How Persistent is Volatility? An Answer with Stochastic Volatility Models with Markov Regime Switching State Equations

Soosung Hwang*
Cass Business School, London, UK

Steve E. Satchell
Cambridge Univeristy, UK

Pedro L. Valls Pereira
Ibmec Business School, BRAZIL

January 2004

Abstract

We introduce SV models with Markov regime changing state equation (SVMRS) to investigate the important properties of volatility, high persistence and smoothness. With the quasi-ML approach proposed in our study, we showed that volatility is far less persistent and smooth than the GARCH or SV models suggest.

Keywords: Stochastic Volatility, Markov Switching, Persistence.

JEL Codes: C13

*Corresponding Author. Faculty of Finance, Cass Business School, 106 Bunhill Row, London, EC1Y 8TZ, UK. Tel: 44 (0)20 7040 0109. Fax: 44 (0)20 7040 8881. Email: s.hwang@city.ac.uk

1 Introduction

One of the most common methods to obtain a proxy measure of volatility is to fit parametric econometric models such as GARCH or stochastic volatility (SV) models, and others include option implied volatility, the intra-day return volatility (Andersen and Bollerslev, 1998) and range volatility (Parkinson, 1980; Garman and Klass, 1980; Alizadeh, Brandt, and Diebold, 2002). In most cases, the results from these parametric or nonparametric methods show that ex-post squared returns or absolute returns are too noisy and volatility is highly persistent and smooth. These results are consistent with the poor forecasting power of GARCH models on ex-post squared returns or absolute returns.

However, we may ask if the persistence and smoothness represent the properties of true volatility. For example, GARCH conditional volatility reflects only lagged information and is not designed to take account of cross-sectional information. If asset returns follow linear factor models such as Fama and French (1992), then there are multiple cross-sectional factors which are not explained by conditional volatility, but are sources of volatility. Campbell, Lettau, Malkiel, and Xu (2001), for example, using cross-sectional decomposition on equity volatility, showed that market and industry volatilities are important components for the explanation of individual asset volatility. Connor and Linton (2001) and Hwang and Satchell (2004) also suggested that there is common heteroskedasticity in asset-specific returns. Therefore, a significant amount of squared returns may not be noise but come from cross-sectional heteroskedasticity in factors and factor loadings, which is not explained by conditional volatility.

Another econometric question is that the persistence and smoothness obtained with well known volatility models such as GARCH or SV models may come from the restrictive nature of the models. For example, Lobato and Savin (1998), Granger and Hyung (1999) and Diebold and Inoue (2001) suggested that structural breaks in the mean of volatility may be a source of persistence. As a second example, Bollerslev's (1986) nonnegativity constraints on coefficients on GARCH model may restrict autocorrelation structure of volatility. Nelson and Cao (1992) showed that the Bollerslev's non-negativity conditions are too restrictive and in some cases negative estimates may be obtained in practice. He and Teräsvirta (1999) further showed that allowing negative parameters in GARCH models can give us various autocorrelation structures of squared returns.

In our study we use SV models with Markov regime changing state equations (SVMRS) to investigate the questions on persistence and smoothness of volatility. Existing models such as So, Lam, and Li (1998), Kalimipalli and Susmel (2001), and Smith (2002) provide such a structure, but our model is more general in that we allow volatility to have regime-dependent means, variances and autoregressive characteristics. SV models are useful for our purpose since they allow us to decompose squared returns into transitory noise and permanent innovation (volatility process). Note that the error (innovation) in the state equation matters over time through a process, whilst the error (noise) in observation equation does not, and it is the innovation term that captures the persistence of the model. In addition, in SV models we do not need nonnegativity restrictions on the parameters.

Furthermore, by allowing regime changes in the parameters of the state equations in SV models, we can investigate properties of the volatility process. As is standard, we assume that the state equation in our SV model follows an AR(1).[1] However, the assumption of volatility process being AR(1) processes seems to be too restrictive, if there are structural breaks. Therefore, we allow the state equation in our SV model to Markov regime switch over time.

We find that the squared returns are better specified with our SVMRS model. More importantly, we find that volatility is far less smooth and persistent. Our results suggest that the conventional SV and GARCH models may be too restrictive for squared returns. In addition, the large proportion of transitory noise in SV

[1] Using ARMA processes in the state equation does not change the persistence and smoothness of the volatility process. See Hwang and Satchell (2000) for example.

models decrease significantly in the SVMRS model and many cases in our study show that there is little transitory noise in squared returns when the SVMRS model is used. We also show that when a AR(1) process follows regime changes, under certain conditions in the transition probabilities, autocorrelation coefficients may show more persistence than the AR parameter suggests. These results are consistent with those of Granger and Hyung (1999) and Diebold and Inoue (2001) for example.

In the next section we introduce our model. We also derive the autocorrelation function of the regime switching AR(1) process. In section 3 using S&P500 and FTSE100 daily and weekly returns, we show estimates of our SVMRS model and compare the conventional SV model. Conclusions follow in section 4.

2 Models

2.1 SV and Markov Regime Switching Models

The stochastic volatility (variance) (SV) model was introduced by Taylor (1986) and Hull and White (1987) and has been further developed by Harvey and Shephard (1993, 1996) and Harvey, Ruiz and Shephard (1994). In the SV model, the log of ε_t^2, where ε_t is typically asset returns (or residuals from a return process), is modelled as a stochastic process:

$$\begin{aligned} \varepsilon_t &= \epsilon_t \sigma \exp(\frac{1}{2} h_t) \\ h_t &= \phi h_{t-1} + \eta_t \end{aligned} \tag{1}$$

where $\epsilon_t \sim N(0,1)$ and independent of $\eta_t \sim N(0, \sigma_\eta^2)$, and σ is a positive scale factor. Squaring (1) and taking logs we have a process

$$\begin{aligned} \ln \varepsilon_t^2 &= \ln \epsilon_t^2 + \ln \sigma^2 + h_t \\ &= E[\ln \epsilon_t^2] + \ln \sigma^2 + h_t + \ln \epsilon_t^2 - E[\ln \epsilon_t^2] \\ &= \mu + h_t + \varphi_t \end{aligned} \tag{2}$$

where $\mu = E[\ln \epsilon_t^2] + \ln \sigma^2$ and $\varphi_t = \ln \epsilon_t^2 - E[\ln \epsilon_t^2]$ is a martingale difference, but not normal. When we replace $\ln \varepsilon_t^2$ with y_t and $\mu + h_t$ with x_t, the SV model in equations (1) and (2) can be written as

$$y_t = x_t + \varphi_t \tag{3}$$

$$x_t - \mu = \phi(x_{t-1} - \mu) + \eta_t \tag{4}$$

SV models are useful to decompose log-squared returns into transitory noise and permanent innovation. This is because the innovation, η_t, matters over time through the AR(1) process, whilst noise, φ_t, does not. Using this concept, Hwang and Satchell (2000) showed that squared daily index returns such as FTSE100 or S&P500 consists of 95% of noise and 5% of unobserved innovation (volatility). This result is asymptotically consistent with the poor forecasting power of GARCH models (see Andersen and Bollerslev (1998)).

Note that the AR(1) process is commonly used in the state equation (volatility process). However, the assumption of the volatility process being an AR(1) process seems to be too restrictive. We may use ARFIMA models to generalise the volatility process. In our study, we focus on structural breaks. Many studies such as Lobato and Savin (1998), Granger and Hyung (1999), and Diebold and Inoue (2001) showed that there are structural breaks in the volatility process and the structural breaks have been blamed as a source of extreme persistence in volatility. Hwang (2004) recently showed that in a mean zero AR(1) process, persistence (or the magnitude of estimated AR coefficient) is a function of structural breaks in the mean as well as in the AR parameter.

It is clear that if there are structural breaks in volatility, the conventional GARCH or SV models are misspecified and we need a model for the structural breaks. In recent years economic time series have been modelled with the assumption that the distribution of the variables is known conditional on a regime or state occurring. The Markov regime switching models introduced by Hamilton (1989) allow the unobserved regime to follow a first order Markov process. The models have been used extensively in macroeconometrics as a means of capturing the different patterns of expected growth in output, see, for example, Filardo (1994) and Goodwin (1993).

Suppose that there is a state variable, s_t, which is unobservable. When we allow regime switching in the fundamental equation, a simple state equation is

$$x_t = \begin{cases} \mu_0 + \eta_{0,t}, & \text{when } s_t = 0, \\ \mu_1 + \eta_{1,t}, & \text{when } s_t = 1, \end{cases}$$

where $\eta_{i,t} \sim N(0, \sigma^2_{\eta_i})$, $i = 0, 1$, and s_t follows a Markov chain. Here x_t does not allow persistence, and thus the simple state equations are not appropriate for volatility process. Note that we can easily allow x_t to follow AR(1) processes, but this assumption can be generalised by the introduction of more lags in the AR component, i.e., AR(p) processes. For simplicity, throughout this study we assume that the state equation follows a regime switching AR(1) process;

$$x_t = \begin{cases} \mu_0 + \phi_0(x_{t-1} - \mu_0) + \eta_{0,t}, & \text{when } s_t = 0, s_{t-1} = 0, \\ \mu_0 + \phi_0(x_{t-1} - \mu_1) + \eta_{0,t}, & \text{when } s_t = 0, s_{t-1} = 1, \\ \mu_1 + \phi_1(x_{t-1} - \mu_0) + \eta_{1,t}, & \text{when } s_t = 1, s_{t-1} = 0 \\ \mu_1 + \phi_1(x_{t-1} - \mu_1) + \eta_{1,t}, & \text{when } s_t = 1, s_{t-1} = 1 \end{cases} . \tag{5}$$

Previous studies introduced regime switching state equations to investigate the effects of structural breaks on the persistence of volatility. See Diebold and Inoue (2001) for example. However, they only allowed μ to have different values over time. Our model is more general in the sense that ϕ and σ^2_η as well as μ are allowed to change.

