Abstract
The purpose of the paper is to investigate the effects of poverty and educational policies on school attendance, child labour and growth. We consider an OLG model, with parental educational choices. It is assumed there is a trade off between child labour and human capital accumulation. If parents don’t choose for quality of education, it is shown that a poverty trap may occur in the presence of a consumption subsistence or when the quality of education is inadequate.

A private education system, where schooling quality is endogeneized can improve growth and reduce child labour, and cycles may occur. A public education system does not generate cycles, but it can generate more easily a poverty trap. In this case, only subsidies would help to reduce poverty and, consequently, child labour.

Key words: education, child labour, consumption subsistence, growth, educational policies.

JEL : I20, J24, O11
1 Introduction

Many economists and international organizations see investment in education as a priority, through the social capabilities it creates\(^1\). An important question is what governments in developing countries can do to raise educational attainment. In many developing countries the number of school years is on average rather low, even if public expenditures may represent an important part of their GDP. This is due to a lack of educational infrastructures and to microeconomic and macroeconomic factors, such as poverty.

The purpose of the paper is to study the impact of poverty on educational choices, when there is a trade-off between human capital accumulation and child labour, and to compare different educational policies.

Child labour is an old and complex problem (see Basu, 1999 for a well documented survey). In 1996, at least 120 million of the world’s children under age 15 did full-time work, according to the ILO, 95 % are in developing countries, with half of these in Asia (excluding Japan). Among children going to school in developing countries, up to one-third of the boys and more than two-fifth of the girls are engaged in economic activities on a part-time basis\(^2\).

<table>
<thead>
<tr>
<th>TABLE 1. CHILD LABOUR: Number of children (below 15 years) working (in thousands)</th>
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<tr>
<td>World</td>
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In the battle against child labour, a set of laws have been tried, on national and supranational levels, through international organizations such as

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\(^1\)See Temple, 1999; Benhabib and Spiegel, 1994 (for its positive impact on technological diffusion)., Abramowitz, 1986.

\(^2\)These figures probably underestimate child labor, because the ILO statistics do not take into account domestic duties (as cooking, child care, ...).
the ILO, the WTO, and UNICEF, but the determinants of child labour are rather difficult to evaluate and the choice of an “international labour standard” is complex. In this context, enforcing restrictions on child labour through the use of trade sanctions can enhance poverty and do not necessarily solve the problem if it comes from bad economic conditions (Basu and Van, 1998)3.

In formal analysis, child labour is generally related to the modeling of household behaviors. Seminar works of Rosenzweig and Evenson (1977) or Parsons and Goldin (1989) tried to explain simultaneously decisions of consumption and child labour, linked to education or fertility, in simple statics framework, aimed to allow empirical testing. These models of child labour assume that parents take decisions for the whole household, by taking explicit account for the child and for the effects of total earnings changes on consumption. But the impact of a trade off between child labour and education on growth is often ignored in the literature, with few exceptions like Glomm (1997) and Baland and Robinson (2000). Baland and Robinson (2000) show that child labour is inefficient. But these authors don’t study the causal links between poverty, schooling and child labour. Glomm (1997) consider two educational regimes, and their respective impact on human capital accumulation, but doesn’t consider the effects on poverty in the process.

The purpose of the paper is to study the dynamic implications of poverty and child labour, and their relations to human capital accumulation. In fact, empirical studies find evidence that poverty can compel parents to keep children away from school (see for example Jensen and Nielsen, 1997). Psacharopoulos (1997), using data from Latin America, shows that child labour and schooling attainment are substitutes. The paper contributes to study how poverty can generate a development trap, by inhibiting schooling. Poverty is considered in the model through the concept of poverty lines, that is, according to the World Bank (1990), the income or budget necessary to purchase a subsistence basket of goods ($1 a day or $2 a day). A subsistence consumption is introduced in the model, that represents a poverty line used to identify the part of the population regarded as absolutely poor. As noted by Steger (2000), subsistence consumption denotes a standard of living that

3The question of child labour regulation and of the choice of an international labour standard is complex. Basu (2000) shows for instance that a minimum wage legislation in developing countries as a form of international labour standard presents the risk to exacerbating child labour. On the contrary, child labour regulation may have positive effects if it reduces fertility, and then the opportunity costs of education (Doepke, 2003, Doepke and Zilibotti, 2003).
allows for the satisfaction of the minimum, physical and mental, basic needs of life. In growth literature, the requirement of subsistence consumption is considered to restrict ability to save. Azariadis (1996) shows that it represents one of the potential causes of poverty traps, and Basu and Van (1998) that it can generate multiple equilibria in a labour market with both child and adult labour.

