Risk Premium and Nominal Rigidities

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This paper investigates implications of nominal rigidities for the risk premium. We use Obstfeld and Rogoff (1998) type monetary dynamic stochastic general equilibrium model with nominal rigidities, imperfect market competitions, internal goods market segmentation, a production sector, and a interest sensitive money demand.

For a monthly frequency, we generate the Fama (1984)'s volatility relations derived from violations of unbiasedness in the forward exchange rate and the autocorrelation of the forward premium observed in the data. However, we fail to obtain the same results when the quarterly decision interval is considered mainly because of time varying uncertainty and the discount factor. We also find that real exchange risks and staggered nominal contracts play a role in the determination of the risk premium as well as in generating the volatility relations while habit persistence and asset market structure do not. Finally, our analysis shows that one should be cautious about measuring the risk premium from the regression of ex post predictable returns.

Keywords: Nominal rigidities, the risk premium, the forward premium, the expected depreciation, monetary shocks, money-in-the-utility function, the exchange rate.

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1 Introduction

Many studies have documented negative values of the slope coefficient from regressions of the exchange rate depreciation on the forward premium. The time varying risk premium has been considered as a possible candidate for the cause of this abnormal exchange rate behavior since Meese and Rogoff (1983) and Fama (1984). In particular, Fama (1984) illustrates that if the market's expectation is rational and equals the statistical prediction of the exchange rate, then the risk premium should be time varying and highly volatile to explain the apparent presence of deviations from uncovered interest parity.[1] Specifically, Fama's regression test suggests that the following two volatility relations must be satisfied: (i) the risk premium should be negatively correlated with the expected depreciation, and (ii) the variance of the risk premium should be greater than the sum of variances of the expected depreciation and the forward premium.

In general, previous general equilibrium studies on the behavior of the risk premium are based on the Lucas (1982) model and keep main assumptions of flexible prices and perfect market competition while modifying preferences and distributions of the underlying exogenous forces of the economy: Macklem (1991) for an exogenously given foreign price level, Dutton (1993) for a constant risk premium, Canova and Marrian (1993) for time varying uncertainty, Backus, Gregory, Telmer (1993) for habit persistence, Bekaert, Hodrick and Marshall (1994) for first order risk aversion, and Bekaert (1996) for habit persistence, consumption duration, and transaction-costs function.[2] In these frameworks, purchasing power parity (PPP) holds because there are no frictions on international goods trade but the uncovered interest parity (UIP) does not hold because of presence of the risk premium. All these studies fail to generate the Fama's volatility relations but some studies find that non-linear preferences and time varying uncertainty increase variations of the risk premium. For example, Backus, Gregory, Telmer (1993) introduce habit persistence in the Lucas model and increase fluctuations of the marginal rate of substitution. On the other hand, Canova and Marrian (1993) can produce variable risk premia although their results rely heavily on the Jensen's inequality terms by assuming that time varying monetary policy and government expenditure shocks follow a GARCH (1,1) process.

This paper deviates from two main assumptions-flexible prices and goods market competition-widely used in previous studies and investigates quantitative implications of nominal rigidities for the risk premium as well as exchange rates. In contrast to flexible price models, the introduction of nominal rigidities into a dynamic stochastic general equilibrium model (DSGE) enables us to produce a channel in which monetary shocks can have real effects on fluctuations of the risk premium as well as the real exchange rate. Recent quantitative studies [Chari, Kehoe, McGrattan (2002), Bergin and Feenstra (2001), and Kollman (2001)] show that models with nominal rigidities can produce variations in the nominal and

[1] On the other hand, systematic forecast errors have been considered as another candidate for explaining violations of unbiasedness in the forward market [e.g., see Froot and Frankel (1987 and 1989) for irrational expectations, and Lewis (1989a and 1989b) and Evans and Lewis (1995) for rational forecast errors due to either learning or a peso problem.].

[2] see, also, Engel (1996) for a survey on this area.

real exchange rates observed in the data. The mechanism that translates persistent money growth rates into highly variable exchange rates is intertemporal links of the interest sensitive money demand. In this framework, the nominal exchange rate is determined in a first order stochastic difference equation like as other asset prices. However, this does not work in a static money demand such as a simple cash-in-advance constraint in which the nominal exchange rate is a ration home to foreign money supplies. Since prices do not much change in response to current shocks in models with nominal rigidities the real exchange rate is also variable and moves closely together with the nominal exchange rate. We investigate whether or not the same driving force that determines the exchange rates in these models is also able to produce highly variable risk premia. By extending Obstfeld and Rogoff (1998)'s model, we further study roles of time varying uncertainty by introducing a GARCH (1,1) process for time varying conditional variances of money growth rates, real exchange rate risks by allowing the failure of the law of one price, persistent real effects of monetary shocks by considering staggered nominal contracts, and habit persistence in the determination of the risk premium and the volatility relations.

Both nominal and real exchange rates are much more volatile than economic fundamentals moving closely together with each other. As can be seen in Table 3, standard deviations of both real and nominal exchange rate depreciations for G7 countries (except Canada) against the US dollar are about as 7 times as that of growth rates of the US GDP for a quarterly frequency. As Bekaert (1996) points out, "highly volatile exchange rates may limit gains from international investments and trade since foreign exchange rate risks cannot be costlessly hedged in the forward market when unbiasedness is violated." Moreover, weak consumption comovements between G7 countries as well as highly variable real exchange rate movements may indicate that even idiosyncratic country specific risks are not significantly diversified. For example, Hollifield and Yaron (2000) find that monetary policy must have large impacts on consumption risks while it should have small impacts on inflation risks and interactions between real and inflation risks. We investigate how real exchange rate risks affect the risk premium and the volatility relations in our model.

One of main differences between one period in advance price setting and staggered nominal contracts for the determination of the risk premium in our DSGE model comes from intertemporal links of interest sensitive money demand. For example, in the case of the static money demand, the risk premium does not change with respect to the length of contract periods because prices do not respond to current monetary shocks and only current shocks matter for the determination of the risk premium. For the case of the interest sensitive money demand, effects of current monetary shocks are translated into two parts marginal utility of current consumption and conditional expected marginal utilities of future consumption. Under one period in advance synchronized price setting only unexpected monetary shocks affect real variables so that households do not expect that current realized monetary shocks would affect their future consumption. Rather, firms completely adjust their next period prices in response to the current monetary shocks in a synchronized way. In contrast, multi-period staggered nominal contracts create a channel in which interactions among price setters can lead to persistent consumption movements via the contract multiplier generated from the pricing equations with both forward and backward looking terms. Under these contracts, economic agents now expect that current monetary shocks would affect future real variables because prices are gradually adjusted over time. Thus, households' perception of risks would change with respect to the degree of persistent real effects of monetary shocks so would the risk premium. We investigate how the risk premium fluctuate with respect to the length of contract periods.

Two different decision intervals, monthly and quarterly, are considered in our quantitative analysis.[3] Selecting a data frequency plays an important role in generating the volatility relations in our model because of time varying uncertainty and the discount factor. When the distribution of monetary shocks is time invariant volatilities of macro economic variables would be proportionally changed with respect to magnitudes of unconditional variances of monetary shocks other things being equal. However, when the distribution of the underlying exogenous forces of the economy is time varying, the unconditional variance of time varying conditional variances is related to the fourth moment of the underlying distribution. Therefore, the size of the underlying monetary shocks would matter as long as effects of time varying second moments on macro variables are significant. For example, models with a lower frequency tend to produce a relatively more variable risk premium other things being equal than those with a higher frequency do because only time varying second order terms are relevant to the determination of the risk premium.

On the other hand, the discount factor plays a significant role in the determination of the forward premium and the expected depreciation in the model with the interest sensitive money demand function. In this framework these two quantities are a function of money growth rates which reflect the response of money demand to nominal interest rates and the risk premium. Especially, the degree of the discount factor is inversely related to effects of money growth rates on these two quantities. This is reasonable in the sense that as forecasting intervals become shorter and shorter the money demand becomes less and less sensitive to change in nominal interest rates. As a consequence, relative effects of money growth rates on the forward premium and the expected depreciation become smaller as the discount factor becomes close to 1. On the other hand, the discount factor is not much relevant to the determination of the risk premium since it is mainly related to comovement between the marginal rate of substitution in consumption and nominal exchange rate depreciation. In general, the unconditional variance of stochastic disturbances in the process of money growth rates tends to be smaller and the discount factor is greater for a monthly decision interval than for a quarterly decision interval. Therefore, our model with the interest sensitive money demand tends to generate the Fama's volatility relations more likely for a higher date frequency than for a lower frequency. One should note that the model tends to reduce variations of both the expected depreciation and the forward premium much more significantly than those of the risk premium as the decision interval becomes shorter.

We also introduce habit persistence in our model because previous studies find that non-

[3] There is conflicting evidence in empirical studies [see, Lewis(1990) and Lewis(1991) for the detail]. For example, Hodrick and Srivastava (1984) and Giovannini and Jorion (1987) reject the restrictions of the model by using one month and one week maturities for foreign exchange markets while Campbell and Clarida (1987) do not reject them for using quarterly data.

linear preferences tend to increase variations of the risk premium. In general, it is known that adding non linear preference specification to an otherwise standard DSGE model allows moderate consumption fluctuations to have large impacts on the marginal rate of substitution between current and future consumption [see, Constantinides (1990), Backus, Gregory, Telmer (1993), Bekaert (1996), and Christiano, Eichenbaum, and Evans (2001)]. However, this mechanism does not work in our model. There are two main differences between previous studies that employ habit persistence and a simple cash-in-advance constraint such as Backus, Gregory, Telmer (1993) and Bekaert (1996) and our study; price stickiness, and intertemproal links of the interest sensitive money demand. When prices are flexible, marginal utility of consumption is not relevant to monetary shocks because prices are immediately adjusted to the response of realized money supplies in order to clear money markets. However, when prices are sticky current monetary shocks affects marginal utilities of current consumption as well as future consumption through intertemporal links of interest sensitive money demand. This channel distinguishes from the model with the static money demand function in the sense that fluctuations of the real exchange rate and the risk premium are independent of degrees of both the relative risk aversion and thus habit persistence. For example, in the case of the static money demand, current monetary only affect marginal utility of current consumption. Since all that matters in the determination of the risk premium is the marginal rate of substitution between current and future consumption but not a consumption growth rate itself, the relative risk aversion coefficient as well as habit persistence coefficient do not play any role. Furthermore, our quantitative results show that habit persistence does not play a significant role in the determination of the risk premium even when the interest sensitive money demand function is introduced into the model.

Recently, Duarte and Stockman (2001) investigate the "exchange rate disconnect puzzle" by considering changes in expectations resulting from rational speculation via a regime change under the combination of incomplete asset markets and production market segmentation in a DSGE with nominal rigidities.[4] While this study investigates the speculation behavior of rational agents our study focuses on patterns of the time varying risk premium.[5]

[4]Mussa (1986), Baxter and Stockman (1989), and Flood and Rose (1995) provide empirical evidence that much more volatile exchange rates are due to changes in exchange rate regimes. Flood and Rose (1995 and 1999) further argue that exchange rate volatility is independent of macroeconomic fundamentals. However, as Duarte and Stockman (2001) pointed out, empirical evidence of a strong relation between risk premium and expected depreciation contrasts with Flood and Rose (1995 and 1999)'s arguments.

[5]Studies related to econometric analyses focused on tests of consumption Euler equations and latent variable models and restrictions on pricing kernels [see, Lewis (1995) and Engel (1996) for a survey of econometric studies]. For example, Hansen and Hodrick (1980), Mark (1985), Kaminsky and Peruga (1990) used aggregate consumption data based on an ICAPM with time additive preferences. In these frameworks, risk premium can vary over time and is due to aggregate consumption risks measured by nominal marginal rates of intertemporal substitution in consumption. However, these models turn out to be unable of generating such a highly variable risk premium since the observed small variability of aggregate consumption data is not reconciled with relatively large volatile excess returns for many risk assets unless risk aversion parameter value is unrealistically high. This result is also consistent with empirical evidence in other asset markets such as the equity premium puzzle of Mehra and Prescott (1985). On the other hand, Hansen and Hodrick (1983) and Hodrick and Srivastava (1984) test the latent variable model while Hansen and Jaganathan (1991) and Bakaert and Hodrick (1992) estimate a lower bound of volatility of the intertemporal marginal

2 The Foreign Exchange Rate Risk Premium in a Model with Nominal Rigidities

In this section we investigate roles of nominal rigidities and goods market imperfection in the determination of the risk premium and derive main driving forces in generating highly variable exchange rates and risk premia, based on Obstfeld and Rogoff (1998, 2000). We further analyze how deviations from PPP and staggered nominal contracts play a role in generating the Fama's volatility relations in the model with intertemporal links of the interest sensitive money demand.

2.1 The Foreign Exchange Rate Risk Premium in a DSGE Model with Nominal Rigidities

There are two countries in the world, home (H) and foreign (F) and the population of each country is normalized to 1. We assume that there exist complete nominal bond markets across countries as well as within each country.

Preference for home individual $n \in [0, 1]$ is given by

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[\frac{1}{1-\gamma}C(n,s^t)^{1-\gamma}+\frac{1}{1-\phi}(\frac{M(n,s^t)}{P(s^t)})^{1-\phi}-\frac{1}{1+\mu}L^{1+\mu}(n,s^t)] \tag{2-1}$$

where γ denotes the relative risk aversion coefficient, ϕ denotes the coefficient value for real money balances, and μ denotes labor supply elasticity. And $C(n)$ is a composite consumption, $\frac{M(n)}{P}$ a real money balance, $L(n)$ a labor supply for home individual n, and $0 < \beta < 1$ denotes the discount factor.

The home household n's budget constraint (denominated in home currency) under contingent nominal bond markets is:$\forall s^t, s^{t+1}$

$$P(s^t)C(n,s^t)+M(n,s^t)+\sum_{s^{t+1}}Q(s^{t+1}|s^t)B_H(n,s^{t+1}|s^t)+\sum_{s^{t+1}}\epsilon(s^t)Q^*(s^{t+1}|s^t)B_F(n,s^{t+1}|s^t)\leq$$
$$\int_0^1 W(n,s^t)L^d(i,n,s^t)di+M(n,s^{t-1})+B_H(n,s^t)+\epsilon(s^t)B_F(n,s^t)+\Pi(n,s^t)+T(n,s^t) \tag{2-2}$$

where $M(n, s^t)$ is nominal money balance held by individual n at s^t, $W(n, s^t)$ is nominal wage for labor type n independent of across firms, $L^d(i, n, s^t)$ is firm i's demand for labor type n, $\Pi(n, s^t)$ represents the share of profits of all home firms for home household n, and $T(n, s^t)$ denotes a nominal transfer paid from home government. We assume that home household n is a monopoly supplier of a differentiated labor input $L(i, n, s^t)$ and sets her nominal wage $W(n, s^t)$ across all home firms indexed by $i \in [0, 1]$ one period in advance. Home household n's labor service is assumed to be transformed into an aggregate labor $L(s^t)$ using

rate substitution in consumption.

the following technology $L(s^t) = (\int_0^1 L(n, s^t)^{\frac{\eta-1}{\eta}} dn)^{\frac{\eta}{\eta-1}}$. Then, firm i's demand for labor type n can be defined by $L^d(i, n, s^t) = [\frac{W(n,s^t)}{W(s^t)}]^{-\eta} L(i, s^t)$.[6] And, $L(s^t) = \int_0^1 L(i, s^t) di$.

Both Home and foreign households can trade state contingent nominal discount bonds denominated in either home or foreign currency. Let $Q(s^{t+1}|s^t)$ denote nominal price (denominated in home currency) of home state contingent bond paying one unit of home currency in s^{t+1} and 0 otherwise. $B_H(n, s^{t+1}|s^t)$ denotes the number of home state contingent bonds held by home household n in s^t. And $B_F(n, s^{t+1}|s^t)$ denotes the number of foreign state contingent bonds held by home household n in s^t. The foreign quantity and price variables are indicated by an asterisk superscript. For example, $B_H^*(n, s^{t+1}|s^t)$ and $B_F^*(n, s^{t+1}|s^t)$ denote the number of both home and foreign state contingent bonds held by foreign household n, respectively.

Let $P(s^t)$ be the home CPI index and $C(n, s^t)$ be a composite real consumption index for home household n defined by $C(n, s^t) = \frac{C_H^h(n,s^t) C_F^{1-h}(n,s^t)}{h^h (1-h)^{1-h}}$.[7] An expenditure share on goods produced in home country is denoted by h which represents home bias in preferences. The home CPI is defined by $P(s^t) = P_H^h(s^t) P_F^{1-h}(s^t)$, where P_H is the home price of the composite home goods and P_F is the home price of the composite foreign goods. The consumption of home household n for the composite home goods is defined by $C_H(n, s^t) = [\int_0^1 C_H^{\frac{\nu-1}{\nu}}(i, n, s^t) di]^{\frac{\nu}{\nu-1}}$, where $C_H(i, n, s^t)$ denotes home individual n's demand for home tradable good i defined by $C_H(i, n, s^t) = (\frac{P_H(i,s^t)}{P_H(s^t)})^{-\nu} (\frac{P_H(s^t)}{P(s^t)})^{-1} h C(n, s^t)$. $P_H(i, s^t)$ is the price of home good i denominated in home currency and ν denotes elasticity of substitution among goods within a country and assumed to be greater than 1.

