On Quantity Competition and Transmission Constraints in Electricity Market*

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Abstract

In this paper we characterize equilibria in a quantity game where symmetric firms face a local demand together with an export-constrained demand. Firms have unlimited access to a local demand but a restricted access to a second market, like in the electricity network where generators compete to satisfy demand but competition is restricted by transmission capacity. We show the existence of an effective demand that is continuous but not differentiable due to the transmission constraint. Three types of equilibria emerge in this context, parametrized by capacity. First, a symmetric equilibrium (unique) when the access to the second market is constrained. Second, a set of continuous and asymmetric equilibria with a fully used link but not constrained; and finally, a symmetric and unique equilibrium in which the link is not fully used. We also show how multiplicity of equilibria tends to disappear as the number of competitors increase.

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1 Introduction

Introducing effective competition in electricity markets presents some challenges that are characteristic of the nature and the usage of electricity. This

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issue is recognized by Joskow and Schmalensee (1983) when they mention that applying mechanisms coming from other industries will not achieve a satisfactory solution. The electricity industry is characterized by a highly variable and inelastic short term demand that leads to a great loss of welfare in case of blackout. At each moment, supply must meet demand and in addition, both are affected by significant transmission constraints. The market equilibrium is also affected by the fact that it is not feasible (economically and technically) to store production. Therefore, inventories cannot be used to mitigate the lack of generation capacity in periods of peak demand; hence it leads to high price. Furthermore, the lack of capacity may also appear in transmission. In effect, most of the transmission lines have not been designed to support the increase in trade when the industry is liberalized. In addition to the lack of capacity, the transmission of electricity creates some network externalities that add complexity to the implementation of a competitive market.

All these reasons can help to explain why many governments pay special attention to the organization of a new industry design with competition (Joskow 1996 and 2001 and Hogan 2000). Basically there are two polar approaches.

In the first one, a central planner collects bids from all participants (generators, consumers -represented by distributors-, large consumers) and it organizes a mandatory dispatch that internalizes the network constraints while maximizing the net consumers' surplus. In case of deviation (i.e. not being available when required), the failing agents are punished.

In the second approach, any agent (generator or distributor) is allowed to trade bilaterally. Since transmission constraints must be satisfied, these agents must bid in a secondary market their willingness to pay for an increase (or decrease) in their trade. Using this information the central planner adjusts trade if necessary.

It is important to remark the lack of a general consensus on the optimal design. In effect, many countries where reforms took place are still redesigning the industry (Wilson, 2002). Understanding how competition takes place was the origin of a large literature in industrial organization.

Schweppe et al. (1981) were the first to describe a mechanism that can be used to price production and consumption, that is, a nodal price system. Hogan (1992) applied this mechanism to decentralize a vertically integrated monopoly under the supervision of a central planner. This central coordinator collects bids for production and consumption and sets the dispatch that maximizes the net consumers' surplus. This dispatch is technically feasible in the sense that it does not overcome the transmission constraint in any line. Hogan also describes a mechanism of financial rights issued with the purpose of protecting users of the transmission system from the volatility of nodal prices but also of determining and remunerating new investment in transmission.

Four years later, Chao and Peck (1996) showed that more decentralization is possible. There is no need to coordinate the market because a system of capacity rights together with bilateral trade is enough to implement the same Pareto optimal allocation as in the Hogan dispatch.

We should remark that an empirical comparison of both organizations is missing up to date (Bower and Bunn, 2000) but as Nasser (1997) points out, examples from finance show that the convergence of both organization types is not complete. In practice, the industry is structured with a mix of centralization and bilateral trade like in England and Argentina, or more recently, in the new PJM and California electricity systems.

There exists now an important literature about the effect of imperfect competition and in some cases, on how the different types of rights affect the dispatch. Oren (1997), Stoft (1997), Nasser (1997), Bushnell (1999), Joskow and Tirole (2000), Willems (2002), Borenstein et al. (2000), Gilbert et al. (2002) are good examples. Most of them will be referred in the next section.

The main objective of this paper is to characterize equilibria in a quantity game where symmetric firms face a local demand together with an export-constrained demand. Firms have unlimited access to a local demand but a restricted access to a second market. We show the existence of an effective demand that is continuous but not differentiable due to the transmission constraint. Three types of equilibria emerge in this context as a function of capacity. First, a symmetric equilibrium (unique) when the access to the second market is constrained. Second, a set of continuous and asymmetric equilibria with a fully used link but not constrained; and finally, a symmetric and unique equilibrium in which the link is not fully used. A similar result was obtained although in a different setting by Stoft (1997) and Borenstein et al. (2000). However, in contrast to the first paper, we fully characterize equilibria and we make explicit the role of the system operator. In addition, we avoid the network externalities created by the effect of counterflows like in the second paper which leads to a simpler description of the strategic behavior of each player. We also show how multiplicity of equilibria tends to disappear as the number of competitors increase.

As mentioned before, the motivation is the electricity industry where each demand represents a region or a country, firms are generators and the technology of transmission is the network. We assume the existence of a central dispatcher, which is in practice the most common way to organize the industry under competition. This dispatcher also fixes the equilibrium prices as a function of the bids made by the Cournot competitors and the capacity of transmission. Generators are not allowed to sale energy directly to consumers, instead, they sale energy into a pool and it is the central planner the agent that allocates the energy among nodes internalizing the network constraint. Therefore, firms do not care about how much of their production is sold in each market, and what matters is the total of energy sold in each market.

The remainder of the paper is organized as follows. Section 2 reviews the literature on price and quantity competition under transmission constraint and Section 3 describes the basic model. Section 4 analyzes two polar cases, that is, perfect competition and monopoly while in section 5 we explore quantity competition. Section 6 briefly describes price competition and section 7 concludes. The Appendix contains most of the proofs.

2 Literature Review on Competition Under Transmission Constraints

Like in the analysis of other industries, electricity competition was largely described by price competition, quantity competition or, in between, the supply function approach.[1] Introducing transmission constraints reduces the set of references but it is still a large number. As mentioned above, Hogan (1992) and Chao and Peck (1996), analyze price competition when firms behave competitively in the context of transmission constraints.

Price competition and transmission constraints were analyzed by Nasser (1997) through auction theory. In his PhD thesis, Nasser was the first to deal with mechanism design theory and transmission constraint. In effect, using the optimal auction mechanism to analyze the unconstrained case and the multi-units auction for the constrained one, Nasser analyses in a three-node

[1] A good survey about electricity competition can be found in Fabra and Harbord (2001)

example how transmission constraints affect the predicted outcome. The network setting consist of two generators and an inelastic demand; each of them is located at different nodes.

When transmission constraints are active, Nasser shows that price cap can be the solution to prevent excessively high prices (but this solution comes at some cost, as the author points out) when implementing the optimal auction. Another source of inefficiency in the allocation process is described but related to the existence of asymmetric information.

The supply function approach under transmission constraint with endogenous transmission charge is not developed yet (to our knowledge).

When the decision variables in competition are quantities, Hogan (1997) and Oren (1997) analyze the case where the transmission is modeled as a constraint in the optimization program of each agent. This means that each agent reduces and coordinates its strategy space with the rival's strategy space. This game is not a standard one defined in Fudenberg and Tirole (1991) but it is a "generalized Nash equilibrium (GNE) game," a "social equilibrium game," or a "pseudo-Nash equilibrium game." Stoft (1997) criticizes this approach:

"This definition of a game allows one player's set of legal moves to depend on the other player's choice of move, even though they both move simultaneously. This allows the definition of a game in which the players' moves automatically satisfy the feasibility constraints. Unlike a standard game, this specification does not model some well-defined procedure but instead is meant to mimic an unexplained negotiation that takes place between the two players".

