Inequality-Driven Growth

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Abstract

This paper presents a simple Cass-Koopmans-Ramsey AK growth model with heterogeneity that explains how policies that increase income inequality may temporarily boost a country's income growth rate. Briefly put, a change in policy that reduces redistributive transfers will free up resources to the households with the highest productivities, resulting in an aggregate growth rate increase that will endure until new limits to differentiated accumulation are found. The unambiguous effect takes place in poor and rich countries alike, arising from productivity heterogeneity and redistribution (although it could also arise from other sources of heterogeneity). The effect is explicitly captured in the aggregate growth equation by the changes of the mean logarithmic deviation (MLD or Theil's second measure) of the income. The model supports the empirical results found in Forbes (AER, 2000). The accelerated growth episodes observed in Brazil from 1968 to 1973 and in China recently are shown to be empirically consistent with the model. If the model predictions are correct, Chinese growth rates may eventually fall, following a pattern that, even if not presenting the same magnitude, could resemble the one observed during the Brazilian slowdown.

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> "The cake has to grow in order to be cut." Delfim Netto, Minister of Finance during the Brazilian "economic miracle" period

> "Draw a cake to satisfy one's hunger." Chinese proverb

1 Introduction

A large body of literature addressing the relations between inequality and growth has been developed across the years. Yet, despite all the theoretical and empirical developments since the pioneering work of Kuznets, much theoretical and empirical disagreement remains. This paper will try to add to the literature by bringing in another possible connection between these two economic variables, one that, to a certain extent, has been disregarded both theoretically and empirically. The connection can be found in accelerated growth episodes that happened at the cost of permanently higher levels of income inequality. Those episodes are defined here as cases of *inequality-driven growth*.

The paper will use a Cass-Koopmans-Ramsey AK growth model based on households with heterogeneous productivity levels. The household production function will incorporate spillovers from public capital and from private capital owned by other households, which will work as a redistribution mechanism.

The combination of heterogeneous productivities with the redistribution mechanism implied by the spillovers, under given conditions, will generate balanced growth income trajectories defined by an equilibrium distribution of income and a unique income growth rate common to all households. A log-linearized version of the model will be aggregated and, as a result, the aggregate growth rate will be decomposed into three parts, which represent the negative time preference effect, the positive aggregate productivity effect, and the positive inequality-driven effect – the latter representing the original contribution of this paper.

This effect should not be confounded with the savings rate effect from the Keynesian literature, which may have an ambiguous sign for poor and rich countries and may affect growth rates temporarily or permanently.[1] The inequality-driven growth effect presented here does not depend on any special assumption regarding the savings behavior. It results exclusively from productivity heterogeneity and aggregation.[2]

The model will then be used to investigate how redistributive policy changes may not only permanently affect growth rates but also generate inequality-driven growth episodes.

The presence of an inequality-driven growth component in the aggregate growth equation lends support to the strong empirical findings of Forbes (2000). As summarized in that paper, "(empirical) results suggest that, in the short and medium term, an increase in a country's level of income inequality has a significant positive relationship with subsequent economic growth. This relationship is highly robust across samples, variable definitions, and model specifications."

Two empirical cases will be discussed: the Brazilian "economic miracle" high-growth period, and the recent Chinese high-growth episode. Data for the two countries will be analyzed under the scope of the previously developed model, and the similarities between the two cases will be considered. The two countries will be shown to possibly present inequality-driven growth dynamics.

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[1] See Gersovitz (1988) for a discussion on the relations between savings and growth.

[2] The inequality-driven growth effect is an aggregation effect and, as such, it can result from any source of inequality. In this paper, productivity heterogeneity is chosen as the source, to ensure that it is not confounded with other effects yet established in the growth literature.

2 A Simple AK Growth Model with Productivity Heterogeneity

2.1 The AK Model

The endogenous growth model that is presented here is based on a Cass-Koopmans-Ramsey framework with an AK production function.[3]

Consider therefore an economy with a large number of households $(N \to \infty)$. Each infinitely lived household maximizes the following CRRA utility function:

$$U_n = \sum_{t=0}^{\infty} e^{-\rho t} \frac{C_{nt}^{1-\theta}}{1-\theta},$$

where

$$\rho > 0, \quad \text{and} \quad 0 < \theta < 1.$$

To keep the representation as simple as possible, and without loss of generality, it is assumed, as in other AK models, that capital is represented by a single variable that encompasses all production factors. The production function depends not only on the household's private capital but also on spillovers from public capital and other household's private capital.

The government appropriates a fixed proportion of each household's total stock of capital in order to make public capital available to every household. Notice that the appropriation mechanism described here does not imply legal expropriation of private property. This representation works exactly like an income tax, with the advantage however of creating a simpler and more tractable description of the economy.