Households are assumed to take prices of goods as given while they are assumed to set their wages one period in advance before monetary shocks are realized. Then, home household n's first order conditions can be derived by maximizing her expected utility defined in equation (2-1) subject to the budget constraint (2-2), and firm i's labor demand function (the optimal conditions for foreign household n can also be derived analogously)

$$Q(s^{t+1}|s^t) = \beta \frac{\pi(s^{t+1})}{\pi(s^t)} \frac{\frac{U_c(n,s^{t+1})}{P(s^{t+1})}}{\frac{U_c(n,s^t)}{P(s^t)}} \tag{2-3}$$

$$\frac{1}{\epsilon(s^t)} Q(s^{t+1}|s^t) = \beta \frac{\pi(s^{t+1})}{\pi(s^t)} \frac{\frac{U_c^*(n,s^{t+1})}{P^*(s^{t+1})}}{\frac{U_c^*(n,s^t)^{-\gamma}}{P^*(s^t)}} \frac{1}{\epsilon(s^{t+1})} \tag{2-4}$$

$$U_m(n, s^t) \frac{1}{P(s^t)} = \frac{U_c(n, s^t)}{P(s^t)} - \beta \sum_{s^t} \pi(s^{t+1}|s^t) [\frac{U_c(n, s^{t+1})}{P(s^{t+1})}] \tag{2-5}$$

[6] Here, we assume that all firms have the same unit nominal wage, that is $W(s^t) = W(i, s^t)$ for all $i \in [0, 1]$. And home household n sets the same nominal wage across all home firms, that is, $W(n, s^t) = W(n, i, s^t)$ for all $i \in [0, 1]$.

[7] In the quantitative analysis, we also consider the case in which elasticity of substitution across countries are greater than unity.

$$U_m^*(n,s^t)\frac{1}{P^*(n,s^t)} = \frac{U_c^*(n,s^t)}{P^*(s^t)} - \beta\sum_{s^t}\pi(s^{t+1}|s^t)[\frac{U_c^*(n,s^{t+1})}{P^*(s^{t+1})}] \tag{2-6}$$

$$W(n,s^t) = \frac{\eta}{\eta-1}\frac{\sum_{s^t}\pi(s^t|s^{t-1})[U_l(s^t)L^d(n,s^\tau)]}{\sum_{s^t}\pi(s^t|s^{t-1})[\frac{U_c(n,s^t)}{P(s^t)}L^d(n,s^t)]} \tag{2-7}$$

$$W^*(n,s^t) = \frac{\eta}{\eta-1}\frac{\sum_{s^t}\pi(s^t|s^{t-1})[U_l^*(n,s^t)L^{*d}(n,s^t)]}{\sum_{s^t}\pi(s^t|s^{t-1})[\frac{U_c^*(n,s^t)}{P^*(s^t)}L^{*d}(n,s^t)]} \tag{2-8}$$

where $\frac{\pi(s^{t+1})}{\pi(s^t)} = \pi(s^{t+1}|s^t)$ is the probability of occurring state s^{t+1} given state s^t, marginal disutility of labor is denoted by $U_l(n,s^t) = L^\mu(s^t)$, and $L^d(n,s^t)$ denotes home aggregate labor demand for labor type n defined by

$$L^d(n,s^t) = \int_0^1 L^d(i,n,s^t)di = \int_0^1 [\frac{W(n,s^t)}{W(s^t)}]^{-\eta}L(i,s^t)di = [\frac{W(n,s^t)}{W(s^t)}]^{-\eta}L(s^t). \tag{2-9}$$

And marginal utility of consumption is defined by $U_c(s^t) = C^{-\gamma}(n,s^t)$.

Equations (2-3) and (2-4) are related to home and foreign nominal intertemporal Euler equations expressed in home currency for each state: the price of state contingent home nominal bond $Q(s^{t+1}|s^t)$ should be equal to the marginal rate of substitution either in home consumption between s^{t+1} and s^t weighted by the change in purchasing power of home money or in foreign consumption weighted by the change in purchasing power of foreign money once converted to home currency. If PPP holds for each state, consumption growth rates should be equal across countries regardless of whether or not prices are flexible. Equations (2-5) and (2-6) represent home and foreign money market clearing conditions: the marginal rate of substitution between consumption and real money balances should be equal to the user costs of holding an extra unit of real balances for one period. Note that equations (2-3)-(2-6) are always binding in both models either with or without nominal rigidities. Both home and foreign household n determine their nominal wages in equations (2-7) and (2-8) before shocks are realized, respectively: home nominal wage $W(n,s^t)$ for type n should be equal to the conditional expected marginal rate of substitution between home consumption and labor for a given state s^{t-1}. These labor leisure trade-off conditions would not be binding after realizations of shocks since households have to supply their labor services in order to meet demands for the same wage in the contract that they made before shocks are realized.

Let $R(s^{t+1}|s^t) = \beta\frac{P(s^t)}{P(s^{t+1})}\frac{U_c(s^{t+1})}{U_c(s^t)}$ and $R^*(s^{t+1}|s^t) = \beta\frac{P^*(s^t)}{P^*(s^{t+1})}\frac{U_c^*(s^{t+1})}{U_c^*(s^t)}$.[8] Then, $\sum_{s^{t+1}}\pi(s^{t+1}|s^t)R(s^{t+1}|s^t) = E_t[R(s^{t+1}|s^t)]$ and $\sum_{s^{t+1}}\pi(s^{t+1}|s^t)R^*(s^{t+1}|s^t) = E_t[R^*(s^{t+1}|s^t)]$ are the inverse of the gross home and foreign nominal interest rates, respectively. Then, using the covered interest parity condition the forward premium can be defined by

$$\frac{F_t}{\varepsilon_t} = \frac{E_t[R^*_{t+1}]}{E_t[R_{t+1}]} \tag{2-10}$$

[8]From now on we ignore an index for individual household since all agents are identical under one period in advance synchronized nominal contracts in the symmetric equilibrium.

where F_t denotes one period ahead forward exchange rate.

Using equations (2-3)-(2-4), one can derive the following optimal risk sharing condition: $\forall s^{t+1}$

$$\frac{P^*(s^{t+1})\epsilon(s^{t+1})}{P(s^{t+1})}\frac{Uc(s^{t+1})}{Uc^*(s^{t+1})} = \frac{P^*(s^t)\epsilon(s^t)}{P(s^t)}\frac{Uc(s^t)}{Uc^*(s^t)} \tag{2-11}$$

This condition holds regardless of price flexibility and assumptions of money demand. As in the case of Chari, Kehoe, and McGrattan (2002), by imposing initial conditions this condition can be further simplified so that the real exchange rate is equal to the marginal rate of substitution between home and foreign consumption. Now, the equation (2-11) can be rewritten using definitions of stochastic discount factors:

$$\varepsilon(s^t) = \frac{R(s^{t+1}|s^t)\varepsilon(s^{t+1})}{R^*(s^{t+1}|s^t)}. \tag{2-12}$$

Since equation (2-12) is log linear, we can first take logs of both sides of equation (2-12), multiply both sides by $\pi(s^{t+1}|s^t)$, and add over all s^{t+1}. By rearranging terms we have

$$E_t[e_{t+1}] - e_t = E_t[r^*_{t+1}] - E_t[r_{t+1}] \tag{2-13}$$

where e_t is the log of nominal exchange rate at time t, $r_{t+1} = \log R_{t+1} = \log\beta + (p_t - p_{t+1}) + \gamma(c_t - c_{t+1})$, and $r^*_{t+1} = \log\beta + p^*_t - p^*_{t+1} + \gamma(c^*_t - c^*_{t+1})$. From now on, we suppress notation for state. Note that under the assumption of complete markets equation (2-13) even holds in certainty for each state.[9]

As in Obstfeld and Rogoff (1998), by assuming that all variables follow log normal distributions one can derive closed form solutions of the model for a special case in which the law of one price price holds, there is no home bias in preferences, elasticity of substitution across countries is unity, and prices and wages are set one period in advance. Since these assumptions restrict us to investigate interesting cases such as real exchange rate risks and effects of the length of nominal contract periods, we take second order approximations wherever a relation is not log linear by following Sutherland (2002) [see, also Woodford (2001)]. Then,

[9] Since stochastic discount factor R_{t+1} is no longer log linear when consumption exhibits habit persistence, as will be seen later, we take second order approximation on equation (2-12).

the second order approximated version of equation (2-10) can be written[10]

$$\widehat{f}_t - \widehat{e}_t = E_t[\widehat{r}^*_{t+1}] - E_t[\widehat{r}_{t+1}] + \frac{1}{2}(Var_t[\widehat{r}^*_{t+1}] - Var_t[\widehat{r}_{t+1}]) \tag{2-14}$$

where $\widehat{r}_{t+1} = \widehat{p}_t - \widehat{p}_{t+1} + \gamma(\widehat{c}_t - \widehat{c}_{t+1})$ and $\widehat{r}^*_{t+1} = \widehat{p}^*_t - \widehat{p}^*_{t+1} + \gamma(\widehat{c}^*_t - \widehat{c}^*_{t+1})$. Here, a hat over a small letter denotes the log deviation from the deterministic steady state. By subtracting the approximated version of equation (2-13) from equation (2-14), we can derive the following foreign exchange rate risk premium:

$$\widehat{f}_t - E_t[\widehat{e}_{t+1}] = rp_t = \frac{1}{2}(Var_t[\widehat{r}^*_{t+1}] - Var_t[\widehat{r}_{t+1}]) \tag{2-15}$$

It is obvious from equation (2-15) that the risk premium is zero in a certainty equivalence setting. Note further that if distributions of the underlying exogenous forces of the economy are time invariant then the risk premium is constant. Equation (2-15) holds true regardless of price flexibility although the interpretation is different across different assumptions. To see this, decompose the exchange rate risk premium in equation (2-15)

$$\begin{aligned}\frac{1}{2}(Var_t[\widehat{r}^*_{t+1}] - Var_t[\widehat{r}_{t+1}]) = \frac{1}{2}(Var_t[\widehat{p}^*_{t+1}] - Var_t[\widehat{p}_{t+1}]) + \frac{1}{2}(Var_t[\gamma\widehat{c}^*_{t+1}] - Var_t[\gamma\widehat{c}_{t+1}]) +\\ (Cov_t[\widehat{p}^*_{t+1}, \gamma\widehat{c}^*_{t+1}] - Cov_t[\widehat{p}_{t+1}, \gamma\widehat{c}_{t+1}]).\end{aligned} \tag{2-16}$$

Here, we omit time t variables since those are constant at time t. When PPP holds, $\widehat{c}_{t+1} = \widehat{c}^*_{t+1}$. Hence, $(Cov_t[\widehat{p}^*_{t+1}, \gamma\widehat{c}^*_{t+1}] - Cov_t[\widehat{p}_{t+1}, \gamma\widehat{c}_{t+1}]) = -Cov_t[\widehat{e}_{t+1}, \gamma\widehat{c}_{t+1}]$. And $Var_t[\widehat{c}^*_{t+1}] - Var_t[\widehat{c}_{t+1}] = 0$. $(Var_t[\widehat{p}^*_{t+1}] - Var_t[\widehat{p}_{t+1}]) = Var_t[\widehat{e}_{t+1}] - 2Cov_t[\widehat{p}_{t+1}, \widehat{e}_{t+1}]$ are related to Jensen's inequalities. We interpret $-Cov_t[\widehat{e}_{t+1}, \gamma\widehat{c}_{t+1}]$ as the true risk premium by following Engel (1992).

When prices are flexible monetary shocks do not affect consumption while real shocks do not affect nominal exchange rates in a simple cash-in-advance constraint. Therefore, the true risk premium is zero in an environment with flexible prices unless monetary shocks are correlated with real shocks [see, also, Engel (1992)]. This implies that it is unlikely that models with flexible prices produces the variable true risk premium because the covariance

[10] equation (2-14) is derived by taking a quadratic Taylor series approximation on equation (2-10). For example, let $U_t = E_t[R_{t+1}]$. Now, take Taylor second order approximation on both sides. Then,

$$U_t = E_t[\overline{R} + R_{t+1} - \overline{R}] = E_t[\overline{R} + \overline{R}(\frac{R_{t+1} - \overline{R}}{\overline{R}})] = E_t[\overline{R} + \overline{R}(\widehat{r}_{t+1} + \frac{\widehat{r}^2_{t+1}}{2})].$$

where $\widehat{r}_{t+1} = \log R_{t+1} - \log \overline{R}$. The second equality uses the relation $exp(x) = 1 + x + \frac{x^2}{2} + \cdot$ and ignores terms higher than second order. That is, $\frac{R_{t+1}}{\overline{R}} = 1 + \widehat{r}_{t+1} + \frac{\widehat{r}^2_{t+1}}{2}$. Since $U_t - \overline{R} = \overline{R}(\widehat{u}_t + \frac{\widehat{u}^2_t}{2})$, we have

$$\widehat{u}_t = E_t[\widehat{r}_{t+1} + \frac{\widehat{r}^2_{t+1}}{2}] - \frac{\widehat{u}^2_t}{2} = E_t[\widehat{r}_{t+1}] - \frac{Var_t[\widehat{r}_{t+1}]}{2}.$$

Again, we ignore terms higher than second order for deriving this equation.

of real shocks with monetary shocks is considered to be small in empirical studies. However, when prices are sticky and PPP holds, home consumption is equally and always positively affected by both home and foreign monetary shocks. Since exchange rates are always positively affected by home monetary shocks and negatively affected by foreign monetary shocks, the true risk premium derived from the model with nominal rigidities depends on both home and foreign monetary shocks and changes its sign consistent with empirical studies.

Further, magnitudes of Jensen's inequality terms are different in models between with nominal rigidities and with flexible prices and wages. To see this, we first have to consider firms' pricing decisions. Suppose that each firm presets its price in its own country's currency for N periods in a staggered way. Under this assumption [producer currency pricing (PCP)], home firm i maximizes its profits by choosing $P_{Ht}(i)$ in Home currency for sales to both Home and Foreign markets subject to $Y(i,s^t) = C_H(i,s^t) + C_H^*(i,s^t) = L(i,s^t)$ and corresponding downward sloping demand functions defined above before time t shocks are realized

$$\max_{\{P_H(i,s^t)\}} \sum_{\tau=t}^{t+N-1} \sum_{s^t} Q(s^\tau|s^{t-1})\{(P_H(i,s^t) - MC(i,s^\tau))[C_H(i,s^\tau) + C_H^*(i,s^\tau)]\}$$

where $MC(i,s^\tau)$ denotes firm i's marginal cost. The optimal price derived from the above problem is:

$$P_H(i,s^t) = \frac{\nu}{\nu-1}\frac{\sum_{\tau=t}^{t+N-1} Q(s^\tau|s^{t-1})[MC(i,s^\tau)(C_H(i,s^\tau) + C_H^*(i,s^\tau))]}{\sum_{\tau=t}^{t+N-1} Q(s^\tau|s^{t-1})[(C_H(i,s^\tau) + C_H^*(i,s^\tau))]}. \tag{2-17}$$

Since the law of one price holds under PCP, the price in foreign currency of home good i is defined by $P_H^*(i,s^t) = \frac{P_H(i,s^t)}{\varepsilon(s^t)}$. Let $P_F^*(i,s^t)$ be the price of foreign good i denominated in foreign currency. Analogously, the foreign price of foreign good i can be defined by

$$P_F^*(i,s^t) = \frac{\nu}{\nu-1}\frac{\sum_{\tau=t}^{t+N-1} Q(s^\tau|s^{t-1})[MC^*(i,s^\tau)(C_F(i,s^\tau) + C_F^*(i,s^\tau))]}{\sum_{\tau=t}^{t+N-1} Q(s^\tau|s^{t-1})[(C_F(i,s^\tau) + C_F^*(i,s^\tau))]}. \tag{2-18}$$

From the law of one price, $P_F^*(i,s^t) = \frac{P_F(i,s^t)}{\varepsilon(s^t)}$. Suppose that the length of contract periods, N, is 1. By taking second order approximations on equations (2-17) and (2-18), we have

$$\widehat{p}_{Ht} = E_{t-1}\widehat{mc}_t + V1_{t-1} \tag{2-19}$$

$$\widehat{p}_{Ft}^* = E_{t-1}\widehat{mc}_t^* + V1_{t-1}^* \tag{2-20}$$

where $V1_{t-1} = \frac{1}{2}Var_{t-1}[\widehat{mc}_t] + Cov_{t-1}[\widehat{mc}_t, \widehat{y}_t] + E_{t-1}[\widehat{y}_t]E_{t-1}[\widehat{mc}_t]$ and $V1_{t-1}^* = \frac{1}{2}Var_{t-1}[\widehat{mc}_t^*] + Cov_{t-1}[\widehat{mc}_t^*, \widehat{y}_t^*] + E_{t-1}[\widehat{y}_t^*]E_{t-1}[\widehat{mc}_t^*]$ are $t-1$ conditional second order moments. Since all firms in the economy preset their prices one period in advance and are assumed to be symmetric, $\widehat{p}_{Ht}(i) = \widehat{p}_{Ht}$ and $\widehat{p}_{Ft}^*(i) = \widehat{p}_{Ft}^*$. Note that both $\widehat{p}_{Ht}$ and $\widehat{p}_{Ft}^*$ contain terms that reflect pricing risks since both home and foreign firms must set their prices before shocks are realized [see Obstfeld and Rogoff (1998, 2000) for economic interpretation of these terms.].[11]

[11] Our results show that these effects are quantitatively small.

