A Fictitious Play of the Nash Demand Game Implements the Nash Bargaining Solution

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Abstract

We study a repeated Nash demand game, where bargainers follow a fictitious play procedure after their one-shot decision on demand in the initial period. In the reduced static game they play at the initial period, all the ϵ-equilibria are clustered around the division corresponding to the Nash bargaining solution when the bargainers are patient. As the bargainers make a more accurate comparison of payoffs and become more patient accordingly, the only equilibrium left is the division of the Nash bargaining solution.

JEL Classification: C71, C72, C78, D83.

Key Words: fictitious play, Nash demand game, ϵ-equilibrium, Nash bargaining solution, Nash program.

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1 Introduction

Since Nash's work in the early 1950's, there have been two different approaches to analyzing bargaining problems: the strategic approach and the axiomatic approach. Nash (1953) claimed that these two approaches should be complementary, and considered himself a strategic model of bargaining to implement his axiomatic solution. The idea of relating axiomatic solutions to equilibria of strategic models is now known as the "Nash program" (see Binmore (1987)). Binmore et al. (1986) also showed that the subgame-perfect equilibrium of the Rubinstein's (1982) alternating offers model approaches the Nash (1950) bargaining solution when the friction becomes smaller. We refer readers to Osborne and Rubinstein (1990) for more results that relate axiomatic solutions to equilibria of strategic models. In this paper, we add another strategic model, which is evolutionary in nature, to implement the Nash bargaining solution.

In Young's (1993) evolutionary model of bargaining, individuals from two populations of bargainers are randomly matched to play the Nash demand game. They make their demands by choosing best replies based on an adaptive play process with incomplete sampling. In our model, a fixed pair of individual bargainers are matched to play the Nash demand game. They make their demands by choosing best replies based on a fictitious play process. Our model explains what bargainers will demand if they expect a long-term relationship with the other bargainer. (A review of recent evolutionary approaches to bargaining to be added here.)

Bounded rationality of the players is incorporated into our model in two ways. In the initial period, the players foresee what will happen following their choices, but allow small differences (Radner's (1986) ϵ-equilibrium concept) when they compare their expected average payoffs. In the subsequent periods, both players use a simple learning rule and make a decision according to a fictitious play.

We show that the fictitious play resolves eventually the inefficiency due to miscoordination. If players miscoordinate in the initial period by demanding too little, they get to coordinate eventually in the way that the player who is less greedy gets a better share. If players miscoordinate in the initial period by demanding too much, they get to coordinate eventually in the way that the player who is greedier gets a worse share.

Our main result is that in the reduced static game they play at the initial period, all the ϵ-equilibria are clustered around the division corresponding to the Nash bargaining solution, when the bargainers are patient (the time discount is small). Furthermore, as the bargainers make a more accurate comparison of payoffs and become more patient (the time discount vanishes) accordingly, the only equilibrium left is the division of the Nash bargaining solution.

2 The strategic model

Two players 1 and 2 are playing the Nash (1953) demand game infinitely many times, starting at time $t = 0$. In each period both players simultaneously announce their demands x and y respectively, where $0 < x, y \leq 1$. That is, the size of the pie has been normalized to 1. If $x+y \leq 1$, they receive $u(x)$ and $v(y)$ in that period respectively. Otherwise, they receive $u(0)$ and $v(0)$ respectively. The utility functions are strictly increasing, concave, and normalized so that $u(0) = v(0) = 0$.

There is a discount of payoff by δ between periods, where $0 \leq \delta < 1$. Alternatively, $(1-\delta)$ can be interpreted as a probability of breakdown, as Binmore, et al. (1986) did. Player 1 receives $u(x_t)I_{[x_t+y_t\leq 1]}(x_t, y_t)$ in each period t, and the average payoff of the infinite sequence of payoffs is

$$\bar{u} \equiv (1-\delta)\sum_{t=0}^{\infty}\delta^t u(x_t)I_{[x_t+y_t\leq 1]}(x_t, y_t),$$

where I is an indicator function. Similarly, player 2 receives $v(y_t)I_{[x_t+y_t\leq 1]}(x_t, y_t)$ in each period t, and the average payoff of the infinite sequence of payoffs is

$$\bar{v} \equiv (1-\delta)\sum_{t=0}^{\infty}\delta^t v(y_t)I_{[x_t+y_t\leq 1]}(x_t, y_t).$$