It is precisely in Stoft (1997), Borenstein et al. (2000) and in this paper where the traditional definition of a static game is applied. That is, the game includes the set of players, a strategy space and the payoffs associated to the strategy space. When transmission constraints become binding, the payoff function recognizes this effect but players are not constrained in their strategy space to the rival's choice. Stoft (1997 and 1998) analyzes a two-node network where generation and consumption do not share the same node. Multiplicity of equilibria is found for a linear demand and constant marginal cost with symmetric generators. Borenstein et al. (2000) modify the network setting by allocating at each node a demand together with a monopoly generator. This case adds a new complexity since it creates some externalities due to the net flow property of electricity. That is, opposite flows in the same link are netted off. In such a case, each agent decides over three regimes of demand. First, she can face its local demand minus the rival exports. Second the opposite case, that is, the local demand plus her exports. Finally she can face an integrated market where there is only one large demand. By offering a certain quantity, conditional on the rival's bid and the link capacity, a generator can enjoy some local market power. The authors also show the competitive effect of a capacity expansion.

Smeers and Wei (1997) split the problem of quantity competition defining two markets: energy and transportation. Different equilibrium concepts are used for each market. In the first one, Nash equilibrium while in the second one, a market-clearing equilibrium are applied. The interaction of both markets results in the aggregate equilibrium of the game. Willems (2002) -using the same setting of Oren (1997), Stoft (1997) and Smeers and Wei (1997)- compares their results using a different game where generators face an exogenous transmission charge. This charge is used to control the trade with respect to the link capacity. Willems also allows for asymmetric generators with constant marginal cost.

Gilbert et al. (2002) analyze the case of import and export market power using a linear demand and constant marginal cost. However, they just consider a situation where the link is constrained since they are also concerned about the value of some congestion and capacity rights.

The equilibrium concept used in this paper is Nash equilibrium where the payoff function has a non-differentiability due to existence of a transmission constraint. In some sense it is similar to the game analyzed by Borenstein et al. (2000) without the externality mentioned above and it makes the description of the agent behavior simpler.[2]

3 The model

Consider a network made of two nodes connected by a capacity of transmission denoted by K. Nodes are labeled $l = a, b$ and at each one, there is a symmetric demand $D_l(p) = D(p)$ with $D'(\cdot) < 0$ and $D''(\cdot) \leq 0$. Suppose demand is bounded in price and quantity, that is, it exists $\overline{p} < \infty$ such that $D(\overline{p}) = 0$ and $D(0) < \infty$. In node a there are two identical generators labeled $i = 1, 2$ with convex cost $C_i(q_i) = C(q_i)$.

[2]The network setting is different in that case.

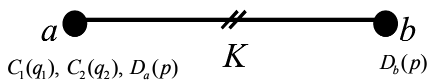

Figure 1: One-line-two-nodes network

In the short term, electricity demand is characterized by a low (or zero) price elasticity. This property can be explained by the absence of substitutes but also by the own characteristics of electricity. On the contrary, in the long term, some consumers can decide to build their own facility with the purpose of obtaining energy or they can choose another technology. Therefore, it is natural to consider some price elasticity. As regards to technology, in the short term, generators face a constant marginal cost up to capacity. However, strictly convex marginal cost can be justified in the long term when generators decide how much capacity they will install. The motivation of this analysis is more related to the effects of long term competition over two regions that are asymmetric from a demand perspective.[3] Even though at node b there is not generation, our model can give some insights in networks like north-south link in UK and the cross-border link between France and UK.

We need more assumptions. Firms must sale their product to a central authority called System Operator (SO). The SO allocates the aggregate offers $Q = q_1 + q_2$ among both demands controlling over the transmission constraint. Generators cannot determine how much of their offers is sold in each market and they are remunerated with a unique price. To be more precise, generators compete a la Cournot bidding quantities to the SO and the latter allocates Q units to both regions through the mechanism described below.

It is clear that consumers located in region b cannot consume more than K units of energy. Therefore and using the fact that SO allocates all the offered units, we can establish a threshold in the aggregate offer $\widetilde{Q}$ such that, for $Q \leq \widetilde{Q}$ consumers in b are not rationed. In effect, this threshold is defined as the maximum aggregate offer such only one price is enough to implement a large market $\widetilde{Q} : \widetilde{Q} - D_a(P(\widetilde{Q})) = K$. Since $P(\widetilde{Q})$ is the same for both market and demand are symmetric, then $D_a(P(\widetilde{Q})) = \widetilde{Q}/2$ and $\widetilde{Q} = 2K$.

We define $D^u(p) = 2D(p)$ for all $Q \leq \widetilde{Q}$ where the superscript u means unconstrained demand. In term of prices, they should not be lower than

[3]The strategy space and the payoff function remain symmetric between generators.

$\widetilde{p} = P(\widetilde{Q})$ to implement the unconstrained demand. Generators receive for their offers $P^u(Q)$ when $Q \leq \widetilde{Q}$.

Otherwise, if $p < \widetilde{p}$ (or $Q > \widetilde{Q}$) demand in b is greater than the capacity of transportation and then consumers in b receive K units at price $P_b(K)$ while consumers in a receive $Q-K$ units at price $P_a(Q-K)$. Since consumers in b are rationed it is consistent to consider that $P_a(Q-K) \leq P_b(K)$ and it occurs when $Q > 2K$. In this case, firms receive $P_a(Q-K)$ for the their sale and we assume that any congestion rents go to consumers as a lump sum transfers or they are used to cover the fixed cost of the transmission operator. We do not consider the case of financial transmission rights. Putting it differently, if there are financial rights, they are owned by agents who are neither consumers nor producers of electricity. The monopoly and perfect competitive case with financial rights have been extensively analyzed by Joskow and Tirole (2000).

We define $D^k(p) = D_a(p) + K$ where the superscript k means constrained demand and it is valid if $p < \widetilde{p}$.

In effect, the effective demand faced by generators is a continuous function of quantity with some non-differentiability at $Q = \widetilde{Q}$. To summarize the energy allocation and the payment rule, we define the effective $D^e(p)$ as:

$$D^e(p) : \begin{cases} D^u(p) = 2D(p) & \text{if } p \geq P(\widetilde{Q}) \\ D^k(p) = D(p) + K & \text{otherwise} \end{cases}$$

In some sense, the mechanism described below resembles a Nodal Price System. However, the main difference resides in the interpretation of quantity bids instead of price bids. A Nodal Price System is a mechanism where production and consumption are priced optimally per node using some objective function (maximization of net consumer's welfare or minimization of dispatch cost when demand is fixed). When generators bid price, the SO uses this information as a proxy of generation cost when it sets the optimal dispatch. Nevertheless, when generators bid quantities, we assume that SO allocates the aggregate offer internalizing the network constraints. In **Appendix A** we show how this allocation is performed optimally through the maximization of consumers' surplus.

