“Inter-temporal Price Discrimination with Time Inconsistent Consumers”

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Abstract

How should a rational monopolist respond to boundedly rational consumers? This paper looks at the inter-temporal price discrimination game that arises when a monopolist faces naive-time-inconsistent consumers. En route to solving this game, we introduce two new solution concepts for dynamic games where some players are time inconsistent. The first solution concept is similar in spirit to the subgame perfect Nash equilibrium, whereas the second one relies on backwards induction. Unlike in standard finite games, these solution concepts are not equivalent, even with perfect information. We then use these solution concepts to solve the inter-temporal pricing game with time inconsistent consumers. We derive implications for monopoly profits, consumer welfare and the path of prices (Coase conjecture). We conclude that the existence of time inconsistency will reduce monopoly profits and the welfare of all consumers, except of the highest valuation ones. Moreover, with time inconsistent consumers the path of prices will approach marginal cost, but at a lower rate.

1 Introduction

How should a rational firm respond to boundedly rational consumers? This paper asks, and then answers, this question in the context of a durable good monopoly in the presence of time inconsistent consumers. We embed partially naive-time-inconsistent consumers in a standard durable goods monopoly and we explore the implications of time inconsistency for monopoly profits, consumer welfare, the path of prices and the Coase conjecture.

Motivation

Consider a standard durable good monopoly problem, as, for example, in Gul, Sonnenschein and Wilson (1986). Typically, both in previous theoretical work and in the real world, the monopolist does not observe the valuation of any individual consumer and, as a result, he can not perfectly price discriminate. However, he knows the distribution of the consumer valuations, thus he can use inter-temporal price discrimination as a screening mechanism. The monopolist's inter-temporal price discrimination problem can be modelled as a game, between the monopolist and the consumers. The monopolist chooses current and future prices, whereas the consumers choose whether to buy now or in some future period, based on the current price and their expectations about future prices.

It can be shown that in equilibrium the monopolist initially charges some relatively high price to attract the higher valuation consumers, for whom it is very costly to wait and buy at lower prices in future periods. With the higher valuation consumers out of the market, the monopolist follows a similar strategy with the remaining consumers. Therefore, as time goes by prices fall and asymptote marginal cost. As the length of each period becomes smaller, price falls to marginal cost (almost) immediately and the Coase conjecture is verified.

To make the inter-temporal pricing problem more realistic, we want to allow for the possibility that consumers have time inconsistent preferences. Time inconsistency is a consequence of the following fact about human cognition: the discount factor for adjacent periods is lower for the immediate future than for the more distant one. In other words, people feel that in the future they will be more patient than they are today. Recall that in the standard case of exponential/geometric discounting, the discount factor is the same for all adjacent periods. This implies that if at time t, a decision maker prefers alternative A to alternative B, she will maintain a preference for alternative A at all future periods as well. On the contrary, when a decision maker mistakenly thinks that in the future she will be more patient, (i.e. her discount factor will be higher), it is possible that in period t she prefers alternative A to alternative B, but reverses her preference in some future period $t+k$. Hence the term *time inconsistency.* [1]

[1] For example, the majority of us would prefer "1000 euros today" to "1020 euros in two

The psychological evidence supporting such preference reversals is overwhelming (see for example Ainslie (1992)), and it is fairly uncontroversial to claim that time inconsistent preferences are a fact of life that economic theory should address.

What is more controversial is the issue of whether people are aware of their time inconsistency. Strotz (1956) recognized that there can be two polar cases. On one extreme, people may be *sophisticated* about their time inconsistency, which means that even though they feel that in the future they will be more patient, they do recognize that once the future rolls over they will be less patient. On the other extreme, people may be *naive* about their time inconsistency, in which case they do not foresee that in the future they will not be as patient as they think they will be. Of course, there is also the intermediate range of partially-naive consumers. Della Vigna and Malmandier (2003) use data on gym memberships and attendance to test whether consumers are sophisticated or naive-time-inconsistent. They conclude in favor of partially naive consumers. We allow for both sophisticated and partially naive consumers, but as we shall see the problem is non-trivial only with partially-naive consumers.[2]

A wide range of questions then arises. How should a rational monopolist price in the presence of time inconsistent consumers? What are the implications of time inconsistency for monopoly profits, consumer welfare and the Coase conjecture? And, how do our answers depend on whether we assume sophisticated versus naive-time-inconsistent consumers?

Some game theory and the main results

In the presence of naive-time-inconsistent consumers, the core of the problem is to figure out an appropriate solution concept that applies to games with time inconsistent players. The seminal papers on time inconsistency of Strotz (1956) and Phelps and Pollak (1968), as well as the more recent contributions of Laibson (1997) and O'Donaghue and Rabin (1999, 2001), solve single-agent decision problems. However, there is no previous work that addresses time inconsistency in the context of a non-cooperative game. Our paper proposes two solution concepts for games with time inconsistent players and then applies them to the durable goods monopoly.

When agents are naive-time-inconsistent, they make mistakes in predicting their future behavior; in period t they think that in period $t+1$ they will choose alternative A, only to discover that when period $t+1$ rolls over they choose some other alternative B. When such agents (consumers) play a game, this means they

weeks", but we would reverse this preference if we were given the choice between "1000 euros in exactly one year from today" and "1020 in exactly one year and two weeks from today".

[2]Our model is cast in the context of fully naive consumers, but, as we shall see, a simple reinterpretation of the parameters handles any intermediate degree of naivete.

also make mistakes in predicting how others (the monopolist) will behave. This is because in period t they think that the monopolist will play a best response to alternative A, but in fact he will play a best response to alternative B. In the context of our pricing game, this means that time inconsistent consumers make two types of mistakes. First, they do not correctly predict when they will buy the good. Second, and this is a consequence of the first, they cannot correctly anticipate future prices. These observations open up two possibilities for a solution concept.

One possibility is to require that consumers play a best response to the prices that the rational monopolist will actually charge. This is "as if" when the market opens in period 1, the monopolist announces all future prices and in each period consumers decide when to buy based on these prices. Consumers take these prices as given and they do not question how and why the monopolist has chosen these prices. In turn, these prices are optimal for the monopolist given how the consumers will actually behave. This is an equilibrium in the usual sense; players are endowed with some beliefs about how others play and when players play a best response to these beliefs the original beliefs are verified. We also impose the restriction that the monopolist is sequentially rational, which guarantees that the optimal pricing policy is subgame perfect. We refer to this solution concept as the *equilibrium* and we argue it is related to the subgame perfect Nash equilibrium (SPNE).

The other possibility is to require that consumers play a best response to the prices that they *think* the monopolist will choose. Consumers arrive at their expectations about future prices by introspection, i.e. by backwards induction. We refer to the prediction that survives this introspective process as *naive backwards induction.*

When consumers are sophisticated about their time inconsistency, what happens in the game coincides with what consumers think will happen and the two solution concepts are equivalent. The first result of the paper shows that for sophisticated consumers the solution also coincides with the solution to the case where consumers are time consistent.[3]. When consumers are (partially) naive, the equilibrium and naive backwards induction lead to different predictions. We first develop the main results in the context of the 3-period model and we then extend to the general T-period model.

It is shown that in the equilibrium, as the degree of time inconsistency rises, monopoly profits and consumer welfare fall for all consumers, except for the high valuation ones. Prices fall over time and approach marginal cost, but at

[3] This result, according to which sophisticated time inconsistency is equivalent to time consistency, is true only in our pricing game and does not hold for general games

a lower rate, relative to the time consistent case. To get an idea about the forces that drive these results, notice that as time inconsistency becomes more pronounced consumers think that they will be more patient in the future. As a result, consumers become more reluctant to buy early when the price is high and fewer people buy at a high price. This hurts the monopolist's profits and to mitigate the negative effect the monopolist has to lower his price in the first period in order to attract some consumers to buy early. In the subgame that follows, there are more potential consumers, since fewer bought in the first period, and this enables the monopolist to increase his prices. The only consumers that gain are the highest valuation ones that buy in the now lower first period price.

The above results come with a caveat. If the degree of time inconsistency is not high enough, then the equilibrium with time inconsistent consumers coincides with the equilibrium of the time consistent case. Therefore, if we adopt the equilibrium as the appropriate solution concept, naive time inconsistency has an effect on the outcome of the game only when the degree of time inconsistency is above some threshold. For the 3-period model, we provide a closed form expression for this threshold, in terms of the model's parameters.

Under naive backwards induction we obtain similar qualitative results, but the intuition is different. Now, the driving force for the results is consumer expectations about future prices. It turns out that as the degree of time inconsistency rises, consumers think that the second period price will be lower. As a result, consumers become more reluctant to buy early at a high price. This triggers the same effects as the ones obtained under the equilibrium. Monopoly profits fall, prices approach marginal cost at a lower pace and welfare falls for all but the highest valuation consumers. These results hold for any degree of time inconsistency.

Related Literature

This paper builds on four strands of literature; the durable good monopoly, time inconsistent preferences, psychology and economics and the game theoretic literature on solution concepts.

Any mention of the durable good monopoly usually goes hand in hand with the Coase conjecture (1972), according to which durability robs the monopolist of his market power because price falls (almost) immediately to marginal cost. In the 1980's a series of papers proved that Coase's conjecture was indeed correct as a limit result, as the length of a period becomes arbitrarily small. See for example, Stokey (1981) and Gul, Sonnenschein and Wilson (1986). Besanko and Winston (1990) solve for the subgame perfect optimal pricing policy for the special case where consumer valuations are uniformly distributed. Waldman (2003) provides a broad survey of the literature on the durable good monopoly, which also includes issues of optimal durability and adverse selection. Notice that the durable-good-monopoly literature pertaining to the Coase conjecture is often recognized by the term *time inconsistency*. However, this terminology refers to the monopolist's inability to commit to future prices and should not be confused with the time inconsistency in our paper, which refers to how consumers discount future gains from trade.

