The performance of the tests of linear and logarithmic transformations for integrated processes with stochastic unit roots

GAWON YOON

Department of Economics, Pusan National University, Pusan, S. Korea, 609-735

E-mail: gyoon@pusan.ac.kr

Summary

This paper shows that the recently proposed tests of linear and logarithmic transformations for integrated processes against each other by Kobayashi and McAleer (1999) are severely biased for alternative hypotheses when the true data generating process is a stochastic unit root. An empirical example with four daily bond yields is also provided.

Keywords: data transformation, (stochastic) unit roots, nonnested tests.

1. INTRODUCTION

Economic variables are routinely transformed into logarithms before they are subject to empirical analysis like unit root tests and cointegration analysis. For instance, in their classic study of unit roots among fourteen major macroeconomic variables, Nelson and Plosser (1982) use all variables in logarithms except one. Corradi and Swanson (2003) show that unit root tests are severely biased when data are incorrectly transformed and propose a new simple randomized procedure to choose between linear and logarithmic transformations. Much discussion has been made on the proper transformations of integrated economic variables and on the properties of the transformed ones. Only partial lists include Granger and Hallman (1991), Ermini and Granger (1993), Corradi (1995), Ermini and Hendry (1995), and Franses and McAleer (1998).

Recently, Kobayashi and McAleer (1999, KM hereafter) develop a nonnested procedure to discriminate between linear and logarithmic transformations for integrated processes against each other, based on the different behavior of correlation coefficients under misspecified models. Their procedure is essentially a test for a specific type of heteroskedasticity occurring under a misspecified model and has potentially wide applications to various economic variables. In this paper, it is shown that when a variable follows a particular time series process, the KM tests are severely biased and that the tests should be used with care. KM already note that their tests are not reliable for ARCH and that they are not designed to detect ARCH-type heteroskedasticity. This paper provides another class of models that has been found to be empirically very plausible, in which the KM tests produce quite unreliable results. The time series model considered here is a stochastic unit root [STUR] process introduced in Granger and Swanson (1997), Leybourne, McCabe, and Tremayne (1996), and McCabe and Tremayne (1993), among others. An earlier discussion on STUR is also contained in Granger (1987). This paper shows that the KM tests are severely biased for alternative hypotheses when the data generating process is STUR. Computer simulations confirm the main arguments of this paper. An empirical example is discussed with four daily bond yields to shed more light on their dynamic properties. In the next section, the KM tests for linear and logarithmic integrated models against each other are briefly reviewed.

2. THE KOBAYASHI AND MCALEER (1999) TESTS FOR LINEAR AND LOGARITHMIC INTEGRATED MODELS

KM propose a nonnested testing procedure that discriminates between linear and logarithmic transformations for integrated variables. Their procedure is derived from the different behavior of correlation coefficients under misspecified models. A linear integrated model for y_t with positive drift is defined as follows:

$$y_t - y_{t-1} = \mu + e_t \tag{1}$$

for $t = 1, \quad ..., \quad n$, $\mu > 0$ and $\alpha(L)e_t = \varepsilon_t$ with $\alpha(L) = 1 - \alpha_1 L - \cdots - \alpha_p L^p$. ε_t is a serially independent random variable, with $E\varepsilon_t = 0$, $Var(\varepsilon_t) = \sigma^2$, $E\varepsilon_t^3 = 0$, and $E\varepsilon_t^4 < \infty$. It is also assumed that $\alpha(L) = 0$ has all roots outside unit circle. A logarithmic integrated model with positive drift is

$$\ln(y_t) - \ln(y_{t-1}) = \eta + u_t \tag{2}$$

where $\eta > 0$ and $\beta(L)u_t = \varsigma_t$ with $\beta(L) = 1 - \beta_1 L - \cdots - \beta_p L^p$. ς_t is a serially independent random variable, with $E\varsigma_t = 0$, $Var(\varsigma_t) = \omega^2$, $E\varsigma_t^3 = 0$, and $E\varsigma_t^4 < \infty$. It is also assumed that $\beta(L) = 0$ has all roots outside unit circle.