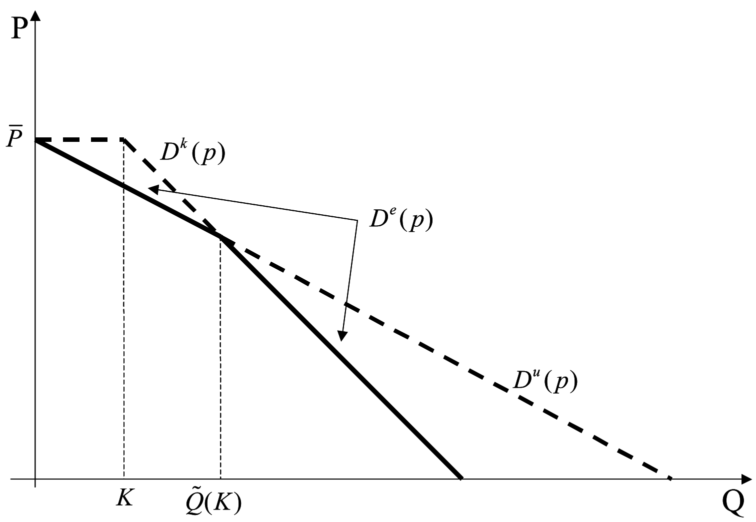

Figure 2: Shape of effective demand

4 Perfect Competition and Monopoly

We begin analyzing these two polar cases since they capture some insights about the effect of competition. Let's assume in this section that $C(Q)$ represents the aggregate cost function. In our framework, a perfect competitive equilibrium is a quantity Q^e_{pc} that solves: $P^e(Q^e_{pc}) = C'(Q^e_{pc})$. This is a general characterization in the sense that Q^e_{pc} does not show when consumers in region b are rationed. A more detailed characterization expresses that $Q^e_{pc} = Q^u_{pc}$ if $P^u(Q^u_{pc}) = C'(Q^u_{pc})$ with $Q^u_{pc} \leq \widetilde{Q}$ or on the contrary, $Q^e_{pc} = Q^k_{pc}$ if $P^k(Q^k_{pc}) = C'(Q^k_{pc})$ with $Q^k_{pc} > \widetilde{Q}$. Note that both equilibria cannot occur simultaneously with $Q^u_{pc} \neq Q^k_{pc}$ since $P^e(\cdot)$ has negative slope and the marginal cost is a monotonic function. Therefore, we can distinguish two types of equilibria: when the link is not constrained (it can be fully used or not) and second, when the link is constrained, that is, when prices differs between regions.

Consider a monopoly and be Q^e_m the equilibrium quantity obtained as: $Q^e_m : \underset{q}{Max}\{P^e(q)q - C(q)\}$. As before, this characterization is general. Using the same argument, if $Q^e_m \leq \widetilde{Q}$ then $Q^e_m = Q^u_m$ with $Q^u_m : \underset{q}{Max}\{P^u(q)q - C(q)\}$. That is, the monopoly outcome occurs in the unconstrained demand. On the contrary, if $Q^e_m > \widetilde{Q}$ then $Q^e_m = Q^k_m$ with $Q^k_m : \underset{q}{Max}\{P^k(q)q -$

$C(q)\}$. In this case the monopoly solution has a maximum on the constrained demand.

Both equilibria cannot occur simultaneously. Note that we do not preclude the possibility that $Q_m^u > \widetilde{Q} > Q_m^k$ since it is certainly possible. It is clear that Q_m^u and Q_m^k cannot be equilibria since they will not be implemented on $P^u()$ and $P^k()$ respectively. The monopolist finds optimal to set $Q_m^e = \widetilde{Q}$. Suppose that she plays a quantity greater than $\widetilde{Q}$. The effective demand is in the constrained region and the optimal quantity for the constrained demand occurs for a quantity lower than $\widetilde{Q}$, therefore, $Q > \widetilde{Q}$ cannot be an optimal outcome. The same argument applies for quantities lower than $\widetilde{Q}$. The intuition of this result is explained by the fact that marginal income is a decreasing function with a discontinuity at $P^e(\widetilde{Q})$.[4,5]

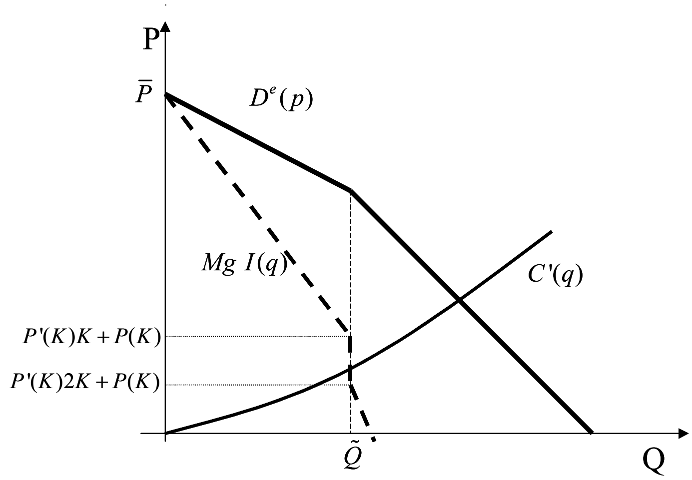

Figure 3: Monopolist Marginal Income (in dashed line)

This discontinuity shows that for any marginal cost that intersects the vertical region of marginal income, the monopoly outcome does not change. A second result is the fact that it is possible to define a set of capacity

[4]It does not happen under perfect competition since agents cannot manipulate the price.

[5]Be the marginal income equal to $P^{e\prime}(q)q + P^e(q)$, but $P^e(\cdot) = \{P^u(\cdot)$ or $P^k(\cdot)\}$ depending on q. If $q = \widetilde{Q}$, then both demands coincide. In addition, we can use the fact that $P^u(Q) = P(Q/2)$ and $P^k(Q) = P(Q-K)$. Therefore, the marginal income evaluated at $q = \widetilde{Q}$ is equal to $P'(K)K + P(K)$ when it is in the unconstrained demand and it is equal to $P'(K)2K + P(K)$ in the constrained demand. It is clear than in the first case, marginal income is greater. Then, a discontinuity exists in marginal income at $q = \widetilde{Q}$.

$(\underline{K}, \overline{K})$ with $\underline{K} < \overline{K}$ such that $Q_m^u > \widetilde{Q} > Q_m^k$. That is, for any value of capacity in such interval, there is not enough capacity to implement an unconstrained equilibrium. On the contrary, there is enough capacity to implement the constrained equilibrium quantity in the unconstrained demand (but in such a case, it is not an equilibrium). Therefore, the equilibrium remains in $Q_m^e = \widetilde{Q}$. It has some consequence in term of congestion rents since there is not different in nodal prices. Therefore, the price differential fails to capture the social valuation for capacity (Nasser 1997, Stoft 1997, Oren 1997, Joskow-Tirole 2000). There is a large set of capacity that fixes the nodal prices different equal to zero while it is socially desirable to expand capacity since it moves the equilibrium to a (still) monopoly unconstrained equilibrium. If expansion is accomplished with some price regulation, the potential benefits are enhanced.

To sum up, when there is perfect competition, the equilibrium can belong to the unconstrained or constrained region. On the contrary, the discontinuity in the monopolist' marginal cost adds the possibility of a new type of equilibrium just at the kink of demand. This third possibility occurs when the link is fully used but not constrained, that is, prices are equal among regions.

We will show in the next section how it affects quantity competition.

5 Quantity Competition

In a short term period, many economists argue that the best way to represent the contest in electricity market is by price competition (Nasser 1997, Klemperer 2000, Fabra et al. 2004). In effect, demand is certainly inelastic and many market architectures allow firms to submit price bids. However in the long term, demand has some elasticity and firms can decide through the installed capacity, how much energy they submit to the market. In this setting of quantity competition we define the profit function of firm i as:

$$\Pi_i(q_i, q_j) = P^e(q_i + q_j)q_i - C(q_i)$$

Note that profit function is a continuous function on q_i, q_j but it is not smooth when demand changes from the unconstrained to the constrained case. It occurs at $q_i + q_j = \widetilde{Q}$.

