Debt-Constraints or Incomplete Markets? A Decomposition of the Wealth and Consumption Inequality in the U.S.

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The large wealth and consumption inequality in the U.S. is usually attributed to two market frictions: debt constraints and incomplete markets. Recent literature has argued that debt constraints are the critical friction while market incompleteness plays only a secondary role. We evaluate the independent role of debt constraints versus market incompleteness to explain U.S. inequality. We introduce full insurance opportunities in a standard model of inequality along the lines of Aiyagari (1994). Debt constraints are the only friction in such model. We find that for a quite standard calibration of the income process, that of Heaton and Lucas (1996), debt constraints alone can explain *none* of the observed inequality. The reason is that the U.S. capital stock would be enough to secure all required contingent debts if markets were completed. Using various non-standard calibrations, we find that debt constraints can play an important role to explain inequality but still market incompleteness remains as the main friction. In particular, debt-constrained models cannot account for the large wealth dispersion and wealth concentration in the top tail of the distribution in the U.S.

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1 Introduction

Wealth in the U.S. is highly concentrated and skewed to the right. According to the 1992 Survey of Consumer Finance, the wealthiest 1% of the population owns over 30% of the nation's wealth while the bottom 50% owns less than 5%. What are the *main* causes of, and potential welfare costs associated with, this large wealth inequality?

A recent stream of papers have studied different possible quantitative explanations of inequality with different degrees of success (papers include Aiyagari 1994; Huggett 1996; Quadrini 1997; Krusell and Smith 1998; Castañeda, Días-Giménez and Ríos-Rull 2003; De Nardi 2003; and Domeij and Heathcote 2004). Those studies regard inequality as the result of primarily two market frictions: market incompleteness and debt constraints. The first constraint states that there are not enough securities to insure against all events, and usually only a riskless bond is assumed; the second constraint states that individuals are unable to borrow as much as they want, usually nothing at all. As a result of these two assumed frictions, income fluctuations translate into wealth and consumption fluctuations and, moreover, into non-degenerated and stationary distributions of wealth and consumption.

This paper, like the papers above, takes the view that inequality is mainly the result of debt limits and market incompleteness. It asks what is the quantitative *contribution* of each friction *alone* in explaining the inequality of consumption and wealth in the U.S. We think this is an important question for at least two reasons. First, each market friction is likely to be the result of different underlying constraints. Debt limits likely arise from commitment and enforcement problems while incomplete markets probably arise from asymmetric information and moral hazard problems. If so, our quantitative exercise should shed light on the underlying main determinants of inequality and, consequently, provide direction for researchers and policymakers.

Second, a recent stream of literature has argued that market incompleteness may not be crucial to explaining important regularities of the data, including wealth and consumption inequality. This literature instead stresses the role of debt constraints *alone* as the main deviation from the Arrow-Debreu framework (Alvarez and Jerman 2000, Kehoe and Levine 2001; Kehoe and Zame 2002, Kehoe and Perri 2002; Krueger and Perri 2002, Lustig 2003, Lorenzoni 2003). Specifically on the issue of inequality, Krueger and Perri (2002) argue that debt-constrained models[2] can better explain the rising inequality in the U.S. during the last 30 years. We explicitly evaluate the role of debt constraints in macroeconomics by assessing how much wealth and consumption inequality they alone can explain.

Our decomposition exercise is as follows. We take as our benchmark model of inequality Aiyagari (1994). He provides a general equilibrium framework to study consumption and wealth inequality based on three elements: income fluctuations, incomplete markets, and debt constraints. Specifically, agents face idiosyncratic earnings shocks but can only save in a riskless asset, capital, and borrowing is prohibited. Moreover, agents are altruistic, ex-ante identical, and the economy is a production economy with capital. We calibrate our benchmark model following closely Aiyagari (1994) and Heaton and Lucas (1996).

In order to assess the sole role of the debt constraint, we modify the baseline model by allowing agents to trade a complete array of Arrow securities, in the spirit of the debt constrained literature. Our modified model has thus only one friction, the debt constraint. This constraint prohibits agents from borrowing in any state of the world. In particular, agents can sell Arrow securities but only up to an amount that is fully secured by the value of their capital so that their net asset position in any state of the world is never negative. Thus, our non-borrowing constraint can be interpreted as a secure debt constraint or collateral constraint as in Kiyotaki and Moore (1997), Córdoba and Ripoll (2004), or Lustig (2001). By comparing the predictions of the

[2] Following Kehoe and Levine (2001), we call these models debt-constrained models.

models with and without Arrow securities, we can assess the independent role of market incompleteness and the debt constraint in creating inequality.

We find that the main cause of inequality is the lack of insurance markets (market incompleteness) rather than the debt constraint. Surprisingly, *all* inequality and inefficiencies of the benchmark economy disappear when markets are completed even when net borrowing is prohibited. This finding is in sharp contrast with the debt-constrained literature and, in particular, with the results of Krueger and Perri (2002) who study the U.S. consumption inequality using a related complete-markets debt-constrained model. In principle, inequality in our complete markets model should be larger than in theirs because our debt limits are seemingly more stringent. They allow for unsecured debt but we do not.

The explanation for our strong result lies in the role of capital stock in the economy. Debt-constrained models typically assume a pure exchange economy[3]. In such economies, our stringent assumption precluding borrowing in any state of nature would result in autarky. Instead, in our production economy with complete markets, capital can make full risk sharing possible. This is because capital serves three purposes. First, it is an input in production. Second, it creates the need and means for aggregate savings. The average individual in a production economy has positive savings and can use them for self-insurance. This reduces the role of borrowing limits relative to a pure exchange economy in which the typical agent has zero savings.

Third, capital enables security markets to work. Agents with positive capital, a non-contingent asset, can use it as collateral for their contingent debts. Moreover, if the modified golden rule level of capital is sufficiently large relative to the economy's income risk, a representative agent could perfectly collateralize all his/her needs for contingent debts and obtain full insurance (See Proposition 7). If so, perfect equality

[3] Lustig (2001, 2003) and Lorenzoni (2003) have a debt constraints similar to ours. As they also assume complete markets, our main results apply to their papers.

becomes a stationary equilibrium.

We find that, for the benchmark calibration, the modified golden rule level of capital is sufficient to secure all contingent debts that a representative agent would require to perfectly smooth consumption. The amount of income risk in the U.S. is just not large enough to exhaust all the collateral in any state of nature. It is important to stress that our baseline calibration of the income process is based on the results of Heaton and Lucas (1996) who use direct evidence on household earnings dynamics from the Panel Study of Income Dynamics (PSID).

In order to check the robustness of our results, we perform sensitivity analysis using two radically different calibrations of the income process. One follows Storesletten, Telmer and Yaron (2003), and the other, Castañeda, Días-Giménez and Ríos-Rull (2003). These authors use indirect evidence and a variety of assumptions to argue that the amount of income risk is much larger than previously thought. In particular, shocks should be much larger and persistent than our baseline.

When we use these alternative calibrations, we find that debt constraints do play a role. Specifically, we find that a substantial degree of inequality remains even after completing the markets. For example, the Gini coefficients of wealth and consumption only slightly decrease when markets are completed. However, the standard deviation of wealth decreases dramatically, between 40% and 70%. The remaining wealth dispersion is far too small to explain the U.S. evidence. The coefficient of variation of wealth is above 6 in the data but below 1.5 in the complete market model. This strongly suggests that debt-constraints alone cannot account for the observed dispersion of wealth for a realistic earning process. Moreover, using a standard equalitarian measure of social welfare, we find that between 55% and 72% of the social welfare costs of inequality could be eliminated by just completing the markets.