The after-tax production function is accordingly defined as

$$f_n\left(\tilde{K}_{gt}, \tilde{K}_{pt}, K_{pnt}\right) = A_n \tilde{K}_{gt}^{\gamma} \tilde{K}_{pt}^{\eta} K_{pnt}^{1-\gamma-\eta},$$

[3] See, for example, Aghion and Howitt (1998) or Jones and Manuelli (1997).

where

$$0 < \tau < 1, \quad 0 \le \eta \le 1, \quad 0 \le \gamma \le 1, \quad 0 \le \gamma + \eta < 1, \quad A_n > 0,$$

$$K_{nt} = K_{pnt} + K_{gnt}, \quad K_{gnt} = \tau K_{nt}, \quad \widetilde{K}_{pt} = \prod_{n=1}^{N} K_{pnt}^{1/N}, \quad \text{and} \quad \widetilde{K}_{gt} = \prod_{n=1}^{N} K_{gnt}^{1/N}.$$

In the equations above, K_{nt} is the household's total capital, which has two components, K_{pnt}, the private capital not appropriated by the government, and K_{gnt}, the part of the household's total capital that is appropriated by the government for public use, which is determined as a fixed proportion τ of the total capital.[4]

The household's production level depends on the productivity parameter A_n (which is heterogeneous across households), on the private capital level K_{pnt}, and on the spillovers from the geometric average of all appropriated capital levels $\widetilde{K}_{gt}$ and from the geometric average of all private capital levels $\widetilde{K}_{pt}$.

In the production function above, government policies can affect two parameters: the *tax rate* τ and the redistribution parameter η. The tax rate τ represents redistributions that affect the amount of public capital available to all households, and, as such, it redistributes wealth following the principle of *equality of opportunities*. Meanwhile the redistribution parameter η represents, at least in part, the government will to socialize final production, and, as such, it redistributes wealth following the principle of *equality of results*. Note that a $\gamma+\eta$ or τ that approaches one implies communism (absence of private property). Values of η between zero and $1-\gamma$ define varying degrees of socialism (equalization of results).

The redistribution parameter η is subject only in part to government control, since it represents any possible spillover that is not related to public capital, such as: government-enforced income transfers, donations, charity,

[4] Under the household's perspective, the capital tax is equivalent to an income tax rate $\pi = 1-(1-\tau)^{1-\gamma-\eta}$, since $Y_{nt}^{\text{after-tax}} = A_n \widetilde{K}_{gt}^{\gamma} \widetilde{K}_{pt}^{\eta} K_{pnt}^{1-\gamma-\eta} = (1-\tau)^{1-\gamma-\eta} A_n \widetilde{K}_{gt}^{\gamma} \widetilde{K}_{pt}^{\eta} K_{nt}^{1-\gamma-\eta} = (1-\pi) Y_{nt}^{\text{before-tax}}$.

crime, epidemics, riots, specialization, trade, or any other positive or negative externality originating from private capital. Given the unbalanced nature of those externalities, a zero-sum restriction on the redistributive transfers will typically not hold.[5]

Finally, notice that households must observe the budget constraint

$$Y_{nt} = f\left(\widetilde{K}_{gt}, \widetilde{K}_{pt}, K_{pnt}\right) \geq C_{nt} + \Delta K_{nt+1}, \tag{2.1}$$

and also that the production function can be rewritten as:

$$Y_{nt} = f_n\left(\widetilde{K}_t, K_{nt}\right) = A_n \tau^{\gamma} (1-\tau)^{1-\gamma} \widetilde{K}_t^{\gamma+\eta} K_{nt}^{1-\gamma-\eta}. \tag{2.2}$$

2.2 First-Order Conditions

Assume now that all necessary conditions for the existence of an interior solution hold. The first-order conditions are thereafter given by:

$$\lambda_{nt} = e^{-\rho t} C_{nt}^{-\theta},$$

and

$$\frac{\lambda_t}{\lambda_{t+1}} = 1 + \phi A_n \left(\frac{\widetilde{K}_t}{K_{nt}}\right)^{\gamma+\eta},$$

where

$$\phi = \tau^{\gamma}(1-\tau)^{1-\gamma}(1-\gamma-\eta), \tag{2.3}$$

which, when combined, lead to the Euler equation

$$\frac{C_{nt+1}}{C_{nt}} = \left\{ e^{-\rho} \left[1 + \phi A_n \left(\frac{\widetilde{K}_t}{K_{nt}}\right)^{\gamma+\eta} \right] \right\}^{\frac{1}{\theta}}.$$

Now, assume that $A_n \phi \left(\widetilde{K}_t / K_{nt}\right)^{\gamma+\eta} << 1$, and take the logarithm of the equation above to find

[5] Note that the values of $\widetilde{K}_{gt}$ and $\widetilde{K}_{pt}$ for an isolated household are zero. A hermit household production function cannot benefit from spillovers from public and private capitals.

$$\Delta c_{nt+1} \approx \frac{1}{\theta}\left[-\rho + \phi A_n \left(\frac{\widetilde{K}_t}{K_{nt}}\right)^{\gamma+\eta}\right], \tag{2.4}$$

where $c_{nt+1} = \ln C_{nt+1}$. This Euler equation describes the household consumption growth rate as an increasing function of the household productivity A_n, as expected. Additionally, from (2.3), it is also a decreasing function of the household discount factor ρ, of the coefficient of relative risk aversion θ, and of the relative wealth level $K_{nt}/\widetilde{K}_t$. The relation between the consumption growth rate and the tax rate τ and the redistribution parameter η can be positive or negative.

