A model of credit constraints and bankruptcy

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Abstract

We examine the costs induced by endogenous credit constraints and bankruptcy in the presence of heterogenous shocks –of different types– and the effect on the main variables of our economy: interest rates, production, consumption; and on the distribution of income. Our main model has a fixed factor, which leads to the existence of rents, as these are necessary for a credit market to arise in the presence of moral hazard. We find that sufficiently small shocks do not trigger the credit constraints and have no effects (even in the presence of labor inflexibilities). However, once the shocks are large enough that some agents become credit constrained, the efficiency of the economy decreases and the interest rate falls. These effects become stronger when some agents go bankrupt. We simulate this model to show the importance of these effects on the efficiency of an economy.

1 Introduction

The ability of an economy to withstand shocks without a significant degradation of its performance is a significant advantage. Macroeconomists have shown that the effects of shocks are often due to the existence of credit constraints. We examine the costs induced by credit constraints and bankruptcies induced by moral hazard on the performance of the economy. We also consider the addition of labor inflexibilities and analyze the interaction between credit constraints and labor inflexibility in response to a shock to the endowments of the agents. Our labor constraints take the form of a legal labor restriction that makes it difficult to adapt the number of workers to the new conditions of the firm.

We begin by showing that in a standard constant returns, two mobile factor, static economic model, each one of these two restrictions by itself has no effect on the performance of the economy. On the other hand, when the two conditions appear, the economy is less efficient and we can have bankruptcies. However, this model is not useful for the analysis of credit constraints, because constrained firms never obtain loans, so by the conditions of the model we get these results.

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We proceed to a more realistic model in which credit is possible, because there are rents. This model introduces a third factor of production that is specific to each firm, and that is lost if the firm goes bankrupt. We modify the model by retaining the assumption of constant returns to scale, but separating capital into two types of capital: capital that is specific to each firm, plus working capital, which is fungible. This is a a realistic assumption, since the loss of specific capital following bankruptcy it introduces cost that cannot be avoided and a reason to try to prevent bankruptcies. Specific capital can be anything that is not easily transferable to a different firm: specific equipment, specific knowledge about markets, distribution channels, etc.

We assume constant returns to scale to all three factors, but since one of the factors is fixed, there are decreasing returns to scale to the other factors of production, leading to the existence of rents (as the residual for the fixed factor). When a firm suffers a negative shock to its working capital, firms try to attain the new optimal factor combination, and if they cannot achieve it with their own resources, will require a loan. However, the endogenous credit constraint implies that the firm may not be able to get a loan that will lead it to the optimal factor market combination, or that it may not even be able to get a loan at all. In the case that the entrepreneur receives a loan that is not sufficient to lead the firm to the efficient production point, it will produce inefficiently. If it gets no loan it has to compare the value of two options: to produce under the restricted conditions, or to go bankrupt, losing the specific capital and offering its working capital in the credit market.

In this model we show that credit constraints, even if unaccompanied by labor restrictions, have real effects, altering the efficiency of the economy, so long as the shocks are sufficiently large. However, as in the case of constant returns to scale, labor restrictions by themselves do not affect the efficiency of the economy, though there is an effect on the distribution of income among entrepreneurs.

We simplify the previous model by assuming a fixed relationship between working capital (including any loan obtained) and labor. In particular, in this model, entrepreneurs must be able to pay in advance for the workers employed, i.e., mobile capital is a form of working capital. In this model we are able to characterize the optimal loan of a firm with different stocks of working capital. We use this information to derive the effect of a shock on the equilibrium of the economy. We show that a sufficiently large shock (large enough to make the credit constraint active for some agents) leads to a rise on the optimal size of the firm, a drop in wages and a drop in the interest rate (it also implies that the capital labor ratio increases for the firms that can attain the optimal capital stock. Moreover, we can show that the effect of the labor constraint on the behavior of the agents is to enhance the effect of the shock (so long as the credit constraint is sufficiently strict). We next show that the second model can be recast so as to have a fixed relationship between capital and labor in the equilibrium, and that this means that it inherits all its properties.

Finally, we simulate this model to examine the importance of the credit constraints in the presence of shocks with non-homogenous effects. We use a simple approach which captures the features of our theoretical model. The results confirm the importance of credit constraints and bankruptcy on the efficiency of the economy.

The literature on credit constraints has been mostly macroeconomic, as for example Kiyotaki and Moore (1997), who developed a model of credit constraints and asset prices, and show how fluctuations can be augmented in this scenario and may become persistent. Aghion et al. (2004) use an exogenous market constraint to examine the behavior of the economy in the presence of shocks and show that economies with an intermediate level of financial development can be more instable. There are several differences between our paper and the macroeconomic literature on credit constraints: first, we consider bankruptcies and not only credit constraints; second, because we analyze heterogenous shocks to initially homogenous agents and because we concentrate on the microeconomic aspects of credit constraints and bankruptcy, while avoiding the complications of dynamic analysis.

Another line of research lies in studying efficiency in bankruptcy procedures as in the papers of Bebchuk (1988), Bebchuk (2001), Aghion et al. (1992), Hart (2000), and Hart et al. (1997), among others. For the case of Chile, we have the report of Bonilla et al. (2003).

2 The general model

We examine a very simple model. There is a first period, which serves as a benchmark –there are no dynamics in the model–, with a symmetric economy having a continuum of identical firms whose owners (and the firms themselves) are identified with $z \in [0,1]$. There is also a continuum of workers $L \in [0,1]$ and each worker has an inelastic supply of labor that equals one.

In the first period each entrepreneur owns assets $\bar{K}$, with $\int_0^1 \bar{K} dz = \bar{K}$. Each entrepreneur maximizes profits by hiring $L_z = 1, \forall z$ workers in this full employment initial economy. We assume a small country with free capital markets are free so that capital flows in and out of the country face no constraints.

Workers receive w for the unit of labor offered and obtain a utility of $U^l(c) = c = w$. Firms produce an homogenous good that is also the numeraire and can be stored as the capital in this economy (we can think of it as working capital). Entrepreneurs are risk neutral and maximize $U(c_z) = c_z = \pi(K,L) + (1+r)(K_z - K)$, where $\mathcal{L} \equiv K_z - K$ is the loan (positive or negative) received by the agent. The production function has either constant or decreasing returns to scale and satisfies:

$$x = F(K,L), F_L, F_K > 0, F_{LL}, F_{KK} < 0, F_{KL} > 0, \; F_K \mid_{K=0} = F_L \mid_{L=0} = +\infty$$

The first order conditions in this case are:

$$\left.\frac{\partial F}{\partial K}\right|_{K=\bar{K}} = 1 + r_0, \qquad \left.\frac{\partial F}{\partial L}\right|_{L=1} = w_0$$

Consider now the situation after the economy receives a shock which alters the assets of the entrepreneurs. At the beginning of the second period, the distribution of capital is given by K_z with $E(K_z - \bar{K}) = 0$. We order the entrepreneurs according to their stock of capital after the shock, so that

$$K_z < K_{z'} \text{ if } z < z'. \tag{1}$$

2.1 Tirole's condition

The demand for loans originates in those entrepreneurs z who have assets such that, given the labor constraint, are not at the optimal capital and labor stocks, (K^*, L^*).

