Mean Reversion of Real Exchange Rates and Purchasing Power Parity in Turkey

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Abstract

The important concept of purchasing power parity (PPP) has a number of practical implications. Our central objective is to examine the stationarity of Turkey's real exchange rates to test for the empirical validity of PPP. Our results from conventional univariate unit root tests fail to support PPP. However, when we use the empirical methodology developed by Caner and Hansen (2001), which allows us to jointly consider non-stationarity and non-linearity, we find evidence of non-linear mean reversion in Turkey's real exchange rates. This implies that PPP holds in one threshold regime but not in another.

JEL Codes

F31, C5

Keywords

Turkey, Europe, purchasing power parity, real exchange rate, unit root, non-linearity

1 Introduction

The concept of purchasing power parity (PPP) remains a cornerstone of exchange rate theory and international macroeconomics. PPP is based on the law of one price and implies that exchange rates should equalize the national price levels of different countries in terms of a common currency. There is a fairly widespread belief among economists that PPP helps to explain exchange rates at least in the long run. Furthermore, estimates of PPP exchange rates are important for some practical purposes, including measuring nominal exchange rate misalignment, determining exchange rate parities, and comparing the national incomes of different countries.

Such practical implications of PPP take on an added significance for Turkey, a developing country with a history of macroeconomic instability. This is because Turkey is currently making a concerted effort to eventually join the European Union (EU) and, by extension, the single-currency euro zone, in the future, in the hopes of achieving more rapid economic growth and politically consolidating its place in Europe. Severe and persistent nominal exchange rate misalignment contributes to macroeconomic instability and thus adversely affects the chances of EU membership. Such misalignment also complicates the task of estimating the appropriate exchange rate parity at which to join the euro.[1] Finally, deviation of the exchange rate from its PPP level creates uncertainty about relative per capita incomes, a relevant issue in light of concerns within the EU about Turkey's lower living standards.[2]

The real exchange rate is the nominal exchange rate adjusted for relative national price levels. According to PPP, any change in relative national price levels between two countries should lead to a corresponding adjustment in their bilateral nominal exchange rate. This suggests that variations in the real exchange rate represent deviations from PPP. Consequently, one avenue for investigating the empirical validity of PPP is to examine the characteristics of the real exchange rate. In particular, since PPP implies the mean reversion of real exchange rates, or their tendency to eventually return to PPP-determined levels in response to any disturbance, whether real exchange rates are stationary or non-stationary becomes an issue of central significance. Stationary real exchange rates imply mean reversion and thus provide empirical support for PPP.

The baseline empirical test of stationarity involves testing for unit roots in real exchange rates using the augmented Dickey Fuller (ADF) test. Rejection of the unit root hypothesis indicates stationarity in real exchange rates. Much of the large empirical literature on this issue fails to reject unit roots in real exchange rates.[3] While one may view such evidence as refuting the empirical validity of PPP, conventional univariate unit root tests such as the ADF test have relatively low power to reject a false null hypothesis of unit roots.[4] Increasing the length of the sample period increases the power of the tests. However, doing so requires a sufficiently long time-series of data, which is not available for Turkey. Aside from the requirement of data availability, using a longer time-series can create additional complications such as lumping together periods of fixed and flexible exchange rate regimes.

Conventional univariate unit root tests such as the ADF test assume absence of non-linearity and this may provide an additional explanation for why the evidence from those tests supports non-stationary real exchange rates. Non-linearity denotes the existence of threshold effects, or distinct threshold regimes with different dynamic properties. In particular, it is theoretically possible that real exchange rates are mean reverting in one regime but unit root processes in another when transactions costs in international arbitrage, such as shipping costs and trade barriers, create a band of no arbitrage for the real exchange rate.[5] A number of empirical studies support such a non-linear adjustment of real exchange rates toward long-run equilibrium. However, those studies generally assume smooth transition between different threshold regimes and focus on developed countries.[6] A discrete transition is likely to be more appropriate for developing countries with a history of macroeconomic instability.

In this paper, we empirically explore the possibility of non-linear mean reversion, or different threshold regimes in terms of stationarity, in Turkey's monthly real exchange rates. To do so, we apply the methodology developed by Caner and Hansen (2001) that allows us to simultaneously investigate non-stationarity and non-linearity under a discrete transition between regimes. Stationary real exchange rates would provide support for the empirical validity of PPP in Turkey whereas non-stationary exchange rates would not. The practical implications of deviations from PPP are especially meaningful for Turkey in the context of its on-going efforts to join the EU. Our findings indicate non-linearity in the stationarity of Turkey's real exchange rates, and hence lend mixed empirical support to PPP.