The literature on time inconsistent preferences has its origins in Strotz (1956) and Phelps and Pollak (1968). Strotz (1956) was also the first to observe the distinction between sophisticated and naive time inconsistency. Phelps and Pollak (1968) proposed a mathematically convenient way to model time inconsistent preferences using a quasi-hyperbolic discount function, referred to as $\beta - \delta$. Laibson (1997) and O'Donaghue and Rabin (1999, 2001) have popularized this quasi-hyperbolic discount function and explored some implications of time inconsistency for single person saving and task completion decisions. O'Donaghue and Rabin (1999, 2001) have also explored some implications of assuming sophistication, naivete or partial naivete.

Time inconsistency is only one of a long list of psychologically- motivated challenges to the neoclassical paradigm. See Rabin (1998) for a survey of psychological biases that are relevant for economic theory, such as overconfidence, framing effects, etc. The overwhelming majority of the early literature on psychology and economics has focused on showing how a theory of bounded rationality based on such psychological biases can explain various economic anomalies. Thaler (1991) offers a collection of articles on this topic. Recently, however, there has been a growing interest in understanding how the existence of bounded rational agents in a market setting affects the welfare and the decisions of rational agents. Sarafidis (2003) asks how a rational agent (politician/employee/advertiser) can exploit the memories of his forgetful assessor (electorate/employer/consumers). Della Vigna and Malmandier (2003) solve the two tariff problem of a monopolist (gym club) facing time inconsistent consumers. Gabaix and Laibson (2004) argue that rational firms can exploit boundedly rational consumers by making their product inefficiently complex. The present paper is most similar in spirit with these last three contributions.

Finally, the solution concepts of *equilibrium* and *naive backwards induction* are motivated by the standard notions of the subgame perfect Nash equilibrium, due to Selten (1965), and backwards induction. Moreover, the concept of rationalizability in extensive form games, introduced by Berheim (1984) and Pearce (1984), is useful in understanding the relative pros and cons of our solution concepts.

The rest of the paper is organized as follows. In section 2, we take a quick look at a standard durable good monopoly problem to which we introduce time inconsistent consumers. In section 3, we propose the two solution concepts of equilibrium and naive backwards induction. Section 4 uses these solution concepts to solve the inter-temporal pricing game and to derive implications for the effect of time inconsistency on profits, prices and welfare. Section 5 considers the arguments for and against each solution concept. Section 6 concludes.

2 The model

Consider a monopolist selling a durable good to a population of consumers. Each consumer can buy at most one unit of the good and let v denote a generic consumer valuation for this one unit. The monopolist can not observe the valuation of any individual consumer, but knows that consumer valuations in the population are uniformly distributed between 0 and x, so that $v \sim U[0, x]$. As a result, the monopolist can not perfectly price discriminate, but he can screen costumers with inter-temporal price discrimination.

Let there be T periods, indexed by $t \in \{1, 2, ..., T\}$, and assume that the monopolist can commit to a price for the duration of a period t. The monopolist's inter-temporal pricing problem consists of choosing a price in each period t, with the objective of maximizing his future stream of profits. We assume that the monopolist discounts future profits at rate $\gamma < 1$, and, for simplicity we let marginal cost be zero.

2.1 Price discrimination with time consistent consumers

Consider first the case where consumers have an exponential/geometric discount factor $\delta < 1$. Besanko and Winston (1990) show how the monopolists' inter-temporal pricing problem can be solved backwards using dynamic programming. Their solution is the benchmark to which we compare our subsequent results with time inconsistency and, in what follows, we give a brief overview of the Besanko-Winston solution.

First, observe that for any expected future prices, if a consumer with valuation v' buys in a given period, so will any consumer with valuation $v > v'$. Let v_t denote the lowest valuation consumer who buys in period t. Then in period t, the state of the market is summarized by the *state variable* v_{t-1}, which indicates that from period t onwards the monopolist faces consumers whose valuations are uniformly distributed on the interval $[0, v_{t-1}]$.

In each period t, the monopolist chooses the price p_t that maximizes his discounted stream of profits from that period onwards, as a function of the state variable v_{t-1}. Let $\Pi_t(v_{t-1})$ denote the maximum profit that can be attained from period t onwards when the state of the market at time t is v_{t-1}. Then, we can write recursively,

$$\Pi_t(v_{t-1}) = \max_{v_t, p_t} (v_{t-1} - v_t)p_t + \gamma \Pi_{t+1}(v_t) \tag{1}$$

$$\text{subject to } v_t - p_t = \delta(v_t - p_{t+1}(v_t)) \tag{2}$$

The first equation is the usual value function of the dynamic program. The second equation says that the last consumer who buys in period t, and whose valuation is defined as v_t, is exactly indifferent between buying in period t or in the next, $t+1$, period. We will refer to this consumer as the *marginal consumer* for period t. For future reference, it is important to understand why the state variable v_t is defined by an indifference condition between periods t and $t+1$, rather than an indifference condition between periods t and some other future period, say $t+2$. To see this consider the following argument. In period $t+1$ the marginal consumer is v_{t+1} and to be such she has to (weakly) prefer to buy in period $t+1$ than in any future period, including the period $t+2$. This implies that every consumer with valuation higher than v_{t+1} will *strictly* prefer to buy in period $t+1$ than in any future period, including period $t+2$. Therefore, the marginal consumer in period t who has valuation $v_t > v_{t+1}$ will *strictly* prefer to buy in period t than in period $t+2$. Therefore, the marginal consumer in period t can not be indifferent between buying in the current period t or in some future period $t+k$, where $k > 1$.

Besanko and Winston (1990) show that in the optimal pricing profile the price falls over time and they compute the rate at which this occurs. Such decreasing pricing policy is referred to as *price skimming*, because it allows the monopolist to screen consumers and charge a higher price to consumers with higher valuations. As the number of periods goes to infinity, the last period price asymptotes marginal cost. Moreover, as the physical duration of a period becomes arbitrarily small, price falls to marginal cost almost immediately. This insight is referred to as the *Coase conjecture*, after Coase (1972) who argued that durability of a good robs the monopolist of his market power.

The solution of Besanko and Winston (1990) can also be thought of as the subgame perfect Nash equilibrium (or alternatively, the backwards induction outcome) of a game that the monopolist plays with the consumers. The monopolist's strategy is to choose a stream of T prices, one for each period. The consumers see the price in period t, p_t, and their strategy is to decide whether or not to buy (provided they have not already done so) based on what expectations they have about future prices. In the SPNE (or the backwards induction outcome) the monopolist's equilibrium strategy is to offer the good at the prices that solve the dynamic program in equations (1) and (2). The consumers' equilibrium strategy is to buy the good in period t, if their valuation is at least as high as v_t. The consumer with valuation exactly equal to v_t is the marginal consumer for period t.

Notice that same pricing policy would also solve the incomplete information variant of the game, where the monopolist faces a single consumer whose valuation v is private information, but distributed as $v \sim U[0, x]$. In this case, the appropriate equilibrium concept is the perfect bayesian equilibrium (PBE). The equilibrium strategies in the PBE are the same, as in the SPNE of the complete information game. These strategies support the posterior beliefs that, if the consumer has not bought at the end of period t, her valuation is distributed as $v \sim U[0, v_t]$. (see for example exercise 9.C.3 in Mas-Collel, Whinston and Green (1995))

For future reference, we conclude our discussion with the solution to the two period model, i.e. $T = 2$. It is straightforward to calculate the following prices and marginal consumers for periods 1 and 2, respectively.

$$p_1(x) = \frac{x}{2}\frac{(2-\delta)^2}{(4-2\delta-\gamma)} \tag{3}$$

$$v_1(x) = x\frac{(2-\delta)}{(4-2\delta-\gamma)} \tag{4}$$

$$p_2(v_1) = v_2(v_1) = \frac{v_1}{2} \tag{5}$$

2.2 Price discrimination with time inconsistent consumers

We now introduce time inconsistency in the benchmark model. For simplicity, suppose that there are only three periods.

We assume that in period 1 consumers feel that between periods 2 and 3 they will be more patient than they will actually be. To model such preferences, we introduce two discount factors for the consumers, δ and δ'. We refer to them as the *true* and the *perceived* discount factor, respectively. The true discount factor δ captures, how in period 1 consumers discount payoffs realized in period 2 and how in period 2 consumers discount payoffs realized in period 3. The perceived discount factor δ' captures how in period 1 consumers expect to discount period 3 payoffs in period 2. In other words, in period 1 the discount function is $\{1, \delta, \delta\delta'\}$, whereas in period 2 onwards the discount function is $\{1, \delta\}$. With such preferences, it is possible that in period 1 a consumer expresses a desire to buy in period 2 at price p_2 rather than in period 3 at price p_3, but when period 2 rolls over she reverses her original preference. Hence, we say that consumers will be time inconsistent.

Moreover, we assume that consumers are *naive* about their time inconsistency. That is, in period 1 they do no understand that when period 2 comes they will not be as patient as they think they will be. Alternatively, we could have assumed that consumers are *sophisticated* about their time inconsistency, in which case, in period 1 they *feel* that between periods 2 and 3 they will be more patient than they are today, but they also realize that when period 2 actually rolls over they will not be as patient as they feel they will be. As we shall later see (proposition 1), an attractive feature of our model is that it can also encompass the case where consumers are partially sophisticated about their time inconsistency.

The monopolist is assumed to be perfectly rational; he is time consistent with discount factor γ, he knows that the consumers think that they will be more patient in the future and he is aware that consumers will not be as patient as they think they will be. Consumers, being naive, do not know that the monopolist knows that they will not be as patient as they think they will be. As a result, we have neither common knowledge of rationality nor common knowledge of irrationality.

