# THE SUPPLY OF PERISHABLE GOODS

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*ABSTRACT*

This paper models the supply of perishable goods within a randon framework. Perishability affects a large group of goods usually traded in the economy such as fruits and vegetables, newspapers, medicine drugs, a.s.o.. Surprisingly, one cannot find in the literature a decision model for suppliers that takes into account the specificity of this kind of goods. The suppliers guess their demand by choosing a probability density function, one at each price level. Then they choose optimal supply functions maximizing their expected profits. Examples of the optimal solution are given for some known demand distribution functions like Pareto and Weibull. The autarchic model is then extended to include nonprice competition among the sellers. Each seller chooses the supply curve that maximizes his expected profit, conditioned by the event that competitors' markets are in equilibrium. The supply of rivals affect the sales for certain to loyal clients, but not the random sales. The autarchic model is then used to analyze the green-pepper market in Rio de Janeiro(1994/7-2000/11). The results give consistency to the rational hypothesis of the model.

**Key-words**: Perishable goods, price-elasticity, Lerner index, Nash equilibrium.

**JEL classification:** Q21, C60, D81, L81.

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# I - Introduction

This paper models the supply of perishable goods within a random framework. Perishability affects a large group of goods usually traded in the economy such as fruits and vegetables, newspapers, medicine drugs, a.s.o.. Surprisingly, one cannot find in the literature a decision model for suppliers that takes into account the specificity of this kind of goods. An important feature of trading perishables is that the market clearing requires a prompt response of the buyers to the supply of sellers. When the amount unsold at a given date cannot be stocked and transferred to subsequent periods, losses may reach up to 100% of overall costs. This suggests to treat the quantity demanded as a random variable rather than a deterministic one.

Formal approaches modelling the market of perishables most often focus on price determination, competition, or price linkages among the production, wholesale and retail sectors. Models of price determination are considered in Gardner (1975) and Heien (1980). Other empirical studies estimate demand and supply curves defined exogenously. Concerns for the effects of uncertainty in trading perishable goods can be found in Fraser(1995). Sexton and Zhang(1996) built a price determination model at farm level which is used to test the assumption of a competitive behavior of buyers in the California iceberg lettuce market. The present paper does not model price determination. In the decision making, the sellers choose quantities by taking input and output prices as given. Grossman(1981) and Klemperer and Meyer(1989) also model supply curves given prices, but they do it in a different framework.

Given the demand distribution function and the level of prices, a relevant problem that a supplier faces is choosing the amount he should supply in order to maximize expected profit. In this regard, the short life-cycle of the good allows to insert the decision rule within a one-period setting. In the present model, the seller takes into account two possible losses: *(i)* the accounting loss incurred with the depreciation of the quantity unsold; *(ii)* the economic loss incurred with a demand in excess of supply, estimated as a proportion of the opportunity cost of the undersupply. The probability densities will depend on product prices. Before choosing his supply level, the seller guesses its actual demand by choosing a function belonging to a family of distributions, one at each price level.

By holding prices and technology constants, our model may appear to resemble the perfect competition model. But in fact, it diverts from the latter at least in two aspects. First, the optimal condition does not require a price-marginal cost equalization, as in the competitive model. Here, the supplier maximizes expected profit by equating the probability of insufficient demand with an increasing function of the price-marginal cost margin. If the margin is null, then it is optimal to supply, at the most, to committed clients only. Second, even if one assumes that the sellers interact with one another, the maximizing behavior will not imply the market clearing. The optimal supply $(S)$ is a deterministic function while the demand $(X)$ remains a random variable.

Our approach shows some interesting features that are not present in previous models of supply determination: a) The optimal supply $S$ in the retail segment is also a demand function in the wholesale segment. Thus, if $c$ and $p$ are the wholesale and retail prices respectively, $S(p, c)$ is an increasing function of $p$ at each $c$ and a decreasing function of $c$ at each $p$. Moreover, the price-elasticity of the supply at retail equates the negative of the price-elasticity of the demand at wholesale : $\epsilon_{Sp}(p; c) = -\epsilon_{Dc}(c; p)$. The supply curve depends on the parameters of the demand distribution; the estimation of the curve also gives us estimates for these parameters; b) Adequate parametrization of the support of the demand distribution allows to evaluate the importance of its deterministic component; c) When the quantities committed to loyal customers are not significant, an analysis of losses can be carried out by estimating the probability of getting negative profits at each period.

As a natural extension to the model, nonprice competition among several suppliers is considered in *section III.* Given the vector of prices, the supply of competitors affect the sales committed to loyal clients, but not the random sales of the supplier. The seller chooses the supply that maximizes his expected profit, conditioned by the event that competitors are in equilibrium. At the Nash equilibrium, the interactive solution is a linear transformation of the optimal supply curves obtained in autarchy.

In order to give an illustration, the autarchic model is used in *section IV* to analyze the green-peppers retail market in Rio de Janeiro 1994/7 to 2000/11.

## II- The autarchic model

Let $S$ be the retailer quantity supplied ($S \geqslant 0$), and $p$ the product price ($p > 0$). The demand $X$ is assumed to be an absolutely continuous random variable with density $f$ defined on the support $[a, b)$, $b > a \geqslant 0$, with finite variance. The density depends on a parametric vector $\xi$ and on the price level $p$. At point $X = x$, it will be noted $f(x; \xi, p)$. Let $C$ be the nonnegative and increasing cost function, defined on the supply space. We assume $C$ is two times continuously differentiable.

Consider the event $A = [X \leqslant S]$ : demand is no greater than supply. Let $\mathbf{1}_A$ be the indicator function of $A : \mathbf{1}_A(x) = 1$ if $x \in A$ and $= 0$ if $\omega \notin A$.