Now, we can derive the following equation from the definition of home CPI

$$\widehat{p}_{t+1} = h\widehat{p}_{H,t+1} + (1-h)\widehat{p}_{F,t+1} = h\widehat{p}_{H,t+1} + (1-h)(\widehat{p}^{*}_{F,t+1} + \widehat{e}_{t+1}). \tag{2-21}$$

Here the second equality is derived by using properties of the law of one price. Since home currency price of home goods $\widehat{p}_{H,t+1}$ preset by home firms and foreign currency price of foreign goods $\widehat{p}^{*}_{F,t+1}$ by foreign firms are known at time t, the Jensen's inequality terms in equation (2-16) depend on the expenditure share. Suppose that there is no home bias in preferences, that is $h = 0.5$. Then, $Var_t[\widehat{e}_{t+1}] - 2Cov_t[\widehat{p}_{t+1}, \widehat{e}_{t+1}] = 0$. Hence, Jensen's inequality terms are zero when there are no idiosyncratic risks in models with nominal rigidities. However, when prices are flexible the conventional risk premium depends entirely on Jensen's inequality terms since the true risk premium is zero

$$Var_t[\widehat{e}_{t+1}] - 2Cov_t[\widehat{p}_{t+1}, \widehat{e}_{t+1}] = -Cov_t[\widehat{pH}_{t+1} + \widehat{pF}^{*}_{t+1}, \widehat{e}_{t+1}]. \tag{2-22}$$

2.2 Aggregate Risks

In this section, we first solve for the model in a rather simplified in which PPP holds so that only aggregate risks matter for the determination of the risk premium. We further assume that all firms set their prices one period in advance and there is no home bias in preferences.

Suppose that the money supply for each country evolves in the following way

$$\frac{M_{t+1}}{M_t} = G_{t+1} \tag{2-23}$$

$$\log G_{t+1} = g_{t+1} = \rho_m g_t + um_{t+1}. \tag{2-24}$$

where um_{t+1} is an *i.i.d.* stochastic disturbance term and g_{t+1} is the log of money growth rate at $t+1$. We further assume that the process of the money supply for each country evolves independently.

A production function is given by $Y(i,s^t) = AL(i,s^t)^{\alpha} Z(i,s^t)^{1-\alpha}$ for each good $i \in [0,1]$. $L(i,s^t) = \int_0^1 L(i,n,s^t)dn$ and $Z(i,s^t)$ denotes labor services and an aggregate intermediate input used by home firm i, respectively. α is the cost share for labor input, and A is the technology shock. For expositional simplicity, we assume that $\alpha = 1$ and $A = 1$. Then, goods market clearing conditions for home and foreign good i are

$$C_{H,t}(i) + C^{*}_{H,t}(i) = (\frac{P_{H,t}(i)}{P_{H,t}})^{-\nu}(\frac{P_{H,t}}{P_t})^{-1}0.5C_t(i) + (\frac{P^{*}_{H,t}(i)}{P^{*}_{H,t}})^{-\nu}(\frac{P^{*}_{H,t}}{P^{*}_t})^{-1}0.5C^{*}_t(i) = Y_t(i) \tag{2-25}$$

$$C_{F,t}(i) + C^{*}_{F,t}(i) = (\frac{P_{F,t}(i)}{P_{F,t}})^{-\nu}(\frac{P_{F,t}}{P_t})^{-1}0.5C_t(i) + (\frac{P^{*}_{F,t}(i)}{P^{*}_{F,t}})^{-\nu}(\frac{P^{*}_{F,t}}{P^{*}_t})^{-1}0.5C^{*}_t(i) = Y^{*}_t(i). \tag{2-26}$$

Since we are concerning a symmetric equilibrium and all firms are simultaneously set their prices only one period in advance, $Y_t = \int_0^1 Y_t(i)di = Y_t(i)$ for each i. Further, consumption is equalized across countries because PPP holds .[12] By using the law of one price and summing over $i \in [0,1]$, equations (2-25) and (2-26) can be simplified

$$(\frac{P_{H,t}}{P_t})^{-1}\frac{C_t^w}{2} = Y_t \tag{2-27}$$

$$(\frac{P_{F,t}}{P_t})^{-1}\frac{C_t^w}{2} = Y_t^*. \tag{2-28}$$

where $C_t^w = \int_0^1 (C_t(i) + C_t^*(i))di = C_t + C_t^*$, $Y_t = \int_0^1 Y_t(i)di$, and $Y_t^* = \int_0^1 Y_t^*(i)di$.

Let $\widehat{l}_t$ and $\widehat{l}_t^*$ be log deviations of home and foreign aggregate labor inputs. Then, we can derive the aggregate world labor $\widehat{lw}_t$ from equations (2-27) and (2-28) and definitions of both home and foreign production functions

$$\widehat{lw}_t = \widehat{l}_t + \widehat{l}_t^* = \widehat{y}_t + \widehat{y}_t^* = \widehat{c}_t + \widehat{c}_t^* = \widehat{cw}_t. \tag{2-29}$$

where $\widehat{cw}_t$ is the log deviation of the world aggregate consumption. From equations (2-19) and (2-20), one could obtain

$$\widehat{pw}_t = \widehat{p}_t + \widehat{p}_t^* = \widehat{p}_{Ht} + \widehat{p}_{Ft}^* = E_{t-1}[\widehat{w}_t + \widehat{w}_t^*] + V1w_{t-1} \tag{2-30}$$

where $\widehat{pw}$ denotes the log deviation of the world aggregate price and $V1w_{t-1} = V1_{t-1} + V1_{t-1}^*$. And $\widehat{mc}_t = \widehat{w}_t$ since the production function is linear in labor. Further, $Var_{t-1}[\widehat{w}_t] = 0 = Var_{t-1}[\widehat{w}_t^*]$ since nominal wages are determined at $t-1$ as in equations (2-7) and (2-8). And $\widehat{pw}_t = E_{t-1}\widehat{pw}_t$ since both $\widehat{p}_{Ht}$ and $\widehat{p}_{Ft}^*$ are known at $t-1$. Now, the world nominal wage can be derived from equations (2-7) and (2-8)

$$\widehat{ww}_t = E_{t-1}[(\mu+\gamma)\widehat{cw}_t + \widehat{pw}_t] + V3_{t-1} + V3_{t-1}^*. \tag{2-31}$$

where $\widehat{ww}_t = \widehat{w}_t + \widehat{w}_t^*$, $\widehat{cw}_t = \widehat{c}_t + \widehat{c}_t^*$, and $V3_{t-1}$ and $V3_{t-1}^*$ are conditional second order moments derived from second order approximations on both home and foreign labor-leisure trade off conditions, respectively. By inserting equation (2-30) into equation (2-31) we can obtain

$$E_{t-1}[\widehat{cw}_t] = \frac{-1}{2(\mu+\gamma)}(V3_{t-1} + V3_{t-1}^* + V1_{t-1} + V1_{t-1}^*). \tag{2-32}$$

Note that the conditional expectation on time t consumption given a time $t-1$ information set is just a function of only time $t-1$ second order moments: money growth rates do not affect conditional expected consumption. This condition implies that only unexpected

[12] As in Cole and Obstfeld (1991) and Obstfeld and Rogoff (1998), the assumption of Cobb-Douglas preferences also makes asset markets redundant.

money growth rates which reflect the response of money demand to nominal interest rates and second moments which reflect firms and households' risks derived from price and wage decisions would have real effects. In other words, households do not expect that current monetary shocks would affect their future consumption. Rather, firms completely adjust their next period prices in response to current shocks in a synchronized way so that these shocks do not exhibit any persistent real effects.

Now, by taking second order approximations on both home and foreign money market clearing conditions (2-5) and (2-6), the log deviation of the world price can be derived

$$\widehat{pw}_t = \frac{\phi\bar{i}}{1+\phi\bar{i}}\widehat{mw}_t - \frac{(1+\bar{i})\gamma}{1+\phi\bar{i}}\widehat{cw}_t + \frac{\gamma}{1+\phi\bar{i}}E_t[\widehat{cw}_{t+1}] + \frac{1}{1+\phi\bar{i}}E_t[\widehat{pw}_{t+1}] + \frac{1}{1+\phi\bar{i}}(rw_t + V5w_t) \tag{2-33}$$

where $\bar{i}$ is steady state interest rate, $rw_t = \frac{1}{2}(Var_t[\widehat{r}^*_{t+1}] + Var_t[\widehat{r}_{t+1}])$, $\widehat{mw}_t = \widehat{m}_t + \widehat{m}^*_t$ denotes the log deviation of the world money supply, and $V5w_t = V5_t + V5^*_t$ is second order terms derived from second order approximations on money market clearing condition in each country. The world price contains forward looking components because of presence of intertemporal links of interest sensitive money demand. Note that this condition holds true regardless of assumptions of currency pricing, contract periods, and asset market structures. By solving equation (2-33) and using $\widehat{c}_t = 0.5*\widehat{cw}_t + 0.5*\widehat{cd}_t$, the marginal rate of substitution in home country while ignoring second order terms can be derived by

$$\begin{aligned}\gamma[\widehat{c}_{t+1} - \widehat{c}_t] &= \frac{1+\phi\bar{i}}{2(1+\bar{i})}[(\frac{1+\phi\bar{i}}{1+\phi\bar{i}-\rho_m})(gw_{t+1} - gw_t) - (\frac{(1+\phi\bar{i})\rho_m}{1+\phi\bar{i}-\rho_m})(gw_t - gw_{t-1})] \\ &= \frac{1+\phi\bar{i}}{2(1+\bar{i})}(\frac{1+\phi\bar{i}}{1+\phi\bar{i}-\rho_m})(uw_{t+1} - uw_t) \qquad (2\text{-}34)\end{aligned}$$

where $uw_t = u_t + u^*_t$. Equation (2-34) shows that persistence of money growth rates plays an important role in increasing variations of the marginal rate of substitution. Further, the marginal rate of substitution is independent of the risk aversion coefficient. This implies that, in contrast to previous studies, the habit persistence coefficient is also independent of the marginal rate of substitution. Rather, the log deviation of the home consumption is inversely related to the risk aversion parameter. Hence, relative standard deviations of real exchange rates to output increase with respect to the risk aversion coefficient.

Now, consider the determination of nominal exchange rates. From home and foreign money market clearing conditions and the optimal risk sharing condition (2-17), we have

$$\begin{aligned}\widehat{e}_t - E_t[\widehat{e}_{t+1}] + \frac{1}{2}(Var_t[\widehat{r}_{t+1}] - Var_t[\widehat{r}^*_{t+1}]) &= \phi\bar{i}(\widehat{md}_t) - \phi\bar{i}(\widehat{pd}_t) - \gamma\bar{i}(\widehat{cd}_t) + V5d_t \\ &= \phi\bar{i}(\widehat{md}_t) + \gamma(\phi - 1)\bar{i}\widehat{cd}_t - \phi\bar{i}(\widehat{e}_t) + V5d_t \qquad (2\text{-}35)\end{aligned}$$

where $\widehat{md}_t = \widehat{m}_t - \widehat{m}^*_t$, $\widehat{cd}_t = \widehat{c}_t - \widehat{c}^*_t$, $\widehat{pd}_t = \widehat{p}_t - \widehat{p}^*_t$, and $V5d_t = V5_t - V5^*_t$. The second equality is derived from the optimal risk sharing condition (2-17). In contrast to a static money demand, the nominal exchange rate is determined in a first order stochastic difference equation like as other asset prices even under complete asset markets due to the presence of intertemporal links of the interest sensitive money demand. We can solve equation (2-35) by assuming no speculative bubbles in money markets and considering the processes of money supply for both home and foreign country

$$\widehat{e}_t = \frac{\phi\bar{i}}{(1+\phi\bar{i})}\sum_{s=0}^{\infty}(\frac{1}{1+\phi\bar{i}})^s\{E_t[\widehat{md}_{t+s}] + E_t[\frac{rp_{t+s}+V5d_{t+s}}{\phi\bar{i}}] + \frac{\gamma(\phi-1)\bar{i}}{\phi\bar{i}}E_t[\widehat{cd}_{t+s}]\}$$

$$= \widehat{md}_t + \frac{\rho_m}{1+\phi\bar{i}-\rho_m}gd_t + \frac{\phi\bar{i}}{(1+\phi\bar{i})}\sum_{s=0}^{\infty}(\frac{1}{1+\phi\bar{i}})^s\{E_t[\frac{rp_{t+s}+V5d_{t+s}}{\phi\bar{i}}] + \frac{\gamma(\phi-1)\bar{i}}{\phi\bar{i}}E_t[\widehat{cd}_{t+s}]\} \tag{2-36}$$

where $\widehat{rp}_t = \frac{1}{2}(Var_t[\widehat{r}^*_{t+1}] - Var_t[\widehat{r}_{t+1}])$ and $gd_t = g_t - g^*_t$. The nominal exchange rate is the expected present value of fundamentals and risk premia as in Obstfeld and Rogoff (1998).[13] As one can see in equation (2-36), volatility of the nominal exchange rate depends critically on the persistence of growth rates of money supply.[14] As money growth rates become persistent, nominal exchange rates become highly volatile even under complete asset markets. This is one of main sources that recent quantitative studies rely on generating highly variable exchange rates.[15] When money enters the utility function the nominal exchange rate also contains conditional expected values of time varying conditional variances. Obstfeld and Rogoff (1998) argue that the level risk premium in equation (2-29) may have significant impacts on both the level and the volatility of exchange rates since the conventional risk premium can be amplified by a factor of $\frac{1}{\phi\bar{i}}$. For a reasonable range of parameter values on the interest elasticity of money demand, this level risk premium can have significant effects on the nominal exchange rates. However, the effectiveness of this depends crucially on the model's ability of generating variable conventional risk premium rp_t in the first place. We investigate magnitudes of this risk premium and its effects on the level and volatility of nominal exchange rates in our quantitative analysis.

Change in nominal exchange rates between time t and $t+1$ can be derived from equation (2-34) and using the property of PPP

$$\widehat{e}_{t+1} - \widehat{e}_t = \frac{1+\phi\bar{i}}{1+\phi\bar{i}-\rho_m}gd_{t+1} - \frac{\rho_m}{1+\phi\bar{i}-\rho_m}gd_t$$

$$+\frac{\phi\bar{i}}{1+\phi\bar{i}}\sum_{s=0}^{\infty}(\frac{1}{1+\phi\bar{i}})^s\frac{E_{t+1}[rp_{t+1+s}+V5d_{t+1+s}] - E_t[rp_{t+s}+V5d_{t+s}]}{\phi\bar{i}}. \tag{2-37}$$

[13]Hodrick(1989) also consider the case in which time varying second order moments affect nominal exchange rates in a model with cash-in-advance constraints and flexible prices.

[14]Note that if $\rho_m = 0$, $\phi = 1$, and the process of money supply is time invariant then one can obtain a closed form solution for money demand function so that the nominal exchange rate is a function of current money supplies.

[15]One should note that even models with flexible prices can also produce high variable nominal exchange rates if money enters the utility function and money growth rates are persistent.

Here the nominal exchange rate does not exactly follow a random walk. However, the auto-correlation of the exchange rate depreciation is near zero because the $i.i.d.$ term, $\frac{1+\phi\bar{i}}{1+\phi\bar{i}-\rho_m}ud_{t+1}$ dominates the persistent effects of $\frac{\phi\bar{i}\rho_m}{1+\phi\bar{i}-\rho_m}gd_t$ and time varying conditional variances.

Now, by using definitions of home and foreign CPI and inserting equations (2-34) and (2-37) into equation (2-15), the risk premium can be obtained:

$$\widehat{f}_t - E_t[\widehat{e}_{t+1}] = rp_t = 0.5 * (Var_t[\widehat{r}^*_{t+1}] - Var_t[\widehat{r}_{t+1}]) = -Cov_t[\widehat{e}_{t+1}, \gamma\widehat{c}_{t+1}] = -Cov_t[\frac{1+\phi\bar{i}}{1+\phi\bar{i}-\rho_m}(g_{t+1} - g^*_{t+1}), \frac{1+\phi\bar{i}}{2(1+\bar{i})}\frac{1+\phi\bar{i}}{(1+\phi\bar{i}-\rho_m)}(g_{t+1} + g^*_{t+1})] \tag{2-38}$$

Since Jensen's inequality terms are zero when PPP holds, equation (2-38) can be interpreted as the true risk premium caused only by world aggregate risks. Again, the relative risk aversion coefficient does not affect magnitudes of the risk premium. Second, like as the determination of the nominal exchange rate, persistence of money growth rates plays an important role in increasing variations of the risk premium. The mechanism that translate persistent money growth rates into highly variable nominal exchange rates as well as the risk premium is intertemporal links of the interest sensitive money demand. However, this mechanism does not exist in a simple cash-in-advance constraint in which the nominal exchange rate is just a ratio of home to foreign money supplies regardless of whether or not money supplies grow.