2 Basic Model and Data

The real exchange rate is calculated by:

$$q = e + p^* - p, \tag{1}$$

where q is the logarithm of the real exchange rate, e is the logarithm of Turkey-United States nominal exchange rate in terms of liras per dollar, p is the logarithm of Turkey's price index, and p^* is the logarithm of the price index of the United States, our numeraire country.

As a first step, we use the univariate ADF tests to examine the unit root null in Turkey's real exchange rates by running regressions on the following equation:

$$\Delta q_t = \mu + \rho q_{t-1} + \sum_{i=1}^{k} c_i \Delta q_{t-i} + \varepsilon_t, \tag{2}$$

where Δq_t is the first-difference of the logarithm of the real exchange rate and k is the number of lagged differences.[7] We determine k according to the recursive procedure proposed by Hall (1994). The null hypothesis is unit roots and the alternative hypothesis is level stationarity. If the coefficient of the lag of the real exchange rate (ρ) is significantly different from zero, we can reject the null hypothesis.

After the univariate ADF test, which implicitly assumes absence of non-linearity, we examine non-stationarity allowing for the possibility of non-linearity. To do so, we use the threshold autoregression (TAR) model described in Caner and Hansen (2001) as our underlying model.[8] The vector of coefficients θ will differ between threshold regimes in the presence of non-linearity.

$$\Delta q_t = \theta'_1 x_{t-1} 1_{\{Z_{t-1}<\lambda\}} + \theta'_2 x_{t-1} 1_{\{Z_{t-1}\geq\lambda\}} + e_t, \tag{3}$$

where $t = 1,.....,T$, $x_{t-1} = (q_{t-1} r'_t\ \Delta q_{t-1} ... \Delta q_{t-k})'$, $1_{\{.\}}$ is the indicator function, e_t is an identical and independently distributed error term, $Z_{t-m} = q_{t-m} - q_{t-m-1}$ for some delay parameter $m \geq 1$, and r_t is a vector of deterministic components including an intercept and possibly a linear time trend. The threshold λ is unknown and it takes on values between λ_1 and λ_2, which are chosen so that the probability that Z_t is less than or equal λ_1 is $\pi_1 > 0$ and the probability that Z_t is less than or equal to λ_2 is $\pi_2 < 1$. It is conventional to treat π_1 and π_2 symmetrically so that $\pi_2 = 1 - \pi_1$.[9] The specific form of the threshold variable Z_{t-1} is not central to our analysis.[10]

It is helpful to partition the vector of coefficients in threshold regime 1 and threshold regime 2, θ_1 and θ_2 respectively, as

$$\theta_1 = \begin{pmatrix} \rho_1 \\ \beta_1 \\ \alpha_1 \end{pmatrix}, \qquad \theta_2 = \begin{pmatrix} \rho_2 \\ \beta_2 \\ \alpha_2 \end{pmatrix}$$

where (ρ_1, ρ_2) are the slope coefficients on q_{t-1} or the lag of the real exchange rate, (β_1, β_2) are the slope coefficients on the deterministic components r_t, and (α_1, α_2) are the slope coefficients on $(\Delta q_{t-1}, \ldots, \Delta q_{t-k})$ in the two regimes.[11] The parameters ρ_1 and ρ_2 are of particular interest to us since they control the stationarity of q_t and correspond to ρ in the univariate ADF tests in (2).

We can estimate the TAR model (3) by least squares (LS). It is helpful to use concentration to implement such LS estimation.[12] For each λ, we estimate by ordinary least squares (OLS):

$$\Delta q_t = \hat{\theta}_1(\lambda)' x_{t-1} 1_{\{Z_{t-1} < \lambda\}} + \hat{\theta}_2(\lambda)' x_{t-1} 1_{\{Z_{t-1} \geq \lambda\}} + \hat{e}_t(\lambda). \tag{4}$$

Let us denote the OLS estimate of variance for fixed λ as $\hat{\sigma}^2(\lambda)$. We can find the LS estimate of the threshold λ, or $\hat{\lambda}$, by minimizing $\hat{\sigma}^2(\lambda)$. We can then find the LS estimates of the other parameters $\hat{\theta}_1 = \hat{\theta}_1(\hat{\lambda})$ and $\hat{\theta}_2 = \hat{\theta}_2(\hat{\lambda})$ by plugging in the point estimate $\hat{\lambda}$ into the vectors of coefficients θ_1 and θ_2 in each threshold regime. The estimated model is then

$$\Delta q_t = \hat{\theta}_1' x_{t-1} 1_{\{Z_{t-1} < \hat{\lambda}\}} + \hat{\theta}_2' x_{t-1} 1_{\{Z_{t-1} \geq \hat{\lambda}\}} + \hat{e}_t, \tag{5}$$

which also defines the LS residuals $\hat{e}_t$ and the residual variance $\hat{\sigma}^2$.