2.3 The Log-Linearized Euler Equation

In order to easily aggregate equation (2.4), a log-linearized version needs to be found. From equation (2.2),

$$\frac{\widetilde{K}_t}{K_{nt}} = \left(\frac{A_n}{\widetilde{A}}\frac{\widetilde{Y}_t}{Y_{nt}}\right)^{\frac{1}{1-\gamma-\eta}}. \tag{2.5}$$

Assume that savings rates change slowly compared to the growth rates of the economy, such that, from (2.1),

$$\Delta c_{nt} = \Delta y_{nt} + \Delta \ln(1 - s_{nt}) \approx \Delta y_{nt}.$$

Applying the assumption above and equation (2.5) to equation (2.4) leads to the approximation

$$\Delta y_{nt+1} \approx \frac{1}{\theta}\left[-\rho + \phi A_n \left(\frac{A_n}{\widetilde{A}} H_{nt}\right)^{\psi}\right], \tag{2.6}$$

where $H_{nt} = \widetilde{Y}_t / Y_{nt}$ represents the relative income gap of the household, and

$$\psi = \frac{\gamma + \eta}{1 - \gamma - \eta}.$$

Log-linearizing equation (2.6) results in the following approximation:

$$\Delta y_{nt+1} \approx \alpha_n + \beta h_{nt}, \tag{2.7}$$

where

$$\alpha_n \approx \frac{1}{\theta}\left[-\rho + \phi A_n + \psi\left(a_n - \overline{a}\right)\right], \quad \beta = \frac{\Psi}{\theta},$$

$$a_n = \ln A_n\,,\ \overline{a} = \frac{1}{N}\sum_{n=1}^{N} \ln A_n\,,$$

and

$$h_{nt} = \ln H_{nt} = y_{nt} - \overline{y}_t$$

is the logarithmic income gap.

Consider now the two parameters in equation (2.7). Parameter β represents the *redistributive effectiveness* of the economy. The higher the β, the more significant will be the transfers of growth rates between households, due to government interventions or due to positive or negative social externalities, and the lower will be the income inequality, as will be shown later.

Parameter α_n, on the other hand, summarizes the household productivity contribution to growth, conditional on economic incentives. It represents innate skills and non-sharable environmental advantages and disadvantages, but also depends on government policies defined by τ and η, since those policies affect the incentive structure of the economy.

2.4 The Aggregation Method

Consider now a simplified version of the log-linear aggregation method presented in Albuquerque (2003). Take I+1 vectors representing the values of I+1 variables for N households at time t,

$$\mathbf{Y}_t = \left[Y_{1t} \quad \cdots \quad Y_{nt} \quad \cdots \quad Y_{Nt}\right]', \qquad \mathbf{X}_{it} = \left[X_{i1t} \quad \cdots \quad X_{int} \quad \cdots \quad X_{iNt}\right]',$$

where

$$Y_{nt} > 0, \quad X_{int} > 0, \quad i = 1,\ldots,I, \quad n = 1,\ldots,N, \quad \forall t\,,$$

If a heterogeneous household function is defined as

$$Y_{nt} = X_{1nt}^{a_1} X_{2nt}^{a_2} \cdots X_{Int}^{a_I}$$

then the relationship among the aggregate variables $\overline{Y}_t$ and $\overline{X}_{it}$ at each period t will be given by

$$\overline{Y}_t = \overline{X}_{1t}^{a_1} \overline{X}_{2t}^{a_2} \cdots \overline{X}_{It}^{a_I} D_t , \tag{2.8}$$

where the aggregate variables are defined as per capita values

$$\overline{Y}_t = \frac{1}{N}\sum_{n=1}^{N} Y_{nt} \, , \; \overline{X}_{it} = \frac{1}{N}\sum_{n=1}^{N} X_{int} \, ,$$

and the term

$$D(\mathbf{Y}_t, \mathbf{X}_{1t}, \ldots, \mathbf{X}_{Nt}) = \exp\left\{ L(\mathbf{Y}_t) - \sum_{i=1}^{I} a_i L(\mathbf{X}_{it}) \right\}$$

represents distributional effects, where

$$L(\mathbf{Y}_t) = \frac{1}{N}\sum_{n=1}^{N} \ln\left(\frac{\overline{Y}_t}{Y_{nt}} \right)$$

is the sample analog of the *mean logarithmic deviation* (MLD), also known as the *Theil's second measure* of $\mathbf{Y}_t$, a measure of income inequality,[6]

$$\mathbf{x}_{it} = \begin{bmatrix} x_{i1t} \\ \vdots \\ x_{iNt} \end{bmatrix} = \begin{bmatrix} \ln(X_{i1t}) \\ \vdots \\ \ln(X_{iNt}) \end{bmatrix}, \quad \text{and } \overline{x}_{it} = \frac{1}{N}\sum_{n=1}^{N} x_{int} \, .$$

Note that all components of D_t represent relative measures of inequality, meaning that D_t is scale invariant.

