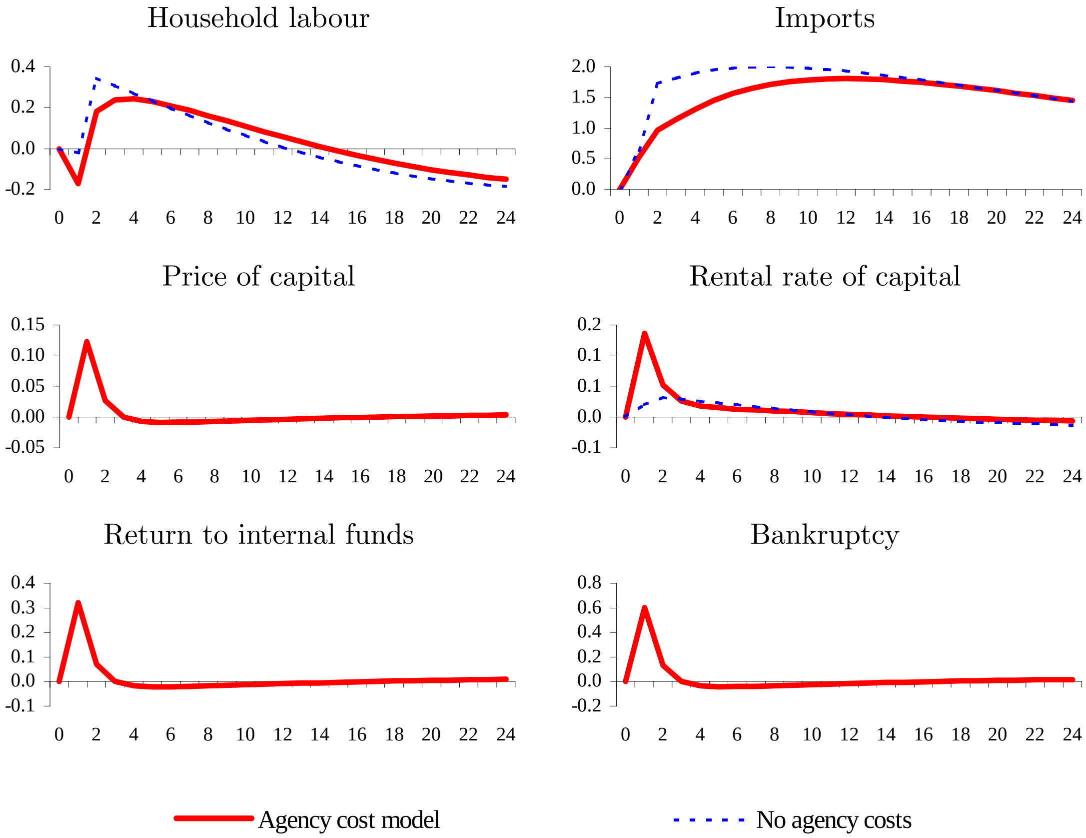
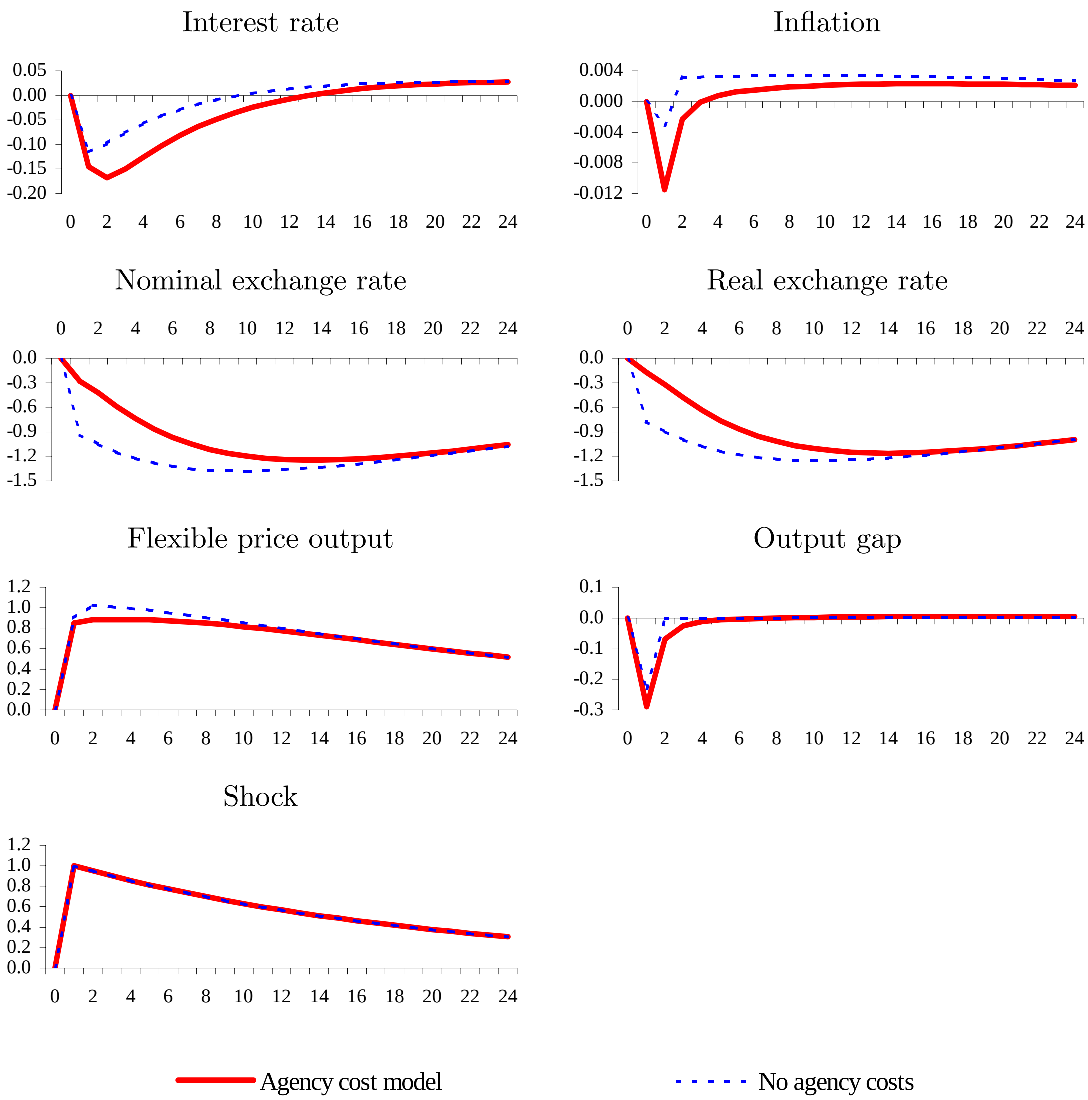


Figure 1 continued



consumption and net worth. As the price and rental rate of capital return to steady state, entrepreneurs' net worth starts increasing, leading to faster capital accumulation and consumption growth than in the model without agency costs.

In both models, the increase in foreign demand leads to excess demand, higher inflation and a tightening in monetary policy. But the output gap and inflation increase by more in the agency cost model than in the model without agency costs. Consequently the interest rate increases by more and stays higher for longer, leading to larger swings in the nominal and real exchange rates. The (real) exchange rate appreciates by more and returns more slowly to steady state in the agency cost model. As a result the price of imports falls by more, leading to higher imports and eventually lower employment in the agency cost model compared to the model without agency costs.

5.3 Foreign interest rate shock

The responses in the agency cost model and the model without agency costs to a foreign interest rate shock are plotted in Figure 3. In both models, an unexpected increase in the foreign interest rate leads to a nominal and real depreciation of the domestic currency. The real depreciation lowers the cost of exports, leading to an increase in foreign demand and exports and a small increase in domestic output, investment, capital, employment and imports. However, in the agency cost model, capital and investment increase less than in the model without agency costs. This is because increased investment demand raises the price and rental rate of capital. In both models, employment rises by more than imports because of a substitution from imports to labour due to the real depreciation of the exchange rate and increase in the cost of imports.

The real depreciation lowers full capacity flexible price output, which together with the increase in output leads to a positive output gap and tightening in monetary policy. Higher domestic interest rates increase the opportunity cost of having to hold demand deposits and lower households' consumption, leading to a fall in employment, investment, imports, capital and output.

The decline in household consumption, employment, investment, imports, capital and output is larger in the agency cost model than in the model without agency costs. The slowing in economic activity is magnified because of the effects of output on entrepreneurs' net worth. The decline in output lowers entrepreneurs' wage, leading to a fall in entrepreneurial net worth and capital accumulation, accelerating further the output decline.

The sharper slow down in economic activity in the agency cost model leads to a larger and more prolonged decline in domestic interest rates. As a result, the (real) exchange rate depreciates by more and remains undervalued for longer than in the model without agency costs.