For these entrepreneurs, credit rationing, embodied in a moral hazard consideration we denote by *Tirole's condition*, is a possibility (see Tirole (2001)). We assume that entrepreneurs are opportunistic, so that, if they receive a loan, they may consume it instead of using it for the project, and in that case they also consume their after-shock capital stock. In this case, the probability that the project is successful is zero.[1] Let F be the fixed cost of running the firm. An entrepreneur z with $K_z < K^*$ receives a loan $\mathcal{L}_z \leq K^* - K_z$ if and only if the following condition is satisfied:

$$\underbrace{f(K_z + \mathcal{L}_z, L_z) - wL_z - (1+r)(K_z + \mathcal{L}_z) - F}_{Standard profits} + (1+r)K_z \geq (K_z + \mathcal{L}_z)(1+r) \tag{2}$$

where the left hand side corresponds to the utility received by the entrepreneur: profits from the firm plus any rent on capital loans; and the right hand side corresponds to the value that the entrepreneur receives by absconding: his own capital plus the loan. In effect, this condition means that a loan must have a marginal return of at least $2(1+r)$, a term that will become important in our analysis. We can rewrite the inequality in the form of rents being higher than the repayment:

$$\pi_z(K_z + \mathcal{L}_z, L_z, w, 1+r; \gamma) \geq (1+r)\mathcal{L}_z \tag{3}$$

Observe that this condition implies, at a minimum, that the borrowing firm must have positive economic profits (or rents) in equilibrium in order to receive a loan. In turn, this implies that an active credit market is incompatible with a competitive equilibrium when there are constant returns to scale. In turn, this implies that in such an environment, if labor constraints impose inefficiencies, the credit market may be unable to correct them.

2.2 Labor inflexibility

We assume that firms may face labor inflexibilities. We assume a very simple form of inflexibilities: the firm may be unable to adjust its labor to the new conditions of the entrepreneur. The inflexibility is described by the parameter $0 \leq \gamma \leq 1$, which describes the maximum allowed reduction in the pre-shock workforce. In the case $\gamma = 1$, the workforce is totally inflexible and firms that cannot keep

[1] We are assuming that entrepreneurs abscond and cannot be punished.

all their workers must close. In the case $\gamma = 0$, the workforce is totally flexible. Intermediate cases require that $L \geq \gamma$.

2.3 The case of constant returns to scale

The case in which the production function has constant returns to scale is not very interesting, because the credit market cannot exist in the presence of Tirole's condition, as there are no rents. However, it is a useful benchmark to analyze the effect of both labor and credit constraints. Note that in this case, any firm that has the optimal capital-labor ratio can produce efficiently.

First, we show that Tirole's condition, in the absence of further distortions, does not distort the economy; the only effect of the shock is to redistribute income among entrepreneurs.

Proposition 1 *When $\gamma = 0$ (no labor inflexibility), the original factor prices (w_0, r_0) remain the equilibrium factor prices for the economy after the shock. Tirole's condition has no effect.*

Proof Let $\bar{k} \equiv \bar{K}/L$ be the first period capital-labor ratio in this economy. Assume that in the second period all entrepreneurs keep their new stock of capital and hire or fire workers so that they keep the original capital-labor ratio $\bar{k}$. Hence, there is no excess supply of capital or labor. By homogeneity, wages, return on capital and output remain the same as before the shock. By homogeneity of the production function, the factor choices of firms are optimal at the original factor prices (w_0, r_0). ∎

This result shows that in the absence of inflexibilities, the shock has no effect on the economy, except for changes in the distribution of income.[2] The result fails when labor inflexibilities are introduced, because firms might need to borrow, and this is only possible in an economy that generates rents. However, observe that if loan contracts are perfectly enforceable, labor inflexibility is not a problem.

Proposition 2 *Under perfect enforceability of loan contracts, the original factor prices (w_0, r_0) remain the equilibrium factor prices for the economy after the shock. Labor inflexibility has no effect.*

Proof Under perfect enforceability, Tirole's condition does not apply, and any entrepreneur can get a loan that allows him to attain $\bar{K}$, if she pays the market interest rate. With that capital stock, there is no need to fire any agents, so labor inflexibility is not a problem, and the original capita-labor ratio is preserved, with identical firms. The loan is repaid at the market interest rate, since factor prices are (w_0, r_0), and r_0 is the marginal productivity of the last unit of capital used by the borrowing entrepreneur. ∎

[2] Of course, if production is non-homogenous, this result does not hold, since agents with different asset levels optimize at different capital-labor ratios.

Both opportunism and labor rigidity are required for the shock to have an effect in this model: if there is sufficient labor inflexibility (given the extent of the shock), so that at least some firms cannot achieve the optimal capital-labor ratio, some firms will close down, as they cannot sustain losses in a competitive environment, and there are no rents that would allow firms to obtain loans. However, since we have assumed perfect mobility of factors, the fact that firms disappear will have no effect on the economy.

Corollary 1 *If agents are opportunistic and there is sufficient labor inflexibility, the shock will lead to firm closures. However, there will be no effect on the efficiency of the economy.*

In order to continue, it is necessary to analyze a model where there are rents. This requires the existence of decreasing returns to scale, which we examine in the next section.

3 A model with specific capital

We assume that each firm owns specific capital T and that the production function of each firm z is given by $F(T, K_z, L_z)$ which has constant returns to scale. Since T is fixed, this means that there are decreasing returns to scale to the remaining factors, and the function of the remaining factors is homothetic.[3]

Example 1

$$F(T, K_z, L_z) = K_z^{\alpha} L_z^{\beta} T^{(1-\alpha-\beta)}, \quad \alpha + \beta < 1,$$

a constant returns to scale production function. If $T = 1$, *the production function can be rewritten as*

$$f(K_z, L_z) = K_z^{\alpha} L_z^{\beta}, \quad \alpha + \beta < 1.$$

In this case, even under competition there are economic rents that accrue to the fixed factor. These rents can be used to sustain credit, because the agent may be willing to sacrifice some of these rents in order to preserve them, since closing the firm now means that the return to specific capital is lost.

The optimal conditions without restrictions are the same as before: $f_K = 1 + r$; $f_L = w$. Because of the homotheticity of the production function, profits are maximized on a specific point (K^*, L^*) along the ray $k^*(w, r; \gamma) \equiv K^*(w, r; \gamma) / L^*(w, r; \gamma)$ in isoquant space. Before the shock, when the stocks of all resources are the same for all firms, the equilibrium occurs at $\bar{K} = K^*$ and $L = L^*$, i.e., each firm is already at the equilibrium.[4]

[3] The specific capital may be specific equipment, specialized knowledge -perhaps about clients preferences-, distribution systems, etc. A function is homothetic (in a cone D) if for any $x, y \in D$ and $t > 0$, $f(x) = f(y) \Rightarrow f(tx) = f(ty)$, i.e., the indifference curves have parallel tangents along a ray from the origin.

[4] Otherwise, since all the firms are identical, if some other point were the optimal K^*, L^*, there would be a strictly positive (negative) excess supply of capital, so the price of that factor would be either infinite or zero, respectively.

