Welfare Costs of Business Cycles in South America*

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Abstract

How large are welfare costs related to economic aggregate fluctuations is a topic of great concern among economists at least since Robert Lucas' well-known and thought-provoking exercise in the late 1980s. Our analysis assesses the magnitude of such costs for 11 countries in South America by means of two approaches: Lucas' classical set-up with deterministic linear trend for consumption, and one in which consumption trend is stochastic and whose implementation is performed using Beveridge-Nelson decomposition. The latter approach is motivated by a substantial theoretical literature and empirical evidence. Our results suggest South American countries have welfare costs associated with economic fluctuations notably higher than the US economy, hence eliminating cyclical variability in consumption to some extent may be desirable in those countries.

Key Words: business cycles; welfare costs; consumption; Beveridge-Nelson decomposition.

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1 Introduction

In an influential work, Lucas (1987) estimated the welfare costs of business cycles to be rather small in the US economy. This result suggested there is no role for marginal counter-cyclical policies, since the upper bound of the welfare gains from such policies (as the calculations were intended) could be easily overwhelmed by their costs. But what can be said about other economies, especially the extremely unstable South American countries? Is it true as well that there is no role for marginal counter-cyclical policies?

The aim of this paper is to appraise the welfare costs of economic aggregate fluctuations in eleven South American countries. Our approach relies not only on the basic set-up in Lucas (1987), but also on the Beveridge and Nelson (1981) approach in decomposing economic time series. The last approach is not usual in doing these calculations, but has recently been used in several papers - see, e.g., Issler and Rocha (2000), Duarte, Issler and Salvato (2003), and Franco, Guillén and Issler (2003).

Our investigation is relevant at least for two reasons. Firstly, to the best of our knowledge, there is no empirical evidence concerning how large are the costs of business cycles in South America vis-à-vis stable economies such as the United States. Previous studies have been mainly concerned with the costs in the US economy (for example, Lucas (1987), Obstfeld (1994), Dolmas (1998)), while a few ones analyzed Europe (Duarte, Issler and Salvato (2003)) and even Africa (Pallage and Robe (2000)). Hence, depending upon our results, one may appraise whether it is desirable to increase the size of counter-cyclical policies in those South American countries.

Secondly, the majority of applied works assumes consumption to be stationary, and the few papers that allow for an integrated ($I(1)$) process impose a priori restrictions on the cycle of the series (e.g., Obstfeld (1994), Dolmas (1998)). Here we do not impose any a priori restrictions on the cycle of consumption. Rather, if we find evidence that $\log c_t \sim I(1)$, then we model $\Delta \log c_t$ by a general stationary ARMA, thereby endogenizing model choice for each country data.

The main results of this paper are that South American countries generally have large welfare losses associated with economic fluctuations if we consider Lucas' classical framework. Moreover, if we take into account the approach which allows for $\log c_t \sim I(1)$, such losses are even larger. Therefore, there is a potential positive role for more effective counter-cyclical policies in poor countries in South America, contrary to what is often claimed about the US economy.

We proceed as follows. In section 2, the economic environment is presented carefully. Data are described in section 3, where we subsequently present the main results and comment upon the limitations of our approach. The conclusion set forth some ideas for further research and summarizes the findings up to this point.

2 Environment

Agents are supposed to live an infinite number of periods and to derive utility from the stream of consumption (c) throughout their lives according to the following utility function:

$$U(c_0, c_1, \ldots) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where E_0 is the expectation operator given the information set at $t = 0$, $\beta \in (0, 1)$ stands for the intertemporal discount factor, and the momentary utility function is represented by

$$u(c) = \frac{c^{1-\phi} - 1}{1 - \phi}, \tag{2}$$

where $\phi \geq 1$.[1] Assume further, as Lucas (1987), that $(c_t)_t$ is log-normal around a deterministic trend, that is:

$$c_t = \alpha_0 \left(1 + \alpha_1\right)^t z_t, \tag{3}$$

where $\log z_t \sim N\left(0, \sigma_z^2\right)$.

In this set-up, it is straightforward to notice that a cycle-free consumption stream is given by $(c_t^*)_t = (Ec_t)_t$. Thus $c_t^* = \alpha_0 \left(1 + \alpha_1\right)^t \exp\left(\frac{\sigma_z^2}{2}\right)$. Every risk-averse consumer (as the one represented by the concave utility function above) prefers a certain stream $(c_t^*)_t$ to an uncertain one, $(c_t)_t$, since both series have the same mean. Therefore, the welfare costs associated with aggregate fluctuations in this economy can be represented by the compensating variation in consumption, λ, which solves:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left((1 + \lambda)c_t\right) = \sum_{t=0}^{\infty} \beta^t u(c_t^*). \tag{4}$$

That is, λ is the compensation in all dates and states of nature that makes a typical agent indifferent between the two streams of consumption previously mentioned. Notice that, the

[1] Notice that, when $\phi \to 1$, $u(c)$ collapses to $\log c$.

higher is λ, the stronger will be an agent's willingness to live in a business cycle-free world instead of a world with aggregate fluctuations.