Overall, we conclude that market incompleteness is the main friction behind inequality. Debt constraints could play a potentially important, but still limited, role. This conclusion is strengthened by another finding about debt-constrained models. They can barely explain the large concentration of wealth in the top tail of the distribution. When markets are complete, rich agents can perfectly smooth consumption. A feature of these models, as stressed by Kehoe and Levine (2001), is that the interest rate is below the rate of time preference. As a result, rich agents will have decreasing profiles of consumption and wealth, which prevents these models from producing a long right tail of the wealth distribution. Empirically, this tail is critical as a large fraction of wealth is concentrated there.

Finally, we want to highlight the analytical contributions of the paper. We derive equilibrium properties of the complete-markets debt-constrained economy when debt constraints are exogenous, as in Aiyagari (1994). Most of the existent work on this topic refers to pure exchange economies with participation constraints under the threat of exclusion (particularly, Kehoe and Levine 2001, Krueger and Perri 2002). Instead, our focus is on production economies with simpler non-borrowing constraints. We think this is an important workhorse model to study issues related to debt constraints and incomplete markets. We use simple methods to show that if the capital intensity of the economy is not sufficiently large relative to the income risk, the equilibrium interest rate is below the rate of time preference, and the capital stock exceeds the modified golden rule level. We devise a simple algorithm to compute the equilibrium, and in particular, to solve the portfolio decision problem with multiple assets. This is an important contribution of the paper because similar problems with multiple assets have been regarded as "computationally too burdensome (Kluber and Schmedders, 2001, footnote 3)."

The remaining part of the paper is divided into six sections. Section 2 describes the model and equilibrium properties of the complete-markets debt-constrained model;

Section 3 describes the baseline calibration and Section 4 reports the results under this calibration; Section 5 performs robustness checks using alternative characterizations of the earning process; and Section 6 concludes.

2 Model economy

We now describe a parsimonious model that can be used to study two different economies: one without insurance markets and one with insurance markets. The first economy is identical to the one studied by Aiyagari (1994). The second economy adds insurance markets into the first economy via competitive markets for Arrow securities. Formally, this economy is a complete-markets debt-constrained economy. We call it complete markets or debt constrained economy for short. Both economies are populated by a continuum of infinitely lived households of mass one and time is discrete.

2.1 Employment opportunities

We assume that every household has a random endowment, e_t, of efficiency labor units. Labor endowments are independently and identically distributed across households. They follow a finite state Markov chain with conditional transition probabilities given by $\pi(e'|e) = \Pr(e_{t+1} = e'|e_t = e)$, where e and $e' \in E = \{e_1, e_2, ..., e_n\}$, and $0 < e_1 < ... < e_n$. There is a unique stationary distribution of endowments, P, with unconditional expected value of 1. As a result, $e_1 < 1$ and $e_n > 1$.

2.2 Preferences

Households's preferences over consumption streams are described by a standard expected utility function ,

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right] \text{ where } 0<\beta<1,$$

where E_t is the mathematical expectation conditional on information available up to time t, c_t is consumption, and function u is identical across agents and satisfies $u' > 0$, $u'' < 0$.

2.3 Production possibilities

There is a freely available constant returns to scale production technology that transforms efficiency units of labor, L, and capital, K, into output according to the function $F(L, K)$. Aggregate capital is obtained by aggregating the capital holdings of every household, and the aggregate labor input is obtained by aggregating the efficiency labor units supplied by every household. Capital depreciates geometrically at a constant rate δ. Define gross output as $f(k) \equiv F(k,1) + (1-\delta)\,k$. We assume that function f satisfies $f(0) = 0$, $f' > 0$, $f'' < 0$.

2.4 Market arrangements

We consider two sequential market arrangements: one without insurance markets and one with insurance markets or complete markets. In both arrangements, households can accumulate wealth in the form of real capital, b_t, a riskless asset. This is the only asset available in the first market arrangement. In the second arrangement, households can also buy or sell a full array of Arrow securities, $a_t(e_i)$. These securities entitle the owner to receive $a_t(e_i)$ units of consumption goods at $t+1$ only if his/her labor endowment at $t+1$ is e_i. A negative $a_t(e_i)$ means that the owner is required to deliver goods rather than to receive.

We further assume that households cannot borrow. This assumption implies that in the first arrangement households cannot hold a negative amount of capital. For the second market arrangement, it implies a secure debt constraint. In particular, sales of Arrow securities, a form of borrowing, are allowed but those promises must be secured by the value of the physical capital so that net borrowing is prevented in every possible state.

Finally, we assume that markets are competitive. Firms rent factors of production from households in competitive spot markets, and insurance contracts are traded in competitive spot markets. These assumptions imply that factor prices are given by the corresponding marginal productivities and that insurance prices are actuarially fair.

2.5 Recursive competitive equilibrium

We now describe a recursive formulation of the household's problem and define the equilibrium concept in recursive terms. Denote r_t the rental price of capital, w_t the wage rate, and $q_t(e_j, e_i)$ the price of an Arrow security that pays one unit of good in state e_j at time $t+1$ if the current state is e_i, where $e_j, e_i \in E$.

2.5.1 Household's problem

At the beginning of period t households observe the realization of their labor endowment, e_t. The endowment determines the household's resources for that period which includes wage earnings, $w_t e_t$, capital holdings including the returns on capital, $(1+r_t)b_{t-1}$, and insurance claims or liabilities, $a_{t-1}(e_t)$. Let x_t denote the total resources available at time t defined as

$$x_t = w_t e_{it} + (1+r_t)b_{t-1} + a_{t-1}(e_t).$$

A household's resources lie in the state space $X \in [0, \infty]$. We can now describe the household's individual state at time t by the couple (x_t, e_t). For the recursive formulation, we drop time subscripts for the current-period variables, and use primes to denote the values of the variables one period ahead. In addition, we use the parameter Γ to switch between the economy with insurance markets ($\Gamma = 1$) and the one without insurance markets ($\Gamma = 0$).

The following is the dynamic program solved by a household whose state is (x, e):

$$(1)\quad V(x,e) = \max_{c\geq 0, b, \{a(e_i)\}_{i=1}^{n}} \left\{ u(c) + \beta \sum_{i=1}^{n} V\left(w'e_i + \Gamma a(e_i) + (1+r')b, e_i\right) \pi(e_i|e) \right\},$$

$$\begin{aligned} \text{subject to } x &\geq c + b + \sum_{i=1}^{n} \Gamma a(e_i) q(e_i, e), \\ \Gamma a(e_i) + (1+r')b &\geq 0 \text{ for } e_i \in E, \end{aligned}$$

where v denotes the household's value function. The first restriction on the problem is the budget constraint that allows accumulation of capital, b, and contingent claims, $a(e_i)$, if $\Gamma = 1$. The second constraint of the problem is the borrowing constraint. It states that the net wealth, defined as $a(e_i) + (1 + r')b$, cannot be negative in any state.

The solution to this problem is a set of functions that map states into choices for consumption, capital accumulation, and insurance claims or promises. We denote this set by $\{c(x, e), b(x, e), a(e_1; x, e), ..., a(e_n; x, e)\}$.