The economic literature commonly models time inconsistent preferences with a quasi-hyperbolic discount function of the form $\{1, \beta\delta, \beta\delta^2, ...\}$. Such preferences are due to Phelps and Pollak (1968) and are referred to as $(\beta-\delta)$ *preferences.* Our model formulation is equivalent to the one implied by the $(\beta-\delta)$ discount function. To see this, notice that our true discount factor δ translates to the term $\beta\delta$ in the Phelps-Pollak formulation. And similarly, our perceived discount factor δ' translates to the parameter δ of Phelps-Pollak. We chosen not to use the original Phelps-Pollak formulation because one of our goals in this paper is to explore how they degree of time inconsistency affects prices, profits and consumer welfare. In the $\beta - \delta$ model, the degree of time inconsistency is captured by the parameter β. As the parameter β falls, the degree of time inconsistent rises. At the same time, however, a decrease in the parameter β also makes the decision maker less patient, because it decreases the discount factor $\beta\delta$ that the decision maker uses for discounting two adjacent periods, say periods 1 and 2. In our formulation the degree of time inconsistency is captured by the perceived discount factor δ'. As the perceived discount factor δ' increases, the degree of time inconsistency increases, but this does not affect the true discount factor between periods 1 and 2. In other words, our formulation allows us to disentangle the effect of increased time inconsistency from increased impatience.

What is the optimal pricing strategy for the monopolist in the presence of naive-time-inconsistent consumers? And what are the implications of time inconsistency for monopoly profits, consumer welfare and the Coase conjecture?

To answer these questions it is first necessary to define a solution concept for dynamic games where some players are naive-time-inconsistent. In the next section we propose two ways to solve such games.

3 Solution concepts

In this section, we propose two solution concepts for dynamic games where one player is rational and the rest are naive-time-inconsistent. Our objective here is to simply present two "sensible" ways to solve such games. We will then apply these two new solution concepts to our motivating inter-temporal pricing problem. We do not have here neither the intention nor the space to discuss technical issues of, say, existence or the epistemic foundations of these solution concepts. Such issues are explored in a companion paper, Sarafidis (2004).[4]

Consider the following game, whose extensive form is depicted in figure 1. Player B (for **B**oundedly rational) moves first in period 1 and she can play *in* or *out*.[5] If she plays *out*, the game ends, but players have to wait until the end of period 3 before they receive their payoffs.[6] If player B plays *in*, player R (**R**ational) gets to play $Left$ or $Right$, still in period 1. Then in period 2, player B gets to play $left$ or $right$. The game ends in period 2, but in case player B plays *right*, she has to wait an extra period, before she gets her payoff of 10 or 8.

Moreover, assume that player B is time inconsistent in the following sense. In period 1, she thinks that in period 2 she will prefer "10 in period 3" to "9 in period 2" and that she will prefer "8 in period 3" to "7 in period 2". However, in period 2 she reverses her preference and she goes for the lower, but immediate payoffs of 9 and 7, respectively. In other words she thinks that she will be patient, but in reality she will be impatient. Moreover, we assume that in period 1 player B is not aware that she will actually not be patient in period 2, and hence she is naive about her time inconsistency. Player R is assumed to be rational; he is time consistent, and he knows that player B is naive-time-inconsistent. Finally, player B, being naive about her time inconsistency, does not know that player R knows that player B will not be as patient as she thinks she will be.

[4]Also, at this stage our objective is not to compare these two solution concepts and ask which one is "better". There are pros and cons for both solution concepts and we believe that the appropriateness of using one over the other depends on the particular economic application we have in mind. We postpone a comparison between our two solution concepts for later, after we have seen how they perform "in practise", in solving the motivating pricing problem. (see section 5)

[5]Player B (resp. R) is the female (resp. male) player.

[6]The asterisk next to the payoffs in figure 1 indicates that these payoff will be realized at the end of period 3. All other payoffs are obtained at the end of period 2.

How would you play this game if you were player B? What if you were player R? In period 1 player B could reason as follows. "If I play *out* I will get 9*. If instead I play *in*, then if player R plays $Left$ I will play *right* and get 10*, whereas if player R plays *Right* I will play *right* and get 8*. Therefore, I should play *in* only if I think that player R will play $Left$. To determine what player R will play, I should put myself in his shoes and reason about the problem from his perspective. Player R will think that if he plays $Left$, I will play *right* and he will end up with nothing. If instead he plays *Right*, he will think that I will play *right* and he will get 1. Therefore, he will play *Right* and I should play *out*."

The reader has probably recognized that the reasoning of player B in the preceding paragraph is a form of backwards induction. According to it, player B will play *out*, because she expects that if she instead plays in, player R will play *Right* and then she will play *right*. This play path is portrayed with the dotted lines in the figure and it describes what player B *thinks* will happen. We will refer to this prediction for the game as *naive backwards induction* (NBI) and we will formalize it shortly. We have added the qualifier *naive* to emphasize that under NBI player B plays a best response to what she naively thinks player R will play.

Here is another sensible prediction about how the game will be played. Assume that player R announces that he will play $Left$ and player B does not worry about how player R has come to the conclusion that he should play $Left$. Now, in period 1 player B reasons as follows. "If I play *out* I get 9*. If I play in, player R will play $Left$, because he said so. I will then play *right* and my payoff will be 10*. Therefore, my best response to his announcement is to play in, and then *right*". As the game develops and player B plays in, player R plays $Left$ and then player B plays $left$. Notice that player R, knowing that player B will be impatient, will indeed find it optimal to play $Left$, as he announced. Our new prediction for the game is $((in, left); Left)$. We will refer to this solution concept as the "equilibrium" and we will formalize it shortly. We have chosen the term *equilibrium* to highlight the similarity of this solution concept to the standard notion of the Nash equilibrium. As in a Nash equilibrium, players are endowed with some beliefs about how others will play. Each player takes these beliefs as given, without questioning how and why other players have chosen to play this way, and chooses a best response. Moreover, the original beliefs are confirmed in equilibrium, because all players actually play as others expect them to play.

It is well known that in dynamic games the Nash equilibrium allows players to take actions that are not sequentially rational. To overcome this problem, we have the equilibrium refinement of subgame perfection, due to Selten (1965). When we will later define the equilibrium formally, we will also impose the requirement of subgame perfection. For the time being, notice that the proposed equilibrium for our motivating example is indeed subgame perfect. Player R plays $Left$ and from his point of view this is indeed a sequentially rational move.

In our example, the equilibrium and NBI predict different outcomes. In finite games of perfect information with time consistent players the concepts of backwards induction and subgame perfect Nash equilibrium are equivalent. Our preceding motivating example shows that in games with time inconsistent players, it is possible to have an equilibrium which is subgame perfect, but fails to survive (naive) backwards induction and the equivalence breaks down.

We can summarize the main features of our two solution concepts as follows. In a NBI the time inconsistent player plays a best response to what he thinks the rational player will play. The attractive feature of NBI is that players can rationalize what they play with a speech of the form: "I play x, because I expect my opponent to play y, because he expects that I play $x^{'}$, because I expect him to play $y^{'}$,...". However, the time inconsistent player will be surprised by how the rational player plays the game, because player R may play contrary to the expectations of player B. In an equilibrium the time inconsistent player plays a best response to what the rational player actually plays. The attractive feature of this concept is that the expectations that players have about how others play the game are confirmed in equilibrium. The downside is that the time inconsistent player can not understand why the rational player plays the way he does. Section 5 provides a more detailed discussion of the relative pros and cons of each solution concept.

Formalizing the solution concepts

We now formalize the two solution concepts. For notational convenience we restrict attention to two-player games.

Let Γ be a finite dynamic two-player game of complete[7] information that lasts for T periods. There are two players, R and B. Player B is naive-time-inconsistent. Player R is time consistent and he knows that player B is naive-time-inconsistent.

We create $T-1$ new time consistent players from player B as follows. Player B_i, for $i = 1, 2, ..., T$ has the preferences of the i-period self of player B and has available the actions that player B has from period i onwards. For example, applying this to our earlier game gives three players, namely B_1, B_2 and B_3. Player B_1 has the preferences of the period 1 self and has available the following strategies: $(out, left, left)$, $(out, left, right)$, $(out, right, left)$, $(out, right, right)$, $(in, left, left)$, $(in, left, right)$, $(in, right, left)$, $(in, right, right)$. Player B_2 has

[7] The same ideas can be modified to handle infinitely repeated games and games of incomplete information, see Sarafidis (2004).

the preferences of the period 2 self and she prefers to play $left$ rather than $right$ at both information sets. Her strategies are $(left, left)$, $(left, right)$, $(right, left)$, $(right, right)$. Player B_3 in this example is immaterial because the game ends after period 2.

Let s_R be a strategy for player R and s_{B_i} be a strategy for the fictitious player B_i. It will be useful to define strategy s_B from each s_{B_i} as: the actual strategy that player B will execute if each i-period incarnation, player B_i, plays according to strategy s_{B_i}. For example, return to our example and let $s_{B_1} = (in, left, left), s_{B_2} = (right, left)$. Then, we can write $s_B = (in, right, left)$. Another way to think of the strategy s_B is to pretend that player B uses a notebook to write down what she plans to do in the game. The notebook has T pages and each page has T lines. On page 1 the fictitious player B_1 starts on line 1 and on each line $i \geq 1$ she writes what she plans to do in period i. Then, on page 2 the fictitious player B_2 starts on line 2 and on line $i \geq 2$ she writes what she plans to do in period i. And so on, for each fictitious player B_i. Then, what we call s_B is the strategy that someone would play if she did what was written down on line 1 of page 1, then line 2 of page 2, line 3 of page 3 and so on.

We can now formalize the two solution concepts.

Definition 1. A strategy profile $s = (s_R, s_{B_1}, ..., s_{B_T})$ is an equilibrium if:

1. strategy s_R is a best response to s_B for player R
2. strategy s_{B_i} is a best response to strategy s_R for each player s_{B_i}
3. Profile s induces an equilibrium in all subgames

Definition 2. A strategy profile $s = (s_R, s_{B_1}, ..., s_{B_T})$ survives naive backwards induction if:

1. strategy s_R is a best response to s_B for player R
2. strategy s_{B_i} is the strategy of player B_i that survives the backwards induction procedure in the game between players R and B_i.