2.2 Stochastic Volatility Models with Markov Regime Switching Equation

It is interesting if we can combine (3) and (5), i.e., a stochastic volatility model with a Markov regime switching state equation (SVMRS). As pointed out in Andersen and Bollerslev (1998) among many others, if squared residuals are too noisy for a proxy volatility process, we need to take out noise from the squared residuals and then investigate the remainder to see if there are structural breaks or persistent.

However, the decomposition of squared returns depends on which models are used for state equations. There are some previous attempts to model with stochastic volatility and regime switching models. For example, in the studies of So, Lam, Li (1998) and Kalimipalli and Susmel (2001), μ is allowed to regime-change. So, Lam, Li (1998), using weekly S&P500 index volatility, found that volatility is far less persistent than that of SV models. Kalimipalli and Susmel (2001) applied their model to explain the behaviour of short-term interest rates. They found that their regime switching model performs better than the GARCH family of models and SV models. On the other hand, Smith (2002) generalised these models and showed that Markov-regime switching or stochastic volatility models need to be improved to explain short-term interest rates.

Our SVMRS model allows all three parameters in (5) to change and thus is a generalised version. That is, there are two regimes and the state equation is assumed to follow an AR(1) process;

$$y_t = x_t + \varphi_t \tag{6}$$

$$x_t = \mu_i + \phi_i(x_{t-1} - \mu_j) + \eta_{i,t}, \text{ when } s_t = i, s_{t-1} = j, \tag{7}$$

where $i = 0, 1$ and $j = 0, 1$, $\varphi_t \tilde{} N(0, \sigma^2_\varphi)$, $\eta_{i,t} \tilde{} N(0, \sigma^2_{\eta_i})$, and the transition probabilities are given by:

$$p^{(i,j)} = \Pr(s_t = i | s_{t-1} = j) \tag{8}$$

and the transition matrix is given by:

$$\mathbf{p} = \begin{bmatrix} p^{(0,0)} & 1 - p^{(1,1)} \\ 1 - p^{(0,0)} & p^{(1,1)} \end{bmatrix} \tag{9}$$

We note that the above SVMRS model has two unobserved variables; x_t which follows different processes according to an unobserved variable s_t. We may allow more states and lags, but the number of cases we should consider for x_t increases rapidly, i.e., $(number\ of\ states)^{lags+1}$.

The SVMRS model treats φ_t as a transitory noise and η_t as a permanent innovation (or volatility process) as suggested by Hwang and Satchell (2000). The treatment provides intuitively interesting perspective since we can decompose any process into just noise which is not explained by the state equation and innovation. In addition, when $\sigma_{\eta_0} = \sigma_{\eta_1}$, the volatility of volatility is unchanged regardless of states and when $\mu_0 = \mu_1$, the unconditional level of volatility is the same across different states. Finally, when $\phi_0 = \phi_1$, x_t has the same persistence.

The SVMRS model above is the generalised version of SV as well as the Hamilton's Markov regime switching model. By restricting parameters in appropriate ways, we can derive these models. Usually this can be achieved when we estimate the SVMRS model and investigate if parameters satisfy some conditions;

- If $\sigma_\varphi^2 = 0$, the SVMRS model becomes Hamilton's Markov regime switching model.
- If $\mu_0 = \mu_1$, $\phi_0 = \phi_1$, $\sigma_{\eta_0} = \sigma_{\eta_1}$, and $\sigma_\varphi^2 \neq 0$, then we have the SV model.
- If $\mu_0 = \mu_1$, $\phi_0 = \phi_1$, $\sigma_{\eta_0} = \sigma_{\eta_1}$, and $\sigma_\varphi^2 = 0$, then we have the AR(1) model.
- If one of σ_{η_i} is zero, then the unobserved process consists of a stochastic process and a deterministic process.

Note that by restricting $\phi_0 = \phi_1$, $\sigma_0 = \sigma_1$, and $\sigma_\varphi^2 = \frac{\pi^2}{2}$ under the assumption that ϵ_t is standard normal in (2), we have a model similar to Smith (2002) which is

$$y_t = x_t + \varphi_t \tag{10}$$

$$x_t = \begin{cases} c_0 + \phi x_{t-1} + \eta_t, & \text{when } s_t = 0, \\ c_1 + \phi x_{t-1} + \eta_t, & \text{when } s_t = 1 \end{cases} \tag{11}$$

where $\varphi_t \tilde{} N(0, \frac{\pi^2}{2})$ and $\eta_t \sim N(0, \sigma_\eta^2)$. The model can be easily generalised so that ϕ and σ_η may have different values for different states. However, the model does not explain the two other cases, i.e., $s_t = 0$ and $s_{t-1} = 1$, and $s_t = 1$ and $s_{t-1} = 0$. If the frequency of inter-state changing is small, the effects of disregarding these two cases may be trivial, and this may be appropriate for most macroeconomic variables where the number of structural breaks is usually less than 1%. See Stock and Watson (1996), Ben-David and Papell (1998), McConnell and Perez-Quiros (2000), Hansen (2001), and Bai, Lumsdaine, and Stock (1998) among many. However, as will be shown later, for volatility which has many structural breaks, Smith's (2002) model may become restrictive.

When we define ξ_{1t} as a random variable that is equal to unity when $s_t = 1$ and zero otherwise, the AR(1) representation of state 1 is

$$\xi_{1,t} = (1 - p^{(0,0)}) + (-1 + p^{(0,0)} + p^{(1,1)})\xi_{1,t-1} + v_{1,t}, \tag{12}$$

where $v_{1,t}$ is a martingale difference sequence of state 1 at time t. The unconditional probability that the process will be in regime 1 at any time, $\overline{p}_1$, is;

$$\overline{p}_1 = E(\xi_{1,t}) = \frac{1 - p^{(0,0)}}{2 - p^{(0,0)} - p^{(1,1)}} \tag{13}$$

Theorem 1 *The autocorrelation function with lag τ, $\rho(\tau)$, of the state equation in (7) which is equivalent to that of Markov regime switching process, is*

$$\rho(\tau) = E\left[\prod_{s=1}^{\tau} (\phi_0 + (\phi_1 - \phi_0)s_{t-s+1})\right].$$

Proof. The state equation in (7) can be represented as

$$\begin{aligned} x_t - [\mu_0 + (\mu_1 - \mu_0)s_t] &= (\phi_0 + (\phi_1 - \phi_0)s_t)\left[x_{t-1} - [\mu_0 + (\mu_1 - \mu_0)s_{t-1}]\right] \\ &\quad + (\sigma_{\eta_0} + (\sigma_{\eta_1} - \sigma_{\eta_0})s_t)\xi_t, \end{aligned} \tag{14}$$

where $\xi_t \tilde{} N(0,1)$. Note that

$$\begin{aligned} E\left[x_t - [\mu_0 + (\mu_1 - \mu_0)s_t]\right] &= E(x_t) - E\left[\mu_0 + (\mu_1 - \mu_0)s_t\right] \\ &= E\left[x_t\xi_{1,t} + x_t(1 - \xi_{1,t})\right] - E\left[\mu_0 + (\mu_1 - \mu_0)s_t\right] \\ &= \mu_1\overline{p}_1 + \mu_0(1 - \overline{p}_1) - \mu_0 - (\mu_1 - \mu_0)\overline{p}_1 \\ &= 0. \end{aligned}$$

Substituting $z_t = x_t - [\mu_1 + (\mu_0 - \mu_1)s_t]$, we have the following mean zero AR(1) process;

$$z_t = (\phi_0 + (\phi_1 - \phi_0)s_t)z_{t-1} + (\sigma_{\eta_0} + (\sigma_{\eta_1} - \sigma_{\eta_0})s_t)\xi_t, \tag{15}$$

and thus

$$\rho(\tau) = E\left[\prod_{s=1}^{\tau}(\phi_0 + (\phi_1 - \phi_0)s_{t-s+1})\right].$$

■

Note that when $\tau = 1$, $\rho(1) = \phi_0 + (\phi_1 - \phi_0)\overline{p}_1$, since $E(s_t) = \overline{p}_1$. However, when $\tau = 2$, we have

$$\begin{aligned}
\rho(2) &= E\left[(\phi_0 + (\phi_1 - \phi_0)s_t)(\phi_0 + (\phi_1 - \phi_0)s_{t-1})\right] \\
&= E\left[\phi_0^2 + (\phi_1 - \phi_0)\phi_0 s_t + (\phi_1 - \phi_0)\phi_0 s_{t-1} + (\phi_0 - \phi_1)^2 s_t s_{t-1}\right] \\
&= \phi_0^2 + 2(\phi_1 - \phi_0)\phi_0\overline{p}_1 + (\phi_1 - \phi_0)^2 E[s_t s_{t-1}].
\end{aligned}$$

Since

$$\begin{aligned}
E[s_t s_{t-1}] &= E[\xi_{1,t}\xi_{1,t-1}] \\
&= E[((1 - p^{(0,0)}) + (-1 + p^{(0,0)} + p^{(1,1)})\xi_{1,t-1} + v_{1,t})\xi_{1,t-1}] \\
&= E[((1 - p^{(0,0)})\xi_{1,t-1} + (-1 + p^{(0,0)} + p^{(1,1)})\xi_{1,t-1}^2] \\
&= (1 - p^{(0,0)})\overline{p}_1 + (-1 + p^{(0,0)} + p^{(1,1)})\overline{p}_1 \\
&= p^{(1,1)}\overline{p}_1,
\end{aligned}$$

we have

$$\begin{aligned}
\rho(2) &= \phi_0^2 + 2(\phi_1 - \phi_0)\phi_0\overline{p}_1 + (\phi_1 - \phi_0)^2 p^{(1,1)}\overline{p}_1 \\
&= [\phi_0 + (\phi_1 - \phi_0)\overline{p}_1]^2 + (\phi_1 - \phi_0)^2[p^{(1,1)}\overline{p}_1 - (\overline{p}_1)^2] \\
&= \rho(1)^2 + (\phi_1 - \phi_0)^2\overline{p}_1[p^{(1,1)} - \overline{p}_1].
\end{aligned}$$

Therefore, Theorem 1 shows that generally $\rho(\tau) \neq \rho(1)^\tau$, unless either $\phi_1 = \phi_0$ or $\overline{p}_1 = 0$ or 1, i.e., there is only one state. Therefore, the autocorrelations of the Markov regime changing AR(1) process may not show the persistence level that the value of $\rho(1)$ suggests.