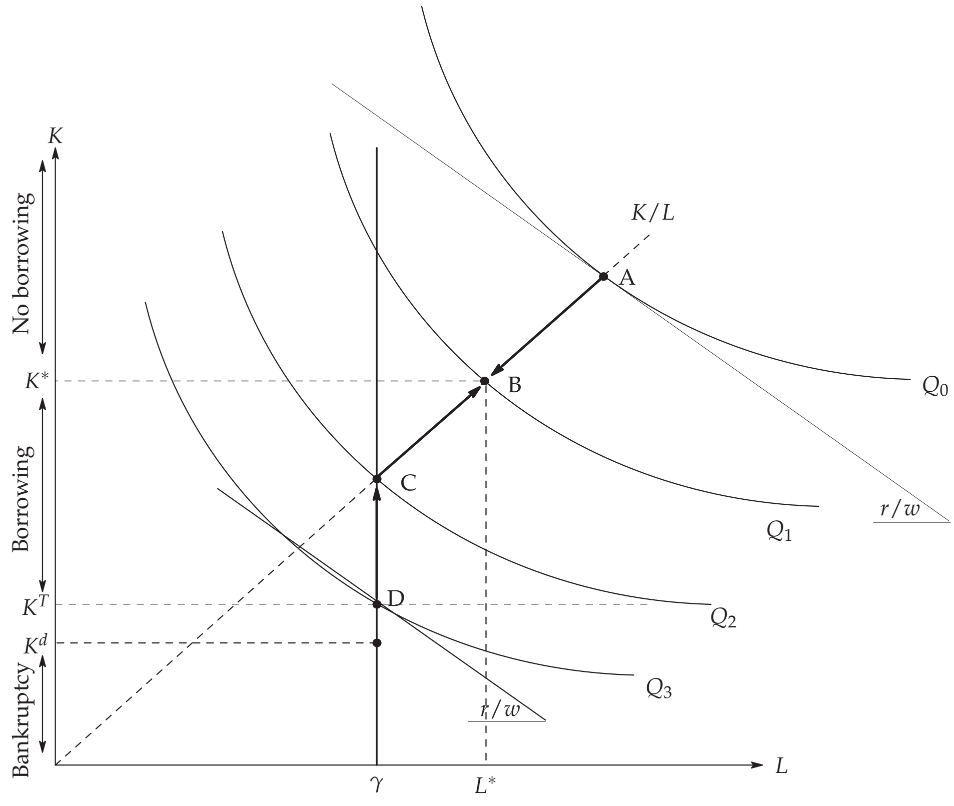


Figure 1: Constrained and unconstrained agents

Proposition 2 continues to hold in this case. If there is a labor market restriction but there is no opportunism, all firms with $K_z < \bar{K}$ can get loans and can hire workers to get to the optimal capital and labor stocks, while those that have more capital and labor shed those factors, because it is not optimal to have them.

On the other hand, Tirole's condition implies real effects of the shock, even if there is no labor market restriction. The problem is that firms that cannot get loans $\mathcal{L} = \bar{K} - K_z$, will not be able to achieve the optimal stock of factors and, on the other hand, there will be firms with capital stocks above the optimal level that need to loan them out, but will receive a lower return on them. Even though all firms may be able to use the optimal capital-labor ratios (because there is no labor constraint), this does not mean optimality, because this condition is attained only at a particular capital and labor stock pair.

Figure 1 shows the behavior of agents in different positions. First, there are agents that have more than the optimal capital and labor (point A) which move towards the optimal point B by shedding capital and labor. Second, the agents that are unconstrained by the labor market restriction, which get loans to move to the optimal point (C to B). Third, those firms that are constrained by the labor market constraint, and that first have to try to remove the restriction by loans. The last agent that can achieve this is agent with stock K^T. However, there may exist yet another class of agents (between K^d and K^T) which prefer to continue to operate even though they do not obtain loans, even if they produce inefficiently.[5] Finally, there is the final type of agents that go bankrupt, lose their specific capital, and can invest their working capital, which is smaller that K^d. These various level of capital are defined as follows:

Definition 1

K^t *is the lowest level of capital that allows loans:*

$$\begin{aligned} K^t &= \min K_z \\ s.t. \quad & f(K_z + \mathcal{L}_z, L_z) - wL_z - (1+r)(K_z + \mathcal{L}_z) - F \geq (1+r)\mathcal{L}_z \\ & \mathcal{L}_Z > 0 \end{aligned}$$

K^d *is the level of capital below which a firm closes down:*

$$\begin{aligned} 0 &= \max_{\mathcal{L}} \pi(K^d + \mathcal{L}, L) \\ s.t. \quad & f(K_z + \mathcal{L}_z, L_z) - wL_z - (1+r)(K_z + \mathcal{L}_z) - F \geq (1+r)\mathcal{L}_z \\ & \mathcal{L}_Z > 0 \end{aligned}$$

which implies that $f(K^d, L) - (1+r)K^d - wL - F = 0$, *because the firm receives no loans.*

K^r *is the lowest level of capital that allows an agent to attain the optimal* (K^*, L^*).

We can immediately obtain one result:

Lemma 1 *The bankruptcy capital stock and the lowest capital that allows a firm to obtain a loan is the same:* $K^d = K^t$.

Proof From the definition of K^t, a firm can ask for loans only if it can obtain positive rents with the loan. A firm is in bankruptcy if it obtains negative or zero profits with any possible loan, i.e., if it cannot get loans. ∎

The next result shows that sol long as the shock is sufficiently large that some agents are constrained, there are real effects.

Proposition 3 *If there is an agent* z *with* $K_z < K^r$ *(there are credit constrained agents), the economy is less productive than before the shock and the interest rate is lower.*

[5] We show below that this class of agents does not exist. Agents that do not get loans prefer to go bankrupt.

Proof Assume first that the optimal capital and labor stocks do not change. Demand for credit is lower because some agents are credit constrained, so they demand less credit. Since the shock has no effect on total capital stocks, at the original optimal capital stock there is an excess supply of capital at the original interest rate. Therefore, the interest rate must fall and the optimal capital stock must increase. The economy is worse off for two reasons: i) there are agents who cannot achieve the optimal capital and labor; ii) the optimal capital stock has increased with respect to the original and in an economy with decreasing returns to scale this reduces productivity.

Finally, in the case in which some agents go bankrupt, these effects are enhanced, because the bankrupt agents will offer their working capital in the market, and they do not demand loans, leading to a fall in the interest rate. Finally, because the specific capital of the firms that go bankrupt is lost, there is a further negative effect. ∎

In order to proceed, we develop a simpler version of this model (we show later that the current model can be translated into the simpler model).