2.5.2 Equilibrium concept

Each period the economy-wide state is a measure of households, J_t, defined over $\mathcal{B}$, an appropriate family of subsets of $\{X \times E\}$. For the purposes of this paper we focus on a stationary situation in which J_t is constant over time. By the law of large numbers, the aggregate labor supply is equal to 1.

Definition: A stationary equilibrium of this economy is a value function, $V(x, e)$; policy functions $\{c(x, e), b(x, e), a(e_1; x, e), ..., a(e_n; x, e)\}$; a probability measure of households, J; factor prices (r, w); security prices $q(e_j, e_i)$ for $i, j = 1, 2, .., n$; and aggregate K such that:

1. $c(x, e)$, $b(x, e)$ and $\{a(e_i; x, e)\}_{i=1}^{n}$ are optimal decision rules given prices, and $V(x, e)$ solves the functional equation.

2. Factor prices are factor marginal productivities:

$$r = F_1(K, 1) - \delta \text{ and } w = F_2(K, 1)$$

3. $q(e_i, e_j) = \pi(e_i, e_j)/\,(1 + r)$ for e_i,$e_j \in E$ (actuarially fair insurance prices)

4. $K = \int_{X \times E} [b(x, e)dJ(x, e) + \sum_{i=1}^{n} \Gamma a(e_i, x, e)q(e_i, e)]\, dJ(x, e)$

5. The measure of households is stationary:

$$J(B) = \int_B \left\{ \int_{X,E} I_{x'=we'+\Gamma a(e';x,e)+(1+r)b(x,e)} \pi(e', e)dJ(x, e) \right\} dx'de'$$

   for all $B \in \mathcal{B}$; I is an indicator function.

2.6 Characterization of the equilibrium

We now provide an analytical characterization of the equilibrium for the cases $\Gamma = 0$ and $\Gamma = 1$. We discuss briefly the case $\Gamma = 0$ since it has been well studied in the literature. We instead focus on the case $\Gamma = 1$. A useful reference point for the analysis is the frictionless case. In that case, the aggregate capital stock, k^*, is determined by the modified golden rule, and the interest rate, r^*, is equal to the rate of time preference, $\rho \equiv 1/\beta - 1$:

$$1 = \beta f'(K^*, 1) \text{ and } r^* = \rho$$

2.7 Incomplete markets case ($\Gamma = 0$)

The case $\Gamma = 0$ is by now well understood and fairly standard. Details about the stationary equilibrium for this economy can be found in Aiyagari (1994) and Hugget (1997). The following are some important properties of this equilibrium. First, some restrictions on preferences or endowments are required to guarantee the existence of a stationary equilibrium. In particular, a stationary equilibrium may not exist if the coefficient of risk aversion is unbounded. Second, the equilibrium displays capital over-accumulation relative to the complete markets economy, and an interest rate below the rate of time preference. Finally, the ergodic set for x is $[we_1, x_{\max}]$, where $x_{max} > we_n$.

To put some of these results in perspective, we show in the next section that when markets are complete, (i) the first restriction is not required; (ii) there is still capital over accumulation and the interest rate is below the rate of time preference provided that an additional restriction is satisfied; (iii) the ergodic set shrinks to $[we_1, we_n]$. This last property means that, at least from the point of view of the range of the stationary distribution, inequality decreases when insurance markets are introduced.

The following proposition collects the results for the case $\Gamma = 0$. Denote by K^a and r^a the equilibrium capital stock and interest rate of this economy.

Assumption 1: $cu(c)''/u'(c)$ is bounded above for all c sufficiently large.

Assumption 2: either $u'(0) > 0$, or $we_1 > 0$.

Proposition 1 *Under Assumptions 1 and 2, the economy with no insurance* ($\Gamma = 0$) *(i) posseses a stationary equilibrium; (ii) the ergodic set for* x *is* $[we_1, x_{\max}]$, *where* $x_{max} > we_n$; *and (iii)* $K^a > K^*$ *and* $r^a < \rho$.

2.8 Complete markets case ($\Gamma = 1$)

This section now characterizes some relevant properties of the equilibria with complete markets. To the extent of our knowledge, most of the results of this section are new in the literature. It is convenient to divide the task into two sections. First, we describe the solution to the household's problem given that the interest rate is below the rate of time preference ($r < \rho$), and insurance prices are actuarially fair. This partial equilibrium problem is an extension of *the income fluctuation problem*, analyzed, among others, by Schechtman and Escudero (1977).

Second, we study the determination of the interest rate. We provide a necessary and sufficient condition so that in fact $r < \rho$ in equilibrium. If such condition holds, we show that the stationary equilibrium exists without further restrictions on the degree of risk aversion.

2.8.1 The income fluctuation problem with complete markets

For analytical convenience, consider the case in which endowments are i.i.d, i.e., $\pi(e', e_i) = \pi(e')$ for all $e_i \in E$. We drop this assumption for the quantitative work. As a result of this assumption, the only relevant individual state variable is now x, since e has no informational content.

Let $\Gamma = 1$ in the household's problem, suppose $r < \rho$, and $q(e_i) = \pi(e_i)/(1+r)$. Define $x_i \equiv we_i + a(e_i) + (1+r)b$. We can then rewrite the household problem as:

$$\begin{aligned} V(x) &= \max_{c \geq 0, \{x_i\}_{i=1}^n} \left\{ u(c) + \beta \sum_{i=1}^n V\left(x_i'\right) \pi(e_i) \right\}, \\ \text{s.t } x &\geq c + \sum_{i=1}^n \frac{x_i' - we_i}{1+r} \pi(e_i), \\ x_i' &\geq \underline{x}_i \text{ for } i = 1, .., n, \end{aligned}$$

where $\underline{x}_i \equiv we_i$. The last constraints are the state contingent borrowing constraints.

We find convenient, for analytical clarity as well as for computational purposes, to rewrite this problem into two simpler subproblems, a deterministic-dynamic problem and a stochastic-static problem. The first subproblem describes the optimal saving-consumption decision, which only requires finding the *total* amount of savings. The second subproblem describes the optimal allocation of those savings across different states (or assets).

Subproblem 1 (Dynamic and deterministic):

$$
\begin{aligned}
V(x) &= \max_{c\geq 0, y} \{u(c) + \beta W(y)\} \\
\text{s.t } x &\geq c + \frac{y - \omega}{1 + r} \\
y &\geq \omega
\end{aligned}
\tag{2}
$$

where $\omega \equiv \sum_{i=1}^{n} we_i \pi(e_i)$.

Subproblem 2 (Stochastic and static):

$$
\begin{aligned}
W(y) &= \max_{x'_i} \sum_{i=1}^{n} W(x'_i)\pi(e_i) \\
\text{s.t. } y &\geq \sum_{i=1}^{n} x'_i \pi(e_i), \\
x'_i &\geq \underline{x}_i \text{ for } i = 1, .., n
\end{aligned}
\tag{3}
$$

Consider first the solution to subproblem (3). Standard arguments show that the first order necessary and envelope conditions of this problem are:

$$
\begin{aligned}
V'(x'^*_i) &\geq \lambda, \text{ with equality if } x'^*_i > we_i, \\
V'(x'^*_i) &= u'(c'),
\end{aligned}
$$

where λ is the multiplier on the resource constraint. Note that absent the borrowing constraints, it will be optimal to smooth resources, and consumptions, completely across states, i.e., $x'^*_i = y$.