If poverty is the major source of child labour, an increase of growth and of family income would reduce child labour. Edmonds and Turk (2002) find for example a strong correlation between living standards improvements and decline in child labour during the 1990s in Vietnam. The question addressed in the paper is how educational policies may contribute to enhance education and reduce child labour, if the economy lies in a poverty trap.

A simple OLG model with educational parental choices is built in order to study the effects of poverty on human capital accumulation. The meaning of the requirement of subsistence consumption for intertemporal consumption decisions is formalized by means of an intertemporal Stone-Geary utility function. All agents are considered to be identical, and two polar educational regimes are considered: a private education regime and a public education regime, in an economy where children are useful as income-earning assets. As noted by the UNESCO (2002) “The goals of expanding education systems and maintaining equitable access to education are inextricably linked to questions of education finance” (p. 12). The level of public and private investment in education varies largely among developing countries, from 1.2 per cent of GDP in Indonesia to 9.9 per cent of GDP in Jamaica or 5.9 in the Philippines, according to the UNESCO (2002).

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1 Concrete actions, like the rise of school quality for example (Glewwe, 2002) or building subsidy programs, like the Food-for-Education program (FFE) established successfully in Bangladesh (Ravallion and Woden, 2000), may contribute to fight against poverty and child labor.
The majority of resources in developing countries are focused on primary and secondary education. Private spending often makes a substantial contribution to overall levels of education spending (see Table 2). In this context, it seems interesting to compare both regimes, and study their efficiency in terms of poverty reducing.

The rest of the paper is organized as follows. In Section 2 the model is described, under the assumption that parents don’t choose the quality of education. In the Section 3, quality of education is introduced and two polar educational regimes are compared: a private education regime and a public one. In Section 4 some concluding remarks are presented.

2 Model

A simple model of schooling choices is developed, in which identical individuals live for two periods. There is assumed that parents make decisions for their children, that seems to be realistic for primary and secondary education, in developing countries. Their objective is to maximize a utility function that has two arguments: consumption of goods and child cognitive skills (i.e. his or her human capital). For simplicity, it is assumed that there are two time periods and only one child per family. The size of population is normalized to one.

In period $t$, a child born and may attend school, work or both. If both, the child first goes to school and works after schooling is completed. When a child works, part or all of the child’s earnings is given to his or her parents. Parents have a Stone-Geary utility function, that involves a consumption
subsistence term $\ubar{c}$. This kind of utility function has been used by Rebelo (1992) in order to explain international differences in rates of growth.

The Stone-Geary utility function is:

$$U(c_t) = \ln(c_t - \ubar{c}) + \gamma \ln h_{t+1}, \quad (1)$$

where $c_t$ denotes consumption when old, $\gamma$ indicates parental tastes for educated children. Parents value educated children for two distincts reasons: educating children can increase parents’ consumption, and educating children directly affects parents’ utility (through $\gamma$). Stone-Geary preferences are a simple generalization of COBB-DOUGLAS preferences. There are strictly monotonic if $c_t > \ubar{c}$.

A simple human capital production function is assumed:

$$h_{t+1} = b\theta_t G(.) h_t^\beta, \quad (2)$$

where $b > 0$ is the “learning efficiency” of the child, $\theta_t$ the time devoted to school (i.e. the years of schooling) and $h_t$ the inherited human capital from parents, $\beta \in ]0, 1]$. The function $G$ is increasing with school quality, assumed to be exogenous in this section.

Family’s consumption in $t$ is given by:

$$c_t = y_t - p_t \theta_t + (1 - \theta_t)\lambda h_t, \quad (3)$$

where $p_t$ is the price of schooling, $y_t$ the parental income in periods $t$ and $t + 1$ respectively, $(1 - \theta_t)h_t$ the child’s incomes when working, and $\lambda$ is the fraction of that income given to the parents\(^5\). If $\lambda < 1$, there are frictions on the job market. Another way to think about $\lambda$ is to consider that a child produces a fraction $\lambda$ of an adult’s labour. In $t$ the family income is: $\Omega_t = y_t + (1 - \theta_t)\lambda h_t$. Educating children has a direct cost for the parents ($p_t \theta_t$) and an opportunity cost due to the time devoted to school by the child (i.e. $\theta_t\lambda h_t$). The results of the paper go through as long as the child income is a significant part of family income ($\lambda > 0$).

Consider the case where school quality is exogenous, so that $\theta_t$ is the only choice variable. From the first order condition, optimal years of schooling

\(^5\)Equation (3) rules out borrowing and saving. In fact, the study is concerned with primary and secondary education, for which credit markets to finance investments in human capital are seldom available (West, 1991). Note that in a general way, the introduction of borrowing and saving reduces parents’ incentive to invest in their children’s education, without completely removing this incentive if parents prefer to diversify their investments (Glewwe, 2002).
\[ \theta_t = \frac{\gamma [(1 + \lambda)h_t - \sigma]}{(1 + \gamma)(p_t + \lambda h_t)}. \]  

(4)

\( \theta_t > 0 \) if and only if \((1 + \lambda)h_t > \sigma\), in other words when child income is not absolutely necessary to get a sufficient income to survive, education is possible. On the contrary, when \((1 + \lambda)h_t \leq \sigma\), there is no room for education and \(\theta_t = 0\).