4 A simplified version of the previous model

Assume that the entrepreneur must have working capital on hand before he can hire workers. This restriction could be interpreted as that fact that in a previous period, workers have labored but have not been paid yet. Alternatively, workers require materials before they can start producing, and the amount of materials required by each worker are constant. Using the first interpretation, working capital must pay the wages of workers, so there is a single factor of production, labor. Since we must have $(1+r)(K_z+\mathcal{L}_z) = wL_z$, the production function can be written as:

$$g(K+\mathcal{L}; w)$$

where $\mathcal{L}$ is the loan. It is important to observe that a reduction in the equilibrium interest rate leads to a fall in wages in this model. A positive loan depends on Tirole's condition being satisfied. We can define $\hat{\mathcal{L}}_z$ as the maximum loan allowed by Tirole's condition for an agent z, i.e.,[6]

$$g(K_z+\hat{\mathcal{L}}_z) - F = (1+r)(K_z+\hat{\mathcal{L}}_z) + (1+r)\hat{\mathcal{L}}_z \tag{4}$$

With these definitions we can characterize the loans received as a function of the capital stock of the entrepreneurs. For agents with less capital than the optimal amount, loans are first increasing and then decreasing as the index of the agent decreases (i.e. as his stock of working capital declines). First, because loans are increasing as $z \in [z^t, z^*]$ falls for agents whose loans allow them to reach the optimal capital stock. Second, that entrepreneurs in $[K^t, K^r]$ always ask for the maximum loan they

[6]Note that we have used the fact that $K_z + \mathcal{L}_z = wL$ for the last term in the RHS.

can get, since $g'(K_z + \hat{\mathcal{L}}) > (1 + r)$.

Further results can be derived from an analysis of the two cases in figure 2. As can be seen in the figure, the two cases are distinguished by whether at the point of bankruptcy (i.e., where $g(K^d) - (1+r)K^d - F = 0$) the slope of the production function $g'(K^d)$ is larger that $2(1+r)$. As we have mentioned before, this slope is crucial: when the production function is steeper, additional capital will increase production by so much that the entrepreneur will not be tempted to abscond with the loan.

In the first case in the figure, the slope at the point of intersection is less than $2(1+r)$, so (by the concavity of g) any positive loan for the agent with K^d will not produce enough additional rents to ensure that the entrepreneur does not abscond. The figure also shows the rents of an entrepreneur z with $K^t = K^d < K_z < K^r$ as the thick line between $g(K_z + \hat{\mathcal{L}}_z) - F$ and the cost of capital and loans, $(1+r)(K_z + \hat{\mathcal{L}}_z)$. In this case, as $K_z \to K^d = K^t$, profits and loans decrease smoothly towards zero at the point of intersection.

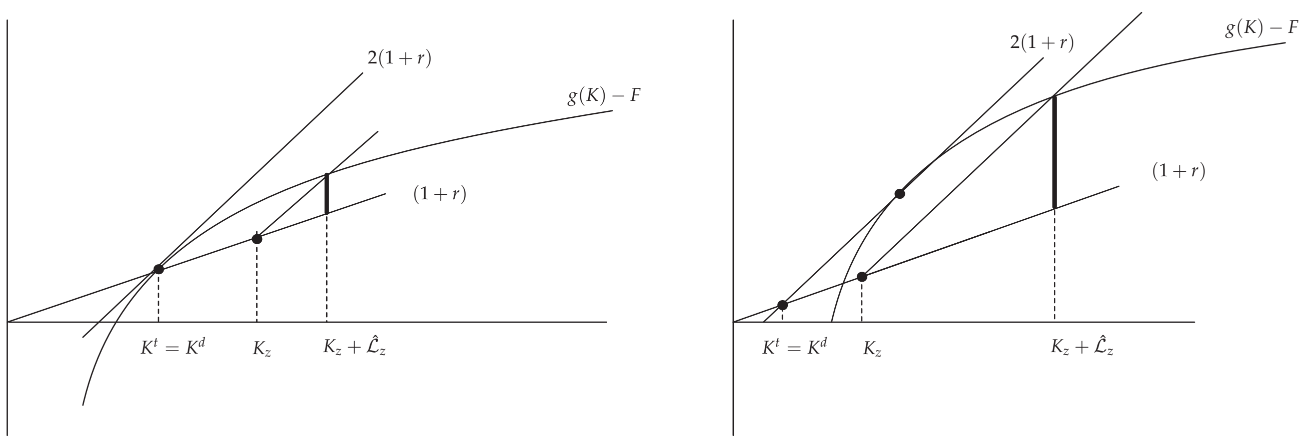


Figure 2: The two possible configurations of the bankruptcy capital stock.

In the second case in the figure, the slope $g'(K_z)$ at the point of intersection between the line $(1+r)K_z$ and the curve $g(K_z) - F$ is larger than $2(1+r)$. This means that at the capital stock of intersection, the firm can ask for strictly positive loans, and therefore the point $K^d = K^t$ must lie to the left of the point of intersection. The lowest level of capital stock that allows loans is the intersection between a line beginning at the origin with slope $(1+r)$ and a line with slope $2(1+r)$ tangent to $g(K_z) - f$. An entrepreneur with that capital stock can receive a strictly positive loan, but any decrease in the capital stock means that no loans are available. In this case, there is a discrete jump (to zero) in the loan function to the left of K^t. For the same reason, rents are strictly positive at K^t and zero to the left. Therefore

Proposition 4 *1. In the case in which $g(K_z) - (1+r)K_z - F = 0$ at a point such that $g'(K_z) < 2(1+r)$, $K_z = K^d = K^t$, loans converge to zero and there are no rents at that point.*

2. *In the case in which $g(K_z) - (1+r)K_z - F = 0$ at a point such that $g'(K_z) > 2(1+r)$, $K_z > K^t$, loans and rents are strictly positive for $K_z' \geq K^t$.*

Proof For part i), notice that to receive a positive loan at K_z, there must be a $\mathcal{L} > 0$ such that Tirole's condition, equation (4), is satisfied. Then, note that

$$(1+r)(K_z + \hat{\mathcal{L}}_z) + (1+r)\hat{\mathcal{L}}_z = g(K_z + \hat{\mathcal{L}}_z) - F < g(K_z) - F + g'(K_z)\mathcal{L}$$

where the first equality comes from Tirole's condition and the inequality is due to the concavity of g. Using the condition $g(K_z) - (1+r)K_z - F = 0$, we have that $2(1+r)\mathcal{L} \leq g'(K_z)\mathcal{L}$, which contradicts the hypothesis of the proposition. Furthermore, at K^t we also have that $g(K_z) - (1+r)K_z - F = 0$, i.e. it is the bankruptcy point when no loans are forthcoming, so $K^t = K^d$. Moreover, totally differentiating equation (4), we get:

$$\frac{d\hat{\mathcal{L}}_z}{dK_z} = \frac{(1+r) - g'(K_z + \hat{\mathcal{L}}_z)}{g'(K_z + \hat{\mathcal{L}}_z) - 2(1+r)} > 0$$

where the numerator is negative in the range $[K^t, K^r]$ (because the firm cannot get loans to get it to K^*) and the denominator is also negative so long as $g'(K_z + \hat{\mathcal{L}}_z) - 2(1+r) < 0$, so loans increase with the capital stock in the range $[K^t, K^r]$. For the same reason, profits tend to zero at K^t.