**The objective function** . We assume that the seller maximizes expected earnings. When $A$ occurs and the seller succeed in recovering a proportion $\delta$ ($0 \leqslant \delta < 1$) of the cost of the excess supply, the profit obtained when the demand is $X$ is given by: $pX - C(S) + \delta(C(S) - C(X))$. When $A^c = [X > S]$ occurs, the opportunity cost of a shortage of supply relative to demand is: $p(X - S) - (C(X) - C(S))$. Let $\tau \geqslant 0$ be the proportion of the opportunity cost that the supplier is willing to deduct from his potential revenue $pS - C(S)$. Then, his net economic earning will be: $pS - C(S) - \tau[p(X - S) - C(X) + C(S)]$. If $L$ indicates the net earning, we have:

$L(X, S; p) \equiv [pX - C(S) + \delta(C(S) - C(X))]\mathbf{1}_A + [\ pS - C(S) - \tau(p(X - S) - C(X) + C(S))]\mathbf{1}_{A^c}$. Since $\mathbf{1}_A + \mathbf{1}_{A^c} = 1$ we obtain, after simplifying:

$$L(X,S;p) = [(1+\tau)pS - \tau pX + \tau C(X) - (1+\tau)C(S)] +$$
$$[(1+\tau)p(X-S) \quad + (\tau+\delta)(C(S) - C(X))]\mathbf{1}_A \qquad (1)$$

The inclusion of the *opportunity cost* in the objective function is also justified because of the possibility of discontinuity in the demand. For example, suppose a consumer only purchases $n$ units. In this case, it may be that sales fall short of $S$ even when $A^c = [X > S]$ occurs. It is not difficult to find numerical examples showing that the present specification may include the accounting profit as well as the effective profit, according to different values of the parameter $\tau$. When $\tau = 0$, the objective function (1) is simply a *profit* function.

Let $\Pi(S;p)$ be the expected earning when supply is $S$ and retail price is $p$. Then:

$$\Pi(S;p) \equiv EL = [(1+\tau)pS - \tau pEX + \tau EC(X) - (1+\tau)C(S)] +$$
$$\int_a^S [(1+\tau)p(x-S) \quad +(\tau+\delta)(C(S) - C(x))] f(x;\xi,p)dx. \qquad (2)$$

**Supply curves.** For meaningful economic values of the supply, *i.e.*, $S \geqslant a$ and $C'(S) \leqslant p$, one can check that $\Pi$ is a concave function of $S$, provided that the cost function $C$ is convex or not too concave for these values of $S$. Under this assumption, the first order equation $\partial\Pi/\partial S = 0$ gives the necessary and sufficient condition for the expected earnings to be globally maximized in $S$. Applying the Leibnitz rule in the derivation of (2) we obtain the first order condition:

$$\int_a^S f(x;\xi,p)dx \;=\; \frac{(1+\tau)[p - C'(S)]}{(1+\tau)[p - C'(S)] + (1-\delta)C'(S)} \qquad (3)$$

The l.h.s. of the above equation gives the probability that demand is not greater than supply. If $F$ stands for the cumulative distribution function of $X$, the l.h.s. is: $F(S) = P(X \leqslant S)$. Now, let $\lambda(S) \equiv [p - C'(S)]/p$ be the price-marginal cost margin (Lerner index). A simpler solution in $S$ is obtained by assuming constant marginal cost: $C' = c > 0$. In this case, the equation (3) simplifies to:

$$F(S) \; = \frac{(1+\tau)\lambda}{(1+\tau)\lambda + (1-\delta)(1-\lambda)} \qquad (4)$$

In order to have a clear interpretation of the optimal condition, assume $\tau = \delta = 0$. In this case (4) reduces to $F(S) = \lambda$. Thus, the optimum requires the amount supplied must be such that the probability of oversupply equates the mark-up $(p - c)/p$. This implies that $S$ will be an increasing function of $p$ and a decreasing function of $c$. Further, if $\lambda = 0$, we obtain $S \leqslant a$. So, in the competitive case $(p = c)$, the seller will supply, at the most, to committed customers only.

Our assumptions ensure that $F$ admits an inverse $F^{-1}$. Therefore,

$$S(\lambda, \tau, \delta) = F^{-1}\left(\frac{(1+\tau)\lambda}{(1+\tau)\lambda + (1-\delta)(1-\lambda)}\right) \qquad (5)$$

Viewed as a function of $\tau$, $S(\tau)$ increases, meaning that the supply maximizing the expected profit is a lower bound for the family of curves that maximize expected earnings: $S(0) \leqslant S(\tau)$. The *rationale* is simple: if supply shortages are also penalized $(\tau > 0)$, the seller is led to supply larger quantities than he would do if only oversupply was penalized $(\tau = 0)$ .

To offer an illustration, the supply curves generated in the *Pareto* and *Weibull* models are given below. Both admit an explicit inverse $F^{-1}$.

**a) Pareto demand distribution.** The probability density function equals 0 for $x < A(p)$ and $f(x;\gamma,A(p)) = \frac{\gamma}{A(p)}[\dfrac{A(p)}{x}]^{\gamma+1}$ for $x \geqslant A(p)$; $\gamma > 2$. $A$ stands for the deterministic component of the demand, to be estimated from the data. When the retail price is $p$, loyal clients commit to buy from the seller $A(p)$ quantities. It seems reasonable to assume that $A$ is a nonincreasing function of prices. The expected demand is: $EX = (\dfrac{\gamma}{\gamma-1})A(p)$. The figure below shows the density of the demand for two price levels: $p < p^1$.

