Nash Equilibrium under Knightian Uncertainty: A Generalization of the Existence Theorem *

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Abstract

Dow and Welang (1994) extended the notion of Nash equilibrium for two-player finite normal games when players are uncertainty on the behavior of his opponents. They showed the existence of equilibrium for any given degree of uncertainty (however constant over all possible events, except the null and the whole event). Using a different definition of support, Marinacci (2000) proved the existence of Nash equilibrium for any given uncertainty aversion function.

In this paper I will extend Dow and Werlang (1994)'s Nash equilibrium under uncertainty using the same definition of support that they used and a parametrical approach, based on the uncertainty aversion function, which enable me to do comparative static exercises in a easy way. I will work with convex capacities that are "squeezes" of (additive) probability measures, as defined in Coimbra-Lisboa (2003).

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1 Introduction

The most well successful definition of Nash equilibrium for two-person normal form games in the presence of Knightian uncertainty is due to Dow and Werlang (1994). With the formalization of Gilboa and Schmeidler they proved the existence of equilibrium for the case of a uniform squeeze (let Σ be the power set of a finite state space, Ω, v be a convex capacity, q be a(n) (additive) probability measure and c a number between 0 and 1, and then, for any $A \in \Sigma$ (except the whole set): $v(A) = (1-c)q(A)$ and in the case of A being the whole set both v and q are 1). Taking a different definition of support from that of Dow and Werlang's paper, Marinacci (2000) extended the proof of the existence for any given uncertainty aversion function.

The purpose of this paper is to extend the proof of the Dow and Werlang's existence theorem of Nash equilibrium under uncertainty, using the same definition of support in their paper (the most useful, general notion and has some advantages over the others definitions). Let Σ, v and q as above defined and $\psi : \Sigma \to [0,1]$ be an uncertainty aversion function i.e., a set-function such that, for any $A \in \Sigma$: $\psi(A) = c(v, A)$ (where the right hand side of this equality is the uncertainty aversion measure of v at event A (see Dow and Werlang (1992)). I will present a restriction over the set of convex capacities, more specifically, I will work with the class of convex capacities that are squeezes of (additive) probability measure (that I will refer as the set $\Theta(\Omega, \Delta)$). Then any capacity $v \in \Theta(\Omega, \Delta)$ can be represented as: $v(A) = (1 - \psi(A))q(A)$, except for the case when A is the whole set (implying v and q are 1). This will enable me to give a parametric approach of the existence result that generalize Dow and Werlang's existence theorem and will be very useful for comparative static exercises over the uncertainty aversion function.

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The paper is organized as follows. The next section introduces the required definitions and basic statements on Knightian uncertainty's decision theory and discuss about the class of convex capacities that are squeezes of (additive) probability measures, a key concept to the extension that I am purposing in this paper. Section 3 I will present an example that will go to motivate the generalization that will be done in this paper. Section 4 gives the definition of Nash equilibrium under uncertainty and present the theorem on the existence of Nash equilibrium that extend the Dow and Werlang (1994)'s existence theorem in a parametric approach based on the uncertainty aversion funcion. Section 5 present some related results and concludes.

2 Set-up and Preliminaries on Capacity Integration

The decision setting that I will use in the paper is developed in a Savage-style (see Savage (1954)). I will assume that the uncertainty a decision maker faces can be described by a non-empty and finite set of states Ω (in this paper: $\Omega = \{\omega_1, ..., \omega_n\}$). Associate with the set of states is the set of events taken to be an algebra of subsets of Ω,denoted by Σ. I assume that for each $\omega \in \Omega$, $\{\omega\} \in \Sigma$. Let χ be a non-empty and finite set of outcomes. Let $\Im$ be the class of all simple acts. A simple act is a finite valued function $f : \Omega \to \chi$ which is measurable with respect to Σ.[1] For $x \in \chi$ I will define $x \in \Im$ to be the constant act such that $x(\omega) = x$ for all $\omega \in \Omega$. So, with slight abuse of notation, I shall let χ also denote the subclass of constant acts in $\Im$.