Remark 1 *Equation (13) suggests that when* $p^{(1,1)} > 1 - p^{(0,0)}$ *and* $\phi_1 \neq \phi_0$*, we have* $[p^{(1,1)} - \overline{p}_1] > 0$ *and thus* $\rho(2) > \rho(1)^2$.

Thus even though (15) has an AR(1) representation, the process does not show the same exponential decay rate as the conventional AR(1) process because of the probability of states. In addition, the difference between $\rho(2)$ and $\rho(1)^2$ is a positive function of persistence difference $\phi_1 - \phi_0$, $\overline{p}_1$, and $p^{(1,1)} - \overline{p}_1$.[2]

2.3 Estimation Procedure

Harvey, Ruiz and Shephard (1994) adopted a procedure based on the Kalman filter to estimate SV models in (3) and (4). However, since the distribution of φ_t is not known, it is not possible to represent the likelihood function in closed form. However, quasi-maximum likelihood (QML) estimators of the parameters can be obtained using the Kalman filter by treating φ_t and η_t as normal. Harvey, Ruiz and Shephard (1994) treated φ_t as though it were $N(0, \pi^2/2)$, and maximized the resulting quasi-likelihood function. Ruiz (1994) suggested that for the kind of data typically encountered in empirical finance, the QML for the SV model has good finite sample properties.

In this study we propose a Quasi-Maximum Likelihood (QML) estimation method using the Kalman filter. The basic concept is that both x_t and conditional probability that are unobserved processes, can be obtained through predicting and updating which was proposed by Smith (2002). That is, we have $(number\ of\ states)^{lags+1}$ state equations, e.g., in our study 4 state equations, and each state equation is updated and predicted in the same way as for the standard Kalman filter. We use the method suggested in Hamilton (1989) to update the conditional probability with a transition probability matrix. A detailed explanation on estimation and smoothing procedure can be found in the Appendix. We could use other methods such as

[2] We only show the cases of $\rho(1)$ and $\rho(2)$. The autocorrelations with larger lags are complicted and we do not discuss them further in this study.

Markov Chain Monte Carlo (MCMC), the generalised method of moments (GMM), the efficient method of moments (EMM) to estimate the SVMRS. However, these methods are more complicated than the QML with the updating procedure proposed in the appendix.

One problem of the Markov regime switching model is that the model is multi-modal. To find out the global maximum of the log-likelihood, we try various starting values. However, the largest ML value does not always guarantee that the estimates are appropriate. Another criterion we use for the SVMRS model is the relative magnitude of σ_{η_i} to σ_{φ} (signal-to-noise ratio). If a model is well specified, then the proportion that is not explained by the model, i.e., the transitory noise in the SVMRS model, should be minimised. Since the true volatility process and thus the amount of transitory noise included in squared returns is not known, a model that explains squared returns as much as possible may be better than a model that does not. Thus, signal-to-noise ratio can be a criterion to differentiate different sets of converged estimates.[3]

3 Empirical Tests

We use two daily indices, i.e., S&P500 and FTSE100, from 27 February 1992 to 27 February 2002. For the sample period, 2548 and 2606 log-returns are obtained for FTSE100 and S&P500 indices, respectively. We also use 522 weekly log-returns from 26 February 1992 to 27 February 2002 for the two indices. To calculate residuals, we simply take the mean returns during the sample from the log-returns.[4]

[3]However, this criterion may be controversial. We may use the Bayesian analysis, but again we need some knowledge of the true volatility process. Empirical results in the next section show that there are not significantly differences in model selection between ML values and signal-to-noise ratio.

[4]Since daily and weekly expected returns are very small, and taking the mean returns from the daily and weekly returns does not have significant effects. In the following we use 'log-squared returns' for the logs of squared de-meaned returns.

Table 1 reports the property of two index returns. As expected, daily returns are negatively skewed and leptokurtic, suggesting non-normal. In addition, autocorrelation coefficients are not significant. For the two weekly returns, we also find similar property, but the magnitude of non-normality is much smaller than that of the daily returns.

On the other hand, the log-squared residuals, as reported in many other studies, are negatively skewed and fat-tailed, and also are persistent. One noticeable difference between the daily and weekly log-squared residuals is that daily log-squared residuals are more persistent than weekly log-squared residuals. The temporal aggregation affects the level of persistence, i.e., autocorrelation structure.

The large negative skewness in the log-squared residuals results from the so-called 'inlier' problem in stochastic volatility models. For the daily returns used in this study, for example, the largest log-squared residuals of 3.559 for the FSTE100 and 3.935 for the S&P500 are within three standard deviations. However, the lowest log-squared residuals are -13.857 for the FTSE100 and -16.237 for the S&P500, respectively, and both of them are outside five standard deviations. These extremely small log-squared returns reflect returns close to zero.

Various methods may be used for inlier adjustment for the squared residuals. Harvey and Shephard (1993) set an arbitrary critical value and trim all values less than the critical value to the arbitrary critical value. These trimmed estimates are better behaved than the untrimmed estimates in their simulations. However, these kinds of inlier adjustments are criticised to be "profoundly suspicious" by Nelson (1994). In this study, we use the following Breidt and Carriquiry (BC) (1996) transformation as used in Harvey and Strieibel (1996):

$$\widehat{\ln \varepsilon_t^2} = \ln(\varepsilon_t^2 + \kappa\sigma_\varepsilon^2) - \kappa\sigma_\varepsilon^2/(\varepsilon_t^2 + \kappa\sigma_\varepsilon^2) \tag{16}$$

The idea behind the BC transformation is as follows. For the zero or extremely small ε_t^2, $\ln(\varepsilon_t^2 + \delta)$, where δ is a small increment, is evaluated. Then, the transformed $\ln(\varepsilon_t^2)$ can be obtained by the linear extrapolation from the point $(\varepsilon_t^2 + \delta, \ln(\varepsilon_t^2 + \delta))$

using the slop of the tangent line, $(\varepsilon_t^2 + \delta)^{-1}$. In the above equation, δ is set to $\kappa\sigma_\varepsilon^2$.

Table 1 reports the property of log-volatility changes for the three different parameter values of κ, i.e., 0.02, 0.05, 0.1. For different time series, different values of κ are required. For example, S&P500 daily log-squared residuals show that when $\kappa = 0.02$, we have the smallest Jarque and Bera statistic, whilst $\kappa = 0.1$ gives the smallest Jarque and Bera statistic for FTSE100 weekly log-squared residuals. Note that when κ is too large, then we may lose information included in the original data. For example, the autocorrelation coefficients increase as κ increases. On the other hand when κ is too small, we still have the inlier problem. In many case, the choice of κ is arbitrary and needs econometricians subjective decision. In this study, we choose $\kappa = 0.05$ to minimise the inlier problem in the stochastic volatility model.

3.1 SV and SVMRS Models

We first estimate the SV model in (3) and (4) for daily data. As in many other previous studies, we find that the unobserved volatility process is highly persistent for both S&P500 and FTSE100 daily log-squared returns. This is a typical result of SV models; the extreme persistence in volatility process. However, the autocorrelation coefficients presented in table 1 does not suggest such a high level of persistence. The difference between the two is usually attributed to high level of noise in squared returns (see Andersen and Bollerslev, 1998). This is supported by the signal-to-noise (SN) ratios, $\sigma_\eta/\sigma_\varphi$, which are 0.018 and 0.049 for S&P500 and FTSE100, respectively. This means that SV models (or asymptotically GARCH models) explain only a small proportion of squared residuals. Figures 1a and 2a show absolute values of residuals and smoothed standard deviation obtained from the SV model. As in most empirical results on SV models, they show the volatility is smooth. However, as discussed in the previous section, the smoothness in the SV model may be achieved by disregarding the structural breaks.

The estimates of our SVMRS model for the daily data are reported in table 2.

We find that the ML values are larger than those of the SV model in both indices, suggesting that the SV model with the MRS state equation better specifies the log-squared residuals. This is also supported by large SN ratios. For S&P500, the SN ratios are 1436 ($s_t = 0$) and 774 ($s_t = 1$), whilst for FTSE100, these ratios are 1.74 ($s_t = 0$) and 0.79 ($s_t = 1$). In particular, the transitory noise for S&P500 from the SVMRS is close to zero. Therefore, the large amount of transitory noise unexplained by the SV model is now explained by switching regimes. In addition, we also notice that state 0 which is the lower level of volatility is more volatile; the SN ratios of state 0 are larger than those of state 1. However, the large volatility of the lower level of log-volatility ($s_t = 0$) may not have significant meaning, because state 0 represents lower level of volatility which is close to zero when transformed back to volatility using the exponential function.

More importantly, the high persistence found with the SV model disappears in the SVMRS model. For example, the estimates of the AR parameter in the SV model are 0.999 and 0.990 for S&P500 and FTSE100, respectively. However, the estimates of the SVMRS model show that the AR parameters are 0.361 ($s_t = 0$) and 0.126 ($s_t = 1$) for S&P500 and 0.899 ($s_t = 0$) and 0.260 ($s_t = 1$) for FTSE100, respectively. These estimates are far from those of the SV model. The significantly different levels of means in states 0 and 1 suggest that structural breaks in mean may be a source of high persistence. Therefore, SV models without considering structural breaks can provide spurious persistence. These results are consistent with recent studies such as Lobato and Savin (1998), Granger and Teräsvirta (1999), Granger and Hyung (1999), Diebold and Inoue (2001), and Hwang (2004).