Take conditional expectations on both sides of equation (2-37) for a given information set at time t in order to investigate the Fama's volatility relations:

$$E_t[\widehat{e}_{t+1}] - \widehat{e}_t = \frac{\phi\bar{i}\rho_m}{1+\phi\bar{i}-\rho_m}gd_t - \frac{1}{1+\phi\bar{i}}(rp_t + V5d_t) + \frac{\phi\bar{i}}{1+\phi\bar{i}}\sum_{s=1}^{\infty}(\frac{1}{1+\phi\bar{i}})^s E_t[rp_{t+s} + V5d_{t+s}]. \tag{2-39}$$

Now, the forward premium can be derived by adding the risk premium to equation (2-39)

$$\widehat{f}_t - \widehat{e}_t = \frac{\phi\bar{i}\rho_m}{1+\phi\bar{i}-\rho_m}gd_t + \frac{\phi\bar{i}}{1+\phi\bar{i}}rp_t - \frac{1}{1+\phi\bar{i}}V5d_t + \frac{\phi\bar{i}}{1+\phi\bar{i}}\sum_{s=1}^{\infty}(\frac{1}{1+\phi\bar{i}})^s E_t[rp_{t+s} + V5d_{t+s}]. \tag{2-40}$$

As one can see in equations (2-39) and (2-40), both the expected exchange rate depreciation and the forward premium are a function of money growth rates and the risk premium once other second moment terms are ignored. As the relation, $f_t - e_t = rp_t + E_t[e_{t+1}] - e_t$, indicates, the only difference between two equations is magnitudes of weights on the risk premium. Since the risk premium is a function of time varying conditional variances of home and foreign money growth rates in the GARCH (1,1) process as in equation (3-4) introduced later, the above equations for the expected depreciation and the forward premium can be rewritten in the following way

$$E_t[\widehat{e}_{t+1}] - \widehat{e}_t = \frac{\phi\bar{i}\rho_m}{1+\phi\bar{i}-\rho_m}gd_t - \frac{1-\rho_c}{1+\phi\bar{i}-\rho_c}rp_t - \frac{1-\rho_c}{1+\phi\bar{i}-\rho_c}V5d_t. \tag{2-41}$$

$$\widehat{f}_t - \widehat{e}_t = \frac{\phi\bar{i}\rho_m}{1+\phi\bar{i}-\rho_m} gd_t + \frac{\phi\bar{i}}{1+\phi\bar{i}-\rho_c} rp_t - \frac{1-\rho_c}{1+\phi\bar{i}-\rho_c} V5d_t. \tag{2-42}$$

where $\rho_c = \rho_h + \rho_u$. Note that equations (2-41) and (2-42) would not be equal due to the presence of the risk premium. That is, our model allows deviations from uncovered interest parity. Further, equation (2-41) shows that the relation between the expected depreciation and the risk premium is negative, which is consistent with one of the results in the Fama (1984)'s regression test. The other volatility relation depends on relative magnitudes between money growth rates and the risk premium. As an extreme case, suppose that money supply follows a random walk. Then, terms for money growth rates are zero so that the model can generate the Fama's volatility relation. However, at the same time, the variation of the risk premium significantly decreases since money growth rates exhibit zero persistence. As money growth rates become persistent, all three quantities become more and more volatile. But, relative effects of money growth rates on the forward premium and expected depreciation also become bigger so that the model has less chance to generate the volatility relation.

As one can see in equations (2-41) and (2-42), the discount factor plays an important role in the determination of the expected depreciation and the forward premium in the presence of intertemporal links of the interest sensitive money demand. As the discount factor becomes close to 1 (or, as the steady state interest rate becomes close to 0) the model has better chance to generate the volatility relations because relative effects of money growth rates on the forward premium and the expected depreciation become much smaller. The intuition behind this is that as forecasting interval becomes shorter and shorter effects of the discount factor become less and less because money demand becomes less sensitive to nominal interest rates. However, the discount factor does not much affect the risk premium since it is related to comovements of marginal rate of substitution and nominal exchange rate depreciation.

2.3 Real Exchange Rate Risks

We, now, consider how real exchange risks affect the foreign exchange rate risk premium. For this purpose, we assume that prices are preset in consumer's currency. Since PPP does not hold under this assumption, even complete asset markets do not deliver risk pooling so that consumption is no longer equalized across countries. And this causes idiosyncratic country specific risks to affect the risk premium.

Suppose that prices are preset one period in advance in consumer's currency. This assumption implicitly implies that international goods markets are segmented so that consumers cannot arbitrage across countries. Under this assumption, a home firm maximizes the expected profit by choosing $P_H(i,s^t)$ in home currency for sales to home market and $P_H^*(i,s^t)$ in foreign currency for sales to foreign market subject to $Y_H(i,s^t) = C_H(i,s^t)$, $Y_H^*(i,s^t) = C_H^*(i,s^t)$, and corresponding downward sloping demand functions defined above

$$\max_{\{P_H(i,s^t),P_H^*(i,s^t)\}} \sum_{t=0}^{\infty}\sum_{s^t} Q(s^t|s^{t-1})\{P_H(i,s^t)C_H(i,s^t) + \varepsilon(s^t)P_H^*(i,s^t)C_H^*(i,s^t) -$$
$$MC(i,s^t)[C_H(i,s^t) + C_H^*(i,s^t)]\}$$

Then, the optimal prices, $P_H(i,s^t)$, $P_H^*(i,s^t)$ for home good i in both home and foreign markets, respectively, are

$$P_H(i,s^t) = \frac{\nu}{\nu-1}\frac{\sum_{s^t} Q(s^t|s^{t-1})[MC(i,s^t)C_H(i,s^t)]}{\sum_{s^t} Q(s^t|s^{t-1})[C_H(i,s^t)]} \tag{2-43}$$

$$P_H^*(i,s^t) = \frac{\nu}{\nu-1}\frac{\sum_{s^t} Q(s^t|s^{t-1})[MC(i,s^t)C_H^*(i,s^t)]}{\sum_{s^t} Q(s^t|s^{t-1})[\varepsilon(s^t)C_H^*(i,s^t)]} \tag{2-44}$$

Analogously one can also derive prices $P_F(i,s^t)$ in home market and $P_F^*(i,s^t)$ in foreign market for foreign good i. Note that under the assumption of the local currency pricing all prices are preset before shocks are realized. Hence this assumption shuts down a channel related to expenditure switching effects since relative prices of home and foreign goods do not respond to changes in nominal exchange rates. Take second order approximation on equations (2-43) and (2-44).

$$\widehat{p}_{Ht}^{L} = E_{t-1}\widehat{mc}_t + V2_{t-1} \tag{2-45}$$

$$\widehat{p}_{Ht}^{*L} = E_{t-1}[\widehat{mc}_t - e_t] + V2^*_{t-1} \tag{2-46}$$

where $V2_{t-1}$ and $V2^*_{t-1}$ are $t-1$ conditional second order moments. In contrast to a certainty equivalence setting, under the assumption of local currency pricing (LCP) even ex ante law of one price may not hold because firms face idiosyncratic risks across different markets which cannot be insured even under complete nominal bond markets. That is, home firms face nominal exchange rate risks on foreign demands (and vise versa foreign firms face nominal exchange rate risks on home demands). Since payoffs are specified in nominal terms asset markets do not deliver complete risk sharing unless PPP holds.

Let $\widehat{rq}_t$ denote the log deviation of the real exchange rate defined by $\widehat{rq}_t = \widehat{p}_t^* + \widehat{e}_t - \widehat{p}_t$. Then, using definitions of home and foreign CPI and from equations (2-45) and (2-46) and foreign analogous prices, we have

$$\widehat{rq}_t = 0.5(\widehat{p}_{Ft}^* - \widehat{p}_{Ht} + \widehat{e}_t) + 0.5(\widehat{p}_{Ht}^* - \widehat{p}_{Ft} + \widehat{e}_t) = \widehat{e}_t - E_{t-1}[\widehat{e}_t] + V6d_{t-1} = -\widehat{mud}_t = \gamma(c_t - c_t^*). \tag{2-47}$$

where $\widehat{mud}_t = \widehat{mu}_t - \widehat{mu}_t^*$. The first equality is derived using definitions of home and foreign CPI indexes and the second equality comes from pricing equations (2-45)-(2-46), and the third equality is derived from the optimal risk sharing condition. Equation (2-47) shows that the log deviation of the real exchange rate at time t is the difference between conditional expected nominal exchange rate and realized nominal exchange rate plus risks carried from firms' pricing decisions. By taking time $t-1$ conditional expectation in equation (2-36) and using optimal risk sharing condition we can derive the real exchange rate under LCP

$$\widehat{rq}_t = \gamma(\widehat{c}_t - \widehat{c}_t^*) = \frac{1+\phi\bar{i}}{(1+\bar{i})}[\frac{1+\phi\bar{i}}{(1+\phi\bar{i}-\rho_m)}(gd_t) - \frac{(1+\phi\bar{i})\rho_m}{(1+\phi\bar{i}-\rho_m)}(gd_{t-1})]$$
$$+\frac{1+\phi\bar{i}}{1+\bar{i}}\frac{1}{1+\phi\bar{i}-\rho_c}(rp_t - rp_{t-1}) + \frac{\gamma(\phi-1)\bar{i}}{1+\bar{i}}E_{t-1}[\widehat{cd}_t] + (V7d_t - V7d_{t-1}). \tag{2-48}$$

$V7d_t$ represents for other second moments. As in the case of the determination of the nominal exchange rate and the risk premium, the volatility of the real exchange rate depends critically on the persistence of money growth rates and is independent of the relative risk aversion coefficient.

Since the log deviation of the world aggregate consumption is same regardless of assumptions on currency pricing and two country consumption difference under LCP is $\frac{\widehat{rq}_t}{\gamma}$, home marginal rate of substitution can be derived from equation (2-32) and (2-48) once other second order terms are ignored

$$\gamma[\widehat{c}_{t+1} - \widehat{c}_t] = \frac{1+\phi\bar{i}}{(1+\bar{i})}[\frac{(1+\phi\bar{i})}{(1+\phi\bar{i}-\rho_m)}(g_{t+1} - g_t) - \frac{(1+\phi\bar{i})\rho_m}{(1+\phi\bar{i}-\rho_m)}(g_t - g_{t-1})] \qquad (2\text{-}49)$$

Note that when prices are preset in consumer's currency, only home monetary shocks affect home marginal rate of substitution.

Real exchange rate becomes constant when PPP holds. However, the international goods market segmentation induces the law of one price to fail across countries and thus causes deviations from PPP which lead to incomplete risk sharing even under complete nominal bond markets. Thus, idiosyncratic country specific risks are not completely diversified and affect the risk premium through terms for real exchange rates in equation (2-50):

$$e_{t+1} - \widehat{e}_t = \frac{\phi\bar{i}\rho_m}{1+\phi\bar{i}-\rho_m} gd_t + \frac{1+\phi\bar{i}}{1+\phi\bar{i}-\rho_m} umd_{t+1} + \frac{1}{1+\phi\bar{i}-\rho_c}(rp_{t+1} - rp_t)$$
$$+ \frac{1}{1+\phi\bar{i}-\rho_c}(V5d_{t+1} - V5d_t) + \frac{\phi\bar{i}}{1+\phi\bar{i}}\sum_{s=0}^{\infty}(\frac{1}{1+\phi\bar{i}})^s(\frac{(\phi-1)}{\phi})(E_{t+1}[rq_{t+1+s}] - E_t[rq_{t+s}]) \qquad (2\text{-}50)$$

Note that the combination of international goods market segmentation and intertemporal links of the interest sensitive money demand is required in order for this channel to work. If either PPP holds or the money demand is static, then this channel is destroyed like as in the previous studies with flexible prices. Since the real exchange rate is positively related to both money growth rates and the risk premium as in equation (2-48), presence of this channel increase variations of the nominal exchange rate and the risk premium.

Under local currency pricing, both $(Var_t[\widehat{p}^*_{t+1}] - Var_t[\widehat{p}_{t+1}])$ related to inflation risk and $(Cov_t[\widehat{p}^*_{t+1}, \widehat{c}^*_{t+1}] - Cov_t[\widehat{p}_{t+1}, \widehat{c}_{t+1}])$ related to interactions between consumption and inflation risks are zero because $t+1$ prices are already known in period t. Only the term related to consumption risks, $\frac{1}{2}(Var_t[\gamma\widehat{c}^*_{t+1}] - Var_t[\gamma\widehat{c}_{t+1}])$, determines the risk premium since PPP does not hold. Therefore, the risk premium now reflects real exchange rate risks. Although this is consistent with Hollifield and Yaron (2000)'s findings, one should note that this decomposition also involves Jensen's inequality terms:

$$\widehat{rp}^L_t = \frac{1}{2}(Var_t[\widehat{c}^*_{t+1}] - Var_t[\widehat{c}_{t+1}]) = \frac{1}{2}Var_t[\widehat{e}_{t+1}] - \gamma Cov_t[\widehat{e}_{t+1}, \widehat{c}_{t+1}] =$$
$$-\frac{1}{2}Var_t[\widehat{e}_{t+1}] - \gamma Cov_t[\widehat{e}_{t+1}, \widehat{c}^*_{t+1}]. \qquad (2\text{-}51)$$

Now using equations (2-49) and (2-50), we can derive the home true risk premium

$$\widehat{Hrp}_t^{trL} = -\gamma Cov_t[\widehat{e}_{t+1}, \widehat{c}_{t+1}] = -(\frac{1+\phi\bar{i}}{1+\bar{i}})^2(\frac{1+\phi\bar{i}}{1+\phi\bar{i}-\rho_m})^2(Var_t[g_{t+1}]) \tag{2-52}$$

Analogously, one can obtain the foreign true risk premium

$$\widehat{Frp}_t^{trL} = -\gamma Cov_t[\widehat{e}_{t+1}, \widehat{c}^*_{t+1}] = (\frac{1+\phi\bar{i}}{1+\bar{i}})^2(\frac{1+\phi\bar{i}}{1+\phi\bar{i}-\rho_m})^2(Var_t[g^*_{t+1}]) \tag{2-53}$$

Compare equation (2-52) with equation (2-53): first, each county true risk premium is determined by only its own monetary shocks so that there is no risk sharing across countries even under complete nominal bonds markets. Second, the sign of home and foreign risk premium is opposite. Fluctuations of the true home risk premium are always negative while those of the true foreign risk premium are always positive. Finally, equations (2-52) and (2-53) imply that absolute values of two risk premiums would be different in each period even though unconditional variances and means of both home and foreign processes of money growth rates are same.

As one can see in the following equations (2-54) and (2-55), when PPP does not hold real exchange rates affect both the forward premium and the expected depreciation

$$\begin{aligned} E_t[e_{t+1} - \widehat{e}_t] = {} & \frac{\phi\bar{i}\rho_m}{1+\phi\bar{i}-\rho_m} gd_t - \frac{1-\rho_c}{1+\phi\bar{i}-\rho_c} rp_t - \frac{(\phi-1)\bar{i}}{1+\phi\bar{i}} \widehat{rq}_t - \frac{1-\rho_c}{1+\phi\bar{i}-\rho_c} V5d_t \\ & + (\frac{\phi\bar{i}}{1+\phi\bar{i}})^2 \sum_{s=0}^{\infty} (\frac{1}{1+\phi\bar{i}})^s \{\frac{(\phi-1)}{\phi} E_t[\widehat{rq}_{t+1+s}]\} \end{aligned} \tag{2-54}$$

$$\begin{aligned} f_t - \widehat{e}_t = {} & \frac{\phi\bar{i}\rho_m}{1+\phi\bar{i}-\rho_m} gd_t + \frac{\phi\bar{i}}{1+\phi\bar{i}-\rho_c} rp_t - \frac{(\phi-1)\bar{i}}{1+\phi\bar{i}} \widehat{rq}_t - \frac{1-\rho_c}{1+\phi\bar{i}-\rho_c} V5d_t \\ & + (\frac{\phi\bar{i}}{1+\phi\bar{i}})^2 \sum_{s=0}^{\infty} (\frac{1}{1+\phi\bar{i}})^s \{\frac{(\phi-1)}{\phi} E_t[\widehat{rq}_{t+1+s}]\}. \end{aligned} \tag{2-55}$$

That is, idiosyncratic country specific risks affect both the forward premium and the expected depreciation via two channels: One is due to changes in the risk premium and the other is due to fluctuations in real exchange rates. Here, relative effects of money growth rates to the risk premium on both the expected depreciation and the forward premium become smaller because real exchange rates are negatively related to these quantities. This relation is reasonable in the sense that increase in the nominal interest rate reduces the money demand and thus increase current prices other things being equal.

2.4 Staggered Nominal Contracts

Staggered nominal contracts tend to increase persistent real effects of monetary shocks through the so called contract multiplier generated from pricing equations with both forward and backward looking terms. In this section, we investigate how the length of contract periods affects the risk premium. Since it is cumbersome to solve for the risk premium when prices are fixed for more than one period in a staggered way and does not give much intuition, we focus on examining where main differences between the two types of contracts come from.

The main departure between one period in advance and multi-period staggered nominal contracts for the determination of the risk premium and the volatility relations comes from intertemporal links of the interest sensitive money demand: how do monetary shocks affect households' expectations on their future consumption? When prices are flexible, it does not matter whether or not money demand is static because monetary shocks do not affect real variables. However, when prices are sticky effects of current monetary shocks on current consumption depend on how money demand responds to current monetary shocks. For example, consider the following static money demand function

$$\widehat{m}_t - \widehat{p}_t = \widehat{mu}_t. \tag{2-56}$$

Here, current realized monetary shocks only affect marginal utility of current consumption because prices are determined in advance. Since only current shocks matter for the determination of the risk premium prices do not affect the risk premium regardless of types of nominal contracts. On the other hand, the nominal exchange rate is determined by difference between home and foreign money supplies and thus the same under the two types of nominal contracts. Hence, the risk premium does not change with respect to the length of contract periods when the money demand is static.

However, for the case of intertemporal links of the interest sensitive money demand effects of current monetary shocks are translated into two parts as in equation (2-33): marginal utility of current consumption and conditional expectations of marginal utilities of future consumption. As one can see in equation (2-32), money growth rates do not affect conditional expectation of consumption under one period in advance price setting because monetary shocks do not exhibit any persistent real effects. However, prices are gradually adjusted under staggered price settings since they are fixed more than one period. Since next period's prices are not fully adjusted to the response of current monetary shocks under staggered nominal contracts, households now expect that current shocks would affect their future consumption. Therefore, households' perception of risks change with respect to the degree of persistent real effects of monetary shocks. For example, this effect is reflected in conditional expectation of marginal utility of future consumption in equation (2-33).