A set-function $v : \Sigma \to \Re$ with $v(\emptyset) = 0$ is called a capacity (also called a non-additive probability) on (Ω, Σ) if it is normalized and monotone, that is: i) normalized: $v(\Omega) = 1$; ii) monotone: For all A, $B \in \Sigma$ such that $A \subseteq B$: $v(A) \leq v(B)$.[2] I will denote by $V(\Omega, \Sigma)$ the class of all capacities on (Ω, Σ).[3] A capacity is convex if, besides (i) and (ii) it also satisfy the following property: (iii) For all A, $B \in \Sigma$: $v(A \cup B) + v(A \cap B) \geq v(A) + v(B)$. In fact is easy to prove that if a set-function $v : \Sigma \to \Re$ with $v(\emptyset) = 0$ satisfy the property (iii) then the property (ii) is also satisfied, i.e., v is monotone. I will denote by $\Lambda(\subset V(\Omega, \Sigma))$ the class of all convex capacities on (Ω, Σ).

A capacity is (finitely) additive (also called a(n) (additive) probability measure) if, besides properties (i) and (ii) it also satisfy the following property: (iii') For all A, $B \in \Sigma$ such that $A \cap B = \emptyset$: $v(A \cup B) = v(A) + v(B)$. I will denote by $\Delta(\subset \Lambda)$ the class of all (additive) probability measures on (Ω, Σ).

The notion of support of a capacity is the first step necessary to understand the conditions under which a convex capacity can be understood as a squeeze of a(n) (additive) probability measure. Let $v \in V(\Omega, \Sigma)$ and $A, B \in \Sigma$. The support of the capacity v is an event B such that: i) $v(\Omega \backslash B) = 0$; ii) For all $A, B \in \Sigma$, $A \subset B$: $v(\Omega \backslash A) > 0$.

If $v \in \Lambda$ is not a(n) (additive) probability measure then there exists at least a pair A, $B \in \Sigma$ such that: $v(A \cup B) + v(A \cap B) > v(A) + v(B)$. In particular, if $B = (\Omega \backslash A)$ then $v(A) + v(\Omega \backslash A)$ may be less than 1, implying that not all probability mass is allocated to an event and its complement. Dow and Werlang (1992) proposed an uncertainty aversion measure of a capacity v at event A. Let $v \in V(\Omega, \Sigma)$ and $A \in \Sigma$. The uncertainty aversion measure of v at event A, is defined by: $c(v, A) = 1 - v(A) - v(\Omega \backslash A)$.[4] Convex capacities are also know as non-additive probabilities reflecting uncertainty aversion.

Marinacci (2002) presented the properties that are satisfied by the uncertainty aversion measure of v at event A if v is a convex capacity. The uncertainty aversion function is a set-function $\psi : \Sigma \to [0, 1]$ that satisfy the same properties of the uncertainty aversion measure associated to a convex capacity (see Marinacci (2000)), i.e.: i) $\psi(\emptyset) = \psi(\Omega) = 0$; ii) For all $A \in \Sigma$: $\psi(A) = \psi(\Omega \backslash A)$; iii) For all $A, B \in \Sigma$: $\psi(A \cup B) + \psi(A \cap B) \leq \psi(A) + \psi(B)$. I will denote by $\Psi(\Omega, \Sigma)$ the class of all uncertainty aversion function on (Ω, Σ). Associate with each uncertainty aversion function $\psi \in \Psi(\Omega, \Sigma)$ there exists a subclass of convex capacities, $v \in \Lambda$, with the property that, for all $A \in \Sigma$, $\emptyset \neq A \neq \Omega$ the uncertainty aversion measure of v is such that: $c(v, A) = \psi(A)$.[5] So, for each $\psi \in \Psi(\Omega, \Sigma)$, I will denote by $\Lambda(\psi)$ $(\subset \Lambda)$[6] the class of all convex capacities on (Ω, Σ) that are associate to the uncertainty aversion function ψ.

[1] A real-valued function, bounded on Ω, $a : \Omega \to \Re$ is said to be Σ-measurable if, for all open set $O \subseteq \Re$, $a^{-1}(O) \in \Sigma$, where $a^{-1}(O) = \{\omega \in \Omega : a(\omega) \in O\}$. I will denote by $B(\Omega, \Sigma)$ the class of all real-valued function, bounded on Ω, that are Σ-measurable. Note that $\Im(\subset B(\Omega, \Sigma))$.

[2] In this paper it will be used the following notation: $A \subseteq B$ means that the set A is not a proper subset of B (i.e., $A = B$ is possible) and $A \subset B$ means that A is a proper set of B (i.e., $A \neq B$ always).