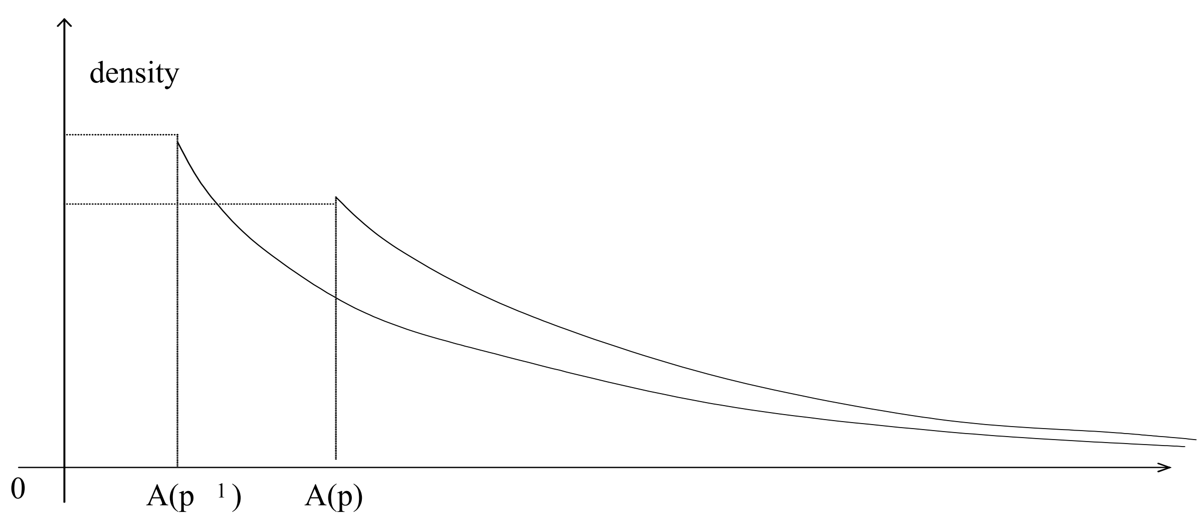


**FIG.1: Pareto demand for two price levels**

The distribution function is: $F(S;\gamma,A(p)) = 1-(\dfrac{A(p)}{S})^{\gamma}$ for $S \geqslant A(p)$ and equal to 0 otherwise. By using equation (5) we obtain the following supply function:

$$S(\lambda, \tau, \delta, A(p)) = A(p).[1 + (\frac{1+\tau}{1-\delta})(\frac{\lambda}{1-\lambda})]^{(1/\gamma)} \qquad (6)$$

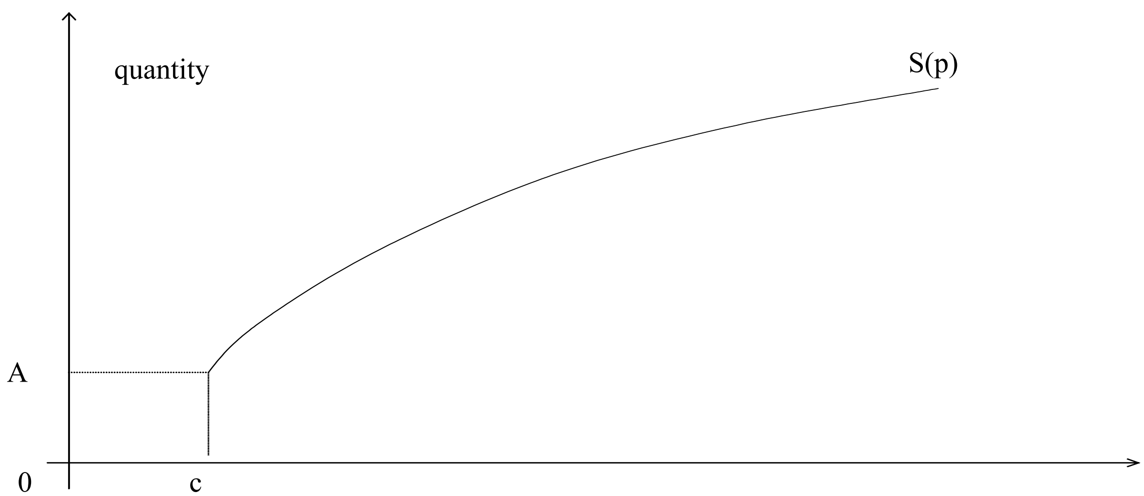


**FIG.2: Supply for Pareto demand distribution**

**b) Weibull demand distribution.** The density function equals 0 for $x < a(p)$ and $f_x(x;\ \alpha, \beta, a(p)) = \beta\alpha[\alpha(x - a(p))]^{\beta-1}e^{-[\alpha(x-a(p)]^{\beta}}$ for $x \geqslant a(p)$, where $\alpha$ e $\beta > 0$ are parameters. The case $\beta = 1$ gives the truncated exponential density function. The expected demand is: $EX = a(p) + \frac{1}{\alpha\beta}\Gamma(\frac{1}{\beta})$, where $\Gamma(\frac{1}{\beta}) = \int_0^\infty x^{\frac{1}{\beta}-1}e^{-x}dx$ is the gamma function. The distribution function is $F_x(x) = 1 - e^{-[\alpha(x-a(p))]^{\beta}}$. By using the inverse $F^{-1}$ according to (5), the supply function is:

$$S(\lambda; \tau, \delta, a(p)) = a(p) + \frac{1}{\alpha}(Ln[1 + (\frac{1+\tau}{1-\delta})(\frac{\lambda}{1-\lambda})])^{(1/\beta)} \qquad (7)$$

If $a = 0$, a reduced form is obtained by taking logarithms on both sides of (7) :

$$LnS = \ -Ln(\alpha) + \frac{1}{\beta}Ln(Ln[1 + (\frac{1+\tau}{1-\delta})(\frac{\lambda}{1-\lambda})]) \qquad (8)$$

A direct derivation of (8) w.r.t. $p$ (or $c$) allows us to obtain the price-elasticity of the supply (demand) in the retail market (wholesale market):

$$\epsilon_{Sp} = \frac{(1+\tau)}{\alpha^{\beta}\beta[(1+\tau)\lambda + (1-\delta)(1-\lambda)](S-a)^{\beta-1}S} \ = -\epsilon_{Dc} \qquad (9)$$