Also, be $\Pi_i(q_i, q_j)$ a concave function with $|\partial^2\Pi/\partial q_i^2| > |\partial^2\Pi/\partial q_i \partial q_j|$

for $i \neq j$ on the relevant range. This assumption guarantees existence of equilibrium[6]. Even that best respond function has slope lower than one in absolute value, the usual uniqueness condition is not sufficient due to the non-differentiability. It has some consequences in terms of equilibria as we show in this paper.

We define three types of quantity games.

Definition 1 *The effective game (G^e) is defined by the set of two players, firm 1 and 2, with strategy space in the non-negative real numbers and payoff functions:*

$$\Pi_i^e(q_i, q_j) = P^e(Q)q_i - C(q_i)$$
$$with$$
$$P^e(Q) : \begin{cases} P^u(Q) & if \quad Q \leq \widetilde{Q} \\ P^k(Q) & otherwise \end{cases}$$

Also be the unconstrained game (G^u) defined for $P^e(Q) = P^u(Q)$ for all $Q \geq 0$. The constrained game (G^k) is defined for $P^e(Q) = P^k(Q)$ for all $Q \geq 0$

The intuition behind the unconstrained and constrained games is the fact that they allow us to better understand the behavior of each player in the effective game despite the existence of transmission constraints and their impact in the payoff function.

Also, be $r^e(q)$, $r^u(q)$ and $r^k(q)$ the effective, unconstrained and constrained best respond function respectively.

Note that in G^u and G^k, the threshold $\widetilde{Q}$ does not play any role since demand is defined on the non-negative real numbers. In effect, it can be the case where $r^u(q) + q > \widetilde{Q}$ (or $r^k(q) + q < \widetilde{Q}$) and the market is constrained (unconstrained). However we will show that it cannot be the case under $r^e(q)$.

With the purpose of characterizing the shape of the $r^e(q)$, we need the following condition.

Condition 1 *: If it exists $\widehat{q} \geq 0$ such that $r^u(\widehat{q}) + \widehat{q} \geq \widetilde{Q}$, then $r^u(\widehat{q}) \geq r^k(\widehat{q})$.*

Proof. See **Appendix B**. ■

[6] Together with $q \in \Re_0^+$. See Tirole (1988) page 224.

Lemma 1 *:The Effective Best respond Function (EBRF) becomes in:*

$$r^e(q):\begin{cases} r^u(q) & \text{if } \exists\ q \geq 0 \text{ such that } \ r^u(q) \leq \widetilde{Q} - q \\ \widetilde{Q} - q & \text{if } \exists\ q \geq 0 \text{ such that } \ r^u(q) > \widetilde{Q} - q > r^k(q) \\ r^k(q) & \text{if } \exists\ q \geq 0 \text{ such that } \ r^u(q) > r^k(q) \geq \widetilde{Q} - q \end{cases}$$

Proof. See **Appendix C.** ■

When the profit function is differentiable, the best respond function is easier of characterizing, however, it is precisely the non-differentiability that introduces some complexities in the characterization. In particular, we use a sort of revealed preference to prove this lemma. In addition, note that the $EBRF$ has non-positive slope and it is continuous function of q.

With the purpose of characterizing the equilibria of the effective game, it will be useful to define the next two thresholds.

Definition 2 *Be* $\widetilde{q}^u : \underset{q\geq 0}{Min}\{r^u(q) + q \geq \widetilde{Q}\}$ *and* $\widetilde{q}^k : \underset{q\geq 0}{Min}\{r^k(q) + q \geq \widetilde{Q}\}$.

When $\widetilde{q}^u$ or $\widetilde{q}^k$ are positive but lower than $\widetilde{Q}$ it means that $r^u(\widetilde{q}^u)$ or $r^k(\widetilde{q}^k)$ intersects the 45° degree line of $\widetilde{Q} - q$. Figure 4 shows the shape of the $EBRF$ of firm 2. Observe that the line $\widetilde{Q} = q_1 + q_2$ divides the space in two regions (unconstrained and constrained). In (a), the transmission capacity is sufficiently small and consequently $r^u(q)$ belongs entirely to the constrained region. Therefore, it cannot be part of the $EBRF$. In (b), the capacity of transmission is greater than the previous case and it allows to some non-differentiabilities in the $EBRF$. In that case, we can appreciate three possible segments that shape the $EBRF$. For any $q \leq \widetilde{q}^u$ or $q \geq \widetilde{q}^k$(played by firm 1), $r^u(q)$ or $r^k(q)$ are part of the relevant function respectively. And for any $q \in (\widetilde{q}^u, \widetilde{q}^k)$, the relevant $EBRF$ is $\widetilde{Q} - q$. Finally in (c), $r^u(q)$ belongs entirely to the unconstrained region and it shapes the $EBRF$.

In the following example we illustrate how the different possibilities emerge as a function of K.

Example 1 *: Be* $D(p) = 1 - p$, *constant marginal cost normalized to zero and* $K \in (0, 1]$. *In that case* $r^u(q) = 1 - q/2$ *and* $r^k(q) = (1 + K - q)/2$. *If* $K \leq 1/3$ *then* $r^e(q) = r^k(q)$. *When* $K \in (\frac{1}{3}; \frac{1}{2}]$, $r^e(q) = \widetilde{Q} - q$ *for any* $q \leq \widetilde{q}^k$ *with* $\widetilde{q}^k = 3K - 1$ *and* $r^e(q) = r^k(q)$ *otherwise. Finally when* $K \in (\frac{1}{2}; 1]$, $r^e(q) = r^u(q)$ *for any* $q \in [0, \widetilde{q}^u]$, $r^e(q) = \widetilde{Q} - q$ *for any* $q \in (\widetilde{q}^u, \widetilde{q}^k)$ *with* $\widetilde{q}^u = 4K - 2$ *and* $r^e(q) = r^k(q)$ *otherwise.*