For part ii), note that since no loans are forthcoming at $K_z < K^t$, $g(K_z) - (1+r)K_z - F < 0$, by the definition of K^t (see figure) and hence $K^t = K^d$. The same differentiation argument as above shows that loans increase with the capital stock in the range $[K^t, K^r]$, since in that range, $g'(K_z) < 2(1+r)$. By definition of $K^t = K^d$ as the intersection of the line beginning at the origin with slope $(1+r)$ and the tangent to $g(K_z) - F$ with slope $2(1+r)$, the maximum loan at K^t is equal to the distance between K^t and the abscissae, which is strictly positive, and rents are also strictly positive. It is also obvious from the figure (or elementary computation), that if $K_z < K^t$, there are no loans. ∎

One of the main results of this paper is the dependency between the shock and the equilibrium prices and quantities in the system. Assume rectangular shocks, i.e., a uniform distribution of shocks centered on the original average (and optimal) capital stock:

Definition 2 *A shock will be a uniform distribution of $K_z \in [K^* - a, K^* + a]$.*

Proposition 5

1. *If the shock is such that $a \leq K^* - K^r$, the only effect is to redistribute income from entrepreneurs with negative shocks to those with positive shocks.*

2. *If the shock is such that $K^* - K^t > a > K^* - K^r$, then r falls and K^* increases.*

Proof The first part is obvious, since all agents can achieve the first best capital stock, there is no efficiency effect due to the shock.

For the second part we proceed by contradiction. Consider the value of the optimal capital stock for two values of the shock $a' > a$. Let K_a^* be the optimal capital stock for the shock of value a. Suppose that K_a^* remains constant when passing from a shock of size a to one of size a'. Then the demand for credit must fall, more agents (in the measure sense) do not have access to the loans they require in order to get to the efficient point. Conversely, the supply of loans increases, since some agents have larger positive shocks than under a. Hence the interest rate must fall to accommodate the credit market. If $K_{a'}^* < K_a^*$, the effect would be even stronger, because some of the agents that were demanding loans before would now have surplus capital and the agents that were offering capital before would have even more to offer (and the agents that require loans will ask for smaller ones). Therefore, in order for supply and demand for capital to reach a new equilibrium, K_a^* must increase. ∎

Thus we have shown that an increase in the size of the shocks leads to higher levels of the optimal capital stocks and lower interest rates, as some firms become rationed in the credit market and reduce their demand for credit. The economy as a whole is less efficient after the shock because of the credit constraint (since the optimal capital after the shock is different from that in the original equilibrium, it must be less efficient). In turn, the fall in interest rates leads to a fall in wages.

Note that we have been careful to restrict the size of the shocks so that no bankruptcies occur. Bankruptcies complicate matters because two effects work in opposite directions. First, firms that close down return their capital stock to the pool of capital and this tends to reduce the interest rate. On the other hand, the larger shock implies that firms that might have been constrained with a smaller positive shock have enough capital to ask for larger loans, increasing the demand for capital. Thus it is not obvious that in this case the interest rate falls. In fact, in the simulations, we observe precisely this phenomenon in those cases in which the fixed cost is high and entry has high costs.

5 Effects of the labor restriction

We have shown that the combination of a sufficiently large shock with the endogenous credit constraint is enough, by itself, to alter the equilibrium of the economy, shifting it to a higher capital-labor ratio, and lowering the interest rate.

The addition of the labor restriction enhances these effects, since it sets a lower limit to the capital stock that allows a firm to continue operating. An entrepreneur z with $K_z < K^r$, such that

$$(K_z + \hat{\mathcal{L}}_z)/w < \gamma, \tag{5}$$

goes bankrupt, even if $K_z > K^t$ (where K^t is the one defined in the absence of labor restrictions). Since this means that additional fixed capital is lost, the inefficiency caused by shocks increases when γ increases, so long as (5) holds (because then the z of the last agent to go bankrupt increases). Using

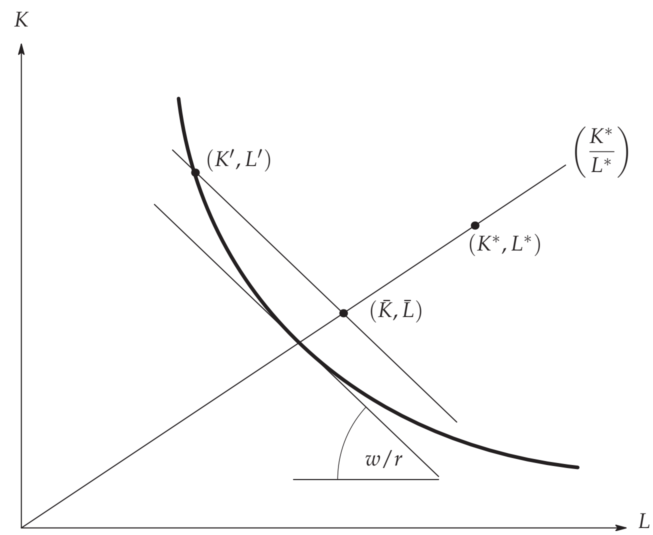


Figure 3: The general model can be reduced to the specific form.

the arguments of proposition 5, an increase in gamma will lead to an increase in K^* and a fall in r.

On the other hand, labor inflexibility has no effect when the shocks are small, i.e., $a \leq K^* - K^r$. In that case, all agents can obtain loans that get them to the optimal point and therefore the labor constraint is non-binding.

Proposition 6 *1. If $a \leq K^* - K^r$, a labor restriction has no effect on the economy.*

2. If for some z with $K^t < K_z < K^r$ we have $(K_z + \hat{\mathcal{L}}_z)/w < \gamma$, an increase in the labor restriction raises K^* and lower r.

5.1 Generalizing the model

Consider the specific factor model of section 3, where working capital and labor do not have fixed proportions. Assume that when factor prices are (w, r), a firm is operating at a point (K', L') off the ray with slope (K^*/L^*) which passes through the optimal point (K^*, L^*), as shown in figure 3. Given the factor prices (w, r) in the equilibrium, the firm would be better off operating at $(\bar{K}, \bar{L})$, which is feasible for the firm, since it lies on an isocost line with slope (w/r) that passes through (K', L'). This means that firms will always be located along the ray (K^*/L^*), and there will be a fixed relationship between the working capital and labor, as in the simplified model. In turn, this implies that the firm will always operate as if it needs a given amount of capital if it wants to hire a certain amount of labor and the analysis and the results of the previous section can be reproduced in this more general model. In particular, propositions 4 and 5 continue to hold.

6 Simulating the model

We use Gams to simulate a simple version of the model. We assume a production function of the type: $g(K) = AK^{\alpha}$, with $0 < \alpha < 1$. We use a uniform distribution of shocks, centered on the original average value of the capital stock, dividing the population of entrepreneurs into N groups. We first compute an equilibrium with loans but without the credit constraint and use the values for the variables interest in this simulation as our benchmark values.

We then check for credit constraints by looking at the difference between profits and repayment of the loan of entrepreneurs and for bankruptcy by checking if rents (excluding repayment of loans) are positive. Observe that entrepreneurs with lower capital stocks are credit constrained or go bankrupt with smaller shocks than those with higher capital after the shock. This means we can proceed iteratively: beginning with the agent with least capital we proceed as follows:

1. If the agent is credit constrained we set his loan at the maximum possible level given the credit constraint (due to decreasing returns to scale, larger loans increase net profits for credit constrained agents) and recalculate the equilibrium.