From the time $t = 1$, both players use a simple learning rule and make a decision according to the fictitious play. For any $t \geq 1$, let $f_t(x)$ denote the relative frequency with which player 1 has chosen x up to time $(t-1)$. Similarly, let $g_t(y)$ denote the relative frequency with which player 2 has chosen y up to time $(t-1)$. According to the fictitious play, players choose x_t and y_t for any $t \geq 1$ as follows:

$$x_t = \arg\max_x \sum_{y:g_t(y)>0} g_t(y)u(x)I_{[x+y\leq 1]}(x, y),$$

$$y_t = \arg\max_y \sum_{x: f_t(x)>0} f_t(x) v(y) I_{[x+y\leq 1]}(x, y).$$

That is, in each period each player chooses his best response to the observed historical frequency of his opponent's choices. For simplicity, we assume that ties are broken in favor of a higher demand.

In the initial period $t = 0$, however, there has been no opponent's action to refer to. In this initial period, they play a one-shot Nash demand game where the payoffs are the average payoff of the infinite sequence of payoffs that they expect in the initial and subsequent periods. Their decisions in this one-shot Nash demand game unambiguously determine the subsequent demands by the fictitious play procedure. In the reduced static game they play at the initial period, the equilibrium concept we employ is Radner's (1986) ϵ-equilibrium. A strategy profile is an ϵ-equilibrium if no player has an alternative strategy that increases his payoff by more than ϵ.

3 The implementation result

Lemma 1 *For any (x_0, y_0), the following hold:*
(1) $x_1 = 1 - y_0$ and $y_1 = 1 - x_0$.
(2) For any $t \geq 2$, x_t must be either x_0 or $(1 - y_0)$, and y_t must be either y_0 or $(1 - x_0)$.
(3) For any $t \geq 1$, $f_t(x_0) + f_t(1 - y_0) = 1$ and $g_t(y_0) + g_t(1 - x_0) = 1$.

Proof. (1) Clearly, $x_1 = \arg\max_x u(x) I_{[x\leq 1-y_0]}(x) = 1 - y_0$, and $y_1 = \arg\max_y v(y) I_{[y\leq 1-x_0]}(y) = 1 - x_0$.
(2) We prove statement (2) by mathematical induction. x_2 must be either x_0 or $(1 - y_0)$ because $x_1 = \arg\max_x[\frac{1}{2}u(x)I_{[x\leq x_0]}(x) + \frac{1}{2}u(x)I_{[x\leq 1-y_0]}(x)]$. Similarly, y_2 must be either y_0 or $(1 - x_0)$.
Suppose that x_t is either x_0 or $(1 - y_0)$, and y_t is either y_0 or $(1 - x_0)$ for any $t \geq 2$ (induction hypothesis). Then x_{t+1} must be either x_0 or $(1 - y_0)$ because $x_{t+1} = \arg\max_x[g_{t+1}(1-x_0)u(x)I_{[x\leq x_0]}(x)+g_{t+1}(y_0)u(x)I_{[x\leq 1-y_0]}(x)]$. Similarly, y_{t+1} must be either y_0 or $(1 - x_0)$.
(3) Statement (3) follows immediately from statements (1) and (2). ■

Let (x^N, y^N) be the division of the Nash bargaining solution, given the utility functions u and v. That is,

$$(x^N, y^N) = \arg\max_{(x,1-x)} u(x)v(1 - x).$$

We define a function ϕ that assigns $y = \phi(x)$ in $[0, 1]$ to each number x in $[0, 1]$ as follows:

- $\phi(x^N) = y^N$.
- If $x \neq x^N$, $\phi(x)$ is the solution of the following equation which is different from $(1 - x)$:

$$u(x)v(1 - x) = u(1 - \phi(x))v(\phi(x)).$$

Since u and v are strictly increasing and concave, $\phi(x)$ is uniquely determined for each $x \in [0, 1]$. The function $\phi(x)$ is strictly increasing and reflects the shape of the Pareto frontier of the feasible alternatives. For example, if u and v are linear then $\phi(x) = x$.