Solving (4) for λ, given (1) to (3), it is easily checked that:

$$\lambda = \exp\left(\frac{\phi\sigma_z^2}{2}\right) - 1, \tag{5}$$

for $\phi \geq 1$.[2] This formula for the welfare costs of aggregate fluctuations has two intuitions. Initially notice that the more volatile is consumption time series, in the sense of a higher variance (σ_z^2), the higher are the costs of business cycles. Furthermore, the welfare costs are also higher for more risk averse agents, that is, λ is increasing in ϕ.

Although Lucas (1987) proposed exactly this analysis, he implemented it in a different way. Instead of estimating σ_z^2 from the residuals associated with the log-linear regression implied by (3), Lucas filtered the logarithm of consumption series using the procedure in Hodrick and Prescott (1997) - HP -, and estimated σ_z^2 from cycle obtained by subtracting the HP-trend from the original series.

In spite of its simplicity, the preceding analysis has a serious drawback: it does not take into account that $\log c_t$ is frequently considered $I(1)$ in several theoretical (as permanent income theory[3]) and empirical studies (e.g., Gomes and Paz (2003)), thereby causing specification error. It is worth noting this error may lead to completely flawed results, despite all the intuition in equation (5). In case $\log c_t \sim I(1)$, the aggregate risk of the economy would be a function of all z_i, $i = 1, 2, \ldots, t$. On the other hand, λ as described by (5) is merely a function of σ_z^2, not of the entire history of the random variable z_t. Thus, if it were the case of $\log c_t$ being $I(1)$, equation (5) probably would underestimate the costs of economic fluctuations.

To deal with this fact, test whether $\log c_t \sim I(1)$ (Augmented Dickey-Fuller). If so, the trend of the series is also stochastic and, given the evidence of a unit root, unconditional mean and variance if the original series are not well-defined. We then redefine $c_t^* = E_0 c_t$ as in Obstfeld (1994), and next apply Beveridge and Nelson (1981) decomposition for $\log c_t \sim I(1)$. The latter have shown that any difference stationary stochastic process can be decomposed as the sum a deterministic term, a random walk trend and a stationary cycle. That is, if

[2] Appendix A.

[3] See Hall (1978), Flavin (1981), and Campbell (1987).

$\Delta \log c_t \sim I(0)$, then:

$$\log c_t = \log \left[\alpha_0 (1+\alpha_1)^t\right] + \log X_t + \log Y_t, \tag{6}$$

where $\log (1+\alpha_1)^t$ is the deterministic trend, $\log X_t = \sum_{i=1}^t \varepsilon_i$ stands for the random walk component, and $\log Y_t = \sum_{i=1}^t \psi_{t-i} \vartheta_i$ represents the stationary cycle. We assume further that permanent shock (ε_t) and the transitory shock (ϑ_t) have a bivariate normal distribution as

$$\begin{bmatrix} \varepsilon_t \\ \vartheta_t \end{bmatrix} \sim IIDN \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \right). \tag{7}$$

In this framework, we define implicitly λ as in (4), except for the fact that $c_t^* = E_0 c_t$. It can be shown that, if $\phi > 1$, then:

$$\lambda = \left(\frac{\sum_{t=0}^{\infty} \left[\beta (1+\alpha_1)^{1-\phi}\right]^t \exp\left(\frac{(1-\phi) w_t^2}{2}\right)}{\sum_{t=0}^{\infty} \left[\beta (1+\alpha_1)^{1-\phi}\right]^t \exp\left(\frac{(1-\phi)^2 w_t^2}{2}\right)} \right)^{\frac{1}{1-\phi}} - 1, \tag{8}$$

where $w_t^2 = t\sigma_{11} + 2\sigma_{12} \sum_{j=1}^t \psi_j + \sigma_{22} \sum_{j=1}^t \psi_j^2$ is the variance of the logarithm of consumption conditional in the information at $t = 0$.[4] When $\phi = 1$, equation (8) amounts to:

$$\lambda = \exp\left((1-\beta) \sum_{t=0}^{\infty} \beta^t \frac{w_t^2}{2} \right) - 1. \tag{9}$$

Notice that, as before, the more volatile is the consumption series, the higher are the costs associated with business cycles. Furthermore, even though those costs now depend upon the growth rate of consumption, they are not functions of the initial level, α_0: linear shifts on the logarithm of consumption series do not affect λ. Therefore, more opulent societies (in the sense of enjoying a higher level of consumption vis-à-vis other countries in every period) do not necessarily have lower costs of cyclical fluctuations.