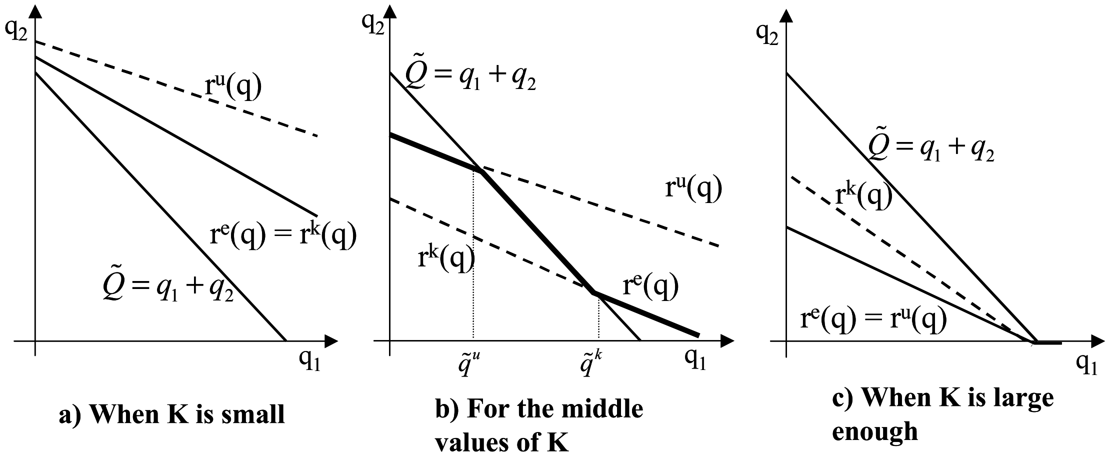

Figure 4: Shape of effective best respond function

The previous example illustrates the effect of transmission capacity in the shape of the effective best respond function. The intuition is the following one: in general, a Cournot duopolist maximizes her own profits over the residual demand. When demand is differentiable, the problem is simpler. It is just to find the optimal trade-off between an increase in the price and a reduction in the offered quantity respect to the marginal cost. However, in our problem, there is a new degree of freedom since the duopolist can chooses on which demand the trade-off is optimal. That is, the firm can bid aggressively by offering a large quantity and since the SO must allocate all offers, the firm gets an extra reduction on the demand elasticity. On the contrary, by bidding not so aggressively, the firm allows for an integration of the two demands in a large market. Moreover, depending on the value of K, it can be the case where the optimal quantity it is located on a region of $P^u()$ (or $P^k()$) that is constrained (unconstrained). In such a case, the optimal (restricted) decision is just to respond $\widetilde{Q} - q$, that is, the maximum (or minimum) quantity without changing the regime. Note in this example how the $EBRF$ changes as a function of K. When capacity is relative small, it is necessary a large reduction in the offered quantity with the purpose of obtaining a large increase in price to unconstrained the market. However it can be non-profitable. As K increases, the possibility to choose between the two types of demand arises depending on the rival decision. Finally, when K is large enough to implement $r^u(2K) = 0$, the problem becomes totally unconstrained.

Before characterizing the equilibria we define the equilibrium concept used in this game

Lemma 2 *A Nash Equilibrium (NE) of the game G^e is a pair (q^e_{1*}, q^e_{2*}) such that:*

$$\begin{array}{ll} \Pi^e_1(q^e_{1*}, q^e_{2*}) \geq \Pi^e_1(q_1, q^e_{2*}) & \textit{for all } q_1 \geq 0 \\ \Pi^e_2(q^e_{1*}, q^e_{2*}) \geq \Pi^e_2(q^e_{1*}, q_2) & \textit{for all } q_2 \geq 0 \end{array}$$

Proof. See Fudenberg and Tirole (1991). ■

In addition, using the same equilibrium concept we can define the pairs (q^u_{1*}, q^u_{2*}) and (q^k_{1*}, q^k_{2*}) as the NE of the game G^u and G^k respectively. Notice by the assumption made in the derivatives of $\Pi()$, in each game, the equilibrium is unique and it is symmetric.

Lemma 3 *The NEs in the game G^e is a pair (q^e_{1*}, q^e_{2*}) characterized by:*

$$(q^e_{1*}, q^e_{2*}) : \begin{cases} q^e_{i*} = q^u_* & \textit{if} \quad 2q^u_* \leq \widetilde{Q} \\ q^e_{i*} = \widetilde{Q} - q \textit{ for } q \in (\underline{q}, \overline{q}) & \textit{if} \quad 2q^u_* > \widetilde{Q} > 2q^k_* \\ q^e_{i*} = q^k_* & \textit{if} \quad 2q^k_* \geq \widetilde{Q} \end{cases}$$

where $\underline{q} = \max\{\widetilde{q}^u; r^k(\widetilde{q}^k)\}$ *and* $\overline{q} = \min\{\widetilde{q}^k; r^u(\widetilde{q}^u)\}$.

Proof. See **Appendix D** ■

When capacity of transmission is large enough, it can be possible to implement a NE over the unconstrained demand. The market is formed by the addition of the two demands. On the contrary, when capacity is relative small, generators find profitable to constraint the market in the local demand plus exports to node b. This local demand also gives the opportunity to increase more the price without a large reduction in the offered quantity respect to the unconstrained demand.

Moreover, when capacity is in between, nor a NE unconstrained can be implemented in $P^u()$ nor a NE constrained can be implemented on $P^k()$ by the SO. The point that balances the incentive to increase the offered quantity in the unconstrained demand or to decrease the offered quantity in the constrained demand is on the 45° degree line of $\widetilde{Q} - q$. For any $q \in (\underline{q}, \overline{q})$ chosen by the rival, the best respond is to produce $\widetilde{Q} - q$ that leads a price equal to $P(\widetilde{Q})$. It is precisely at the kink of the effective demand where the usual condition of uniqueness in Cournot equilibrium is violated. Therefore, as consequence of multiplicity, even that generators are symmetric, equilibria can be asymmetric.

We introduce a last characterization of NEs as a function of the capacity of transmission.

Lemma 4 *: A characterization of the NEs through the capacity of transmission yields in:* (q^e_{1*}, q^e_{2*}) :

$$(q^e_{1*}, q^e_{2*}) : \begin{cases} q^e_{i*} = q^u_* & \text{if} \quad K \geq \overline{K} \\ q^e_{i*} = \widetilde{Q} - q \text{ for } q \in (\underline{q}, \overline{q}) & \text{if} \quad K \in (\underline{K}, \overline{K}) \\ q^e_{i*} = q^k_* & \text{if} \quad K \in (0, \underline{K}] \end{cases}$$

with $\widehat{K} : \min\{K : r^u(2K) = 0\}, \underline{K} : \max\{K : 2q^k_* \geq \widetilde{Q}\}$ *and* $\overline{K} : \min\{K : 2q^u_* \leq \widetilde{Q}\}$.

Proof. The proof follows the same argument as in the previous lemma. $\widehat{K}$ is the minimum quantity of transmission such that all $r^u(q)$ belong to the unconstrained region. $\underline{K}$ is the maximum capacity of transmission such that the constrained symmetric equilibrium is in the unconstrained region while $\overline{K}$ is just the opposite, that is, the minimum capacity of transmission such that the unconstrained symmetric equilibrium is in the constrained region.

Observe that $\widehat{K} \geq \overline{K} \geq \underline{K}$ since q^k_* cannot be greater than q^u_* when $2q^u_* \geq \widetilde{Q}$ due to Condition 1. ■

We can compute the NEs of the last example.

Example 2 *For capacity* $K \geq \frac{2}{3}$, $q^e_* = q^u_* = \frac{2}{3}$. *When* $K \in (\frac{1}{2}, \frac{2}{3})$, $q^e_{i*} = \widetilde{Q} - q$ *for* $q \in (\max\{4K-2, 1-K\}, \min\{3K-1, 2-2K\})$ *and finally, for* $K \in (0, \frac{1}{2})$, $q^e_* = q^k_* = \frac{1+K}{3}$.

The existence of multiple equilibria and the validity of Nash Equilibrium concept deserves some comments. In effect, as Mas Collel et al. (1997) point out, "*Nash equilibrium as a necessary condition if there is a unique predicted outcome of the game*". When there are several equilibria this concept may fail to explain how coordination among all the possibilities will be. Pareto optimality, focal points, reputation or some dynamics concept can be used sometimes to better explain equilibrium. A good analysis of the validity of NE can be found in Mailath (1998).

No focal points or Pareto optimal equilibria exist in this game and the multiplicity survives even if agents are allowed to play making some mistake (Trembling-Hand Perfection). However, among the set of multiple equilibria, there is a unique symmetric equilibrium $q_1 = q_2 = K$ that can survive in a dynamic game if we assume the existence of some adjustment cost of quantities per period[7]. That is, firms prefer to coordinate "collude" on certain level of production. On the contrary, if there is not such a cost, another strategy is to bid in period t, $q_i = \widetilde{Q} - \underline{q}$ and $q_{-i} = \underline{q}$ while in period $t+1$, $q_i = \widetilde{Q} - \overline{q}$ and $q_{-i} = \overline{q}$. That is, a bid rotating strategy.