The logarithmic version of (2.8) is

$$y_t = a_1 x_{1t} + a_2 x_{2t} + \cdots + a_I x_{It} + d(\mathbf{Y}_t, \mathbf{X}_{1t}, \ldots, \mathbf{X}_{Nt}), \tag{2.9}$$

where

$$y_t = \ln \overline{Y}_t, \quad x_{it} = \ln \overline{X}_{it}, \quad \text{and} \quad d(\cdot) = \ln D(\cdot).$$

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[6] It is illustrative to reproduce here the properties of this measure according to Bourguignon (1979): "That the inequality measure L has seldom been used in applied works on income distribution is somewhat surprising because it has very much to commend it. Besides the fact that it is decomposable … and satisfies the basic properties of an inequality measure, L lends itself to a very simple interpretation in terms of social welfare. In the utilitarian framework, the social welfare function is the sum of identical concave individual utility function. If we choose the logarithm form for those utility functions, L is simply the difference between the maximum social welfare for a given total income, which corresponds to the equalitarian distribution, and the actual social welfare."

2.5 Aggregating the Model

A literal solution to the aggregation problem can now be provided. From equation (2.7):

$$y_{nt+1} = y_{nt} + \alpha_n + \beta h_{nt} . \tag{2.10}$$

Equation (2.9) can be applied to equation (2.10), resulting in the following per household aggregate income equation:

$$\Delta y_t = \overline{\alpha} + \Delta L(\mathbf{H}_t), \tag{2.11}$$

where

$$\overline{\alpha} = -\frac{\rho}{\theta} + \frac{\phi}{\theta}\overline{A} , \tag{2.12}$$

$$\overline{A} = \frac{1}{N}\sum_{n=1}^{N} A_n \ , \quad \text{and} \quad \mathbf{H}_t = \begin{bmatrix} H_{1t} & \cdots & H_{nt} & \cdots & H_{Nt} \end{bmatrix}' ,$$

since

$$h_t = \ln \overline{H}_t = \overline{h}_t + L(\mathbf{H}_t) = L(\mathbf{H}_t),$$

and

$$d(\mathbf{H}_{t+1}) = \Delta L(\mathbf{H}_{t+1}) - \beta L(\mathbf{H}_t).$$

2.6 Balanced Growth

Assume now that the income distribution converges to some relative income profile under balanced growth such that $\Delta c_n^* = \Delta y_n^*$ and, from equations (2.4) and (2.5),

$$\Delta y_n^* = \frac{1}{\theta}\left[-\rho + \phi A_n \left(\frac{A_n}{\widetilde{A}} H_n^* \right)^{\psi} \right],$$

where $H_n^* = \lim_{t \to \infty} \widetilde{Y}_t / Y_{nt}$ represents the balanced growth relative income gap of the household.

Balanced growth is thereafter feasible if and only if

$$\frac{\partial \Delta y_n^*}{\partial H_n^*} = \frac{\phi}{\theta} \psi A_n \left(\frac{A_n}{\tilde{A}} \right)^{\psi} H_n^{*\psi-1} > 0 \,,$$

or

$$\gamma + \eta > 0, \quad \forall n \,. \tag{2.13}$$

This AK model may represent therefore a society that accepts the existence of an arbitrary level of income inequality, but that does not accept income inequality divergence. The redistribution parameter η and the tax rate τ guarantee that the aggregate economy growth engine work for every household, at least in the long run, as long as condition (2.13) holds. The model is able to capture thereafter the Hirschman and Rothschild (1973) tunnel effect hypothesis.

From equations (2.7) and (2.11), balanced growth is defined as a set of household income growth trajectories where

$$\Delta y^* = \Delta y_n^* = \overline{\alpha}, \quad \forall n \,, \tag{2.14}$$

with household income distributed according to a vector of logarithmic income gaps $\mathbf{h}^* = \begin{bmatrix} h_1^* & \cdots & h_n^* & \cdots & h_N^* \end{bmatrix}'$ where

$$h_n^* = \frac{\overline{\alpha} - \alpha_n}{\beta} \,. \tag{2.15}$$

According to equation (2.15), the relative income distribution under balanced growth will depend on the distribution of the household productivity parameter A. Ceteris paribus, the more unequal the values of A, the higher the income inequality. On the other hand, the higher the redistributive effectiveness β, the lower the income inequality. Finally, the effect of the tax rate τ on inequality can be both positive and negative, since an increase of the tax rate may lead either to an increase or to a reduction of the variability of α.

2.7 Growth Rate Structural Decomposition

Equation (2.11) reveals that the per household income growth rate can be divided into two components: $\overline{\alpha}$, which is related to mean values, and $\Delta L(\mathbf{H}_t)$, which represents distribution effects.

Component $\overline{\alpha}$ is a constant encompassing the negative *time preference effect* $-\rho/\theta$ plus the positive *aggregate productivity effect* $\phi\overline{A}/\theta$. Component $\Delta L(\mathbf{H}_t)$, on the other hand, represents the *inequality-driven growth effect* on aggregate growth.