The following proposition gives sufficient conditions for the convergence of summations in (8) and (9) and will be useful in empirically computing values for the compensating variation in consumption.

[4] All calculations are presented in Appendix A, jointly with a discussion on how to identify relevant parameters.

Proposition 1 *Assume* $\psi_t \to 0$ *as* $t \to \infty$ *(which is often the case in our set-up: see (A14) below). Then* ***(i)*** *(9) always converges, and* ***(ii)*** *(8) converges whenever* $\alpha_1 \geq 0$ *and* $\beta\left(1+\alpha_1\right)^{1-\phi}\exp\left(\frac{(1-\phi)^2\sigma_{11}}{2}\right) < 1$.

Proof. *Appendix B.* ■

Whereas the above proposition gives sufficient conditions in order to guarantee the convergence of λ, it is by no means necessary. Fortunately, in every case those conditions did not hold in our empirical implementation, the computed value of λ was significantly large so as to allow us to infer that the costs of business cycles diverged to ∞. Besides, in most relevant cases (i.e., $\phi \in \{1, 2, 5\}$), those conditions were frequently satisfied.

Given the theory laid out in this section, we turn to the description of our empirical investigation in the next one.

3 Empirical Results

The directions to empirically implement the computations of the welfare costs of cyclical fluctuations are as follows. With respect to Lucas' framework, given the specification in (3), all one must do is to run a linear least squares regression of the logarithm of consumption in a time trend and a constant, and then store the standard deviation of residuals as an input for (5). An analogous exercise accounts for the calculations using the HP filter: σ_z is computed using the deviation of the original series from the HP trend.

Regarding the second approach, we proceeded as follows for each country: 1^{st}) Test whether the series has a unit root. If so, then go on the next steps. 2^{nd}) Model the "best" stationary ARMA process for $\Delta \log c_t$.[5] 3^{rd}) Follow the procedure described in Appendix A on how to identify each component of equations (8) and (9), and then evaluate λ.

Our data set consists of constant price annual information from 1951 to 1999 for per capita consumption in eleven South American countries and US, and it was obtained from Penn World Table - Summers, Heston, and Aten (2002). South American countries are: Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Guyana, Peru, Paraguay, Uruguay and Venezuela. Although we lack specific information for non-durable consumption as it would be preferable, our data are widely used in econometric studies, and they are also directly comparable.

[5] See section 3 for the meaning of "best".

3.1 Lucas' classical set-up

Equation (3) was separately estimated via ordinary least squares. All country estimates were significative at 5%.[6] Our benchmark case is $\beta = 0.95$ and $\phi = 2$, and the results in this case are presented in table 1 (below).

As it was already expected, the costs associated with business in the American economy are quite small, both with linear trend specification and HP filter. The costs in South American are, however, quite large vis-à-vis US: they typically average 10 times the corresponding estimate for US with HP filter, and 20 times in the linear trend case.

3.2 Modified framework: Beveridge-Nelson decomposition

The first step consisted in testing for unit root. Only for Bolivia we have found evidence that $\log c_t \sim I(0)$ (see table 2). In the second step, the estimation of the "best" stationary ARMA(p, q) for $\Delta \log c_t$ when $\log c_t \sim I(1)$, lag length was selected so as to minimize Schwarz information criterion and do not fail diagnostic testing. In particular, we firstly checked if the Ljung-Box Q-statistics associated with partial and autocorrelegram of first difference of consumption were not significative. If so, we merely modelled the demeaned series as an innovation. To the contrary, whenever the previously mentioned statistic was significative for any lag, we have chosen the best ARMA(p, q) in order to minimize information criterion.

[6] Given evidence of serial correlation in residuals, we have used Newey-West covariance estimator.

Table 1 - Welfare Costs of Business Cycles

$\lambda(\%)$ when $\beta = 0.95$ and $\phi = 2$			
Country	Beveridge-Nelson	Hodrick-Prescott	Linear Trend
USA	0.48	0.04	0.10
Argentina	3.68	0.25	0.76
Brazil	4.15	0.24	2.34
Bolivia	–	0.20	0.64
Chile	24.39	0.88	2.59
Colombia	0.19	0.07	0.29
Ecuador	0.26	0.08	0.97
Guyana	212.86	1.54	4.12
Paraguay	1.97	0.35	0.70
Peru	2.73	0.37	5.94
Uruguay	2.53	0.36	1.47
Venezuela	7.66	0.53	3.87

Results of the benchmark case are reported in table 2. If we do not consider the extremely large costs in Chile and Guyana, South American countries have costs associated with cyclical fluctuations 6 times the corresponding estimate for the US economy in average. In absolute terms, if we once more exclude Chile and Guyana from the sample, the welfare costs of aggregate fluctuations using the Beveridge-Nelson approach average 2 times the costs in the linear trend case, and 9 times the costs with HP. As it was expected, to impose that consumption is $I(0)$ when they are $I(1)$ instead leads to a underestimation of λ.

