

Estimation of Credit and Default Spreads: An Application to CDO Valuation

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Abstract

Many securities are, to a certain extent, subject to credit risk in one way or another. Both the financial institutions and regulators are keen to have their credit risk exposures well managed. In order to fulfill their needs, the market for credit derivatives has become one of the fast growing securities markets in the last several years. In particular, the credit risk on a corporate balance sheet has become an important topic. Along with this growing importance of credit risk, the development of credit risk models has received much attention from both practitioners and academia. This paper addresses the impact of default rate modeling on the risk analysis and market valuation of credit derivatives products.

I. Introduction

Many securities are, to a certain extent, subject to credit risk in one way or another. Both the financial institutions and regulators are keen to have their credit risk exposures well managed. In order to fulfill their needs, the market for credit derivatives has become one of the fast growing securities markets in the last several years. In particular, the credit risk on a corporate balance sheet has become an important topic. Along with this growing importance of credit risk, the development of credit risk models has received much attention from both practitioners and academia. This paper addresses the impact of default rate modeling on the risk analysis and market valuation of credit derivatives products.

Researchers attempt to model credit risk and valuation by using either one of the following two approaches: structural models or reduced form models. The first group, like KMV, mainly follows Merton (1974) by relating a firm's credit risk to the variation of its asset and liability value through equity value. A default event occurs when its asset value is lower than certain threshold liability level. However, to implement this approach, firms' assets need to be estimated, and the complex structure of payoffs of all the liabilities needs to be specified. For multi-factor models, given the complexity in modeling default as a first passage time event, it is difficult to obtain closed-form solutions. The second group, like Credit Metrics, Jarrow and Turnbull (1995), and Duffie and Singleton (1997), works on the credit migration and default probability based on the historical credit transition probability or market credit spreads without explicitly taking account of firms' underlying asset values. Duffie and Singleton (1997) used the affine class of term structure models to capture both the cross-sectional and time-series properties.

Delianedis and Geske (2000), Elton, Gruber et al (2001) described the discrepancy between the default spread and credit spread due to taxes, liquidity and market risk factors. For default rate modeling, there is little empirical evidence with which to differentiate the parameters guiding the dynamics of risk-neutral and actual default intensities. Jarrow, Lando, and Turnbull (1997) provided some methods for calibrating risk-neutral default intensities from ratings-based transition data and bond-yield spreads. Duffee (1998) provided some empirical estimates for the actual dynamics of risk-neutral default intensities for relatively low-risk corporate bond issuers, based on time-series data on corporate bonds. However, both studies provide hazard rate models for credit spreads not for default spreads. In this paper, we separately estimate the processes for default spreads and credit spreads combining the above two approaches.

Since default process is an important factor in measuring credit risk and pricing credit derivatives, if there are big gaps between credit spreads and default spreads, it would make more sense to estimate default rate models using default spreads data rather than using credit spreads data. We find that default spreads account for only a small portion of credit spreads for investment grade firms and they display quite different volatility structure.

In section II and III we describe a model for constructing credit spreads, default spreads and correlations. In section IV we describe the estimation procedures and section V shows how we model credit event generator. Section VI shows the estimation results and the impact of default correlation on the market value of Collateralized Debt Obligations (CDO).

II. Credit and Default Spread Model

In this section we describe an empirical approach to measure actual and risk neutral default probability. First we start with structural approach for default probability and bond pricing. Then we describe a parametric model for default process to compute credit and default spread using the market bond data and the default probability from the structural approach.

A. Structural Approach

Let V_i be the time t value of the firm i 's assets and it follows, under the risk neutral measure,

$$dV_{i,t} = rV_{i,t} + \sigma_{i,v}V_{i,t}dZ_{i,t} \quad (1)$$

where r is the risk-free interest rate. Under the actual measure, it can be written as follows:

$$dV_{i,t} = \mu_i V_{i,t} + \sigma_{i,v} V_{i,t} dZ_{i,t} \quad (2)$$

where $\mu_i = r + \lambda_i \sigma_{i,v}$ and λ_i is the price of risk.

Merton (1974) modeled the option to default by considering the stock in a leveraged firm as a call option on the assets of the firm with an exercise price equal to the face value of the total debt:

$$S_i = V_i N(k_i + \sigma_{i,v} \sqrt{T-t}) - M^{-r(T-t)} eN(k_i) \quad (3)$$

where

$$k_i = \frac{\ln(V_i / M_i) + (\mu_i - 0.5\sigma_{i,v}^2)(T-t)}{\sigma_{i,v} \sqrt{T-t}}$$

and S_i is the current market value of the stock, M_i is the face value of debt.