Figure 2: Impulse responses to a foreign demand shock

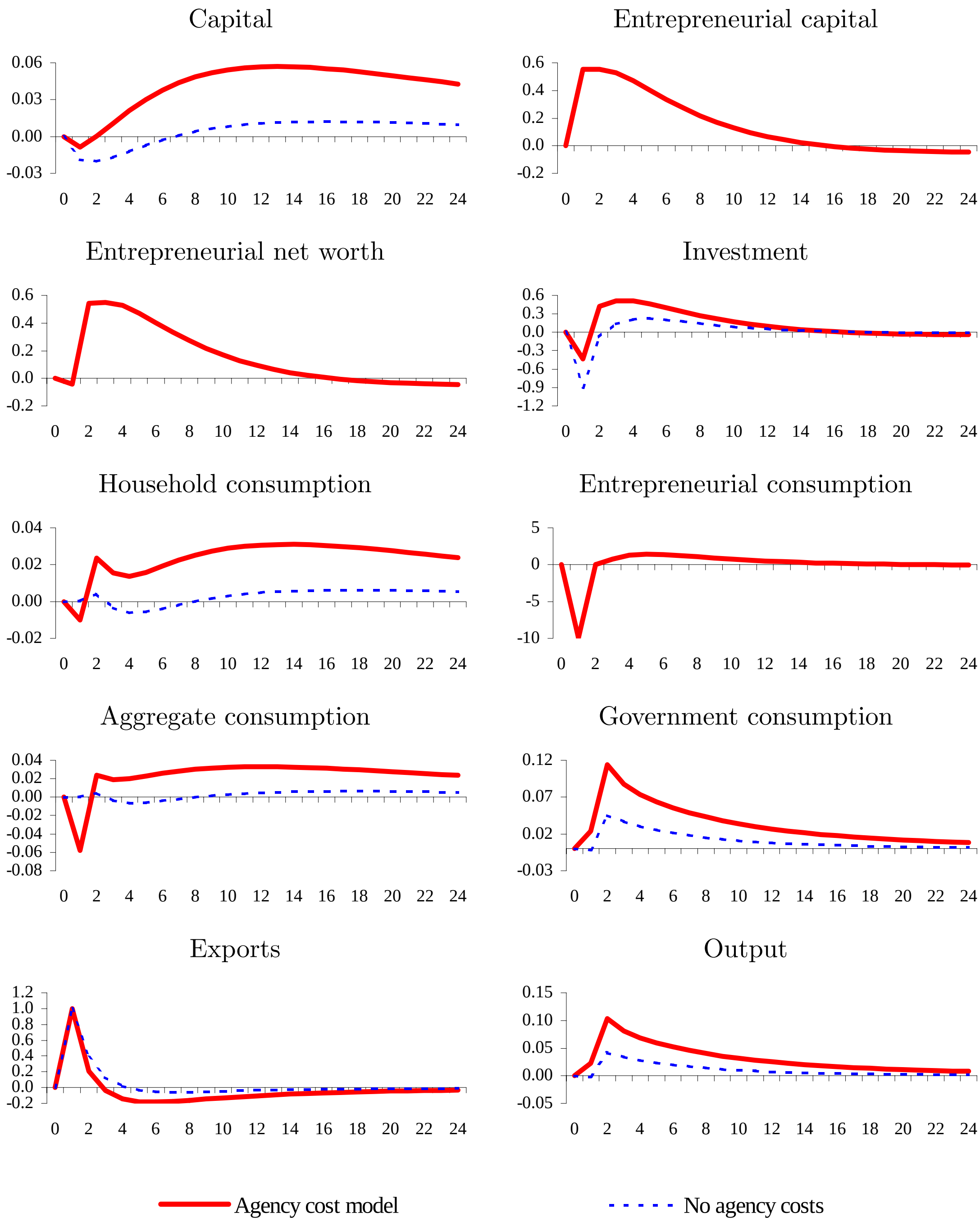


Figure 2 continued

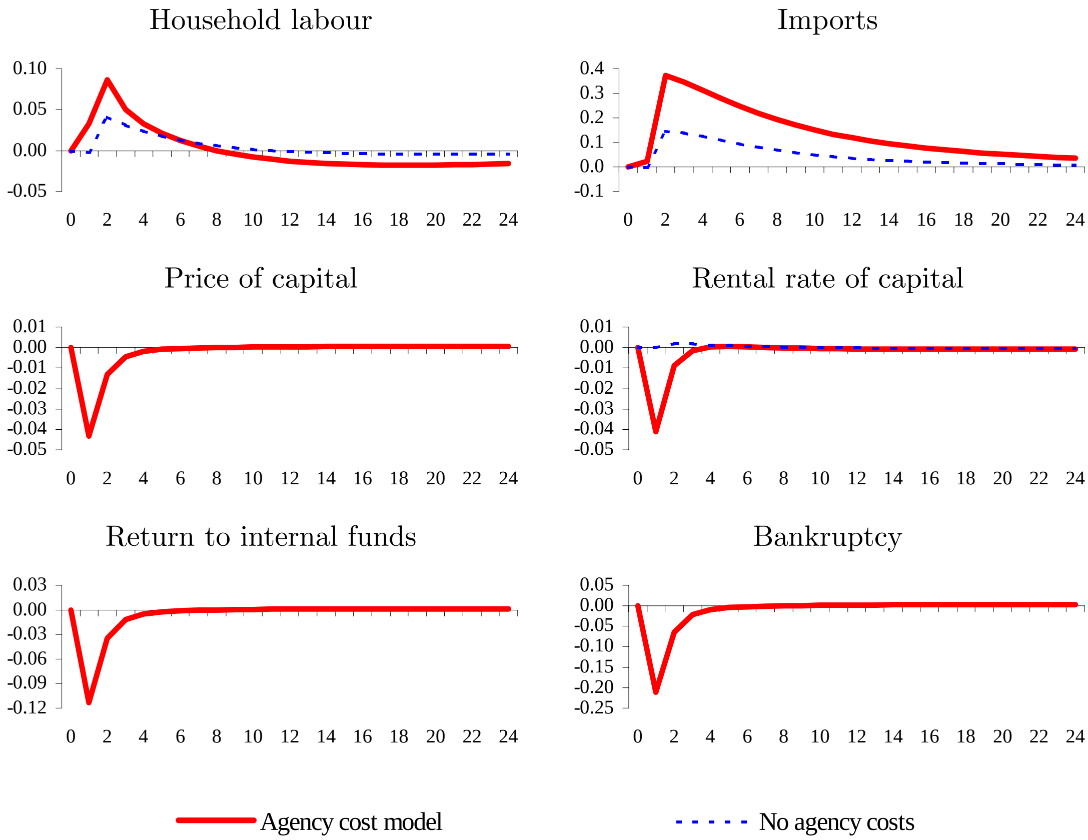


Figure 2 continued

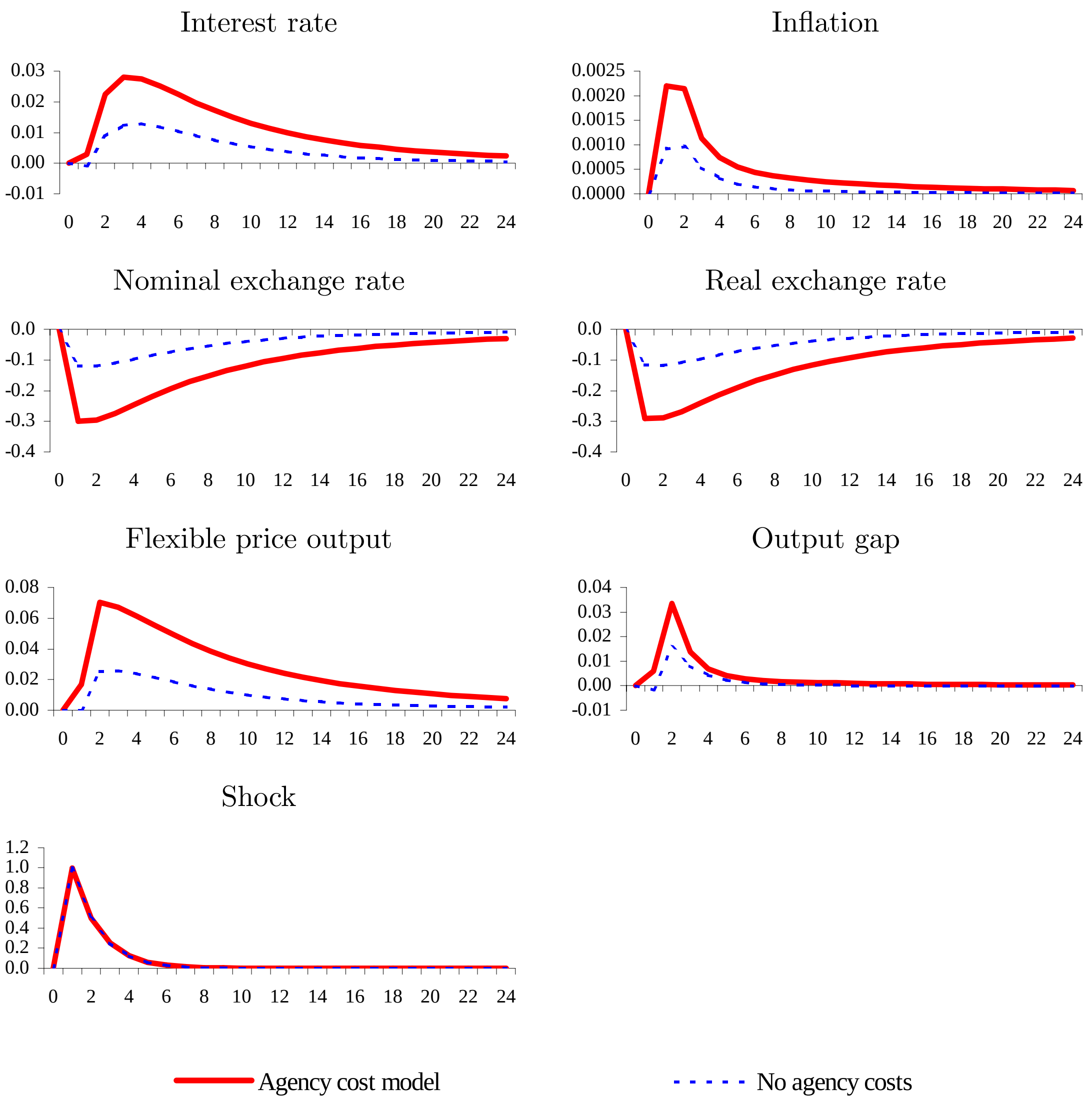


Figure 3: Impulse responses to a foreign interest rate shock

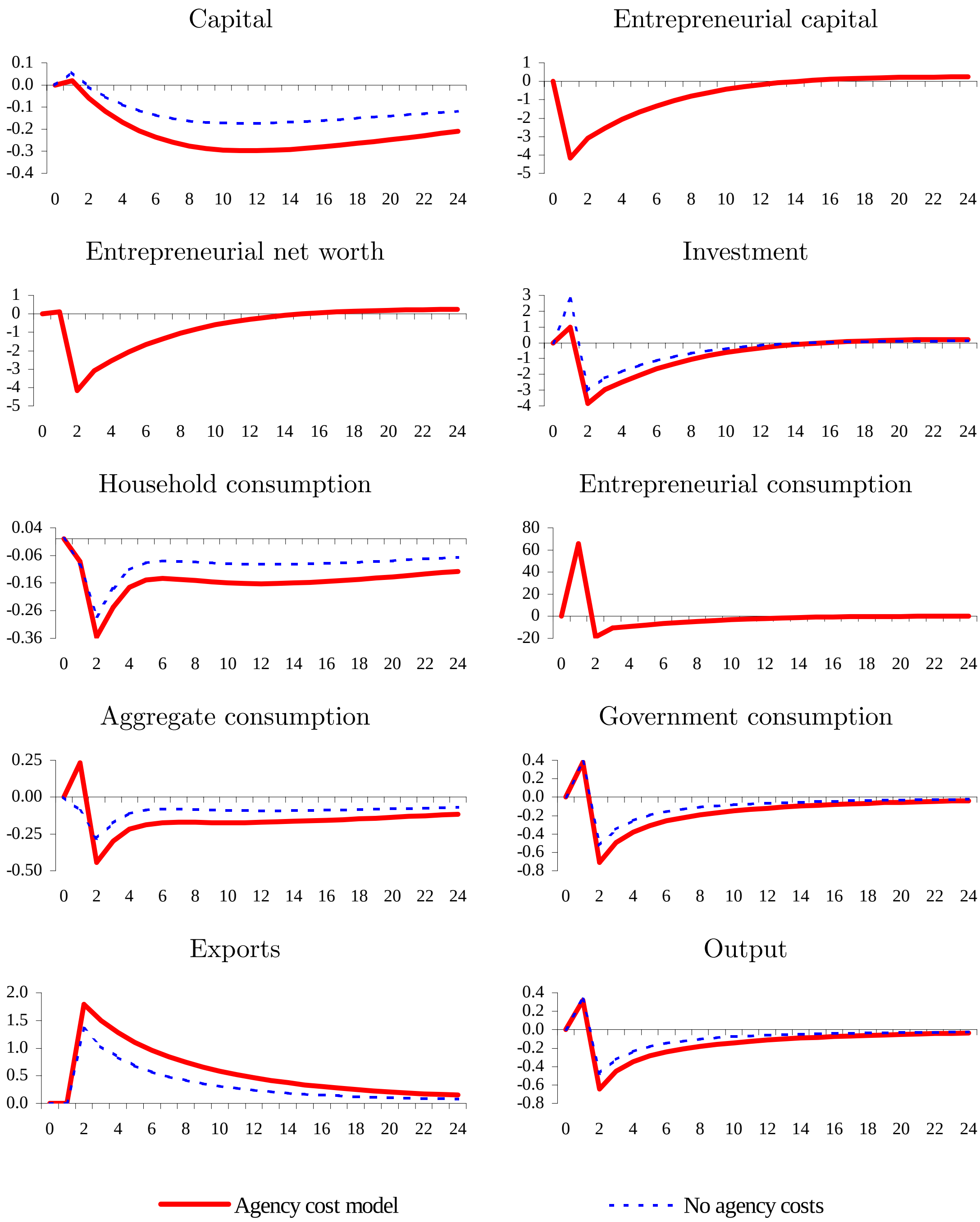


Figure 3 continued

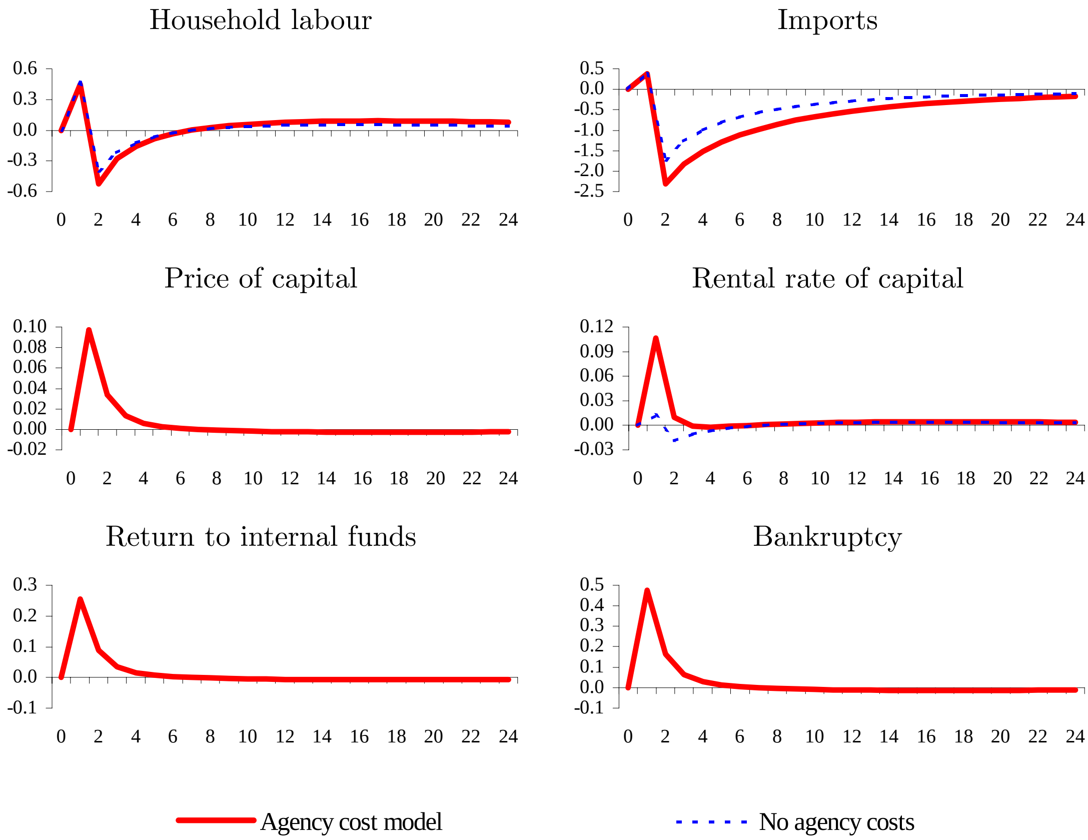
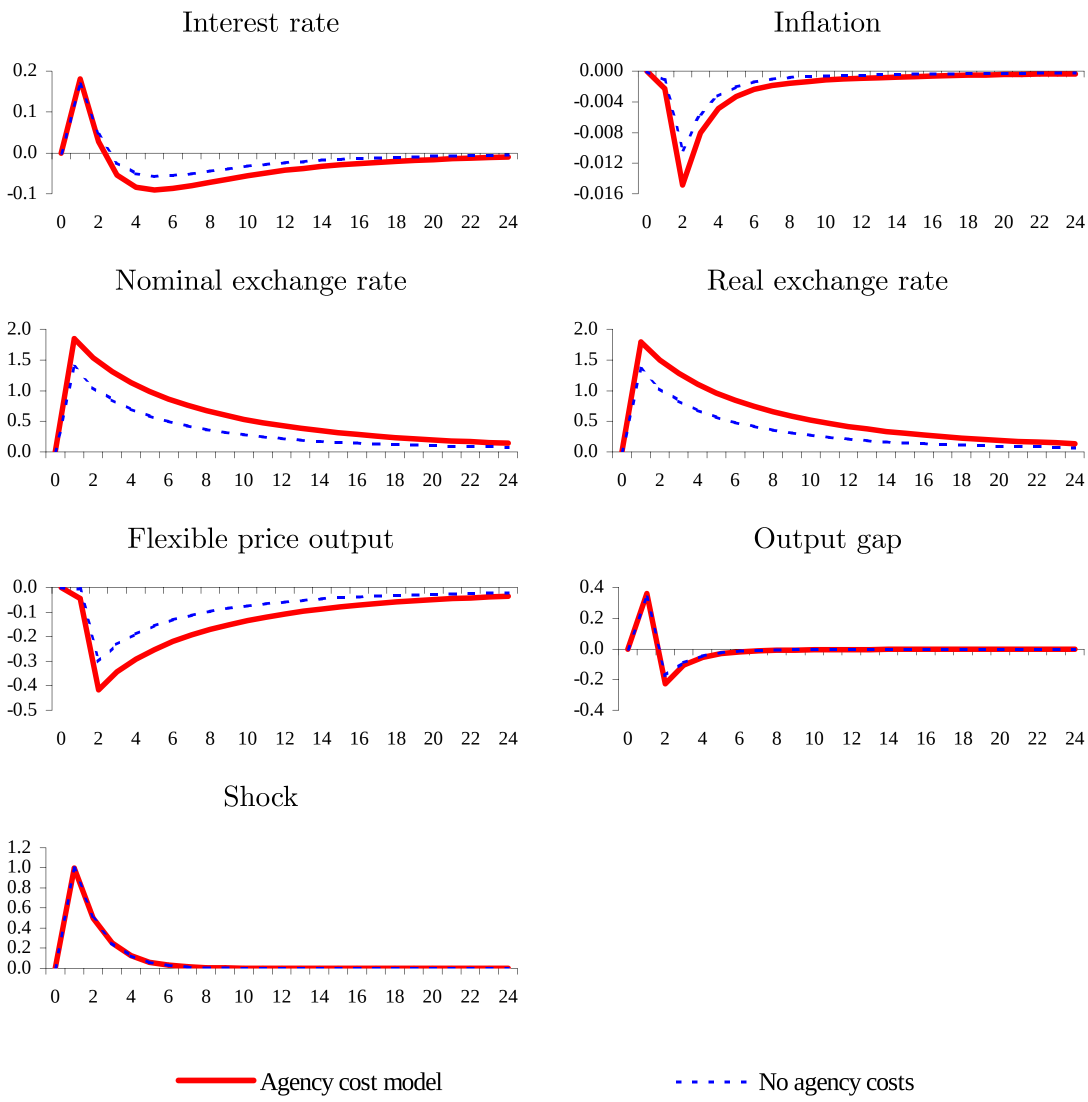


Figure 3 continued



5.4 Monetary policy shock

Figure 4 shows the impulse responses of the two models to an unanticipated tightening in monetary policy that increases the domestic interest rate. In both models, the initial increase in the domestic interest rate is similar. However, in the agency cost model, the interest rate stays higher for longer, leading to a larger appreciation of the (real) exchange rate. The larger real appreciation in the agency cost model raises the price of exports by more than without agency costs, leading to a larger decline in foreign demand and exports.

In both models, output falls following the tightening in monetary policy, lowering the demand for household labour, imports and capital. However, capital and investment fall slightly less in the agency cost model because of a fall in the price of capital. Aggregate consumption falls in the agency cost model (and increases slightly in the model without agency costs) as a decline in entrepreneurial consumption more than offsets an initial increase in households' consumption. Entrepreneurial consumption falls due to the lower price of capital, a lower rental rate of capital and a fall in entrepreneurs' wage. Household consumption initially increases because the monetary policy shock only lasts for one period and the interest rate starts returning to steady state immediately after the shock. The decline in the interest rate lowers the opportunity cost of having to hold demand deposits, leading to higher consumption.

The monetary policy shock reveals some interesting dynamics in the open economy. In both models, the fall in output is quickly reversed. This is because the real appreciation of the domestic currency lowers the cost of imports, leading to higher imports, capital and output. Eventually the economy returns to steady state. Household labour also increases after an initial decline, but then falls again as output slowly returns to steady state. Employment falls faster than imports because the real appreciation leads to a substitution from labour to imports.

5.5 Summary of results

The main results from the impulse response analysis can be summarised as follows. The adjustment paths following a shock to the economy differ in the agency cost model and the model without agency costs. Following a temporary positive productivity shock, for example, output rises more gradually in the agency cost model compared to the model without agency costs. This is because of a delayed response in investment due to net worth. In contrast, following a temporary shock to foreign demand or the foreign interest rate the effects on output are magnified in the agency cost model. Output increases (decreases) faster and remains above (below) steady state for longer compared to the model without agency costs. As a result, the interest rate increases (decreases) by more and stays higher for longer, leading to larger swings in the nominal and real exchange rates.[27]

[27] The results are robust to different specifications of the model. Sensitivity analyses are available upon request.