Table 2

Unit Root Test (ADF) - Logarithm of Consumption (H_0 : series has unit root)		
Country	Level (p-value)	1^{st} difference (p-value)
USA	0.29	0.00
Argentina	0.35	0.00
Brazil	1.00	0.00
Bolivia	0.02	-
Chile	0.49	0.00
Colombia	0.99	0.04
Ecuador	0.89	0.00
Guyana	0.08	0.00
Paraguay	0.06	0.00
Peru	0.14	0.00
Uruguay	0.90	0.00
Venezuela	0.24	0.00

Given the results (see Appendix C for a detailed report) outlined above, we classify South American countries and US in three groups according to the magnitude of their welfare costs. The first group is labeled "small costs" and consists of the US economy and Bolivia, Colombia, and Ecuador (see figure 1). Even though those countries are not alike with respect to economic performance, their consumption series is indeed quite smooth. This support our previous claim (section 2) that welfare costs of aggregate fluctuations are necessarily correlated with good economic outcomes (income, equity, etc.).

Brazil is included in the second group, which we name "medium costs" and comprises five additional countries (viz Argentina, Paraguay, Peru, and Uruguay). Although Issler and Rocha (2000) data set is different from ours and their Beveridge-Nelson approach is not completely in line with our procedures, one must notice that, for the benchmark $\beta = 0.95$ the estimates are not too discrepant. For instance, for $\phi = 5$, Issler and Rocha's results give $\lambda = 3.17\%$, whereas our estimate is 5.17%.

Lastly, the third group consists of three countries (Chile, Guyana and Venezuela), whose consumption series behavior is exceedingly volatile (see figure 3).

4 Concluding Remarks

This study was concerned with the welfare of aggregate fluctuations in developing South American countries. Our results suggest that, as opposed to the US economy, many countries in South America have substantial welfare costs associated with economic fluctuations. This finding is quite intuitive and give rise to more careful evaluation of the size of counter-cyclical policies in those nations. But it is only a claim for additional counter-cyclical policies if government were in fact able to smooth economic cycles; to the contrary, the claim does not apply.

One important contribution of this work was to shed some light on possibly misleading results arising from misespeficication of the consumption series. In this sense, it was shown that, when we endogenize the reduced-form of consumption, the costs of the aggregate fluctuations are no longer substantially small as the underestimates (of at least one order of magnitude) suggest when we impose $\log c_t \sim I(0)$.

Finally, with respect to the US economy, our empirical findings may be compared with Obstfeld (1994), who performs an estimation using total consumption data in PWT. In this case, our estimates are slightly larger, which might be explained by the fact that, when one endogenizes the stochastic process driving the business cycles, underestimates hardly arise. But, at the same time, our estimates of the costs in US allow us to infer they are small relatively to South American countries.

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A Appendix

A.1 Lucas' λ

From (4),

$$E\left(E_0\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda)c_t\right)\right)=E\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda)c_t\right)=\sum_{t=0}^{\infty}\beta^t u(c_t^*). \tag{A1}$$

Given the functional assumption (2) and the stochastic process for consumption (3), (A1) yields:

$$(1+\lambda)^{1-\phi}\sum_{t=0}^{\infty}\beta^t Ec_t^{1-\phi}=\alpha_0^{1-\phi}\exp\left(\frac{(1-\phi)\sigma_z^2}{2}\right)\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\phi}\right]^t. \tag{A2}$$

Notice the left-hand side of (A2) can be simplified by using log-Normal's usual properties:

$$Ec_t^{1-\phi}=\left(\alpha_0(1+\alpha_1)^t\right)^{1-\phi}\exp\left(\frac{(1-\phi)^2\sigma_z^2}{2}\right). \tag{A3}$$

Then, (5) follows from (A2) and (A3) after some straightforward algebra.