We can use the estimates in (5) to make inferences concerning the parameters of (3) using standard Wald and t statistics. Although the statistics are standard, the underlying sampling distributions are nonstandard, due to the presence of potential unidentified parameters and non-stationarity.

Our central objective is to examine stationarity in the possible presence of non-linearity. Therefore, in model (3), the two issues of interest to us are whether or not there is a threshold effect and whether the process q_t is stationary or not. Turning to the first issue, the threshold effect disappears under the joint hypothesis H_0 : $\theta_1 = \theta_2$, in which case the vectors of coefficients θ are identical between regimes and hence there is no non-linearity. The test of H_0 is the standard Wald statistic W_T for this restriction.[13] Large values of W_T and correspondingly low $p-$values would support the presence of threshold effects.

Turning to the second issue, the stationarity of the process q_t in model (3) depends on the parameters ρ_1 and ρ_2, which are the slope coefficients on q_{t-1} or the lag of the real exchange rate. For regime 1, we can reject the null hypothesis of unit roots in favor of the alternative hypothesis of level stationarity if ρ_1 is significantly different from zero, and likewise for regime 2 if ρ_2 is significantly different from zero. If the joint hypothesis H_0 : $\rho_1 = \rho_2 = 0$ holds, the real exchange rate has unit roots in both regimes.[14] The natural alternative to H_0 is $H_1 : \rho_1 < 0$ and $\rho_2 < 0$, in which case the real exchange rates are stationary.[15] In the intermediate partial unit root case $H_2 : \rho_1 < 0$ and $\rho_2 = 0$ or $\rho_1 = 0$ and $\rho_2 < 0$, the real exchange rate behaves like a stationary process in one regime, but a unit root process in the other regime.

The Wald statistic $R_{2T} = t_1^2 + t_2^2$, where t_1 and t_2 are the t ratios for $\hat{\rho}_1$ and $\hat{\rho}_2$ from the OLS regression in (5), is the standard test for H_0 against the unrestricted alternative $\rho_1 \neq 0$ or $\rho_2 \neq 0$. However, since the alternatives H_1 and H_2 are one-sided, we also consider the one-sided Wald statistic $R_{1T} = t_1^2 1_{\{\hat{\rho}_1 < 0\}} + t_2^2 1_{\{\hat{\rho}_2 < 0\}}$, which tests H_0 against the one-sided alternative $\rho_1 < 0$ or $\rho_2 < 0$. A statistically significant R_{1T} or R_{2T} can both justify the rejection of the unit root hypothesis. However, neither can discriminate between the stationary case H_1 and partial unit root case H_2. This calls for examining the individual t statistics t_1 and t_2. If only one of $-t_1$ or $-t_2$ is significant, this would be consistent with the partial unit root case, which allows us to distinguish between the three hypotheses. We look at the negative of the t statistics to retain the convention that H_0 should be rejected for large values of the test statistic.

Determining statistical significance requires the sampling distribution of R_{1T} and R_{2T} under the null hypothesis H_0. Note that the null of a unit root $(\rho_1 = \rho_2 = 0)$ is compatible with the threshold λ being either identified or unidentified. Using simulations, Caner and Hansen find bootstrap methods to be superior to asymptotic approximations. The bootstrap distributions of R_{1T} and R_{2T} differ in the identified and unidentified cases.[16] Caner and Hansen compare the simulated performance of the two bootstrap methods and recommend the unidentified threshold bootstrap for the calculation of p-values.[17] Significantly, their simulations also show that their threshold unit root tests have good power relative to conventional ADF unit root tests in the presence of threshold effects.

We apply the above methodology to simultaneously test for the non-linearity and non-stationarity of Turkey's monthly real exchange rates from January 1973 to July 2002. Our total number of raw observations is thus 355. Our data source for monthly consumer price index (CPI) and end-of-month nominal exchange rate is the International Financial Statistics (IFS).

3 Empirical Results

The result of the univariate ADF test for Turkey's real exchange rates cannot reject the null hypothesis of unit roots. Using Hall's recursive procedure, we determine the number of lagged differences k to be zero. Our estimate of the coefficient of the lag of real exchange rate (ρ) is –0.018 and not significantly different from zero.[18] Our finding is consistent with previous studies based on univariate ADF tests, which generally find evidence of unit roots in the current floating period. Our finding also implies a lack of empirical support for the validity of PPP for Turkey during the sample period. However, if there are non-linearities in Turkey's real exchange rates, then it is not appropriate to use univariate unit root tests, which implicitly assume the absence of non-linearities.