## III- The interactive model: $n$ sellers

A natural extension of the model is to allow for interactions among two or more suppliers of close related goods. In real world, often it is observed significant changes in the quantity supplied by the firms that are not followed by significant changes in market prices. These changes can be accounted for advertising practises, exogenous changes in the horizontal characteristics of the goods or trading facilities, like best locations or greater number of retail outlets. Spillover effects of marketing strategies adopted by the individual firms lead to the enlargement of the market and/or to predation (*business-stealing* effect). In the extended model, sellers go on choosing supply quantities maximizing their expected profits at each price level $p$. However, they now take into account that exogenous shifts in the quantities supplied by rivals may affect, positively or negatively, the sales to their loyal clients. By considering such supply interactions, we intend to examine the conditions under which the optimal sales of the autarchic regime could be either expanded or contracted by introducing nonprice interactions among the sellers. The demand side is modelled by a conditional probability function for each producer.

There are $n$ suppliers of perishable goods, each one facing the conditional demand density $f_i(x, \xi_i | X_1 = S_1; ...; X_{i-1} = S_{i-1}; X_{i+1} = S_{i+1} ... X_n = S_n)$, where $\xi_i$ is a parameter vector. Each supplier looks to his demand density assuming that the supply of the other matches their demand. All sellers have loyal clients, so that the probabilities are positive if $X_i > A_i > 0$. Marginal costs are constant. The supplier $i$ chooses the supply $S_i$ that maximizes his expected profit function $L(X; S_i, \xi_i)$ defined according to (1), with $\tau_i = 0$. Let $X_{-i}$, $S_{-i}$ be the demand and supply vectors obtained by deleting the $i^{th}$ component of $X$ and $S$. The equilibrium condition (4) holds for each producer so that:

$$F_i(S_i; \xi_i | X_{-i} = S_{-i}) = \frac{\lambda_i}{\lambda_i + (1 - \delta_i)(1 - \lambda_i)} \quad , \ i = 1, ..., n. \tag{10}$$

The optimal supply curves are obtained by solving the system (10). Such a solution depends parsimoniously on the specification of the conditional distribution. We assume that the supplies of the competitors affect only the sales for certain of the supplier, not his random sales. More precisely, a linear relationship is considered:

$$A_i(p_i; S_{-i}) = a_i(p_i) + \textstyle\sum_{j=1(j \neq i)}^{n} k_{ij} S_j; \quad i = 1, 2, ..., n \tag{11}$$

A coefficient $|k_{ij}| < 1$ gives the spillover effect of the supply of $j$ over the demand committed to $i$. Since prices are constant, when $k_{ij} > 0$ any market policy generating an expansion of the $j's$ supply induces also an increase in the sales for sure of the good $i$. This causes the $i$'s supply curve to shift upward. If $k_{ij} < 0$, a predatory effect is in force, the expansion of the $j$'s supply causes the seller's $i$ market to reduce. If $k_{ij} = 0 = k_{ji}$, the supplies are independent so that the interactive solution matches the autarchic solution.

In order to improve the visibility of the interactive solution we use a multiplicative structure for the coefficients: $k_{ij} = \phi_i \zeta_j$. If $\phi$ and $\zeta$ indicate the column-vectors of these coefficients, we write the $n \times n$ matrix of these coefficients as $[k_{ij}] = \phi\zeta'$. Note now the diagonal matrix $\Delta \equiv Diag(\phi\zeta')$. Thus, the equation system (11) has the following matrix representation:

$$A(p, S) = a(p) + (\phi\zeta' - \Delta)S \qquad (12)$$

In this system, the $j^{th}$ component of vectors $a(p)$ and $S$ are $a_j(p_j)$ and $S_j$.

**Nash equilibrium.** Assume, without loss of generality, the truncate Weibull model and note: $B_i(p_i) \equiv (Ln[1 + \frac{\lambda_i}{(1-\delta_i)(1-\lambda_i)}])^{(1/\beta_i)}$. As shown by equation (7), the $i^{th}$ equilibrium equation is: $S_i(p_i, S_{-i}) = A_i(p_i, S_{-i}) + \frac{1}{\alpha_i} B_i(p_i)$. Substituting $A_i(p_i, S_{-i})$ in the previous equation by its value given in (11) and recalling that the optimal supply of the good $i$ in autarchy is $S_i^a(p_i) = a_i(p_i) + \frac{1}{\alpha_i} B_i(p_i)$, we arrive to: $S_i(p_i; S_{-j}) = \sum_{j=1(j\neq i)}^{n} k_{ij} S_j + S_i^a(p_i)$, $i = 1, 2, ..., n$. To this system we can give the following matrix representation: $S(p) = (\phi\zeta' - \Delta)S(p) + S^a(p)$. Thus, the interactive optimal supplies $S_i^o(p)$ are linear combinations of the supplies in autarchy:

$$S^o(p) = (I + \Delta - \phi\zeta')^{-1} S^a(p) \qquad (13)$$

For example, with two sellers ($n = 2$), the vectorial equation (13) gives:

$$S_i^o(p_i, p_j) = \frac{1}{1 - k_{ij}k_{ji}} S_i^a(p_i) + \frac{k_{ij}}{1 - k_{ij}k_{ji}} S_j^a(p_j) \ , \ i, j = 1, 2.$$