A.2 Beveridge-Nelson Approach

Initially, we demonstrate (8). Our task is to find λ such that, given (1), (2), (6) and (7), it solves:

$$E_0\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda)c_t\right)=\sum_{t=0}^{\infty}\beta^t u(E_0c_t). \tag{A4}$$

It is easily verified that (A4) simplifies to:

$$(1+\lambda)^{1-\phi}\sum_{t=0}^{\infty}\beta^t E_0c_t^{1-\phi}=\alpha_0^{1-\phi}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\phi}\right]^t\left(E_0X_tY_t\right)^{1-\phi}, \tag{A5}$$

where $E_0X_tY_t = E_0\exp\left(\sum_{i=1}^t\varepsilon_i+\sum_{i=1}^t\psi_{t-i}\vartheta_i\right)$. Let $\zeta_t=\sum_{i=1}^t\varepsilon_i+\sum_{i=1}^t\psi_i\vartheta_{t-i}$. Then its conditional distribution is $\zeta_t\sim N\left(0,w_t^2\right)$, and $w_t^2=t\sigma_{11}+2\sigma_{12}\sum_{j=1}^t\psi_j+\sigma_{22}\sum_{j=1}^t\psi_j^2$. Hence, the right-hand side of (A5) is:

$$RHS=\alpha_0^{1-\phi}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\phi}\right]^t\exp\left(\frac{(1-\phi)\,w_t^2}{2}\right). \tag{A6}$$

Since $\zeta_t \sim N(0, w_t^2)$, the left-hand side of (A5) can be further simplified:

$$LHS = (1+\lambda)^{1-\phi} \alpha_0^{1-\phi} \sum_{t=0}^{\infty} \left[\beta (1+\alpha_1)^{1-\phi}\right]^t \exp\left(\frac{(1-\phi)^2 w_t^2}{2}\right). \tag{A7}$$

Therefore, after some algebra, (A6) and (A7) imply (8).

If $\phi = 1$, λ must solve:

$$\sum_{t=0}^{\infty} \beta^t E_0 \log((1+\lambda)c_t) = \sum_{t=0}^{\infty} \beta^t \log E_0 c_t. \tag{A8}$$

Notice that $E_0 \log c_t = \log \alpha_0 (1+\alpha_1)^t$ and $E_0 c_t = \alpha_0 (1+\alpha_1)^t \exp\left(\frac{w_t^2}{2}\right)$. Then (A8) yields:

$$\frac{1}{1-\beta} \log(1+\lambda) + \sum_{t=0}^{\infty} \beta^t \log \alpha_0 (1+\alpha_1)^t = \sum_{t=0}^{\infty} \beta^t \log \alpha_0 (1+\alpha_1)^t + \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t^2}{2}\right)$$
$$\Longrightarrow \lambda = \exp\left((1-\beta) \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t^2}{2}\right)\right) - 1, \tag{A9}$$

exactly as in (9).

A.3 Identification

Let $\Delta \log c_t \sim I(1)$. According to Beveridge and Nelson (1981), $\log c_t$ can be decomposed as:

$$\log c_t = \log \alpha_0 + t \log(1+\alpha_1) + \log X_t + \log Y_t,$$

where $\log X_t = \sum_{i=1}^t \varepsilon_i$, $\log Y_t = \sum_{i=1}^t \psi_i \vartheta_{t-i}$ and $(\varepsilon_t, \vartheta_t)'$ has a bivariate normal distribution as in (7). Using the Wold decomposition, Beveridge and Nelson have demonstrated that

$$\Delta \log c_t = \log(1+\alpha_1) + \mu_t + \nu_1 \mu_{t-1} + \nu_2 \mu_{t-2} + \nu_3 \mu_{t-3} + \ldots$$

implies

$$\Delta \log X_t = \left(\sum_{i=0}^{\infty} \nu_i\right) \mu_t \tag{A10}$$

and

$$\log Y_t = -\left(\left(\sum_{i=1}^{\infty} \nu_i\right) \mu_t + \left(\sum_{i=2}^{\infty} \nu_i\right) \mu_{t-1} + \left(\sum_{i=3}^{\infty} \nu_i\right) \mu_{t-2} + \ldots\right). \tag{A11}$$

Therefore, the definition of $\log X_t$ combined with (A10) yields:

$$\varepsilon_t = \left(\sum_{i=0}^{\infty} \nu_i\right) \mu_t. \tag{A12}$$

Moreover, it is easily checked that the definition of $\log Y_t$ and (A11) imply:

$$\vartheta_t = \mu_t \tag{A13}$$

$$\begin{aligned} \psi_0 &= -\sum_{i=1}^{\infty} \nu_i \\ \psi_1 &= -\sum_{i=2}^{\infty} \nu_i \\ &\vdots \\ \psi_{t-1} &= -\sum_{i=t}^{\infty} \nu_i \\ &\vdots \end{aligned} \tag{A14}$$

Thus, (7) and (A12) to (A14) imply:

$$\sigma_{11} = \left(\sum_{i=0}^{\infty} \nu_i\right)^2 \text{var}(\mu_t), \tag{A15}$$

$$\sigma_{12} = \left(\sum_{i=0}^{\infty} \nu_i\right) \text{var}(\mu_t), \tag{A16}$$

$$\sigma_{22} = \text{var}(\mu_t). \tag{A17}$$

We must, then, obtain $\text{var}(\mu_t)$ and $(\nu_i)_{i=1}^{\infty}$ as a means to identify the relevant parameters in our model, (A15)-(A17) and α_1. This is indeed a straightforward question inasmuch as, by estimating an ARMA(p, q) for $\Delta \log c_t$, $(\mu_t)_t$ is consistently estimated by the residuals and, inverting the AR polynomial, we also find $(\nu_i)_{i=1}^{\infty}$. Lastly, α_1 is a function of the constant in the ARMA(p, q) and the coefficients of the AR polynomial.