This result can be summarized by the following proposition:

Proposition 1: *Under the assumption of a simple log-linear structural growth model, the aggregate growth rate of an economy can be decomposed into three additive terms: the negative time preference effect* $-\rho/\theta$*, the positive aggregate productivity effect* $\phi\overline{A}/\theta$*, and the inequality-driven effect* $\Delta L(\mathbf{H}_t)$.

The most important feature of Proposition 1 is that it reveals a component of aggregate growth rates that unambiguously depends on inequality, which can be explicitly measured through MLD (Theil's second measure) changes, and which is mostly disregarded in the current inequality and growth literature. This component of aggregate growth rates should appear in any aggregated log-linear growth model based on heterogeneous households subject to redistribution mechanisms, since the component arises not at the structural level, but at the aggregation procedure level. [7]

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[7] To understand the effect captured by this growth component, a parallel can be made with the case of a locomotive pulling a caboose by means of an elastic cable. The locomotive represents high-productivity households, while the caboose represents low-productivity households. The elastic cable represents the redistribution mechanism. Even if no structural parameter is changing (productivity levels, time preference, risk aversion coefficients – the power sources and the frictions), once the cable is made more elastic, the result is a temporary acceleration of any reference point near the locomotive (the equivalent of the per capita income), at the cost of permanently higher inequality levels (the cable will stretch

For example, in Albuquerque (2003) a simple nonstructural heterogeneous log-linear growth model presenting asymmetric productivity shocks for skilled and unskilled households is used to explain, theoretically and empirically, some features the American "new economy" accelerated productivity growth episode in the nineties. In that model, an increase in productivity inequality is what causes the inequality-driven effect. The inequality-driven effect can be generally interpreted therefore as the result of an aggregation "growth identity," obtained from equation (2.9), rather than the result of particular structural model hypotheses.

3 Redistributive Policies and the Creation of Inequality-Driven Growth

3.1 Permanent Effects of Redistribution

In equation (2.12) and in Proposition 1, the time preference component of $\overline{\alpha}$ does not depend on government policies. The aggregate productivity component of $\overline{\alpha}$, on the other hand, represents the aggregate productivity conditional on economic incentives and externalities, capturing two possible effects of redistribution on the aggregate growth rate that reproduce aspects found in the inequality and growth literature.

3.1.1 The Economic Interactions Effect

This economic spillover effect arises from two sources. The first source reflects the benefits of economic interactions through the formation of public capital. Without these economic interactions, production is not possible. Government uses taxes to appropriate a part of each household's private

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further). Finally, when the cable is again fully stretched, the locomotive will fall down to the previous speed and acceleration, since it will be subject once more to the deadweight and additional friction of the caboose.

capital in order to provide society with a certain level of public capital, and by doing this, up to a certain point, it not only raises production levels, but also the balanced growth rate of the economy.[8] From (2.12):

$$\frac{\partial\overline{\alpha}}{\partial\tau} > 0 \text{ when } \tau < \tau^* = \gamma,$$

where $\tau^* = \gamma$ is the optimal tax rate. Excessive appropriation of private capitals, on the other hand, is inefficient, leading to reductions in the production level and aggregate growth rate of the economy:

$$\frac{\partial\overline{\alpha}}{\partial\tau} < 0 \text{ when } \tau > \tau^*.$$

The second source reflects private capital spillovers. A net benefit from economic specialization and trade arises when an economy of hermits is transformed by trade and specialization into an economy with private capital spillovers. The private and public capital spillovers, during the first stages of economic development, imply a positive effect of redistribution on aggregate wealth and growth.

3.1.2 The Political Economy Effect

Once society achieves balanced growth under optimal taxation, it cannot benefit anymore from the spillovers. The allocative distortions of the redistribution mechanism – the *political economy* effect discussed for example in Barro (2000) and Asano (2002) – may dominate the relation between inequality and growth. Political pressure for inequality reductions will imply that the higher the redistribution levels (the higher the values of β), the more important the allocative distortions that reduce the equilibrium growth rate of the economy and the level of wealth. From (2.12):

$$\frac{\partial\overline{\alpha}}{\partial\beta} < 0.$$

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[8] A survey on the effects of redistributive government spending on growth can be found in Carneiro et al. (2002).

Consider therefore the following proposition, which summarizes the model permanent effects of inequality on growth:

Proposition 2: *A country will benefit most from the economic interactions effect (spillovers from public and private capitals) during the early stages of development, when its tax rate is low. Higher taxation levels during this stage will increase wealth and growth but also increase income inequality. However, as the country reaches balanced growth with optimal taxation, the political economy effect becomes the most important, and allocation distortions ensue if redistribution is enforced. Increases in public capital levels, obtained through increases in the tax rate* τ, *will reduce income inequality at the cost of permanently lower growth rates. Increases in redistribution effectiveness through a higher* β *will have the same positive effect on income distribution but negative effect on growth.*

Notice that, somewhat paradoxically, in early stages of development with low taxation levels, higher levels of taxation and higher levels of public capital will lead to an *increase* in income inequality. This happens because the increase in taxation and public capital, up to a certain level, allows high-productivity households to unlock their dormant skills, and, as such, to differentiate themselves from low-productivity households.