Despite all the critics about the existence of multiplicity of equilibria, it is possible to show that the set of capacity that support this type of equilibrium tends to be empty as the number of competitors increases. Therefore, only an unique symmetric equilibrium exists and it belongs to the unconstrained or constrained region.

Lemma 5 *The set of multiple equilibria is reduced as the number of competitors increase.*

Proof. Consider in Lemma 3 when the equilibrium is $q_{i*}^e = \widetilde{Q} - q$ for $q \in (\underline{q}, \overline{q})$. The condition for such a type of equilibrium is $2q_*^u > \widetilde{Q} > 2q_*^k$ or $Q_*^u > \widetilde{Q} > Q_*^k$. A well known result in the standard Cournot game is when the number of competitors increase the aggregate output becomes more close to the perfect competitive outcome. In our case $Q_*^u(n) \to Q_{pc}^u$ and $Q_*^k(n) \to Q_{pc}^k$ when n (number of competitors) tends to infinity. Therefore, if there is a continuum of equilibria it means that $Q_{pc}^u > \widetilde{Q} > Q_{pc}^k$. Indeed, we know that under perfect competition, the equilibrium is unique. Assume $Q_{pc}^e = Q_{pc}^u$ but then, it implies $Q_{pc}^u \leq \widetilde{Q}$ and it is certainly not possible when there is a continuum of equilibria. The same argument applies when $Q_{pc}^e = Q_{pc}^k$.

To sum up, the set of multiple equilibria is reduced as the number of competitors increase. ■

Increasing competition has two effects. It reduces the oligopolistic rents together with a reduction in the set of multiple equilibria.

[7] That is, a subgame perfect equilibrium in an infinitely repetead game. Note in such a case that the interpretation of long-term competition is less clear when the game becomes dynamic.

6 Price Competition

The results obtained under quantity competition does not remain valid when competition is by price. Suppose that firms have enough capacity of production to satisfy both demands and they bid the minimum price that they are willing to receive up to their capacity. In effect, the consequence of the "winner take all" gives not active role to transmission capacity in the strategy played by firms. For simplicity, suppose constant marginal cost equal to c. If $c \geq P^u\left(\widetilde{Q}\right)$ there is a unique NE with both firms bidding at marginal cost with a not constrained link. Otherwise the equilibrium still remains in price equal to marginal cost but the link is constrained. It is not profitable for any firm to increase its bid beyond the marginal cost since the rival, by undercutting this bid by epsilon, it gets the entire demand with a positive profit.

Consider the case where firms know that they are technologically different in marginal cost $c_1 < c_2$. Suppose that $c_2 > P^u\left(\widetilde{Q}\right) > c_1$. In such a case firm one may have some degree of freedom choosing the best strategy (the market can be constrained or unconstrained).

7 Conclusion and Extensions

Competition in the electricity market was broadly studied in the economic literature. Price competition thought auction theory, the supply function equilibrium and quantity competition are the most common approach used in the analysis. The study becomes more difficult when transmission constraints have to be satisfied. In addition, network externalities become the problem more complex.

The economic literature provides some models under auction theory and quantity competition. Several equilibrium concepts are implemented. However, most of this literature is related to a certain network setting and it does not give a complete characterization of the problem. In addition, assumptions like the role of the SO are not enough clear. Nasser (1997), Borenstein et al. (2000) and Willems (2004) are few exceptions.

In this paper we characterize equilibria in a quantity game where symmetric firms face a local demand together with an export-constrained demand. Firms have unlimited access to a local demand but a restricted access to a second market. We show the existence of an effective demand that is continuous but not differentiable due to the transmission constraint. Three types of equilibria emerge in this context parametrized by capacity. First, a symmetric equilibrium (unique) when the access to the second market is constrained. Second, a set of continuous and asymmetric equilibria with a fully used link but not constrained; and finally, a symmetric and unique equilibrium in which the link is not fully used. A similar result was obtained although in different setting by Stoft (1997) and Borenstein et al. (2000). We also show how multiplicity of equilibria tends to disappear as the number of competitors increase.

We are also analyzed as an extension of this paper, how competition is affected when the congestion rents are used to pay financial rights in hands of generators or consumers. The monopoly and perfect competitive case with financial rights have been extensively analyzed by Joskow and Tirole (2000) but quantity competition with endogenous transmission charge was not study yet.

Removing symmetry (in generation and demand) will be another extension of this paper. In addition, we will analyze the effect of network externalities in a three-nodes network.

Appendix

A. Justifying Cournot Behavior and Nodal Price Dispatch

We adapt Nodal Price Dispatch when generators bid quantity. The SO in the role of Transmission Operator receives the quantity bids q_1, q_2. Let's denote the consumers' surplus at nodes a and b as $S_l(d_l)$ where d_l represents the quantities consumed.

We assume that the SO maximizes welfare, that is, it maximizes the consumers' surplus allocating the offered bids subject to the network constraint.

$$\underset{d_a,d_b}{Max} \qquad S_a(d_a)+S_b(d_b)$$
$$st.$$
$$q_1+q_2-d_a \leq K \qquad (\eta)$$
$$q_1+q_2 \geq d_a+d_b \qquad (\lambda)$$

and using the fact that $S'()=P()$, the FOCs become:

$$P_a(d_a)=\lambda-\eta, \qquad P_b(d_b)=\lambda$$
$$q_1+q_2-d_a \leq K, \qquad \eta\left(q_1+q_2-d_a-K\right)=0, \qquad \eta \geq 0$$
$$q_1+q_2 \leq d_a+d_b, \qquad \lambda \geq 0$$

Consider the case where $q_1+q_2-d_a<K$ (and it implies $\eta=0$). Then $P_a(d_a)=P_b(d_b)$ and $d_a=d_b=\left(q_1+q_2\right)/2$. That is, the line is not congested and the SO allocates the offered quantity on equal basis. Only one price for both regions is required to implement this optimal dispatch. Therefore only one price is enough to integrate both regions in a large *unconstrained* market. Suppliers face a total demand equal to $D^u(p)=2D(p)$ if the line is not congested. As a consequence of the Pool mechanism and since both offers are perfect substitutes, generators do not care if their energy is sale in region a or b.

Suppose now $\eta>0$ ($q_1+q_2-d_a=K$). The first consequence is that region b is constrained in the quantity imported up to K units (and region a, $d_a=q_1+q_2-K$). Then, a rationing using different prices it is optimal when the line is congested. The price in market b will be $P_b(K)$ while in region a, $P_a(d_a)=P_b(K)-\eta$ or $P_a(d_a)=P_a(Q-K)$. This price is used to remunerate generators for their bids. In such a case, the constrained demand faced by generators can be written as $D^k(p)=D_a(p)+K$.

Therefore, if the aggregate offer is greater than certain threshold defined below, the line is congested and it changes the shape of effective demand.

Condition 2 *Given $K\in(0,D(0))$, exists $\widetilde{Q}=2K$ such that if $p\geq P(\widetilde{Q})$ the relevant demand faced by generators in region a is $D^u(p)$. Otherwise $D^k(p)$ is the relevant one.*

Proof. We restrict the set of capacity. In effect, if capacity is greater than $D(0)$, then, for any $Q\leq D(0)$ the line is not congested.