Figure 4: Impulse responses to a monetary policy shock

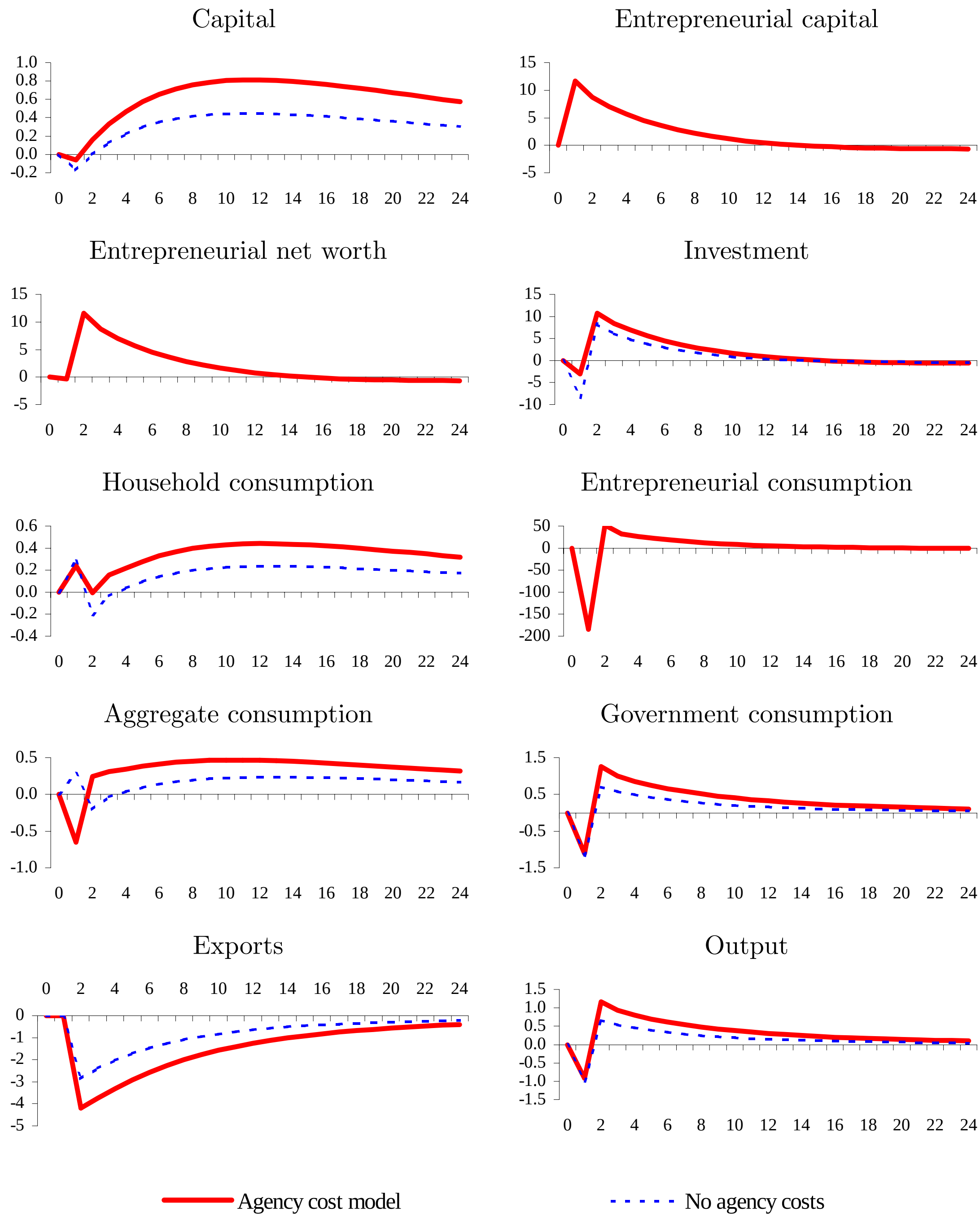


Figure 4 continued

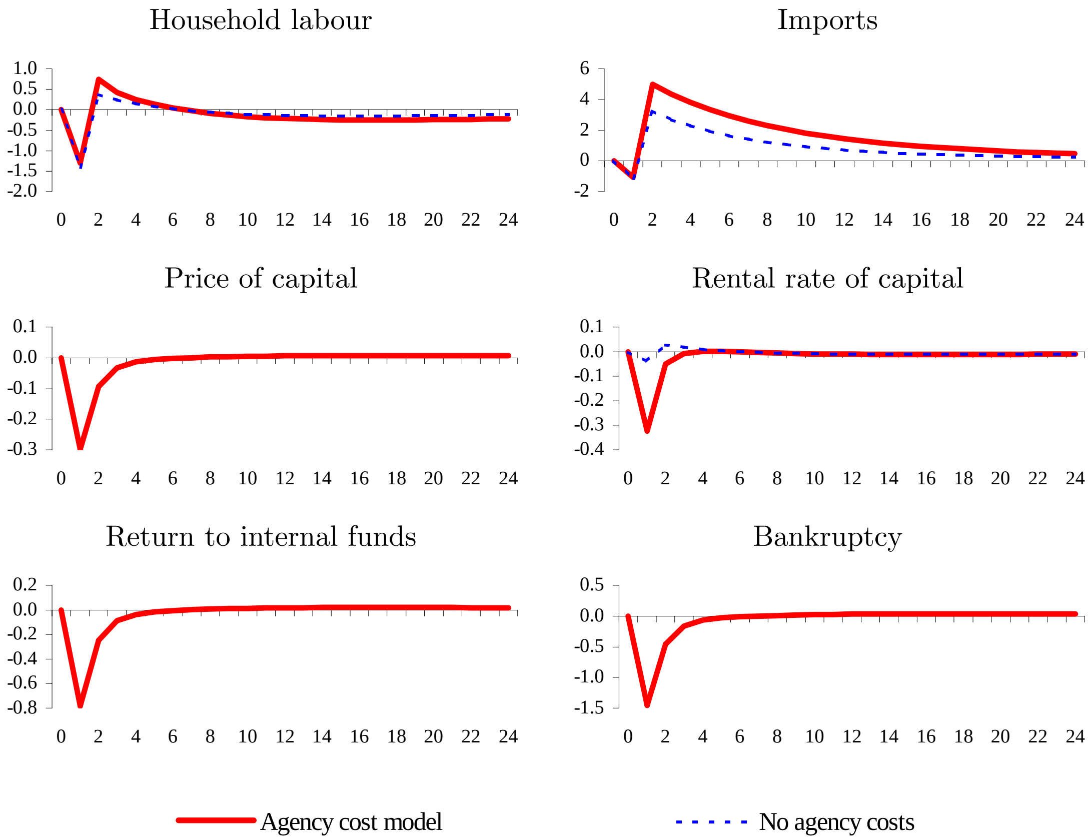
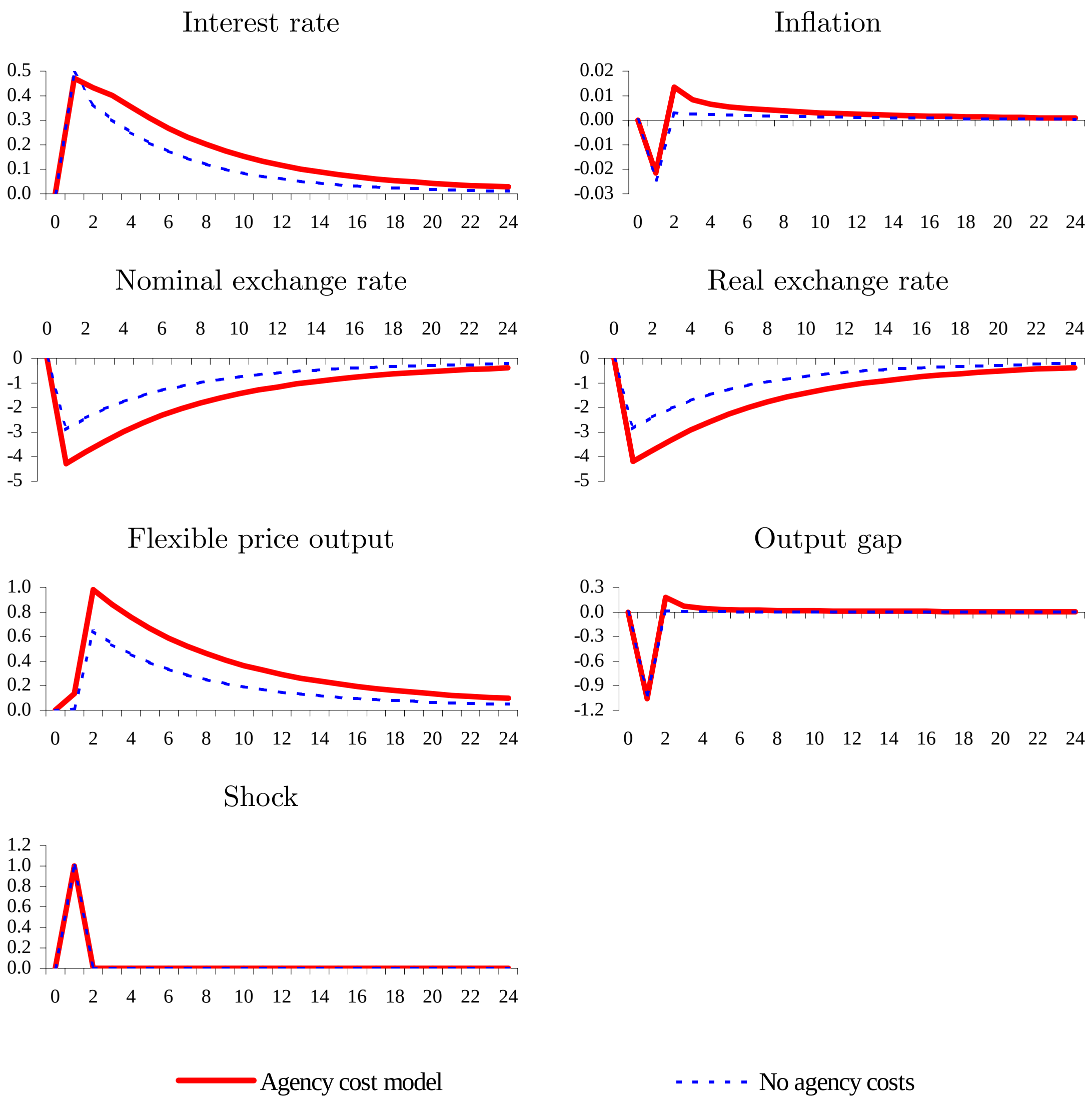


Figure 4 continued



The adjustment paths of the open economy model (with and without agency costs) to a temporary productivity shock are similar to those in Carlstrom and Fuerst's (1997) closed economy model. But the dynamic responses to a monetary policy shock are different from Carlstrom and Fuerst's (2000) closed economy. This is because in this model imports are a production input. Following an unexpected tightening in monetary policy output falls. But the decline in output is quickly reversed and output increases when imports are a production input. This is because the real appreciation of the domestic currency following the tightening leads to a decline in the cost of imports and hence production. In a semi-closed economy with a near-zero coefficient on imports in firms' production function, this does not occur. Output falls following an unexpected tightening and stays below steady state before gradually returning to equilibrium. The result highlights the importance of incorporating a foreign sector and exchange rate into macroeconomic models.

6 Concluding remarks

This paper developed a theoretical model of a small open economy to assess the effects of endogenous agency costs and asymmetric information in credit markets. The basis of the analysis followed Carlstrom and Fuerst's (1997) closed economy model. The Carlstrom and Fuerst model was extended to an open economy with a floating exchange rate. Moreover, slowly adjusting goods prices, a government and an inflation targeting monetary authority were introduced. Agency costs arose because of an ex post information asymmetry between borrowers and lenders. Only borrowers could costlessly observe actual returns after project completion. The imperfect information led to a moral hazard problem and lowered the probability that a loan would be repaid. Financial intermediaries helped overcome the information asymmetry by lending to entrepreneurs via a debt contract and monitoring entrepreneurs who default on their debt.

The model was calibrated for New Zealand. The steady states with and without agency costs were derived and the effects of these costs on business cycle fluctuations were assessed. The analysis showed that a decline in the information asymmetry between borrowers and lenders lowers agency costs and increases the long-run level of steady state investment, capital and output. The presence of agency costs also affects the business cycle and the adjustment paths of interest and exchange rates. Agency costs exacerbate movements in the exchange rate following a shock from the foreign sector.

The differences in the adjustment paths of the model with and without agency costs following a shock to the economy provide evidence of important effects of endogenous agency costs and information asymmetries in credit markets. This suggests that macroeconomic models that do not explicitly account for asymmetric information between borrowers and lenders provide an incomplete description of the economy. It also underlines the importance of a well-functioning financial system. Financial intermediaries and markets help overcome an information asymmetry in credit market and improve the allocation of resources. If financial systems do not function well, the economy cannot operate efficiently and economic activity will be negatively affected. Finally, the results showed important differences in the adjustment paths of the open economy compared to the closed economy case. This finding underlines the importance of incorporating a foreign sector and exchange rate into macroeconomic models.