Figure 1 illustrates the solution of the constrained problem when $n = 4$. If savings is at its lowest possible level, $y = y_1 \equiv \omega$, then the only possible allocation is $x'^*_i = \underline{x}_i$, i.e., no insurance is purchased at all. As savings increase above its minimum level, it is optimal to allocate all additional savings to the poorest state, the one with the highest marginal utility of resources, while keeping the resources for other states at their minimum. This is accomplished by purchasing insurance only for the worst possible state ($a(e_1) > 0$).

There is a level of savings, y_2, that allows agents to purchase enough insurance so that the marginal utility of the worst possible state equates to the one of the second worst possible state. Such a level of savings is given by $y_2 = \underline{x}_2 \pi(e_1) + \sum_{i=2}^{n} e_i \pi(e_i)$. As savings increase above y_2, it is optimal to insure the two worst states so that their marginal utilities are equal. This is accomplish by choosing $x_1'^* = x_2'^*$, and $x_i'^* = \underline{x}_i$ for $i > 2$. One can proceed in this fashion to find the solution for successively higher savings. The following proposition summarizes the solution described in Figure 1.

Proposition 2 *Define*

$$
\begin{aligned}
y_i &\equiv \underline{x}_i \sum_{j=1}^{i} \pi_j + \sum_{j=i+1}^{n} \underline{x}_j \pi(e_j) \text{ for } i = 1, .., n-1 \\
y_n &\equiv \underline{x}_n
\end{aligned}
$$

Then, (i) for $y_i \leq y \leq y_{i+1}$

$$
\begin{aligned}
x_j'^* &= x_j(y) \equiv \frac{y - \sum_{h=i+1}^{n} \pi_h \underline{x}_h}{\sum_{h=1}^{i} \pi_h} \text{ for all } j \leq i \\
x_j'^* &= x_j(y) \equiv \underline{x}_j \text{ for } i < j \leq n
\end{aligned}
$$

and (ii) for $y > y_n$, $x_j'^* = y$ *for all* j.

This closed form solution is very convenient for analytical and computational purposes since, at least for i.i.d endowments, the savings problem with many assets is not more complicated as the one with a single asset[4]. The following corollary of the previous proposition is used below.

[4] Thus, there is no additional computational burden as suspected by Kubler and Schmedders (2001).

Corollary 3 *$x_{t+1} \geq \underline{x}_n$ requires $y \geq y_n$.*

Consider now the solution to the first problem (2). A household's optimal total savings, y, is characterized by the Euler equation:

$$u'(c) \geq \beta(1+r)Eu'(c') \tag{4}$$

Denote $y = g(x)$ the solution for this problem. It is standard to show that g is an increasing function of x. The following proposition states that the maximum sustainable level of resources is $\underline{x}_n$.

Proposition 4 *Suppose $r < \rho$. Then, (i) if $x_t \geq \underline{x}_n$, then $x_{t+1} < x_t$; and (ii) if $x_t \leq \underline{x}_n$ then $x_{t+1} \leq \underline{x}_n$.*

Proof. *(i) Suppose, by contradiction, that $x_{t+1} \geq x_t \geq \underline{x}_n$. By the previous Corollary (3), $y_t = g(x_t) \geq y_n = \underline{x}_n$. Proposition (2) then implies that $x'^{*}_{jt+1} = y_t$ for all j. Thus, $x_{t+1} = y_t$.Therefore, c_{t+1}, a sole function of x_{t+1}, is equal in all possible states. For unconstrained agents, condition (4) then reads $u'(c_t) = \beta(1+r)u'(c_{t+1})$. Since $r < \rho$, then $c_{t+1} < c_t$, which requires $x_{t+1} < x_t$. A contradiction. (ii) Suppose by contradiction that $x_{t+1} > \underline{x}_n \geq x_t$. A similar argument to part (i) implies that $x_{t+1} < x_t$, a contradiction.* ■

The following Proposition states $\underline{x}_1$ is the minimum sustainable level of resources.

Proposition 5 *Suppose $r < \rho$. Then, $g(\underline{x}_1) = y_1$ and $x'_1 = \underline{x}_1$ with probability π_1.*

Proof. *Suppose, by contradiction that $g(\underline{x}_1) > y_1$, which implies $x'_1 > \underline{x}_1$. From f.o.c. with respect to x'_1, $u(c(\underline{x}_1)) = \beta(1+r)u(c(x'_1))$, with equality because $x'_1 > \underline{x}_1$. Then $u(c(\underline{x}_1)) < u(c(x'_1))$, or $c(\underline{x}_1) > c(x'_1)$, or $\underline{x}_1 > x'_1$, a contradiction.* ■

Figure 2 provides a graphical description of the last three propositions and some implications. For an initial sufficiently large level of resources (above $\widetilde{x} > \underline{x}_n$, in the graph), agents fully smooth resources for next period, so that x' is a deterministic function of x, and $x' < x$. Below $\widetilde{x}$, agents do not fully insure. In particular, they only insure against the worst possible states. Thus with some positive probability, $x' = \underline{x}_n$ for all $x < \widetilde{x}$. However, $x' > \underline{x}_1$ a.s. if some insurance is purchased. If x falls below a certain level $\widehat{x}$, agents purchase no insurance at all and $x' = \underline{x}_1$ with some positive probability. Thus, we have established that the ergodic set for x is $[\underline{x}_1, \underline{x}_n]$.

2.8.2 General Equilibrium

We now show that $r < \rho$ in a stationary equilibrium provided that a necessary and sufficient condition is met. We drop the assumption that endowments are i.i.d in this Section. The following results are well known.

Proposition 6 *If borrowing constraints are not binding for any positive mass of agents in equilibrium, then the stationary level of capital is determined by the modified golden rule, individual consumptions are constant over time, and the distribution J is not unique. In particular, a perfectly equalitarian distribution of consumptions and riskless assets is an equilibrium.*

Consider the amount that a representative agent, and agent with average level of consumption and average holdings of riskless asset, would need to purchase (or sell) of contingent assets to sustain a constant consumption profile. Below we show that under perfect risk sharing $a(e)$ for such agent is given by $-w \cdot \theta(e)$, where[5] $\theta(e) = \sum_{s=0}^{\infty} \beta^s \left(E_0[e_s - 1|e]\right)$. Notice that $\theta(e)$ is the expected present value of labor endowment above the average endowment. If $\theta(e) > 0$ then a representative agent

[5] Note that the vector θ can be easily computed using the formulas

$$\theta = \frac{1}{1-\beta}1 - \left(I - \beta\Pi\right)^{-1} e$$

where 1 is a column vector of ones, Π is the transition matrix, and e is a vector of possible values for the endowments.

would like to make $a(e)$ negative to compensate for the relatively good stream of labor income expected in state e. In exchange, the representative agent would like to make $a(e)$ positive for those states with a low expect stream of labor income. Thus, borrowing will occur against states with relatively high θ. Denote $\overline{\theta} = \max_e \{\theta(e)\}$. The following Proposition states whether or not such a level of borrowing could be secured by the capital stock of the economy.