Interestingly we find that the AR coefficients of $\xi_{0,t}$ in equation (12) are all small negative and thus not persistent at all; i.e., the probability of state 0 has the AR coefficient of $-1 + p^{(0,0)} + p^{(1,1)}$ which is close to zero. Therefore, structural breaks do not have memory and there are many structural breaks in the state process. These results seem to be inconsistent with Granger and Hyung (1999) who found a small probability (less than 1%) of structural breaks in squared returns in a long memory volatility model. However, the difference between our approach and other previous studies including Granger and Hyung (1999) is that in our model, all three components, i.e., the level of volatility, AR coefficient and the volatility of permanent error, are allowed to change.

Note that the unconditional probability that $s_t = 1$, $E(s_t = 1)$, is

$$E(s_t = 1) = \frac{1 - p^{(0,0)}}{2 - p^{(0,0)} - p^{(1,1)}}$$

from (13). Using this equation, we obtain the unconditional probability of $s_t = 1$ for the daily S&P index, $\widehat{\overline{p}}_1 = 0.703$ (and we have 0.759 for FTSE100 index). This means that around 70% of cases, volatility is in higher state with mean of -1.006 and AR parameter of 0.126. The AR coefficient of the Markov regime switching state equation for the S&P500 is

$$\widehat{\phi}_0 + (\widehat{\phi}_1 - \widehat{\phi}_0)\widehat{\overline{p}}_1 = 0.196$$

from theorem 1. Using the same method we find that the estimated AR coefficient for the FTSE100 is 0.414. Thus when we remove the transitory noise and allow regimes, the AR coefficients estimated are much smaller than those with the SV model which show extreme persistence. In addition, the estimated transition probabilities in table 2 (and weekly cases in table 3) show that $p^{(1,1)} < 1 - p^{(0,0)}$ in all four cases, suggesting $\rho(2) < \rho(1)^2$. Therefore, at the second lag the autocorrelation coefficient decays faster than the ordinary AR(1) process whose autocorrelation coefficient at lag 1 is equivalent to $\rho(1)$.

Table 3 and Figure 3 report the results of weekly data. As in daily data, we find extreme persistence in volatility processes and small SN ratios from the estimates of the SV model; the estimated AR parameters are 0.992 and 0.969 and the SN ratios are 0.04 and 0.06 for the S&P500 and the FTSE100, respectively. In addition, the AR coefficient of x_t calculated from the estimates of the SVMRS model as in (12) are 0.192 and 0.114 for the S&P500 and the FTSE100, respectively. Again there is clear difference in persistence between SV and SVMRS models.

We plot squared residuals, smoothed volatility and probability in figures 1 to 3. Figures 1b, 2b and 3b clearly show that when we allow two different regimes for the mean, the AR parameter and the volatility of log-volatility, we have much more volatile smoothed volatility. For the S&P500, since the transitory noise is close to zero, most squared residuals are now explained by the two regimes. On the other hand, for the FTSE100, we still have a significant portion of squared returns which is not explained by the SVMRS model.

Tables 2 and 3 and Figures 1 to 3 suggest that the large amount of transitory noise unexplained by the SV model is now explained by either one of the two states. Asymptotically SV models are equivalent to GARCH models, and thus these results can also be applicable to GARCH models; when we allow structural breaks, the persistence level is reduced and the explanatory power of the model will increase.

3.2 Some Other Considerations in SVMRS Models

The SVMRS model proposed in this study can be used to investigate various different cases. Here we investigate two cases; when the mean is regime changing and when the state equation follows (11) as in Smith (2002).

The former is useful to investigate if structural breaks in mean are sources of persistence.[5] The results with daily and weekly data when $\phi_0 = \phi_1$ and $\sigma_{\eta_0} = \sigma_{\eta_1}$ are reported in tables 2 and 3. First of all, even though we allow regime changes in mean, the results still show extreme persistence; the estimated AR coefficients are all around from 0.97 to 0.99. Interestingly the ML values of this restricted model have larger ML values than the unrestricted model of SVMRS (except for weekly FTSE100 case). However, in all four cases, the transitory noise is much larger than the signal. This restricted model may not be appropriate if a good model should explain observed time series as much as possible. Figures 1c, 2c and 3c show an

[5]See Diebold (1986), Lamoureux and Lastrapes (1990), Chu (1995), Lobato and Savin (1998) Lobato and Savin (1998), Granger and Teräsvirta (1999), Granger and Hyung (1999), and Diebold and Inoue (1999) for example.

interesting pattern. Since the permanent innovation is much less than the noise, the two states (the common high and low volatility process) move smoothly over time. However, because of frequently changing probability, the volatility becomes highly volatile; despite the high persistence, the volatility process is far less persistent or smooth.

The figures for the restricted SVMRS model show that the lower volatility process is dominated by close to zero volatility. When we accept that volatility is a proxy measure of risk, we are less interested in small volatilities. In order to avoid econometric difficulties from inliers and to concentrate on large volatilities we use BC method in (16) with $\kappa = 0.05$. Table 2 reports that we still have a similar high AR coefficient for the volatility process for the modified log-squared returns. However, there is significant reduction of transitory noise and the lower level of log-volatility is now significantly shifted upward. Figures 1d, 2d and 3d show that the volatility process is now more concentrated and the difference between higher and lower volatility is reduced. However, we still find that smoothed volatility is highly volatile because of the frequently changing state. For example, the AR coefficients of $\xi_{1,t}$, the probability of state 1 at time t, are -0.012 and -0.023 for the S&P500 and FTSE100 daily modified log-squared residuals, respectively.

The second case we consider in this study is the model proposed by Smith (2002) as in (11). The last two columns of tables 2 and 3 report the estimates and their standard deviations. Interestingly we find that the estimated AR coefficients show that the volatility process is not persistent; all of them are less than 0.11. Comparing the levels of persistence with the SV model, these estimates are significantly small. This may be further evidence that volatility may be far less persistent. Note that the estimated ML values and SN ratios of Smith's model are much larger than those of the SV model. However, as explained in the previous section, Smith's model does not consider the two other cases, i.e., $s_t = 0, s_{t-1} = 1$ and $s_t = 1, s_{t-1} = 0$ and thus is more restrictive than the SVMRS model.

Figure 4 plots the four cases of Smith (2002) model in (11) for the weekly S&P500;

i.e., SV model, SVMRS model, SVMRS model with $\phi_0 = \phi_1$ and $\sigma_{\eta_0} = \sigma_{\eta_1}$ (see the last two columns of table 3a), and SVMRS model with $\phi_0 = \phi_1$ and $\sigma_{\eta_0} = \sigma_{\eta_1}$ for BC modified log-squared returns with $\kappa = 0.05$.[6] As in figure 3, figure 4 confirms that the volatility is much more volatile and less persistent.

4 Conclusions

This paper has presented a SV model with regime-dependent mean, variance, and autocorrelation that generalises existing SV regime-dependent models. We estimate our model using generalisations of the Kalman filter methods of Harvey, Ruiz, and Shephard (1994). Our results show that squared returns are better specified by our SVMRS models. In addition, a broad pattern we have found seems to be that the regime-dependent estimates are less persistent (and more volatile) than single regime estimates provided by other authors. This suggests that ignoring regime switching increases the estimate of persistence.

[6] The estimates that are not reported in the paper can be obtained from authors upon request. We also estimated these models using ten years monthly data. The results are similar and can be obtained from authors upon request.

Appendix

A. Estimation Procedure

We need some notation for the prediction of the state vector and also for its variance depending upon which regime is being used in the conditional set. Prediction equation ($x_{t|t-1}^{(i,j)}$), mean squared error associated with $x_{t|t-1}^{(i,j)}$ ($M_{t|t-1}^{(i,j)}$), prediction error ($v_t^{(i,j)}$), prediction variance ($f_t^{(i,j)}$) and updating equations ($x_{t|t}^{(i,j)}, M_{t|t}^{(i,j)}$) can be obtained using the following procedure; For $s_t = i$ and $s_{t-1} = j$, $i, j = 0, 1$, we have

$$
\begin{aligned}
x_{t|t-1}^{(i,j)} &= E[x_t | s_t = i, s_{t-1} = j, I_{t-1}] && (17) \\
&= (\mu_i - \mu_j \phi_i) + \phi_i x_{t-1|t-1}^{(j)}, \\
M_{t|t-1}^{(i,j)} &= E[(x_t - x_{t|t-1}^{(i,j)})^2 | s_t = i, s_{t-1} = j, I_{t-1}] && (18) \\
&= \phi_i^2 M_{t-1|t-1}^{(j)} + \sigma_i^2, \\
v_t^{(i,j)} &= y_t - x_{t|t-1}^{(i,j)} && (19) \\
&= x_t - x_{t|t-1}^{(i,j)} + \varphi_t \\
f_t^{(i,j)} &= M_{t|t-1}^{(i,j)} + \sigma_\varphi^2 && (20) \\
x_{t|t}^{(i,j)} &= x_{t|t-1}^{(i,j)} + M_{t|t-1}^{(i,j)} \left(f_t^{(i,j)} \right)^{-1} v_t^{(i,j)} && (21) \\
M_{t|t}^{(i,j)} &= M_{t|t-1}^{(i,j)} - M_{t|t-1}^{(i,j)} \left(f_t^{(i,j)} \right)^{-1} M_{t|t-1}^{(i,j)} && (22)
\end{aligned}
$$

Note that as in Hamilton (1989), the filtered transition probability, $\Pr(s_t = i, s_{t-1} = j | I_{t-1})$, is updated with transition probability, $\Pr(s_t = i | s_{t-1} = j)$, and conditional probability, $\Pr(s_{t-1} = j | I_{t-1})$ as follows;

$$
\Pr(s_t = i, s_{t-1} = j | I_{t-1}) = \Pr(s_t = i | s_{t-1} = j) \Pr(s_{t-1} = j | I_{t-1}). \quad (23)
$$

The conditional probability updated with information at time t are given by:

$$
\begin{aligned}
\Pr(s_t &= i, s_{t-1} = j | I_t) = \frac{f(y_t, s_t = i, s_{t-1} = j | I_{t-1})}{f(y_t | I_{t-1})} && (24) \\
&= \frac{f(y_t | s_t = i, s_{t-1} = j, I_{t-1}) \Pr(s_{t-1} = j | I_{t-1})}{\sum_{i=0}^{1} \sum_{j=0}^{1} f(y_t | s_t = i, s_{t-1} = j, I_{t-1}) \Pr(s_{t-1} = j | I_{t-1})}.
\end{aligned}
$$