Assume symmetric spillovers: $\phi_i \zeta_j \ = k$ . In this case, the inverse matrix in (13) can be performed explicitly, which gives:

$$S^o(p) = (\frac{1}{1+k})[I + \frac{k}{1-(n-1)k}\mathbf{1}\mathbf{1}']S^a(p) \qquad (14)$$

where $\mathbf{1}$ is the vector of ones. From (14) it is easily checked that:

$$\mathbf{1}'S^o(p) = \frac{1}{1-(n-1)k}\mathbf{1}'S^a(p) \qquad (15)$$

The optimal supplies are well defined only for $k < \dfrac{1}{n-1}$. Therefore, if $0 < k < \dfrac{1}{n-1}$, we have $\mathbf{1}'S^o(p) > \mathbf{1}'S^a(p)$, and the aggregate supply is greater than that of autarchy. If $k = 0$ we have $\mathbf{1}'S^o(p) = \mathbf{1}'S^a(p)$ and if $k < 0$ the interactive supply is smaller than that of autarchy. If $k > 0$, symmetric positive spillovers are in force. The marketing strategies of the individual sellers stimulate one another to supply greater quantities. Since price changes are not allowed, for the aggregate supply to rise above the autarchy level, the spillover effect cannot be too strong. For example, a $k = 1/10$ ensures an interactive supply higher than the autarchic level only if $n \leqslant 10$. If $k < 0$, negative spillovers create a *business stealing effect* that shifts the supply curves downward. The ultimate aggregate supply is lower than that obtained in autarchy.

**Estimation.** If data on prices $p$ and $c$, and individual supplies $S_i$ are available during $T$ periods, the two-step procedure described below can be used to estimate the unknown parameters of the model. In the $\mathbf{1}^{st}$ stage, the optimal autarchy supplies $S_i^a$

are estimated separately, for each seller $i$, as it is shown ahead, in *section IV.* This also provides the estimates $\widehat{a}_i(p)$ for the deterministic demand. In the $\mathbf{2}^{nd}$ stage, the coefficients $k_{ij}$ from equation (13) are estimated by using the observed supplies $S_i$ and $\widehat{a}_i(p)$. For this aim, the following $n$-equation system is to be considered: $\widehat{a}_i(p_i) = A_i - \sum_{j=1(j\neq i)}^{n} k_{ij}S_j$ , $i = 1, 2, ..., n$. We may view that as a seemingly unrelated equation system. The parameters $A_i$ and $-k_{ij}$ can be estimated by the SUR method (*Seemingly Unrelated Regression*). Once the estimators $\widehat{k}_{ij}$ $(i \neq j)$ are obtained, the solution vector $S^o$ is found by using the equivalent of the equation $(13)$ : $S^o(p) = (I - \widehat{K})^{-1}S^a(p)$, where $\widehat{K} = [\widehat{k}_{ij}]$ with $\widehat{k}_{ii} \equiv 0$.

**Meaning of the estimates.** By assuming that all supplies $S_i^o$ are positive, the sign of the estimates $\widehat{k}_{ij}$ allows the supplier $i$ to know the predatory $(\widehat{k}_{ij} < 0)$, neutral $(\widehat{k}_{ij} = 0)$ or beneficial $(\widehat{k}_{ij} > 0)$ effect generated by the marketing practices of each of his partners $j = 1, 2, ..., n$ $(j \neq i)$. On the other hand, at each observed price vector $p$, the comparison of the actual position $S_i$ with the optimal supplies under autarchy and nonprice competition provides to a seller $i$ useful information for evaluating his own market position. The following cases may occur:

**a)** $S_i \leqslant S_i^a \leqslant S_i^o$ or $S_i \leqslant S_i^o \leqslant S_i^a$ : the seller $i$ is a *poor-performer* and there is an indication that the *status quo* is the worst position to him; **b)** $S_i^a \leqslant S_i^o \leqslant S_i$ or $S_i^o \leqslant S_i^a \leqslant S_i$ : the seller $i$ is a *good-performer* and there is an indication that the *status quo* is the best position to him. In the two other cases, $S_i^o \leqslant S_i \leqslant S^a$ $(S_i^a \leqslant S_i \leqslant S_i^o)$ the seller $i$ might implement his market position with less(more) nonprice competition.

# IV- Application to the green-pepper market

The autarchic model presented in *Section II* is used to analyze the market of green-peppers in Rio de Janeiro(Brasil): 1994/July-2000/November.

## 4.1 Data and estimated equations

The monthly quantities demanded and prices payed by the retailers in the wholesale market are collected from CEASA-Irajá, a state-owned supermarket serving about 70% of the fruit and vegetables market in the metropolitan Rio area. The retail prices were provided by the Price Service of the City Government of Rio de Janeiro. Wholesale and retail prices were deflated by using the wholesale price index (IPA-DI), calculated by the Getulio Vargas Foundation(FGV), and the national consumer price index (INPC), published by the National Bureau of Geography and Statistics(IBGE). The equations are estimated under several random assumptions: Uniform, Pareto, Exponential and Weibull. The marginal cost borne by the retailers is assumed to be constant. Wholesalers usually discriminate prices by granting discounts to customers buying higher quantities. This reduces the unit costs and the total cost becomes a concave function of the quantities. However, a nonlinear price assumption would introduce needless mathematical difficulties, making it uneasy to explicit the supply functions from equation (3). Further, the concavity of wholesale prices could destroy the concavity of the expected profit function, a property that is necessary to obtain the maximizing solution. Fortunately, estimations run with aggregate data attenuate the bias of the linearity assumption.

### 4.2 Value-added and gross profitability in the retail sector

The mean quantities traded monthly in CEASA-Rio during the period 1994/7-2000/11 is 1806.07 tons of pepper. Part of the aggregate demand that wholesalers face is formed by orders issued from hotels, hospitals, restaurants and other institutions. The larger part of total demand(about 90%) is formed by middle-traders supplying for secondary markets or by final retailers that sell the goods directly to final customers. During the period, the traded quantities show a modest increase of 0.29%, *i.e.* about 5.23 tons *per* month. The value-added is measured by the difference between the value of the quantities traded at retail prices and its value at the wholesale prices. It is a gross trading margin for the retail sector, including profits and trading costs. Its value increases about 0.6% monthly during the sample period, and the mean price-cost margin is $\lambda \equiv 0.76$. The high value of $\lambda$ is an indication that the trading channel between wholesalers and consumers could be rather long, implying the existence of secondary markets for the product.