Moreover if

\[ \gamma(h_t - \sigma) - \lambda h_t < (1 + \gamma)p_t. \]  

(5)

the optimal schooling time, given by (4), is bounded above by one.

Note that if \(\gamma < \lambda\), (5) is always satisfied. In this case, the altruism term is not big enough to avoid child labour, when a part of the child income is useful to the household.

When (5) is not satisfied, \(\theta_t = 1\) : in this case, parents income are sufficient high, relating to \(p, \sigma\) and \(\gamma\), to abolish child labour, and the corner solution occurs. These results are conform to empirical facts, that show that the major cause of child labour is poverty, and that an increase in standards of living may reduce child labour (see Edmonds and Turk, 2002, for a study on Vietnam).

When \((1 + \lambda)h_t > \sigma\), the optimal years of schooling are an increasing function of \(\gamma\), the parents’ taste for educated children, and of parental human capital \(h_t\); \(\theta_t\) is an decreasing function of \(\sigma\) and of \(p_t\).

Related to this last result, when \(\sigma + p_t > h_t\) note that the optimal years of schooling \(\theta_t\) rises when parents expect to receive a larger proportion \((\lambda)\) of their children’s income from working; it means that a high child contribution to the family income enables part-time labour and not full-time labour; otherwise when \(\sigma + p_t < h_t\), \(\theta_t\) is an decreasing function of \(\lambda\), in this case, child income is not necessary to the family to survive, and it is more required when it becomes more attractive.

2.1 Dynamics

To study the steady-state and the dynamics of the model without educational policy, let’s consider that \(G(.)\) and \(p_t\) are constant over time and respectively equal to \(Q\), an index of educational quality, and \(p\). These assumptions would be relaxed in Section 3. Parents offer one unit of human capital over the period. Human capital is the only engine of growth in the model. There are multiple steady-state trajectories, that depend upon the subsistence consumption and of the initial values of the different parameters.
Following Glomm (1997), the different cases are dissociated in function of education parental choices. Three main cases can be distinguished, according to the trade-off between education and child labour.

Let \( H_1 = \frac{c(1+\gamma)}{\lambda} \) and \( H_2 = \frac{(1+i)\delta \gamma + \gamma c}{(\gamma - \lambda)} \). If \( H_1 < H_2 \), the dynamics of human capital is

\[
\begin{align*}
  h_{t+1} &= H_1, \quad H_1 \geq h_t \\
  h_{t+1} &= bQ \frac{c(1+\lambda)h_t - \gamma c}{(1+i)(\delta \gamma + \gamma c)} h_t^\beta, \quad \text{if } H_2 > h_t > H_1 \\
  h_{t+1} &= Qh_t^\beta, \quad \text{if } H_2 \leq h_t
\end{align*}
\]

if not the dynamics is given by

\[
\begin{align*}
  h_{t+1} &= bQ \frac{c(1+\lambda)h_t - \gamma c}{(1+i)(\delta \gamma + \gamma c)} h_t^\beta, \quad \text{if } h_t > H_1 \\
  h_{t+1} &= H_1, \quad \text{other.}
\end{align*}
\]

Let us consider that \( H_2 > H_1 \)

**Proposition 1**

- a) If \( bQH_2^{\beta-1} < 1 \), the following situations may occur:

  - If \( H_2 < x_2 \); there is one unique steady state \( H_3 \) which is semi-stable.

  - If \( H_2 > x_2 \); there are three steady states: \( H_1, H^* \in [H_1, H_2] \) and \( H^{**} \in [H_1, H_2] \) with \( H^* < H^{**} \); \( H_1 \) is stable, \( H^* \) is unstable and \( H^{**} \) is stable.

where

\[
x_2 = \frac{-\left( \lambda \bar{c} (2-\beta) + (1+\lambda)\beta p \right) - \sqrt{\left( \lambda \bar{c} (2-\beta) + (1+\lambda)\beta p \right)^2 + 4(\beta - 1)^2(1+\lambda)\lambda \bar{c} \beta p}}{2(\beta - 1)(1+\lambda)\lambda}
\]

- b) If \( bQH_2^{\beta-1} > 1 \), there are three steady states: \( H_1, H^* \in [H_1, H_2] \) and \( H_4 = (Qb)^{\frac{1}{\lambda\beta-1}} \in [H_2, \infty[ \). \( H_1 \) and \( H_4 \) are stable, \( H^* \in [H_1, H_2] \) is unstable.