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A Expected shares of net capital output in the agency cost model

This appendix derives the expected shares of net capital output received by the borrower and lender, $f(\varpi)$ and $g(\varpi)$. The random variable ω is distributed lognormal, i.e. $\ln(\omega) \sim N\left(\tilde{\mu},\ \tilde{\sigma}^2\right)$. The probability density function for the log normal distribution is

$$\phi(\omega) = \frac{1}{\omega\tilde{\sigma}\sqrt{2\pi}} \exp\left(-\frac{(\ln(\omega)-\tilde{\mu})^2}{2\tilde{\sigma}^2}\right) \tag{92}$$

This distribution is normalised, since letting $\upsilon \equiv \ln(\omega)$ gives $d\upsilon = \frac{d\omega}{\omega}$ and $\omega = \exp(\upsilon)$, so

$$\int_0^\infty \phi(\omega)\, d\omega = \frac{1}{\tilde{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{(\upsilon-\tilde{\mu})^2}{2\tilde{\sigma}^2}\right) d\upsilon = 1 \tag{93}$$

The n^{th} moment of ω for $n \geq 1$ is given by

$$\begin{aligned}
E\left[\omega^n\right] &= \int_0^\infty \omega^n \phi(\omega)\, d\omega \\
&= \frac{1}{\tilde{\sigma}\sqrt{2\pi}} \int_0^\infty \omega^{n-1} \exp\left(-\frac{(\ln(\omega)-\tilde{\mu})^2}{2\tilde{\sigma}^2}\right) d\omega \\
&= \frac{1}{\tilde{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} \omega^n \exp\left(-\frac{(\upsilon-\tilde{\mu})^2}{2\tilde{\sigma}^2}\right) d\upsilon \\
&= \frac{1}{\tilde{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\exp(\upsilon))^n \exp\left(-\frac{(\upsilon-\tilde{\mu})^2}{2\tilde{\sigma}^2}\right) d\upsilon \\
&= \frac{1}{\tilde{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\exp(n\upsilon)) \exp\left(-\frac{(\upsilon-\tilde{\mu})^2}{2\tilde{\sigma}^2}\right) d\upsilon \\
&= \exp\left(n\tilde{\mu} + \frac{n^2\tilde{\sigma}^2}{2}\right)
\end{aligned} \tag{94}$$

Therefore, the mean and variance of ω are

$$\mu = E(\omega) = \exp\left(\tilde{\mu} + \frac{\tilde{\sigma}^2}{2}\right) \tag{95}$$

and

$$\begin{aligned}
\sigma^2 = Var(\omega) &= E\left[\omega^2\right] - (E[\omega])^2 \\
&= \exp\left(2\left(\tilde{\mu} + \tilde{\sigma}^2\right)\right) - \left(\exp\left(\tilde{\mu} + \frac{\tilde{\sigma}^2}{2}\right)\right)^2 \\
&= \exp\left(2\left(\tilde{\mu} + \tilde{\sigma}^2\right)\right) - \exp\left(2\tilde{\mu} + \tilde{\sigma}^2\right) \\
&= \exp\left(2\tilde{\mu} + \tilde{\sigma}^2\right)\left(\exp\left(\tilde{\sigma}^2\right) - 1\right)
\end{aligned} \tag{96}$$

The mean and standard deviation of ω are assumed to be unity and σ, i.e.

$$\begin{aligned}
\exp\left(\tilde{\mu} + \frac{\tilde{\sigma}^2}{2}\right) &= 1 \\
\tilde{\mu} &= -\frac{\tilde{\sigma}^2}{2}
\end{aligned} \tag{97}$$

and

$$\begin{aligned}&\exp\left(2\tilde{\mu}+\tilde{\sigma}^2\right)\left(\exp\left(\tilde{\sigma}^2\right)-1\right)=\sigma^2\\&\tilde{\sigma}^2=\ln\left(1+\sigma^2\right)\end{aligned} \tag{98}$$

Substituting (98) into (97) yields

$$\tilde{\mu}=-\frac{\ln\left(1+\sigma^2\right)}{2} \tag{99}$$

The expected share of net capital output received by the firm, $f(\varpi)$, can then be derived as follows

$$\begin{aligned}f(\varpi)&=\int_{\varpi}^{\infty}\omega d\Phi(\omega)-\left[1-\Phi(\varpi)\right]\varpi\\&=\int_{\varpi}^{\infty}\omega d\Phi\left(\omega|\tilde{\mu},\ \tilde{\sigma}^2\right)-\left[1-\Phi\left(\varpi|\tilde{\mu},\ \tilde{\sigma}^2\right)\right]\varpi\\&=1-\Phi\left(\varpi|\tilde{\mu}+\tilde{\sigma}^2,\ \tilde{\sigma}^2\right)-\left[1-\Phi\left(\varpi|\tilde{\mu},\ \tilde{\sigma}^2\right)\right]\varpi\end{aligned} \tag{100}$$

where

$$\begin{aligned}&\int_{\varpi}^{\infty}\omega d\Phi\left(\omega|\tilde{\mu},\ \tilde{\sigma}^2\right)\\&=\exp\left(\tilde{\mu}+\frac{\tilde{\sigma}^2}{2}\right)\int_{\varpi}^{\infty}d\Phi\left(\omega|\tilde{\mu}+\tilde{\sigma}^2,\ \tilde{\sigma}^2\right)\\&=1-\Phi\left(\varpi|\tilde{\mu}+\tilde{\sigma}^2,\ \tilde{\sigma}^2\right)\end{aligned} \tag{101}$$

$f'(\varpi)$ is given by

$$f'(\varpi)=-\phi\left(\varpi|\tilde{\mu}+\tilde{\sigma}^2,\ \tilde{\sigma}^2\right)-\left[1-\Phi\left(\varpi|\tilde{\mu},\ \tilde{\sigma}^2\right)\right]+\phi\left(\varpi|\tilde{\mu},\ \tilde{\sigma}^2\right)\varpi \tag{102}$$

Using (3) the share of net capital output received by the lender, $g(\varpi)$, is derived as

$$\begin{aligned}g(\varpi)&=1-f(\varpi)-\alpha\Phi(\varpi)\\&=1-f(\varpi)-\alpha\Phi\left(\varpi|\tilde{\mu},\ \tilde{\sigma}^2\right)\\&=1-f(\varpi)-\alpha\Phi\left(\varpi|\tilde{\mu},\ \tilde{\sigma}^2\right)\end{aligned} \tag{103}$$

and $g'(\varpi)$ is given by

$$g'(\varpi)=-f'(\varpi)-\alpha\phi\left(\varpi|\tilde{\mu},\ \tilde{\sigma}^2\right) \tag{104}$$

B Proof of proposition ?? and solution of the matrix quadratic equations

The proof follows Uhlig (1997).

Proof. Use the recursive equilibrium law of motion (equations 82 and 83) to replace $\mathbf{x}_{t+1}$ and $\mathbf{y}_t$ in equation (79). This yields equation

$$(AP + CR + B)\,\mathbf{x}_{t+1} + (AQ + CS + D)\,\mathbf{z}_t = 0 \tag{105}$$

which holds for arbitrary $\mathbf{x}_{t+1}$ and $\mathbf{z}_t$. Thus, the coefficient matrices on $\mathbf{x}_t$ and $\mathbf{z}_t$ are zero. Plugging equations (82) and (83) into equation (80) twice and using (81) yields

$$\begin{aligned} 0 &= ((FP + JR + G)\,P + KR + H)\,\mathbf{x}_{t+1} \\ &+ ((FQ + JS + L)\,N + (FP + JR + G)\,Q + KS + M)\,\mathbf{z}_t \end{aligned} \tag{106}$$

Again, the coefficient matrices on $\mathbf{x}_{t+1}$ and $\mathbf{z}_t$ need to be zero. Taking the columnwise vectorisation of the coefficient matrix on $\mathbf{z}_t$ in equations (105) and (106) and collecting terms in $vec\,(Q)$ and $vec\,(S)$ yields the formula for Q and S.

To find P and thus R, rewrite the coefficient matrix on $\mathbf{x}_{t+1}$ in equation (105) as

$$\begin{aligned} R &= -C^{+}\,(AP + B) \\ 0 &= C^{0}AP + C^{0}B \end{aligned} \tag{107}$$

where the matrix $\left[(C^{-1})', (C^{0})'\right]$ is nonsingular and $C^{0}C = 0$. Using (107) to replace R in the coefficient matrix on $\mathbf{x}_{t+1}$ in (106) yields (85). Note that the stability of the equilibrium is determined by the stability of P, since N has stable roots by assumption. ■

To solve the matrix quadratic equations (84) and (85) for P, write them as

$$\Upsilon P^{2} - \Gamma P - \Theta = 0 \tag{108}$$

where

$$\begin{aligned} \Upsilon &= \begin{bmatrix} 0_{l-n,m} \\ F - JC^{+}A \end{bmatrix} \\ \Gamma &= \begin{bmatrix} C^{0}A \\ JC^{+}B - G + KC^{+}A \end{bmatrix} \\ \Theta &= \begin{bmatrix} C^{0}B \\ KC^{+}B - H \end{bmatrix} \end{aligned}$$

l is the number of deterministic equations, n is the number of endogenous variables and m the number of endogenous state variables.

Equation (108) can be solved by turning it into a generalised eigenvalue and eigenvector problem, where the generalised eigenvalue λ and eigenvector s of a matrix Ξ with respect to matrix Δ are defined to be a vector and a value satisfying

$$\lambda \Delta s = \Xi s \tag{109}$$

A standard eigenvalue problem is obtained if Δ is the identity matrix. More generally, the generalised eigenvector problem can be reduced to a standard one if Δ is invertible by calculating standard eigenvalues and eigenvectors for $\Delta^{-1}\Xi$ instead.

Theorem 1 *To solve the quadratic matrix equation*

$$\Upsilon P^2 - \Gamma P - \Theta = 0 \tag{110}$$

for the $m \times m$ matrix P, given the $m \times m$ matrices Γ and Θ, define the $2m \times 2m$ matrices Ξ and Δ as

$$\Xi = \begin{bmatrix} \Gamma & \Theta \\ I_m & 0_{m,m} \end{bmatrix} \tag{111}$$

and

$$\Delta = \begin{bmatrix} \Psi & 0_{m,m} \\ 0_{m,m} & I_m \end{bmatrix} \tag{112}$$

where I_m is the identity matrix of size m, and $0_{m,m}$ is a $m \times m$ matrix with only zero entries.

1. *If $\mathbf{s}$ is a generalised eigenvector and λ the corresponding generalised eigenvalue of Ξ with respect to Δ, then $\mathbf{s}$ can be written as $\mathbf{s}' = \left[\lambda \mathbf{x}', \mathbf{x}'\right]$ for some $\mathbf{x} \in \mathbf{R}^m$.*

2. *If there are m generalised eigenvalues $\lambda_1, ..., \lambda_m$ together with $s_1, ..., s_m$ of Ξ with respect to Δ, written as $s_i' = \left[\lambda_i x_i', x_i'\right]$ for some $x_i \in \mathbf{R}^m$, and if $(x_1, ..., x_m)$ is linearly independent, then*

$$P = \Omega \Lambda \Omega^{-1} \tag{113}$$

is a solution to the matrix quadratic equation (108), where $\Omega = [x_1, ..., x_m]$ and $\Lambda = diag\,(\lambda_1, ..., \lambda_m)$. The solution P is stable if $|\lambda_i| < 1$ for all $i = 1, ...m$. Conversely, any diagonalisable solution P to equation (108) can be written this way.

Proof. Verify that, for the last m rows of equation (109), any eigenvector s for some eigenvalue λ of the matrix Ξ with respect to Δ can indeed be written as

$$\mathbf{s} = \begin{bmatrix} \lambda \mathbf{x} \\ \mathbf{x} \end{bmatrix}$$

for some $\mathbf{x} \in \mathbf{R}^m$ because of the special form of Ξ and Δ. For the first m rows, show that

$$\lambda^2 \Upsilon \mathbf{x} - \lambda \Gamma \mathbf{x} - \Theta \mathbf{x} = 0$$

It follows that

$$\Upsilon \Omega \Lambda^2 - \Gamma \Omega \Lambda^2 - \Theta \Omega = 0$$

and hence

$$\Upsilon P^2 - \Gamma P - \Theta = 0$$

after multiplying with Ω^{-1}.