The probability that the firm will be insolvent and default on its debt obligation at date T is:

$$APD_i = 1 - N(k_i) \quad (4)$$

where $N(\cdot)$ is the cumulative normal distribution function. The risk neutral probability that the firm will be insolvent and default on its debt obligation at date T is¹:

$$\begin{aligned} RPD_i &= N\left(-k_i + \frac{(u_M - r)}{\sigma_M} R_i \sqrt{T-t}\right) \\ &= N\left(N^{-1}(APD_i) + \frac{(u_M - r)}{\sigma_M} R_i \sqrt{T-t}\right) \end{aligned} \quad (5)$$

Under the risk-averse assumption, the expected return on the firm must exceed the risk free rate. Since the actual and risk neutral distributions of the firm have the same variance and actual distribution must have a mean greater than the risk free rate, then the risk neutral distribution will have the larger default probability.

To compute the above default probabilities, we need to estimate the asset value and asset volatility. In addition to equation (3), we need the following condition to estimate them²:

$$\sigma_S = \sigma_V \left(\frac{V}{S} \frac{\partial V}{\partial S} \right) \quad (6)$$

Using twelve months of daily data, we estimate the asset value and asset volatility iteratively until the asset volatility converges. This process is repeated every end of month.

B. Reduced Form Approach

Following Duffie and Singleton (1999), we describe the default-adjusted rate R_t as a sum of default-free interest rate r_t and hazard rate g_t multiplied by loss given default L_t , i.e., $R_t = r_t + g_t L_t$. Generally, g_t and L_t are not separately identifiable. In this paper we assume constant L_t , equal to L . Since g_t and L_t are not separately identifiable, the validity of this assumption will not affect the model performance in fitting the corporate bond prices. Moreover, an empirical study conducted by Skinner and Diaz (2000)

¹ The firm i 's beta is $\beta_i = R_i \frac{\sigma_{i,V}}{\sigma_M}$ where R_i is the correlation between the firm i 's return and the market return. Hence, the equation (5) is expressed in terms of the market price of risk instead of the firm i 's price of risk.

² Under certain conditions, the relationship can be expressed as follows (see CreditGrades):

$\sigma_V = \sigma_S \frac{S}{S + \bar{M}}$. This implies that, for a stable asset volatility, the equity volatility increases with declining stock price consistent with volatility skew in equity option market.

suggests that allowing for time dependent variation in L_t is of secondary importance in pricing defaultable claims.

We assume the default-free interest rate is defined as follows:

$$r_t = \alpha_r + s_{1t} + s_{2t}, \quad (7)$$

and the two factors, s_{1t} and s_{2t} , follow the CIR process:

$$\begin{aligned} ds_{it} &= (\kappa_{ir}\theta_{ir} - \kappa_{ir}s_{it})dt + \sigma_{ir}\sqrt{s_{it}}dz_{it} \\ ds_{it} &= (\kappa_{ir}\theta_{ir} - (\kappa_{ir} + \lambda_{ir})s_{it})dt + \sigma_{ir}\sqrt{s_{it}}dz_{it} \end{aligned} \quad (8)$$

where (8) holds under the actual and risk neutral measure, respectively. The value of a default-free bond, $G(t, T)$, for maturity $T - t$ is given by

$$\begin{aligned} G(t, T) &= E_t^Q \left[\exp\left(-\int_t^T (r_s) ds\right) \right] \\ &= A_{1r}(t, T) A_{2r}(t, T) \exp(-\alpha_r(T-t) - B_{1r}(t, T)s_{1t} - B_{2r}(t, T)s_{2t}) \end{aligned} \quad (9)$$

where, for $i = 1$ and 2 ,

$$\begin{aligned} A_{ir}(t, T) &= \left[\frac{2\gamma_{ir} \exp((a_{ir} + \gamma_{ir})(T-t)/2)}{(a_{ir} + \gamma_{ir})(\exp(\gamma_{ir}(T-t)) - 1) + 2\gamma_{ir}} \right]^{2\kappa_{ir}\theta_{ir}/\sigma_{ir}^2} \\ B_{ir}(t, T) &= \left[\frac{2(\exp(\gamma_{ir}(T-t)) - 1)}{(a_{ir} + \gamma_{ir})(\exp(\gamma_{ir}(T-t)) - 1) + 2\gamma_{ir}} \right] \\ a_{ir} &= \kappa_{ir} + \lambda_{ir} \\ \gamma_{ir} &= \sqrt{a_{ir}^2 + 2\sigma_{ir}^2}. \end{aligned} \quad (10)$$