[3] It is easy to prove that if $v \in V(\Omega, \Sigma)$ then, for all $A \in \Sigma$: $v(A) \in [0, 1]$.

[4] It is easy to prove that: i) if $v \in \Lambda$ then, for all $A \in \Sigma$: $c(v, A) \in [0, 1]$; and ii) if $v \in \Delta$ then, for all $A \in \Sigma$: $c(v, A) = 0$.

[5] In fact, given $\psi \in \Psi$ it is easy to prove that $v \in \Lambda(\psi)$ if and only if $v \in \Lambda$ is such that, for all $A \in \Sigma$, $\emptyset \neq A \neq \Omega$, $c(v, A) = \psi(A)$.

[6] Note that $\Lambda = \{v \in \Lambda(\psi); \psi \in \Psi(\Omega, \Sigma)\}$.

Given $p \in \Delta$ and $\psi \in \Psi(\Omega, \Sigma)$, $v \in V(\Omega, \Sigma)$ is said to be a squeeze of p associate to ψ if it is true that $v(A) = (1 - \psi(A))p(A)$ for all $A \in \Sigma$, except for the case of A being the whole set (in which p and v are 1). Let $\emptyset \neq D = supp\ p \subseteq \Omega$ and $C, C', D, E \in \Sigma$. Coimbra-Lisboa (2003) shows that if ψ also satisfy the following properties:

iv) For all $\emptyset \neq C, C' \subset D$, with $C \cup C' \subset D$ and $C \cap C' = \emptyset : \psi(C) = \psi(C') \geq \psi(D)$; and
v) For all $\emptyset \neq C \subset D$ and all $E \subseteq (\Omega \backslash D) : \psi(CUE) = \psi(C)$

and if $v \in V(\Omega, \Sigma)$ is defined by: $v(A) = (1 - \psi(A))p(A)$ if $A \neq \Omega$ and $v(\Omega) = 1$, then $v \in \Lambda(\psi)$ and also satisfy the following properties:

iv) For all $\emptyset \neq C, C' \subset D$, with $C \cup C' \subset D$ and $C \cap C' = \emptyset : v(C \cup C') = v(C) + v(C')$; and
v) For all $\emptyset \neq C \subset D$ and all $E \subseteq (\Omega \backslash D) : v(C \cup E) = v(C)$

If any $v \in \Lambda$, with $\emptyset \neq D = supp\ v \subseteq \Omega$ also satisfy proprerties iv) and v) above then I will say that v is a squeeze of a(n) (additive) probability measure $p \in \Delta$ (also with the same support) associate to some $\psi \in \Psi(\Omega, \Sigma)$. Let Δ be the class of (additive) probability measures and $\Psi(\Omega, \Sigma)$ be the class of uncertainty aversion function. I will denote by $\Theta(\Delta, \Psi)$ the class of convex capacities that are squeezes of (additive) probability measures associate to some ψ.

Since that capacities can be a non-additive measure I can't use an integral in the sense of Lebesgue. The appropriate notion of integral is due to Choquet (1953). For any given real-valued function, bounded on Ω, $a \in B(\Omega, \Sigma)$, the Choquet integral of a with respect to a capacity $v \in V(\Omega, \Sigma)$ is defined as follows:

$$\int a dv \equiv \int_{-\infty}^{0} [v(\omega \in \Omega : a(\omega) \geq \alpha) - 1] d\alpha + \int_{0}^{\infty} v(\omega \in \Omega : a(\omega) \geq \alpha) d\alpha$$ [7]

where the right hand side is a well defined integral in the sense of Riemann (because a is bounded and v is monotone).[8]

A utility function $u : \Im \to \Re$ defined on the class of simple acts is said to be affine if for any pair of simple acts $f,\ g \in \Im$, and any $\alpha \in (0,1) : u(\alpha f + (1-\alpha)g) = \alpha u(f) + (1-\alpha)u(g)$. Fix an affine utility function $u : \Im \to \Re$, a convex capacities that is squeeze of (additive) probability measures (i.e., $v \in \Theta(\Delta, \Psi)$) and any simple act $f \in \Im$ defined such that: $f(\omega_1) \leq f(\omega_2) \leq ... \leq f(\omega_n)$. Coimbra-Lisboa (2003) showed that the Choquet expected utility of the act f with respect to u and v can be represented as:[9]

$$\int u(f) dv \equiv \psi_1 u(f)(\{\omega_1\}) + (1 - \psi_1) \int u(f) dp + \sum_{j=3}^{n} (\psi_1 - \psi_{j,\dots,n}) \left(\sum_{i=j}^{n} p(\{\omega_i\}) \right) [u(f)(\omega_j) - u(f)(\omega_{j-1})]$$ [10]

Throughout this paper I will restrict attention to convex capacities that are squeezes of (additive) probability measures associated to some uncertainty aversion function and I will use the Choquet expected utlity formula above.