Assuming normality the density for y_t conditional on s_t, s_{t-1} and I_{t-1} is given by:

$$f(y_t|s_t = i, s_{t-1} = j, I_{t-1}) = \frac{1}{\sqrt{2\pi f_t^{(i,j)}}} \exp\left\{ -\frac{\left(v_t^{(i,j)}\right)^2}{2f_t^{(i,j)}} \right\}. \tag{25}$$

and

$$\Pr(s_t = i|I_t) = \sum_{j=0}^{1} \Pr(s_t = i, s_{t-1} = j|I_t).$$

Note that

$$\begin{aligned} \Pr(s_t &= 0|s_{t-1} = 1) = 1 - \Pr(s_t = 1|s_{t-1} = 1) \\ \Pr(s_t &= 1|s_{t-1} = 0) = 1 - \Pr(s_t = 0|s_{t-1} = 0). \end{aligned}$$

We also obtain

$$\begin{aligned} x_{t|t}^{(i)} &= E[x_t|s_t = i, I_t] \\ &= \frac{\sum_{j=0}^{1} \Pr(s_t = i, s_{t-1} = j|I_t) x_{t|t}^{(i,j)}}{\Pr(s_t = i|I_t)}, \\ M_{t|t}^{(i)} &= E[(x_t - x_{t|t}^{(j)})^2|s_t = i, I_t] \\ &= \frac{\sum_{j=0}^{1} \Pr(s_t = i, s_{t-1} = j|I_t) M_{t|t}^{(i,j)}}{\Pr(s_t = i|I_t)}. \end{aligned}$$

The above procedure should be repeated from t=1 to T to calculate the log likelihood:

$$\pounds(\mathbf{y}|\boldsymbol{\theta}) = \sum_{t=1}^{T} \log[f(y_t|I_{t-1})]$$

where $\boldsymbol{\theta} = \{\mu_0, \mu_1, \phi_0, \phi_1, \sigma_0^2, \sigma_1^2, \Pr(s_t = 0|s_{t-1} = 0), \Pr(s_t = 1|s_{t-1} = 1)\}$. We choose different initial value sets of $\boldsymbol{\theta}$ to find the global maximum likelihood value, since Markov regime switching models usually have many local maxima. The starting values may be given by

$$
\begin{aligned}
x_{0|0}^{(i)} &= \mu_i \\
M_{0|0}^{(i)} &= \frac{\sigma_i^2}{1-\phi_i^2} \\
P(s_0 &= 1|I_0) = \frac{1}{2}.
\end{aligned}
$$

B. Smoothing Procedure

As before, in order to extract the volatility we need the smoother component. In order to get theses estimates we need some notation

$$
\begin{aligned}
x_{t|T}^{(k,i)} &= E[x_t|s_{t+1}=k, s_t=i, I_T] && (26)\\
M_{t|T}^{(k,i)} &= E[(x_t - x_{t|T}^{(k,i)})^2|s_{t+1}=k, s_t=i, I_T] && (27)\\
\Pr(s_{t+1} &= k, s_t = i|I_T) = \Pr(s_{t+1}=k|I_T)\Pr(s_t=i|I_t)\frac{\Pr(s_{t+1}=k|s_t=i)}{\Pr(s_{t+1}=k|I_t)}, && (28)
\end{aligned}
$$

where

$$\Pr(s_{t+1}=k|I_T) = \sum_{l=0}^{1}\Pr(s_{t+2}=l, s_{t+1}=k|I_T),$$

and

$$
\begin{aligned}
\Pr(s_t &= i|I_t) = \sum_{j=0}^{1}\Pr(s_t=i, s_{t-1}=j|I_t), \\
\Pr(s_{t+1} &= k|I_t) = \sum_{i=0}^{1}\Pr(s_{t+1}=k, s_t=i|I_t)
\end{aligned}
$$

from the estimation procedure in Appendix A. The smoothing equations are

$$
\begin{aligned}
x_{t|T}^{(k,i)} &= x_{t|t}^{(i)} + J_t^{(k,i)}(x_{t+1|T}^{(k)} - x_{t+1|t}^{(k,i)}) \\
M_{t|T}^{(k,i)} &= M_{t|t}^{(i)} + J_t^{(k,i)}(M_{t+1|T}^{(k)} - M_{t+1|t}^{(k,i)})J_t^{(k,i)}
\end{aligned}
$$

where

$$J_t^{(k,i)} = M_{t|t}^{(i)} \phi_k (M_{t+1|t}^{(k,i)})^{-1}.$$

Note that

$$\begin{aligned}
x_{t|T}^{(i)} &= \frac{\sum_{k=0}^{1} \Pr[s_{t+1} = k, s_t = i|I_T] x_{t|T}^{(k,i)}}{\Pr(s_t = i|I_T)} \\
M_{t|T}^{(i)} &= \frac{\sum_{k=0}^{1} \Pr[s_{t+1} = k, s_t = i|I_T] M_{t|T}^{(k,i)}}{\Pr(s_t = i|I_T)} \\
x_{t|T} &= \sum_{k=0}^{1} \sum_{i=0}^{1} \Pr[s_{t+1} = k, s_t = i|I_T] x_{t|T}^{(k,i)} \\
M_{t|T} &= \sum_{k=0}^{1} \sum_{i=0}^{1} \Pr[s_{t+1} = k, s_t = i|I_T] M_{t|T}^{(k,i)}
\end{aligned}$$

where $t = T-1, T-2, ..., 1$. For $t = T$, we use the last values from estimation procedure; these are $x_{T|T}^{(k,i)}$, $x_{T|T}^{(i)}$, $M_{T|T}^{(k,i)}$, $M_{T|T}^{(i)}$, $\Pr(s_T = k, s_{T-1} = i|I_T)$, $\Pr(s_T = k|I_T)$.

References

Alizadeh, S., M. Brandt, and F. X. Diebold, 2002, Range-Based Estimation of Stochastic Volatility Models, Journal of Finance 57, 2002, 1047-1091.

Andersen, T. and T. Bollerslev, 1998, Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts, *International Economic Review* 39(4), 885-905.

Bai, J., R. L. Lumsdaine, and J. H. Stock, 1998, Testing for and Dating Common Breaks in Multivariate Time Series, *Review of Economic Studies* 63, 395-432.

Ben-David, D. and D. H. Papell, 1998, Slowdowns and Meltdowns: Postwar Growth Evidence from 74 Countries, *Review of Economics and Statistics*, 80, 561-571.

Breidt, F., and A. Carriquiry (1996), Improved Quasi-Maximum Likelihood Estimation for Stochastic Volatility Models, Modelling and Prediction: Honoring Seymour Geisser, J. Lee, W. Johnson, and A. Zellner (eds), Springer-Verlag.

Chu, C. J., 1995. Detecting parameter shift in GARCH models. *Econometric Reviews*, 14(2), 241-266.

Diebold, F. X., 1986. Modeling the persistence of conditional variances: comment. *Econometric Reviews* 5, 51-56.

Diebold, F. X., Inoue, A., 2001. Long Memory and Regime Switching, Journal of Econometrics 105, 131-159.

Filardo, A. J., 1994, Business-cycle phases and their transitional dynamics, *Journal of Business and Economic Statistics* 12(3), 299-308.

Goodwin, T. H., 1993, Business-cycle analysis with a Markov switching model, *Journal of Business and Economic Statistics* 11(3), 331-339.

Granger, C. W. J. and Hyung, N., 1999. Occasional Structural Breaks and Long Memory. Discussion Paper 99-14, Department of Economics, University of California, San Diego.

Granger, C. W. J. and Teräsvirta, T., 1999. A Simple Nonlinear Time Series Model with Misleading Linear Properties. *Economics Letters* 62, 161-165.

Hamilton, J.D., 1989, A new approach to the economic analysis of nonstationary time-series and the business cycle, *Econometrica* 57, 357-384.

Hamilton, J. D., 1994, *Time series analysis*, Princeton.

Hamilton, J. D. and R. Susmel, 1994, Autoregressive Conditional Heteroscedasticy and Changes in Regime, *Journal of Econometrics* 64, 307-333.

Hansen, B. E., 2001, The New Econometrics of Structural Change: Dating Breaks in U.S. Labor Productivity, mimeo, University of Wisconsin.

Harvey, A. C. and Shephard, N., 1993. Estimation and testing of stochastic variance models. Econometrics discussion paper EM/93/268, London School of Economics.

Harvey, A. C. and Shephard, N., 1996. Estimation of an asymmetric stochastic volatility model for asset returns. *Journal of Business and Economic Statistics* 14, 4, 429-34.

Harvey, A. C., Ruiz, E. and Shephard, N., 1994. Multivariate stochastic variance models. *Review of Economic Studies* 61, 247-264.

Harvey, A. C., and M. Streibel (1996), Testing for a Slowly Changing Level with Special Reference to Stochastic Volatility, mimeo.

Hwang, S., 2004. Structural Breaks in AR(1) Process and Persistence. Mimeo. (Faculty of Finance, Cass Business School, London)

Hwang, S. and Satchell, S. E., 2000. Market risk and the concept of fundamental volatility: measuring volatility across asset and derivative markets and testing for the impact of derivatives markets on financial markets. *Journal of Banking and Finance* 24(5), 759-785.

Hwang, S. and Satchell, S. E., 2004, GARCH Model with Cross-Sectional Volatility: GARCHX Models, forthcoming in *Applied Financial Economics*.

Hull, J., and A. White, 1987, The pricing of options on assets with stochastic volatilities, *Journal of Finance* 42, 281-300.

Kalimipalli, M. and R. Susmel, 2001, Regime-Switching Stochastic Volatility and Short-Term Interest Rates, C. T. Bauer College of Business, University of Houston, Working Paper.