To examine stationarity in the possible presence of non-linearities, we apply the Caner and Hansen methodology described above. All our results in this section are based on $\pi_1 = .15$ and $\pi_2 = .85$, which, according to Andrews (1993), provides an optimal trade-off between various relevant factors.[19] These include the power of the test and the ability of the test to detect the presence of a threshold effect. Each regime must also have enough observations to identify the parameters.

The first issue we must address is the presence of threshold effects and hence non-linearity. The appropriate test statistic for this purpose is the Wald test W_T we discussed earlier. In the first four columns of Table 1 below, we report the Wald tests W_T, 1% bootstrap critical values, and bootstrap $p-$values for threshold variables of the form $Z_{t-1} = q_{t-m} - q_{t-m-1}$ for delay parameters m from 1 to 12. All our bootstrap tests in this section are based on 10,000 replications. Many of the statistics are significant, which supports the presence of threshold effects.

[Insert Table 1 here]

Let us now make m endogenous to address the criticism that the results of Table 1 depend on m even though m is generally unknown. The least squares estimate of m is the value that minimizes the residual variance, which is the value that maximizes W_T since W_T is a monotonic function of the residual variance. This estimate is $\hat{m} = 1$, and the corresponding W_T and $p-$values in Table 1 are 119.91 and 0.000, respectively. When we recalculate the bootstrap $p-$value allowing for the estimation of m, we still obtain a bootstrap $p-$value of only 0.004, lending very strong support for a TAR model and hence the presence of threshold effects.

The second issue of interest is unit roots. We calculate the threshold unit root test statistics R_{1T}, t_1 and t_2 for each delay parameter m from 1 to 12, and report both their asymptotic and bootstrap $p-$values in the last six columns of Table 1 above. We do not report the R_{2T} test results since they are almost identical to the R_{1T} test. We calculate both types of $p-$values under the assumption of unidentified thresholds, for reasons mentioned earlier. The most relevant R_{1T} statistic is that for the $m=1$ case, which has a bootstrap $p-$value of 0.001. In addition, for $m=1$, the bootstrap $p-$values for the individual t ratios t_1 and t_2 are 0.000 and 0.937, respectively. This suggests that we can reject the unit root hypothesis in favor of $\rho_1 < 0$ but we are unable to reject $\rho_2 = 0$. Our results thus seem to indicate that Turkey's real exchange rates behave like a stationary process in one threshold regime, but a unit root process in the other regime.

We report the LS parameter estimates for our preferred $m=1$ specification in Table 2 below. The point estimate of the threshold $\hat{\lambda}$ is –0.032. Therefore, the TAR splits the regression function depending on whether our threshold variable $Z_{t-1} = q_{t-2} - q_{t-3}$ is greater or less than –0.032. The first regime is when $Z_{t-1} < -0.032$, which occurs when the real exchange rate has fallen by more than –0.032 points over a one-month period. The second regime is when $Z_{t-1} > -0.032$, which occurs when the real exchange rate has fallen by less than –0.032 points, remained constant, or has risen over a one-month period. Around 15% of the observations belong to the first regime and around 85% belong to the second regime.

[Insert Table 2 here]

In Table 2 above we also report tests for the pair-wise equality of individual coefficients, and bootstrap $p-$values based on the null of no threshold. An examination of the results in Table 2 suggests that the coefficients on Δq_{t-1} through Δq_{t-8} are driving the threshold model, while the coefficients on Δq_{t-9} through Δq_{t-12} are either less important or do not vary between the two regimes. Imposing the constraint of equality of the coefficients on Δq_{t-9} through Δq_{t-12}, we re-estimate the model and report the results in Table 3 below. As expected, the results are qualitatively similar to those in Table 2. In particular, the threshold estimate $\hat{\lambda}$ is identical in the constrained and unconstrained models, which implies that the division of the data into the two threshold regimes is also identical.

[Insert Table 3 here]

Figure 1 below shows the estimated division of our data into the two threshold regimes. Notice that Turkey's real exchange rates follow a discrete trend rather than a smooth trend. This lends further support to our choice of the TAR model, which is appropriate for non-linear time-series involving discrete transition between regimes, rather than the STAR (Smooth Transition Autoregressive) model, which is appropriate for non-linear time-series involving smooth transition.[20]

[Insert Figure 1 here]

Since the constrained model has fewer parameters than the unconstrained model, its threshold and unit root tests may be more precise. We report those results in Table 4 below. The first four columns address the issue of threshold effects. In line with the results for the unconstrained model in Table 1, many of the W_T statistics are significant, which supports a threshold model. When we recalculate the bootstrap

$p-$value on the basis of the least squares estimate of $\hat{m}=1$, we still obtain a bootstrap $p-$value of only 0.002, providing strong support for a TAR model.