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[7] From now on we will use the following simplification: $v(a \geq \alpha) = v(\{\omega \in \Omega : a(\omega) \geq \alpha\})$. so the Choquet integral can be re-writer as:

$$\int a dv \equiv \int_{-\infty}^{0} (v(a \geq \alpha) - 1) d\alpha + \int_{0}^{\infty} v(a \geq \alpha) d\alpha$$

[8] If v is a(n) (additive) probability measure then the integral is equal to a standard (additive) integral.

[9] I will use the following simplification:
$\psi_1 = \psi(\{\omega_1\})$; $\psi_{1,\dots,i} = \psi(\{\omega_1, ..., \omega_i\})$; and $\psi_{j,\dots,n} = \psi(\{\omega_j, ..., \omega_n\})$

[10] If, for all $A \in \Sigma$, $\emptyset \neq A \neq \Omega$, $\psi(A) = c \in [0,1]$ then it is easy to prove that this formula collapses with the formula of Choquet expected utility with uniform squeeze, i.e.:

$$\int u(f) dv \equiv c u(f)(\{\omega_1\}) + (1 - c) \int u(f) dp$$

3 Nash Equilibrium under Uncertainty

Let $\Gamma = (A_1, A_2, u_1, u_2)$ be a two-person finite game, (also known as a bi-matrix game) where the A_i's are pure strategy sets and u_i's are utilities (payoffs). This will be called the primitive game, or game without: uncertainty. Let us now generalize the definition of Nash equilibrium under uncertainty. The point: of departure will be a well known definition of mixed strategy in standard theory: an additive probability on the space of pure strategies of the player. In the standard theory, a mixed strategy Nash equilibrium can be defined as follows. Let (μ_1, μ_2) be a pair of (additive) probability measures and let *supp* (μ_i) denote the support of μ_i. In Nash equilibrium, every $a_1 \in$ *supp* (μ_1) is a best response to μ_2, i.e. a_1 maximizes the expected utility of player 1 given that player 2 is playing the mixed strategy μ_2, conversely, every $a_2 \in$ *supp* (μ_2) is a best response to μ_1. A subjective interpretation can be given to the Nash equilibrium: the mixed strategy of player I, μ_1, may be viewed as the beliefs that player 2 has about the pure strategy play of player 1. Conversely, the mixed strategy of player 2, μ_2, may be viewed as the beliefs player 1 has about the pure strategy play of player 2.

Now, under uncertainty, what happens is that each player no longer views the strategy of the other player as an additive, but as convex capacity the other player's strategy space. Moreover, I will restrict attention to convex capacities that are squeezes of (additive) probability measures.

Definition 1 *For each player $i \in \{1, 2\}$ let $\psi^i \in \Psi^i(A_i, 2^{A_{-i}})$, where $\psi^i : 2^{A_{-i}} \to [0, 1]$ be the uncertainty aversion function of $v_{-i} \in \Theta(\Delta, \Psi^{-i})$.*[11]

A pair (v_1, v_2) of convex capacities that are squeezes of (additive) probability measures ($v_i \in \Theta(\Delta, \Psi^i)$), v_1 over A_1 and v_2 over A_2 is a Nash equilibrium under uncertainty if there exists a support of v_1 and a support of v_2 such that:

i) for all a_1 in the support of v_1, a_1 maximizes the Choquet expected utility player 1, given that player 1 beliefs about the strategies of player 2 are v_2, and conversely;

ii) for all a_2 in the support of v_2, a_2 maximizes the Choquet expected utility player 2, given that player 2 beliefs about the strategies of player 1 are v_1.[12]

The following theorem that generalize Dow and Werlang (1994)'s result with the use of a uncertainty aversion function of each player $i \in \{1, 2\}$ as a parameter in the game (instead of a constant uncertainty aversion function as they used).