Proposition 7 *A stationary equilibrium with perfect risk sharing exists if and only if*

$$\frac{K^*}{f(K^*)} > \frac{\beta\overline{\theta}}{1+\overline{\theta}}$$

Proof. *Suppose perfect risk sharing is a stationary equilibrium. Then, using the budget constraint, $a_t(e_t)$ satisfies*

$$a_t(e_t) = c + b - we_t - (1+r)b + \sum \beta a(e_{t+1})\pi(e_{t+1}|e_t)$$

where consumption and holdings of riskless assets are constant, and $q(e', e) = \beta\pi(e_{t+1}|e_t)$ has been used. Iterating on this equation, one finds that

$$a_t(e_t) = \frac{c - rb}{1-\beta} - w\sum_{s=0}^{\infty}\beta^s E_t\left[e_{t+s}|e_t\right]$$

Without loss of generality, consider a representative agent of this economy who holds $b = K^*$ and $c = f(K^*) - K^*$. Then, $c - rb = w$ and

$$a_t(e_t) = w\sum_{s=0}^{\infty}\beta^s\left(1 - E_t\left[e_{t+s}|e_t\right]\right) = -w\theta(e_t)$$

Since perfect risk sharing is possible, then $a_t(e_t) > -(1+r)K^* = K^*/\beta$ for all e_t, or $-w\theta > K^*/\beta$. Using the result $w = f(K^*) - K^*/\beta$, one finds that $\frac{K^*}{f(K^*)} > \frac{\beta\overline{\theta}}{1+\overline{\theta}}$. For sufficiency, one can go backwards and construct a stationary equilibrium with a representative agent. ■

Proposition 7 states that a stationary equilibrium with full insurance exists only if the modified golden rule level of the capital-output ratio is sufficiently large relative to a measure of income dispersion, θ. To illustrate this condition, consider two cases. First, suppose shocks are i.i.d. In that case, $\theta(e) = e - 1$ and perfect risk sharing becomes a stationary equilibrium if $\frac{K^*}{f(K^*)} > \beta \frac{e_n - 1}{e_n}$. This formula shows that perfect risk sharing is possible if the best possible endowment is not extremely large relative to the capital stock. Intuitively, agents would like to borrow against their best possible endowment. If such endowment is large relative to the average, then capital may not be enough to secure all the required borrowing.

Second, suppose shocks are persistent. In particular, suppose $E_t[e_t - 1|e] = \tau^t (e-1)$ so that $\tau \in [0,1]$ captures the degree of persistence[6]. In that case $\theta(e) = \frac{e-1}{1-\beta\tau}$ and perfect risk sharing is obtained if $\frac{K^*}{f(K^*)} > \beta \frac{e_n - 1}{e_n - \beta\tau}$. This expression shows that perfect risk sharing could occur if shocks are not extremely persistent.

The following proposition states that two basic properties of incomplete markets economies still hold for the complete markets economy provided that the condition of previous proposition does not hold. Specifically, a stationary equilibrium is characterized by capital over-accumulation and an interest rate below the rate of time preference. Denote by K^b and r^b the equilibrium capital stock and interest rate of the complete markets economy.

Proposition 8 *Suppose* $\frac{K^*}{f(K^*)} < \frac{\beta\overline{\theta}}{1+\overline{\theta}}$. *Then* $K^b > K^*$ *and* $r^b < \rho$.

Proof. *From the first order conditions we obtain*

$$u'(c) \geq \beta(1+r^b)Eu'(c')$$

If $\beta(1+r^b) > 1$ *then* $K^b < K^*$ *and* $M_t \geq E_t M_{t+1}$, *where* $M_t \equiv \beta^t(1+r^b)^t u'(c_t)$. M_t *is a non-negative supermartingale and therefore converges. Since* $\beta(1+r^b) > 1$, *then* u' *must converge to zero. Thus,* $c \to \infty$ *which violates feasibility since* $K^b < K^*$. *If* $\beta(1+r^b) = 1$ *then* $u'(c) \geq Eu'(c')$. *Thus,* $u'(c)$ *converges. If* c *converges to a*

[6]This assumption do not fit our assumption of a discrete Markov process, but it helps to develop some intuition.

finite constant, then full insurance is obtained which contradicts Proposition 7. Thus, $c \to \infty$ *which violates feasibility. Then* $\beta(1 + r^b) < 1$*, which requires* $K^b > K^*$. ■

3 Baseline calibration

Our baseline calibration closely follows Aiyagari (1994) and Heaton and Lucas (1996). The model period is assumed to be one year. We assume that households preferences take an isoelastic form, $u(c) = \frac{c^{1-\mu}}{1-\mu}$, with $\mu = 3$, a value that is within the range (1-3) that is standard in the literature. The technology is assumed to be Cobb-Douglas, $F(K, L) = K^\alpha L^{1-\alpha}$, with $\alpha = 0.36$ which roughly matches the share of capital income in the U.S. The rate of depreciation, δ, is set at a standard value of 0.08. The discount factor, β, is assumed to be 0.96.

For the labor endowments, Aiyagari (1994) follows Heaton and Lucas (1996). Given the importance of the income process for our results, we describe in some detail the procedure employed by Heaton and Lucas. They adopt an autoregressive representation for the logarithm of the labor endowments of the form

$$\log(e_t^i) = \overline{e}^i + \gamma \log(e_{t-1}^i) + \sigma(1-\gamma^2)^{1/2}\epsilon_t^i, \ \epsilon_t \sim Normal(0,1), \tag{5}$$

where i refers to a particular household, σ is the coefficient of variation, γ is the correlation coefficient and $\overline{e}^i$ captures permanent differences in relative labor endowments and labor income. The process is then estimated using a longitudinal panel of data from the PSID that includes 860 families with income data spanning 1969-84. The average value of γ across households is 0.53 and the average value of $\sigma(1-\gamma^2)^{1/2}$ is 0.25, or $\sigma = 0.35$.

Aiyagari approximates this process using a seven state Markov Process, according to the procedure described by Deaton (1991) and Tauchen (1988). The approximation takes the state space of $\log(e_t)$, excluding permanent differences, to be the finite set

$\{-3\sigma, -2\sigma, -\sigma, 0, \sigma, 2\sigma, 3\sigma\}$ so that the state space for e_t^i spans from $\exp(-3\sigma)$ to $\exp(3\sigma)$. Since endowments are log-normal distributed, this interval should include around 99.9% of the population. The transition probabilities are then computed by numerical integration.

4 Results

It is easy to show that for the baseline parametrization all inequality[7] would disappear if markets were completed. We only need to check if the condition provided by Proposition 7 is satisfied. If it is, we can then construct a stationary equilibrium in which debt constraints are not binding, and individual consumptions and capital holdings are constants and equal.

The baseline calibration implies a modified golden rule level of capital-output ratio, K^*/Y^*, equal to 0.79. Remember that Y^* here includes undepreciated capital. The corresponding capital output ratio excluding undepreciated capital from output is 2.96, which is relatively low compared with standard values used in the literature, usually above 3.

Table 1 shows the levels of e_i, we_i, $\theta(e_i)$ and $a(e_i)$ implied by the baseline parametrization. Labor endowments range from 0.32 to 2.66, and labor income from 0.39 to 3.14, a ratio of more than 8. States with lower endowments require larger initial levels of contingent assets, while states with large endowments require initial negative contingent asset positions. For example, if the worst endowment is realized so that $we_1 = 0.39$, the level of contingent assess for that state of 1.87 will allow a representative agent to keep a constant consumption equal to 1.41 permanently in spite of the fact that more low wage income realizations are expected to follow.

[7] Permanent income differences are excluded from the model. If they were include, then all inequality associated to those differences will remain.