The preceding definitions highlight the difference between the two solution concepts that we have already discussed. In an equilibrium the time inconsistent player plays a best response to what the rational player will actually do. When players play according to NBI the time inconsistent player plays a best response to what she thinks the rational player will do. Finally notice that condition 3 in the definition of an equilibrium imposes the restriction of subgame perfection.

4 Solving the model

We now apply the two solution concepts of equilibrium and NBI to the motivating inter-temporal pricing problem. Recall that in an equilibrium (see definition 1) the boundedly rational players play a best response to what the rational player actually does. In our case, this means that in an equilibrium the consumers decide when to buy based on the prices that the monopolist will actually charge. This is "as if" in period 1 the monopolist announces a price for period 1 and for all future periods.[8] In contrast, with NBI the consumers decide when to buy based on the future prices that they think the monopolist will charge. Consumers calculate these future prices via backwards induction, without recognizing that in the future they will not be as patient as they think they will be.

When consumers are sophisticated about their time inconsistency, the prices that consumers expect coincide with the prices that the monopolist actually charges. It is straightforward to conclude that the two solution concepts (equilibrium and naive backwards induction) are equivalent when consumers are sophisticated. One can also show that with sophisticated consumers the equilibrium to our pricing game coincides with the solution of Besanko and Winston (1990) for time consistent consumers. In other words, the prices that solve the price discrimination problem when consumers are time consistent with discount factor δ, also solve the problem when consumers are time inconsistent but sophisticated, with true discount factor δ. We formalize this result in the following proposition.

Proposition 1. In the price discrimination game with sophisticated time inconsistent consumers and true discount factor δ, both the equilibrium and the backwards induction outcome coincide with the SPNE of the game with time consistent consumers and discount factor δ.

It is important to note that this equivalence between time consistency and sophisticated time inconsistency is a feature particular to our pricing game and does not hold in general. For a counterexample, revisit the game in figure 1 and replace the payoff to player B if she plays *out* to "10 in period 3", rather than "9 in period 3". Consider first the case where player B is time consistent and suppose that she discounts future payoffs at $\delta = \frac{1}{2}$. Then, backwards induction predicts the outcome $((in, left); Left)$. Now, assume that player B is time inconsistent. In period 1 she discounts period 2 payoffs by δ and period 3 payoffs by $\delta\delta'$, where $\delta' > \delta$ and δ' close to 1. In period 2 she discount period 3 payoffs by δ. With sophisticated time inconsistency, backwards induction predicts that player B plays *out*.

[8] Condition 3 of definition 1 requires that these prices that the monopolist announces in period 1 are also subgame perfect.

As a corollary to proposition 1, one can see that by reinterpreting the parameters of our model we can also encompass the case of partially-naive consumers. Suppose, for example, that the true discount rate is 0.9, but consumers feel that in the future they will be more patient and their future discount factor will be 0.95. Moreover, assume that consumers realize that they will not be as patient as they think they will be. However, being only partially aware of their time inconsistency they think that the future discount rate will be 0.92, still greater than the true value of 0.9. Such partially naive consumers are equivalent to totally naive consumers who think that the future discount factor will be 0.92. This is because, as proposition 1 shows, that part of time inconsistency that the consumers are aware of is immaterial.

4.1 Equilibrium

We now return to our motivating case of naive-time-inconsistent consumers and we compute the *equilibrium* of the pricing game.

As we have seen, in an equilibrium the prices that consumers anticipate for the future coincide with the ones that the monopolist will actually charge. One candidate equilibrium is for the monopolist to announce the same path of prices, (p_1, p_2, p_3), as in the Besanko-Winston solution when the time consistent consumers have discount factor δ. Let us now explore whether this path of prices can be an equilibrium when consumers are naive-time-inconsistent.

We have already argued that with time consistent consumers in each period $t < 3$ there is a marginal consumer with valuation v_t who is exactly indifferent between buying in periods t or $t+1$. If (p_1, p_2, p_3) is the SPNE path of prices, the indifference conditions for the marginal consumers are

$$v_1 - p_1 = \delta(v_1 - p_2) \tag{6}$$

$$v_2 - p_2 = \delta(v_2 - p_3) \tag{7}$$

Moreover, since $v_1 > v_2$, equations (6) and (7) imply that

$$v_1 - p_1 > \delta^2(v_1 - p_3) \tag{8}$$

which says that the marginal consumer from period 1 would strictly prefer to buy in period 1 than in period 3.

When consumers are naive-time-inconsistent with true discount factor δ and the path of prices is the same, (p_1, p_2, p_3), in period 2 a consumer with valuation v_2 is again the marginal consumer, as she is indifferent between buying in periods 2 or 3. Also, in period 1 a consumer with valuation v_1 is indifferent between buying in periods 1 and 2. However, she may no longer strictly prefer to buy in period 1 (or 2) than in period 3, and therefore she will no longer be the marginal consumer. This is because the discount factor between periods 1 and 3 is no longer δ^2, but $\delta\delta'$. If the discount factor δ' is close enough to δ, then a consumer with valuation v_1 will strictly prefer to buy in period 1 (or 2) than in in period 3, as before. On the other extreme, if the perceived discount factor δ' is close to 1, then she will prefer to buy in period 3. Therefore, as long as the time inconsistency is not very pronounced, i.e. δ' is close to δ, the equilibrium with time consistent consumers is also an equilibrium in the presence of naive-time-inconsistent consumers. We formalize this in the following proposition.

Proposition 2. Let δ and δ' be the true and the perceived discount factors in the price discrimination game with naive-time-inconsistent consumers. If the perceived discount factor δ' is less than the threshold $\frac{4-2\gamma-\delta^2}{6-2\gamma-3\delta}$, then the SPNE of the game with time consistent consumers and discount factor δ is also an equilibrium in the game with naive-time-inconsistent consumers.

Straightforward differentiation shows that the threshold $\frac{4-2\gamma-\delta^2}{6-2\gamma-3\delta}$ is increasing in the discount factor δ and decreasing in the monopolist's discount factor γ. It is also always greater than δ and less than 1.

The more interesting case is when the degree of time inconsistency is high and we have $\delta' > \frac{4-2\gamma-\delta^2}{6-2\gamma-3\delta}$. In this case, the marginal consumer in period 1 is defined by an indifference condition between periods 1 and 3. Then, the period 1 equilibrium price and marginal consumer, p_1 and v_1 respectively, solve the following program.

$$\max_{p_1,v_1}(x-v_1)p_1+\gamma\Pi_2(v_1) \tag{9}$$

$$\text{subject to } v_1-p_1=\delta\delta'(v_1-p_3(v_1)) \tag{10}$$

The expression $\Pi_2(v_1)$ in equation (9) is the equilibrium profit of the monopolist in the subgame that begins in period 2. The price p_3 in (10) is the price that the monopolist will choose in the subgame that starts in period 3. To see how these are calculated, notice that in the subgame that begins in period 2 we have only two periods left and, as a result, the time inconsistency will no longer matter. Therefore, the equilibrium prices and marginal consumers for periods 2 and 3 will coincide with those of the SPNE of the $T=2$ game with time consistent consumers and initial state $x=v_1$. Using equations (3)-(5) we can write

$$p_2(v_1) = \frac{v_1}{2}\frac{(2-\delta)^2}{(4-2\delta-\gamma)} \tag{11}$$

$$v_2(v_1) = v_1\frac{(2-\delta)}{(4-2\delta-\gamma)} \tag{12}$$

$$p_3(v_2) = v_3(v_2) = \frac{v_2}{2} \tag{13}$$

Substituting these expressions into equations (9) and (10) and using the fact that $\Pi_2(v_1) = \frac{v_1}{2}p_2(v_1)$ results to a standard two variable constrained maximization problem. With $\Pi_1(x)$ we denote the value of the objective function at the maximum. The expressions for the price p_1 and the marginal consumer v_1 that solve the problem are too complicated to be of any practical use (see appendix). Nevertheless, it is possible to prove qualitative results for the monopoly profits and the path of prices.

In particular we are interested in how the presence and the degree of time inconsistency affect monopoly profits and the path of prices. In our model time inconsistency enters through the perceived discount factor δ'. The higher the perceived discount factor δ' is, the more pronounced time inconsistent is. The next result shows how the degree of time inconsistency affects monopoly profits.

Proposition 3. Let $\delta' > \frac{4-2\gamma-\delta^2}{6-2\gamma-3\delta}$. *The equilibrium monopoly profits fall as the degree of time inconsistency rises* ($\frac{d\Pi_1(\delta')}{d\delta'} < 0$).

Therefore, monopoly profits are at their highest level when the monopolist faces time consistent consumers, i.e. ($\delta' = \delta$). The intuition for the result is as follows. We know that in the time consistent case an increase in the discount factor δ hurts the monopolist. This is because when the discount factor rises, consumers become more patient and they prefer to wait and buy in the future when prices will be lower. As the degree of time inconsistency rises, consumers *think* that they will be more patient. Some consumers who would otherwise buy in period 1 at a high price, postpone now their purchases in the expectation that they will buy in period 3 and this hurts monopolist profits. This is despite the fact that once period 2 rolls over, some of these consumers will not wait until period 3, but they will buy in period 2.

The next result shows how the degree of time inconsistency affects the first period price, p_1, and the valuation of the marginal consumer, v_1.