B Appendix

Proof of Proposition 1. Firstly, we check (i). Let $\zeta_t = \beta^t \frac{w_t^2}{2}$. By applying D'Alembert's convergence test for infinite series, it suffices to assure that $\lim_{t\to\infty} \frac{\zeta_{t+1}}{\zeta_t} = c < 1$. Toward this end, notice initially that

$$\frac{1}{\beta}\frac{\zeta_{t+1}}{\zeta_t} = \frac{\left(\frac{t+1}{t}\right)\sigma_{11} + 2\sigma_{12}\frac{\sum_{j=1}^{t+1}\psi_j}{t} + \sigma_{22}\frac{\sum_{j=1}^{t+1}\psi_j^2}{t}}{\sigma_{11} + 2\sigma_{12}\frac{\sum_{j=1}^{t}\psi_j}{t} + \sigma_{22}\frac{\sum_{j=1}^{t}\psi_j^2}{t}}. \tag{B1}$$

For any sequence $(x_t)_t$ such that $\lim_{t\to\infty}(x_{t+1} - x_t) = 0$, it is true that $\frac{x_t}{t} \to 0$ as $t \to \infty$. Therefore, if we set $x_t = \sum_{j=1}^{t}\psi_j$ or $\sum_{j=1}^{t}\psi_j^2$, then $x_{t+1} - x_t = \psi_t$ or ψ_t^2 and the result applies. An analogous argument also holds for $\sum_{j=1}^{t+1}\psi_j$ and $\sum_{j=1}^{t+1}\psi_j^2$. Thus, given σ_{11}, σ_{12} and $\sigma_{22} < \infty$, the right-hand side of (B1) converges to 1, implying that $\lim_{t\to\infty}\frac{\zeta_{t+1}}{\zeta_t} = \beta < 1$.

In order to verify the second claim, (ii), define $\tilde{\zeta}_t = \left[\beta(1+\alpha_1)^{1-\phi}\right]^t \exp\left(\frac{(1-\phi)w_t^2}{2}\right)$. It is easily checked that

$$\frac{\tilde{\zeta}_{t+1}}{\tilde{\zeta}_t} = \beta(1+\alpha_1)^{1-\phi}\exp\left(\frac{(1-\phi)\left(\sigma_{11} + 2\sigma_{12}\psi_{t+1} + \sigma_{22}\psi_{t+1}^2\right)}{2}\right). \tag{B2}$$

Using the fact that $\psi_t \to 0$ as $t \to \infty$, (B2) gives: $\lim_{t\to\infty}\frac{\tilde{\zeta}_{t+1}}{\tilde{\zeta}_t} = \beta(1+\alpha_1)^{1-\phi}\exp\left(\frac{(1-\phi)\sigma_{11}}{2}\right)$. Hence, given $\phi > 1$ and $\alpha_1 \geq 0$, the numerator in (8) is clearly convergent. A similar line of reasoning implies that the denominator in the same equation is convergent if $\beta(1+\alpha_1)^{1-\phi}\exp\left(\frac{(1-\phi)^2\sigma_{11}}{2}\right) < 1$. ■