3 Calibration and Estimation

3.1 Calibration

All parameter values used in our benchmark model are reported in Table 1.

We first choose parameter values for the utility function specified in equation (2-1). The discount factor β is set to be $0.96^{1/N}$ so that an annualized steady state interest rate is to be

0.04 for $N = 1$. We set $N = 4$ for a quarterly frequency and $N = 12$ for a monthly frequency. The parameter μ is set at 2, which is consistent with the empirical labor literature. The coefficient of the relative risk aversion γ is set at 6 by following Chari, Kehoe, McGrattan (2002). This value is relatively high compared to the parameterization in real business cycle literature. As we already discussed in the previous section, however, our main results do not depend on this parameter value. Rather, we choose the high value of the risk aversion coefficient in order to match with fluctuations of output and consumption in the data. Since preference is separable between real consumption and real money balance in our model, parameterization of consumption elasticity of money demand is tied with that of interest elasticity of money demand. We first set consumption elasticity of money demand $\frac{\gamma}{\phi}$ to be 1 by following the estimate of Mankiw and Summers (1986). Then, the degree of interest elasticity of money demand $\frac{1}{\phi}$ can be derived from both combination of choices on the risk aversion parameter and the degree of consumption elasticity of money demand. However, estimates for interest elasticity of money demand range widely from 0.02 in Mankiw and Summers (1986), through 0.25 in Helliwell, Conkerline, and Lafrane (1990) to 0.39 in Chari, Kehoe and McGrattan (2002). Further, Since ϕ is one of main determinants in the risk premium and the nominal exchange rates, we conduct sensitivity analysis by varying values of ϕ.

The labor share parameter α in the production function $Y = AL^{\alpha}Z^{1-\alpha}$ is set to 0.15 so that the share of aggregate intermediate goods is 0.85 by following Basu (1995) and Bergin and Feenstra (2001). The steady state share of intermediate goods in output is derived by combining goods market clearing conditions with the condition for expenditure share $\frac{PZ}{WL} = \frac{1-\alpha}{\alpha}$ and price determination equations:

$$\frac{\overline{Z}}{\overline{Y}} = (1-\alpha)\frac{\nu - 1}{\nu} \tag{3-1}$$

where $(1-\alpha)$ is cost share for a composite of intermediate goods and $\frac{\nu-1}{\nu}$ is the steady state markup over marginal costs. We set 10 for elasticity of substitution within a country ν so that a steady state markup is 0.11 as is standard in the literature. Further, elasticity of substitution across countries is set at $\theta = 1$. This is lower than Backus, Kehoe, and Kydland (1994)'s parameterization ($\theta = 1.5$). Hence, we also conduct sensitivity analysis on this parameter. Although we do not provide the formal analysis in this paper, our analytical analysis in previous version of this paper shows that this parameter value does not have significant effects on exchange rates and consumption in the transmission mechanism of monetary shocks.

Other things being equal, we consider the following three cases related to the degree of home bias h and currency pricing in order to investigate relative contribution between real exchange rate risks and aggregate risks to fluctuations of the risk premium. For the first case, we set $h = 0.5$ and assume that prices are set in producer's currency so that only aggregate risks affect the risk premium (PCP). For the second case, we set $h = 0.85$ while still assuming that the law of one price holds for each good i so that we can investigate relative effects of aggregate risks to real exchange rate risks due to home bias in preferences

(Home). For the third case, we assume that firms set their price in consumers' currency and $h = 0.85$ so that we can analyze how violations of the law of one price affect the determination of the risk premium. When prices are preset in consumer's currency, the degree of home bias in preferences is irrelevant in the transmission mechanism of monetary shocks. We consider local currency pricing as our benchmark currency pricing. Finally, we consider two types of nominal contracts with respect to the length of contract periods: one period in advance synchronized nominal contracts versus three period staggered nominal contracts. We consider three period as our benchmark contract periods.

Money supplies for home and foreign countries are assumed to follow a univariate process of the form

$$\frac{M_{t+1}}{M_t} = G_{t+1}$$
$$\frac{M^*_{t+1}}{M^*_t} = G^*_{t+1}, \tag{3-2}$$

where G_{t+1} and G^*_{t+1} are period $t+1$ stochastic growth rates for home and foreign country, respectively, and evolve by

$$g_{t+1} = \rho_m g_t + um_{t+1}$$
$$g^*_{t+1} = \rho_m g^*_t + um^*_{t+1}. \tag{3-3}$$

where $\log G_{t+1} = g_{t+1}$. We further assume that conditional variances of money growth rates are time varying and follow a univariate GARCH (1,1) process:

$$hm_t = \zeta_m + \rho_h hm_{t-1} + \rho_u um^2_{t-1} hm^*_t = \zeta_m + \rho_h hm^*_{t-1} + \rho_u um^{*2}_{t-1} \tag{3-4}$$

where ρ_h denotes a persistent coefficient of conditional variance shocks and ρ_h denotes a kurtosis coefficient, hm_t denotes conditional variance of home monetary shocks given time t information, and $\rho_h + \rho_u < 1$. The intersection term is assumed to be $\zeta_m = var(um)(1 - \rho_h - \rho_u)$, where $var(um)$ is the unconditional variance of stochastic disturbances so that the unconditional mean of conditional variances is equal to the unconditional variance of stochastic disturbances.

As can be seen in Table 2, both the US monthly and quarterly M1 growth rates highly serially correlated and contain strong ARCH components that support our specification for the process of time varying conditional variances of money growth rates. Until a fairly long lag lengths, we continue to reject the hypothesis of no serial correlation in money growth rates [see, the Ljung-Box test statistic Q(k) and LM test statistic LM(k) in Table 2]. Further, we also reject the hypothesis of conditional homoskedasticity for both frequencies. Finally, we consider two conditional distributions along with the GARCH conditional variance model by following Baillie and Bollerslev (1989) in order to account for high degree of fat-tailedness in the unconditional distribution of monthly M1 growth rates: one is the t distribution with degrees of freedom 5 and the other is the normal distribution. We consider the t distribution as our benchmark distribution but also report results obtained from the normal distribution.

Parameter values in equations (3-3) and (3-4) are jointly estimated using both quarterly and monthly US data for M1 between January 1959 and December 2002, obtained from Board of Governors of the Federal Reserve System. Our estimation results show that M1 growth rates for both frequencies exhibit strong GARCH effects. For example, $\widehat{\rho}_h = 0.56$ and $\widehat{\rho}_u = 0.25$ for a quarterly frequency while $\widehat{\rho}_h = 0.68$ and $\widehat{\rho}_u = 0.29$ for a monthly frequency. From the estimation we choose $\rho_m = 0.83$ for a quarterly frequency while $\rho_m = 0.76$ for a monthly frequency. Unconditional variances of stochastic disturbances are set to be 0.01^2 and 0.0057^2 for a quarterly and a monthly frequency, respectively. We assume that processes of both home and foreign money supply are same.

Our simulation follows Sutherland (2002)'s two step procedure by taking full second approximations of the model, considering time varying distributions of shocks (an univariate GARCH (1,1) for the process of home and foreign money growth rates), and ignoring moments higher than order two: First, use the first order dynamic system of the model to generate the path of conditional second moments. Then, use these conditional second order moments and the second order dynamic system to obtain statistics of the risk premium, the forward premium, and the expected depreciation.

3.2 Estimation of Predictable Returns

Empirical studies find that ex ante predicted returns from currency speculation not only change their sign but also display large variations [see, also, Hodrick (1987), Lewis (1995), and Engel (1996) for a survey of empirical evidence]. In this section, we also estimate these excess returns from currency speculation using monthly and quarterly exchange rates data between January 1976 and December 1994 taken from Chris Telmer's webpage (originally from the Harris Bank's Weekly Review: International Money Markets and Foreign Exchange). Monthly observations are values of last Friday of the month and quarterly observations are values of last Friday of the quarter. Summary statistics of spot and forward exchange rates for the US dollar versus the Canadian dollar, the French franc, the German mark, the Japanese yen, and the British pound are provided in Table 3.

Panel A and B in Table 3 are concerned with ex post returns from currency speculations and exchange rate depreciations, respectively for both a monthly and quarterly frequency. In general, ex post returns and exchange rate depreciations of currencies against the US dollar (except the Canada dollar) are highly variable, move closely together, and have similar magnitudes of volatilities: standard deviations of these two time series are about 9 times greater than those of the US consumption growth rate for a quarterly frequency. Cross correlations between two series are on average 0.99 close to 1 for a both monthly and quarterly frequency. Further, autocorrelations of ex post returns and depreciations are less than 0.2 for all five currencies. This implies that most variations of these two times series are due to fluctuations of future spot exchange rates. In other words, neither past depreciation rates or forward premiums have predictability for these two series. Panel B and D are concerned with variations of nominal and real exchange rate depreciations, respectively. For a quarterly frequency, growth rates of the two exchange rates move closely together and highly variable.

This implies that movements in nominal exchange rates are independent of movements in international goods prices at least for a quarterly frequency. Finally, panel C in Table 2 is concerned with forward premium. For each currency, the forward premium exhibit highly persistence while volatility is much less than other series in Table 2.

Estimation results for monthly and quarterly ex ante predictable returns for five currencies are reported in Table 4. By following Cumby (1988), Canova and Marrinan (1993), and Backus, Gregory, and Telmer (1993), we directly compute these returns by calculating standard deviations of fitted values from the regression (3-5) and interpret them as one measuring the theoretical risk premium.[16] Ex ante predictable returns from currency speculation can be estimated from the regression of the form

$$f_t - e_{t+1} = a_1 + a_2(f_t - e_t) + u1_{t+1} \tag{3-5}$$

where f_t is either one month or three months ahead log of forward exchange rate at t, e_t denotes the log spot exchange rate at time t, and $u1_{t+1}$ denotes error terms which are not correlated with $(f_t - e_t)$.[17] Since $u1_{t+1}$ is not a forecast error, the fitted values from the regression (3-5) may not correctly measure the theoretical risk premium. We will discuss this point in detail later. Nevertheless, if uncovered interest parity hypothesis holds and the market's expectation equals the statistical prediction then both the slope coefficient a_2 and the intersection term a_1 should be zero. Non-zero estimates of a_2 imply that the forward premium $f_t - e_t$ contains predictable components.

As reported in Table 4, we find that all estimates for a_2 are strictly positive and greater than 1, and statistically significant except France and Germany for a quarterly frequency. These results are consistent with previous studies. Estimates of the slope coefficient range from 1.39 for the Canadian dollar to 2.35 for the Japanese yen for a quarterly frequency while those range from 0.95 for Japanese Yen to 3.21 for British pound for a monthly frequency. Further, standard deviations of fitted values for each regression range from 0.0049 for the Canadian dollar to 0.0259 for Japanese Yen for quarterly frequency data while those range 0.0027 for the Canadian dollar to 0.0087 for British pound for monthly data. Like as the forward premiums, autocorrelations of predictable returns for each currency are highly persistent. We interpret these predictable returns as measuring the theoretical foreign exchange rate risk premium and use them for comparing with quantities generated from the model.

We also run the following complementary regression in order to investigate whether or not the forward exchange rate is an unbiased estimator of the future spot exchange rate

$$e_{t+1} - e_t = a_3 + a_4(f_t - e_t) + u2_{t+1} \tag{3-6}$$

[16] Although there are several econometric ways of estimating predictable returns [e.g., Ito (1988), Diebold and Nason (1990), and Canova (1991)] we directly estimate these returns by projecting ex post returns on a constant and the forward premium.

[17] Following previous studies, we also consider the following form of regression:

$$\frac{F_t - \varepsilon_{t+1}}{\varepsilon_t} = a_0 + a_1\left(\frac{F_t - \varepsilon_t}{\varepsilon_t}\right) + U1_{t+1}.$$

However, results are almost same in both cases. Hence, we use equation (3.5) to be consistent with our analytical analysis.

where $u2_{t+1}$ is an error term. As reported in Table 4, all estimates for a_4 are negative but most currencies are not statistically significant.

4 Results

4.1 Main Results

Our main results from several experiments based on extensions of the benchmark model are reported in Table 5 and 6.

First, our benchmark model with assumptions of international goods market segmentation, no habit persistence, 3 period staggered price setting, and complete asset markets can generate the Fama's volatility relations for a monthly frequency but not a quarterly frequency. The variance of the risk premium generated from the model is greater than the sum of variances of the forward premium and the expected depreciation for a monthly frequency. However, the magnitudes are much smaller than those in the data. For a monthly frequency, variances of the risk premium, the forward premium, and the expected depreciation in the benchmark model are 0.0049E(-4), 0.0009E(-4), 0.0025E(-4), respectively, while those are 0.1089E(-4), 0.0225E(-4), 0.0196E(-4), respectively, for the Canadian dollar in our sample. However, for a quarterly frequency, the model generates quantities whose variations are much closer to those in the data although the model now cannot generate the volatility relations. For a quarterly frequency, those quantities generated from the benchmark model are 0.0441E(-4), 0.0676E(-4), 0.1156E(-4), respectively, while those are 0.2401E(-4), 0.1681E(-4), 0.0049E(-4) for the Canadian dollar.

As we already discussed our model tends to have better chance to produce one of two volatility relations, $[Var(rp_t) > Var(f_t - e_t) + Var(E_t(e_{t+1} - e_t)]$, for a monthly frequency other things being equal since the discount factor is greater for a monthly frequency than for a quarterly frequency. Our results show that effects of the discount factor on both the expected depreciation and the forward premium become smaller as decision intervals become shorter because relative effects of money growth rates to the risk premium on these quantities become smaller. However, the discount factor do not affect the risk premium. Hence, other things being equal, our model with higher data frequency tends to generate the volatility relation by reducing variations of these two quantities. Now, let us discuss why variances of these quantities for a quarterly frequency are much greater than those for a monthly frequency even though the variance of stochastic disturbances in the process of money growth rates for a monthly frequency is about one fourth of that for a quarterly frequency. This is mainly due to the assumption of the time varying distribution of money growth rates. When the distribution is time varying the unconditional variance of the risk premium is not proportionally related to the unconditional variance of stochastic disturbances in the process of money growth rates. Rather, it is related to the fourth moments of the underlying distribution. This implies that magnitudes of the underlying distribution matter for the determination of the variances of the quantities such as the risk premium, the forward premium, and the expected depreciation which are mainly determined by time varying second moments of the distribution, other things being equal.

Our model can also produce another volatility relation derived by Fama, negative correlation between the expected depreciation and the risk premium, as can be seen in equations (2-39) and (2-54). The cross correlations between these two quantities generated from the benchmark model for both monthly and quarterly frequencies are -0.53 and -0.78, respectively. As can be seen in Tables 5 and 6, since other results hold true for both frequencies our analysis focuses on the quarterly frequency.

Second, our quantitative results show that real exchange rate risks significantly affect the volatility relation in our model with intertemporal links of the interest sensitive money demand by raising variations of the risk premium and by reducing variations of the forward premium and the expected depreciation for both frequencies. Standard deviations of the risk premium, the forward premium, the expected depreciation are 0.0021, 0.0026, 0.0034, respectively, in the benchmark model while those are 0.0014, 0.0052, 0.0054, respectively, when there exist only aggregate risks. When the law of one price is violated because firms set their prices in consumer's currency, the mechanism under complete nominal bonds markets that shares idiosyncratic risks across countries does not work. The failure of this mechanism increase variations of the risk premium. Further, this failure is not relevant to the degree of home bias in preferences because impart prices do not change with respect to change in nominal interest rates. One should note that real exchange rate risks significantly reduce variations of the forward premium and the expected depreciation.

Our results also show that real exchange rate risks tend to raise significantly variations of Jensen's inequality terms so that standard deviations of both Jensen's inequality terms and the true risk premium are similar [note that Jensen's inequality terms are zero in a model without country specific risks as in third and fourth column in Table 5]: the standard deviation of Jensen's inequality terms is 0.0018 in the benchmark model. As the degree of home bias increases Jensen's inequality terms become more fluctuate under PCP while they are independent of the degree of home bias under LCP. Although real exchange rate risks increase variations of Jensen's inequality terms as well as the true risk premium, the standard deviation of sum of the two terms does not increase because there is negative relationship between these two terms as can be seen in equation (2-51): the standard deviation of the sum of these two terms is 0.0021.

Third, staggered nominal contracts tend to increase variations of the risk premium. Standard deviations of the true risk premium are 0.0020 under LCP and 0.0014 under PCP, respectively, when prices are set in a staggered way for three periods while they are 0.0016 and 0.0012, respectively, when prices are set one period in advance. One of main differences between one period in advance price setting and staggered nominal contracts is how current monetary shocks affect marginal utilities of current consumption and conditional expected future consumption for a given current information set. For example, as we already discussed in the previous section, there is no difference with respect to the length of contract periods in the case of the static money demand because the impacts of current monetary shocks on current marginal utilities are same for any length of contract periods. However, for the case of the interest sensitive money demand, effects of current monetary shocks are translated into two parts as one can see in money market clearing conditions (2-5) and (2-6), marginal utilities of consumption and conditional expected future consumption. Under one period in advance price setting, current monetary shocks do not affect conditional expected consumption since all future prices are adjusted in a synchronized way to the response of current shocks. However, prices are gradually adjusted under staggered price settings so that current monetary shocks affect conditional future expected consumption. Thus, other things being equal, current world aggregate monetary shocks affect current world consumption both by affecting current marginal utilities and conditional future expected consumptions. This relation also holds for consumption differences between two countries in an analogous way. Therefore the risk premium tends to fluctuate more under staggered contracts than under one period synchronized contracts.