Lobato, I. N. and Savin, N. E., 1998, Real and spurious long-memory properties of stock-market data. *Journal of Business and Economic Statistics* 16(3), 261-268.

McConnell, M. M. and G. Perez-Quiros, 2000, Output Fluctuations in the United States: What Has Changed Since the Early 1980's? *American Economic Review* 90(5), 1464-1476.

Nelson, D. B., 1994, Comment on Jacquier, Polson, and Rossi's "Beyesian Analysis of Stochastic Volatility Models", *Journal of Business and Economic Statistics* 12, 403-406.

Smith, D. R., 2002, Markov-Switching and Stochastic Volatility Diffusion Models of Short-Term Interest Rates, Forthcoming in *Journal of Business and Economic Statistics.*

So, M. K. P., K. Lam and W. K. Li, 1998, A stochastic volatility model with Markov switching, *Journal of Business and Economic Statistics* 16, 244-253.

Stock, J. H. and M. W. Watson, 1996, Evidence on structural instability in macroeconomic time series relations, Journal of Business and Economic Statistics 14, 11-30.

Table 1 Properties of Log-Squared Residuals

A. S&P500 Index Volatility

		Returns (%)	Log- Squared Residuals (Returns-Mean)	Log- Squared Residuals Modified with Breidt and Carriquiry (1996) Method (κ=0.02)	Log- Squared Residuals Modified with Breidt and Carriquiry (1996) Method (κ=0.05)	Log- Squared Residuals Modified with Breidt and Carriquiry (1996) Method (κ=0.10)
Daily Returns (A total number of 2548 returns from 27 February 1992 to 27 February 2002)	Mean	0.046	-1.945	-1.624	-1.459	-1.290
	Standard Deviation	0.983	2.606	1.956	1.739	1.551
	Skewness	-0.307	-1.120	-0.051	0.208	0.434
	Excess Kurtosis	4.937	2.101	-0.868	-0.867	-0.708
	Jarque and Bera Statistics	2687.326	1024.458	82.962	100.341	136.093
	Ljung and Box (10)	21.335	304.897	545.184	630.323	697.345
	Ljung and Box (50)	95.914	1371.533	2328.800	2606.691	2807.545
	Autocorrelation (Lag 1)	0.005	0.092	0.117	0.126	0.133
	Autocorrelation (Lag 5)	-0.034	0.127	0.178	0.190	0.199
	Autocorrelation (Lag 10)	0.043	0.133	0.171	0.183	0.191
	Autocorrelation (Lag 15)	0.016	0.099	0.137	0.143	0.147
	Autocorrelation (Lag 20)	-0.008	0.094	0.122	0.126	0.130
	Autocorrelation (Lag 30)	-0.010	0.109	0.142	0.150	0.154
	Autocorrelation (Lag 40)	0.012	0.084	0.121	0.131	0.138
	Autocorrelation (Lag 50)	-0.013	0.088	0.110	0.114	0.115
Weekly Returns (A total number of 522 returns from 26 February 1992 to 27 February 2002)	Mean	0.227	-0.110	0.130	0.259	0.397
	Standard Deviation	2.090	2.395	1.856	1.664	1.492
	Skewness	-0.421	-1.237	-0.251	0.010	0.244
	Excess Kurtosis	1.743	2.207	-0.701	-0.829	-0.786
	Jarque and Bera Statistics	81.463	239.005	16.166	14.957	18.610
	Ljung and Box (10)	16.357	51.645	76.889	88.010	98.154
	Ljung and Box (50)	41.056	190.159	296.735	332.606	361.150
	Autocorrelation (Lag 1)	-0.094	0.099	0.117	0.123	0.128
	Autocorrelation (Lag 5)	-0.015	0.035	0.067	0.083	0.098
	Autocorrelation (Lag 10)	0.031	0.051	0.069	0.079	0.087
	Autocorrelation (Lag 15)	0.017	0.098	0.121	0.119	0.115
	Autocorrelation (Lag 20)	0.043	0.064	0.128	0.146	0.158
	Autocorrelation (Lag 30)	0.032	0.101	0.123	0.128	0.132
	Autocorrelation (Lag 40)	-0.020	0.006	0.037	0.047	0.055
	Autocorrelation (Lag 50)	0.045	0.097	0.121	0.126	0.130

B. FTSE100 Index Volatility

		Returns (%)	Log- Squared Residuals (Returns-Mean)	Log- Squared Residuals Modified with Breidt and Carriquiry (1996) Method (κ=0.02)	Log- Squared Residuals Modified with Breidt and Carriquiry (1996) Method (κ=0.05)	Log- Squared Residuals Modified with Breidt and Carriquiry (1996) Method (κ=0.10)
Daily Returns (A total number of 2606 returns from 27 February 1992 to 27 February 2002)	Mean	0.041	-1.556	-1.339	-1.212	-1.074
	Standard Deviation	1.003	2.321	1.852	1.666	1.496
	Skewness	-0.115	-1.084	-0.234	0.016	0.236
	Excess Kurtosis	2.163	1.591	-0.744	-0.873	-0.830
	Jarque and Bera Statistics	502.327	767.686	82.045	81.106	96.734
	Ljung and Box (10)	41.900	283.552	389.479	443.857	502.353
	Ljung and Box (50)	77.288	894.607	1297.436	1478.730	1658.872
	Autocorrelation (Lag 1)	0.059	0.110	0.116	0.121	0.126
	Autocorrelation (Lag 5)	-0.011	0.123	0.136	0.141	0.148
	Autocorrelation (Lag 10)	0.011	0.105	0.118	0.124	0.130
	Autocorrelation (Lag 15)	-0.008	0.072	0.101	0.110	0.117
	Autocorrelation (Lag 20)	0.006	0.089	0.107	0.113	0.120
	Autocorrelation (Lag 30)	0.000	0.083	0.108	0.117	0.124
	Autocorrelation (Lag 40)	-0.033	0.037	0.041	0.044	0.047
	Autocorrelation (Lag 50)	0.040	0.051	0.082	0.093	0.101
Weekly Returns (A total number of 522 returns from 26 February 1992 to 27 February 2002)	Mean	0.199	0.047	0.293	0.417	0.548
	Standard Deviation	2.147	2.429	1.854	1.660	1.488
	Skewness	0.091	-1.416	-0.410	-0.169	0.039
	Excess Kurtosis	1.152	2.722	-0.730	-0.947	-0.990
	Jarque and Bera Statistics	29.572	335.635	26.225	21.989	21.438
	Ljung and Box (10)	9.657	16.255	27.094	33.236	40.110
	Ljung and Box (50)	59.082	64.692	95.459	109.816	124.071
	Autocorrelation (Lag 1)	-0.079	0.009	0.039	0.048	0.056
	Autocorrelation (Lag 5)	-0.015	0.040	0.049	0.056	0.064
	Autocorrelation (Lag 10)	0.067	0.052	0.069	0.080	0.091
	Autocorrelation (Lag 15)	-0.053	0.104	0.110	0.120	0.128
	Autocorrelation (Lag 20)	-0.014	0.039	0.053	0.058	0.062
	Autocorrelation (Lag 30)	-0.035	-0.029	-0.011	-0.002	0.007
	Autocorrelation (Lag 40)	-0.029	0.014	0.034	0.033	0.030
	Autocorrelation (Lag 50)	0.023	0.032	0.034	0.027	0.022

Table 2 Estimates of SV, SV with Regime Changing Mean and SV with MRS State Equation for Daily Volatility

A. S&P500 Index Volatility

	SV Model		SVMRS Model		SVMRS Model with Restriction of $\phi_1 = \phi_2$ and $\sigma_{1\eta}=\sigma_{2\eta}$				Smith's (2002) Model	
	Log-Squared Residuals		Log-Squared Residuals		Log-Squared Residuals		BC Log-Volaitlity (κ=0.05)		Log-Squared Residuals	
	Estimates	STD	Estimates	STD	Estimates	STD	Estimates	STD	Estimates	STD
μ_0	-1.976	0.455	-4.602	0.278	-9.241	0.676	-2.860	0.265	-6.546	0.229
μ_1			-1.006	0.099	-3.796	0.624	-0.334	0.268	-1.075	0.064
ϕ_0	0.999	0.001	0.361	0.141	0.993	0.002	0.996	0.003	0.081	0.019
ϕ_1			0.126	0.036						
$\sigma_{\eta 0}$	0.044	0.016	2.872	0.108	0.105	0.026	0.066	0.020	1.829	0.051
$\sigma_{\eta 1}$			1.548	0.042						
σ_{φ}	2.465	0.054	0.002	0.003	1.668	0.048	0.991	0.022	0.000	0.000
$p^{(0,0)}$			0.268	0.050	0.132	0.020	0.486	0.021	0.134	0.024
$p^{(1,1)}$			0.691	0.036	0.877	0.012	0.502	0.021	0.871	0.013
ML Values	-6071.1		-5950.9		-5854.5		-4886.6		-6002.452	

Notes: A total number of 2606 returns from 27 February 1992 to 27 February 2002 is used.

B. FTSE100 Index Volatility

	SV Model		SVMRS Model		SVMRS Model with Restriction of $\phi_1 = \phi_2$ and $\sigma_{1\eta}=\sigma_{2\eta}$				Smith's (2002) Model	
	Log-Squared Residuals		Log-Squared Residuals		Log-Squared Residuals		BC Log-Volaitlity (κ=0.05)		Log-Squared Residuals	
	Estimates	STD	Estimates	STD	Estimates	STD	Estimates	STD	Estimates	STD
μ_0	-1.549	0.205	-4.814	1.537	-7.690	0.467	-3.383	0.212	-6.042	0.248
μ_1			-0.941	0.397	-2.706	0.407	-0.825	0.208	-0.871	0.060
ϕ_0	0.990	0.005	0.899	0.241	0.985	0.005	0.986	0.005	0.079	0.018
ϕ_1			0.260	0.146						
$\sigma_{\eta 0}$	0.107	0.027	1.881	0.605	0.142	0.026	0.120	0.018	1.653	0.036
$\sigma_{\eta 1}$			0.856	0.156						
σ_{φ}	2.196	0.042	1.084	0.200	1.490	0.036	0.909	0.019	0.000	0.046
$p^{(0,0)}$			0.217	0.053	0.157	0.023	0.382	0.019	0.135	0.022
$p^{(1,1)}$			0.752	0.151	0.892	0.012	0.595	0.020	0.897	0.012
ML Values	-5670.3		-5494.9		-5423.9		-4674.3		-5540.448	

Notes: A total number of 2548 returns from 27 February 1992 to 27 February 2002 is used.