C Figures and Tables

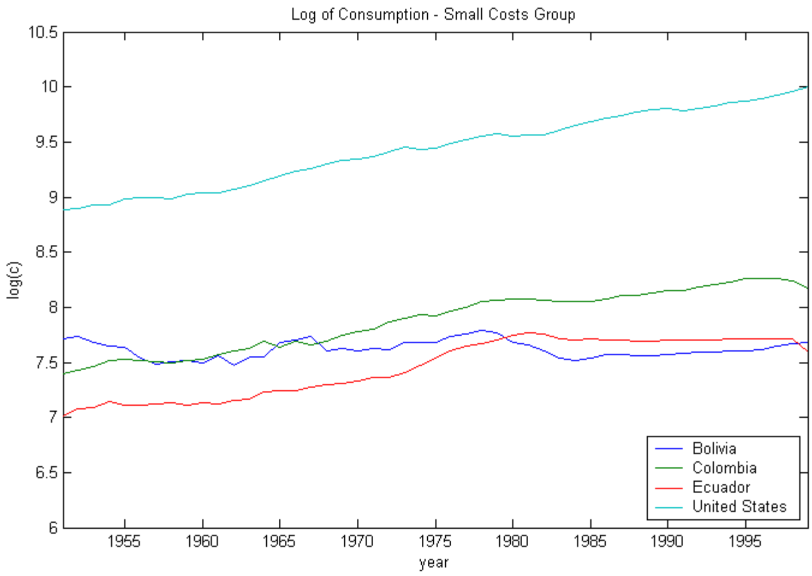

Figure C1

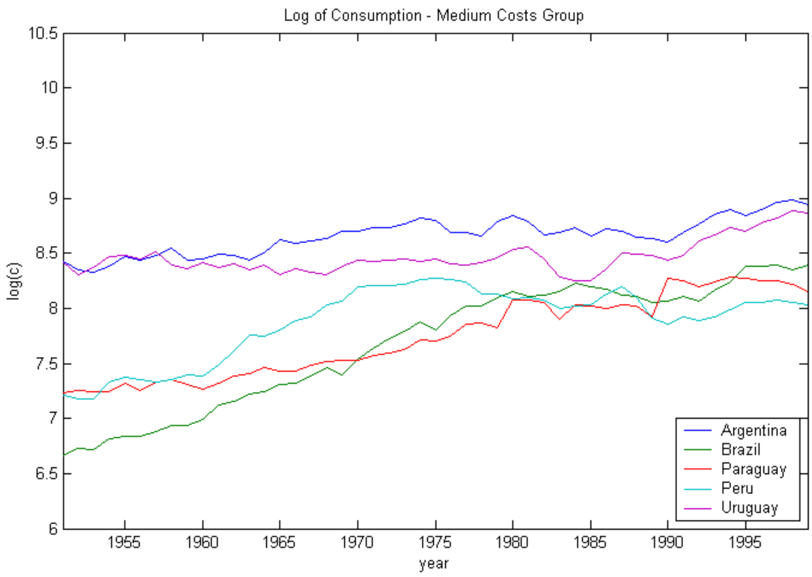

Figure C2

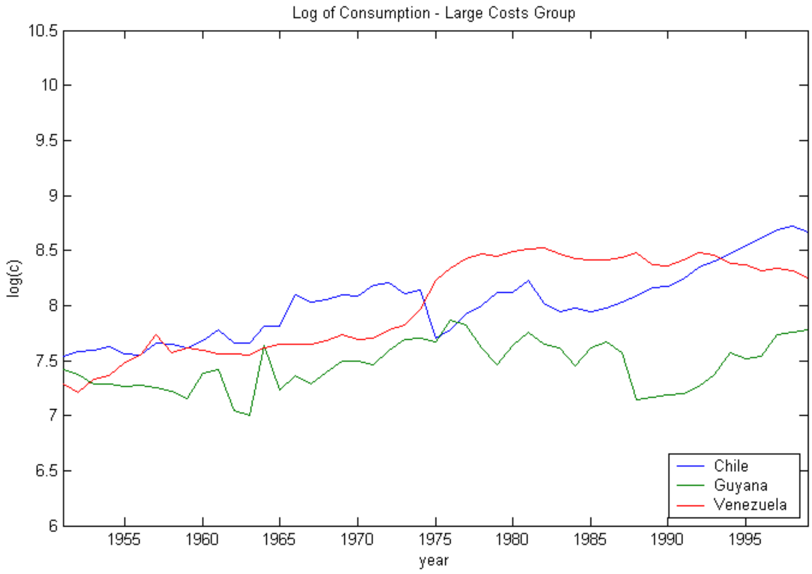

Figure C3

Table C1

Welfare Costs of Business Cycles ($\lambda\%$) - Argentina							
	Beveridge-Nelson					Linear	HP
ϕ \ β	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.54	0.85	1.82	3.24	6.45	0.38	0.13
2	1.08	1.72	3.68	6.58	13.34	0.76	0.25
5	2.82	4.63	10.90	23.41	∞	1.92	0.64
10	7.47	16.19	∞	∞	∞	3.88	1.28
20	∞	∞	∞	∞	∞	7.90	2.57

Table C2

Welfare Costs of Business Cycles (λ%) - Bolivia		
ϕ	Linear (for all β)	HP (for all β)
1	0.32	0.10
2	0.64	0.20
5	1.60	0.51
10	3.23	1.02
20	6.56	2.05

Table C3

Welfare Costs of Business Cycles (λ%) - Brazil							
	Beveridge-Nelson					Linear	HP
$\phi \ \backslash \ \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	1.14	1.71	3.45	6.03	11.98	1.16	0.12
2	1.90	2.61	4.15	5.51	7.06	2.34	0.24
5	3.37	4.08	5.17	5.82	6.36	5.95	0.60
10	5.07	5.74	6.62	7.08	7.42	12.26	1.20
20	9.23	10.45	12.18	13.17	13.97	26.03	2.42