Reversing the steps shows that any diagonalisable solution P to (108) can be written this way. ■

C Log-linearisation of the agency cost model

$$\left(\bar{Y}_t\right)^{\nu} = \eta_l \left(Z_t \bar{L}_t^h\right)^{\nu} + \eta_k \left(\bar{K}_t\right)^{\nu} + \eta_{im} \left(I\bar{M}_t\right)^{\nu}$$
$$+ \left(1 - \eta_l - \eta_k - \eta_{im}\right) \eta^{\nu}$$
$$\bar{Y}^{\nu} \left(1 + \nu \bar{y}_t\right) = \eta_l \left(\bar{Z}\bar{L}^h\right)^{\nu} \left(1 + \nu z_t + \nu \bar{l}_t\right) + \eta_k \bar{K}^{\nu} \left(1 + \nu \bar{k}_t\right) \tag{114}$$
$$+ \eta_{im} I\bar{M}^{\nu} \left(1 + \nu i\bar{m}_t\right) + \left(1 - \eta_l - \eta_k - \eta_{im}\right)$$
$$\bar{y}_t = \eta_l \left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu} z_t + \eta_l \left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu} \bar{l}_t + \eta_k \left(\frac{\bar{K}}{\bar{Y}}\right)^{\nu} \bar{k}_t + \eta_{im} \left(\frac{I\bar{M}}{\bar{Y}}\right)^{\nu} i\bar{m}_t$$

$$\gamma - \frac{\eta_l (Z_t)^{\nu} \left(\frac{\bar{Y}_t}{\bar{L}_t^h}\right)^{1-\nu}}{\frac{\theta}{\theta - 1}} = 0$$
$$\gamma - \frac{\eta_l \bar{Z}^{\nu} \left(\frac{\bar{Y}}{\bar{L}^h}\right)^{1-\nu}}{\frac{\theta}{\theta - 1}} \left(1 + \nu z_t + (1-\nu)\, \bar{y}_t - (1-\nu)\, \bar{l}_t^h\right) = 0 \tag{115}$$
$$\nu z_t + (1-\nu)\, \bar{y}_t - (1-\nu)\, \bar{l}_t^h = 0$$
$$\bar{l}_t^h = \bar{y}_t + \frac{\nu}{1-\nu} z_t$$

$$R_t - \frac{\eta_k \left(\frac{\bar{Y}_t}{\bar{K}_t}\right)^{1-\nu}}{\frac{\theta}{\theta-1}} = 0$$
$$\bar{R}\left(1 + r_t\right) - \frac{\eta_k \left(\frac{\bar{Y}}{\bar{K}}\right)^{1-\nu}}{\frac{\theta}{\theta-1}} \left(1 + (1-\nu)\, \bar{y}_t - (1-\nu)\, \bar{k}_t\right) = 0 \tag{116}$$
$$r_t - (1-\nu)\, \bar{y}_t + (1-\nu)\, \bar{k}_t = 0$$
$$\bar{k}_t = \bar{y}_t - \frac{1}{1-\nu} r_t$$

$$Q_t = \frac{\eta_{im} \left(\frac{\bar{Y}_t}{I\bar{M}_t}\right)^{1-\nu}}{\frac{\theta}{\theta-1}}$$
$$Q_t = \frac{\eta_{im} \left(\frac{\bar{Y}_t}{I\bar{M}_t}\right)^{1-\nu}}{\frac{\theta}{\theta-1}} \tag{117}$$
$$\bar{Q}\left(1 + q_t\right) = \frac{\eta_{im} \left(\frac{\bar{Y}}{I\bar{M}}\right)^{1-\nu}}{\frac{\theta}{\theta-1}} \left(1 + (1-\nu)\, \bar{y}_t - (1-\nu)\, i\bar{m}_t\right)$$
$$i\bar{m}_t = \bar{y}_t - \frac{1}{1-\nu} q_t$$

$$
\begin{aligned}
&\bar{y}_t=\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}z_t+\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}\bar{l}_t+\eta_k\left(\frac{\bar{K}}{\bar{Y}}\right)^{\nu}\bar{k}_t+\eta_{im}\left(\frac{I\bar{M}}{\bar{Y}}\right)^{\nu}\bar{im}_t\\
&\bar{y}_t=\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}z_t+\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}\left(\bar{y}_t+\frac{\nu}{1-\nu}z_t\right)\\
&+\eta_k\left(\frac{\bar{K}}{\bar{Y}}\right)^{\nu}\left(\bar{y}_t-\frac{1}{1-\nu}r_t\right)+\eta_{im}\left(\frac{I\bar{M}}{\bar{Y}}\right)^{\nu}\left(\bar{y}_t-\frac{1}{1-\nu}q_t\right)\\
&\left(1-\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}-\eta_k\left(\frac{\bar{K}}{\bar{Y}}\right)^{\nu}-\eta_{im}\left(\frac{I\bar{M}}{\bar{Y}}\right)^{\nu}\right)\bar{y}_t\\
&=\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}\left(\frac{1}{1-\nu}\right)z_t-\eta_k\left(\frac{\bar{K}}{\bar{Y}}\right)^{\nu}\left(\frac{1}{1-\nu}\right)r_t-\eta_{im}\left(\frac{I\bar{M}}{\bar{Y}}\right)^{\nu}\left(\frac{1}{1-\nu}\right)q_t\\
&\left(1-\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}-\eta_k\left(\frac{\bar{K}}{\bar{Y}}\right)^{\nu}-\left(1-2\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}-\eta_k\left(\frac{\bar{K}}{\bar{Y}}\right)^{\nu}\right)\right)\bar{y}_t\\
&=\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}\left(\frac{1}{1-\nu}\right)z_t-\eta_k\left(\frac{\bar{K}}{\bar{Y}}\right)^{\nu}\left(\frac{1}{1-\nu}\right)r_t-\eta_{im}\left(\frac{I\bar{M}}{\bar{Y}}\right)^{\nu}\left(\frac{1}{1-\nu}\right)q_t\\
&\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}\bar{y}_t=\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}\left(\frac{1}{1-\nu}\right)z_t-\eta_k\left(\frac{\bar{K}}{\bar{Y}}\right)^{\nu}\left(\frac{1}{1-\nu}\right)r_t\\
&-\eta_{im}\left(\frac{I\bar{M}}{\bar{Y}}\right)^{\nu}\left(\frac{1}{1-\nu}\right)q_t\\
&\bar{y}_t-\frac{1}{1-\nu}z_t+\frac{\eta_k\left(\frac{\bar{K}}{\bar{Y}}\right)^{\nu}}{\eta_l(1-\nu)\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}}r_t+\frac{\eta_{im}\left(\frac{I\bar{M}}{\bar{Y}}\right)^{\nu}}{\eta_l(1-\nu)\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^{\nu}}q_t=0
\end{aligned}
\tag{118}
$$

$$
\begin{aligned}
&\frac{1}{\gamma C_t^h}-\frac{\left(1+\frac{I_t}{1+\Pi_t}\right)}{\frac{(1-\tau)\eta_l(Z_t)^{\nu}\left(\frac{Y_t}{L_t^h}\right)^{1-\nu}}{\xi_t}}=0\\
&(1-\tau)\,\eta_l\,(Z_t)^{\nu}\left(\frac{Y_t}{L_t^h}\right)^{1-\nu}-\gamma\xi_t C_t^h\left(1+\frac{I_t}{1+\Pi_t}\right)=0\\
&(1-\tau)\,\eta_l\bar{Z}^{\nu}\left(\frac{\bar{Y}}{\bar{L}^h}\right)^{1-\nu}\left(1+\nu z_t+(1-\nu)\,y_t-(1-\nu)\,l_t^h\right)\\
&-\frac{\gamma\theta\bar{C}^h}{\theta-1}\left(1+\frac{\bar{I}}{1+\bar{\Pi}}\right)\left(1+\ln\xi_t+c_t^h+i_t-\pi_t\right)=0\\
&(1-\nu)\,y_t-(1-\nu)\,l_t^h-c_t^h+\nu z_t-\ln\xi_t-i_t+\pi_t=0
\end{aligned}
\tag{119}
$$

Using the relationship $\frac{\xi_t}{\frac{\theta}{\theta-1}}=\frac{\xi_t MC_t}{\frac{\theta}{\theta-1}MC_t}=\frac{P_t}{P_t'(i)}$, where MC_t denotes the aggregate marginal cost, equation (119) can be written as

$$
(1-\nu)\,y_t-(1-\nu)\,l_t^h-c_t^h+\nu z_t+\frac{1}{\theta}\left(y_t-\bar{y}_t\right)-i_t+\pi_t=0 \tag{120}
$$

$$
\begin{aligned}
&K_{t+1}-(1-\delta)\,K_t-\eta IN_t\left(1-\alpha\Phi\left(\varpi_t\right)\right)=0\\
&K_{t+1}-(1-\delta)\,K_t-\eta IN_t+\eta IN_t\alpha\Phi\left(\varpi_t\right)=0\\
&\bar{K}\left(1+k_{t+1}\right)-(1-\delta)\,\bar{K}\left(1+k_t\right)-I\bar{N}\left(1+in_t\right)\\
&+I\bar{N}\alpha\Phi\left(\bar{\varpi}\right)\left(1+in_t+\frac{\phi(\bar{\varpi})\bar{\varpi}}{\Phi(\bar{\varpi})}\boldsymbol{\varpi}_t\right)=0\\
&\bar{K}k_{t+1}-(1-\delta)\,\bar{K}k_t-I\bar{N}\left(1-\alpha\Phi\left(\bar{\varpi}\right)\right)in_t\\
&+I\bar{N}\alpha\phi\left(\bar{\varpi}\right)\bar{\varpi}\boldsymbol{\varpi}_t=0
\end{aligned}
\tag{121}
$$

$$
\begin{array}{l}
1+(1-\tau)\,R_t-(1-\delta)\,\hat{\Psi}_t-\frac{(1-\tau)\eta_k\left(\frac{Y_t}{K_t}\right)^{1-\nu}}{\xi_t}=0\\
\left(1+(1-\tau)\,\bar{R}\right)(1+r_t)-(1-\delta)\,\bar{\Psi}\,(1+\psi_t)\\
-\frac{(1-\tau)\eta_k\left(\frac{\bar{Y}}{\bar{K}}\right)^{1-\nu}}{\frac{\theta}{\theta-1}}\left(1+(1-\nu)\,y_t-(1-\nu)\,k_t-\ln\xi_t\right)=0\\
\left(1+(1-\tau)\,\bar{R}\right)r_t-(1-\delta)\,\bar{\Psi}\psi_t-\frac{(1-\tau)(1-\nu)\eta_k\left(\frac{\bar{Y}}{\bar{K}}\right)^{1-\nu}}{\frac{\theta}{\theta-1}}y_t\\
+\frac{(1-\tau)(1-\nu)\eta_k\left(\frac{\bar{Y}}{\bar{K}}\right)^{1-\nu}}{\frac{\theta}{\theta-1}}k_t+\frac{(1-\tau)\eta_k\left(\frac{\bar{Y}}{\bar{K}}\right)^{1-\nu}}{\frac{\theta}{\theta-1}}\ln\xi_t=0\\
\frac{(1-\tau)(1-\nu)\eta_k\left(\frac{\bar{Y}}{\bar{K}}\right)^{1-\nu}}{\frac{\theta}{\theta-1}}y_t+(1-\delta)\,\bar{\Psi}\psi_t-\frac{(1-\tau)(1-\nu)\eta_k\left(\frac{\bar{Y}}{\bar{K}}\right)^{1-\nu}}{\frac{\theta}{\theta-1}}k_t\\
+\frac{(1-\tau)\eta_k\left(\frac{\bar{Y}}{\bar{K}}\right)^{1-\nu}}{\frac{\theta^2\bar{\Psi}}{\theta-1}}\left(y_t-\bar{y}_t\right)-\left(1+(1-\tau)\,\bar{R}\right)r_t=0
\end{array}
\tag{122}
$$