KM suggest four different tests, the V_1, V_2, U_1, and U_2 tests. A heuristic discussion of their tests is provided below with $p = 0$. For more details, the original work should be consulted. Under the logarithmic integrated model (2), $\Delta y_t \equiv y_t - y_{t-1} = y_{t-1}(\exp(\eta + u_t) - 1)$, so that $Var(\Delta y_t \mid I_{t-1})$ is proportional to y_{t-1}^2. I_t denotes information available at time t. Hence, y_{t-1}^2 and $(\Delta y_t)^2$ have a positive correlation under (2). Under the linear model (1), however, y_{t-1}^2 and $(\Delta y_t)^2$ are not correlated. Therefore, the correlation coefficient between y_{t-1}^2 and $(\Delta y_t)^2$ could be used as a test statistic for the linear integrated model. Because of a better asymptotic approximation to the asymptotic distribution, KM base the V_1 and U_1 tests on the correlation between y_{t-1} and $(\Delta y_t)^2$. Formally, the V_1 test takes the linear model (1) as a null hypothesis against the logarithmic model (2) with positive drift, while the U_1 test assumes no drift. The test statistic V_1

$$V_1 = n^{-3/2}\sum_{t=1}^{n} y_{t-1}\left(z_t^2 - s^2\right)\Big/\left(s^4 m^2/6\right)^{1/2}$$

has a standard normal distribution asymptotically, where z_t is the residual from regressing Δy_t on 1, Δy_{t-1}, ..., Δy_{t-p}, $z_t \equiv \Delta y_t - a_0 - a_1 \Delta y_{t-1} - \ldots - a_p \Delta y_{t-p}$, $s^2 \equiv n^{-1}\sum_{t=1}^{n} z_t^2$ is the sample variance of z_t, and $m \equiv a_0 \big/ \left(1 - a_1 - \ldots - a_p\right)$. a_i is the least squares estimate of α_i, $i = 1, \ldots, p$. If $\mu = 0$ in (1), the test statistic U_1 is

$$U_1 = n^{-1}\sum_{t=1}^{n} y_{t-1}\left(z_t^2 - s^2\right)\Big/\left(2^{1/2} a(1)^{-1} s^3\right)$$

where $z_t \equiv \Delta y_t - a_1 \Delta y_{t-1} - \ldots - a_p \Delta y_{t-p}$. KM show that the U_1 test has a nonstandard distribution and provide its critical values. The $U_{\bullet}$ tests utilize small-σ expansion employed in Bickel and Doksum (1981).

When the data are generated by (1) with $p = 0$, note that $\Delta \ln(y_t) = \ln\left(1 + (\mu + e_t)/y_{t-1}\right)$ and $\exp\{\Delta \ln(y_t)\} = 1 + (\mu + e_t)/y_{t-1}$. Hence, $Var\left(\exp\{\Delta \ln(y_t)\} \mid I_{t-1}\right)$ increases with $1/y_{t-1}^2$ and consequently $Var\left(\Delta \ln(y_t) \mid I_{t-1}\right)$ tends to increase with $-\ln(y_{t-1})$. Under the logarithmic model (2), however, $\Delta \ln(y_t)$ has a constant variance that does not depend on $-\ln(y_{t-1})$. From this observation, the correlation between $(\Delta \ln y_t)^2$ and $-\ln(y_{t-1})$ can be used as a test statistic for the logarithmic integrated model. Formally, the V_2 test takes the logarithmic model (2) as a null hypothesis against the linear model (1) with positive drift, while the U_2 test assumes no drift. The test statistic V_2

$$V_2 = n^{-3/2}\sum_{t=1}^{n}\left(-\ln y_{t-1}\right)\left(v_t^2 - w^2\right)\Big/\left(w^4 h^2/6\right)^{1/2}$$

has a standard normal distribution asymptotically, where v_t is the residual from regressing $\Delta\ln y_t$ on 1, $\Delta\ln y_{t-1}$, ..., $\Delta\ln y_{t-p}$, $v_t \equiv \Delta\ln y_t - b_0 - b_1\Delta\ln y_{t-1} - \ldots - b_p\Delta\ln y_{t-p}$, $w^2 \equiv n^{-1}\sum_{t=1}^{n} v_t^2$ is the sample variance of v_t, and $h \equiv b_0\big/\left(1 - b_1 - \ldots - b_p\right)$. b_i is the least squares estimate of β_i. When $\eta = 0$ in (2), the test statistic U_2 is