- c) If \( bQH_2^{\beta-1} = 1 \), there are 2 steady states: \( H_1 \) and \( H_2 \). \( H_1 \) is stable, \( H_2 \) is semi-stable.

**PROOF:**
1. Steady states: To study the steady states coming from the part a) of the dynamics, let \( f(x) = \frac{\lambda x - \bar{c}}{\mu + \lambda x} x^{\beta - 1} \) and solve equation

\[
 bQ^\gamma \frac{\gamma}{\gamma + 1} f(x) = 1,
\]

\[
 f'(x) = \frac{x^{\beta - 2}}{(p + \lambda x)} \left( \frac{(1 + \lambda)p + \lambda \bar{c}}{p + \lambda x} x + ((1 + \lambda)x - \bar{c}) (\beta - 1) \right)
\]

The sign of \( f'(x) \) is the same as the sign of \( P(x) = (\beta - 1) (1 + \lambda) \lambda x^2 + x \left( \frac{\lambda \bar{c} (2 - \beta) + (1 + \lambda) \beta p}{(1 + \lambda) \beta p} \right) - \bar{c} (\beta - 1) p \).

Let \( \Delta = \left( \frac{\lambda \bar{c} (2 - \beta) + (1 + \lambda) \beta p}{(1 + \lambda) \beta p} \right)^2 + 4 (\beta - 1)^2 (1 + \lambda) \lambda \bar{c} p > 0 \) and \( x_2 \) be the positive root of \( P(x) \);

\[
 x_2 = \frac{\left( \frac{\lambda \bar{c} (2 - \beta) + (1 + \lambda) \beta p}{(1 + \lambda) \beta p} \right) + \sqrt{\left( \frac{\lambda \bar{c} (2 - \beta) + (1 + \lambda) \beta p}{(1 + \lambda) \beta p} \right)^2 + 4 (\beta - 1)^2 (1 + \lambda) \lambda \bar{c} p}}{2 (1 - \beta) (1 + \lambda) \lambda}
\]

The graph of \( f(x) \) is the following:

\[
 \begin{array}{ccc}
 bQ^\gamma & f'(x) & H_1 \\
 bQ^\gamma f(x) & 0 & \backslash_0
 \end{array}
\]

Let us distinguish the following cases:

- If \( H_2 < x_2 \)
  
  we then have to distinguish the following cases:

  - if \( bQ^\gamma H_2^{\beta - 1} < 1 \); there are no steady states in \( [H_1, H_2] \)
  
  - if \( bQ^\gamma H_2^{\beta - 1} > 1 \); there is a unique steady state \( H^* \) in \( [H_1, H_2] \).

- If \( H_2 > x_2 \)
  
  we then have to distinguish the following cases:

  - If \( f(x_2) < 1 \); there are no steady states in \( [H_1, H_2] \)

  - If \( f(x_2) > 1 \); there is a unique steady state \( H^* \) in \( [H_1, H_2] \).
If \( f(x_2) > 1; \)
we then have to distinguish the following cases:

- If \( bQH_2^{\beta-1} > 1; \) there is a unique steady state \( H^* \) in \( [H_1, H_2] \).
- Soit \( bQH_2^{\beta-1} < 1; \) there are two steady states \( H^* \) and \( H^{**} \) with \( H^* < H^{**} \) in \( [H_1, H_2] \).

To study the second part of the dynamics (b) just notice that \((Qb)^{1-\gamma}\) is the only solution of \( Qbx^{\beta} = x; \) \((Qb)^{1-\gamma} > H_2 \) if and only if \( bQH_2^{\beta-1} > 1 \).

2. Dynamics: To study the dynamics, just notice that \( bQ\frac{(1+\lambda)x^{-\gamma}}{(\rho+\lambda)x^{\gamma}}x^{\beta} \)
and \( Qbx^{\beta} \) are increasing function of \( x \)

**Corollary 2 Poverty trap and no education.** When \( bQH_2^{\beta-1} < 1 \), the family income is below the poverty line, and there is no room for education. The economy lies in a poverty trap, and no human capital accumulation is possible \((\theta = 0)\).

In this case, parents income are not sufficient enough to pay for education, because they are too close from the poverty line. Child labour is a crucial source of income for poor family \((\theta_t = 0)\), and the economy remains in a poverty trap. If there are no, or few, borrowing possibilities for schooling investments, as in most developing countries, a public policy against such a poverty trap would be a distribution of food subsidies \( ^6 \), like the Food-for-Education Program built in Bangladesh (Ravallion and Wodon, 2000).

The introduction of a subsistence consumption in the model and of a price for education \((p)\) modifies the standard results of OLG model with education in the logarithmic case.