Let's define $Q = \widetilde{Q}$ as the maximum aggregated offers such that the net supply in node a is equal to K. That is $\widetilde{Q} - D_a(P^u(\widetilde{Q})) = K$. Note when $Q = \widetilde{Q}$, $P^u(\widetilde{Q}) = P^k(\widetilde{Q})$ and then, if we use the fact that $P^u(Q) = P(Q/2)$ and $P^k(\widetilde{Q}) = P(Q - K)$ we obtain $P(\widetilde{Q}/2) = P(\widetilde{Q} - K)$ or $\widetilde{Q} = 2K$.

By construction, we know when $Q \leq \widetilde{Q}$ the line should be uncongested. We need to show if $Q > \widetilde{Q}$ then, $D^u(p)$ cannot be part of the effective demand faced by the generators. Assume the contrary, that is $Q > \widetilde{Q}$ and $D^u(p)$ is part of the effective demand. If both demands are integrated into a large market $d_a = Q/2$ and the transmission constraint is not binding: $Q - d_a \leq K$. Replacing $K = \widetilde{Q}/2$ and d_a we find $Q \leq \widetilde{Q}$, a contradiction. ■

To sum up, if the aggregate bid lead in a price p not lower than $P(\widetilde{Q})$, the effective demand is $D^u(p)$, on the contrary it will be $D^k(p)$.

B. Proof of Condition 1

The idea of the proof is the following one. We determine first the minimum quantity of transmission ($\widehat{K}$) such that all $r^u(q)$ belong to the unconstrained region. For such a value we show in that case $r^k(2\widehat{K}) \leq 0$. Second, for any amount of capacity lower than $\widehat{K}$, we prove that $r^u(\widetilde{q}^u)$ must be not lower than $r^k(\widetilde{q}^k)$ if $\widetilde{q}^u, \widetilde{q}^k < 2\widehat{K}$. Finally, in a third step, we show that it does not exist intersection in the constrained region between $r^u(q)$ and $r^k(q)$. Therefore, it is not possible to find $r^k(q) > r^u(q)$ for some q.

Suppose that $\widehat{K} = \min\{K : r^u(2K) = 0\}$. That is, for any $q \leq 2\widehat{K}$, $r^u(q) \geq 0$. Then, $r^k(2\widehat{K})$ cannot be positive. To prove this statement, consider the FOC when demand is $P^u()$ evaluated at $x = r^u(2\widehat{K})$:

$$x \left[P^{u'}(x + 2\widehat{K})x + P^u(x + 2\widehat{K}) - C'(x) \right] = 0$$

and we can use the fact that $P^u(Q) = P(Q/2)$ to replace in the previous equations

$$x \left[P' \left(\frac{x}{2} + \widehat{K} \right) x + P \left(\frac{x}{2} + \widehat{K} \right) - C'(x) \right] = 0$$

and the optimal x is equal to zero by construction of $\widehat{K}$.

Consider the FOC when demand is $P^k()$ evaluated at $y = r^k(2\widehat{K})$:

$$y \left[P^{k'}(y + 2\widehat{K})y + P^k(y + 2\widehat{K}) - C'(y) \right] = 0$$

and using the fact that $P^k(Q) = P(Q - K)$ to replace in the previous equations we get:

$$y\left[P^{'}(y+\widehat{K})q + P(y+\widehat{K}) - C'(y)\right] = 0$$

Since marginal income is a decreased function of quantities then: $P^{'}\left(\frac{x}{2}+\widehat{K}\right)x + P\left(\frac{x}{2}+\widehat{K}\right) \geq P^{'}(y+\widehat{K})q + P(y+\widehat{K})$ give that $\frac{x}{2}+\widehat{K} \leq y+\widehat{K}$. Therefore, if $x = 0$, it implies $y = r^k(2\widehat{K}) = 0$.

Now, suppose that $K \in (0, \widehat{K}]$. Be $\widetilde{q}^u, \widetilde{q}^k$ the minimum quantities characterized in Definition 2. Then, $r^u(\widetilde{q}^u) \geq r^k(\widetilde{q}^k)$. To prove this statement, consider the opposite. We can write the $FOCs$ for both cases evaluated at $\widetilde{q}^u, \widetilde{q}^k$:

$$\begin{aligned}
P^{u'}(r^u(\widetilde{q}^u)+\widetilde{q}^u)r^u(\widetilde{q}^u) + P^u(r^u(\widetilde{q}^u)+\widetilde{q}^u) - C'(r^u(\widetilde{q}^u)) &= 0 \\
P^{k'}(r^k(\widetilde{q}^k)+\widetilde{q}^k)r^k(\widetilde{q}^k) + P^k(r^k(\widetilde{q}^k)+\widetilde{q}^k) - C'(r^k(\widetilde{q}^k)) &= 0
\end{aligned}$$

We can use again the fact that $P^u(Q) = P(Q/2)$ and $P^k(Q) = P(Q-K)$ to simplify both conditions. But since we are evaluating both demands at $Q = \widetilde{Q} = 2K$, then $P(Q/2) = P(Q-K)$ and after some manipulation of both $FOCs$ we get:

$$P^{'}(K)\left[r^k(\widetilde{q}^k) - \frac{r^u(\widetilde{q})}{2}\right] = C'(r^k(\widetilde{q}^k)) - C'(r^u(\widetilde{q}^u))$$

Certainly, the left hand side is negative while the right hand side is positive if $r^k(\widetilde{q}^k) > r^u(\widetilde{q}^u)$, a contradiction.

Finally, we have shown that for $q \in \{\widetilde{q}^u; 2\widehat{K}\}$, $r^u(q) \geq r^k(q)$. Consider any $\widehat{q} \in [\widetilde{q}^u; 2K]$ with $K \in (0, \widehat{K})$ and suppose that exists $r^u(\widehat{q}) = r^k(\widehat{q}) = r(\widehat{q}) > 0$, that is, both functions have an intersection. Again we use both $FOCs$ evaluated at $r(\widehat{q})$:

$$\begin{aligned}
P^{u'}(r(\widehat{q})+\widehat{q})r(\widehat{q}) + P^u(r(\widehat{q})+\widehat{q}) - C'(r(\widehat{q})) &= 0 \\
P^{k'}(r(\widehat{q})+\widehat{q})r(\widehat{q}) + P^k(r(\widehat{q})+\widehat{q}) - C'(r(\widehat{q})) &= 0
\end{aligned}$$

and replacing $P^u(Q) = P(Q/2)$ and $P^k(Q) = P(Q-K)$ after some manipulation we obtain:

$$P\left(\frac{r(\widehat{q})+\widehat{q}}{2}\right)-P(r(\widehat{q})+\widehat{q}-K)=\left[P^{'}\left(r(\widehat{q})+\widehat{q}-K\right)-\frac{P^{'}\left(\frac{r(\widehat{q})+\widehat{q}}{2}\right)}{2}\right]r(\widehat{q})$$

Note that left hand side is positive if $r(\widehat{q})+\widehat{q}\geq 2K=\widetilde{Q}$ and this is valid by assumption. However, the right hand side is negative. It simple to show that when $P''()=0$. Consider just the term into the brackets with the following modification: $P^{'}\left(r(\widehat{q})+\widehat{q}-K\right)-P^{'}\left(\frac{r(\widehat{q})+\widehat{q}}{2}\right)$. It is negative if $r(\widehat{q})+\widehat{q}\geq 2K$. Since $P'\left(\frac{r(\widehat{q})+\widehat{q}}{2}\right)<0$, then $P'\left(\frac{r(\widehat{q})+\widehat{q}}{2}\right)<P'\left(\frac{r(\widehat{q})+\widehat{q}}{2}\right)/2$. Finally, it shows that the left hand side is negative, therefore, a contradiction about the existence of a positive intersection.