$$U_2 = n^{-1}\sum_{t=1}^{n}\left(-\ln y_{t-1}\right)\left(v_t^2 - w^2\right)\Big/\left(2^{1/2} b(1)^{-1} w^3\right)$$

where $v_t \equiv \Delta\ln y_t - b_1\Delta\ln y_{t-1} - \ldots - b_p\Delta\ln y_{t-p}$. The U_2 test has a nonstandard distribution and its critical values are provided in KM. It should be also added that the KM tests may yield inconclusive results in that they rejects or accepts both null hypotheses of a linear integrated model and a logarithmic integrated model at the same time. These $V_\cdot$ and $U_\cdot$ tests will be applied to stochastic unit root processes, which are discussed in the next section.

3. STOCHASTIC UNIT ROOT PROCESSES

In this section, the following simple time series model is considered:

$$y_t = (1 + a_t) y_{t-1} + \varepsilon_t \tag{3}$$

for $t = 1, \ldots, n$, with $a_t \sim i.i.N(0, \quad \sigma_a^2)$ and $\varepsilon_t \sim i.i.N(0, \quad \sigma_\varepsilon^2)$. The normality assumption is only for convenience. a_t and ε_t are assumed to be independent. Given that $Ea_t = 0$, y_t has a unit root only on average and is an example of STUR discussed in Granger and Swanson (1997) and Leybourne et al. (1996). While other functional forms are possible for modeling STUR, (3) is most convenient for the discussion in this section. When $\sigma_a^2 = 0$, y_t becomes a standard (fixed) unit root process. Granger and Swanson (1997) show that a STUR process is very hard to tell from a standard (fixed) unit root process by standard unit root tests. A STUR model is especially useful in modelling financial time series because y_t behaves like a martingale on average, while Δy_t is conditionally heteroskedastic;

$$Var(\Delta y_t \mid I_{t-1}) = \sigma_a^2 y_{t-1}^2 + \sigma_\varepsilon^2. \tag{4}$$

There is strong belief that stochastic unit roots are prevalent among economic variables. For example, Granger (2000) notes, “most economic variables that appear to be $I(1)$ are better described as STUR.” Empirical evidence for STUR is found in Bleaney et al. (1999) for exchange rates and in Sollis et al. (2000) for stock price indices. See also Leybourne, McCabe, and Mills (1996) and Leybourne et al. (1996) for additional evidence. Abadir (2004) provides economic rationale for STUR. Interestingly, Marriott et al. (2003) recently report STUR-like behavior in consumer prices, velocity, and bond yields among the Nelson-Plosser data set, using a Bayesian graphical analysis.

It is the purpose of this paper to show that the KM tests are biased for alternative hypotheses when the data generating process is STUR. It is easy to infer from (4) that the correlation between $(\Delta y_t)^2$ and y_{t-1}^2 is positive for STUR.[1] This is different from the result for a (standard) linear integrated process with $\sigma_a^2 = 0$, in which $(\Delta y_t)^2$ and y_{t-1}^2 have no correlation. Recall that the V_1 and U_1 tests of KM are based on the different behavior of the correlation coefficients between $(\Delta y_t)^2$ and y_{t-1}^2 under linear and logarithmic integrated models. The correlation is positive for a logarithmic integrated process (2), as already shown in the previous section. Therefore, the V_1 and U_1 tests will tend to find that a STUR process (3) in level is a logarithmic integrated process. Now suppose instead that $\ln(y_t)$ follows STUR:

$$\Delta \ln(y_t) = \varphi_t \ln(y_{t-1}) + \zeta_t . \tag{5}$$

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[1] In fact, the correlation becomes positive for processes more general than STUR. For instance, consider $y_t = (\varpi + a_t) y_{t-1} + \varepsilon_t$, where ϖ is not necessarily equal to 1. Given that this paper regards STUR as a plausible alternative to a standard (fixed) unit root process, only STUR with $\varpi = 1$ will be treated here. When $\varpi > 1$, the process is what Granger (2000) calls an explosive stochastic root process.