$$
\begin{array}{l}
\frac{\Psi_t}{C_t^h\left(1+\frac{(1-\tau)I_t}{1+\Pi_t}\right)}-E_t\left[\frac{\beta(1+(1-\tau)R_{t+1})}{C_{t+1}^h\left(1+\frac{(1-\tau)I_{t+1}}{1+\Pi_{t+1}}\right)}\right]=0\\
\frac{\bar{\Psi}}{\bar{C}^h\left(1+\frac{(1-\tau)\bar{I}}{1+\bar{\Pi}}\right)}\left(1+\psi_t-c_t^h-i_t+\pi_t\right)\\
-\frac{\beta\left(1+(1-\tau)\bar{R}\right)}{\bar{C}^h\left(1+\frac{(1-\tau)\bar{I}}{1+\bar{\Pi}}\right)}\left(1+E_t\left[r_{t+1}\right]-E_t\left[c_{t+1}^h\right]-i_{t+1}+E_t\left[\pi_{t+1}\right]\right)=0\\
c_t^h+i_t-\pi_t-\psi_t+E_t\left[r_{t+1}\right]-E_t\left[c_{t+1}^h\right]-i_{t+1}+E_t\left[\pi_{t+1}\right]=0
\end{array}
\tag{123}
$$

$$
\begin{array}{l}
Y_t-(1-\eta)\,C_t^h-\eta C_t^e-G_t-EX_t-\eta IN_t=0\\
\bar{Y}\,(1+y_t)-\frac{(1-\eta)\bar{C}^h}{\eta}\left(1+c_t^h\right)-\bar{C}^e\,(1+c_t^e)-\bar{G}\,(1+g_t)\\
-\bar{EX}\,(1+ex_t)-\bar{IN}\,(1+in_t)=0\\
\frac{(1-\eta)\bar{C}^h}{\eta}c_t^h+\bar{C}^e c_t^e+\bar{G}g_t+\bar{EX}ex_t+\bar{IN}in_t-\bar{Y}y_t=0
\end{array}
\tag{124}
$$

$$
\begin{array}{l}
\hat{\Psi}_t-\frac{1}{1-\alpha\Phi(\varpi_t)+\frac{\alpha\phi(\varpi_t)f(\varpi_t)}{f'(\varpi_t)}}=0\\
\frac{1}{\hat{\Psi}_t}-1+\alpha\Phi\left(\varpi_t\right)-\frac{\alpha\phi(\varpi_t)f(\varpi_t)}{f'(\varpi_t)}=0\\
\frac{1}{\bar{\Psi}}\left(1-\psi_t\right)-1+\alpha\Phi\left(\bar{\varpi}\right)\left(1+\frac{\phi(\bar{\varpi})\bar{\varpi}}{\Phi(\bar{\varpi})}\boldsymbol{\varpi}_t\right)\\
-\frac{\alpha\phi(\bar{\varpi})f(\bar{\varpi})}{f'(\bar{\varpi})}\left(1+\left(\frac{\phi'(\bar{\varpi})\bar{\varpi}}{\phi(\bar{\varpi})}+\frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}-\frac{f''(\bar{\varpi})\bar{\varpi}}{f'(\bar{\varpi})}\right)\boldsymbol{\varpi}_t\right)=0\\
\frac{1}{\bar{\Psi}}\psi_t+\frac{f(\bar{\varpi})}{f'(\bar{\varpi})}\left(\frac{\phi'(\bar{\varpi})}{\phi(\bar{\varpi})}-\frac{f''(\bar{\varpi})}{f'(\bar{\varpi})}\right)\alpha\phi\left(\bar{\varpi}\right)\bar{\varpi}\boldsymbol{\varpi}_t=0
\end{array}
\tag{125}
$$

where

$$
f''\left(\varpi\right)=-\phi'\left(\varpi|\tilde{\mu}+\tilde{\sigma}^2,\ \tilde{\sigma}^2\right)+2\phi\left(\varpi|\tilde{\mu},\ \tilde{\sigma}^2\right)+\phi'\left(\varpi|\tilde{\mu},\ \tilde{\sigma}^2\right)\varpi
\tag{126}
$$

$$
\begin{array}{l}
\phi'\left(\omega|\tilde{\mu},\ \tilde{\sigma}^2\right)=\frac{d\left(\frac{1}{\omega\tilde{\sigma}\sqrt{2\pi}}\exp\left(-\frac{(\ln(\omega)-\tilde{\mu})^2}{2\tilde{\sigma}^2}\right)\right)}{d\omega}\\
=-\frac{1}{\omega^2\tilde{\sigma}\sqrt{2\pi}}\exp\left(-\frac{(\ln(\omega)-\tilde{\mu})^2}{2\tilde{\sigma}^2}\right)\left(1+\frac{\ln(\omega)-\tilde{\mu}}{\tilde{\sigma}^2}\right)\\
=-\frac{\phi\left(\omega|\tilde{\mu},\ \tilde{\sigma}^2\right)}{\omega}\left(1+\frac{\ln(\omega)-\tilde{\mu}}{\tilde{\sigma}^2}\right)
\end{array}
\tag{127}
$$

and

$$\begin{array}{l}\phi'\left(\omega|\tilde{\mu}+\tilde{\sigma}^2,\ \tilde{\sigma}^2\right)=\frac{d\left(\frac{1}{\omega\tilde{\sigma}\sqrt{2\pi}}\exp\left(-\frac{\left(\ln(\omega)-\left(\tilde{\mu}+\tilde{\sigma}^2\right)\right)^2}{2\tilde{\sigma}^2}\right)\right)}{d\omega}\\ =-\frac{1}{\omega^2\tilde{\sigma}^3\sqrt{2\pi}}\exp\left(-\frac{\left(\ln(\omega)-\left(\tilde{\mu}+\tilde{\sigma}^2\right)\right)^2}{2\tilde{\sigma}^2}\right)\left(\ln\left(\omega\right)-\tilde{\mu}\right)\\ =-\frac{\phi\left(\omega|\tilde{\mu}+\tilde{\sigma}^2,\ \tilde{\sigma}^2\right)\left(\ln(\omega)-\tilde{\mu}\right)}{\omega\tilde{\sigma}^2}\end{array}\tag{128}$$

$$\begin{array}{l}IN_t-\frac{NW_t}{1-g(\varpi_t)\hat{\Psi}_t}=0\\ IN_t-\frac{IR_tNW_t}{f(\varpi_t(i))\hat{\Psi}_t}=0\\ \bar{IN}\left(1+in_t\right)-\frac{\bar{IR}\bar{NW}}{f(\bar{\varpi})\bar{\Psi}}\left(1+ir_t+nw_t-\frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}\boldsymbol{\varpi}_t-\psi_t\right)=0\\ ir_t+nw_t-\psi_t-\frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}\boldsymbol{\varpi}_t-in_t=0\end{array}\tag{129}$$

$$\begin{array}{l}NW_t-\frac{(1-\tau)(1-\eta_l-\eta_k-\eta_{im})(Y_t)^{1-\nu}}{\xi_t}-\left(1+\left(1-\tau\right)R_t\right)K_t^e=0\\ \bar{NW}\left(1+nw_t\right)-\frac{(1-\tau)(1-\eta_l-\eta_k-\eta_{im})\bar{Y}^{1-\nu}}{\frac{\theta}{\theta-1}}\left(1+\left(1-\nu\right)y_t-\ln\xi_t\right)\\ -\left(1+\left(1-\tau\right)\bar{R}\right)\bar{K}^e\left(1+r_t+k_t^e\right)=0\\ \frac{(1-\tau)(1-\nu)(1-\eta_l-\eta_k-\eta_{im})\bar{Y}^{1-\nu}}{\frac{\theta}{\theta-1}}y_t+\frac{(1-\tau)(1-\eta_l-\eta_k-\eta_{im})\bar{Y}^{1-\nu}}{\frac{\theta^2}{\theta-1}}\left(y_t-\bar{y}_t\right)\\ +\left(1+\left(1-\tau\right)\bar{R}\right)\bar{K}^ek_t^e+\left(1+\left(1-\tau\right)\bar{R}\right)\bar{K}^er_t-\bar{NW}nw_t=0\end{array}\tag{130}$$

$$\begin{array}{l}NW_t\left(i\right)\left(1+IR_t\right)-C_t^e-\hat{\Psi}_tK_{t+1}^e=0\\ \bar{NW}\left(1+\bar{IR}\right)\left(1+nw_t+ir_t\right)-\bar{C}^e\left(1+c_t^e\right)\\ -\bar{\Psi}\bar{K}^e\left(1+\psi_t+k_{t+1}^e\right)=0\\ \bar{NW}\left(1+\bar{IR}\right)nw_t+\bar{NW}\left(1+\bar{IR}\right)ir_t-\bar{C}^ec_t^e\\ -\bar{\Psi}\bar{K}^e\psi_t-\bar{\Psi}\bar{K}^ek_{t+1}^e=0\end{array}\tag{131}$$

$$
\begin{aligned}
&1+IR_t-\frac{f(\varpi_t)\hat{\Psi}_t}{1-g(\varpi_t)\hat{\Psi}_t}=0\\
&\frac{1}{1+IR_t}-\frac{1}{f(\varpi_t)\hat{\Psi}_t}+\frac{g(\varpi_t)}{f(\varpi_t)}=0\\
&\frac{1}{1+\bar{IR}}\left(1-ir_t\right)-\frac{1}{f(\bar{\varpi})\bar{\Psi}}\left(1-\frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}\boldsymbol{\varpi}_t-\psi_t\right)\\
&+\frac{g(\bar{\varpi})}{f(\bar{\varpi})}\left(1+\left(\frac{g'(\bar{\varpi})\bar{\varpi}}{g(\bar{\varpi})}-\frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}\right)\boldsymbol{\varpi}_t\right)=0\\
&-\frac{1}{1+\bar{IR}}ir_t+\frac{1}{f(\bar{\varpi})\bar{\Psi}}\psi_t\\
&+\left(\frac{g(\bar{\varpi})}{f(\bar{\varpi})}\left(\frac{g'(\bar{\varpi})\bar{\varpi}}{g(\bar{\varpi})}-\frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}\right)+\frac{1}{f(\bar{\varpi})\bar{\Psi}}\frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}\right)\boldsymbol{\varpi}_t=0\\
&-\frac{1}{1+\bar{IR}}ir_t+\frac{1}{f(\bar{\varpi})\bar{\Psi}}\psi_t\\
&+\left(\frac{g'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}-\frac{g(\bar{\varpi})}{f(\bar{\varpi})}\frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}+\frac{1}{f(\bar{\varpi})\bar{\Psi}}\frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}\right)\boldsymbol{\varpi}_t=0\\
&-\frac{1}{\bar{IR}}ir_t+\frac{1}{f(\bar{\varpi})\bar{\Psi}}\psi_t\\
&+\left(\frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}\left(\frac{g'(\bar{\varpi})}{f'(\bar{\varpi})}-\frac{g(\bar{\varpi})}{f(\bar{\varpi})}+\frac{1}{f(\bar{\varpi})\bar{\Psi}}\right)\right)\boldsymbol{\varpi}_t=0\\
&-\frac{1}{1+\bar{IR}}ir_t+\frac{1}{f(\bar{\varpi})\bar{\Psi}}\psi_t+\left(\frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}\left(\frac{g'(\bar{\varpi})}{f'(\bar{\varpi})}+\frac{1-g(\bar{\varpi})\bar{\Psi}}{f(\bar{\varpi})\bar{\Psi}}\right)\right)\boldsymbol{\varpi}_t=0\\
&-\frac{1}{1+\bar{IR}}ir_t+\frac{1}{f(\bar{\varpi})\bar{\Psi}}\psi_t+\left(\frac{g'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}+\frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}\frac{1-g(\bar{\varpi})\bar{\Psi}}{f(\bar{\varpi})\bar{\Psi}}\right)\boldsymbol{\varpi}_t=0\\
&ir_t-\frac{1}{1-g(\bar{\varpi})\bar{\Psi}}\psi_t-\frac{\left(\frac{g'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}+\frac{f'(\bar{\varpi})\bar{\varpi}}{f(\bar{\varpi})}\frac{1-g(\bar{\varpi})\bar{\Psi}}{f(\bar{\varpi})\bar{\Psi}}\right)}{\frac{1-g(\bar{\varpi})\bar{\Psi}}{f(\bar{\varpi})\bar{\Psi}}}\boldsymbol{\varpi}_t=0\\
&\frac{1}{1-g(\bar{\varpi})\bar{\Psi}}\psi_t+\left(\frac{g'(\bar{\varpi})\bar{\Psi}}{1-g(\bar{\varpi})\bar{\Psi}}+\frac{f'(\bar{\varpi})}{f(\bar{\varpi})}\right)\bar{\varpi}\boldsymbol{\varpi}_t-ir_t=0
\end{aligned}
\tag{132}
$$