Fourth, although our model fails to generate enough volatility of the risk premium, we are able to produce variable nominal and real exchange rate depreciation observed in the data: standard deviations of nominal and real exchange rates in the benchmark model are 7.21 and 8.1 percent, respectively, while the median values of standard deviations of nominal and real exchange rates in our sample are 6.38 (Japanese yen) and 6.47 percent (French frac), respectively. These values are a bit higher than those in the data. In contrast to a cash in advance constraint, volatilities of the nominal and real exchange rates and the risk premium in our model depend crucially on the persistence of money growth rates via intertemporal links of the interest sensitive money demand. We also investigate whether or not the level risk premium in Obstfeld and Rogoff (1998) significantly affects the volatility of the nominal exchange rate. As can be seen in equation (4-5), variations of the level risk premium depends on sum of the persistent coefficient of conditional variance shocks expected discounted future values of risk premia combined with the multiplication factor affect the nominal exchange rate in a model with the interest sensitive money demand under the assumption of time varying conditional variances of monetary shocks. Our quantitative results under a GARCH (1,1) model show that this effect on fluctuations of exchange rates is present but relatively small. Rather, most volatility effects on exchange rates are related to first order monetary shocks so that volatility effects of the level risk premium are dominated by those of first order monetary shocks.

Finally, both the risk premium and the forward premium generated from our benchmark model are highly persistent. Autocorrelations of the two time series are 0.681 and 0.921, respectively in the benchmark model while those are 0.765 and 0.777, respectively for the Canadian dollar. These statistics are in the range for other currencies in our sample. The risk premium inherits to the high degree of persistence in the GARCH(1,1) process of time varying conditional variances while the autocorrelation of the forward premium depends on persistence of money growth rates as well as time varying conditional variances.

4.2 Habit Persistence

In this section, we investigate implications of the habit persistence for the risk premium in the model with nominal rigidities and market imperfections. One of main advantages for adopting habit persistence is known as a small variations of consumption fluctuations can have a large effect on the marginal rates of substitution between current and future consumption for a reasonable range of values of the risk aversion coefficient. For example, Backus, Gregory, Telmer (1993) introduce habit persistence in a Lucas type two country general equilibrium model with flexible prices and a cash-in-advance constraint in order to increase volatilities of the risk premium.[18]

By following Christiano, Eichenbaum, and Evans (2001), the preference for home individual $n \in [0,1]$ is considered when consumption exhibits habit persistence

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) [\frac{1}{1-\gamma}(C(n,s^t) - bC(n,s^{t-1}))^{1-\gamma} + \frac{1}{1-\phi}(\frac{M(n,s^t)}{P(s^t)})^{1-\phi} - \frac{1}{1+\mu} L^{1+\mu}(n,s^t)]. \tag{4-1}$$

Parameter b indicates habit persistence or consumption durability: if $b = 0$, the preference is time additive, if $b > 0$ consumption exhibits habit persistence, while if $b < 0$ consumption is durable. In our quantitative study b is set to be 0.63 as in Christiano, Eichenbaum, and Evans (2001).

In contrast to previous studies with a cash in advance constraint [e.g. Backus, Gregory, Telmer (1993)], our quantitative study shows that habit persistence does not play any role in the determination of the risk premium in our model with nominal rigidities and the money-in-the-utility function. As can be seen in fifth column in Tables 5, standard deviations of the risk premium are almost the same between time separable preferences and habit persistence. This is mainly because, other things being equal, the effects of monetary shocks on the marginal utility are always the same regardless of values of the risk aversion coefficient and/or the degree of habit persistence when money enters in the utility function. For example, the marginal rate of substitution between real money balances and consumption is always constant when money demand is static. Since realized monetary shocks are not completely translated to the price term when prices are preset in producer's currency (or there is no translation at all under the assumption of local currency pricing), the marginal utility of consumption should be changed in order to clear money markets regardless of whether or not consumption exhibits habit persistence. Since all that matters in the determination of the risk premium is the magnitude of marginal utilities of consumption but not consumption itself. The degree of habit persistence does not affect results.[19] This mechanism works even in the general case with an intertemporal link of the interest sensitive money demand in which monetary shocks are translated into both marginal utilities of consumption and nominal interest rates. As we already discussed, monetary shocks do not have persistent effects on real variables when either prices or wages or both are preset one period in advance. In order words, current monetary shocks do not affect the conditional expected consumption for a

[18] On the other hand, Boldrin, Christiano, and Fisher (2000) introduce habit persistence into the standard RBC model with only real shocks and investigate equity premium puzzle and low risk free interest rate puzzle.

[19] There is a possibility that different preference specifications can change relative effects of monetary shocks between on price and consumption. However, this effect is negligible. For example, if prices are preset in consumer's currency, the result is independent of assumptions on preferences.

given current information set. Hence, the presence of the expected future consumption term under habit persistence would not affect the risk premium at all as long as only monetary shocks are concerned.

4.3 Incomplete Asset Markets

In this section we investigate roles of incomplete asset markets in the determination of the risk premium. There is mixed evidence on roles of incomplete asset markets in models with nominal rigidities. For example, Bacchetta and Wincoop (2000) and Chari, Kehoe, and McGrattan (2002) report that asset market structure does not quantitatively affect results. On the other hand, Devereux and Engel (2001) consider incomplete asset markets in order to produce exchange rates that are more variable than fundamentals in the economy with noise traders. And Duarte and Stockman (2001) emphasize roles of incomplete asset markets combined with international product market segmentation in order to obtain deviations from uncovered interest parity. Here, we consider the case in which there exist two uncontingent nominal bonds issued by each country.

The home household n's budget constraint (denominated in home currency) under uncontingent nominal bond markets is now:

$$\begin{aligned} P_t C(n,t) + M_t(n) + B_{H,t+1}(n)Q_{t+1} + \epsilon_t Q^*_{t+1} B_{F,t+1}(n) \leq \\ \int_0^1 W_t(n) L^d_t(i,n) di + M_{t-1}(n) + B_{H,t}(n) + \epsilon_t B_{F,t}(n) + \Pi_t(n) + T_t(n) \end{aligned} \tag{4-2}$$

Then, one can derive condition for optimal risk sharing under incomplete asset markets from both home and foreign intertemporal Euler equations

$$\varepsilon_t = \frac{E_t[R_{t+1}\varepsilon_{t+1}]}{E_t[R^*_{t+1}]} = \frac{E_t[R_{t+1}]}{E_t[\frac{R^*_{t+1}}{\varepsilon_{t+1}}]}. \tag{4-3}$$

By using the same method as in the previous section, we derive the following log deviations of variables in equation (2-62) when preferences are time additive:

$$\begin{aligned} \widehat{e}_t + \widehat{p}^*_t - \widehat{p}_t = \gamma(\widehat{c}_t - \widehat{c}^*_t) + E_t[\widehat{e}_{t+1} + \widehat{p}^*_{t+1} - \widehat{p}_{t+1} - \gamma(\widehat{c}_{t+1} - \widehat{c}^*_{t+1})] \\ -\frac{1}{2}(Var_t[\widehat{r}^*_{t+1}] - Var_t[\widehat{r}_{t+1}]) + \frac{1}{2}Var_t(\widehat{e}_{t+1}) + Cov_t[\widehat{r}_{t+1}, e_{t+1}]. \end{aligned} \tag{4-4}$$

where $\widehat{e}_t + \widehat{p}^*_t - \widehat{p}_t$ is the log deviation of the real exchange rate. Under complete asset markets, $E_t[\widehat{e}_{t+1} + \widehat{p}^*_{t+1} - \widehat{p}_{t+1} - \gamma(\widehat{c}_{t+1} - \widehat{c}^*_{t+1})]$ is always zero and $\frac{1}{2}(Var_t[\widehat{r}^*_{t+1}] - Var_t[\widehat{r}_{t+1}]) = \frac{1}{2}Var_t(\widehat{e}_{t+1}) + Cov_t[\widehat{r}_{t+1}, \widehat{e}_{t+1}]$. However, this may not be necessarily true when asset markets are incomplete and PPP does not hold. If PPP holds then $-\frac{1}{2}(Var_t[\widehat{r}^*_{t+1}] - Var_t[\widehat{r}_{t+1}]) + \frac{1}{2}Var_t(\widehat{e}_{t+1}) + Cov_t[\widehat{r}_{t+1}, \widehat{e}_{t+1}]$ should be zero even under incomplete asset markets since the real exchange rate is constant. Second moments terms appear in the determination of the real exchange (2-63) due to the combination of incomplete asset markets and deviations from PPP. We quantitatively investigate magnitudes of these terms and effects on fluctuations of real exchange rates since the theory does not impose further restriction on these second order terms.

Our results show that asset market structures do not have significant impacts on the determination of the risk premium: standard deviations of the true risk premiums for different types of currency pricing are almost the same between complete and incomplete asset markets. When the law of one price holds and a composite consumption index is Cobb-Douglas, asset market structure is irrelevant as Cole and Obstfeld (1991) point out [see, also, Corsetti and Pesenti (1998) and Obstfeld and Rogoff (1998)]. Our quantitative results also show that this is true even when the law of one price fails and elasticity substitution across countries is greater the unity. This result is also consistent with findings of Chari, Kehoe and McGrattan (2002). Although theoretically incomplete asset markets play a role in leading to wealth redistributions across countries so that idiosyncratic country specific risks can have large effects on the risk premium, quantitatively, they do not affect results. To see this, solve for $\widehat{e}_t + \widehat{p}_t^* - \widehat{p}_t - \gamma(\widehat{c}_t - \widehat{c}_t^*)$ in equation (4-4)

$$\widehat{e}_t + \widehat{p}_t^* - \widehat{p}_t - \gamma(\widehat{c}_t - \widehat{c}_t^*) = \lim_{T \longrightarrow \infty} E_t[\widehat{e}_{t+T} + \widehat{p}_{t+T}^* - \widehat{p}_{t+T} - \gamma(\widehat{c}_{t+T} - \widehat{c}_{t+T}^*)]$$

$$- \sum_{s=1}^{\infty} \{\frac{1}{2}(Var_t[\widehat{r}_{t+1+s}^*] - Var_t[\widehat{r}_{t+1+s}]) + \frac{1}{2} Var_t(\widehat{e}_{t+1+s}) + Cov_t[\widehat{r}_{t+1+s}, \widehat{e}_{t+1+s}]\}. \qquad (4\text{-}5)$$

Equation (4-5) implies that $\widehat{e}_t + \widehat{p}_t^* - \widehat{p}_t - \gamma(\widehat{c}_t - \widehat{c}_t^*)$ may not be zero because of presence of second order terms even if $\lim_{T \longrightarrow \infty} E_t[\widehat{e}_{t+T} + \widehat{p}_{t+T}^* - \widehat{p}_{t+T} - \gamma(\widehat{c}_{t+T} - \widehat{c}_{t+T}^*)] = 0$. Note that $\frac{1}{2}(Var_t[\widehat{r}_{t+1}^*] - Var_t[\widehat{r}_{t+1}]) + \frac{1}{2}Var_t(\widehat{e}_{t+1}) + Cov_t[\widehat{r}_{t+1}, \widehat{e}_{t+1}] = 0$ for each time t under complete asset markets. Our quantitative results show that the presence of second order moments in equation (4-5) does not have significant effects on fluctuations of real exchange rates. As a consequence, our results are virtually the same between the two market structures.

4.4 The US Dollar Pricing

We consider another type of currency invoicing called the US dollar pricing (USP) based on empirical evidence that about 92 percent of the US exports and about 81 percent of the US imports are invoiced in US dollars in 1995 [Bekx (1998)]. Under this assumption, home monopoly (a US producer) i sets its price $P_{Ht}(i)$ in home currency (the US dollar) for sales to both home and foreign markets. Foreign monopoly i sets its price $P_{Ft}^*(i)$ in foreign currency for sales to foreign market and $P_{Ft}(i)$ in home currency (the US dollar) for sales to home market. Under this assumption only home currency (the US dollar) is used for international trade. Thus, this currency invoicing practice modifies international transmission of monetary shocks in a different way; expenditure switching effects in foreign country which is absent under LCP and exchange rates disconnect in home country which is absent under PCP since the law of one price holds for each home good $i \in [0,1]$, $P_{Ht}(i) = P_{Ht}(i)^* \varepsilon_t$ while it does not for each foreign good i. That is, relative prices between home and foreign goods do not change in home country under this assumption while there is complete exchange rate pass-through to foreign country. We investigate how this asymmetry caused from international goods markets can affect the determination of the risk premium, especially for home and foreign risk premium.

As can be seen in Table 5 and 6, adding asymmetry in currency pricing does not qualitatively change our main results. Quantitatively, it performs a bit better than the case with PCP and a bit less than the case with LCP; For example, the standard deviation of the true risk premium under USP is 0.18 percent for a quarterly frequency while they are 0.17 percent under PCP with home bias and 0.20 percent in the benchmark model. Further, standard deviations and the means of absolute values of both home and foreign risk premium are same even though there is asymmetry in the goods markets.

4.5 Sensitivity Analysis

In this section, we conduct sensitivity analysis to investigate how the risk premium fluctuates as we vary values of key parameters, add real shocks, and consider the normal distribution for the conditional variance of money growth rates.

As we analytically investigate in the previous section, the degree of the risk aversion coefficient is independent of real exchange rate determination and the risk premium since real money balances and marginal utility of consumption is a one to one relation in the money-in-utility function model. Rather, the degree of the intertemporal elasticity of substitution is related to fluctuations of consumption: as households want to smooth consumption more and more volatilities of consumption become smaller and smaller. Our quantitative results show that relatively high value of the risk aversion coefficient, $\gamma = 6$ is needed to obtain volatility of consumption observed in the data. Further, the degree of elasticity of substitution across countries θ does not have significant effects on fluctuations of international macro variables such as the risk premium, exchange rates, and consumption in the transmission mechanism of monetary shocks [see ninth column in Tables 5 and 6].[20]

Equations (2-32) and (2-34) show that, in addition to the persistence of money growth rates, the degree of the interest elasticity of money demand plays a role in determining volatilities of macro variables such as exchange rates and the the risk premium: as ϕ increases, volatilities of the nominal and real exchange rate depreciation and consumption decrease while those of the forward premium and the expected premium increase. For example, the standard deviation of the nominal exchange rate in the benchmark model is 6.44 percent for a quarterly frequency when $\phi = 10$ while they are 7,59 for $\phi = 2.5$ and 7.82 percent for $\phi = 1$. On the other hand, standard deviations of the forward and expected depreciation are 0.37 and 0.43 percent when $\phi = 10$ while they are 0.15 and 0.26 percent in the case with $\phi = 2.5$. Hence, volatilities of the forward premium and the expected exchange rate depreciation tend to be sensitive to this parameterization. However, the standard deviation of the risk premium does not vary with respect to the degree of the interest elasticity of money demand: the standard deviation of the risk premium is 0.2 percent when when $\phi = 10$ while it is 0.21 percent when $\phi = 1$. Interestingly, as the value of ϕ decreases, the standard deviation of the risk premium tend to be greater than that of the forward premium even for a quarterly

[20]The magnitude of elasticity matters in the propagation of real shocks.

frequency. However, overall these effects do not change our main results on roles of real exchange rate risks, habit persistence, the length of contract periods, and the asset market structure. The quantitative results can be provided upon request.

As one can see in equation (2-48), real exchange rates are independent of the degree of home bias in preferences when prices are set in consumers' currency. Hence, the determination of the risk premium is not relevant to the degree of the home bias in the presence of violations of the law of one price. This result is consistent with findings of recent quantitative studies which typically assume local currency pricing. In contrast to LCP, when prices are preset in producer's currency, fluctuations of real exchange rate risks and the risk premium are sensitive to the degree of home bias because the only source for deviations from PPP is the degree of home bias.

Since the volatility relations on the nominal exchange rate depreciation, the risk premium, and the expected depreciation are closely related to the magnitude of the discount factor, we investigate whether or not our results are sensitive to the parameterization on β. Hence, we set $\beta = 0.92$ so that the annualized steady state of nominal interest is 8 percent. This can be interpreted as a lower bound for the parameterization of β. Our result show that the volatility relation does not change: when a monthly decision interval is considered standard deviations of nominal exchange rate depreciation, the risk premium, and the expected depreciation is 2.58, 0.07, and 0.06 percent when $\beta = 0.92^{1/12}$ while those are 2.69, 0.07, and 0.05 percent when $\beta = 0.96^{1/12}$. Further, the standard deviation of the forward premium is 0.05 percent when $\beta = 0.92^{1/12}$ and it is 0.03 percent when $\beta = 0.96^{1/12}$. Although, the lower value of β increases standard deviations of both the forward premium and the expected premium our results on the volatility relations continue to hold.

Finally, we investigate whether or not our results are sensitive to the assumption of the conditional distribution. We report our results in Table 7 when normal distribution is considered as the conditional distribution along with the GARCH conditional distribution. Qualitatively, all results are preserved under the normal distribution. Quantitatively, second moments of most variables are not significantly changed. However, the volatility of the risk premium is less when normal distribution is considered than when the t-distribution is considered: standard deviations of the risk premium in the benchmark model for a monthly and quarterly frequency are 0.07 and 0.21 percent, respectively while those are 0.06 and 0.13 percent respectively, under the normal distribution.