The last six columns of Table 4 address the issue of unit roots. We calculate the threshold unit root test statistics R_{1T}, t_1 and t_2 for each delay parameter m from 1 to 12, and report both the asymptotic and bootstrap $p-$values for R_{1T}, t_1 and t_2. The most relevant R_{1T} statistic is that for the $m=1$ case, which has a bootstrap $p-$value of 0.001. Furthermore, for $m=1$, the bootstrap $p-$values for the individual t ratios t_1 and t_2 are 0.000 and 0.938, respectively. Those results, which are very similar to those of the unconstrained model, again suggest that Turkey's real exchange rates are stationary in one regime but characterized by unit roots in the other regime.

[Insert Table 4 here]

4 Concluding Remarks

Purchasing power parity (PPP), an important concept in exchange rate theory and international macroeconomics, has a number of practical implications, including the measurement of nominal exchange rate misalignment, the determination of exchange rate parities and the international comparison of national incomes. Such implications take on additional significance for Turkey, a developing country that has experienced a lot of macroeconomic instability in the past and is currently making efforts to join the European Union (EU). Our central objective is to examine the empirical validity of PPP in Turkey under the current float.

Our empirical analysis is based on investigating whether Turkey's real exchange rates are stationary or non-stationary. Stationarity would provide support for mean reversion and hence PPP. Using the conventional univariate augmented Dickey Fuller (ADF) test, we fail to find evidence of stationarity in Turkey's real exchange rates. However, using the empirical methodology developed by Caner and Hansen (2001), which is more appropriate than the ADF test in the presence of non-linearity or threshold effects, we find that Turkey's real exchange rates are non-linear in the sense that they are stationary in one regime but non-stationary in the other. Therefore, we find somewhat stronger evidence of PPP in Turkey when we allow for the possibility of non-linearity than when we do not.

Our findings from Turkey suggest that a more complete empirical analysis of the stationarity of real exchange rates in developing countries should consider the possibility of non-linear mean reversion in real exchange rates. The existing empirical literature on non-linear mean reversion and more generally, non-linearities in real exchange rates is focused on developed countries. A simultaneous investigation of non-stationarity and non-linearity of real exchange rates will give us a more accurate indication of the empirical validity of PPP in developing countries just as it does for developed countries.

Notes

[1] Joining the euro at the appropriate parity is important even for a large developed country. For example, many economists attribute Germany's current economic difficulties to having joined at too high an exchange rate. The adverse consequences of joining at an inappropriate rate are likely to be even higher for a developing country such as Turkey.

[2] A large income gap is likely to cause a higher migration from Turkey into the EU. Additional EU concerns about Turkey's prospective membership include its large and predominantly Muslim population, along with a poor human rights record.

[3] Please refer to Rogoff (1996) for a comprehensive survey of the empirical literature on PPP.

[4] See, for example, Campbell and Perron (1991) and Lothian and Taylor (1997).

[5] Please refer to Taylor (2003) for a more comprehensive discussion of transactions costs in international arbitrage. Examples of theoretical models of non-linear real exchange rates based on transactions costs include O'Connell (1997), Dumas (1992) and Sercu, Uppal and Van Halle (1995).

[6] See, for example, Taylor, Peel and Sarno (2001) and Sarantis (1999).

[7] In accordance with the concept of long-run PPP, we exclude the time trend.

[8] The Caner and Hansen model is a variant of the threshold autoregressive (TAR) model, which was pioneered by Tong (1978) and is widely used for analyzing non-linear time-series involving discrete transition between regimes. The transition between real exchange rate regimes is likely to be discrete in developing countries. The main contribution of the Caner and Hansen model is that it allows for simultaneous testing for non-stationarity and non-linearity for TAR models.

[9] Doing so imposes the restriction that each regime has at least $\pi_1\%$ of the total

sample. The specific choice of π_1 is necessarily arbitrary to some extent, but the sample for each regime should be sufficiently large to identify the regression parameters.

[10] What is required is that Z_{t-1} be predetermined, strictly stationary, and ergodic with a continuous distribution function.

[11] ρ_1 and ρ_2 are scalar, β_1 and β_2 have the same dimension as r_t, and α_1 and α_2 are k-vectors. An important issue in applications of TAR is how to specify the deterministic component r_t. If the series q_t is non-trended, it is natural to set $r_t = 1$, as we do in our study. Please refer to Caner and Hansen (2001) for additional assumptions and parameter restrictions in the model as well as the motivations for those assumptions and restrictions.