$$
\begin{aligned}
&\hat{\Psi}_t-E_t\left[\frac{\zeta\beta(1+(1-\tau)R_{t+1})f(\varpi_{t+1})\hat{\Psi}_{t+1}}{1-g(\varpi_{t+1})\hat{\Psi}_{t+1}}\right]=0\\
&\hat{\Psi}_t-E_t\left[\zeta\beta\left(1+(1-\tau)R_{t+1}\right)IR_{t+1}\right]=0\\
&\bar{\Psi}\left(1+\psi_t\right)-\left(\zeta\beta\left(1+(1-\tau)\bar{R}\right)\bar{IR}\right)\left(1+E_t\left[r_{t+1}\right]+E_t\left[ir_{t+1}\right]\right)=0\\
&E_t\left[r_{t+1}\right]+E_t\left[ir_{t+1}\right]-\psi_t=0
\end{aligned}
\tag{133}
$$

$$
\begin{aligned}
&Y_t-(\eta_l\left(Z_tL_t^h\right)^\nu+\eta_k\left(K_t\right)^\nu+\eta_{im}\left(IM_t\right)^\nu\\
&+\left(1-\eta_l-\eta_k-\eta_{im}\right)\left(L_t^e\right)^\nu)^{\frac{1}{\nu}}=0\\
&\left(Y_t\right)^\nu-\eta_l\left(Z_tL_t^h\right)^\nu-\eta_k\left(K_t\right)^\nu-\eta_{im}\left(IM_t\right)^\nu\\
&-\left(1-\eta_l-\eta_k\right)\left(\eta\right)^\nu=0\\
&\bar{Y}^\nu\left(1+\nu y_t\right)-\eta_l\left(\bar{Z}\bar{L}^h\right)^\nu\left(1+\nu z_t+\nu l_t^h\right)\\
&-\eta_k\bar{K}^\nu\left(1+\nu k_t\right)-\eta_{im}\bar{IM}\left(1+\nu im_t\right)\\
&-\left(1-\eta_l-\eta_k\right)=0\\
&\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^\nu z_t+\eta_l\left(\frac{\bar{Z}\bar{L}^h}{\bar{Y}}\right)^\nu l_t^h+\eta_k\left(\frac{\bar{K}}{\bar{Y}}\right)^\nu k_t\\
&+\eta_{im}\left(\frac{\bar{IM}}{\bar{Y}}\right)^\nu im_t-y_t=0
\end{aligned}
\tag{134}
$$

$$
\begin{array}{l}
\tau\left(\hat{W}_t^h L_t^h + \hat{W}_t^e L_t^e + \frac{\eta_k (Y_t)^{1-\nu} K_t \left(\frac{1}{K_{t-1}}\right)^{-\nu}}{\xi_t}\right) - G_t = 0 \\
\tau\left(\frac{\eta_l \left(Z_t L_t^h\right)^{\nu} (Y_t)^{1-\nu}}{\xi_t} + \frac{\eta (1-\eta_l-\eta_k-\eta_{im}) (Y_t)^{1-\nu}}{\xi_t} + \frac{\eta_k (Y_t)^{1-\nu} (K_t)^{\nu}}{\xi_t}\right) - G_t = 0 \\
\tau\left(\frac{\eta_l (Z_t)^{\nu} \left(\frac{Y_t}{L_t^h}\right)^{1-\nu} L_t^h}{\xi_t} + \frac{(1-\eta_l-\eta_k-\eta_{im}) (Y_t)^{1-\nu} \eta}{\xi_t} + \frac{\eta_k \left(\frac{Y_t}{K_t}\right)^{1-\nu} K_t}{\xi_t}\right) - G_t = 0 \\
\frac{\tau\left(\frac{Y_t}{\eta}\right)^{1-\nu}}{\xi_t}\left(\eta_l (Z_t)^{\nu} \left(\frac{L_t^h}{\eta}\right)^{\nu} + (1-\eta_l-\eta_k-\eta_{im}) \eta^{\nu} + \eta_k \left(\frac{K_t}{\eta}\right)^{\nu}\right) \\
-\frac{G_t}{\eta} = 0 \\
\frac{\tau\left(\frac{Y_t}{\eta}\right)}{\xi_t} - \frac{\tau \eta_{im} \left(\frac{Y_t}{\eta}\right)^{1-\nu} \left(\frac{IM_t}{\eta}\right)^{\nu}}{\xi_t} - \frac{G_t}{\eta} = 0 \\
\frac{\tau \bar{Y}}{\frac{\theta}{\theta-1}} \left(1 + y_t - \ln \xi_t\right) - \frac{\tau \eta_{im} \bar{Y}^{1-\nu} \bar{IM}^{\nu}}{\frac{\theta}{\theta-1}} \left(1 + (1-\nu) y_t + \nu i m_t - \ln \xi_t\right) \\
-\bar{G}\left(1 + g_t\right) = 0 \\
\frac{\tau\left(\bar{Y} - \eta_{im} (1-\nu) \bar{Y}^{1-\nu} \bar{IM}^{\nu}\right)}{\frac{\theta}{\theta-1}} y_t - \frac{\tau\left(\bar{Y} - \eta_{im} \bar{Y}^{1-\nu} \bar{IM}^{\nu}\right)}{\frac{\theta}{\theta-1}} \ln \xi_t - \frac{\tau \nu \eta_{im} \bar{Y}^{1-\nu} \bar{IM}^{\nu}}{\frac{\theta}{\theta-1}} i m_t \\
-\bar{G} g_t = 0 \\
\frac{\tau\left(\bar{Y} - \eta_{im} (1-\nu) \bar{Y}^{1-\nu} \bar{IM}^{\nu}\right)}{\frac{\theta}{\theta-1}} y_t + \frac{\tau\left(\bar{Y} - \eta_{im} \bar{Y}^{1-\nu} \bar{IM}^{\nu}\right)}{\frac{\theta^2}{\theta-1}} \left(y_t - \bar{y}_t\right) - \frac{\tau \eta_{im} \nu \bar{Y}^{1-\nu} \bar{IM}^{\nu}}{\frac{\theta}{\theta-1}} i m_t \\
-\bar{G} g_t = 0
\end{array}
\tag{135}
$$

$$
\begin{array}{l}
EX_t - (Q_t)^{\kappa} \left(Y_t^*\right)^{\varsigma} = 0 \\
\bar{EX}\left(1 + ex_t\right) - \bar{Q}^{\kappa} \left(\bar{Y}^*\right)^{\varsigma} \left(1 + \kappa q_t + \varsigma y_t^*\right) = 0 \\
ex_t - \kappa q_t - \varsigma y_t^* = 0
\end{array}
\tag{136}
$$

$$
\begin{array}{l}
Q_t - \frac{\eta_{im} \left(\frac{Y_t}{IM_t}\right)^{1-\nu}}{\xi_t} = 0 \\
\xi_t Q_t - \eta_{im} \left(\frac{Y_t}{IM_t}\right)^{1-\nu} = 0 \\
\frac{\theta \bar{Q}}{\theta-1} \left(1 + q_t + \ln \xi_t\right) - \eta_{im} \left(\frac{\bar{Y}}{\bar{IM}}\right)^{1-\nu} \left(1 + (1-\nu) y_t - (1-\nu) i m_t\right) = 0 \\
q_t + \ln \xi_t - (1-\nu) y_t + (1-\nu) i m_t = 0 \\
q_t - (1-\nu) y_t - \frac{1}{\theta} \left(y_t - \bar{y}_t\right) + (1-\nu) i m_t = 0
\end{array}
\tag{137}
$$

$$
\begin{array}{l}
1 + I_{t+1} - \mu_1 \left(\Pi_t - \Pi^T\right) - \mu_2 \left(Y_t - \bar{Y}_t\right) - \mu_3 \left(1 + I_t\right) = 0 \\
1 + I_{t+1} - \mu_1 \Pi_t + \mu_1 \Pi^T - \mu_2 Y_t + \mu_2 \bar{Y}_t - \mu_3 \left(1 + I_t\right) = 0 \\
\left(1 + \bar{I}\right) \left(1 + i_{t+1}\right) - \mu_1 \bar{\Pi} \left(1 + \pi_t\right) + \mu_1 \Pi^T - \mu_2 \bar{Y} \left(1 + y_t\right) \\
+\mu_2 \left(1 + \bar{y}_t\right) - \mu_3 \left(1 + \bar{I}\right) \left(1 + i_t\right) = 0 \\
i_{t+1} - \frac{\mu_1 \bar{\Pi}}{1+\bar{I}} \pi_t - \frac{\mu_2 \bar{Y}}{1+\bar{I}} \left(y_t - \bar{y}_t\right) - \mu_3 i_t = 0
\end{array}
\tag{138}
$$

$$
\begin{array}{l}
\left(1 + I_{t+1}\right) - E_t \left[\left(1 + I_{t+1}^*\right) \frac{S_{t+2}}{S_{t+1}}\right] = 0 \\
\left(1 + \bar{I}\right) \left(1 + i_{t+1}\right) - \left(1 + \bar{I}^*\right) \left(1 + \bar{\Delta S}\right) \left(1 + E_t \left[i_t^*\right]\right. \\
\left. + E_t \left[s_{t+2}\right] - E_t \left[s_{t+1}\right]\right) = 0 \\
i_{t+1} - E_t \left[i_{t+1}^*\right] - E_t \left[s_{t+2}\right] + E_t \left[s_{t+1}\right] = 0
\end{array}
\tag{139}
$$

$$
\begin{aligned}
& E_t\left[\frac{Q_{t+2}}{Q_{t+1}}\right]-E_t\left[\frac{\frac{S_{t+2}}{S_{t+1}}\frac{P^*_{t+2}}{P^*_{t+1}}}{\frac{P_{t+2}}{P_{t+1}}}\right]=0 \\
& \left(1+E_t\left[q_{t+2}\right]-E_t\left[q_{t+1}\right]\right) \\
& -\frac{\left(1+\bar{\Delta S}\right)\left(1+\bar{\Pi}^*\right)}{1+\bar{\Pi}}\left(1+E_t\left[s_{t+2}\right]-E_t\left[s_{t+1}\right]+E_t\left[\pi^*_{t+2}\right]-E_t\left[\pi_{t+2}\right]\right)=0 \\
& E_t\left[q_{t+2}\right]-E_t\left[q_{t+1}\right]-E_t\left[s_{t+2}\right]+E_t\left[s_{t+1}\right]+E_t\left[\pi_{t+2}\right]-E_t\left[\pi^*_{t+2}\right]=0
\end{aligned} \tag{140}
$$