If the best wage income of 3.14 is realized, a representative agent would like to reduce his wealth by -3.45 to keep a constant consumption of 1.41, in exchange for more wealth in bad states. Since the value of the capital stock, $(1+r^*)K^*$, is equal to 5.67 in the baseline calibration, the representative agent can credibly commit to pay this amount if the best endowment is realized. In terms of Proposition (7), the coefficient $\frac{\beta\overline{\theta}}{1+\overline{\theta}}$ equals 0.63 which is well below the capital labor ratio of 0.79. Thus, the capital stock in the benchmark economy is more than enough to secure all contingent debts required to perfectly smooth consumption. According to the baseline parametrization, the level of income risk is not enough to eat up all capital holdings of a representative consumer.

Table 1: Required levels of Arrow securities for perfect risk sharing in the benchmark model

$(1+r^*)k^* = 5.67$

e_i	0.32	0.46	0.66	0.93	1.33	1.88	2.66
we_i	0.39	0.55	0.78	1.10	1.56	2.21	3.14
$\theta(e_i)$	-1.59	-1.20	-0.70	-0.09	0.68	1.66	2.93
$a(e_i)$	1.87	1.40	0.82	0.11	-0.80	-1.96	-3.45

We conducted two alternative experiments using the benchmark. First, given the level of persistence of endowments estimated by Heaton and Lucas, what would be the maximum level of volatility of earnings that could still allow full insurance in the complete market model? We find that for $\sigma = 0.5$, $\frac{K^*}{f(K^*)} \approx \frac{\beta\overline{\theta}}{1+\overline{\theta}}$. This implies a ratio of almost 20 times between the best and worst possible endowment, which is more than twice what Heaton and Lucas documented. Second, given the level of volatility (fixing σ at 0.35), what is maximum level of persistence consistent with perfect risk sharing? We find that for $\gamma = 0.75$, $\frac{K^*}{f(K^*)} \approx \frac{\beta\overline{\theta}}{1+\overline{\theta}}$. This implies that shocks could still be substantially more persistent and still full risk sharing would be possible if markets were completed.

According to these results, there is strong evidence from the PSID data to conclude that if markets were complete, perfect risk sharing would be possible. Debt limits would be irrelevant, as they would not bind, in an economy that can use its capital to secure contingent debts. To further check the robustness of our results, we now turn to other extremely different calibrations of the earning process.

5 Alternative earning processes

According to the results of the previous section, debt constraints will still be binding in the complete markets model if earnings were significantly more volatile and persistent that what Heaton and Lucas estimated. At least two recent papers have argued that this is in fact the case. In this section we analyze the implication of assuming much larger persistence and volatility of earnings along the lines of these recent papers. We argue that even under these seemingly extreme parametrizations, our main result holds. Incomplete markets remain the main friction in explaining inequality and its welfare costs. We find that if markets were complete, the dispersion of wealth will be far too small compared with the data, and most of the tail of the wealth distribution would disappear. Moreover, most of the potential welfare gains of eliminating inequality could be realized by just completing the markets.

In this section we evaluate the role of debt constraints using two radically different parametrizations. The first one follows the recent work of Storesletten, Telmer and Yaron (STY, 2002). They argue that the persistence of the earning process is much larger, at around 0.97, than what Heaton and Lucas found, at around 0.53. They argue that the almost linearly increasing cross- sectional variance of earnings with age requires a very large, almost unit root, degree of persistence. To formally support this claim, they make strong assumptions about finiteness and initial conditions.

Our first alternative parametrization is similar to our benchmark but with two differences. First, in line with STY findings we pick $\gamma = 0.96$. Additionally, we increase the volatility of the labor endowment by choosing $\sigma = 0.58$. In this way, our calibration of the earning process is similar to the one employed by Krueger and Perri (2002) for an endowment economy. We can thus compare our results. Table 2 presents results analogous to Table 1 for this alternative parametrization.

Table 2: Required levels of Arrow securities for perfect risk sharing: STY parameters

$(1 + r^*)K^* = 5.67$

e_i	0.14	0.24	0.43	0.77	1.37	2.46	4.39
we_i	0.16	0.29	0.51	0.91	1.62	2.89	5.17
$\theta(e_i)$	-16.1	-13.4	-9.2	-2.7	7.11	22.0	42.2
$a(e_i)$	18.9	15.8	10.8	3.2	-8.3	-25.9	-49.7

It is clear from this parametrization that the modified golden rule capital stock can only secure a fraction of the contingent debt required for full risk sharing. In particular, a representative agent could only secure around 11% of the payments that he/she would like to promise if the best state of nature is realized. Notice that the ratio between the best and worst possible endowments is now 31 times.

Our second alternative parametrization is similar to the one reported by Castañeda, Días-Giménez and Ríos-Rull (CDR, 2003). Instead of looking directly at evidence on earnings dynamics, they calibrate the earning process to match the U.S. Lorenz curves of earnings and wealth using a model based approach[8]. Their model allows for different realistic features of the U.S. economy regarding taxation, social

[8] See Cordoba (2001, 2003) for a related reverse engineering exercise for the case of city size distributions.

security, aging and retirement. They find the earning process to be very persistent and extremely volatile.

To resemble their results as much as we can with our simpler model, we replicate their parametrization. In particular, we choose $\beta = 0.924$, $\mu = 1.5$, $\alpha = 0.376$, and $\delta = 0.059$. In addition, we use their endowment process for the working population as described by Table 3.

Table 3: The relative endowments of efficiency labor units, e_i, the stationary distribution of working-age households, $p(e_i)$, and the transition probabilities - CDR:

e_i	0.3103	0.9775	3.0349	329.2501
$p(e_i)$	0.6227	0.2254	0.1510	0.0004
		To e'		
From e	$e' = e_1$	$e' = e_2$	$e' = e_3$	$e' = e_4$
$e = e_1$	0.9843	0.0117	0.0040	0.0001
$e = e_2$	0.0314	0.9648	0.0038	0.0000
$e = e_3$	0.0153	0.0044	0.9800	0.0002
$e = e_4$	0.1090	0.0050	0.0625	0.8235

The most notable feature of this earning process is the extremely high value of the largest labor endowment. This value is required to explain the long tail in earnings and wealth implied by the Lorenz curves. Labor endowments are also extremely persistent. For completeness, Table 4 shows the analogous of Table 2 and 3 for this calibration. In this case, the modified golden rule level of capital can only secure 0.34% of the required debt in the best possible state. In addition, the ratio of best to worst possible endowments is now 1,061 (!).

Table 4: Required levels of Arrow securities for perfect risk sharing: CDR parameters

$(1+r^*)K^* = 5.19$

e_i	0.31	0.98	3.03	329.2
we_i	0.35	1.10	3.41	370.6
$\theta(e_i)$	-5.9	-1	23.4	1,376
$a(e_i)$	6.7	1.2	-26.3	-1,548

5.1 Findings

We now report some relevant predictions of four model economies. For each set of parameters we compute two models, one with incomplete markets and one with complete markets, as explained in Section 2. Both models preclude net borrowing.

Wealth Distributions: Table 5 reports Gini indexes and selected points of the Lorenz curve of wealth in the U.S. and two incomplete markets models. The data for the U.S. is taken from Castañeda, Días-Giménez et al. (2003). Overall, the CDR parametrization better accounts for the U.S. wealth distribution in terms of Gini coefficients, and in terms of the amount of wealth owned by the wealthiest group. The STY parametrization accounts better for the 95-99 group.