[12] Please see Caner and Hansen.

[13] Caner and Hansen find that W_T has a nonstandard asymptotic null distribution with critical values that cannot be tabulated. Hence they propose two bootstrap approximations to the asymptotic distribution of W_T – one based on the restriction of a unit root, and the other based on unrestricted estimates. We are sometimes interested in the equality of only a subset of the coefficients of θ. In this case, Caner and Hansen find that the correct asymptotic distribution and bootstrap method depend on the unknown true properties of the coefficients.

[14] In this case, we can rewrite the model (3) as a stationary threshold autoregression in the variable Δq_t so q_t could be described as unit roots.

[15] Please see Chan and Tong (1985).

[16] Since Caner and Hansen find that the asymptotic distribution of R_{1T} and R_{2T} differs substantially depending on whether the threshold is identified or not, so does

the bootstrap distribution.

[17] This is primarily because Caner and Hansen find that the rejection rates using the unidentified threshold model are less sensitive to the nuisance parameters. Also, the one-sided Wald test R_{1T} generally has somewhat better power than the two-sided test R_{2T}. The individual t ratio tests help us to effectively distinguish between the pure unit root, partial unit root, and stationary cases.

[18] The t-statistic is –1.744 and the critical values according to MacKinnon (1991) are –2.574, –2.870 and –3.353 at the 10%, 5% and 1% significance level, respectively.

[19] We also experimented with $[\pi_1, \pi_2] = [.10,.90]$ and $[\pi_1, \pi_2] = [.05,.95]$, but the results are qualitatively the same, and hence we do not report them here.

[20] For example, Taylor, Peel and Sarno (2001) and Sarantis (1999) use STAR models in their empirical analysis.

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Table 1
Threshold and Unit Root Tests Unconstrained Model

Bootstrap Threshold Test				Unit Root Tests, $p-$Values					
				R_{1T}		t_1		t_2	
m	W_T	1% C.V.	$p-$ Value	Asym.	Boot.	Asym.	Boot.	Asym.	Boot.
1	119.91	72.27	0.000	0.000	0.001	0.000	0.000	0.909	0.937
2	61.77	71.02	0.002	0.552	0.377	0.450	0.202	0.875	0.526
3	34.34	71.94	0.187	0.363	0.254	0.203	0.097	0.919	0.932
4	26.29	70.84	0.390	0.977	0.868	0.893	0.563	0.952	0.722
5	47.72	71.18	0.064	0.367	0.259	0.254	0.121	0.906	0.581
6	52.47	71.00	0.046	0.924	0.749	0.769	0.413	0.959	0.765
7	47.03	71.11	0.069	0.252	0.188	0.960	0.841	0.132	0.080
8	44.74	72.18	0.084	0.714	0.510	0.493	0.235	0.939	0.907
9	56.02	71.78	0.035	0.714	0.513	0.961	0.819	0.492	0.234
10	115.50	71.03	0.001	0.367	0.256	0.206	0.106	0.934	0.921
11	46.07	75.06	0.076	0.123	0.114	0.062	0.045	0.953	0.731
12	34.59	74.36	0.191	0.303	0.224	0.950	0.892	0.164	0.087

Note: m denotes delay parameter, W_T denotes the Wald statistic for threshold effects, and 1% C.V. denotes the critical value at the 1% significance level. R_{1T} denotes the one-sided Wald statistic $t_1^2 1_{\{\hat{\rho}_1<0\}} + t_2^2 1_{\{\hat{\rho}_2<0\}}$, which tests $H_0:\ \rho_1 = \rho_2 = 0$ against the one-sided alternative $\rho_1 < 0$ or $\rho_2 < 0$. To discriminate further between the stationary case and the partial unit root case, we have to look at the individual t statistics t_1 and t_2. If only one of $-t_1$ or $-t_2$ is significant, this would be consistent with the partial unit root case. Asym. denotes the asymptotic p-values and boot. denotes the bootstrap p-values for the threshold unit root test statistics.