With a low initial stock of human capital (when \( H_2 > h_t > H_1 \)), and for some parameters values, the equilibrium law of motion of \( h_{t+1} \) is convex in \( h_t \), and the economy may have both child labour and education on the long run \((ie 0 < \theta < 1)\), as the following corollary shows it:

**Corollary 3 Education and child labour.** If \( bQH_2^{\beta-1} > 1 \) there exists an unstable steady state where education and child labour can both exist. Convergence to the steady state with only education or only child labour depends on the initial amount of human capital.

\( ^6\)This assumption is explored in Section 3.
If \( b Q H^{\beta-1}_2 < 1 \) there may exists a stable steady state where education and child labour can both exist (i.e. \( 0 < \theta < 1 \)).

In the case, only where the initial stock of human capital is high and when \( b Q H^{\beta-1}_2 > 1 \), there is not child labour anymore on the long run (\( \theta = 1 \)), otherwise child labour will persist permanently (\( 0 < \theta < 1 \)). This case is little studied in the literature\(^7\), but is frequent in developing countries. The model shows that for bad initial economic conditions and especially for an inadequate quality of education, this case will persist over time.

**Corollary 4 Education without child labour.** When \( b Q H^{\beta-1}_2 > 1 \) and when initial stock of human capital is high, there is no child labour anymore (\( \theta_t = 1 \)).

How is it possible to reduce child labour, to favour schooling and to increase growth? One solution is to expect a rise of income, in order to be in the third case. Edmonds and Turk (2002) find evidence from Vietnam that an increase of income may rise education participation and to a certain degree diminish child labour. But the model shows that, at times, even if there is an income rise, a poverty trap could emerge. An other solution can be to act on the quality of education, to converge to the third case (where \( \theta = 1 \)). In this condition, which kind of educational policy may be set up to enhance schooling participation and to reduce child labour? As Glewwe (2002) has pointed it out, one major problem of education in developing countries lies in the bad quality of their schooling system. In next sections, this last point is investigated. The possibility for parents to choose school quality is introduced in order to compare different educational policies and to find some practical solutions to rise schooling attainment.

### 3 Child labour, education and school quality

School attendance (or conversely child labour) of children in a low developing country may be explained by poverty, like in the last section, but also by the low quality of schooling that may lead households to substitute work for schooling (see empirical evidence from Zambia in Jensen and Nielsen, 1997).

\(^7\)Perotti (1993) for example find that \( \theta = 0 \) or \( \theta = 1 \). The intermediate case (\( 0 < \theta < 1 \)) is not considered, whereas the ILO statistics show that it is a general case in developing countries.
The model can be extended to allow parents to choose school quality in the following way. Like in Glewwe (2002), it is assumed that higher quality implies higher price:

\[ p_t = p_0 Q_t, \]  
\[(6)\]

where \( p_0 \) is the “base” price of schooling. \( Q_t \) may be interpreted as an index of expenditures on quality, and a convenient assumption for a functional form of \( G \) is that \( G(Q_t) = Q_t^\alpha \) (see Glewwe 2002 for a use of this specification for example). Two polar educational regimes are successively considered: a private education regime, where all schooling decisions are made individually by the parent, i.e. each parent chooses the child’s school time and the quality of school; a public education regime, where parents only choose their child’s school time, and education quality depends on taxes.

3.1 Human capital accumulation under private regime

In the private education regime all schooling decisions are made individually by parents, i.e. parents choose the child’s school time and the quality of the schools. The family budget constraint at time \( t \) is:

\[ c_t + p_0 Q_t = \Omega_t \]

The problem of parents becomes:

\[ \text{Max} \ln(c_t - \bar{c}) + \gamma \ln h_{t+1}, \]

s.t.

\[ h_{t+1} = b \theta_t Q_t^\alpha h_t^\beta, \]

\[ c_t = h_t - p_0 Q_t \theta_t + (1 - \theta_t) \lambda h_t. \]

From the First Order Condition the optimal quality of education is,

\[ Q_t = \frac{\alpha \lambda h_t}{p_0 (1 - \alpha)}. \]

A sufficient condition to have \( Q_t > 0 \) is that \( \alpha < 1 \).

**Hypothesis 1.** It is assumed that \( \alpha < 1 \).

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\(^8\)As noted by Glewwe (2002) “School quality is likely to be endogenous. Even in rural areas of low-income countries, where villages often have only one school and are too far apart from children to attend school in a neighbouring village, parents may be able to influence school quality of the sole local school through the parent-teacher association”, p. 443.
The quality of schooling desired by parents is an increasing function of their own human capital $h_t$ and of and $\lambda$. It decreases with $p_0$, the “base” price of schooling, in other words, if the price of education is rather high, parents don’t want to pay more to improve quality of education.