2. If, in the new equilibrium, the agent has negative profits, the agent is bankrupt, he stops producing and loans his capital stock. We then recalculate the equilibrium. [7].

After this step, we look at the agent with second to last capital, and repeat the procedure, until there are no agents with more capital stock than the agent of the previous iteration that are credit constrained, or until all agents are credit constrained. When the procedure converges, we register the important variables in the solution: total production, interest rate, failed and credit constrained firms. This procedure is implemented in Gams and appears in the appendix.

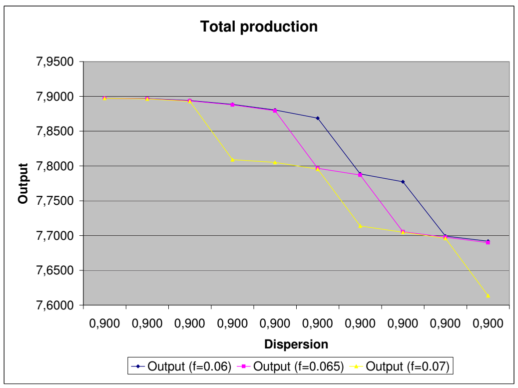
The results for representative runs of the model appear in the figures 4 and in the tables in the appendix. We observe that the effects predicted by the theoretical model are present:

1. As the dispersion increases, and the firms that receive the largest negative shocks become, first constrained, and then go bankrupt, interest rates fall, with a big fall each time there is a bankruptcy.

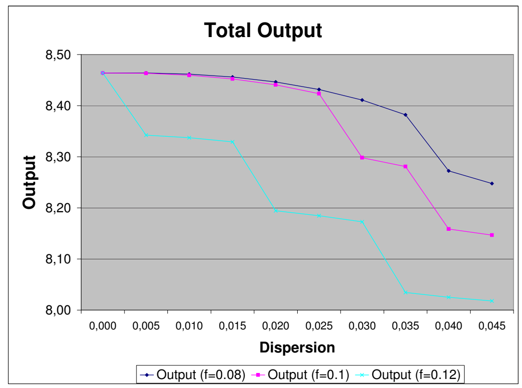
2. Similarly, the economy becomes less efficient, and produces less. First, because constrained firms produce away from the efficient point and second, when firms fail, because the rents associated to the fixed factor are lost.[8]

[7]Note that the assumption that maximum loan under the Tirole condition maximizes profits is due to the fact that these are credit constrained agents, so they cannot reach the optimal capital stock with loans, so any increase in the size of the loan increases profits. See figure ??

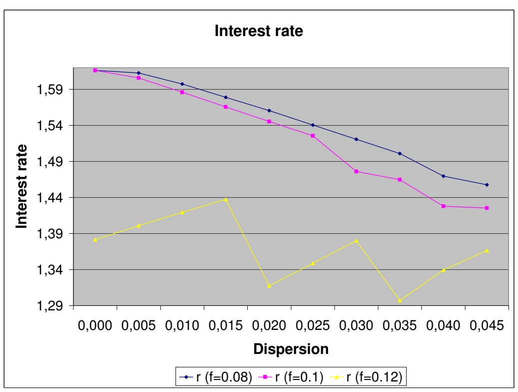
[8]The case of $\alpha + \beta = 0.9$ and fixed cost of $f = 0.12$ is special because in this case all firms are constrained in the original homogenous equilibrium, and in fact one firm needs to go bankrupt in order to release sufficient capital for interest rates to fall in order for the remaining firms to survive (and make profits).



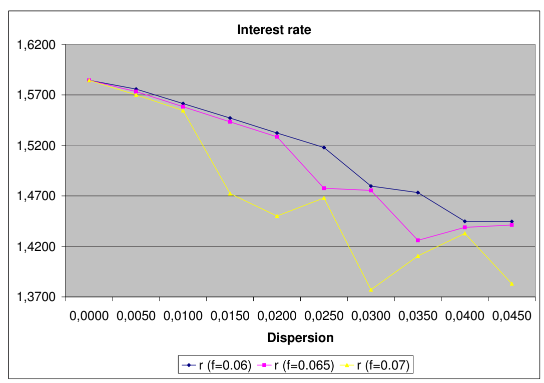
(a) $\alpha + \beta = 0.85$



(b) $\alpha + \beta = 0.9$



(c) $\alpha + \beta = 0.85$



(d) $\alpha + \beta = 0.90$

Figure 4: effects of different size shocks

3. The extent of the effects is not large, but it is significant: the fall in output due to the increased dispersion reached 3-5% of total output, and the interest rates can fall almost 13%.

4. The number of constrained firms can run from 6 to 11, and the number of bankrupt firms in our simulations can run up to 3, and these can be noted in the figures by the large falls in production.

7 Conclusions and extensions

The results of this paper show the importance of credit constraints and bankruptcies on the performance of an economy that is subject to shocks. In our model, neither credit constraints nor bankruptcies are costless. Credit constraints imply that some entrepreneurs do not operate efficiently. Bankruptcies lead to losses because the specific factor (human capital, investment in learning by doing, etc) associated to the firm is lost, even though workers and working capital can move to other occupations. As the size of the shocks increases, more firms become credit constrained and then go bankrupt, and this leads to lower interest rates and lower wages and a loss in output.

Our theoretical propositions are that active credit markets can only exist in economies with decreasing returns, as they require rents in the presence of moral hazard. For this type of economies, we examine shocks that affect agents differentially. We have shown that small shocks may be absorbed without effects by the economies, but as they become larger, they reduce the efficiency of the economy and lower interest rates and wages. These results are confirmed by the simulation results, which also show that the effects are not negligible.

Two interesting extensions are the simulation of the effects of different types of labor rigidities on the economy (for example, rather than prohibiting the firm from reducing its employees beyond a certain amount, employees could be fired at a cost). and trying to incorporate dynamic aspects into the model. As a theoretical result, we have shown that if the labor restrictions are not to constraining, they will have no effect, but that the effects are important once rigidities surpass a certain level.