Theorem 1 ***Existence of Nash Equilibrium Uncertainty***

Let $\Gamma = (A_1, A_2, u_1, u_2)$ be a two-person finite game.

Let, for each $i \in \{1, 2\}$: $\psi^i \in \Psi^i(A_i, 2^{A_{-i}})$, where $\psi^i : 2^{A_{-i}} \to [0, 1]$ is the uncertainty aversion function of $v_{-i} \in \theta(\Delta, \Psi^{-i})$. Then, for all (Ψ^1, Ψ^2) there exists a Nash equilibrium (v_1, v_2), where both v_1 and v_2 exhibt uncertainty aversion measure with the properties of convex capacities that are squeezes.

Proof. My proof is the same, in spirit, to Dow and Werlang (1994)'s existence proof.

We now that if $v_i \in \theta(\Delta, \Psi^i)$ then for some $p \in \Delta$ it is true that:

$$v(A) = \begin{cases} \left(1 - \psi^i(A)\right) p(A) & \text{if } A \neq \Omega \\ 1 & \text{if } A = \Omega \end{cases}$$

Thus, Choquet integral has the form:

$$\int f dv \equiv \psi_1 f(\{\omega_1\}) + (1 - \psi_1) \int f dp + \sum_{j=3}^{n} (\psi_1 - \psi_{j,\ldots,n}) \left(\sum_{i=j}^{n} p(\{\omega_i\}) \right) (f(\omega_j) - f(\omega_{j-1}))$$

Suppose, without loss of generality, that player $-i$ has n pure strategies. So we can order the payoffs of player $i \in \{1, 2\}$ to each pure strategy, $a_i \in A_i$, as:

$$u_i^1(a_i) \leq u_i^2(a_i) \leq \ldots \leq u_i^n(a_i)$$

[11]The reason for the ointerchange in the subscripts is that v_{-i} is what player i thinks player $-i$ is going to do, so that the uncertainty aversion of v_{-i} is a characteristic of player i, for $i \in \{1, 2\}$.

[12]The definition above, reduces to the standard definition of Nash equilíbrium, whenever there is no uncertainty (which means that the Ps are additive). Then, when there is no uncertainty the pair (v_1, v_2) reduces to a pair of (additive) probability measures and the Choquet expedted utility reduces to a subjective expected utility. Clearly, a standard mixed strategy Nash equilibrium is also a Nash equilibrium under uncertainty.

where $u_i^j(a_i) = u_i(a_i, a_j)$ is the j-position $(j = 1, ..., n)$.

We modify the original game Γ to $\Gamma_{(\Psi^1,\Psi^2)} = (A_1, A_2; u_1', u_2')$, where if $j = 1, 2$:

$$u_i'(s_i, s_j) = \psi_1^i u_i^1(s_i) + (1 - \psi_1^i) u_i^j(s_i)$$

and, if $j = 3, ..., n$:

$$u_i'(s_i, s_j) = \psi_1^i u_i^1(s_i) + (1 - \psi_1^i) u_i^j(s_i) + \sum_{k=3}^{j} (\psi_1^i - \psi_{k,...,n}^i)(u_i^k(s_i) - u_i^{k-1}(s_i))$$

Note that ψ_1^i is the uncertainty aversion function associate to the strategy of player $-i$ who gives the worst payoff to player $i (\in \{1, 2\})$ when his choice is the pure strategy s_i.

Let (p_1, p_2) be a standard mixed strategy Nash equilibrium of the modified game. We will show that the pair (v_1, v_2), where

$$v_1(A) = \begin{cases} \left(1 - \psi^2(A)\right) p_1(A) & \text{if } A \neq A_1 \\ 1 & \text{if } A = A_1 \end{cases}$$

and

$$v_2(A) = \begin{cases} \left(1 - \psi^1(A)\right) p_2(A) & \text{if } A \neq A_2 \\ 1 & \text{if } A = A_2 \end{cases}$$

is a Nash equilibrium under uncertainty for the original game, with the specified uncertainty aversion functions associated.