This result, on the other hand, agrees with the Kuznets inverted-U hypothesis.[9] If a country, during its development history, goes from a low taxation level (below the optimal level) to a high taxation level (above the optimal level), as it should normally be the case, inequality levels will increase during early stages of development, and decrease during final stages. This paper's model is consistent therefore with a public capital accumulation explanation of the inverted-U hypothesis.

[9] The Kuznets inverted-U hypothesis proposes that income inequality increases during the initial development stages and decreases during the final development stages of a country. See Kuznets (1955).

3.2 Transitory Effects of Redistribution: Inequality-Driven Growth

During the transition from one balanced growth with lower inequality to another with higher inequality, the value of the component $\Delta L(\mathbf{H}_t)$ in equation (2.11) raises from zero. The positive values of $\Delta L(\mathbf{H}_t)$ temporarily boost the aggregate growth rates of the economy.

Eventually, the economy will reach the new balanced growth with higher levels of inequality, and the permanent growth rate will return to a lower level. This transitory effect applies to poor and rich countries alike and is unambiguous. It should not be confounded with the saving rates effect found in the Keynesian literature. Consider then the following proposition:

Proposition 3: *A reduction in redistribution effectiveness or a tax rate decrease that promotes higher inequality levels will lead, ceteris paribus, to an unambiguous transitory increase of the growth rate for poor and rich countries alike. The growth rate boost will endure until a new balanced growth, characterized by permanently higher levels of inequality, is eventually reached. Episodes characterized by this type of dynamics are defined here as inequality-driven growth periods.*

Proposition 3 reveals that government policies can lead to transitory increases of aggregate growth rates at the cost of permanently higher levels of income inequality. Policy makers may thereafter be tempted to temporarily stimulate aggregate growth by allowing inequality to grow, independently of the permanent effects on growth rates, which, according to the inequality and growth literature, may be positive or negative.[10]

The inequality-driven effect, represented by component $\Delta L(\mathbf{H}_t)$ in equation (2.11), lends support to the strong empirical findings of Forbes (2000). In that paper, panel data methods are applied to a data set

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[10] See, for example, Barro (2000).

representing 45 countries and 180 observations. As summarized by Forbes, "results suggest that, in the short and medium term, an increase in a country's level of income inequality has a significant positive relationship with subsequent economic growth. This relationship is highly robust across samples, variable definitions, and model specifications."

To see that Forbes' empirical results are supported by this paper's model, notice that equation (2.11) can be rewritten as

$$\Delta y_t - \Delta y^* = \Delta L(\mathbf{H}_t),$$

where $\Delta y^* = \overline{\alpha}$ represents the steady state aggregate growth rate. Changes in inequality have to be therefore strongly correlated with growth rate departures from an equilibrium value. This theoretical result holds under somewhat general conditions, and as such may serve as a theoretical foundation for Forbes' empirical findings.[11]

In the next two sections, two empirical examples will be shown to possibly represent inequality-driven growth episodes.

4 A Cake Waiting to Be Cut: The Brazilian "Economic Miracle"

From 1968 to 1973, Brazil experimented a period of high growth rates that came to be known as the Brazilian "economic miracle" period. This period, according to the usual interpretation, was the result, among other things, of high levels of foreign savings, mostly based on government borrowing in foreign capital markets, of an increase in mandated domestic savings, of the achievement, during the previous years, of fiscal discipline, and of central-planned measures that ranged from managed trade policies to

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[11] Forbes' results were based on the use of the Gini coefficient as inequality measure, what would imply, according to this paper's model, a specification error. Yet, the specification error is probably not very significant, since changes in the Gini coefficients tend to be highly correlated with changes in the MLD.

an omnipresent system of subsidies and government credit. See, for example, Fishlow (1972), Sjaastad (1974), Fields (1977), Ahluwalia et al. (1980), Beckerman and Coes (1980), Fields (1980), Fishlow (1980), and Fox (1983) for additional details.

The period was marked by exceptionally high yearly growth rates and substantial increases in income inequality, as can be seen in Appendix 1, Graph 1, and in the following table:

Period	Real GDP per Worker, Yearly Logarithmic Growth	MLD, Yearly Change
1961-1967	0.029	0.003
1968-1973	0.073	0.017
1974-1980	0.032	-0.010
1981-1989	0.011	0.015

The real GDP data in this table comes from Heston et al. (2002), and the data for income inequality comes from the “high quality” WIID databank based on Deininger and Squire (1997). The MLD values were calculated using the databank income distribution quintiles.[12]

It is easy to notice from the table above that the “economic miracle” period (shadowed) was exceptional when compared to all others. The income inequality, measured by the MLD, grows faster than in any other period. The yearly real GDP growth rate per worker during the “miracle” period is approximately 4.2% higher than the rates that prevailed during the preceding and succeeding periods. Notice, however, that near half of this growth rate boost is explained by changes of income inequality levels. The inequality-driven effect is approximately equal to 1.7% per year.