5 Properties of Predictable Returns

In this section, we analyze properties of simulated spot and forward exchange rates by investigating whether or not our model of rational expectation foreign exchange rate risk premium also produces negative values of the slope coefficient from the regression of the exchange rate depreciation on the forward premium. Further, since ex ante predictable returns which are interpreted as the counterpart of the theoretical risk premium are estimated from the regression (3-5), we check whether or not these excess returns are a reasonable candidate for measuring the theoretical risk premium. By using artificial data that reflect unobservable expectations we can decompose the estimate of the slope coefficient from the regression (3-5) into two parts: one is mainly derived from the covariance of theoretical risk premium with the forward premium and the other is derived from a small sample bias. A small sample bias is entirely due to sampling variations in all experiments in this paper since investors' information sets are assumed to be same as that of the econometrician. Hence, this decomposition enables us to investigate roles of the risk premium in the regression while controlling for other factors such as the rationality of expectations and a small sample bias.

Let $\widehat{a}_2$ be the estimate of a_2 in the regression (3-5). Then, we have

$$\begin{aligned}\widehat{a}_2 = \frac{\widehat{Cov}[f_t - e_t, f_t - e_{t+1}]}{\widehat{Var}[f_t - e_t]} &= \frac{\widehat{Cov}[f_t - e_t, f_t - e_t - E_t[e_{t+1} - e_t] - (e_{t+1} - E_t[e_{t+1}])]}{\widehat{Var}[f_t - e_t]} \\ &= \frac{\widehat{Cov}[f_t - e_t, E_t[e_{t+1}] - e_{t+1}] + \widehat{Var}[rp_t] + \widehat{Cov}[E_t[e_{t+1}] - e_t, rp_t]}{\widehat{Var}[f_t - e_t]}. \quad (5\text{-}1)\end{aligned}$$

If the expectation is rational, $e_{t+1} - e_t = E_t[e_{t+1}] - e_t + v_{t+1}$. v_{t+1} is a forecasting error and not correlated with $E_t[e_{t+1}] - e_t$. From equation (2-15), we know that $E_t[e_{t+1} - e_t] = f_t - e_t - rp_t$, where $rp_t = f_t - E_t[e_{t+1}]$. We can rewrite equation (5-1) in the following way

$$\widehat{a}_2 = \widehat{a}_2^{rp} + \widehat{a}_2^{ss} \quad (5\text{-}2)$$

where $\widehat{a}_2^{rp} = \frac{\widehat{Cov}[rp_t, E_t[e_{t+1} - e_t]] + \widehat{Var}[rp_t]}{\widehat{Var}[f_t - e_t]}$ and $\widehat{a}_2^{ss} = \frac{\widehat{Cov}[f_t - e_t, E_t[e_{t+1}] - e_{t+1}]}{\widehat{Var}[f_t - e_t]}$. If $\widehat{a}_2$ is the consistent estimator, $\widehat{a}_2^{ss}$ becomes zero as the sample size increases. The relation between $\widehat{a}_2$ and $\widehat{a}_4$ from the complementary regression is $\widehat{a}_4 = 1 - \widehat{a}_2$. Hence, if $\widehat{a}_2$ is zero then the estimator of the slope coefficient from the regression of exchange rate depreciation on the forward premium should be one. If it is greater than one then the estimator of the slope coefficient should be negative. It is possible that the estimator $\widehat{a}_2$ from actual data can affected either entirely or partially by $\widehat{a}_2^{ss}$ since $\widehat{a}_2$ consists of $\widehat{a}_2^{rp}$ and $\widehat{a}_2^{ss}$ [see, Lewis (1989a and 1989b) and Evans and Lewis (1995)]. Since considering this possibility is beyond the scope of the paper, we only consider the case in which $\widehat{a}_2^{ss}$ should be related to sampling variations and should approach to zero as the sample size increases.

Now, let us examine the relation between the fitted values from the regression (3-5) and the theoretical risk premium since we measure the standard deviation of the fitted values $|\widehat{a}_2| sd(f_t - e_t)$ as the counterpart of the theoretical risk premium. Suppose for a moment $\widehat{a}_2^{ss} = 0$. Then, the covariance between fitted values in regression (3-5) and the risk premium is $\widehat{a}_2^{rp} Cov[(f_t - e_t), rp_t]$. As one can see equation (4-4), the forward premium contains the risk premium as well as the difference between home and foreign money growth rates. Since the risk premium is negatively related to gd_t in equation (4-4), the correlation between the fitted values and the theoretical risk premium could be less than 1. Further, since $\widehat{a}_2^{ss}$ is uncorrelated with the risk premium, the correlation would be much smaller in the presence of sampling variations. Furthermore, the following equation shows that the two complementary regressions (3-5) and (3-6) may not be correctly specified:

$$\frac{Var[fitv_t]}{Var[fitve_t]} = \left(\frac{\widehat{a}_2^{rp}}{1 - \widehat{a}_2^{rp}}\right)^2 = \frac{\widehat{Var}[rp_t] + \widehat{Cov}[rp_t, E_t[e_{t+1} - e_t]]}{\widehat{Var}[E_t[e_{t+1} - e_t]] + \widehat{Cov}[rp_t, E_t[e_{t+1} - e_t]]} \quad (5\text{-}3)$$

where $fitv$ represents fitted values from the regression (3-5) and $fitve$ represents fitted values from the regression (3-6). If either $\widehat{Cov}[rp_t, E_t[e_{t+1} - e_t]] \neq 0$ or $\widehat{Var}[E_t[e_{t+1} - e_t]] \neq \widehat{Var}[rp_t]$ then $\frac{Var[fitv_t]}{Var[fitve_t]}$ is not equal to the theoretical counter parts $\frac{Var[rp_t]}{Var[E_t[e_{t+1}-e_t]]}$.

We now investigate how much the variance of the fitted values in the regression (3-5) differs from that of the theoretical risk premium. The variance of the fitted values can be written in the following way

$$Var[fitv_t] = Var[\widehat{a}_1 + \widehat{a}_2(f_t - e_t)] = \widehat{a}_2^{2rp} Var[f_t - e_t] = \widehat{a}_2^{rp}[Cov[rp_t, E_t[e_{t+1} - e_t]] + Var[rp_t]] \tag{5-4}$$

where $fitv$ represents fitted values and $\widehat{a}_2^{ss}$ is assumed to be zero. Then, the difference between the variance of the fitted values and the risk premium is

$$\begin{aligned} Var[fitv_t] - Var[rp_t] &= \widehat{a}_2^{rp}(\widehat{a}_2^{rp} - 1)Var[f_t - e_t] + Cov[rp_t, E_t[e_{t+1} - e_t]] \\ &= \frac{Cov[rp_t, E_t[e_{t+1} - e_t]]^2 - Var[rp_t]Var[E_t[e_{t+1}] - e_t]}{Var[f_t - e_t]} \end{aligned} \tag{5-5}$$

where rp_t represents the theoretical risk premium and the second inequality in equation (5-5) is derived using the definition of $\widehat{a}_2^{2rp}$. If the regression (3-5) is correctly specified then $Var[fitv_t] - Var[rp_t]$ should always be zero. It is possible if the risk premium is zero. However, in the presence of the risk premium, $Cov[rp_t, E_t[e_{t+1} - e_t]]$ should be greater than or equal to 0 when $0 \leq \widehat{a}_2^{rp} \leq 1$ in order for the theoretical risk premium to be exactly estimated. Hence, it is possible that the risk premium can be either underestimated or overestimated if the regression (3-5) is not correctly specified. For example, if the expected depreciation $Var[E_t[e_{t+1} - e_t]]$ is zero then $\widehat{a}_2^{2rp} = 1$ because $Cov[rp_t, E_t[e_{t+1} - e_t]]$ is zero. However, the reverse is not necessarily true: $\widehat{a}_2^{rp} = 1$ does not mean $Var[E_t[e_{t+1} - e_t]] = 0$. Rather, it only implies that $Var[E_t[e_{t+1} - e_t]] = -Cov[rp_t, E_t[e_{t+1} - e_t]]$. This equality can be achieved either because the expected depreciation is zero or the covariance of the risk premium and the expected depreciation is negative. Therefore, it is possible that the risk premium will be either underestimated or overestimated depending on the sign of $Cov[rp_t, E_t[e_{t+1} - e_t]]$. Equation (5-5) says that the theoretical risk premium would be underestimated if $0 \leq \widehat{a}_2^{rp} \leq 1$ and $Cov[rp_t, E_t[e_{t+1} - e_t]] < 0$. Our quantitative results below show that this is most likely since $Cov[rp_t, E_t[e_{t+1} - e_t]] < 0$ in our model, it may be underestimated. But it is possible that the theoretical risk premium is overestimated if $\widehat{a}_2^{ss}$ is included:

$$\begin{aligned} Var[fitv_t] - Var[rp_t] = \widehat{a}_2^{rp}(\widehat{a}_2^{rp} - 1)Var[f_t - e_t] + Cov[rp_t, E_t[e_{t+1} - e_t]] \\ + 2\widehat{a}_2^{rp}\widehat{a}_2^{ss}Var[f_t - e_t] + \widehat{a}_2^{2ss}Var[f_t - e_t]. \end{aligned} \tag{5-6}$$

$2\widehat{a}_2^{rp}\widehat{a}_2^{ss}Var[f_t - e_t] + \widehat{a}_2^{2ss}Var[f_t - e_t]$ is positive as long as $\widehat{a}_2^{rp}$ and $\widehat{a}_2^{ss}$ have the same sign. Since our simulation inevitably involves sampling variations we quantitatively investigate how severe these variations are.

For this purpose, we first simulate 100 times the benchmark model with 100 periods for a quarterly frequency and with 300 periods for a monthly frequency in order to generate artificial forward and spot exchange rates. Then, we run the regression (3-6) 100 times independently and decompose the estimator into two parts, $\widehat{a}_2^{rp}$ and $\widehat{a}_2^{ss}$. We, further, simulate 100 times the model now with 10000 time periods for both a quarterly frequency and a monthly frequency in order to reduce sampling variations. The estimator reported in Table 7 is the average of these 100 estimators for each case. Data used in these regressions are generated from our benchmark model with the 4 different combinations on sample sizes and data frequencies: the first two columns represent quantities with different number of time periods, 300 vs 10000, respectively, for a monthly frequency while the rest two columns represent quantities, 100 vs 10000, respectively for a quarterly frequency.

All results related to issues in this section are reported in Table 7. In general, our benchmark model can generate negative values of the slope coefficient from the regression (3-6) except the case in which the quarterly decision interval is considered in a large sample. When data are generated in the benchmark model with a monthly frequency, averages of 100 estimators of the slope coefficients from the regression (3-6) are -3.09 when the sample size is 300 and -4.35 when it is 10000, respectively. For a quarterly frequency means of 100 estimators are -0.50 when the sample size is 100 and 0.47 when it is 10000. However, these results are significantly affected by sampling variations. Further, in general, $\widehat{a}_2^{rp}$ is strictly less than one in our experiments except cases with the monthly frequency.[21] Consequently, we fail in most cases to reject the null hypothesis that the slope coefficient is one when data are generated quarterly while we reject in most cases for a monthly frequency: we fail to reject 91 times for a small sample and 81 times for a large sample of 100 regressions, respectively, for a quarterly frequency while 65 times for a small sample and only 7 times for a large sample, respectively, for a monthly frequency. As one can see in the definition of $\widehat{a}_2^{rp}$, if $\widehat{a}_2^{rp}$ is less than 0.5 then $Var(rp_t) < Var(E_t[e_{t+1} - e_t])$ while if $\widehat{a}_2^{rp}$ is greater than 0.5 then $Var(rp_t) > Var(E_t[e_{t+1} - e_t])$. Our quantitative results predict these volatility relations: when monthly decision intervals are considered $Var(rp_t) > Var(E_t[e_{t+1} - e_t])$ while quarterly decision intervals are considered $Var(rp_t) < Var(E_t[e_{t+1} - e_t])$. As one can see in Table 7, the magnitude of $\widehat{a}_2^{ss}$ is not significantly reduced even for a large sample size when the monthly decision interval is considered because the variance of the forward premium is too small to make $\widehat{a}_2^{ss}$ be zero even if the covariance of the forward premium and forecasting errors is close to zero.

Our results show that the risk premium is generally not closely related to the fitted values from the regression (3-5): the correlations between two quantities are 0.21 for a monthly frequency and 0.02 for a quarterly frequency when the sample size is small while it is 0.08 for a quarterly frequency when the sample size is large. However, the correlation significantly increases when the sample size increases for a monthly frequency: the correlation is 0.75 in this case. Since these correlations could be contaminated by sampling variations we also report correlations of the risk premium with the decomposition of the fitted values: correlations between the two quantities when sampling variations are eliminated are 0.49 for a monthly frequency and 0.19 for a quarterly frequency when the sample size is small while those are 0.78 for a monthly frequency and 0.13 for a quarterly when the sample is large.

[21] The average of $\widehat{a}_2^{rp}$ is a bit less than 1 in the case for the combination of monthly frequency with a small sample. This is because $\widehat{a}_2^{rp}$ vary a lot across each regression in the direction that the average is downward.

Although these values are much greater than those in the presence of sampling variations, the relation between the theoretical risk premium and the fitted values are still weak, in particular when the sample size is small. Consequently, fitted values from the regression (3-5) tend to overestimate the theoretical risk premium: the standard deviation of predicted returns from the regression (3-5) is 0.25 percent for a monthly frequency while it is 0.65 for a quarterly frequency when the sample size is small. And these values are much greater than those of the theoretical risk premium because of sampling variations and close to those obtained from the regressions using actual data. As one can see in Panel E in Table 7 most differences between two quantities are due to sampling variations. Note that when sampling variations are excluded, the fitted values tend to underestimate the risk premium because $Cov[rp_t, E_t[e_{t+1} - e_t]] < 0$ as well as $0 \leq \widehat{a}_2^{rp} \leq 1$. As we already discussed above, this is possible if the regression (3-5) is not correctly specified: when $0 \leq \widehat{a}_2^{rp} \leq 1$, the sign of $Cov[rp_t, E_t[e_{t+1} - e_t]] < 0$ could be either negative or positive.

One thing to be noted is that as the sample size increases magnitudes of the risk premium, the forward premium, and the expected depreciation significantly increase compared to those of exchange rate depreciation and consumption. Those are closely matched with data. As can be seen in Table 7, all above results holds true when the distribution of stochastic disturbances of money growth rates is assumed to be normal.

6 Summary and Concluding Remarks

This paper investigates the behavior of the foreign exchange rate risk premium in a DSGE model with nominal rigidities, goods market imperfections, and a money-in-the-utility function model by introducing the same driving force that generates highly volatile nominal and real exchange rates in recent quantitative studies. Our study provides evidence that nominal rigidities play a role in generating variable risk premium as well as real exchange rate and improving other aspects on patterns of exchange rates and the risk premium. Our model under assumptions of staggered nominal contracts and deviations from PPP can produce the volatility relations obtained from Fama's regression test: for a monthly frequency the variance of risk premium is greater than the sum of variances of the expected depreciation and the forward premium, although variation of the forward premium is much smaller than those observed in the data. Further, our model can generate the negative covariance of the expected depreciation and the risk premium. However, we fail to produce one of two necessary conditions for obtaining negative values of the slope coefficient in the regression for a quarterly frequency. When the time varying monetary shocks are considered, selecting a data frequency plays an important role: standard deviations of the time varying risk premium, the forward premium, and the expected depreciation are about three times, nine times, and seven times greater for a quarterly frequency than those for a monthly frequency. Further, volatilities of the forward premium and the expected depreciation are much more reduced than that of the risk premium because increase in the degree of the discount factor reduce relative effects of money growth rates on these variables.

Our results show that the standard sticky price DSGE model also performs better than do flexible prices models in explaining the patterns of the risk premium in other dimensions. First, the sticky price model generates more variable risk premium, which does not rely on Jensen's inequality terms, than do flexible price models. As Engel (1992) pointed out, under the assumption of independence of monetary shocks and real shocks in flexible price models, the risk premium relies entirely on Jensen's inequality terms since monetary shocks do not affect consumption while real shocks do not affect nominal exchange rates. Since empirical studies report that magnitudes of Jensen's inequality terms are smaller than that of the risk premium to be explained, variations of the true risk premium which are not relying on Jensen's inequality terms should be found from the covariance between real shocks and monetary shocks in models with flexible prices. However, empirical studies also document that the magnitude of these covariance terms tend to be too small to explain fluctuations of the risk premium observed in the data. Our model with nominal rigidities avoids this problem. Second, in contrast to flexible price models, we explicitly examine effects of deviations of PPP on the risk premium: real exchange rate risks in our model play a role in the determination of the risk premium, the forward premium, and the expected depreciation. They raise variations of the risk premium but reduce those of the expected depreciation and the forward premium. Third, under the assumptions of nominal rigidities, we can also investigate roles of staggered nominal contracts in the determination of these quantities. Our results show staggered nominal contracts significantly affect variations of these three quantities in the same way as real exchange rate risks are present. However, when PPP holds, staggered contracts do much affect variations of both the forward premium and the expected depreciation.

We also show that the regression of ex post excess returns on the forward premium in the literature may not be correctly specified. Our experiments using simulated data report that the relation between the fitted values from the regression of predictable returns and the theoretical risk premium is not closely related with each other even while controlling for sampling variations. Consequently, standard deviations of fitted values from the regression of predictable returns on the forward premium are much different from those of the risk premium: standard deviations of fitted values tend to be big in our simulations because of sampling variations.