The optimal education choice is,

$$\theta_t = \frac{\gamma(1 - \alpha) [(1 + \lambda)h_t - \overline{c}]}{(\gamma + 1) \lambda h_t}, \quad (7)$$

$\theta_t > 0$ if $(1 + \lambda)h_t > \overline{c}$. This condition is also the same than the one in the previous case.

Note that $\theta_t$ is a decreasing function of $\alpha$ and an increasing function of $h_t$ and of $\lambda$.

$\theta_t$ must be bounded by one, hence (7) applies when,

$$\frac{\overline{c}}{1 + \lambda} < h_t < \frac{(1 - \alpha) \gamma}{\gamma [1 - \alpha (1 + \lambda)] - \lambda} \overline{c} \quad (8)$$

When $\gamma < \frac{\lambda}{1 - \alpha (1 + \lambda)}$, (8) is always verified, and child labour would not decline.

If $h_t > \frac{(1 - \alpha) \gamma}{\gamma [1 - \alpha (1 + \lambda)] - \lambda} \overline{c}$, it is optimal for parents to choose the maximal amount of schooling (i.e. the corner solution occurs). This “threshold” increases with $\overline{c}$, and $\lambda$.

Let $H_1 = \frac{\overline{c}}{1 + \lambda}$ and $H_2 = \frac{(1 - \alpha) \gamma}{\gamma [1 - \alpha (1 + \lambda)] - \lambda} \overline{c}$. The law of motion of human capital is

$$\begin{cases}
    h_{t+1} = H_1, \text{ if } H_1 > h_t \\
    h_{t+1} = b \frac{\gamma (1 - \alpha)}{(\gamma + 1) \lambda} \left[ \frac{\alpha}{p_0 (1 - \alpha)} \right]^\alpha (1 + \lambda) h_t - \overline{c} \right] h_t^{\alpha + \beta - 1}, \text{ if } H_1 < h_t < H_2 \\
    h_{t+1} = b \left[ \frac{\gamma \alpha}{p_0 (1 + \gamma)} \right]^\alpha (h_t - \overline{c})^\alpha h_t^\beta, \text{ if } h_t > H_2.
\end{cases}$$

Let us consider steady states in $[H_1, H_2]$

**Proposition 5**

- if $\alpha + \beta - 1 < 0$, then

  $$-\frac{bH_2^{\beta - 1}}{\frac{\gamma (1 - \alpha (1 + \lambda)) - \lambda}{p_0}} < 1,$$

  the following situations may occur:

  * there is one unique steady state $H_1$ which is semi stable.
* There are three steady states in \([H_1, H_2]\): \(H_1, H^* \in ]H_1, H_2]\) and \(H^{**} \in ]H_1, H_2]\) with \(H^* < H^{**}\); \(H_1\) is stable, \(H^*\) is unstable and \(H^{**}\) is semi-stable.

- If \(\frac{bH_2^{\beta-1}}{p_0 \gamma/(1-\alpha(1+\lambda))} > 1\), there are two steady states in \([H_1, H_2]\): \(H_1, H^* \in ]H_1, H_2]\). \(H_1\) is stable, \(H^* \in ]H_1, H_2]\) is unstable.

- If \(\frac{bH_2^{\beta-1}}{p_0 \gamma/(1-\alpha(1+\lambda))} = 1\), there are 2 steady states: \(H_1\) and \(H_2\). \(H_1\) is stable, \(H_2\) is semi-stable.

- If \(\frac{bH_2^{\beta-1}}{p_0 \gamma/(1-\alpha(1+\lambda))} < 1\), \(H_1\) is the only steady state in \([H_1, H_2]\).

- If \(\frac{bH_2^{\beta-1}}{p_0 \gamma/(1-\alpha(1+\lambda))} > 1\), there are two steady states in \([H_1, H_2]\): \(H_1, H^* \in ]H_1, H_2]\). \(H_1\) is stable, \(H^* \in ]H_1, H_2]\) is unstable.

- If \(\frac{bH_2^{\beta-1}}{p_0 \gamma/(1-\alpha(1+\lambda))} = 1\), there are 2 steady states in \([H_1, H_2]\). \(H_1\) is stable, \(H_2\) is semi-stable.

**Proof:** Let \(F = b\gamma(1-\alpha)/(1+\lambda)((\alpha\gamma/p_0(1-m))\times)\) Steady states in \([H_1, H_2]\) are solutions of \(F[(1+\lambda)h_t - \tau] x^{\alpha+\beta-1} = x\).

Let \(m(x) = [(1+\lambda)h_t - \tau] x^{\alpha+\beta-1}\)

If \(\alpha + \beta - 1 < 0\), \(m(x)\) is a concave increasing function; if \(\alpha + \beta - 1 > 0\), \(m(x)\) is a convex increasing function.