with $\varphi_t \sim i.i.N\left(0,\ \ \sigma_\varphi^2\right)$ and $\zeta_t \sim i.i.N\left(0,\ \ \sigma_\zeta^2\right)$. φ_t and ζ_t are assumed to be independent. It follows that $Var\left(\Delta \ln\left(y_t\right) | I_{t-1}\right) = \sigma_\varphi^2\left(\ln\left(y_{t-1}\right)\right)^2 + \sigma_\zeta^2$ and that the conditional variance tends to increase with $-\ln\left(y_{t-1}\right)$. As already shown in the previous section, this relation is what is expected from a linear integrated process (1) and the V_2 and U_2 tests for logarithmic integrated models are derived from this observation. Therefore, the V_2 and U_2 tests will tend to conclude that y_t in (5) is a linear integrated process, even though in fact it follows STUR in logarithm.

Computer simulations confirm the above arguments. A simulation design is discussed first. STUR processes (3) and (5) are generated with $\sigma_a^2 = 0.02^2$, $\sigma_\varphi^2 = 0.1^2$, and $\sigma_\varepsilon^2 = \sigma_\zeta^2 = 1$ for a sample of $\{50,\ 100,\ 200,\ 500,\ 1000,\ 2000,\ 4000,\ 6000\}$ after discarding initial 100 observations. The simulation is repeated 10,000 times. The rejection frequency of the $V_\bullet$ and $U_\bullet$ tests is reported in table 1 for a nominal size of 5% with $p = 1$. Similar results are also found with different values of p and other significance levels. When the true data generating process is STUR in level (3), the V_1 and U_1 tests are calculated. The second and third columns of table 1 show that the V_1 and U_1 tests are consistent for (3); the rejection frequency approaches 1, as sample size increases. For instance, when $n \geq 1000$, the V_1 and U_1 tests reject the null of a linear integrated model almost all the time. Moreover, the tests are severely biased for the alternative hypothesis of a logarithmic integrated model, when the possibility of STUR is not considered; the tests indicate that the data should be modeled as a logarithmic integrated process, when in fact they are STUR in level. For the true data generating process of STUR in logarithm, (5), the V_2 and U_2 tests are applied. The last two columns of table 1 show that both the V_2 and U_2 tests reject the null hypothesis of a logarithmic integrated model for the alternative of a linear integrated model about 50% of the simulations for large sample sizes. For instance, with $n = 6000$, the rejection frequency is 0.48 and 0.45 for the V_2 and U_2 tests, respectively. It is not clear from the simulation results whether the V_2 and U_2 tests are consistent for (5), as it appears that the rejection frequency increases only slowly as sample size increases. In sum, the KM tests of linear and logarithmic transformations for integrated processes are biased for the alternative hypothesis when the true data generating process is STUR. There is indeed certain evidence that some economic variables are better characterized as STUR than as a standard (fixed) unit root process. The results presented in this section indicate that the KM tests should be used care, especially when STUR is a serious candidate for modelling economics variables. In an ideal situation where a user is confident that the state of affairs is only either (1) or (2), the use of the $V_\cdot$ and $U_\cdot$ tests should be fine. In practice, however, one may wish to use pre-tests to examine if the process of interest is a unit root or a STUR process. The impact of such pre-tests on the size and power properties on the $V_\cdot$ and $U_\cdot$ tests is not clear, especially when it is not very easy to distinguish between a standard unit root and a STUR process. Developing testing procedures for linear and logarithmic transformations for integrated processes robust to STUR would be interesting and is left for future research. Moreover, testing procedures for linear and logarithmic models for STUR against each other, see (3) and (5), à la the KM tests would be also useful and be a complementary tool to the KM tests. An empirical example is presented in the next section with daily bond yields, for which evidence for STUR was previously found.