We also assume that the expected loss rate is defined as:

$$g_t L = (\alpha_h + h_t + \beta_1 s_{1t} + \beta_2 s_{2t}) L \quad (11)$$

where $L = 1 - \delta$ is the loss rate and δ is the recovery rate, and the hazard rate h_t follows the CIR process:

$$dh_t = (\kappa_h \theta_h - \kappa_h h_t)dt + \sigma_h \sqrt{h_t} dz_{ht} \quad (12)$$

$$dh_t = (\kappa_h \theta_h - (\kappa_h + \lambda_h)h_t)dt + \sigma_h \sqrt{h_t} dz_{ht} \quad (13)$$

where (12) and (13) holds under the actual and risk neutral measure, respectively.

The value of a risky zero-coupon bond, $P(t, T)$, for maturity $T - t$, with constant recovery rate δ , is given by

$$\begin{aligned} P(t, T) &= E_t^Q \left[\exp\left(-\int_t^T (r_s + g_s L) ds\right) \right] \\ &= G_h(t, T) \{ \exp(-L\alpha_h(T-t)) A_h(t, T) \exp(-B_h(t, T)Lh_t) \} \end{aligned} \quad (14)$$

where

$$\begin{aligned} A_h(t, T) &= \left[\frac{2\gamma_h \exp((a_h + \gamma_h)(T-t)/2)}{(a_h + \gamma_h)(\exp(\gamma_h(T-t)) - 1) + 2\gamma_h} \right]^{2\kappa_h \theta_h / \sigma_h^2} \\ B_h(t, T) &= \left[\frac{2(\exp(\gamma_h(T-t)) - 1)}{(a_h + \gamma_h)(\exp(\gamma_h(T-t)) - 1) + 2\gamma_h} \right] \\ a_h &= \kappa_h + \lambda_h \\ \gamma_h &= \sqrt{a_h^2 + 2\sigma_h^2}, \end{aligned} \quad (15)$$

and $G_h(t, T)$ is $G(t, T)$ with $\beta_1 s_{1t} L$ and $\beta_2 s_{2t} L$ instead of s_{1t} and s_{2t} , respectively.

III. Default Intensity and Default Correlation

Using the default probability computed from the Merton model, we can price risky bond as follows:

$$P(t, T) = (1 - L) \cdot G(t, T) + L \cdot G(t, T) \cdot (1 - CRPD(t, T)), \quad (16)$$

Similar to Jones, Mason, and Rosenfeld (1984), risky bond can also be priced as follows:

$$\begin{aligned} P(t, T) &= G(t, T) [N(k(M, T-t)) \\ &+ VG(t, T)^{-1} N(-k(\delta, T-t)) + \delta \{N(k(\delta, T-t)) - N(k(M, T-t))\}] \end{aligned} \quad (17)$$

where

$$k(x, \tau) = \frac{\ln\left(\frac{V}{xM}\right) + (r - 0.5\sigma_v^2)\tau}{\sigma_v \sqrt{\tau}}$$

Once the time series of default spread or risky bond price with only default premium is estimated, similar to the hazard rate modeling in the above section, we can use a parsimonious Markov model for each obligor's (or sector's) default probabilities, while varying the correlation among different obligor's default times.

Duffie and Garleanu (1999) and Finger (2002) assumed that default times have default intensity processes ζ_i , which follow the CIR process described in equation (13) with parameters $(\kappa_i, \theta_i, \sigma_i, \lambda_i)$ i.e.,

$$d\zeta_{it} = (\kappa_i \theta_i + \delta_i \bar{\zeta}_i - (\kappa_i + \lambda_i) \zeta_{it}) dt + \sigma_i \sqrt{\zeta_{it}} dz_{it} \quad (18)$$

under the risk neutral measure and where $\bar{\zeta}_i$ denotes the average default intensity.