C. Proof of Lemma 1

Basically, the intuition behind this proof is to construct the $EBRF$ from the best respond function of games G^u and G^k. In effect, we determine the domain of $r^u(q)$ and $r^k(q)$ that make them part of $r^e(q)$. However, as we show below, in some cases, nor $r^u(q)$ nor $r^k(q)$ are part of $r^e(q)$.

Many cases are possible and they depend on the parametrization, in particular in the value of K and consequently on $\widetilde{Q}$.

In the first case, if we can find a set of non negative q such that $r^u(q)\leq\widetilde{Q}-q$ then $r^e(q)=r^u(q)$. We should prove that for the same set of q, a generator will not find profitable a deviation to $r^k(q)$ if $r^k(q)+q>\widetilde{Q}$. Note that it cannot be possible due to Condition 1. In effect, for all $\widehat{q}$ such that $r^u(\widehat{q})+\widehat{q}\geq\widetilde{Q}$, then $r^u(\widehat{q})\geq r^k(\widehat{q})$, but note that for all $q<\widehat{q}$ it implies $r^k(q)<\widetilde{Q}-q$. In such a case, it is clear that $r^k(q)$ cannot be part of the $EBRF$ when $q<\widehat{q}$.

Suppose that we are in the second case where $r^e(q)=\widetilde{Q}-q$ if $r^u(q)>\widetilde{Q}-q>r^k(q)$ for some non negative q. If this case is possible, $r^u(q)$ and $r^k(q)$ are not part of the $EBRF$. Suppose that we play $\widetilde{Q}-q-\epsilon$ (ϵ is a positive small number). Since $\widetilde{Q}-q-\epsilon<r^u(q)$ and profit function is concave we can increase profit by increasing the quantity offered. Then $\widetilde{Q}-q-\epsilon$ cannot be part of the $EBRF$. Suppose that we play $\widetilde{Q}-q+\epsilon$. Using the same argument as before, $\widetilde{Q}-q+\epsilon>r^k(q)$ and we can increase profit by a reduction of the quantity. To conclude, $r^e(q)=\widetilde{Q}-q$.

The last case is when $r^e(q) = r^k(q)$ if we can find non negative q such that $r^u(q) > r^k(q) \geq \widetilde{Q} - q$. It is probable when K is not so large to obtain a best respond function unconstrained (implemented over $P^u(Q)$). It is clear that $r^u(q)$ is not part of the $EBRF$. But since $r^k(q) \geq \widetilde{Q} - q$ and due to the concavity of the profit function, then $\widetilde{Q} - q$ cannot be part of the $EBRF$.

D. Proof of Lemma 3

It is not a surprise that in games G^u or G^k there is a unique and symmetric NE since by assumption, their best respond functions have slope lower than one in absolute value and firms are symmetric. And in G^e when demand is $P^e = P^u(\cdot)$, q_*^u is the NE if $2q_*^u \leq \widetilde{Q}$ (or $q_*^u \leq K$) since it can be implemented by the SO on $P^u()$. In addition, since $q_*^u \leq K$ and $r^u(q)$ has negative slope, using Condition 1 we can establish that q_*^k cannot satisfies $2q_*^k > \widetilde{Q}$ (or $q_*^k > K$). To sum up, q_*^u is the unique and symmetric

In the effective game is clear than q_*^k is the unique and symmetric NE when $2q_*^k > \widetilde{Q}$. As consequence of Condition 1, it shows that necessarily $q_*^u > q_*^k$ and therefore q_*^u cannot be an equilibrium over $P^e(\cdot)$.

The most interesting part of this lemma is when $2q_*^u > \widetilde{Q} > 2q_*^k$ or $q_*^u > K > q_*^k$. Nor q_*^u nor q_*^k can be implemented over $P^u(\cdot)$ and $P^k(\cdot)$ respectively in game G^e. Certainly, both $EBRFs$ intercept over the 45° degree line of $\widetilde{Q} - q$ defined in Lemma 1. They cannot intercept in another region. Consider the opposite case and suppose that it occurs on the unconstrained region. Using the fact that firms are symmetric, it means that exist a symmetric equilibrium different than q_*^u but it is not possible due to the uniqueness of equilibrium of each demand. The same argument applies over a possible equilibrium in the constrained region. We need to show that the intersection of both $EBRFs$ occurs in the interval $(\underline{q}, \overline{q})$ of $r^e(q)$ with $\underline{q} < K < \overline{q}$.

First, we show that $\underline{q} < K < \overline{q}$. Assume $\underline{q} = \widetilde{q}^u$ and suppose $\widetilde{q}^u > K$. In such a case, there is not $q_*^u = r^u(q) > \widetilde{q}^u$. Note that by construction $r^u(\widetilde{q}^u) - K = K - \widetilde{q}^u$ and the right hand side is negative (or $r^u(\widetilde{q}^u) < \widetilde{q}^u$). Therefore, if we increase q from $\widetilde{q}^u > K$ since $r^u(q)$ has negative slope, we cannot find $q_*^u = r^u(q)$. However, for $q < \widetilde{q}^u$, it is possible to find $q = r^u(q) = q_*^u$ but it implies that $q_*^u < K$. Note that it is a contradiction with the assumption $2q_*^u > \widetilde{Q}$.

Consider $\underline{q} = r^k(\widetilde{q}^k)$ and suppose $r^k(\widetilde{q}^k) > K$. In such a case, there is not $q_*^k = r^k(q) < \widetilde{q}^k$. Consider by construction $r^k(\widetilde{q}^k) - K = K - \widetilde{q}^k$ and note that the left hand side is positive (or $r^k(\widetilde{q}^k) > \widetilde{q}^k$) . Therefore, decreasing q from $\widetilde{q}^k < K$ since $r^k(q)$ has negative slope, we cannot find $q_*^k = r^k(q)$. However, for $q > \widetilde{q}^k$, it is possible to find $q = r^k(q) = q_*^k$ but it implies that $q_*^k > K$. Note that it is a contradiction with the assumption $2q_*^k < \widetilde{Q}$. To sum up, be $\underline{q}$ equal to $\widetilde{q}^u$ or $r^k(\widetilde{q}^k)$, it should be lower than K. The same argument can be used to show $\overline{q} > K$.

Finally suppose that agent i chooses a quantity to play $q_i < \underline{q}$. If $\underline{q} = \widetilde{q}^u$ it is clear from Lemma 1 that $r_j^e(q_i) = r_j^u(q_i)$ of agent $j \neq i$, nervelessness, it is not over the 45° degree line of $\widetilde{Q} - q$. On the other side, note that $r_i^k(q)$ intersects the 45° degree line of $\widetilde{Q} - q$ in $r_i^k(\widetilde{q}_j^k)$. We can use the fact that agents are symmetric and then $r_j^k(\widetilde{q}_i^k) = r_i^k(\widetilde{q}_j^k) = r^k(\widetilde{q}^k)$. Therefore, when $\underline{q} = r^k(\widetilde{q}^k)$ if agent i chooses a quantity to play $q_i < \underline{q}$ it means that $r_i^e(q) = r_i^k(q)$ but it is not over the 45° degree line. In such a case, the lower bound in the support of equilibrium is expressed by $\underline{q} = \max\{\widetilde{q}^u; r^k(\widetilde{q}^k)\}$. The same argument can be used to prove the upper bound. To conclude, there is a continuum of equilibria and they can be asymmetric.

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