4. AN EMPIRICAL EXAMPLE WITH FOUR DAILY BOND YIELDS

KM apply their testing procedure to the daily U.S. bond yields available in Mills (1993).[2] In fact, three more bond yields are available additionally in Mills (1993) for the U.K., Japan, and West Germany. These bond yields, with less than 5 years to maturity, are previously tested for STUR in Leybourne et al. (1996). In this section, all four bond yields will be studied and they will be denoted as r_t^{US}, r_t^{UK}, r_t^{JP} and r_t^{WG}, respectively, from now on. The sample period is from April 1, 1986 to December 29, 1989, a total of 960 observations. Figure 1 shows the data series in levels and their first differences. The large negative values in the first differenced yields around the 390th ~ 400th

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[2] KM also study the quarterly observations on U.S. real private consumption of durable goods. Since the data series is short with only 97 observations, it will not be discussed here.

observations correspond to the stock market crash in October 1987. Standard *ADF* tests conclude that the daily bond yields are all difference stationary both in levels and in logarithms. Since they are well known, the *ADF* test results are not reported; see Mills and Mills (1991) or Mills (1993) for details.

STUR testing procedures developed in Leybourne et al. (1996) and Leybourne et al. (1996) are briefly reviewed here with the following model:

$$y_t = (1+\delta_t)\, y_{t-1} + \varepsilon_t \tag{6}$$

where $\delta_0 = 0$ and $\delta_t = \rho\delta_{t-1} + \eta_t$ with $|\rho| \le 1$. It is also assumed that $\varepsilon_t \sim iiN(0,\ \ \sigma_\varepsilon^2)$ and $\eta_t \sim iiN(0,\ \ \sigma_\eta^2)$ and that ε_t and η_t are independent. If $\sigma_\eta^2 > 0$, y_t has a unit root only on average. (6) might be generalized to $y_t^* = (1+\delta_t)\, y_{t-1}^* + \varepsilon_t$ where $y_t^* = y_t - \lambda_t - \sum_{i=1}^{p}\phi_i y_{t-i}$ and λ_t is a polynomial in *t*. The lag polynomial $\phi(L) = 1 - \phi_1 L - \ldots - \phi_k L^p$ is assumed to have all roots outside unit circle. Different procedures are available to test the null H_0: $\sigma_\eta^2 = 0$ against H_1: $\sigma_\eta^2 > 0$. Leybourne, McCabe, and Tremayne (1996) test the null hypothesis with $|\rho| < 1$ and with either $\lambda_t = \beta + \gamma t + \xi t(t+1)/2$ or $\lambda_t = \beta + \gamma t$. The test statistics are denoted Z_1 and Z_2 for different choice of λ_t. Leybourne, McCabe, and Mills (1996) instead test the null hypothesis with $\rho = 1$. For the different choice of λ_t, their test statistics are E_1 and E_2, respectively. More details on the testing procedures are available in Taylor and van Dijk (2002).

STUR test results are shown in table 2 for the levels and logarithms of the data, respectively, with the $Z_{\bullet}$ tests. Some evidence for STUR is found in the bond yields. For instance, according to the Z_1 statistics, r_t^{JP} and r_t^{WG} have STUR both in levels and in logarithms. Also, r_t^{US} is found to be STUR in level with the Z_2 test at the 10% significance level. r_t^{UK} is found to be a standard (fixed) unit root process both in level and in logarithm. These STUR test results are already available in Leybourne et al. (1996). However, it turns out that for West Germany Leybourne et al. (1996) report results with $p=6$, not $p=3$ as claimed in their table VII. Results are not changing much if different values of *p* are used, however. In this paper, p is selected by testing the significance of the last lag included, as discussed in Ng and Perron (1995) for instance. Very similar results for STUR in the bond yields are also reported in Marriott and Yoon (2003) with an explanatory data analysis based on a Bayesian graphical approach.

The KM tests for linear and logarithmic integrated models are also applied to the four bond yields and the results are reported in table 3 with the $U_{\bullet}$ tests. As the bond yields do not have apparent trends, the results from the $V_{\bullet}$ tests are not reported. However, very similar results are also found with the $V_{\bullet}$ tests. The tests find that r_t^{US} is a logarithmic integrated process, while r_t^{UK} and r_t^{JP} are linear integrated processes. For r_t^{WG}, the test results are not conclusive, because both null hypotheses of a linear integrated model and a logarithmic integrated one are rejected by the U_1 and U_2 tests, respectively.