Proposition 4. Let $\delta' > \frac{4-2\gamma-\delta^2}{6-2\gamma-3\delta}$. *The equilibrium first period price falls as the degree of time inconsistency rises* ($\frac{dp_1(\delta')}{d\delta'} < 0$). *Moreover, the equilibrium valuation* *of the marginal consumer in period 1 rises as the degree of time inconsistency rises* ($\frac{dv_1(\delta')}{d\delta'} > 0$)*, and as a result fewer consumers buy in period 1.*

To see the intuition, let v_1^L be the valuation of the marginal consumer in equilibrium when the degree of time inconsistency δ' is at some relatively low level. Assume now that the degree of time inconsistency, δ', rises to a higher level. Now, consumers with valuations slightly higher than v_1^L no longer want to buy in period 1. Instead, they prefer to wait and buy in period 3 when the price is lower. In order to induce some of them to buy in period 1 the monopolist has to decrease the price in period 1.

Finally, since fewer consumers buy in period 1 (v_1 rises), by inspecting equations (11), (12) and (13) it is immediate to see that the period 2 and 3 prices, p_2 and p_3, as well as the valuations of the marginal consumers, v_2 and v_3 will rise with the perceived discount factor δ'. Intuitively, there are more potential buyers in the subgame that starts in period 2 and this allows the monopolist to raise prices in the last two periods.

To summarize, the key points of the preceding discussion have been that time inconsistency hurts monopoly profits and leads to a flatter decline in the path of prices over time, relative to the time consistent case.

4.2 Naive Backwards Induction

Next we explore the effect of time inconsistency using the solution concept of naive backwards induction.

Recall that in a NBI outcome players play a best response to what they think other players will do. In the context of our pricing game, this means that consumers time their purchases based on the current price and the prices that they anticipate for the future. Consumers think that in the future they will be more patient than they will actually be and they also think that the monopolist thinks that they will be more patient. Therefore, the prices that the consumers anticipate will not coincide with the prices that the monopolist will actually charge.

To see how the game will unfold, let players play according to NBI. In period 1 there will be a marginal consumer who is indifferent between buying in period 1 at price p_1 or in period 2 at the anticipated period 2 price.[9] To calculate this anticipated future price for period 2 consumers will have to reason about the subgame that begins in period 2 with v_1 as the state of the market. On one

[9] An argument identical to the one we used for the time consistent case establishes that the marginal consumer in period 1 is indifferent between buying in periods 1 or 2, rather than between periods 1 and 3.

hand, being good game theorists, consumers will realize that the subgame will be played according to equations (11)-(13). On the other hand, being naive about their time inconsistent, consumers will incorrectly set the true discount factor δ in equations (11)-(13) equal to the perceived discount factor $\delta^{'}$. Therefore, consumers anticipate that the price in period 2 will be given by $p_2(v_1, \delta^{'})$ as in equation (14), whereas the actual price will be determined by $p_2(v_1, \delta)$ as in equation (11).

$$p_2(v_1, \delta^{'}) = \frac{v_1}{2} \frac{(2 - \delta^{'})^2}{(4 - 2\delta' - \gamma)} \tag{14}$$

In period 1 the monopolist solves

$$\max_{p_1, v_1} (x - v_1) p_1 + \gamma \Pi_2(v_1) \tag{15}$$

$$\text{subject to } v_1 - p_1 = \delta(v_1 - p_2(v_1, \delta^{'})) \tag{16}$$

Equation (16) says that the marginal consumer is indifferent between buying in period 1 at the price p_1 or buying in period 2 at the *anticipated* price $p_2(v_1, \delta^{'})$. Being naive about her time inconsistency, the marginal consumer does not realize that the price she anticipates will not correspond to the actual price that the monopolist will charge in period 2. As before, $\Pi_2(v_1)$ denotes the continuation profit in the subgame that begin in period 2 when the state of the market is v_1. Also, $\Pi_1(x)$ will denote the value of the period 1 objective function at the maximum.

The price p_1 and the marginal consumer v_1 that solve the NBI problem will be different from the equilibrium ones, that we obtained solving (9) subject to (10). Nevertheless, we obtain the same qualitative results for the effect of time inconsistency on the monopoly profits and the path of prices.

Proposition 5. Under naive backwards induction, as the degree of time inconsistency rises, monopoly profits and the first period price fall ($\frac{d\Pi_1(\delta^{'})}{d\delta'} < 0$ and $\frac{dp_1(\delta^{'})}{d\delta'} < 0$). Moreover, the valuation of the marginal consumer in period 1 rises ($\frac{dv_1(\delta^{'})}{d\delta'} > 0$), and as a result fewer consumers buy in period 1.

The intuition for these results, however, is different than before. Let v_1^L be the valuation of the marginal consumer under NBI when the degree of time inconsistency, $\delta^{'}$, is at some relatively low level. Assume now that the degree of time inconsistency, $\delta^{'}$, rises. Straightforward differentiation of equation (11) with respect to the discount factor δ shows that as the discount factor rises the second period price falls ($\frac{dp_2(v_1)}{d\delta} < 0$). Therefore, as the perceived discount factor

δ' increases, consumers think that the second period price will be lower. As a result, consumers with valuations slightly higher than v_1 will no longer want to buy in period 1 and they would rather wait until period 2. In order to induce some of them to buy now, the monopolist has to lower the price in period 1. In other words, under NBI time inconsistency hurts the monopolist because it leads consumers to anticipate a lower price in the second period. Contrast this to our story when we solved for the equilibrium. There, time inconsistency hurts the monopolist because consumers erroneously think that they will be patient in period 2 and hence some of them think it is worthwhile to wait until period 3.

Another difference between equilibrium and NBI is that the time inconsistency lowers the equilibrium monopoly profits and the equilibrium price in period 1 *only as long as* $\delta' > \frac{4-2\gamma-\delta^2}{6-2\gamma-3\delta}$, i.e. when the degree of time inconsistency is high enough. For example, if the discount factors for the monopolist and the consumers are equal to 0.9, as long as the perceived discount factor stays below 0.9267, the equilibrium of the time inconsistent case is unchanged and coincides with the SPNE of the time consistent game. With naive backwards induction, the outcome of the game will be different from the time consistent one, even for the slightest degree of time inconsistency.

To conclude the description of NBI, we need to see what happens in the subgame that starts in period 2. When there are only two periods to go, time inconsistency will not come into play and the subgame will unfold according to equations (11)-(13). As time inconsistency becomes more pronounced, fewer consumers buy in period 1 (v_1 increases) and this enables the monopolist to raise prices in periods 2 and 3.

To summarize, with NBI time inconsistency lowers monopoly profits and the price in period 1. It increases period 2 and 3 prices, p_2 and p_3, and the valuations of the marginal consumers, v_2, v_3. Consequently, the path of prices is flatter, relative to the time consistent case.

An alternative NBI

So far, we assumed that it is common knowledge that the monopolist is time consistent. A possible objection to this assumption is that, if consumers erroneously think that they, themselves, will be more patient in the future, they may also erroneously think that the monopolist will also be more patient in the future. In an equilibrium, it will not matter whether the consumers realize that the monopolist is time inconsistent or not. However, with naive backwards induction it will matter.

To see this, assume that it is common knowledge that the monopolist and the consumers are, *and will always be*, equally patient. Let the true discount factor for both be $\delta = \gamma$. If consumers erroneously think that in period 2 they will be more patient, they may also erroneously think that the monopolist will also be more patient. If this is the case, in period 1 consumers will anticipate that the price in period 2 will be given by $p_2(v_1, \delta^{'}, \delta^{'})$ as in equation (17) below

$$p_2(v_1, \delta^{'}, \delta^{'}) = \frac{v_1}{2} \frac{(2-\delta^{'})^2}{(4-2\delta^{'}-\delta^{'})} = \frac{v_1}{2} \frac{(2-\delta^{'})^2}{(4-3\delta^{'})} \tag{17}$$

whereas the true period 2 price will be determined by the function $p_2(v_1, \delta, \gamma)$ as in (11).[10] The difference arises, because now consumers think that both they *and* the monopolist will be more patient in the future. Consequently, the first period price, p_1, and the valuation of the marginal consumer, v_1, under NBI will now solve (15) subject to

$$v_1 - p_1 = \delta(v_1 - p_2(v_1, \delta^{'}, \delta^{'})) \tag{18}$$

Equation $p_2(v_1, \delta^{'}, \delta^{'}) = \frac{v_1}{2} \frac{(2-\delta^{'})^2}{(4-3\delta^{'})}$ is a convex parabola that attains a minimum at $\delta^{'} = \frac{2}{3}$. This means that if we restrict attention to the case where both the discount factors δ and $\delta^{'}$ are greater than $\frac{2}{3}$, an increase in the degree of time inconsistency will *increase* the period 2 price that consumers anticipate. As a result, in period 1 the monopolist can get away by charging a higher price which increases his profit. This leads to the following result.

Proposition 6. Suppose that the price that consumers anticipate for period 2 is increasing in the degree of time inconsistency. Then, under naive backwards induction, as the degree of time inconsistency rises, monopoly profits and the first period price increase ($\frac{d\Pi_1(\delta^{'})}{d\delta^{'}} > 0$ *and* $\frac{dp_1(\delta^{'})}{d\delta^{'}} > 0$*). Moreover, the valuation of the marginal consumer in period 1 falls (*$\frac{dv_1(\delta^{'})}{d\delta^{'}} < 0$*), and as a result more consumers buy in period 1.*

This is in contrast to what we saw earlier, where time inconsistency lowers the price that consumers anticipate for period 2 and this leads the monopolist to lower his period 1 price. Therefore, under NBI, the effect of time inconsistency on monopoly profits and the path of prices will depend on how time inconsistency affects the price that consumers anticipate for period 2. When time inconsistency causes consumers to anticipate a lower price in period 2, monopoly profits fall and the path of prices is flatter. In the opposite case, monopoly profits rise and the path of prices is steeper, relative to the time consistent case.

[10]Recall that in our original NBI the price that consumers anticipate is given by $p_2(v_1, \delta^{'}, \gamma)$.

4.3 Consumer Welfare

We now explore the effect of time inconsistency on consumer welfare.