Table 3 Estimates of SV, SV with Regime Changing Mean and SV with MRS State Equation for Weekly Volatility

A. S&P500 Index Volatility

	SV Model		SVMRS Model		SVMRS Model with Restriction of $\phi_1 = \phi_2$ and $\sigma_{1\eta}=\sigma_{2\eta}$				Smith's (2002) Model	
	Log-Squared Residuals		Log-Squared Residuals		Log-Squared Residuals		BC Log-Volaitlity (κ=0.05)		Log-Squared Residuals	
	Estimates	STD	Estimates	STD	Estimates	STD	Estimates	STD	Estimates	STD
μ_0	-0.050	0.396	-2.770	0.474	-5.973	0.416	-1.930	0.447	-4.870	0.617
μ_1			0.738	0.144	-0.682	0.732	0.504	0.456	0.526	0.139
ϕ_0	0.992	0.007	0.426	0.253	0.995	0.004	0.994	0.009	0.109	0.064
ϕ_1			0.104	0.052						
$\sigma_{\eta 0}$	0.088	0.036	2.673	0.219	0.050	0.032	0.069	0.069	1.225	0.835
$\sigma_{\eta 1}$			1.368	0.079						
σ_{φ}	2.280	0.109	0.001	0.005	1.561	0.116	1.015	0.056	1.121	0.867
$p^{(0,0)}$			0.256	0.086	0.123	0.031	0.390	0.047	0.105	0.047
$p^{(1,1)}$			0.718	0.057	0.886	0.047	0.615	0.053	0.886	0.030
ML Values	-1179.4		-1133.8		-1125.6		-968.0		-1145.273	

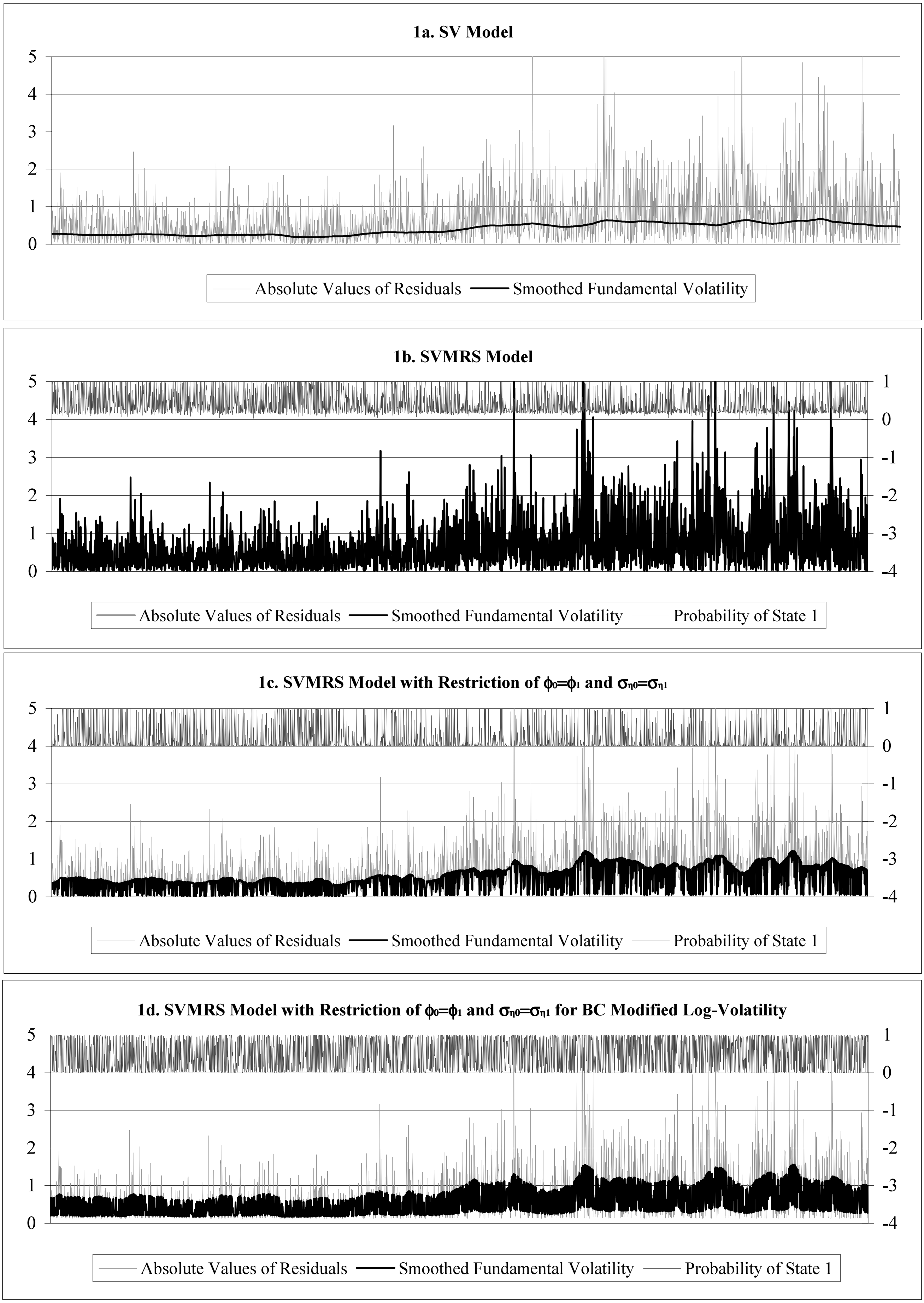
Notes: A total number of 522 returns from 26 February 1992 to 27 February 2002 is used.

B. FTSE100 Index Volatility

	SV Model		SVMRS Model		SVMRS Model with Restriction of $\phi_1 = \phi_2$ and $\sigma_{1\eta}=\sigma_{2\eta}$				Smith's (2002) Model	
	Log-Squared Residuals		Log-Squared Residuals		Log-Squared Residuals		BC Log-Volaitlity (κ=0.05)		Log-Squared Residuals	
	Estimates	STD	Estimates	STD	Estimates	STD	Estimates	STD	Estimates	STD
μ_0	0.050	0.217	-2.224	0.682	-5.881	0.406	-1.658	0.210	-5.186	0.530
μ_1			1.075	0.194	-0.070	0.612	1.092	0.208	0.654	0.100
ϕ_0	0.969	0.019	0.303	0.290	0.972	0.012	0.972	0.014	0.009	0.031
ϕ_1			0.021	0.038						
$\sigma_{\eta 0}$	0.142	0.058	2.795	0.207	0.164	0.039	0.112	0.030	1.646	0.081
$\sigma_{\eta 1}$			1.229	0.156						
σ_{φ}	2.357	0.113	0.001	0.399	1.541	0.078	0.893	0.037	0.000	0.014
$p^{(0,0)}$			0.255	0.083	0.118	0.020	0.323	0.036	0.120	0.048
$p^{(1,1)}$			0.631	0.145	0.899	0.047	0.625	0.043	0.899	0.021
ML Values	-1198.0		-1126.8		-1130.6		-958.8		-1142.167	

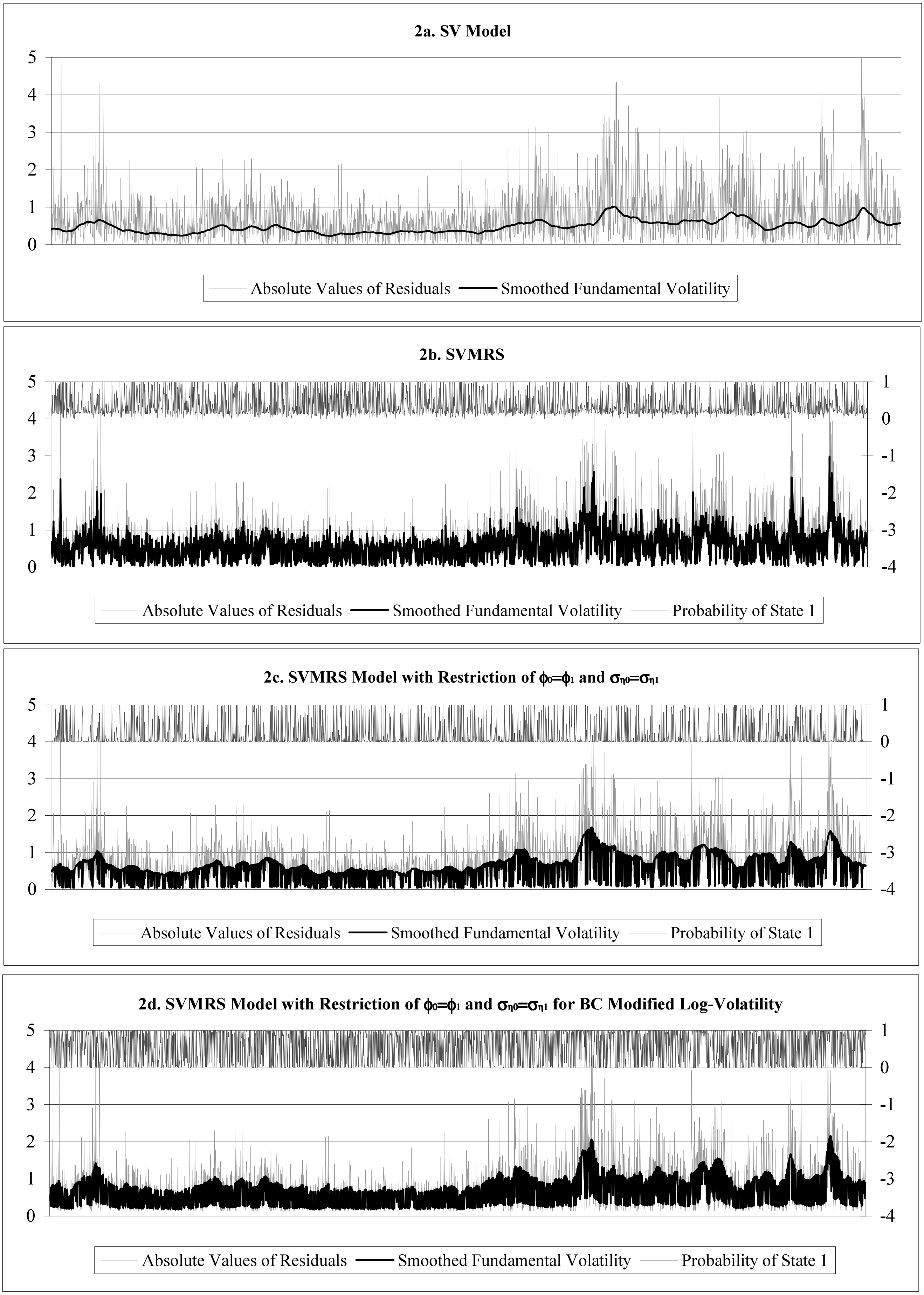
Notes: A total number of 522 returns from 26 February 1992 to 27 February 2002 is used.