The “economic miracle” period easily qualifies as an inequality-driven growth episode. To confirm this conclusion, the following OLS regression is

[12] The inequality levels may be somewhat underestimated due to the quintile approximation, but notice that the yearly changes are less affected by this kind of approximation bias.

run, with the caveat that inequality data for poor countries tend to be subject to a high amount of noise, and that the number of observations in the regression is small. Having that in mind, extensive testing is avoided, and only the main regression results are shown:

$$y_t - y_{t-5} = \underset{(9.00)}{0.214} + \underset{(3.82)}{0.809} \cdot \left(L(\mathbf{H}_t) - L(\mathbf{H}_{t-5})\right) - \underset{(-4.63)}{0.035} \cdot DEBT,$$
$$N = 20, \overline{R}^2 = 0.63, F = 16.9.$$

The regression sample ranges from 1970 to 1989 – the years before 1970 do not present variation in inequality changes, due to the inexistence of intraperiod observations (inequality data in unobserved years were linearly interpolated), and as such they were discarded from the sample. The dependent variable represents the five-year logarithmic growth rate of the real GDP per worker, and the independent variables represent the five-year change of the MLD and a dummy variable (*DEBT*) that takes care of the debt shock of 1981. The values between parentheses and under the estimated parameters represent *t*-statistics. All parameters are significant at a significance level of 1%. The regression explains 63% of the dependent variable variations. The regression fit is presented in Appendix 1, Graph 2. As it can be seen, the fit is surprisingly good.

Interestingly, the hypothesis that the inequality change parameter in the regression above is equal to one cannot be rejected. This result should not be seen as expected. From Proposition 2, it can be seen that this result is only possible if the inequality change component is statistically independent from other growth rate components. It may be the case, however, that this Brazilian episode represents exactly such an extreme case of inequality-driven growth, with enough variation in inequality to enable the model to reveal the aggregate growth "identity" given by equation (2.11).

Someone could argue that the regression is flawed because income growth and inequality changes could be nonstationary variables, and a cointegration model should be considered. To evaluate this possibility, a cointegration test was applied to the series (under the risk of running into overfitting), with the following results:

Johansen Cointegration Rank Test [13]

Series	p	LR Max	LR Min	Estimated Cointegration Vector
$\left[\Delta y_t, \quad \Delta L(\mathbf{H}_t), \quad \text{constant}\right]'$	4	34.8**	1.88	$\left[1, \quad \underset{(-4.71)}{-1.10}, \quad \underset{(-8.68)}{-0.038}\right]'$

The cointegration test rejects the hypothesis of noncointegration, and does not reject the hypothesis of a unique cointegration vector. The hypothesis that the long-run parameter relating income growth to inequality changes is equal to one cannot be rejected, as in the OLS regression. It looks thereafter that the Brazilian experience cannot be rejected as an inequality-driven growth episode.

5 Drawing a Cake: High Growth in China

The Chinese high growth episode, although much more protracted than the Brazilian, has in common the same exceptionally high growth rates and income inequality increases. This topic has been extensively described in previous studies, with a few examples represented by Khan and Riskin

[13] * significant at 5%; ** significant at 1%; Johansen likelihood ratio (LR) cointegration rank test, trace statistic, intercept in cointegration equation, dummy for the debt shock in 1981; a significant LR Max statistic indicates rejection of the null hypothesis of cointegration rank equal to zero (rejection of noncointegration); a significant LR Min statistic indicates rejection of the null hypothesis of cointegration rank lower than or equal to one (rejection of noncointegration and of cointegration with one cointegrating vector); critical values come from Osterwald-Lenum (1992); p represents the number of lags as in Johansen and Juselius (1990); The Akaike lag-selection criterion (AIC) was employed in order to find p.

(1998), Yao (1999), Xu and Zou (2000), Meng (2001), Galbraith and Wang (2002), Park et al. (2002), and Zhang and Harvie (2002).

Unfortunately, the series for China are relatively short. Additionally, it should be noted that there is much dispute about the comparability of Chinese data with data from other countries, as discussed for example in Gibson et al (2001). Yet, the trends are clear, as summarized in Appendix 2, Graph 3, and in the following table, which uses, as in the Brazilian case, data from Heston et al. (2002) and from the "high quality" WIID income inequality databank:

Period	Real GDP per Worker, Yearly Logarithmic Growth	MLD, Yearly Change
1981-1984	0.064	-0.012
1985-1992	0.046	0.016

The period between 1981 and 1984 is atypical, with very high growth rates and decreasing inequality levels. However, the years between 1985 and 1992 (shadowed) can be seen as another case of inequality-driven growth episode. From the total yearly growth rate of 4.7%, approximately 1.6% can be attributed to inequality changes – what could represent a large part of a possible growth boost over a long-run growth trend. According to some of the studies cited above, the trend of inequality increase may have accelerated after 1992, meaning that the effect may have become even more significant afterwards.

Naturally, much of the high growth in China could be explained by structural changes captured by the other components described in Proposition 2. China also has one advantage over Brazil: it started its inequality-driven growth episode from much lower levels of inequality. In that sense, someone could argue that there is much leeway yet in Chinese society for income inequality increases. On the other hand, it could also be argued that the Chinese culture of equality is rooted deeper than in other countries, meaning that the limits to income inequality increases may be reached sooner than expected.