The intersection of the graph of \(m(x)\) with the line \((p+\lambda x)\) admits at most two solutions when \(m(x)\) is concave, almost one when \(m(x)\) is convex.

**Corollary 6 Poverty trap and education.** When \(\alpha + \beta - 1 > 0\), the steady state, when it exists with education and child labour is unstable.

**Remark 7** As \(n(x) = (x - \tau)^\alpha x^\beta\) is not monotonous on \([H_2, \infty]\), there may occur cycles when \(H_2 < \frac{\gamma}{\alpha+\beta}, \alpha + \beta - 1 > 0\)

and \(b \left[ \frac{\gamma}{p_0(1+\alpha+\beta)} \right]^{\alpha} \gamma^{(\beta-\alpha)\alpha} \left( \frac{\gamma}{\alpha+\beta} \right)^{\beta-1} < 1\)

\(\gamma m(x) = [(1+\lambda) (\alpha + \beta) x - \tau(\alpha + \beta - 1)] x^{\alpha+\beta-2} > 0\)

\(\alpha m(x) = [(1+\lambda) (\alpha + \beta) x - \tau(\alpha + \beta - 2)] (\alpha + \beta - 1) x^{\alpha+\beta-3}\)
When the initial human capital is superior to $H_2$, cycles could emerge with a private education regime in a growing economy without child labour. This result relies on the subsistence consumption term, and is very particular to the private educational system where rich parents choose jointly for quantity and quality of education.

The mechanism describes in the paper contributes to show that a private education regime may create good incentives for schooling and may help to reduce child labour, except if the economy lies in a poverty trap. In this case, other educational policies are necessary.

3.2 Human capital accumulation under public education

In the public education regime, the government is assumed to collect income taxes at uniform rate from the labour income of the old. Income of the young is supposed to exempt from taxation. Tax revenues are used to finance education and all children have access to the same quality of education at price $p_0$, such as $p_0Q_t$.

Under these assumptions the family budget constraint becomes:

$$c_t = (1 - \tau_t)y_t - p_0Q_t\theta_t + (1 - \theta_t)\lambda h_t.$$ 

The quality of education to which all children have access becomes:

$$Q_t = \tau_tY_t,$$

where $Y_t$ denotes the aggregate (average) variables. Since the population is normalized to one, it comes that $Y_t = h_t$. Under public education each individual, when old, solves the following maximization problem:

$$\text{Max}_{\theta_t, c_t} \ln(c_t - \bar{c}) + \gamma \ln h_{t+1},$$

s.t.

$$c_t = (1 - \tau_t)y_t - p_0\tau_t Y_t \theta_t + (1 - \theta_t)\lambda h_t,$$

$$h_{t+1} = b\theta_t \tau_t^{\alpha} h_t^{\alpha + \beta},$$

Parents, reasonably, consider the tax rate $\tau_t$ as given. Consequently, the only choice variable is $\theta_t$.

From the F.O.C.,

$$\theta_t = \frac{\gamma [(1 - \tau_t + \lambda)h_t - \bar{c}]}{(1 + \gamma)[p_0\tau_t + \lambda]h_t}.$$ 

Hypothesis 2. The proportional tax is inferior to $1 + \lambda$.

This assumption is necessary to avoid a fiscal burden that would inhibit schooling and growth.
Discussion: \( \theta_t > 0 \) if and only if \((1 - \tau_t + \lambda)h_t > \bar{\pi} \). This opens the possibility to make subsidies and not a fiscal policy (when \( \tau_t < 0 \)).

\( \theta_t < 1 \) when,

\[
\frac{\bar{\pi}}{1 - \tau_t + \lambda} < h_t < \frac{\gamma \bar{\pi} + (1 + \gamma)p_0\tau_t}{\gamma(1 - \tau_t) - \lambda}.
\]

The human capital threshold to avoid child labour depends on \( \bar{\pi}, p_0 \) and \( \tau_t \): it rises with \( \bar{\pi} \) and \( p_0 \).

Under Hypothesis 2, the optimal years of schooling are an increasing function of the parental human capital \( h_t \).

Besides, they fall with the tax rate if \( h_t > \frac{p_0(1+\gamma)}{\lambda}\) and \( \tau_t < 2\lambda \). Hence, with a low level of tax, it would be possible to enhance growth, through its positive impact on education quality and the incentive motives it creates\(^{10}\).