Lemma 2 *(1) If $x_0 + y_0 = 1$, then $\bar{u} = u(x_0)$ and $\bar{v} = v(y_0)$.*
(2) If $x_0 + y_0 < 1$ and $y_0 = \phi(x_0)$, then

$$(1 - \delta)u(x_0) < \bar{u} < (1 - \delta + \delta^2)u(x_0), \quad \lim_{\delta \to 1} \bar{u} = \frac{u(x_0)^2}{u(1 - y_0)},$$

$$(1 - \delta)v(y_0) < \bar{v} < (1 - \delta + \delta^2)v(y_0), \quad \text{and} \quad \lim_{\delta \to 1} \bar{v} = \frac{v(y_0)^2}{v(1 - x_0)}.$$

(3) If $x_0 + y_0 > 1$ and $y_0 = \phi(x_0)$, then

$$(1 - \delta)\delta u(1 - y_0) < \bar{u} < \delta u(1 - y_0), \quad \lim_{\delta \to 1} \bar{u} = \frac{u(1 - y_0)^2}{u(x_0)},$$

$$(1 - \delta)\delta v(1 - x_0) < \bar{v} < \delta v(1 - x_0), \quad \text{and} \quad \lim_{\delta \to 1} \bar{v} = \frac{v(1 - x_0)^2}{v(y_0)}.$$

(4) If $x_0 + y_0 < 1$ and $y_0 > \phi(x_0)$, then

$$(1 - \delta)u(x_0) + \delta^T u(1 - y_0) \leq \bar{u} \leq (1 - \delta^T)u(x_0) + \delta^T u(1 - y_0) \quad \text{and}$$

$$(1 - \delta + \delta^T)v(y_0) \leq \bar{v} \leq v(y_0) \quad \text{for some positive integer } T.$$

If δ is sufficiently large, as $\delta \to 1$, $\bar{u}$ monotonically increases towards

$$\lim_{\delta \to 1} \bar{u} = u(1 - y_0)$$

and $\bar{v}$ monotonically increases towards

$$\lim_{\delta \to 1} \bar{v} = v(y_0).$$

Similarly, if $x_0 + y_0 < 1$ and $y_0 < \phi(x_0)$, then

$$(1 - \delta + \delta^T)u(x_0) \leq \bar{u} \leq u(x_0) \quad \textit{and}$$

$$(1 - \delta)v(y_0) + \delta^T v(1 - x_0) \leq \bar{v} \leq (1 - \delta^T)v(y_0) + \delta^T v(1 - x_0)$$

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(5) If $x_0 + y_0 > 1$ and $y_0 > \phi(x_0)$, then

$$(1 - \delta + \delta^T)u(x_0) \leq \bar{u} \leq u(x_0) \quad \textit{and}$$

$$(1 - \delta)v(y_0) + \delta^T v(1 - x_0) \leq \bar{v} \leq (1 - \delta^T)v(y_0) + \delta^T v(1 - x_0)$$

for some positive integer T.

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Proof. (1) Statement (1) follows from Lemma 1 because $x_0 + y_0 = 1$.
(2)-(5) We prove only statement (2) (the case of $[x_0 + y_0 < 1$ and $y_0 = \phi(x_0)]$) and the first part of statement (4) (the case of $[x_0 + y_0 < 1$ and $y_0 > \phi(x_0)]$) omitting the tedious repetition for the other cases.