In figure 2 we compare consumer welfare with and without time inconsistency. The bottom axis shows the valuations of the marginal consumers in the SPNE of the time consistent case, when the discount factor of the consumers and the monopolist are equal to 0.9, i.e. $\delta = \gamma = 0.9$. The upper axis shows the valuations of the marginal consumers under NBI for the time inconsistent case where $\delta = \gamma = 0.9$ and $\delta' = 0.99$. As predicted by our earlier results, these valuations are higher for the time inconsistent case. Had we solved for the equilibrium of the game instead, these marginal valuations would have been different, but they would still be greater than the time consistent ones. Also, notice that the first period price is lower with time inconsistency. The opposite is true for the prices in periods 2 and 3.

Figure 2 also depicts 7 regions (rectangles). Counting from left to right we will refer to them as regions $1, 2, ..., 7$. Region 6, for example, contains these consumers who buy in period 1 in the time consistent case, but they buy in period 2 in the time inconsistent case. Region 7 contains consumers who buy in period 1 in both cases. And so on, for all other regions.

Evaluating the change in the welfare of consumers in regions 7, 5, 3, 2 and 1 is straightforward. Consumers in region 7, with or without time inconsistency, buy in period 1 and they are better off in the time inconsistent case, because they pay a lower price. Consumers in region 5 (resp. 3) always buy in period 2 (resp. 3) and time inconsistency makes them worse off, because they pay a higher price in period 2 (3 resp.). Consumers in region 2 are worse off with time inconsistency, because they no longer make a purchase. Finally, the lowest valuation consumers in region 1, never make a purchase and the presence of time inconsistency leaves them indifferent.

Furthermore, in the time consistent case, consumers in region 6 prefer to buy in period 1 than in period 2 at price p_2. With time inconsistency, these consumers buy in period 2 at some higher period 2 price. Therefore, time inconsistency leaves them worse off. A similar argument shows that time inconsistency also reduces the welfare of consumers in region 4.

We can conclude that time inconsistency reduces the welfare of all consumers, except for the highest valuation ones. Moreover, in light of proposition 1, which shows that the time consistent case is equivalent to the sophisticated time inconsistent case, we can rephrase our conclusion as follows: *consumers' unawareness of their time inconsistency reduces the welfare of all, but the highest valuation, consumers.*[11]

[11]Earlier, we saw that time inconsistency also reduces the monopoly profits. It would be

4.4 The T-period model

We conclude our description of the pricing game by looking at the general T-period model. As we shall see, inductive versions of our previous arguments generalize most of our earlier results.

We now assume that in every period t the discount factor between periods t and $t+1$ is δ. Furthermore, in every period t consumers think that from period $t+1$ onwards their discount factor will be $\delta^{'} > \delta$, which means that consumers erroneously think that they will be more patient in the future. Consequently, the discount function from period t onwards evolves as $\{1, \delta, \delta\delta^{'}, \delta\delta^{'2}, ...\}$.

Equilibrium in the T-period model

We first describe the equilibrium of the T-period model. Recall that in an equilibrium it is "as if" the monopolist announces the complete path of prices in period 1 and the consumers make their purchasing decisions taking these prices as given. One candidate equilibrium is for the monopolist to announce the same prices that he would charge in the SPNE of the time consistent case with discount factor δ. Recall that in the SPNE of the time consistent game, in each period T there is a marginal consumer with valuation v_t who is indifferent between buying in periods t or $t+1$. Moreover, time inconsistency guarantees that this marginal consumer will strictly prefer to buy in periods t or $t+1$ than buy in some future period, say $t+2$. In period t, if a consumer is indifferent between buying in periods t or $t+1$ in the time consistent case, she would also be indifferent in our candidate equilibrium with time inconsistency. However, with time inconsistency there is no guarantee that this consumer prefers to buy in periods t (or $t+1$) than wait and buy in some future period $t+1+k$ and our candidate equilibrium may break down. For example, in the extreme case where the perceived discount factor is equal to 1 (i.e. consumers think that in the future they will be infinitely patient) a consumer who is indifferent between buying in periods t and $t+1$, will strictly prefer to wait until the last period T, when the price will be at its lowest level.

Therefore, in an equilibrium the valuation of the marginal consumer in period t may be defined by an indifference condition between periods t and any future period $t+k$, depending on the degree of time inconsistency. The complexity of the T-period problem is such that it is not possible to find conditions similar to the one in proposition 2, that would tell us which indifference condition defines the marginal consumers in all periods. However, we can generalize our previous qualitative results from the 3-period model.

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wrong to deduce that this implies that the overall consumer welfare increases, because the game between the monopolist and the consumers is not zero sum.

Proposition 7. In the T-period model the equilibrium monopoly profits and the first period price (weakly) fall as the degree of time inconsistency rises; that is $\frac{d\Pi_1(\delta')}{d\delta'} \leq 0$ and $\frac{dp_1(\delta')}{d\delta'} \leq 0$. Moreover, the equilibrium valuation of the marginal consumer in period 1 (weakly) rises as the degree of time inconsistency rises, i.e. $\frac{dv_1(\delta')}{d\delta'} \geq 0$, and as a result fewer consumers buy in period 1.

The qualifier "weakly" is needed because for low levels of time inconsistency, the equilibrium coincides with the SPNE of the time consistent game. As in the 3-period model, consumers think that they will be more patient in the future and this hurts the monopolist in the same way that that an increase in the true discount factor δ hurts the monopolist in the time consistent case.

What about the effect of time inconsistency on prices and the valuations of marginal consumers in the subsequent subgames? There are two effects. First, since fewer people buy in period 1, the state of the market is higher in the subsequent subgame and this will increase both the price and the valuation of the marginal consumer in period 2. Second, keeping the state of the market fixed, proposition 7 implies, by induction, that in the subgame that starts in period 2, time inconsistency will lower the second period price and increase the valuation of the period 2 marginal consumer. As far as the valuation of the marginal consumer, both of these effects work in the same direction and the valuation of the period 2 marginal consumer increases as time inconsistency becomes more pronounced. As far as the second period price, these effects work in opposite directions and the net effect is ambiguous. Inductively, it is immediate to see that the valuations of the marginal consumer in all periods will increase with time inconsistency. The effect of time inconsistency on the path of prices between periods 2 and $T-2$ is ambiguous. It is possible, however, to predict the effect of time inconsistency for the price in the last two periods. In the subgame that starts in the penultimate $T-1$ period, there are only two periods left and time inconsistency does not matter anymore. As a result, the second of the effects we described is absent and both the $T-1$ and T period prices will necessarily increase with time inconsistency as in the 3-period model.

NBI in the T-period model

In the NBI case, we know that in each period t the marginal consumer is indifferent between buying in periods t at the current price or in period $t+1$ at the price she anticipates for that period. Inductively, we can write the monopolist's problem in period t as

$$\max_{p_t, v_t} (v_{t-1} - v_t)p_t + \gamma \Pi_{t+1}(v_t) \tag{19}$$

$$\text{subject to } v_t - p_t = \delta(v_t - p^*_{t+1}) \tag{20}$$

where p^*_{t+1} is the price that consumers anticipate for period $t+1$. Notice that in the T-period model the true price in period $t+1$ will be a function of the state of the market v_t, the true discount factor δ, the perceived discount factor $\delta^{'}$ and the discount factor of the monopolist γ. We can thus write the true period $t+1$ price as $p_{t+1}(v_t, \delta, \delta^{'}, \gamma)$. Being good game theorists, consumers will correctly calculate this function $p_{t+1}(v_t, \delta, \delta^{'}, \gamma)$, but because they are time inconsistent they will set the true and the perceived discount factors equal. Then, in period t the price that consumers will anticipate for period $t+1$ is $p^*_{t+1} = p_{t+1}(v_t, \delta^{'}, \delta^{'}, \gamma)$.[12] The discrepancy between the true and the anticipated price arises, because in period t consumers think that the constraint of the monopolist's problem in period $t+1$ will be

$$v_{t+1} - p_{t+1} = \delta^{'}(v_{t+1} - p_{t+2}(v_{t+1}, \delta^{'}, \delta^{'}, \gamma)) \tag{21}$$

rather than the correct one given by

$$v_{t+1} - p_{t+1} = \delta(v_{t+1} - p_{t+2}(v_{t+1}, \delta^{'}, \delta^{'}, \gamma))$$

Using the same inductive arguments as in the 3-period model, one can prove the following result for the effect of time inconsistency on profits and the path of prices.

Proposition 8. Consider the t-period model under naive backwards induction. As the degree of time inconsistency rises, monopoly profits and the first period price fall ($\frac{d\Pi_1(\delta^{'})}{d\delta^{'}} < 0$ and $\frac{dp_1(\delta^{'})}{d\delta^{'}} < 0$). Moreover, the valuation of the marginal consumer in period 1 rises ($\frac{dv_1(\delta^{'})}{d\delta^{'}} > 0$), and as a result fewer consumers buy in period 1.

An argument identical to the one we presented earlier, during our discussion for the equilibrium of the T-period model, shows that in the subgames that follow time inconsistency will increase the valuations of all marginal consumers. The effect of time inconsistency on future prices is ambiguous, with the exception of the prices in the last two periods, which unambiguously increase as the degree of time inconsistency rises.

Finally, with NBI we can use the recursive structure of the problem to obtain a closed form solution for the NBI prices and valuations of marginal consumers.

[12] Alternatively, in a NBI outcome we can assume that consumers erroneously think that the monopolist will also be time inconsistent. If, for example, it is common knowledge that consumers and the monopolist are (and will always be) equally patient, in period t consumers anticipate that the price in period $t+1$ will equal $p_{t+1}(v_t, \delta^{'}, \delta^{'}, \delta^{'})$. In this case, the results in proposition 8 are reversed, as long as $\frac{\partial p_{t+1}(\delta^{'})}{\partial \delta^{'}} > 0$.