Let \( \bar{H}_1 = \frac{\bar{\pi}}{1 - \tau_t + \lambda} \) and \( \bar{H}_2 = \frac{\gamma \bar{\pi} + (1 + \gamma)p_0\tau_t}{\gamma(1 - \tau_t) - \lambda} \). The law of motion of human capital is

\[
\begin{cases}
\begin{align*}
&h_{t+1} = br^\alpha \left[ \frac{\gamma[(1 - \tau_t + \lambda)h_t - \bar{\pi}]}{(1+\gamma)p_0\tau_t + \lambda} \right] h_t^{\alpha + \beta - 1}, \text{ if } \bar{H}_1 < h_t < \bar{H}_2 \\
&h_{t+1} = h_t + 1 = br^\alpha h_t^{\alpha + \beta}, \text{ if } h_t > \bar{H}_2.
\end{align*}
\end{cases}
\]

There is a multiplicity of steady state trajectories, according to the initial value of \( h_t \). To study them, let’s assume that the income tax rate is constant on the long run. The dynamics in the case where \( \bar{H}_1 < h_t < \bar{H}_2 \) is quite similar to the second case of the dynamics of the private education system. But the in this case, the poverty trap is more difficult to avoid for a poor economy, because of the tax (\( \bar{H}_1 > H1 \)).

For \( h_t < \bar{H}_1 \), the economy is too poor and lies in a poverty trap. Note that the poverty trap occurs more easily in this case than in the case with the private regime. This is due to the tax, that diminishes the family income. In this case, only subsidies \( (s_t = -\tau t) \) would help the economy to get out the trap.

\(^{10}\)Note that if it is assumed that \( p_t = p_0 \theta_t \), that is less homogenous with the private regime, it comes that \( \theta_t = \frac{\gamma h_t}{(1+\gamma)p_0\tau_t + \lambda} \) : in this case time schooling is always decreasing in the tax rate, but the main results on the long run are unchanged.
For insufficient initial stocks of human capital \((H_1 < h_t < H_2)\), the effect of the income tax is ambiguous, because of its impact on the trade-off between education and child labour: for \(h_t < \frac{\rho \bar{c}}{\bar{p} - p_0} + \lambda\) and \(\tau_t < 2\lambda\), it appears that the optimal years of education are an increasing function of the tax rate, and this policy contributes then to enhance growth. Otherwise, a too high level of tax may inhibit growth when the economy is in the first case.

When \(h_t > H_2\), there is no child labour anymore and \(\theta = 1\). In this case, if \(\alpha + \beta < 1\), there is one unique steady state which is stable and which rises with the tax level. If \(\alpha + \beta = 1\), there is a standard endogenous growth regime, and the growth rate increases with the tax level. The major difference with the private regime is that a high initial stock of human capital generates a potentially higher growth with a public education regime, that does not rely on \(\bar{c}\). Consequently, no cycles may occur with a public regime and it is possible to

Nevertheless, when an economy is poor and above the poverty line \((\bar{H}_1 < h_t < \bar{H}_2)\), the private regime appears to be more efficient to fight against child labour, because the poverty trap occurs less easily. This is coherent with empirical facts in developing countries, where private education systems are rather developed, and are often financed at community levels. When an economy is too poor, the public system may create not enough incentives; moreover when households are very poor, they cannot bear a too heavy fiscal burden and only a redistributive policy could be efficient. More generally, in this case, only a public aid would enable the financement of the educational system.

This framework defined in the paper can be used to study the behavior of a social planner, that would set simultaneously fiscal policy and optimal years of school that maximize the households utility function. When the economy is too poor, that means when \(\frac{\bar{c}}{1 - \tau_t + \lambda} < h_t\), only subsidies would help to get out the poverty trap.

4 Concluding remarks

The model contributes to highlight the link between poverty, education and child labour. Under bad economic conditions, it appears that parents may decide to keep children away from school. In such a context, trade sanctions or repressive laws seem not to be the right solution to fight against child labour. Government policies have to act mainly on growth and on poverty.

Different education policies are considered in the paper and a subsistence
consumption is introduced to define a poverty line.

When the quality of education is too low or when the households income are below the poverty line, a low-development trap occurs. An improvement of the quality of education through a private education regime or a public education regime may enhance human capital growth and reduce child labour.

Nevertheless, if the stock of human capital is low and above the poverty line, the private regime may be more efficient that the public education regime through the incentives it creates, linked to the quality of education. Besides in this case, a public aid can be necessary to finance a public education system, in order to avoid a too heavy fiscal burden that would induce a poverty trap. But if the economy is not too poor, the public regime may be as good as the private system one, or better if the educational quality is high enough.

These results are coherent with empirical facts and may give explanations to the relatively important development of private schooling systems in developing countries at community or local levels.

It is also shown that when the economy is below the poverty line, none of the education regimes considered in the paper enables to get out the low-development trap. In this case only subsidies policy would contribute to enhance growth.

It would be interesting to compare both regimes with heterogeneous agents. Besides, savings should be introduced in the model in order to study how a social protection system may contribute to reduce child labour. This would be done in further work.

References


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