### 4.3 Econometric estimations

The parameters of different supply equations have been estimated for values $\tau = 1$ and $\tau = 0$ of the opportunity cost parameter. The estimates obtained for both values of $\tau$ are very similar, so that only the case $\tau = 0$ is presented. This implies that the suppliers are maximizing accounting profits. The joint estimation of $\delta$ and the other parameters by nonlinear methods is complicated by the singularity of the matrix of the data. Additional variables or extra-sample information would be required to estimate all parameters. Therefore, three hypothesis on the efficiency level of the liquidation market have been considered: $\delta = 0$, 0.5 and 0.9. Preliminary *unit-root tests* for all series used in the regressions were performed. Fortunately, in all cases, ADF and Phillips-Perron statistics lead to the rejection of the null hypotheses, so that the dependent variable and the regressor are stationary. Thus, the Least Squares method provides unbiased and consistent estimators for the parameters. The Weibull model with $a = 0$, performed a better adjustment to the data. The sample shows low wholesale trading levels during June, July and August. In order to capture seasonal effects, a dummy variable was introduced in the supply equation (8), with coefficient $\eta$ as appearing in *Table 1.* The estimated mean of sales to final demand is 1507. 5 tons/month, for $\delta = 0$. The difference w.r.t. the observed mean is 294.5 tons/month, or about 16.34% of the supply. The underestimation may be explained by the non significance of the committed sales in the estimated model. *Table 1* also shows the mean value of the price-elasticity of the supply, noted $\overline{\epsilon}$, which is calculated according to the expression (9).

*Table 1*: **Supply of green-pepper for Weibull demand**

*Econometric estimations (94July/2000Nov.)*$^{(*)}$

| | $Ln(1/\alpha)$ | $1/\beta$ | $\eta$ | $\overline{\epsilon}$ | $R^2$ |
|---|---|---|---|---|---|
| $\delta = 0$ | 7.43 | 0.3302 | $-$ 0.0679 | 0.29 | 0.407 |
| | (468.9) | (6.99) | (2.76) | | |
| $\delta = 0.5$ | 7.27 | 0.4028 | $-$ 0.0677 | 0.29 | 0.407 |
| | (206.6) | (6.984) | (2.76) | | |
| $\delta = 0.9$ | 6.75 | 0.6344 | $-$ 0.0680 | 0.28 | 0.407 |
| | (62.8) | (6.98) | (2.76) | | |

(*) Values in parenthesis are t-Student statistics.

**1.** The 1% significance points of the $Durbin - Watson$ statistics show the same value 1.88 in all cases, which leads to accept the hypothesis of noncorrelated residuals. The $p - value$ for the $F$ statistics in White heteroskedasticity test (with cross terms) is 0.45, indicating that we can accept the homoskedasticity assumption.

**2.** The results are particularly stable w.r.t. the efficiency of liquidation markets for peppers. Figure 3 depicts the estimated curves (8) for $\delta = 0, 0.5, 0.9$. Surprisingly, for retail prices beyond R$1.000/ton. they do indeed coincide at each level of the recovering costs. The same feature is also observed by assuming Pareto or uniform distributions, so that the constancy of the supply does not seems specific to the present demand assumption. For other vegetables traded in the same market, like potatoes, tomatoes and chayote, the estimated supply and density curves shift upward and to the left, respectively, as $\delta$ increases. However, unlike for green-peppers, the committed sales are significant for these vegetables.

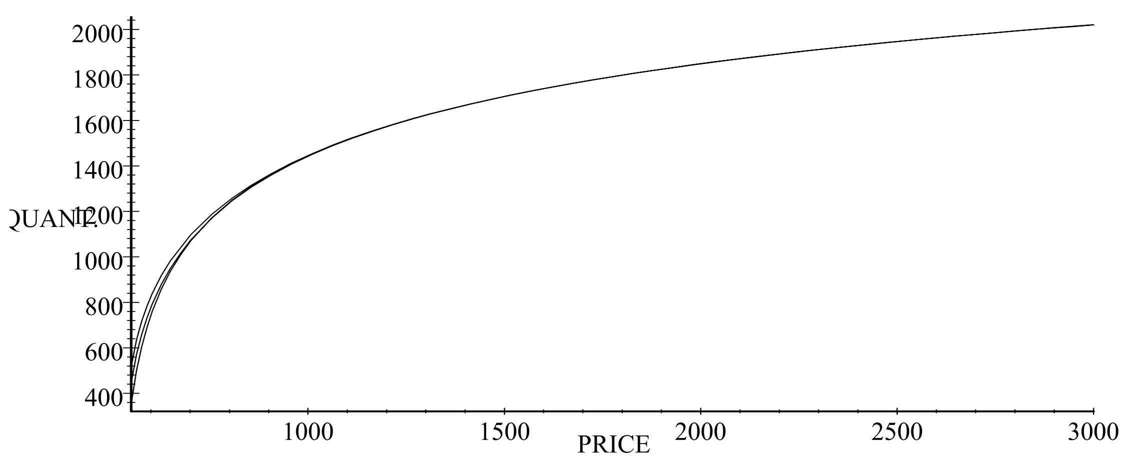


**FIG.3: Supply of green-pepper for Weibull demand.** $\delta = 0,\ 0.5\ , 0.9$.

*Figure 4* below depicts the estimated density of the demand for pepper for $\delta = 0$ (on the right); $\delta = 0.5$ (central curve) and $\delta = 0.9$ (asymmetric curve on the left).