From the simulation results reported in table 1 on the behavior of the KM tests under STUR, the following (very tentative) observations might be provided to the empirical results reported in tables 2 and 3 on the time series behavior of the daily bond yields. From the evidence found in table 2 that r_t^{US} is STUR in level, the simulation results in table 1 indicate that r_t^{US} should be found as a logarithmic integrated process by the KM tests. Indeed, the U_1 test concurs; it is significant at the 1% level, as reported in table 3. However, this finding from the U_1 test might be spurious due to the possible presence of STUR in r_t^{US} in level. Of course, if one believes that r_t^{US} is a logarithmic integrated process, he would conclude that both the STUR and $U_{\bullet}$ tests produce the same findings. However, the main purpose of this paper is to show that the KM tests are biased for alternative hypotheses when the data generating process is better characterized as STUR. Further, it is safe to assume that r_t^{UK} is a linear integrated process, and not STUR, from the results in tables 2 and 3. r_t^{JP} is found be STUR both in level and logarithm. From the simulation results in table 1, if r_t^{JP} is STUR in level, the U_1 test is most likely to conclude that it is a logarithmic integrated process. However, table 3 shows that the null of logarithmic integratedness is strongly rejected for r_t^{JP}. Hence, it might be tentatively concluded that r_t^{JP} is STUR in logarithm. For r_t^{WG} the test results are not concrete enough to give a definite answer whether it is a standard (fixed) unit root or

STUR in level or in logarithm. Of course, these interpretations are only tentative and are based on the strong belief that STUR should be seriously considered as a possible candidate for time series models for the daily bond yields.

5. CONCLUSIONS

Correct transformations of economics variables are important to properly understand their dynamic behavior. Recently, Kobayashi and McAleer (1999) propose a nonnested testing procedure to choose between linear and logarithmic models against each other for integrated processes. This paper shows that their testing procedure is not robust and is severely biased for alternative hypotheses when data follow a stochastic unit root process. There is certain evidence that stochastic unit root processes are prevalent among economic variables, which would be very hard to tell from standard unit root processes. Therefore, the new tests of Kobayashi and McAleer (1999) should be used with care.

Acknowledgements

This work was done during the author's visit to the University of York, whose hospitality, especially Karim Abadir, is gratefully appreciated. Karim Abadir, Lesley Godfrey, Tae-Hwan Kim, Chirok Han, and Namwon Hyung provided helpful comments. Of course, all remaining errors are mine. Earlier versions were presented at the 2003 annual conference of New Zealand Econometrics Study Group at Wellington and the 2004 annual meeting of the Korean Association of Applied Economics in Seoul. This research was supported by the Pusan National University Research Grant.

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Table 1. Rejection frequency of the $V_{\cdot}$ and $U_{\cdot}$ tests for stochastic unit root processes

Sample size	V_1 Null: linear	U_1 Null: linear	V_2 Null: logarithmic	U_2 Null: logarithmic
50	0.35	0.13	0.21	0.05
100	0.53	0.26	0.21	0.05
200	0.81	0.51	0.22	0.06
500	0.99	0.87	0.31	0.15
1000	1.00	0.99	0.38	0.25
2000	1.00	1.00	0.46	0.37
4000	1.00	1.00	0.48	0.43
6000	1.00	1.00	0.48	0.45

Rejection frequency is reported in each cell at the 5% nominal size. For the V_1 and U_1 tests, the data generating process is a stochastic unit root process in level (3) with $\sigma_a^2 = 0.02^2$ and $\sigma_\varepsilon^2 = 1$. For the V_2 and U_2 tests, the data generating process is a stochastic unit root process in logarithm (5) with $\sigma_\varphi^2 = 0.1^2$ and $\sigma_\zeta^2 = 1$. The simulation is repeated 10,000 times, after discarding initial 100 observations.