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A Simulation results

Table 1: Simulations for $\alpha + \beta = 0.85$

$\alpha + \beta$	dispersion	Total quantity	Interest rate	Consumption ratio	Bankruptcies	Constrained firms	Fixed cost
0,0850	0,0000	8,4641	1,6162	1,0000	0,0000	0,0000	0,0800
0,0850	0,0050	8,4640	1,6126	0,9654	0,0000	3,0000	0,0800
0,0850	0,0100	8,4617	1,5974	0,9366	0,0000	5,0000	0,0800
0,0850	0,0150	8,4560	1,5790	0,9124	0,0000	6,0000	0,0800
0,0850	0,0200	8,4463	1,5604	0,8918	0,0000	6,0000	0,0800
0,0850	0,0250	8,4318	1,5405	0,8741	0,0000	7,0000	0,0800
0,0850	0,0300	8,4111	1,5207	0,8588	0,0000	7,0000	0,0800
0,0850	0,0350	8,3820	1,5009	0,8453	0,0000	7,0000	0,0800
0,0850	0,0400	8,2723	1,4696	0,8342	1,0000	8,0000	0,0800
0,0850	0,0450	8,2475	1,4576	0,8233	1,0000	8,0000	0,0800
0,0850	0,0000	8,4641	1,6162	1,0000	0,0000	0,0000	0,1000
0,0850	0,0050	8,4634	1,6057	0,9645	0,0000	5,0000	0,1000
0,0850	0,0100	8,4597	1,5861	0,9353	0,0000	7,0000	0,1000
0,0850	0,0150	8,4524	1,5655	0,9108	0,0000	7,0000	0,1000
0,0850	0,0200	8,4407	1,5453	0,8900	0,0000	7,0000	0,1000
0,0850	0,0250	8,4236	1,5255	0,8722	0,0000	8,0000	0,1000
0,0850	0,0300	8,2980	1,4761	0,8583	1,0000	8,0000	0,1000
0,0850	0,0350	8,2810	1,4647	0,8445	1,0000	8,0000	0,1000
0,0850	0,0400	8,1585	1,4277	0,8338	2,0000	9,0000	0,1000
0,0850	0,0450	8,1468	1,4253	0,8224	2,0000	9,0000	0,1000
0,0850	0,0000	8,4641	1,3813	1,0000	0,0000	0,0000	0,1200
0,0850	0,0050	8,3423	1,4008	0,9643	1,0000	11,0000	0,1200
0,0850	0,0100	8,3373	1,4194	0,9350	1,0000	11,0000	0,1200
0,0850	0,0150	8,3290	1,4373	0,9106	1,0000	11,0000	0,1200
0,0850	0,0200	8,1945	1,3174	0,8912	2,0000	11,0000	0,1200
0,0850	0,0250	8,1845	1,3486	0,8734	2,0000	11,0000	0,1200
0,0850	0,0300	8,1727	1,3801	0,8578	2,0000	11,0000	0,1200
0,0850	0,0350	8,0344	1,2969	0,8456	3,0000	11,0000	0,1200
0,0850	0,0400	8,0251	1,3394	0,8332	3,0000	11,0000	0,1200
0,0850	0,0450	8,0178	1,3665	0,8220	3,0000	10,0000	0,1200

Table 2: Simulations for $\alpha + \beta = 0.90$

$\alpha + \beta$	**dispersion**	**Total quantity**	**Interest rate**	**Consumption ratio**	**Bankruptcies**	**Constrained firms**	**Fixed cost**
0,9000	0,0000	7,8973	1,5846	1,0000	0,0000	0,0000	0,0600
0,9000	0,0050	7,8967	1,5759	0,9643	0,0000	6,0000	0,0600
0,9000	0,0100	7,8940	1,5615	0,9348	0,0000	7,0000	0,0600
0,9000	0,0150	7,8886	1,5471	0,9101	0,0000	8,0000	0,0600
0,9000	0,0200	7,8804	1,5323	0,8890	0,0000	8,0000	0,0600
0,9000	0,0250	7,8688	1,5180	0,8709	0,0000	8,0000	0,0600
0,9000	0,0300	7,7887	1,4797	0,8563	1,0000	9,0000	0,0600
0,9000	0,0350	7,7774	1,4734	0,8422	1,0000	9,0000	0,0600
0,9000	0,0400	7,6993	1,4449	0,8307	2,0000	10,0000	0,0600
0,9000	0,0450	7,6918	1,4446	0,8194	2,0000	9,0000	0,0600
0,9000	0,0000	7,8973	1,5846	1,0000	0,0000	0,0000	0,0650
0,9000	0,0050	7,8965	1,5734	0,9641	0,0000	7,0000	0,0650
0,9000	0,0100	7,8935	1,5584	0,9345	0,0000	8,0000	0,0650
0,9000	0,0150	7,8878	1,5433	0,9097	0,0000	8,0000	0,0650
0,9000	0,0200	7,8792	1,5286	0,8885	0,0000	8,0000	0,0650
0,9000	0,0250	7,7967	1,4776	0,8716	1,0000	10,0000	0,0650
0,9000	0,0300	7,7870	1,4754	0,8557	1,0000	9,0000	0,0650
0,9000	0,0350	7,7054	1,4260	0,8425	2,0000	9,0000	0,0650
0,9000	0,0400	7,6977	1,4389	0,8031	2,0000	10,0000	0,0650
0,9000	0,0450	7,6898	1,4412	0,8188	2,0000	9,0000	0,0650
0,9000	0,0000	7,8973	1,5846	1,0000	0,0000	0,0000	0,7000
0,9000	0,0050	7,8963	1,5700	0,9639	0,0000	8,0000	0,7000
0,9000	0,0100	7,8929	1,5545	0,9342	0,0000	8,0000	0,7000
0,9000	0,0150	7,8091	1,4724	0,9091	1,0000	10,0000	0,7000
0,9000	0,0200	7,8052	1,4501	0,8891	1,0000	11,0000	0,7000
0,9000	0,0250	7,7955	1,4680	0,8709	1,0000	11,0000	0,7000
0,9000	0,0300	7,7138	1,3771	0,8560	2,0000	11,0000	0,7000
0,9000	0,0350	7,7048	1,4105	0,8419	2,0000	11,0000	0,7000
0,9000	0,0400	7,696	1,4329	0,8295	2,0000	10,0000	0,7000
0,9000	0,0450	7,6136	1,3830	0,8191	3,0000	11,0000	0,7000