We first prove the first part of statement (4). Let $f^* \equiv \frac{u(x_0)}{u(1-y_0)}$ and $g^* \equiv \frac{v(y_0)}{v(1-x_0)}$. For any $t \geq 1$,

$$f_{t+1}(x_0) = \begin{cases} \frac{t f_t(x_0)}{t+1} & \text{if } g_t(y_0) \geq f^* \\ \frac{t f_t(x_0)+1}{t+1} & \text{if } g_t(y_0) < f^*, \end{cases}$$

and

$$g_{t+1}(y_0) = \begin{cases} \frac{t g_t(y_0)}{t+1} & \text{if } f_t(x_0) \geq g^* \\ \frac{t g_t(y_0)+1}{t+1} & \text{if } f_t(x_0) < g^*. \end{cases}$$

We define the following four states regarding the pair of relative frequencies $(f_t(x_0), g_t(y_0))$:

- state $[>>]$: $f_t(x_0) \geq g^*$ and $g_t(y_0) \geq f^*$,
- state $[><]$: $f_t(x_0) \geq g^*$ and $g_t(y_0) < f^*$,
- state $[<>]$: $f_t(x_0) < g^*$ and $g_t(y_0) \geq f^*$,
- state $[<<]$: $f_t(x_0) \geq g^*$ and $g_t(y_0) \geq f^*$.

We define state $[>>]^*$ and $[<<]^*$ as follows:

- state $[>>]^*$: $f_t(x_0) \geq g^*$, $g_t(y_0) \geq f^*$, and $f_t(x_0) = g_t(y_0)$,
- state $[<<]^*$: $f_t(x_0) < g^*$, $g_t(y_0) < f^*$, and $f_t(x_0) = g_t(y_0)$.

Note that $0 < f^* < g^* < 1$ because $x_0 + y_0 < 1$ and $y_0 > \phi(x_0)$. At $t = 1$, $f_1(x_0) = g_1(y_0) = 1$ and therefore the pair of relative frequencies is in the state [>>]*. At $t = 2$, $f_2(x_0) = g_2(y_0) = \frac{1}{2}$ and the state can be either [>>]*, [<>], or [<<]* depending on the values of f^* and g^*. However, it cannot be [><] because $f^* < g^*$. We can establish the following regarding the transition between states:

- If the current state is [<>], the next state is always [<>] (Borrowing a term from the Markov chain theory, the state [<>] is an absorbing state).
- If the current state is [>>]*, the next state must be either [>>]*, [<>], or [<<]*.
- If the current state is [<<]*, the next state must be either [>>]*, [<>], or [<<]*.

Therefore, the state must be either [>>]*, [<>], or [<<]* for any $t \geq 2$. Furthermore, the state becomes [<>] eventually because the change in the relative frequency between two periods becomes smaller than $(g^* - f^*)$ eventually. That is, an oscillation between the states [>>]* and [<<]* cannot last for ever.

Let T_1 be the number of periods when the state is [>>]* before the state becomes [<>] eventually, and T_2 the number of periods when the state is [<<]* before the state becomes [<>] eventually. The numbers T_1 and T_2 are nonnegative integers. By taking $T \equiv T_1 + T_2 + 1$, we have

$$(1-\delta)u(x_0) + \delta^T u(1-y_0) \leq \bar{u} \leq (1-\delta^T)u(x_0) + \delta^T u(1-y_0) \quad \text{and}$$

$$(1-\delta+\delta^T)v(y_0) \leq \bar{v} \leq v(y_0) \quad \text{for some positive integer } T.$$

If δ is sufficiently large, as $\delta \to 1$, $\bar{u}$ monotonically increases towards

$$\lim_{\delta \to 1} \bar{u} = u(1-y_0)$$

and $\bar{v}$ monotonically increases towards

$$\lim_{\delta \to 1} \bar{v} = v(y_0).$$

This ends the proof of the first part of statement (4). Now, we prove statement (2).