The curves shift to the left as the liquidation market becomes more efficient. However, quantity adjustments are implied but only on the demand side, not on the supply side. Does this legitimate the presumption that noncompetitive practices are in force in the retail segment? Only extra-sample information might confirm such a hypothesis. One possible explanation is that sellers manipulate prices to clear the secondary markets in any circumstances. So, when there is an excess (insufficiency) of supply, they avoid losses by cutting (rising) prices in order to adjust the demand to the quantities available. Obviously, this behavior is replicated throughout the distribution channel so that, at the price payed by the consumers, the same aggregate supply is optimal, no matter how efficient the liquidation market is.

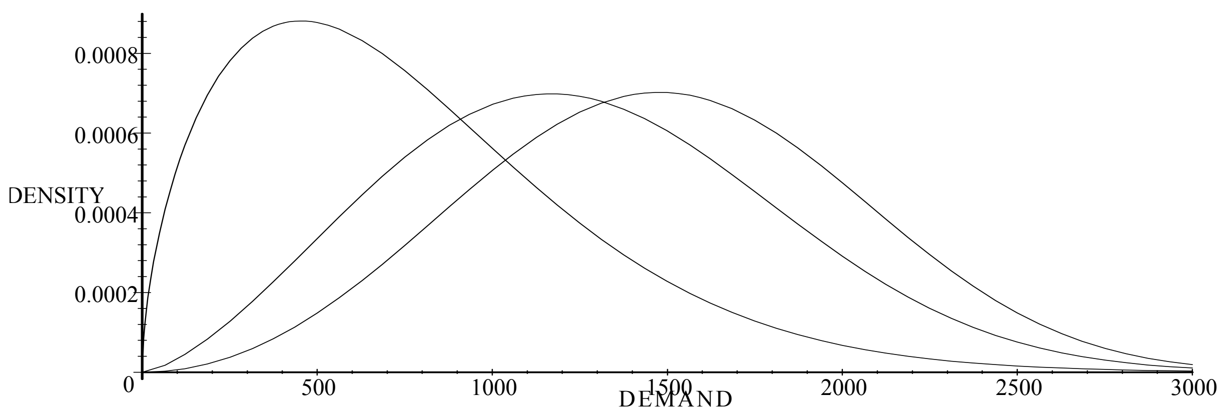


**FIG.4: Estimated Weibull demand of green-peppers.** $\delta = 0$ (curve on right); $\delta = 0.5$ (central); $\delta = 0.9$ (left).

**3.** The long-run price-elasticity of the supply, calculated from (9) is estimated at 0.29. Since $\overline{\epsilon}_{Sp} = -\overline{\epsilon}_{Dc}$, a 10% increase in the retail( wholesale) price increases(reduces) the supply at retail(the demand at wholesale) about 2.9%.

**4.** *International comparisons***.** Few estimations of supply curves for green-peppers have some international visibility. Málaga *et alii* (2001) obtain values 0.08 and 0.12 for the elasticity of the producers' supply of bell-peppers in Mexico (export market) and USA (domestic market), respectively. These values are lower than that obtained here at retail (0.29). With yearly data on vegetables traded in the American market ($1960-1993$), You *et alii*(1997) use the Almost Ideal Demand System(AIDS) to estimate price-elasticities of the demand of several perishables. For peppers, the value obtained is $-0.13$. However, our finding ($-0.29$) is not directly comparable, because it is calculated for the wholesale segment. Further, the $AIDS$ model allows for substitution, whereas we estimate an univariate demand equation.

### 4.4 Profitability and losses

The nontruncate Weibull distribution function provided the best fitting of the data. Since sales for certain to loyal clients are estimated to be 0, it is natural to ask if the random factors affecting the demand at retail are strong enough to causes significant losses in the green-pepper distribution chain. In order to answer this question, we estimate the probability that the profitability rate is nonpositive, at each prices and supply levels. The profitability rate is defined by: $R \equiv (pX - cS)/cS$. The term $pX$ is the gross revenue of the retailers when the aggregate demand is $X$, while $cS$ is the gross revenue of the wholesalers when the quantity sold is $S$. So, $R$ is a gross rate of return in the retail segment. The sample estimate of the demand distribution is used to estimate the distribution of $R$. So, $F_R(r) = 0$ for $r < -1$ and for, $r \geqslant -1$:

$$F_R(r) \equiv P[R \leqslant r] = P[X \leqslant (1-\lambda)S(r+1)] = 1 - \exp\{-(\alpha(1-\lambda)S(r+1))^{\beta}\}$$

The estimated expected value of $R$ is:

$$ER = \tfrac{1}{501.58}(\widehat{1/\beta})(1/\widehat{\alpha})\Gamma(\widehat{1/\beta}) - 1 = 1.111\Gamma(0.33025) - 1 = 2.005\,3.$$

At the estimated expected value, the revenue of the retailers is about 3 times the gross revenue of the wholesalers. The figure below depicts the estimated density of $R$ for $\delta = 0$, by using the values $\widehat{\alpha} = 0.0005926$ and $\widehat{\beta} = 3.028$ obtained from the estimates of $\log(1/\alpha)$ and $1/\beta$ given in the first row of *Table 1*. Three curves are represented, according to different sample values of $(1-\lambda)S$ : the curve on the left side is calculated with the maximum value ($= 824.827$), for periods of low sales; the curve in the middle is calculated with the mean value ($= 501.58$), for periods of normal sales; the curve on the right side is calculated with the minimum value ($= 238.93$), for periods of high sales.