There is evidence that, during the seventies, the tunnel effect presented itself in Brazil, finally leading society to pressure the authoritarian government for effective redistributive policies, in a mechanism discussed for example in Iglesias (1998). One proof of the importance of this phenomenon is the extensive literature concerning income inequality in Brazil, which was produced mainly during the seventies (a parallel can be made with the current sprout of literature concerning income inequality in China). To make things worse, while social pressures for redistribution and democracy were mounting, Brazil had to face significant negative economic shocks, like the oil shock and the debt crisis. All those factors together explain the sudden and significant reduction of growth rates in Brazil.

Given the similarities between the two growth episodes, it is reasonable to assume that Chinese growth rates may eventually present the same falling pattern, even if not as severe as the one observed in Brazil.

6 Conclusions

This paper presents a simple Cass-Koopmans-Ramsey AK growth model with heterogeneity that explains how policies that increase income inequality may temporarily boost a country's income growth rate. Briefly put, a change in policy that reduces redistributive transfers will free up resources to the agents with the highest productivities, resulting in an aggregate growth rate increase that will endure until new limits to differentiated accumulation are found.

The unambiguous effect takes place in poor and rich countries alike, arising from productivity heterogeneity and redistribution (although it could also arise from other sources of heterogeneity), and therefore should not be confounded with the savings rate effect found in the Keynesian framework. The effect is explicitly captured in the aggregate growth equation by the changes of the mean logarithmic deviation (MLD or Theil's second measure) of the income.

This component of aggregate growth rates may appear in any aggregated log-linear growth model based on heterogeneous households subject to redistribution mechanisms, since the component arises not at the structural level, but at the aggregation procedure level. In this sense, the inequality-driven effect may be better interpreted as an aggregation growth identity than as a structural component resulting from particular model hypotheses.

Due to the existence of this effect, policymakers may be tempted to temporarily accelerate aggregate growth rates by cutting on redistributive policies, at the cost however of permanently higher inequality levels and, in some cases, at the cost of lower long-run growth rates.

The inequality-driven effect found in this paper lends support to the strong empirical findings of Forbes (2000). In that paper, panel data methods are applied to a data set representing 45 countries and 180 observations. As summarized by Forbes, "results suggest that, in the short and medium term, an increase in a country's level of income inequality has a significant positive relationship with subsequent economic growth. This relationship is highly robust across samples, variable definitions, and model specifications."

The accelerated growth episodes observed in Brazil during the seventies and in China recently were shown to be empirically consistent with the model. If the model predictions are correct, Chinese growth rates may eventually fall, following a pattern that, even if not presenting the same magnitude, could resemble the one observed during the Brazilian slowdown.

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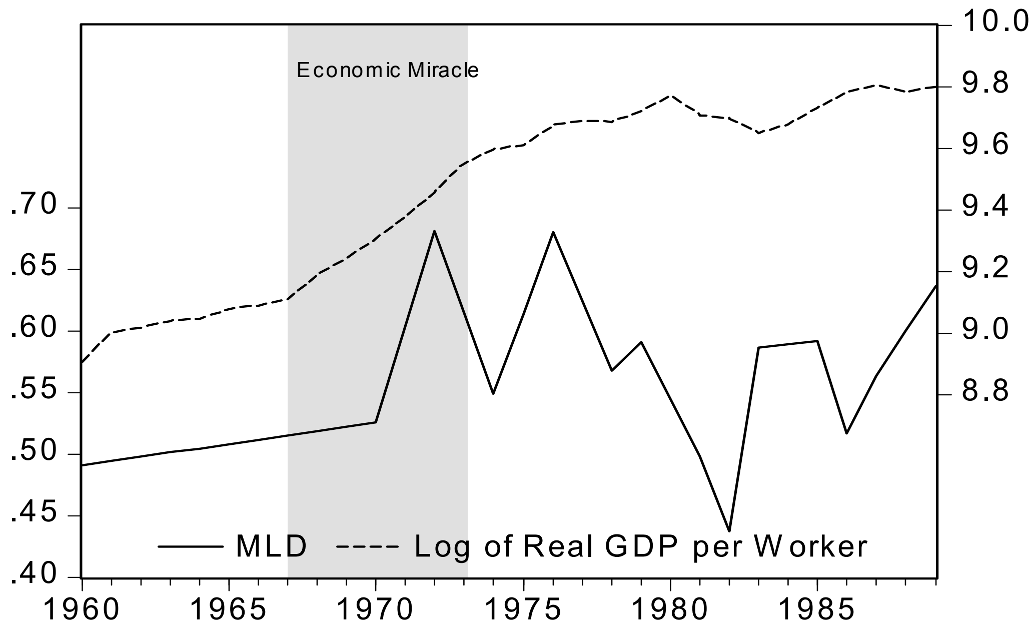
Appendix 1

Graph 1 – Brazil: Income and Inequality



Graph 2 – Real GDP per Worker, 5-Year Growth Rates

Appendix 2

Graph 3 – China: Income and Inequality