If $x_0 + y_0 < 1$ and $y_0 = \phi(x_0)$, then $f^* = g^*$. Therefore, the state must be either $[>>]^*$ or $[<<]^*$ for any $t \geq 1$. The state oscillates between the states $[>>]^*$ and $[<<]^*$ for ever, and the sequence $\{f_t(x_0)\}$ converges to $\frac{u(x_0)}{u(1-y_0)}$ $(= \frac{v(y_0)}{v(1-x_0)})$. Therefore, we obtain

$$(1-\delta)u(x_0) < \bar{u} < (1-\delta+\delta^2)u(x_0), \quad \lim_{\delta \to 1} \bar{u} = \frac{u(x_0)^2}{u(1-y_0)},$$

$$(1-\delta)v(y_0) < \bar{v} < (1-\delta+\delta^2)v(y_0), \quad \text{and} \quad \lim_{\delta \to 1} \bar{v} = \frac{v(y_0)^2}{v(1-x_0)}.$$

This ends the proof of the first part of statement (2). ■

Therefore, in this model, the perpetual miscoordination, as in Young (1993) p. 152, does not happen generically. The limits for the linear utility case were studied by He (2004). To see how long it takes for the demands to reach the limit in the linear utility case, we refer readers to He.

Theorem 1 *For any $\epsilon > 0$, there exists $\delta^*(\epsilon) < 1$ such that ϵ-equilibria are clustered around (x^N, y^N) for any $\delta \geq \delta^*(\epsilon)$. As $\epsilon \to 0$ and $\delta^*(\epsilon) \to 1$ accordingly, the only equilibrium left is the division of the Nash bargaining solution.*

Proof. Using the limit average payoffs $\lim_{\delta \to 1} \bar{u}$ and $\lim_{\delta \to 1} \bar{v}$ that we have obtained in Lemma 2, we can get the best response correspondences for player 1 (illustrated in Figure 1)

$$x^*(y) = \begin{cases} [0, \phi^{-1}(y)) \cup [1-y, 1] & \text{if } y < y^N \\ [0,1] & \text{if } y = y^N \\ \emptyset & \text{if } y > y^N, \end{cases}$$

and for player 2

$$y^*(x) = \begin{cases} [0, \phi(x)) \cup [1-x, 1] & \text{if } x < x^N \\ [0,1] & \text{if } x = x^N \\ \emptyset & \text{if } x > x^N. \end{cases}$$

Note that the average payoff functions are continuous except at the points of $y = \phi(x)$. One can see easily that the only pure-strategy Nash equilibrium in this case is (x^N, y^N).

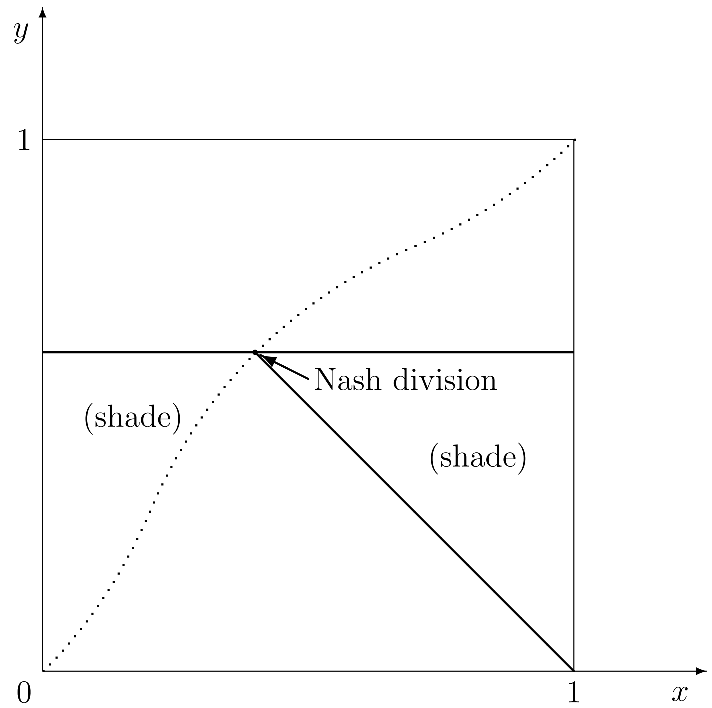

Figure 1: Limit-Average-Payoff Best Response Correspondence for Player 1

Using the limit average payoffs $\lim_{\delta\to 1}\bar{u}$ and $\lim_{\delta\to 1}\bar{v}$ that we have obtained in Lemma 2, we can get the ϵ-best response correspondence for player 1 (illustrated in Figure 2)