Alternatively, using the time series of risk neutral default probabilities, we can estimate the default intensities as follows:

$$\Delta \zeta_{it} = (\kappa_i \theta_i + \delta_i \bar{\zeta}_i - (\kappa_i + \lambda_i) \zeta_{it-1}) \Delta t + \sigma_i \sqrt{\zeta_{it-1}} * \Delta t \varepsilon_{it} \quad (19)$$

where $\zeta_{it}(\tau) = -\ln(1 - RPD_{it}(\tau))$. Then the correlation between the default intensities of firm i and j is computed as the correlation between ε_{it} and ε_{jt} over the sample period.

We can also achieve the same goal using the time series of default spreads and the following relationship:

$$\begin{aligned} e^{-s_{it}\tau} &= 1 - CRPD_{it}(\tau) * L \\ &= 1 - \{1 - (1 - RPD_{it}(\tau))^\tau\} * L \end{aligned} \quad (20)$$

where s_{it} denotes default spreads³.

IV. Estimation

Month-end prices of US Treasury bills, notes, bonds and corporate bonds are extracted from Bridge over the period of beginning July 1995 and ending February 2002. We restrict our sample to corporate bonds issued by U.S. firms. The bonds under consideration have semi-annual fixed rate coupons and principal at maturity. We also

³ This implies that the default intensity is computed as: $\zeta_{it}(\tau) = \frac{1}{\tau}(s_{it} * \tau + \ln(L))$.

exclude bonds with call options, put options and sinking fund provisions. To estimate the term structure parameters, we select 2669 bonds from 231 companies by choosing median credit curves.

For structural approach, we use two sources of data, Compustat and DRI. Compustat database provides quarterly observations of each firm's capital structure and S&P ratings. DRI database provides the daily stock price data and the number of shares outstanding.

Given data on the firm's stock price, the number of shares outstanding, the liabilities, and the interest rates, we can solve the current market value of the firm and volatility, (V, σ_v) , using the Merton equation (3) and the following equation:

$$\sigma_s = \frac{\partial S}{\partial V} \frac{V}{S} \sigma_v \quad (21)$$

Using twelve months of daily data, we estimate the asset value and asset volatility iteratively until the asset volatility converges. This process is repeated every end of month. Once we compute the firms' default probability, we use the above bond information to compute structural bond prices. Then we apply these structural bond prices to estimate the default process.

To estimate the term structure parameter in section II, we use non-linear Kalman filter estimation. The measurement equation for equations (7) through (9), or (12) through (14), can be defined as

$$D_t = Z(s_{1t}, s_{2t}; \Theta) + \varepsilon_t \quad (22)$$

where $D_t = \{p_{1,t}, \dots, p_{m,t}\}$ denotes observed bond prices of m maturities and the Z_t maps the state variables s_{1t}, s_{2t} and parameters Θ into m theoretical prices. Since Z_t is not linear in the state variables, we use extended Kalman filter (EFK) technique following Lund (1997). Using a first-order Taylor series expansion bond prices can be expressed as,

$$Z(s_t; \Theta) = Z(\hat{s}_{t|t-1}; \Theta) + \left[\frac{\partial Z(s_t; \Theta)}{\partial s} \right]' (s_t - \hat{s}_{t|t-1}) \quad (23)$$

After defining the cross-sectional price relationship with measurement equation (22), we can describe the time-series relationship of state variables, which is called transition equation, as follows:

$$s_t = F s_{t-1} + v_t \quad (24)$$

For example, the transition equation for equation (8) and (9) is expressed as

$$s_{it+\tau} = \theta_{ir} (1 - e^{-\kappa_{ir}\tau}) + s_{it} e^{-\kappa_{ir}\tau} + \eta_{it+\tau} \quad (25)$$

where $\tau = 1/12$ and η_{it} has zero mean and the following variance

$$\text{var}(\eta_{it+\tau}) = s_{ir} \frac{\sigma_{ir}^2}{\kappa_{ir}} (e^{-\kappa_{ir}\tau} - e^{-2\kappa_{ir}\tau}) + \theta_{ir} \frac{\sigma_{ir}^2}{2\kappa_{ir}} (1 - e^{-\kappa_{ir}\tau})^2 \quad (26)$$

The log-likelihood function is computed as,

$$l(\tilde{P}_1, \dots, \tilde{P}_T; \Theta) = -\frac{T * N}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T \{ \ln|\Omega_t| + \mu_t' \Omega_t^{-1} \mu_t \} \quad (27)$$

where the prediction errors, μ_t , and their covariance matrices, Ω_t , are updated through the Kalman filter.