Table 2
Least Squares Estimates Unconstrained Threshold Model

Regressor	Estimates $\hat{m}=1,\ \hat{\lambda}=-0.032$				Tests for Equality of Individual Coefficients	
	$Z_{t-1}<\hat{\lambda}$		$Z_{t-1}\geq\hat{\lambda}$		Wald Statistics	Bootstrap $p-$Value
	Estimate	s.e.	Estimate	s.e.		
Constant	1.784	0.292	-0.093	0.123	35.086	0.000
q_{t-1}	-0.168	0.027	0.009	0.012	35.729	0.000
Δq_{t-1}	0.251	0.352	-0.074	0.054	0.832	0.445
Δq_{t-2}	-0.217	0.078	0.129	0.074	10.332	0.026
Δq_{t-3}	-0.148	0.077	0.096	0.073	5.254	0.095
Δq_{t-4}	-0.046	0.091	-0.029	0.067	0.022	0.906
Δq_{t-5}	0.374	0.241	-0.047	0.051	2.924	0.201
Δq_{t-6}	0.075	0.106	-0.040	0.059	0.888	0.467
Δq_{t-7}	1.037	0.158	-0.051	0.053	42.829	0.001
Δq_{t-8}	-0.910	0.311	-0.029	0.051	7.822	0.047
Δq_{t-9}	0.401	0.189	0.001	0.053	4.172	0.124
Δq_{t-10}	0.024	0.128	0.061	0.058	0.068	0.836
Δq_{t-11}	-0.114	0.137	-0.017	0.055	0.430	0.599
Δq_{t-12}	0.109	0.070	0.123	0.077	0.016	0.918

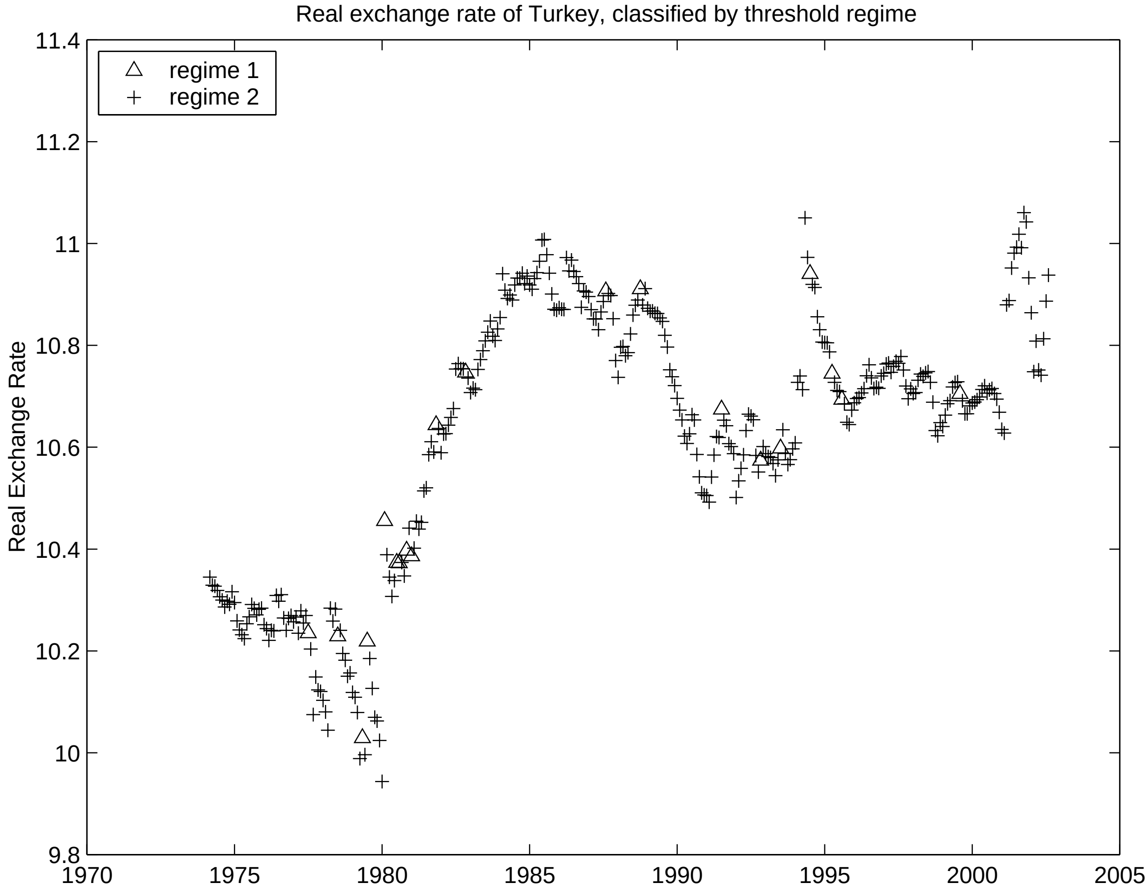
Note: $\hat{m}$ refers to the least squares estimate of m or delay parameter. $\hat{\lambda}$ refers to the point estimate of the threshold. The threshold autoregression (TAR) splits the regression function depending on whether our threshold variable $Z_{t-1}=q_{t-2}-q_{t-3}$ is greater or less than $\hat{\lambda}$. Estimate denotes the least squares estimate of the coefficient and s.e. denote its standard error. The last two columns contain the Wald statistic for the equality of individual coefficients in the two regimes and its bootstrap p-value.