Table 2. Stochastic unit root tests for daily bond yields

Country	p	In level		In logarithm	
		Z_1	Z_2	Z_1	Z_2
U.S.	3	.093	.296*	.040	.139
U.K.	3	.044	.053	.047	.195
Japan	5	.312***	-.304	.276***	.051
West Germany	6	.201**	-.291	.218**	.211

Sample period: April 1, 1986 ~ December 29, 1989, a total of 960 observations. *** (**, *) denotes that the test statistic is significant at the 1% (5%, 10%) significance level. p denotes an autoregressive truncation lag.

Table 3. The Kobayashi and McAleer tests for daily bond yields

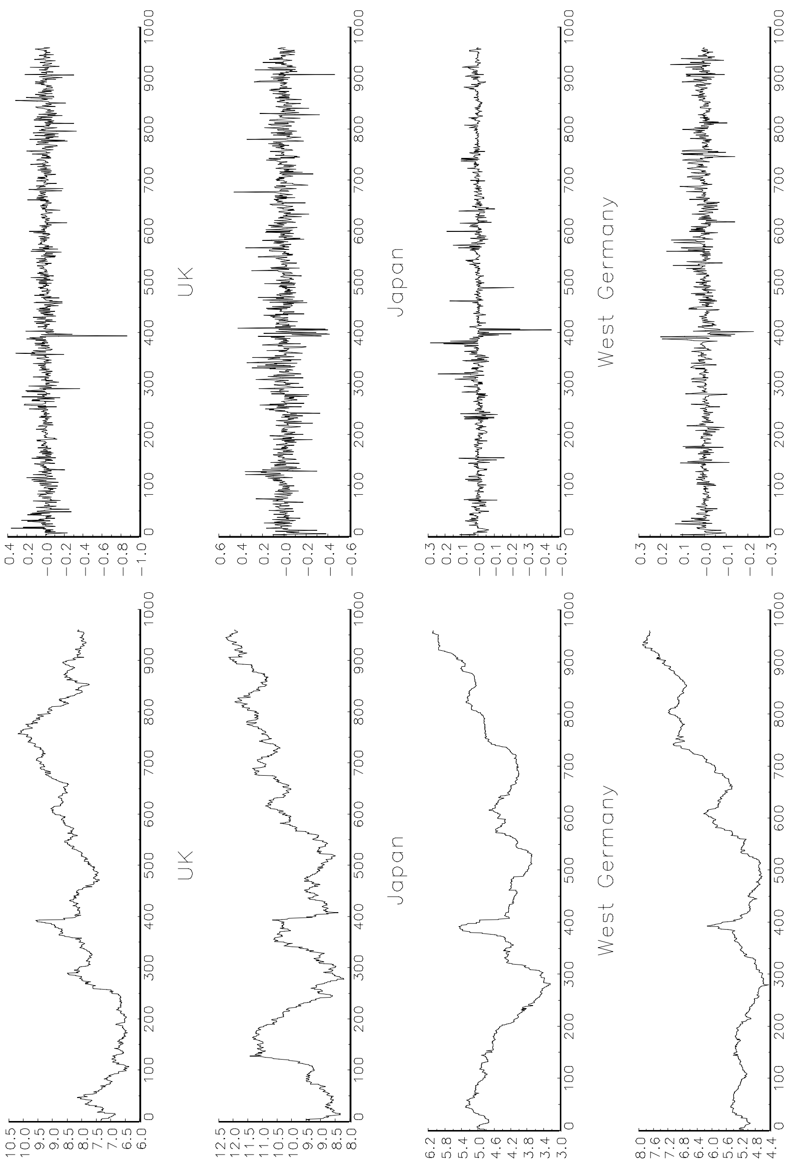
Country	U_1 Null: linear	U_2 Null: logarithmic
U.S.	1.921***	-0.270
U.K.	-0.416	1.816***
Japan	0.373	1.410***
West Germany	2.258***	1.204***

Results are reported with an autoregressive lag $p=1$. Results are not changing much if different values of p are used. Critical values are 0.477 (10%), 0.664 (5%), and 1.116 (1%) for both tests, which are available in Kobayashi and McAleer (1999). *** denotes that the test statistic is significant at the 1% level.

Caption for figure

Figure 1. Daily bond yields of U.S., U.K., Japan, and West Germany and their first differences

Sample period: April 1, 1986 ~ December 29, 1989


US
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