$$x^*_\epsilon(y) = \begin{cases} [0, \phi^{-1}(y)) \cup [u^{-1}(u(1-y)-\epsilon), 1] & \text{if } y < y^{(1)} \\ [0, \phi^{-1}(y)) \cup (\phi^{-1}(y), 1] & \text{if } y^{(1)} \leq y < y^{(2)} \\ [0, 1] & \text{if } y^{(2)} \leq y \leq y^{(3)} \\ [0, \phi^{-1}(y)) \cup (\phi^{-1}(y), 1] & \text{if } y^{(3)} < y \leq y^{(4)} \\ [u^{-1}(u(\phi^{-1}(y))-\epsilon), \phi^{-1}(y)) & \text{if } y > y^{(4)}, \end{cases} \tag{1}$$

where

$$y^{(1)} \text{ is the solution of } \phi^{-1}(y) = u^{-1}(u(1-y)-\epsilon),$$

$$y^{(2)} \text{ is the solution of } \frac{u(\phi^{-1}(y))^2}{u(1-y)} = u(1-y)-\epsilon,$$

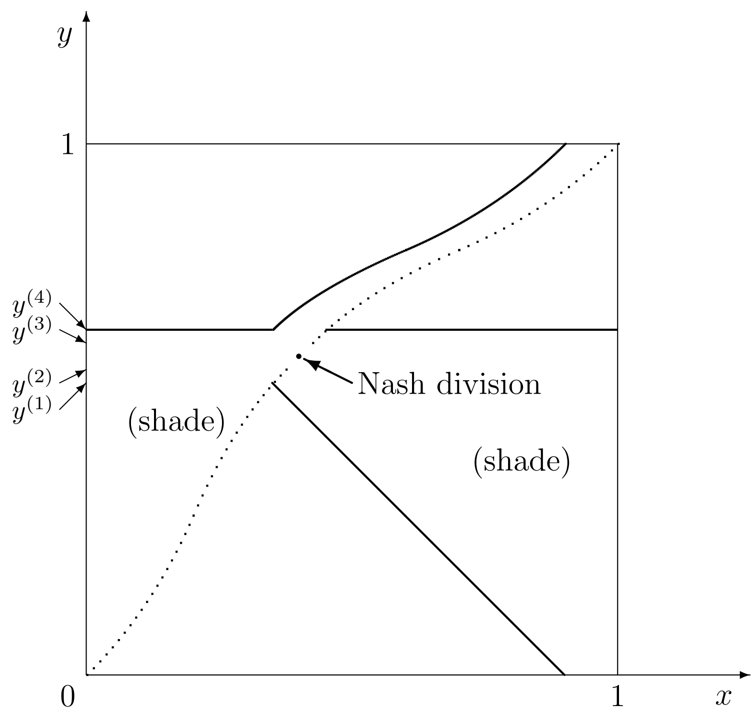

Figure 2: LIMIT-AVERAGE-PAYOFF ϵ-BEST RESPONSE CORRESPONDENCE FOR PLAYER 1

$$y^{(3)} \text{ is the solution of } \frac{u(1-y)^2}{u(\phi^{-1}(y))} = u(\phi^{-1}(y)) - \epsilon, \text{ and}$$

$$y^{(4)} \text{ is the solution of } \phi^{-1}(y) = u^{-1}(u(1-y)+\epsilon).$$

Similarly, the ϵ-best response correspondence for player 2 is

$$y^*_\epsilon(x) = \begin{cases} [0,\phi(x)) \cup [v^{-1}(v(1-x)-\epsilon), 1] & \text{if } x < x^{(1)} \\ [0,\phi(x)) \cup (\phi(x), 1] & \text{if } x^{(1)} \leq x < x^{(2)} \\ [0,1] & \text{if } x^{(2)} \leq x \leq x^{(3)} \\ [0,\phi(x)) \cup (\phi(x), 1] & \text{if } x^{(3)} < x \leq x^{(4)} \\ [v^{-1}(v(\phi(x))-\epsilon), \phi(x)) & \text{if } x > x^{(4)}, \end{cases} \tag{2}$$