B The gams program for credit constraints and bankruptcy

```
 *Paper on Bankruptcy: Case050704
*Date: 05/07/04

$EOLCOM !
$OFFSYMXREF OFFSYMLIST
OPTION LIMROW=0, LIMCOL=0;
Set i groups /1*11/;
Set j groups /1*11/;

* Model parameters

Scalars
a               Production function parameter /0.45/
b               Production function parameter /0.4/
d               Distribution parameter /0.0/
gamma           Gamma labor restriction  /0.000001/
S               scale of production /2.5/
Criteria        Criteria /0.000001/
f               costo fijo /0.12/
count           conteo
count2          conteo
niter           numero de iteraciones
;

*Ex post capital allocations are exogenous

Parameters
k(i)            capital allocation per group;

Parameter a parametro while;

k(i) =  1/4+(6-ord(i))*d;

* Endogenous variables

Variables

qt       Total q
r        Interest rate
mu(i)    Shadow price of labor restriction  on i
kf(i)    Final capital in group i
k1       Auxiliar
kd(i)    Loan of group i
q(i)     Production of group i
rep(i)   Repayment of loan of group i
pi(i)    Profits of group i
c(i)     Consumption of group i
t(i)     Test for Tirole group i
```

```
*aux      a+b
cratio   Ratio of max to min consumption
z        Dummy for solver;

* Lower bounds for some variables, to avoid division by zero.

kf.lo(i)=0.00001;
r.lo=0.000001;
q.lo(i)=0.000;
qt.lo = 0.0001;

* Initial values

r.l=0.1;
kf.l(i)=.11;
kd.l(i)=.11;
q.l(i)=.1;
rep.l(i)=.1;
qt.l=1;
mu.l(i)=.0001;
pi.l(i)=.0011;
count = 12;
count2=12;
*aux.l=a+b;
cratio.l=1;

*
* Here are the equations
*

Equations

Capitaleq1(i)            FOC for capital with no Tirole.
Capitaleq2(i)            Equation for capital with Tirole
Complementarityeq(i)     Complementarity restriction on labor.
Loaneq1(i)               Loan equation
mktcl                    Market clearing
production1(i)            Production of group i
production2(i)           produccion de los que quiebran
totalq                   Total production
Profits(i)               Profits of group i
Consumption(i)           Consumption group i
Tirole(i)                Test for Tirole
RazonC                   Razon Consumos max a min
dummy                    Dummy equation for solver;

****************************************************************

Capitaleq1(i)$(ord(i) lt count)..
        S*kf(i)**(a+b-1)*(a+b)-(1+r)+mu(i)=e=0;
Capitaleq2(i)$(ord(i) ge count and ord(i) lt count2)..
```

```
        pi(i)=e=(1+r)*kd(i);
Complementarityeq(i)$(ord(i) lt count2)..
        mu(i)*(kf(i)-gamma)=e=0;
Loaneq1(i)$(ord(i) lt count2)..
        kf(i)=e=k(i)+kd(i);
mktcl..
        sum(i,kd(i))=e=0;
production1(i)$(ord(i) lt count2)..
        q(i)=e=s*kf(i)**(a+b);
production2(i)$(ord(i) ge count2)..
        q(i)=e=0;
*totalq..
*       sum(i$(ord(i)<count2.l),q(i))=e=qt;
totalq..
        sum(i,q(i))=e=qt;
Profits(i)$(ord(i) lt count2)..
        pi(i)=e=q(i)-(1+r)*kf(i)-f;
Consumption(i)$(ord(i) lt count2)..
        c(i)=e=pi(i)+(1+r)*k(i);
Tirole(i)$(ord(i) lt count2)..
        t(i) =e= pi(i)-(1+r)*kd(i);
RazonC..
        cratio =e= c("3")/(c("1")+Criteria);
Dummy..
        z=e=1000;

*****************************************************

Model Case050704
/
Capitaleq1
Capitaleq2
Complementarityeq
Loaneq1
mktcl
production1
production2
totalq
Profits
Consumption
Tirole
RazonC
Dummy
/ ;

SOLVE  Case050704 USING NLP MAXIMIZING z;

******************************************************
* Setting options to save results
* Results will be saved in Results file
```

```
 file results /resultmg.txt/;
 put results;
 results.nd=4;
 put "RESULTADOS MODELO DE QUIEBRAS" /;
 put "Fecha:" system.date /;
 put "Hora:"     system.time //;

 put " - Parametros iniciales" //;
 put @5 "a+b:", @15 (a+b) /;
 put @5 "Par. Distr:", @15 d /;
 put @5 "Costo fijo:", f //;

 niter = 0;

 put " - Resultados iniciales" //;

 put @5 "--------------------------------------------------"/;
 put @5 "Iteracion", @15, ":", @25 niter /;
 put @5 "--------------------------------------------------"//;
 put @5 "Count",                @15, ":", @25 count /;
 put @5 "Count2",        @15, ":", @25 count2/;
 put @5 "Cratio",        @15, ":", @25 cratio.l/;
 put @5 "qt",            @15, ":", @25 qt.l/;
 put @5 "r",                    @15, ":", @25 r.l//;
 put @5 "i", @15, ":";
 loop(i,       put ord(i));
 put /;
 put @5 "ki", @15, ":";
 loop(i, put k(i));
 put /;
 put @5 "kd", @15, ":";
 loop(i,put(kd.l(i)));
 put /;
 put @5 "kf", @15, ":";
 loop(i, put kf.l(i));
 put /;
 put @5 "pi", @15, ":";
 loop(i,put(pi.l(i)));
 put /;
 put @5 "q", @15, ":";
 loop(i,put(q.l(i)));
 put /;
 put @5 "t", @15, ":";
 loop(i,put(t.l(i)));
 put //;

*****************Entrada al ciclo**********************

 if (smin(i,t.l(i)) gt Criteria,
         count=0;
 else
```

```
        count=count-1
);

put " - Resultados Finales" //;

while(count gt 0,

        niter = niter+1;
        SOLVE  Case050704 USING NLP MAXIMIZING z;
        put @5 "--------------------------------------------------"/;
        put @5 "Iteracion", @15, ":", @25 niter /;
        put @5 "--------------------------------------------------"//;
        put @5 "------------------------"/;
        put @5 " Pre Evaluacion Profits"/;
        put @5 "------------------------"/;
        put @5 "Count",                 @15, ":", @25 count /;
        put @5 "Count2",        @15, ":", @25 count2/;
        put @5 "Cratio",        @15, ":", @25 cratio.l/;
        put @5 "qt",            @15, ":", @25 qt.l/;
        put @5 "r",                     @15, ":", @25 r.l//;
        put @5 "i", @15, ":";
        loop(i,        put ord(i));
        put /;
        put @5 "ki", @15, ":";
        loop(i, put k(i));
        put /;
        put @5 "kd", @15, ":";
        loop(i,put(kd.l(i)));
        put /;
        put @5 "kf", @15, ":";
        loop(i, put kf.l(i));
        put /;
        put @5 "pi", @15, ":";
        loop(i,put(pi.l(i)));
        put /;
        put @5 "q", @15, ":";
        loop(i,put(q.l(i)));
        put /;
        put @5 "t", @15, ":";
        loop(i,put(t.l(i)));
        put //;

********************Entrada al ciclo quiebras*******************

        if (sum(i$(ord(i) eq count2-1), (pi.l(i))) lt Criteria,
                count2 = count2-1;
                kd.fx(i)$(ord(i) ge count)  = -k(i);
                pi.fx(i)$(ord(i) ge count2) = 0;
                q.fx(i)$(ord(i) ge count2)  = 0;
                kf.fx(i)$(ord(i) ge count2) = 0;
                SOLVE  Case050704 USING NLP MAXIMIZING z;
```

```
                c.fx(i)$(ord(i) ge count2)  = (1+r.l)*k(i)+f;
        );
        if (sum(i$(ord(i) eq count-1), t.l(i)) lt Criteria,
                count=count-1;
        else
                count=0;
        );
        put @5 "------------------------"/;
        put @5 " Post Evaluacion Profits"/;
        put @5 "------------------------"/;
        put @5 "Count",                 @15, ":", @25 count /;
        put @5 "Count2",        @15, ":", @25 count2/;
        put @5 "Cratio",        @15, ":", @25 cratio.l/;
        put @5 "qt",            @15, ":", @25 qt.l/;
        put @5 "r",                     @15, ":", @25 r.l//;
        put @5 "i", @15, ":";
        loop(i,        put ord(i));
        put /;
        put @5 "ki", @15, ":";
        loop(i, put k(i));
        put /;
        put @5 "kd", @15, ":";
        loop(i,put(kd.l(i)));
        put /;
        put @5 "kf", @15, ":";
        loop(i, put kf.l(i));
        put /;
        put @5 "pi", @15, ":";
        loop(i,put(pi.l(i)));
        put /;
        put @5 "q", @15, ":";
        loop(i,put(q.l(i)));
        put /;
        put @5 "t", @15, ":";
        loop(i,put(t.l(i)));
        put //;
);
```