Proposition 9. Under naive backwards induction, the sequence of prices, $\{p_t\}_{t=1}^T$, *and the sequence of valuations of marginal consumers,* $\{v_t\}_{t=1}^T$, *evolve as follows.*

$$p_t = v_{t-1} A_t \tag{22}$$

$$v_t = v_{t-1} B_t \tag{23}$$

$$A_t = \frac{\left(1 - \delta + \delta A_{t+1}(\delta', \delta')\right)^2}{2 - 2\delta + 2\delta A_{t+1}(\delta', \delta') - \gamma A_{t+1}(\delta, \delta')} \tag{24}$$

$$B_t = \frac{1 - \delta + \delta A_{t+1}(\delta', \delta')}{2 - 2\delta + 2\delta A_{t+1}(\delta', \delta') - \gamma A_{t+1}(\delta, \delta')} \tag{25}$$

$$A_T = \frac{1}{2} \tag{26}$$

$$B_T = \frac{1}{2} \tag{27}$$

In figure 3 we plot the path of prices with time consistent and time inconsistent consumers, when there are $T = 10$ periods and the discount factors are $\delta = \gamma = 0.9, \delta' = 0.99$. The prices in the first two periods are higher when consumers are time consistent (or equivalently sophisticated-time-inconsistent). However, prices in the subsequent periods are higher when consumers are naive-time-inconsistent. In both cases, prices fall to marginal cost (as the Coase conjecture predicts), but the price path is flatter with naive-time-inconsistent consumers.

5 Equilibrium versus NBI in games with naive-time-inconsistent players

In this section, we discuss the relative advantages and disadvantages of the *equilibrium* and *naive backwards induction*.

As we have seen, the two notions of equilibrium and NBI are the natural extensions of the SPNE and backwards induction, respectively, when some players are time inconsistent. It is well known that in finite games of perfect information the SPNE and backwards induction are equivalent. However, from a methodological point of view, these solution concepts are very different. Recall that in a Nash equilibrium each player is endowed with some beliefs about how others will play the game. Each player takes these beliefs as given, without questioning how and why others have decided to play as she expects them to. Then, each player chooses an action that maximizes her expected payoff and in a Nash equilibrium the players' original beliefs are confirmed by others' actions. The SPNE is a refinement of the Nash equilibrium that guarantees that in a dynamic game equilibrium actions are also sequentially rational. With backwards induction players still have some beliefs about how others will play the game, but each player arrives at these beliefs by putting herself in the shoes of other players who put themselves in the shoes of other players and so on.

We have shown that with naive-time-inconsistent players the equivalence between equilibrium and naive backwards induction breaks down. To understand why, notice that when players are naive-time-inconsistent, they make two types of mistakes. First, fixing some beliefs about how others play, time inconsistent players cannot correctly anticipate their own future actions. Second, if a time inconsistent player erroneously thinks that in the future she will take action x, when in fact she will take action y, she will also erroneously expect that others will play a best response to action x, when in fact they will play a best response to action y. Therefore, time inconsistent players cannot correctly anticipate other players' future actions as well. The solution concept of equilibrium renders this second type of mistake (i.e. incorrect predictions of others' actions) irrelevant, because in an equilibrium beliefs about how others will play are exogenous. With naive backwards induction, beliefs about how others will play are determined by introspection and this leaves players vulnerable to the second type of mistake (i.e. not correctly anticipate others' actions).

A desirable property for a solution concept is that players should be able to defend their choices with an argument of the form: "I play x, because he will play y, because I will play z,...". The game theoretic literature refers to strategies that can be defended in this way as *rationalizable strategies*. The strategies that survive NBI are indeed rationalizable and this is an attractive feature of NBI as a solution concept. The *equilibrium* strategies, however, are not rationalizable. In our pricing problem, for example, if we were to ask consumers to reason about why the monopolist is charging these particular equilibrium prices, they would not be able to come up with a consistent story.

Another desirable property for a solution concept, however, is that players should also be able to defend their choices with a different argument of the form: "I play x, because I think he plays y. In fact, every time he moves he plays according to y, so my theory that he plays according to y holds water." This property is the standard Nash equilibrium requirement that players' beliefs are confirmed by the equilibrium actions. Strategies that survive NBI do not satisfy this property, because players' theories about what other players will do are discredited by other players' actions. By, definition, the equilibrium strategies, satisfy this property.[13]

[13] Notice that under both solution concepts, as the game evolves over time, the time inconsistent players discover that they do not play as they planned they would. This raises the issue

We believe that choosing between the equilibrium and NBI, as the most appropriate solution concept for games with time inconsistent players, should depend on the particular application we have in mind. In our pricing game for example, our choice of one over the other solution concept should depend on what we think is the most realistic way to model how consumers form expectations about future prices. Suppose that a new electronic gadget goes on the market. Do boundedly rational consumers (i.e. real life consumers) form expectations about future prices by putting themselves in the shoes of the seller? Probably not. Or, do consumers simply use their past experiences from how this seller, or similar ones, priced in the past? We believe that the second scenario is more realistic and, as a result, the equilibrium should be a better predictive tool. In other instances NBI may be more appropriate. For example, consider a bargaining game or a Cournot duopoly between two naive-time-inconsistent firms. In this case, it may be more realistic to assume that players decide how to play by putting themselves in the shoes of the other player. In fact, if there are no precedents, players may have no other way to form beliefs about how others will play and using NBI may be more appropriate.

6 Conclusion

We solved the inter-temporal pricing problem of a durable good monopolist facing time inconsistent consumers. In particular, we have shown that as the degree of time inconsistency rises, monopoly profits fall, prices fall to marginal cost following a flatter path and consumer welfare falls for all but the highest valuation consumers.

Before arriving at these conclusions, it was necessary to propose two new solution concepts for games with time inconsistent players. We have chosen the terminology of *equilibrium* and *naive backwards induction* to highlight the fact that these concepts are closely related to the SPNE and backwards induction. Unlike in standard finite games of perfect information, these solution concept are not equivalent. Nevertheless, in our price discriminating game they predict similar qualitative results for profits, consumer welfare and the path of prices.

We hope that the above results contribute to the economic literature in two ways. First, the existing literature has addressed the issue of time inconsistency only in single-agent decision problems. The price discrimination problem that our paper addresses is the first attempt to introduce time inconsistency in a non-cooperative game. Second, even though bounded rationality has been embraced

that players may eventually start questioning what they take for granted in the game and they may eventually become aware of their time inconsistency. We leave this as an open question for future research.

by various economics sub-fields, it was not until very recently that it found its way into industrial organization. The present paper complements the recent contributions of Della Vigna and Malmandier (2003) and Gabaix and Laibson (2003) in spelling out some implications of consumer bounded rationality for rational firms.

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7 Appendix

Proof of proposition 2. Consider the 3-period model with time consistent consumers. The seller solves,

$$\max_{p_1,v_1}(x - v_1)p_1 + \gamma\Pi_2(v_1) \tag{28}$$

$$\text{subject to } v_1 - p_1 = \delta(v_1 - p_2(v_1)) \tag{29}$$

Substitute in $\Pi_2(v_1) = \frac{v_1}{2}p_2(v_1)$ and $p_2(v_1) = \frac{v_1}{2}\frac{(2-\delta)^2}{(4-2\delta-\gamma)}$. Straightforward differentiation shows that the optimal first period price, p_1 and the valuation of the first period marginal consumer, v_1, are given by

$$v_1 = \frac{8 - 8\delta - 2\gamma + 2\delta\gamma + \delta^3}{16 - 16\delta - 8\gamma + 8\delta\gamma + 2\delta^3 - \gamma\delta^2}$$

$$p_1 = \frac{1}{2}\frac{(8 - 8\delta - 2\gamma + 2\delta\gamma + \delta^3)^2}{(4 - 2\delta - \gamma)(16 - 16\delta - 8\gamma + 8\delta\gamma + 2\delta^3 - \delta^2\gamma)}$$

Moreover, using equations (11)-(13), we can express all subsequent period prices, p_2 and p_3, as functions of v_1. To verify that this solution is an equilibrium for the game with time inconsistent consumers and parameters δ and δ', we need to check that the consumer with valuation v_1 prefers to buy in period 1 than in period 3. This is true as long as $v_1 - p_1 \geq \delta\delta'(v_1 - p_3(v_1))$. Tedious algebra shows that this holds as long as $\delta' \leq \frac{4-2\gamma-\delta^2}{6-2\gamma-3\delta}$.

Assume now that the monopolist maximizes (9) subject to (10). Then, the solution is given by

$$v_1 = \frac{8 - 4\delta - 2\gamma - 6\delta\delta' + 3\delta^2\delta' + 2\delta\delta'\gamma}{16 - 8\delta - 8\gamma - 12\delta\delta' + 6\delta^2\delta' + 4\delta\delta'\gamma + 4\delta\gamma - \gamma\delta^2}$$

$$p_1 = \frac{1}{2}\frac{\left(8 - 4\delta - 2\gamma - 6\delta\delta' + 3\delta^2\delta' + 2\delta\delta'\gamma\right)^2}{(16 - 8\delta - 8\gamma - 12\delta\delta' + 6\delta^2\delta' + 4\delta\delta'\gamma + 4\delta\gamma - \gamma\delta^2)(4 - 2\delta - \gamma)}$$

Equations (11)-(13) allow us to express the equilibrium prices p_2, p_3 and the marginal consumer valuation v_2 as functions of v_1. To verify that this is indeed an equilibrium in the game with time inconsistent consumers, we need to check that in period 1 the consumer with valuation v_1 prefers to buy in period 1 (or 3) than in period 2. This is true as long as $v_1 - p_2 \leq \delta'(v_1 - p_3(v_1))$. Tedious algebra shows that this holds as long as $\delta' \geq \frac{4-2\gamma-\delta^2}{6-2\gamma-3\delta}$.