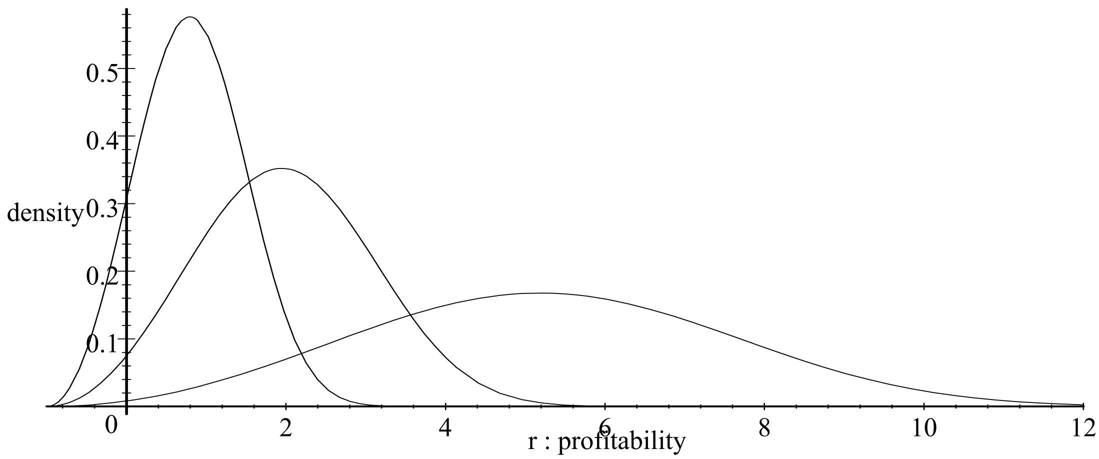


**FIG.5: Estimated density of the profitability rate in the green-pepper market.** Periods of **low sales (** left**); normal sales (**central), **high sales (**right)

The probability of losses of trading downstream in the pepper distribution chain is evaluated by $F_R(0) = P[R \leqslant 0]$. In *Figure* 5 above this probability is calculated by the area below the density for $-1 < r \leqslant 0$. The values of the three areas corresponding to the periods of low, normal and high sales are: 0 .108 ; 0.02 5 and 0.0027, respectively. Figure 6 below depicts the estimated probability along the sample period for $\delta = 0$.

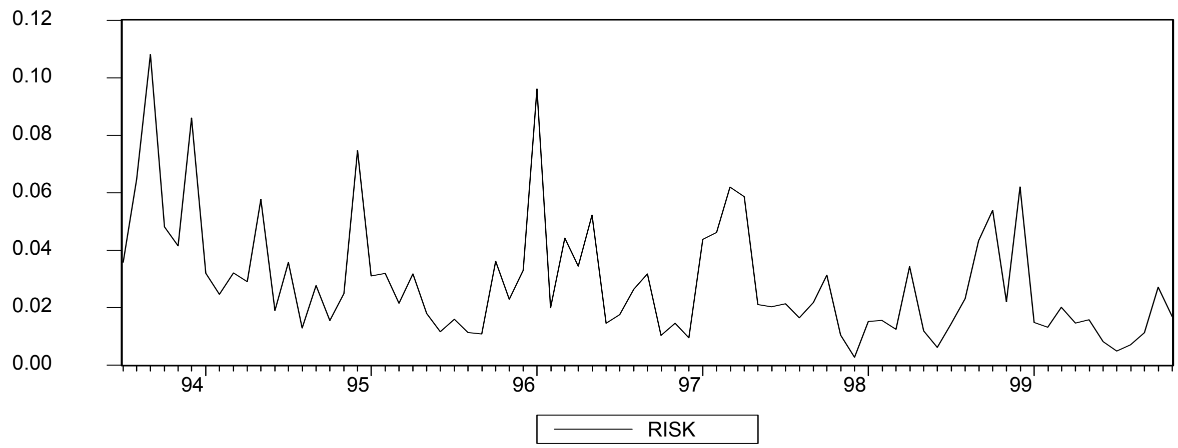


**FIG.6: Estimated probability of losses in the green-pepper market**

The sample probability mean is 2.9%. Of course, a much higher value might be obtained if profitability was measured net of trading costs (transportation + taxes + administrative costs). The calculated probability of losses decreases consistently during the sample period and its variance also decreases as time goes on. By using a linear trend, the probability decreases about 0.00037 points per month, which corresponds to a fall of around 0.86 percent/month during the sample period. This can be explained by the Real stabilization plan established by the Brazilian government in 1994, which succeeded in reducing the monthly inflation rate(IPC/FGV) from 32.1% in 1994/July to 0.87% in 2000/November. Since most losses in the retail sector should be caused by oversupply, the estimated mean may provide a better approximation of the rate of *waste* than that calculated from the estimated expected demand (16.34%

of the mean supply). The value 2.9% is roughly similar as that obtained for tomatoes (2.07%), and corresponds to 52 tons/month of peppers.

## VI- Final comments

We hope the application made above has given a suggestive view of the empirical possibilities opened by the present model. The sellers' objective function takes into account that the quantities unsold cannot be carried on from one period to another. The demand is modelled by a probability distribution function $F$ depending on prices. This assumption does not preclude consumers' cost minimization of utility maximization. If $X_i(p;\theta_i)$ stands for the optimal demand of the individual $i$ with preferences' parameter $\theta_i$, $F(x,p;\theta)$ indicates the proportion of consumers buying at the most $x$ quantities of the good, when the price vector is $p$. Thus, the parameter $\theta$ of the distribution function is related to the individual preferences $\theta_i$. Once $F$ is well-defined, the preferences' parameters can be estimated from the data on quantities supplied rather than on quantities demanded. This is an empirical aspect very useful, since quantities supplied are normally better observed than quantities demanded are. Further, the model is very general and can be applied to a large class of goods like fruits, vegetables, medicines, newspapers, etc. Extensions were made to the autarchic model in order to encompass nonprice interactions among the sellers. For each supplier, it was assumed that the committed sales are significant. Indications were given on how to estimate the spillover coefficients $k_{ij}$. These estimations allow each producer to know the predatory, neutral or beneficial effects of the nonprice marketing practices of each of his partners. They require, of course, firm-level data.

In order to extend the present analysis, more ambitious efforts could be taken in two directions. The first would be to assume price competition in the second stage. The other one would be to focus on multiproduct sellers. Multivariate probability distributions are required to model the demand in this case.

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