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Table 1
The Benchmark Parameter Values

$\beta = 0.96^{1/N}$	($N = 4$ for 1 quarterly frequency and $N = 12$ for a monthly frequency)
$\alpha = 0.15$	Labor share
$\nu = 10$	Elasticity of substitution within each country
$\theta = 1.0$	Elasticity of substitution across countries
$\mu = 2.0$	Elasticity of labor supply
$\zeta = 4$	Elasticity of substitution between labor skills
$\gamma = 6.0$	Risk aversion coefficient
$\phi = 6.0$	Inverse of interest elasticity of money demand
$h = 0.85$	Home bias in preferences
$b = 0.63$	habit persistence

Table 2
Diagnostic Tests on the US Monthly and Quarterly M1 Growth Rates

	Monthly	Quarterly
	$g_{t+1} = \rho_m g_t + um_{t+1}$	
ρ_m	0.59	0.76
	(0.04)	(0.05)
Skewness	-2.00	0.16
Kurtosis	35.21	0.38
$Q(20)$	90.36	87.43
$LM(12)$	88.96	42.44
$ARCH(12)$	127.42	50.77
$Q^2(20)$	115.88	83.75

Estimations for the US Monthly and Quarterly M1 Growth Rates

	Monthly	Quarterly
ρ_m	0.76	0.83
SE	(0.03)	(0.05)
Variance Equation		
constant	0.00	0.00
SE	(0.00)	(0.00)
ARCH	0.29	0.25
SE	(0.09)	(0.09)
GARCH	0.68	0.56
SE	(0.12)	(0.15)
Adjusted R^2	-0.07	0.12
Sd(um)	0.0057	0.01
No. obs	528	176

The money supply processes in equations (3.3) and (3.4) are used for the estimations for monthly and quarterly M1 data. $Q(20)$ represents the Ljung and Box test statistic for 20th-order serial correlation, $LM(12)$ represents Lagrange Multiplier static used for testing serial correlation, $ARCH(12)$ represents ARCH LM test statistic for testing autoregressive conditional heteroskedasticity (ARCH), and $Q^2(20)$ computed based on the squared residuals is used for testing ARCH in the residuals.

Table 3
Summary Statistics for Forward and Spot Exchange Rate

	Quarterly					Monthly				
Statistic	Canada	France	Germany	Japan	UK	Canada	France	Germany	Japan	UK
Panel A: Return from Currency Speculation $f_t - e_{t+1}$										
Mean	0.0001	0.0043	0.0022	0.0082	0.0040	0.0000	-0.0011	-0.0006	-0.0026	-0.0012
Std deviation	0.0221	0.0582	0.0608	0.0674	0.0694	0.0128	0.0334	0.0346	0.0339	0.0356
Autocorrelation	0.069	0.228	0.075	0.244	0.039	0.082	0.013	-0.013	0.091	0.102
Panel B: The Log of Depreciation Rate $e_{t+1} - e_t$										
Mean	0.0040	0.0018	-0.0073	-0.0152	0.0029	-0.0015	-0.0008	0.0023	0.0049	-0.0011
Std deviation	0.0212	0.0571	0.0597	0.0644	0.0674	0.0125	0.0331	0.0343	0.0333	0.0351
Autocorrelation	0.071	0.207	0.042	0.220	-0.002	0.063	-0.006	-0.026	0.091	0.079
Panel C: The Log of Forward Premium $f_t - e_t$										
Mean	0.0041	0.0061	-0.0051	-0.0070	-0.0069	-0.0014	-0.0022	0.0017	0.0024	-0.0023
Std deviation	0.0038	0.0079	0.0087	0.0077	0.0079	0.0015	0.0031	0.0030	0.0027	0.0027
Autocorrelation	0.876	0.777	0.927	0.903	0.840	0.916	0.773	0.968	0.954	0.943
Panel D: Change in the Log of Real exchange rates $rq_{t+1} - rq_t$										
Std deviation	0.0216	0.0614	0.0663	0.0647	0.0669					
Autocorrelation	0.0778	0.1138	-0.0124	0.1462	-0.0295					

Table 4
Estimations for Predicted Returns

	Quarterly					Monthly				
Statistic	Canada	France	Germany	Japan	UK	Canada	France	Germany	Japan	UK
Panel A: $f_t - e_{t+1} = a + b(f_t - e_t)$										
a	-0.007	-0.005	0.009	0.035	-0.015	0.003	0.002	-0.003	-0.010	0.006
	(0.004)	(0.008)	(0.008)	(0.010)	(0.010)	(0.001)	(0.003)	(0.003)	(0.003)	(0.003)
b	1.842	1.465	1.394	3.842	2.730	2.183	1.449	1.653	3.291	2.954
	(0.639)	(0.839)	(0.797)	(0.924)	(0.983)	(0.551)	(0.701)	(0.758)	(0.811)	(0.842)
Adjusted R^2	0.090	0.027	0.027	0.180	0.083	0.061	0.014	0.016	0.064	0.048
No. obs	75	75	75	75	75	227	227	227	227	227
Panel B: $e_{t+1} - e_t = a + b(f_t - e_t)$										
a	0.007	0.005	-0.009	-0.035	0.015	-0.003	-0.002	0.003	0.010	-0.006
	(0.004)	(0.008)	(0.008)	(0.010)	(0.010)	(0.001)	(0.003)	(0.003)	(0.003)	(0.003)
b	-0.842	-0.465	-0.394	-2.842	-1.730	-1.183	-0.449	-0.653	-2.291	-1.954
	(0.639)	(0.839)	(0.797)	(0.924)	(0.983)	(0.551)	(0.701)	(0.758)	(0.811)	(0.842)
Adjusted R^2	0.010	-0.010	-0.010	0.103	0.028	0.016	-0.003	-0.001	0.023	0.019
No. obs	75	75	75	75	75	227	227	227	227	227
Panel C: Fitted values from regression (3-5): $\widehat{a}_1 + \widehat{a}_2(f_t - e_t)$										
Std deviation	0.0070	0.0116	0.0122	0.0295	0.0215	0.0033	0.0046	0.0050	0.0089	0.0081
Autocorrelation	0.754	0.692	0.909	0.820	0.737	0.844	0.693	0.959	0.922	0.908
Panel D: Fitted values from regression (3-6); $\widehat{a}_3 + \widehat{a}_4(f_t - e_t)$										
Std deviation	0.0032	0.0037	0.0034	0.0218	0.0136	0.0018	0.0014	0.0020	0.0062	0.0054
Autocorrelation	0.913	0.719	0.985	0.866	0.745	0.913	0.742	0.982	0.947	0.912
Panel E: Forward Premium $f_t - e_t$										
Std deviation	0.0038	0.0079	0.0087	0.0077	0.0079	0.0015	0.0031	0.0030	0.0027	0.0027
Autocorrelation	0.876	0.777	0.927	0.903	0.840	0.916	0.773	0.968	0.954	0.943

Table 5
Quarterly Results

Statistics	PCP1	PCP3	Bench	Habit	Home	USP3	INC1	Low disc	High elas
				Panel A: Standard Deviation					
rp w/oJ	0.0012	0.0014	0.0020	0.0020	0.0017	0.0018	0.0011	0.0016	0.0019
Jensen	0.0000	0.0000	0.0018	0.0018	0.0012	0.0017	0.0012	0.0016	0.0018
rp_t	0.0012	0.0014	0.0021	0.0020	0.0018	0.0020	0.0012	0.0018	0.0020
Forward Premium	0.0052	0.0052	0.0026	0.0025	0.0031	0.0028	0.0069	0.0041	0.0027
Expected Depreciation	0.0053	0.0054	0.0034	0.0033	0.0036	0.0034	0.0071	0.0045	0.0035
Consumption	0.0078	0.0072	0.0108	0.0064	0.0088	0.0097	0.0097	0.0058	0.0107
NER	0.0640	0.0640	0.0721	0.0723	0.0692	0.0706	0.0587	0.0670	0.0717
RER	0.0000	0.0000	0.0810	0.0810	0.0547	0.0676	0.0829	0.0752	0.0811
				Panel B: Autocorrelation					
Forward Premium	0.783	0.783	0.921	0.921	0.924	0.943	0.730	0.902	0.919
Rp (w/o Jensen)	0.697	0.697	0.681	0.681	0.687	0.684	0.680	0.681	0.681
NER	0.040	0.040	-0.009	-0.009	0.045	-0.002	-0.013	-0.013	-0.010
RER	Na	Na	-0.041	-0.042	-0.048	-0.043	-0.492	-0.044	-0.044
				Panel C: Cross correlation					
$(E_t[e_{t+1} - e_t], rp_t)$	-0.158	-0.186	-0.527	-0.533	-0.418	-0.479	-0.144	-0.349	-0.508
(NER, RER)	Na	Na	0.892	0.894	0.879	0.886	0.712	0.892	0.885
				Panel D: Mean					
Hrp w/o Jensen	0.0000	0.0000	-0.0021	-0.0021	-0.0013	-0.0017	-0.0013	-0.0017	-0.0021
Frp w/o Jensen	0.0000	0.0000	0.0021	0.0021	0.0014	0.0017	0.0013	0.0017	0.0021
Jensen	0.0000	0.0000	0.0027	0.0027	0.0017	0.0026	0.0018	0.0023	0.0026
				Panel E: Fitted Values from regression (3-5)					
Std	0.0074	0.0074	0.0064	0.0064	0.0075	0.0072	0.0060	0.0059	0.0064

Unconditional variance of stochastic disturbances of home and foreign monetary shocks is set to be 0.01^2. 'rp' represents the time varying risk premium and (w/oJ) means excluding Jensen's inequality terms. 'Hrp' represent the time varying home risk premium while 'Frp' represent time varying foreign risk premium. All benchmark parameter values are used in these experiments. 'PCP' means the case in which PPP holds ('1' for one period in advance nominal contract and '3' for three period staggered nominal contracts). 'Bench' means the benchmark model and 'Habit' means b=0.63. 'Home' means the cases in which there exists home bias in preferences (h=0.85) while prices are set for 3 periods in a staggered way. 'USP' means the case in which only the US dollars are used in international goods trade. 'INC' represent the case in which asset markets are incomplete. 'Low disc' represents $\beta = 0.92$ and 'High elas' represents $\theta = 3$. Standard deviations of predictable returns and the forward premium for the Canadian dollar against the US dollar are used in the data while standard deviations of nominal and real exchange rates are the median value of the currencies in the sample.

Table 6
Monthly Results

Statistics	PCP1	PCP3	Bench	Habit	Home	USP3	INC1	Low disc	High elas
Panel A: Standard Deviation									
rp w/oJ	0.0004	0.0005	0.0006	0.0006	0.0006	0.0006	0.0005	0.0006	0.0006
Jensen	0.0000	0.0000	0.0006	0.0006	0.0004	0.0006	0.0006	0.0006	0.0006
rp_t	0.0004	0.0005	0.0007	0.0007	0.0006	0.0007	0.0006	0.0007	0.0007
Forward Premium	0.0006	0.0006	0.0003	0.0003	0.0003	0.0003	0.0005	0.0005	0.0003
Expected Depreciation	0.0006	0.0007	0.0005	0.0005	0.0005	0.0005	0.0006	0.0006	0.0005
Consumption	0.0030	0.0025	0.0041	0.0024	0.0033	0.0037	0.0043	0.0039	0.0041
NER	0.0250	0.0252	0.0269	0.0269	0.0263	0.0265	0.0255	0.0258	0.0268
RER	0.0000	0.0000	0.0302	0.0301	0.0208	0.0254	0.0362	0.0290	0.0303
Panel B: Autocorrelation									
Forward Premium	0.752	0.754	0.893	0.891	0.921	0.937	0.705	0.871	0.892
Rp (w/o Jensen)	0.857	0.857	0.859	0.859	0.861	0.860	0.858	0.860	0.859
NER	0.004	0.003	-0.015	-0.015	-0.010	-0.013	-0.014	-0.016	-0.015
RER	Na	Na	-0.047	-0.048	-0.057	-0.051	-0.508	-0.047	-0.049
Panel C: Cross correlation									
$(E_t[e_{t+1} - e_t], rp_t)$	-0.266	-0.296	-0.776	-0.785	-0.668	-0.756	-0.463	-0.512	-0.753
(NER, RER)	Na	Na	0.896	0.898	0.888	0.893	0.712	0.895	0.889
Panel D: Mean									
Hrp w/o Jensen	0.0000	0.0000	-0.0003	-0.0003	-0.0002	-0.0002	-0.0003	-0.0003	-0.0003
Frp w/o Jensen	0.0000	0.0000	0.0003	0.0003	0.0002	0.0002	0.0003	0.0003	0.0003
Jensen	0.0000	0.0000	0.0004	0.0004	0.0003	0.0004	0.0004	0.0004	0.0004
Panel E: Fitted Values from regression (3-5)									
Std	0.0019	0.0019	0.0025	0.0025	0.0023	0.0025	0.0019	0.0022	0.0024

Unconditional variance of stochastic disturbances of home and foreign monetary shocks is set to be 0.01^2. 'rp' represents the time varying risk premium and (w/oJ) means excluding Jensen's inequality terms. 'Hrp' represent the time varying home risk premium while 'Frp' represent time varying foreign risk premium. All benchmark parameter values are used in these experiments. 'PCP' means the case in which PPP holds ('1' for one period in advance nominal contract and '3' for three period staggered nominal contracts). 'Bench' means the benchmark model and 'Habit' means b=0.63. 'Home' means the cases in which there exists home bias in preferences (h=0.85) while prices are set for 3 periods in a staggered way. 'USP' means the case in which only the US dollars are used in international goods trade. 'INC' represent the case in which asset markets are incomplete. 'Low disc' represents $\beta = 0.92$ and 'High elas' represents $\theta = 3$. Standard deviations of predictable returns and the forward premium for the Canadian dollar against the US dollar are used in the data while standard deviations of nominal and real exchange rates are the median value of the currencies in the sample.

Table 7

Properties of Predicted Returns (t distribution with df 5)

Statistics	Monthly		Quarterly	
	300 periods	10000 periods	100 periods	10000 periods
Panel A: Standard Deviation				
rp w/oJ	0.0006	0.0015	0.0020	0.0034
Jensen	0.0006	0.0018	0.0018	0.0030
rp_t	0.0007	0.0020	0.0021	0.0034
Forward Premium	0.0003	0.0007	0.0026	0.0029
Expected Depreciation	0.0005	0.0014	0.0034	0.0042
Consumption	0.0041	0.0050	0.0108	0.0113
NER	0.0269	0.0356	0.0725	0.0741
Panel B: $f_t - e_{t+1} = a_1 + a_2(f_t - e_t) + v_{t+1}$				
$std(fitv)$	0.0025	0.0041	0.0065	0.0010
$\widehat{a}_2$	4.0899	5.3478	1.5017	0.2316
$\widehat{a}_2^{rp}$	0.9398	2.0084	0.0483	0.1924
$\widehat{a}_2^{ss}$	3.1501	3.3393	1.4533	0.0392
Panel C: $e_{t+1} - e_t = a_3 + a_4(f_t - e_t) + u_{t+1}$				
$std(fitve)$	0.0023	0.0034	0.0060	0.0023
Panel D: Cross Correlation				
$(E_t[e_{t+1} - e_t], rp_t)$	-0.7768	-0.9403	-0.5284	-0.6944
$(fitv, rp_t)$	0.2052	0.7537	0.0219	0.0836
$(\widehat{a}_2^{rp}(f_t - e_t), rp_t)$	0.4927	0.7840	0.1916	0.1317
$(\widehat{a}_2^{ss}(f_t - e_t), rp_t)$	0.1715	0.5801	-0.0074	0.0146
Panel E: $Var(fitv) - Var(rp_t)$				
$\widehat{a}_2^{rp}$	-0.0481E-5	-0.0729E-5	-0.7285E-5	-1.3526E-5
$\widehat{a}_2^{ss}$	1.3579E-5	2.1633E-5	7.0251E-5	0.0833E-5
Properties of Predicted Returns (Normal distribution)				
Panel A: Standard Deviation				
rp w/oJ	0.0005	0.0013	0.0013	0.0016
Jensen	0.0006	0.0013	0.0012	0.0014
rp_t	0.0006	0.0015	0.0013	0.0016
Forward Premium	0.0003	0.0006	0.0027	0.0029
Expected Depreciation	0.0005	0.0010	0.0030	0.0032
Consumption	0.0043	0.0050	0.0121	0.0111
NER	0.0293	0.0340	0.0728	0.0733
Panel B: $f_t - e_{t+1} = a_1 + a_2(f_t - e_t) + v_{t+1}$				
$std(fitv)$	0.0021	0.0023	0.0058	0.0007
$\widehat{a}_2$	4.6193	3.6197	1.2473	0.0857
$\widehat{a}_2^{rp}$	0.9705	1.8107	0.0097	0.0380
$\widehat{a}_2^{ss}$	3.6478	1.8090	1.2376	0.0477
Panel C: $e_{t+1} - e_t = a_3 + a_4(f_t - e_t) + u_{t+1}$				
$std(fitve)$	0.0020	0.0018	0.0054	0.0026
Panel D: Cross Correlation				
$(E_t[e_{t+1} - e_t], rp_t)$	-0.7795	-0.9403	-0.3834	0.0212
$(fitv, rp_t)$	0.2128	0.7537	0.0242	0.0212
$(\widehat{a}_2^{rp}(f_t - e_t), rp_t)$	0.5204	0.7840	0.1979	0.1312
$(\widehat{a}_2^{ss}(f_t - e_t), rp_t)$	0.1512	0.5801	0.0088	-0.0007
Panel E: $Var(fitv) - Var(rp_t)$				
$\widehat{a}_2^{rp}$	-0.0246E-5	-0.0073E-5	-0.2098E-5	-0.0966E-5
$\widehat{a}_2^{ss}$	0.8669E-5	2.1633E-5	5.8035E-5	1.6650E-5