Table C4

Welfare Costs of Business Cycles (λ%) - Chile							
	Beveridge-Nelson					Linear	HP
$\phi \ \backslash \ \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	3.69	5.59	11.49	20.62	43.68	1.29	0.44
2	7.52	11.50	24.39	46.12	113.52	2.59	0.88
5	26.72	64.43	∞	∞	∞	6.61	2.22
10	∞	∞	∞	∞	∞	13.66	4.49
20	∞	∞	∞	∞	∞	29.17	9.18

Table C5

Welfare Costs of Business Cycles (λ%) - Colombia							
	Beveridge-Nelson					Linear	HP
ϕ \ β	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.03	0.06	0.12	0.22	0.44	0.15	0.04
2	0.06	0.09	0.19	0.29	0.43	0.29	0.07
5	0.11	0.16	0.26	0.33	0.41	0.73	0.18
10	0.15	0.20	0.28	0.33	0.37	1.46	0.35
20	0.17	0.21	0.26	0.29	0.31	2.95	0.70

Table C6

Welfare Costs of Business Cycles (λ%) - Ecuador							
	Beveridge-Nelson					Linear	HP
ϕ \ β	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.11	0.12	0.13	0.13	0.13	0.49	0.04
2	0.23	0.24	0.26	0.26	0.27	0.97	0.08
5	0.57	0.61	0.65	0.66	0.67	2.45	0.20
10	1.15	1.22	1.30	1.33	1.35	4.97	0.40
20	2.34	2.48	2.62	2.69	2.73	10.18	0.81

Table C7

Welfare Costs of Business Cycles (λ%) - Guyana							
	Beveridge-Nelson					Linear	HP
ϕ \ β	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	20.33	30.94	68.70	143.50	450.38	2.04	0.77
2	45.17	73.11	212.86	1318.86	∞	4.12	1.54
5	∞	∞	∞	∞	∞	10.63	3.90
10	∞	∞	∞	∞	∞	22.39	7.95
20	∞	∞	∞	∞	∞	49.79	16.53

Table C8

Welfare Costs of Business Cycles (λ%) - Paraguay							
	Beveridge-Nelson					Linear	HP
$\phi \ \backslash \ \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.25	0.43	0.98	1.78	3.59	0.35	0.18
2	0.50	0.86	1.97	3.60	7.32	0.70	0.35
5	1.29	2.26	5.40	10.70	27.89	1.76	0.89
10	3.03	5.82	25.04	∞	∞	3.56	1.78
20	∞	∞	∞	∞	∞	7.24	3.60

Table C9

Welfare Costs of Business Cycles (λ%) - Peru							
	Beveridge-Nelson					Linear	HP
$\phi \ \backslash \ \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.41	0.65	1.36	2.40	4.74	2.93	0.18
2	0.82	1.30	2.73	4.85	9.72	5.94	0.37
5	2.12	3.42	7.69	15.19	46.79	15.52	0.93
10	5.11	9.49	∞	∞	∞	33.44	1.86
20	∞	∞	∞	∞	∞	78.06	3.75

Table C10

Welfare Costs of Business Cycles (λ%) - Uruguay							
	Beveridge-Nelson					Linear	HP
$\phi \ \backslash \ \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.41	0.63	1.26	2.17	4.22	0.73	0.18
2	0.83	1.26	2.53	4.38	8.61	1.47	0.36
5	2.14	3.29	6.98	13.28	35.58	3.70	0.90
10	4.96	8.51	∞	∞	∞	7.54	1.81
20	∞	∞	∞	∞	∞	15.66	3.65

Table C11

Welfare Costs of Business Cycles (λ%) - Venezuela							
	Beveridge-Nelson					Linear	HP
$\phi \ \backslash \ \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	2.18	2.71	3.76	4.79	6.57	1.92	0.26
2	4.42	5.49	7.66	9.82	13.57	3.87	0.53
5	11.87	14.95	21.38	28.28	43.38	9.96	1.32
10	30.97	40.17	63.35	∞	∞	20.92	2.66
20	∞	∞	∞	∞	∞	46.22	5.40

Table C12

Welfare Costs of Business Cycles (λ%) - United States							
	Beveridge-Nelson					Linear	HP
$\phi \ \backslash \ \beta$	0.85	0.90	0.95	0.971	0.985	(for all β)	(for all β)
1	0.11	0.17	0.34	0.59	1.15	0.05	0.02
2	0.20	0.29	0.48	0.67	0.92	0.10	0.04
5	0.38	0.48	0.65	0.75	0.85	0.25	0.09
10	0.56	0.65	0.77	0.83	0.88	0.50	0.18
20	0.79	0.86	0.93	0.97	0.99	1.01	0.36