To check that this is a Nash equilibrium under uncertainty, note that (except in case where, for each $i \in \{1, 2\}$ and for all $A \in 2^{A_i}$, $\emptyset \neq A \neq A_i$: $\psi^i(A) = 1$) the support of v_i is unique and coincides with the support of p_i for each player $i \in \{1, 2\}$. Since (p_1, p_2) is a standard mixed strategy Nah equilibrium for the modified game, it follows that any $a_i \in supp\ p_i$ is a best response to p_{-i} (for the modified utility u_i'). In other words, si maximizes the following expression over $s \in S_i$:

$$\begin{aligned}
\int u_i'(a_i, \cdot) dp_{-i} &= p_{-i}^1 \left(\psi_1^i u_i^1(a_i) + (1 - \psi_1^i) u_i^1(a_i)\right) + ... + \\
&+ p_{-i}^k \left[\left(\psi_1^i u_i^1(a_i) + (1 - \psi_1^i) u_i^k(a_i)\right) + \sum_{l=3}^{k} (\psi_1^i - \psi_{l,...n}^i) \left(u_i^l(a_i) - u_i^{l-1}(a_i)\right)\right] + ... + \\
&+ p_{-i}^n \left[\left(\psi_1^i u_i^1(a_i) + (1 - \psi_1^i) u_i^n(a_i)\right) + \sum_{l=3}^{n} (\psi_1^i - \psi_{l,...n}^i) \left(u_i^l(a_i) - u_i^{l-1}(a_i)\right)\right] \\
&= \psi_1^i u_i^1(a_i) + (1 - \psi_1^i) \int u_i(a_i, \cdot) dp_{-i} + \sum_{l=3}^{n} (\psi_1^i - \psi_{l,...n}^i) \left(\sum_{r=1}^{n} p_{-i}^r\right) \left(u_i^l(a_i) - u_i^{l-1}(a_i)\right) \\
&= \int u_i(a_i, \cdot) dv_{-i}
\end{aligned}$$

Thus, a_i is also a best response in the original game. In case where, for each player $i \in \{1, 2\}$ and for all $A \in 2^{A_i}$, $\emptyset \neq A \neq A_i$: $\psi^i(A) = 1$ any singleton $\{a_i\}$ is a support of v_i . Therefore any best response for player i is in a support.

Thus (v_1, v_2) is a Nash equilibrium under uncertainty for the original game. ■

The following corollary shows that this theorem generalizes Dow and Werlang (1994)'s existence theorem:

Corollary 1 *Every Dow and Werlang (1994)'s Nash equilibrium under uncertainty is Nash equilibrium under uncertainty as our definition.*

Proof. Consider, for each player $i \in \{1, 2\}$: $\psi^i \in \Psi^i(A_i, 2^{A_{-i}})$, where $\psi^i : 2^{A_{-i}} \to [0, 1]$ is the uncertainty aversion function defined such that, for all $A \in 2^{A_{-i}}$, $\emptyset \neq A \neq A_{-i}$:

$$\psi^i(A) = c_i,\ c_i \in [0, 1]$$

Thus defined, ψ^i exhibits constant uncertainty aversion. So, for each player $i \in \{1, 2\}$, v_i is an uniform squeeze of p_i. These beliefs (v_1, v_2) form Nash equilibrium under uncertainty as in our Definition 1. ■

4 Related Results and Conclusions

The definition of Nash equilibrium under uncertainty provided here, that extends the one presented in Dow and Werlang (1994)'s paper is related with the extension presented in Marinacci (2000)'s paper which proved the existence of Nash equilibrium under uncertainty for any given uncertainty aversion function.

The most important result of this paper is to present the uncertainty aversion function as an explicit parameter in the description of the game, which enable me to do static comparative static exercises in an easy way.

It remains an extension to n players.

References

[1] Choquet, G. (1953): "Theory of Capacities", Ann. Inst. Fourier, 5, 131-295.

[2] Coimbra-Lisboa, P. C. (2003): "Integral Representation with Convex Capacities that are Squeeze of (Additive) Probability Measures", mimeo.

[3] Dow, J. and S. R. C. Werlang (1994): "Nash Equilibrium under Knightian Uncertainty: Breaking Down Backward Induction", Journal of Economic Theory, 64, 305-324.

[4] Marinacci, M. (2000): "Ambiguous Games", Games and Economic Behavior, 31, 191-219.

[5] Savage, L. J. (1954): The Foundations of Statistics,. New York: John Willey. ((Second Edition) 1972, New York: Dover)