Figure 1 Smoothed Volatility and State Probabillity for S&P500 Index Daily Volatility



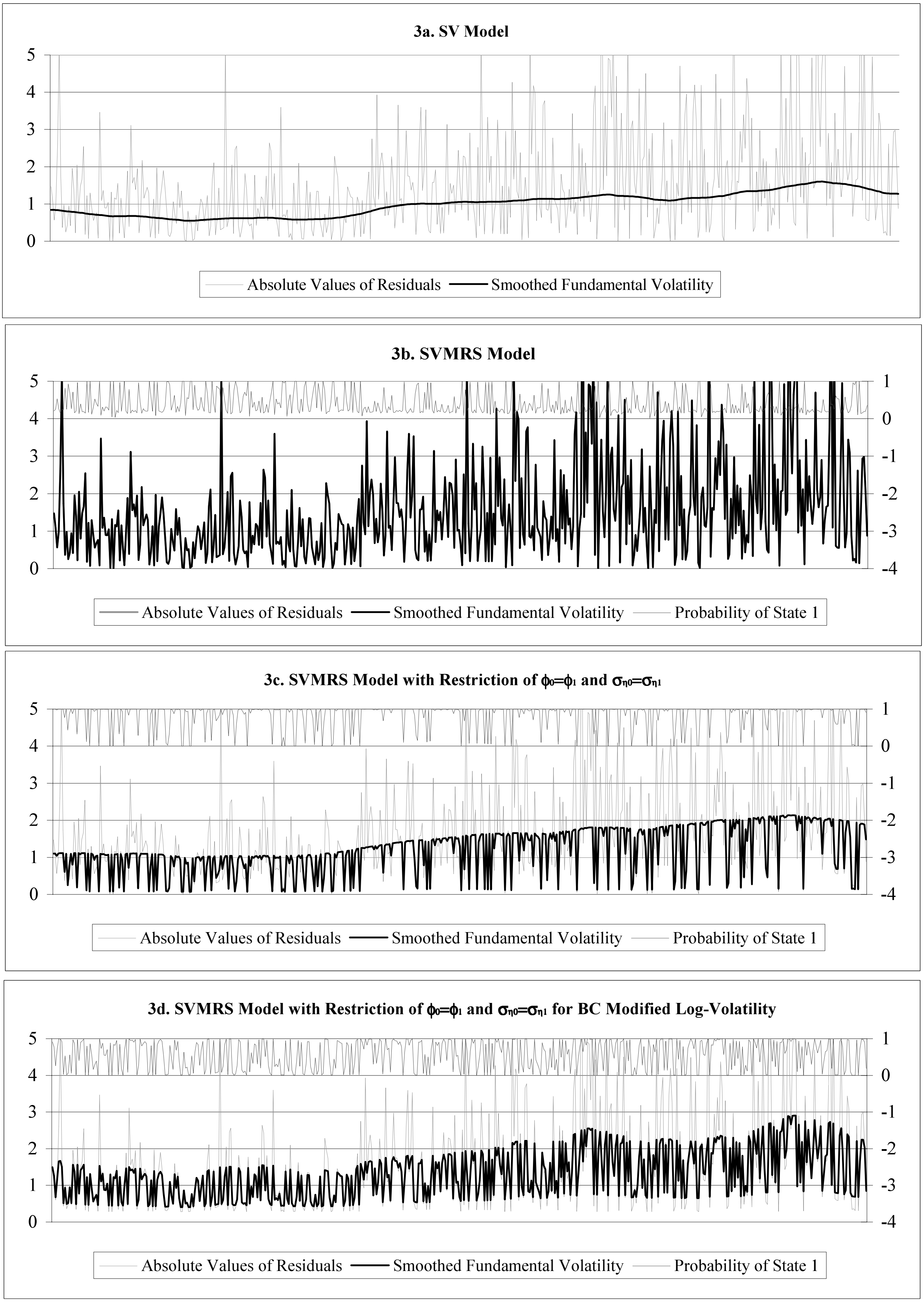
Notes: A total number of 2606 returns from 27 February 1992 to 27 February 2002 is used.

Figure 2 Smoothed Volatility and State Probabillity for FTSE100 Index Daily Volatility



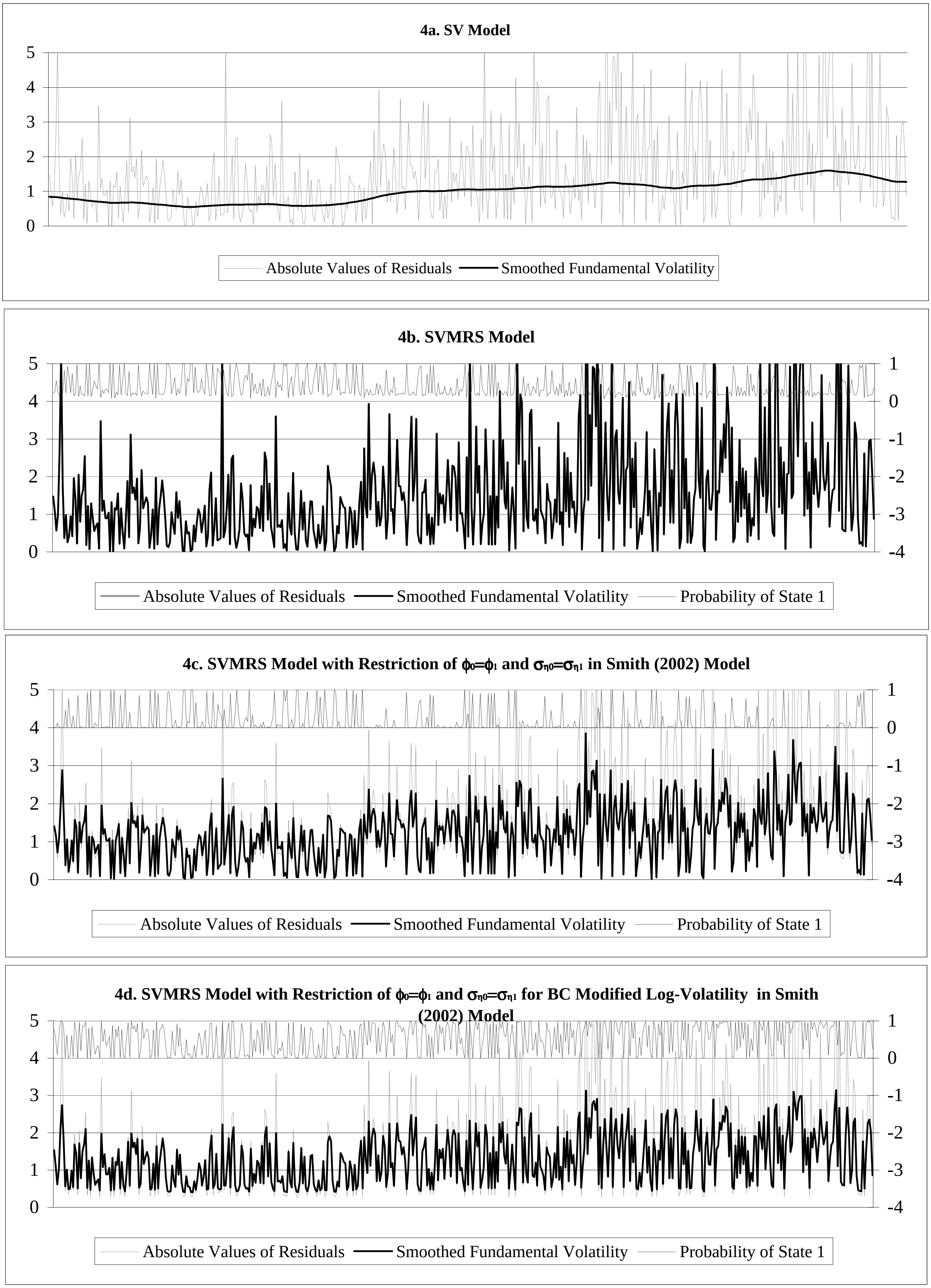
Notes: A total number of 2548 returns from 27 February 1992 to 27 February 2002 is used.

Figure 3 Smoothed Volatility and State Probabillity for S&P500 Index Weekly Volatility



Notes: A total number of 522 returns from 26 February 1992 to 27 February 2002 is used.

Figure 4 Smoothed Volatility and State Probabillity for S&P500 Index Weekly Volatility with Smith (2002) Model



Notes: A total number of 522 returns from 26 February 1992 to 27 February 2002 is used.