where

$$x^{(1)} \text{ is the solution of } \phi(x) = v^{-1}(v(1-x)-\epsilon),$$

$$x^{(2)} \text{ is the solution of } \frac{u(x)^2}{u(1-\phi(x))} = v(1-x) - \epsilon,$$

$$x^{(3)} \text{ is the solution of } \frac{u(1-\phi(x))^2}{u(x)} = v(\phi(x)) - \epsilon, \text{ and}$$

$$x^{(4)} \text{ is the solution of } \phi(x) = v^{-1}(v(1-x)+\epsilon).$$

The set of ϵ-equilibria based on the limit average payoffs is illustrated in Figure 3. This set is a subset of

$$\{x : 1 - v^{-1}(v(y^{(4)})+\epsilon) \leq x \leq \phi^{-1}(v^{-1}(v(y^{(4)})+\epsilon))\}\times$$
$$\{y : 1 - u^{-1}(u(x^{(4)})+\epsilon) \leq y \leq \phi(u^{-1}(u(x^{(4)})+\epsilon))\}. \tag{3}$$

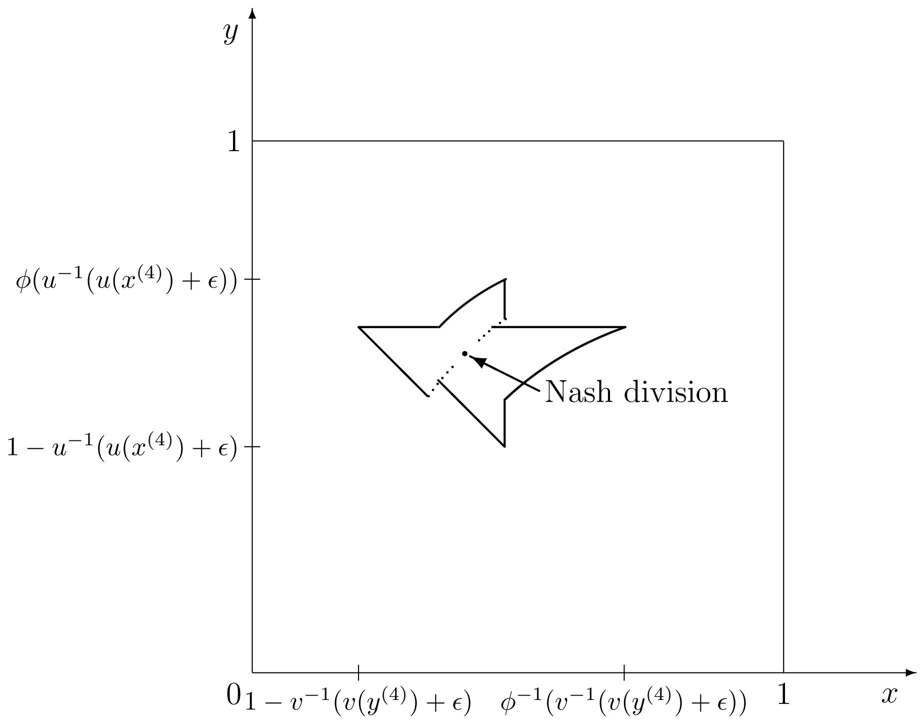

Figure 3: Limit-Average-Payoff ϵ-Equilibria

If we choose a sufficiently large $\delta^*(\epsilon) < 1$, then the set of ϵ-equilibria will be a subset of (3) above for any $\delta \geq \delta^*(\epsilon)$. As $\epsilon \to 0$,

$$1 - v^{-1}(v(y^{(4)})+\epsilon) \to x^N,$$

$$\phi^{-1}(v^{-1}(v(y^{(4)})+\epsilon)) \to x^N,$$

$$1 - u^{-1}(u(x^{(4)}) + \epsilon) \to y^N, \text{ and}$$
$$\phi(u^{-1}(u(x^{(4)}) + \epsilon)) \to y^N.$$

This implies that as $\epsilon \to 0$ and $\delta^*(\epsilon) \to 1$ accordingly, only (x^N, y^N) remains as the limit of ϵ-equilibria. ∎

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