V. Credit Event Generation

In the next section we try to gauge the impact of default correlation on CDO valuation. In order to measure the impact of default correlation on valuation of credit derivatives, we need to generate credit events to simulate a distribution.

In our framework, the probability of default within n years under the risk neutral measure is given by:

$$CRPD(n) = 1 - (1 - RPD(n))^n \quad (27)^4$$

The probabilities of transition within one year period is described by a matrix of the form:

$$\begin{bmatrix} p_{ND,ND} & p_{ND,R} & p_{ND,D} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (28)$$

where $p_{ND,ND} + p_{ND,R} + p_{ND,D} = 1$. We assume that each obligor has a set of transition matrices, a matrix per one-year period. Then the probability of default by the n -th year conditional on survival of the beginning of the n -th year is computed as:

⁴ With KMV model, we can simply replace RPD with QEDF.

$$p_{ND,D}(n) = \frac{CRPD(n) - CRPD(n-1)}{1 - CRPD(n-1)} \quad (29)$$

The probability of restructuring is assumed to be zero i.e., $p_{ND,R} = 0$. We present the following two ways to simulate default events: we can either generate asset returns using a joint normal distribution with asset correlations or generate default intensity residuals using a joint normal distribution. For example, for the second approach, assume that the observed default probability and the intensity are denoted as \hat{p}_i and $\hat{\lambda}_i$, respectively. We generate random normal number $(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N)$ for each obligor from the standard normal distribution with covariance matrix Σ , which was estimated from section V. For the obligor i , let $\lambda_{i,j}$ be the intensity simulated at the j -th time, then the default time, $t_{i,j}$, is computed by solving $p_j = 1 - \exp(-\hat{\lambda}_i t_j)$, where $p_j = 1 - \exp(-\lambda_{i,j})$.

VI. Empirical Study

Table 1 reports the parameters for US Treasury and corporate curves, AAA to BB, and their pricing errors. Table 2 reports the parameters for default curves, AAA to BB and their pricing errors. From Table 2, it looks like we need to include jump factors to fit the default curves better since the default spreads seems to be more volatile than the credit spreads.

In Table 3, we show median fitted credit and default spreads for each credit rating. For AAA credit rating, default spreads account for only 6.5% of the AAA credit spreads. For BB credit rating, default spreads explain for 34.2% of the BB credit spreads. The standard deviation suggests that default spreads are more volatile than credit spreads.

Table 4 shows correlation among credit spreads and default spreads. The default spreads show lower correlation among the group than the credit spreads. For correlation between AAA and BB, credit spreads produce 0.84 and default spreads produce 0.47.

Figure 1 shows median of the time series of the fitted credit spreads and Figure 2 shows the time series of 1-year fitted credit and default spreads.

To gauge the impact of default correlation we study a cash-flow CDO under different correlation structure. CDO is an asset-backed security whose underlying collateral is typically a portfolio of bonds or bank loans. A CDO cash-flow structure allocates interest income and principal repayments from a collateral pool of different debt instruments to a prioritized collection of CDO securities (tranches). A standard prioritization scheme is simple subordination: Senior CDO notes are paid before mezzanine and lower-subordinated notes are paid, with any residual cash flow paid to equity notes.

The uncertainty regarding interest and principal payments to CDO tranches is determined mainly by the number and timing of defaults of the collateral securities. In the Basel

Committee on Banking Supervision (BCBS) document of November 2001, asset correlations were assumed to be a decreasing function of firm probability of default. Lopez (2002) reported the similar results.

We set up a CDO with 5 tranches invested on 77 obligors. We assume that there is no reinvestment and liquidation. In the first set, 3 obligors are rated as A, 6 obligors are rated as BBB, 68 obligors are rated as B with constant asset correlation of 0.3. In the second set, 3 obligors are rated as A with correlation 0.6, 6 obligors are rated as BBB with correlation of 0.6, 68 obligors are rated as B with correlation of 0.1. In the third set, 6 obligors are rated as A, 14 obligors are rated as BBB, 57 obligors are rated as B with constant asset correlation of 0.23. In the fourth set, 6 obligors are rated as A with correlation 0.6, 14 obligors are rated as BBB with correlation of 0.6, 57 obligors are rated as B with correlation of 0.1. Thus asset correlations are decreasing as firm probability of default increases with average correlation around 0.23.