Table 5: The distribution of wealth in the U.S., and two incomplete markets models

	Gini	Quintiles					Top Groups		
Economy		1st	2nd	3rd	4th	5th	90-95	95-99	99-100
U.S.	0.78	-0.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
STY	0.64	0.02	3.0	9.38	21.78	65.67	16.35	20.18	7.98
CDR	0.77	0.01	0.07	4.20	16.87	78.85	16.38	13.58	25.98

Table 6 reports the same statistics as Table 5 but for the complete markets version of the model economies. A significant amount of wealth inequality still remains due to the large amount of labor income risk relative to the capital holdings. Gini indexes fall but not substantially. The concentration of wealth in the top percentile is, however, significantly reduced in both models. These results reveal one of the main limitations of debt-constrained models in explaining the U.S. wealth distribution. They cannot account for the large concentration of wealth in the wealthiest group, even if they can produce particularly high Gini coefficients.

Table 6: The distribution of wealth in the U.S., and in two complete markets models

	Gini	Quintiles					Top Groups		
Economy		1st	2nd	3rd	4th	5th	90-95	95-99	99-100
U.S.	0.78	-0.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
STY	0.60	0.06	2.85	10.89	24.60	61.02	16.03	17.01	6.06
CDR	0.65	0.81	3.67	6.52	18.53	70.47	18.53	18.47	6.68

The distribution of consumption: Table 7 reports Gini indexes and selected points of the Lorenz curve of consumption in the U.S. and two incomplete markets models. The data for the U.S. is taken from Castañeda, et al. (2003). It is important to highlight that data on consumption is less reliable than data on wealth (see Attanasio *et al.* 2004 for a recent discussion). In terms of consumption, the STY model performs extremely well and does a better job than the CDR model in all respects. The CDR model produces too much concentration of consumption in the top tail. We conclude that while the CDR parametrization provides a better explanation for the U.S. wealth inequality, the STY parametrization provides a better explanation for the U.S. consumption inequality.

Table 7: The distribution of consumption in the U.S., and two incomplete markets models

	Gini	Quintiles					Top Groups		
Economy		1st	2nd	3rd	4th	5th	90-95	95-99	99-100
U.S.	0.30	7.19	12.96	17.80	23.77	38.28	9.43	9.69	3.77
STY	0.32	6.98	12.43	19.92	24.44	39.24	10.04	9.96	3.23
CDR	0.52	5.41	5.51	10.98	18.18	59.93	14.97	11.99	11.59

Table 8 reports the same statistics as Table 7 but for the complete markets versions of the model economies. As with the distribution of wealth, Gini coefficients do not change substantially. In terms of Lorenz curves and Gini coefficients, the STY model is still very successful in accounting for the U.S. evidence. In terms of the distribution of consumption, complete market models perform extremely well but not better than incomplete markets models.

Table 8: The distribution of consumption in the U.S. and two complete markets models

The Distribution of Consumption									
	Gini	Quintiles					Top Groups		
Economy		1st	2nd	3rd	4th	5th	90-95	95-99	99-100
U.S.	0.30	7.19	12.96	17.80	23.77	38.28	9.43	9.69	3.77
STY	0.26	9.51	13.32	18.67	22.78	35.72	8.45	9.37	2.51
CDR	0.38	7.19	9.52	13.46	24.80	45.03	10.49	8.40	6.28

Other statistics: Tables 9 and 10 show other relevant predictions of the incomplete and complete-markets debt-constrained models. The first two columns assess the amount of precautionary savings implied by the models. The excess of savings relative to a frictionless economy, measured by $\frac{\delta k}{y} - \frac{\delta k^*}{y^*}$, is between 4.46% and 11.6%

in the incomplete markets economies considered. This is a dramatic increase relative to what Aiyagari (1994) estimated using Heaton and Lucas (1996) computations. The amount of precautionary savings in the complete markets models is significantly lower but still larger than previously found.

Columns three and four in the tables report the coefficient of variation of wealth and income. According to Rodríguez et al (2002, Table 1), the coefficient of variation of wealth is 6.53. The only model that comes close to account for this large variation is the CRD model with incomplete markets. Other models fall substantially short in producing significant wealth variation. The complete markets models considered can at most explain 24% of the observed variability while the incomplete markets models can explain up to 92%.

Table 9: Some statistics from two incomplete markets models

Economy	$\frac{k-k^*}{k^*}$	$\frac{\delta k}{y}-\frac{\delta k^*}{y^*}$	Coef.Var[k]	Coef.Var[c]	Welfare Gains (θ)
STY	86.6%	11.6%	2.63	0.60	53%
CDR	49.5%	4.46%	6.01	2.16	79%

Finally we compute a standard equalitarian measure of welfare costs of inequality. It is obtained as the solution for θ from the following formula:

$$\frac{U\left[c^*\right]}{1-\beta}=\frac{1}{1-\beta}\int U\left[(1+\theta)c(x,e)\right]dJ^*(x,e).$$

The left hand side of this formula is the welfare of a representative agent in a frictionless economy. The right hand side of the economy is a measure of social welfare in one of the distorted economies where all agents have the same weight, and all consumptions are increased proportionally by the rate θ. θ thus measures the social gains from eliminating all inequality in terms of permanent proportional consumption increases. Table 9 shows that the potential welfare gains are quite large, between 53% and 79%. These are quite sizable gains compared, say, with the welfare gains of eliminating business cycles (Lucas, 1988). Most importantly for our purposes, Table 10 shows that most gains could be realized by completing the markets. In particular, if market were complete, the remaining potential welfare gains would be between 14.6% and 35%.

Table 10: Some statistics from two complete market models

Economy	$\frac{k-k^*}{k^*}$	$\frac{\delta k}{y} - \frac{\delta k^*}{y^*}$	Coef.Var[k]	Coef.Var[c]	Welfare Gains (θ)
STY	30.6%	4.41%	1.22	0.48	14.6%
CDR	19.8%	1.87%	1.53	2.16	35%

6 Concluding comments

Kehoe and Levine (2001) favors the use of debt-constrained models over incomplete-markets debt-constrained models because they are simpler and produce similar results. For example, in both models, the interest rate is below the rate of time preference and capital is overaccumulated. Kehoe and Zame (2002) argue that market incompleteness does not matter if agents are sufficiently patient and there is no aggregate uncertainty. Krueger and Perri (2002) argue that debt-constrained models better explain the evidence of U.S. inequality during the last 30 years. In contrast with this literature, we show that market incompleteness matters substantially, more so than debt constraints, to explain the large concentration of wealth in the U.S., and its large dispersion. We also show that the welfare costs of incomplete markets are substantial for two arguably realistic calibrations of the earning process.

Our debt constraints are exogenous and simple. Net borrowing is precluded. Other debt-constrained models derive endogenous debt limits that prevent default in pure exchange economies when traders can be excluded from spot markets. The threat of exclusion provides a role for unsecured debt. We do not analyze the role of unsecured debt in our model because it can only strengthen our main point at the cost of unnecessary complications. If unsecured debt were allowed on the top of secured debt, debt constraints alone would be able to explain even less of the observed wealth and consumption inequality. This is because more relaxed debt limits will further expand the risk-sharing opportunities and prevent larger dispersion of consumptions and wealth.

One could argue that if capital cannot be used as collateral for debts then debt-constrained models could still be able to account for the main features of the data. We do not think this is the case. In the paper, we studied earning processes that were extremely risky so that the amount of collateral in the economy plays only a minor role. We find that in those cases debt constrained-models can produce neither significant wealth dispersion nor wealth concentration in the top tail. Moreover, we think that the most realistic assumption is to allow capital to serve as collateral since most debts in U.S. are secured[9].