Table 3
Least Squares Estimates Constrained Threshold Model

Regressor	Estimates $\hat{m}=1,\ \hat{\lambda}=-0.032$			
	$Z_{t-1}<\hat{\lambda}$		$Z_{t-1}\geq\hat{\lambda}$	
	Estimate	s.e.	Estimate	s.e.
Constant	1.652	0.259	-0.093	0.123
q_{t-1}	-0.156	0.024	0.009	0.012
Δq_{t-1}	0.166	0.305	-0.071	0.054
Δq_{t-2}	-0.143	0.070	0.132	0.074
Δq_{t-3}	-0.164	0.072	0.094	0.073
Δq_{t-4}	-0.111	0.081	-0.030	0.067
Δq_{t-5}	0.276	0.209	-0.045	0.051
Δq_{t-6}	0.094	0.103	-0.040	0.059
Δq_{t-7}	1.055	0.147	-0.051	0.053
Δq_{t-8}	-0.984	0.292	-0.027	0.051
	Estimate		s.e.	
Δq_{t-9}	0.029		0.051	
Δq_{t-10}	0.054		0.053	
Δq_{t-11}	-0.021		0.051	
Δq_{t-12}	0.111		0.050	

Note: The coefficients on Δq_{t-9} through Δq_{t-12} are constrained to be equal in the two regimes. $\hat{m}$ refers to the least squares estimate of m or delay parameter. $\hat{\lambda}$ refers to the point estimate of the threshold. The threshold autoregression (TAR) splits the regression function depending on whether our threshold variable $Z_{t-1}=q_{t-2}-q_{t-3}$ is greater or less than $\hat{\lambda}$. Estimate denotes the least squares estimate of the coefficient and s.e. denote its standard error.

Figure 1



Note: Regime 1 refers to the real exchange rate falling by more than –0.032 points over a one-month period. Regime 2 refers to the real exchange rate falling by less than –0.032 points, remaining constant, or rising over a one-month period. Around 15% of the observations fall into regime 1 and around 85% of the observations fall into regime 2.

Table 4
Threshold and Unit Root Tests Constrained Model

Bootstrap Threshold Test				Unit Root Tests, $p-$Values					
				R_{1T}		t_1		t_2	
m	W_T	1% C.V.	$p-$ Value	Asym.	Boot.	Asym.	Boot.	Asym.	Boot.
1	115.12	57.01	0.000	0.000	0.001	0.000	0.000	0.910	0.938
2	38.76	56.61	0.048	0.018	0.036	0.009	0.016	0.938	0.660
3	26.94	57.42	0.166	0.130	0.120	0.063	0.046	0.909	0.935
4	21.71	56.25	0.304	0.789	0.592	0.666	0.336	0.905	0.581
5	23.32	56.86	0.250	0.745	0.549	0.529	0.251	0.959	0.773
6	29.21	57.32	0.133	0.549	0.395	0.342	0.174	0.961	0.797
7	35.44	55.25	0.076	0.025	0.047	0.955	0.871	0.011	0.021
8	42.41	59.94	0.040	0.711	0.522	0.489	0.245	0.916	0.931
9	48.21	59.04	0.027	0.599	0.432	0.952	0.883	0.384	0.191
10	98.93	58.15	0.000	0.212	0.173	0.109	0.070	0.916	0.936
11	40.40	60.18	0.050	0.142	0.130	0.070	0.049	0.958	0.770
12	26.24	60.85	0.195	0.242	0.197	0.943	0.901	0.126	0.084

Note: m denotes delay parameter, W_T denotes the Wald statistic for threshold effects, and 1% C.V. denotes the critical value at the 1% significance level. R_{1T} denotes the one-sided Wald statistic $t_1^2 1_{\{\hat{\rho}_1<0\}} + t_2^2 1_{\{\hat{\rho}_2<0\}}$, which tests H_0 : $\rho_1 = \rho_2 = 0$ against the one-sided alternative $\rho_1 < 0$ or $\rho_2 < 0$. To discriminate further between the stationary case and the partial unit root case, we have to look at the individual t statistics t_1 and t_2. If only one of $-t_1$ or $-t_2$ is significant, this would be consistent with the partial unit root case. Asym. denotes the asymptotic p-values and boot. denotes the bootstrap p-values for the threshold unit root test statistics.