Table 5 reports the change of market value of the CDO under different default correlations. In both cases, as we switch from constant correlation to decreasing default correlation, the market value of mezzanine increases while the market value of lower tranches decreases.

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Table 1. Estimated Parameters of Treasury and Credit Curves (July 1995 – February 2002)

	Treasury			AAA		AA		A		BBB		BB	
	Coef	Std		Coef	Std	Coef	Std	Coef	Std	Coef	Std	Coef	Std
κ_{1r}	0.423	0.064	κ_h	0.021	0.012	0.014	0.012	0.015	0.012	0.012	0.013	0.026	0.013
θ_{1r}	0.982	0.019	θ_h	0.597	0.042	0.593	0.034	0.595	0.033	0.595	0.028	0.594	0.026
σ_{1r}	0.017	0.004	σ_h	0.008	0.004	0.007	0.003	0.008	0.004	0.008	0.004	0.016	0.007
λ_{1r}	-0.001	0.200	λ_h	-0.487	0.541	-0.489	0.530	-0.490	0.508	-0.491	0.472	-0.491	0.434
κ_{2r}	0.028	0.062	α_h	-0.052	0.271	-0.051	0.262	-0.050	0.231	-0.049	0.198	-0.049	0.222
θ_{2r}	0.144	0.312	σ_ε	3.684	0.197	3.813	0.206	3.093	0.163	2.329	0.116	1.764	0.067
σ_{2r}	0.024	0.003	β_1	-0.511	0.214	-0.510	0.209	-0.509	0.181	-0.509	0.167	-0.508	0.196
λ_{2r}	-0.123	2.506	β_2	-0.762	0.167	-0.765	0.148	-0.765	0.161	-0.765	0.194	-0.761	0.268
α_r	-1.058	0.021											
σ_ε	2.651	0.130											
RMSE(%)	1.670			1.420		1.410		1.390		1.440		1.650	

Table 2. Estimated Parameters of Treasury and Default Curves (July 1995 – February 2002)

	Treasury			AAA		AA		A		BBB		BB	
	Coef	Std		Coef	Std	Coef	Std	Coef	Std	Coef	Std	Coef	Std
κ_{1r}	0.423	0.064	κ_h	0.043	0.020	0.078	0.016	0.056	0.007	0.064	0.013	0.017	0.066
θ_{1r}	0.982	0.019	θ_h	0.641	0.143	0.602	0.041	0.711	0.045	0.626	0.054	0.591	15.308
σ_{1r}	0.017	0.004	σ_h	0.006	0.002	0.032	0.009	0.030	0.006	0.031	0.007	0.001	0.008
λ_{1r}	-0.001	0.200	λ_h	-0.413	0.521	-0.373	0.388	-0.076	0.116	-0.284	0.285	-0.487	16.352
κ_{2r}	0.028	0.062	α_h	-0.063	0.259	-0.069	0.269	-0.083	0.225	-0.082	0.248	-0.053	15.706
θ_{2r}	0.144	0.312	σ_ε	4.277	0.243	1.187	0.059	1.100	0.060	1.156	0.059	10.451	1.732
σ_{2r}	0.024	0.003	β_1	-0.522	0.169	-0.526	0.240	-0.541	0.215	-0.539	0.235	-0.513	0.849
λ_{2r}	-0.123	2.506	β_2	-0.719	0.150	-0.771	0.132	-0.753	0.147	-0.668	0.158	-0.755	0.357
α_r	-1.058	0.021											
σ_ε	2.651	0.130											
RMSE(%)	1.670			1.360		3.720		1.830		3.300		11.990	

Table 3. Fitted Credit and Default Spreads (July 1995 – February 2002)

	Credit spread		Default spread		Ratio
	Median	Std	Median	Std	
AAA	62.50	27.38	4.04	3.66	6.47%
AA	66.00	31.76	5.79	4.98	8.77%
A	87.00	38.23	12.39	9.76	14.24%
BBB	118.50	49.84	22.65	16.64	19.11%
BB	238.00	87.01	81.31	28.27	34.16%

Table 4. Credit and Default Spread Correlations (July 1995 – February 2002)