[9] For example, Canner et. al. (1995, Figure 1) find that more than 75% of the household debt in the U.S. is mortage debt, which can be considered secured debt. A large fraction of the remaining part is also secured because it includes loans for automobiles, mobile homes, trailers, etc.

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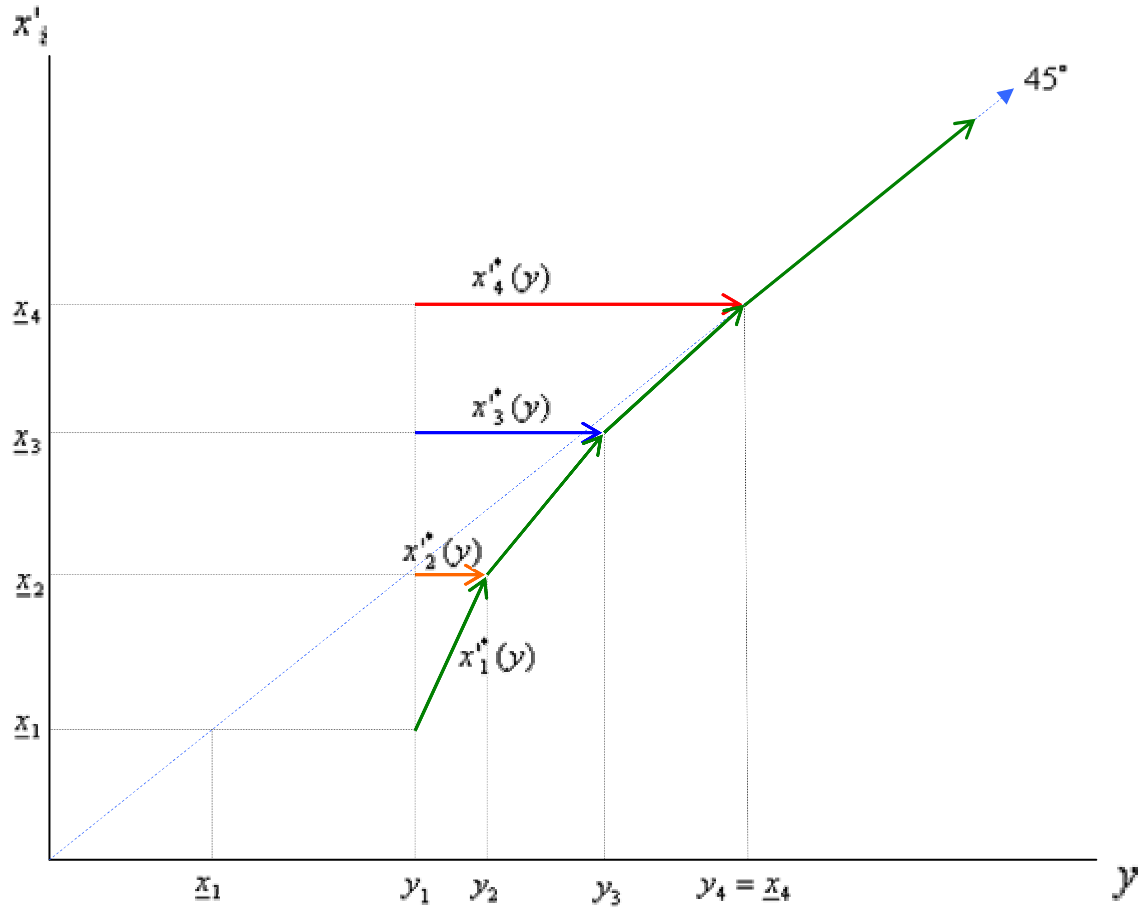

Figure 1: Distribution of savings y, i.i.d. case

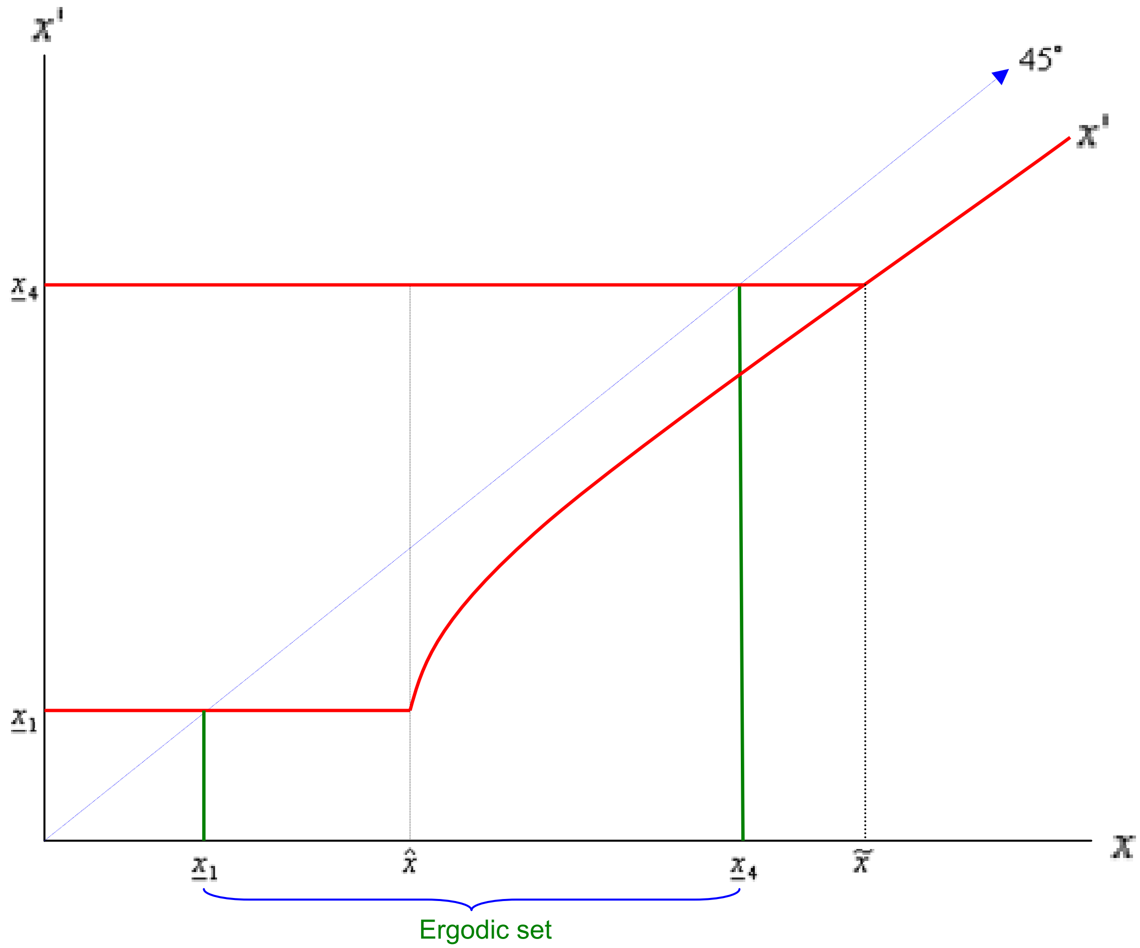

Figure 2: Evolution of resources x , i.i.d case