Credit Spread Correlations

	AAA	AA	A	BBB	BB
AAA	1.00	0.98	0.93	0.91	0.84
AA	0.98	1.00	0.95	0.93	0.89
A	0.93	0.95	1.00	0.95	0.94
BBB	0.91	0.93	0.95	1.00	0.95
BB	0.84	0.89	0.94	0.95	1.00

Default Spread Correlations

	AAA	AA	A	BBB	BB
AAA	1.00	0.89	0.85	0.77	0.47
AA	0.89	1.00	0.84	0.72	0.43
A	0.85	0.84	1.00	0.84	0.65
BBB	0.77	0.72	0.84	1.00	0.84
BB	0.47	0.43	0.65	0.84	1.00

Table 5. Impact of default correlation on cash-flow CDO's market values.

A (3), BBB (6), B & B- (68)

Tranches	TYPE	PV (0.3,0.3)	Std	PV (0.6,0.1)	Std	Book Value	PV_ratio	std_ratio
note.A	Float	414.41	6.19	416.17	5.23	426.00	0.9958	1.1836
note.B-1	Fixed	13.55	1.28	14.46	0.72	11.50	0.9367	1.7774
note.B-2	Float	29.98	4.83	33.09	2.38	36.50	0.9058	2.0258
note.C-1	Fixed	30.88	3.88	31.47	4.00	28.50	0.9814	0.9719
note.C-2	Float	10.34	2.32	10.56	2.32	15.00	0.9789	0.9966
note.D-1	Fixed	30.44	3.07	28.09	1.96	25.50	1.0839	1.5646
note.D-2	Float	11.99	1.90	10.47	1.16	15.00	1.1453	1.6463
note.E	Equity	46.42	6.00	44.66	2.88	42.00	1.0396	2.0870

A (6), BBB (14), B & B- (57)

Tranches	TYPE	PV (0.23,0.23)	Std	PV (0.6,0.1)	Std	Book Value	PV_ratio	std_ratio
note.A	Float	416.54	1.51	416.71	1.73	426.00	0.9996	0.8744
note.B-1	Fixed	14.69	0.47	14.91	0.02	11.50	0.9850	25.5706
note.B-2	Float	34.26	3.55	35.07	0.61	36.50	0.9771	5.8705
note.C-1	Fixed	34.29	6.97	35.26	4.84	28.50	0.9726	1.4396
note.C-2	Float	12.29	4.61	12.79	3.32	15.00	0.9608	1.3889
note.D-1	Fixed	31.65	6.18	30.42	4.21	25.50	1.0405	1.4682
note.D-2	Float	12.69	4.21	11.89	2.95	15.00	1.0672	1.4245
note.E	Equity	40.64	8.77	39.41	3.64	42.00	1.0311	2.4106

Figure 1. Term Structure of Median Fitted Credit Spreads

July 1995 – February 2002

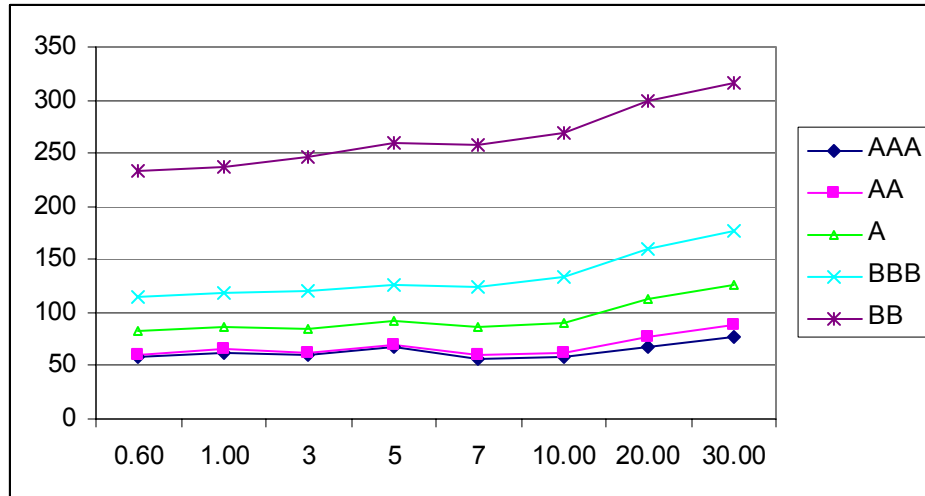


Figure 2. Fitted Credit and Default Spreads (July 1995 – February 2002)