Proof of Proposition 3. Let $\delta_L^{'}$ and $\delta_H^{'}$ be two values of the perceived discount factor $\delta^{'}$, with $\delta_L^{'} < \delta_H^{'}$. Assume that the pair (v_1^L, p_1^L) solves (9) subject to (10) when the perceived discount factor is $\delta_L^{'}$. Similarly, assume that the pair (v_1^H, p_1^H) solves (9) subject to (10) when the perceived discount factor is $\delta_H^{'}$. Let p_1^{HL} be the first period price that would leave the consumer with valuation v_H indifferent between buying in period 1 or period 3 provided that a) the perceived discount factor is $\delta_L^{'}$ and b) from period 2 onwards the monopolist will price optimally. Notice that in the 3-period model the continuation profit $\Pi_2(v_1)$ is not a function of $\delta^{'}$. Then, we can write

$$(x - v_1^L)p_1^L + \gamma\Pi_2(v_1^L) > (x - v_1^H)p_1^{HL} + \gamma\Pi_2(v_1^H) > (x - v_1^H)p_1^H + \gamma\Pi_2(v_1^H)$$

Thus, profit is higher when the perceived discount factor is lower. The first inequality holds by revealed preference. The second inequality holds because $p_1^{HL} > p_1^H$ (when the perceived discount factor is higher you have to charge a lower first period price to make a consumer with a particular valuation indifferent between buying in periods 1 and 3).

A useful observation

For the rest of the proofs it will be useful to make the following observation. Consider the maximization problem

$$\max_{v,p}(x - v)p + \gamma\frac{v^2}{2}c_2 \tag{30}$$

$$\text{subject to } (v - p) = \delta\delta^{'}(v - vc_1) \tag{31}$$

where c_1 and c_2 are some positive constants less than 1, that could depend on the parameters δ and $\delta^{'}$. Let $\Pi(x)$ be the value of the objective function at the maximum. Then, straightforward differentiation establishes the following results for the optimal choices of the variables p and v .

$$p(x) = xp(1) \tag{32}$$

$$v(x) = xv(1) \tag{33}$$

$$\Pi(x) = \frac{1}{2}xp(x) = \frac{1}{2}x^2p(1) \tag{34}$$

Equation (34) is particularly useful because it implies that in our problem the first period price is increasing in a particular parameter if and only if the overall monopoly profits are increasing in the same parameter.

Proof of proposition 4. Equation (34) establishes that $\frac{dp_1(\delta')}{d\delta'}$ has the same sign as $\frac{d\Pi_1(\delta')}{d\delta'}$, and by our previous result, both are negative.

To see how the variable v_1 changes with the perceived discount factor δ', consider the monopolist's maximization problem in period 1.

$$\max_{v}(x - v_1)p_1 + \gamma\frac{v_1^2}{2}p_2(1)$$
$$\text{subject to } v - p_1 = \delta\delta'(v - vp_3(1))$$

Notice how we used equations (32)-(34) to substitute for $\Pi_2(v_1)$ and $p_3(v_1)$. The optimal choice for the variable v_1 satisfies $v_1(\delta') = x\frac{1-\delta\delta'+\delta\delta' p_3}{2-2\delta\delta'+2\delta\delta' p_3-\gamma p_2}$. Differentiating with respect to the perceived discount factor $\delta\prime$ yields

$$\frac{dv_1(\delta')}{d\delta'} = -x\delta\,(-1 + p_3)\,\gamma\frac{p_2}{(2 - 2\delta\delta' + 2\delta\delta' p_3 - \gamma p_2)^2} > 0$$

Proof of proposition 5. We mimic the proof for proposition 3 to show that profits fall as the degree of time inconsistency rises. Let δ'_L and δ'_H be two values of the perceived discount factor δ', with $\delta'_L < \delta'_H$. Assume that the pair (v_1^L, p_1^L) solves (15) subject to (16) when the perceived discount factor is δ'_L. Similarly, assume that the pair (v_1^H, p_1^H) solves (15) subject to (16) when the perceived discount factor is δ'_H. Let p_1^{HL} be the first period price that would leave the consumer with valuation v_H indifferent between buying in period 1 (at price p_1^{HL}) or period 2 (at the incorrect price[14] $p_2(v_H, \delta'_L)$) provided that a) the perceived discount factor is δ'_L and b) from period 2 onwards the monopolist will price optimally. Notice that in the 3-period model the continuation profit $\Pi_2(v_1)$ is not a function of δ'. Then, we can write

$$(x - v_1^L)p_1^L + \gamma\Pi_2(v_1^L) > (x - v_1^H)p_1^{HL} + \gamma\Pi_2(v_1^H) > (x - v_1^H)p_1^H + \gamma\Pi_2(v_1^H)$$

Thus, profit is higher when the perceived discount factor is lower. The first inequality holds by revealed preference. The second inequality holds because $p_1^{HL} > p_1^H$ (when the perceived discount factor is higher, consumers think that the second period price will be lower and you have to charge a lower first period price to make a consumer with a particular valuation indifferent between buying in periods 1 and 2).

[14]Recall that the correct price is $p_2(v_H, \delta)$.

Since profits fall as the degree of time inconsistency rises, so does the first period price, by equation (34). To see how the variable v_1 changes with the perceived discount factor $\delta^{'}$, consider the monopolist's maximization problem in period 1.

$$\max_{v}(x - v_1)p_1 + \gamma\frac{v_1^2}{2}p_2(1, \delta)$$
$$\text{subject to } v - p_1 = \delta(v - vp_2(1, \delta^{'}))$$

Notice how we used equations (32)-(34) to substitute in for $\Pi_2(v_1)$ and $p_2(v_1, \delta^{'})$. Straightforward differentiation shows that the optimal choice for the variable v_1 satisfies $v_1(\delta) = x\frac{1-\delta+\delta p_2(1,\delta)}{2-2\delta+2\delta p_2\left(1,\delta^{'}\right)-\gamma p_2(1,\delta)}$. It is immediate to see that $\frac{\partial v_1(\delta^{'})}{\partial \delta^{'}} > 0$ because $\frac{\partial p_2\left(1,\delta^{'}\right)}{\partial \delta^{'}} < 0$.

The proof of proposition 6 is similar and is omitted. (use reverse arguments)

Proof of proposition 8. Let $\delta^{'}_L$ and $\delta^{'}_H$ be two values of the perceived discount factor $\delta^{'}$, with $\delta^{'}_L < \delta^{'}_H$. Assume that the pair (v_1^L, p_1^L) solves the objective function in (19) subject to (20) when the perceived discount factor is $\delta^{'}_L$ (and when $t = 1, v_0 = x$). Similarly, assume that the pair (v_1^H, p_1^H) solves (19) subject to (20) when the perceived discount factor is $\delta^{'}_H$. Let p_1^{HL} be the first period price that would leave the consumer with valuation v_H indifferent between buying in period 1 (at price p_1^{HL}) or period 2 (at the incorrect price $p_2(v_H, \delta^{'}_L)$) provided that a) the perceived discount factor is $\delta^{'}_L$ and b) from period 2 onwards the monopolist will price optimally.

Unlike in the 3-period model, now the function $\Pi_2(v_1)$ is also a function of $\delta^{'}$. We know, however, that $\Pi_2(v_1)$ is decreasing in the perceived discount factor $\delta^{'}$ when $T = 3$. By extension, we also know that the the first period price, p_1, is also decreasing in the perceived discount factor $\delta^{'}$ when $T = 3$ (by equation (34)). Assume that the same is true for all T up to K. The following succession of inequalities shows that the result is true for $T = K + 1$.

$$(x - v_1^L)p_1^L + \gamma\Pi_2(v_1^L, \delta^{'}_L) > (x - v_1^H)p_1^{HL} + \gamma\Pi_2(v_1^H, \delta^{'}_L) > (x - v_1^H)p_1^H + \gamma\Pi_2(v_1^H, \delta^{'}_H)$$

The first inequality holds by revealed preference. The second inequality holds because $p_1^{HL} > p_1^H$ and because $\Pi_2(v_1^H, \delta^{'}_L) > \Pi_2(v_1^H, \delta^{'}_H)$. The latter statement $(\Pi_2(v_1^H, \delta^{'}_L) > \Pi_2(v_1^H, \delta^{'}_H))$ holds by induction. The former statement $(p_1^{HL} > p_1^H)$ holds if and only if the second period price that consumers anticipate decreases when the perceived discount factor increases. This is also true by our induction hypothesis. These arguments show that profits (and by extension the first period price as well) decrease as the perceived discount factor increases.

The proof of proposition 7 is similar and we omit it.

Proof of proposition 9. Consider the maximization problem

$$(x-v)p+\gamma\frac{v^2}{2}c(\delta,\delta^{'}) \tag{35}$$

$$\text{subject to } v-p=\delta(v-vc(\delta^{'},\delta^{'})) \tag{36}$$

Differentiating with respect to the variables v and p yields

$$v = x\frac{1-\delta+\delta c\left(\delta^{'},\delta^{'}\right)}{2-2\delta+2\delta c\left(\delta',\delta'\right)-\gamma c\left(\delta,\delta'\right)}$$

$$p = x\frac{\left(1-\delta+\delta c\left(\delta^{'},\delta^{'}\right)\right)^2}{2-2\delta+2\delta c\left(\delta',\delta'\right)-\gamma c\left(\delta,\delta'\right)}$$

Under naive backwards induction, in each period t the monopolist solves the preceding maximization problem, where $v=v_t$, $p=p_t$ and $c(\delta,\delta^{'})=p_{t+1}(1,\delta,\delta^{'},\gamma)$. Notice that the price in period $t+1$ is given by the function $p_{t+1}(1,\delta,\delta^{'},\gamma)$, whereas consumers erroneously think that it is given by $p_{t+1}(1,\delta^{'},\delta^{'},\gamma)$. The result follows inductively, by inspection.

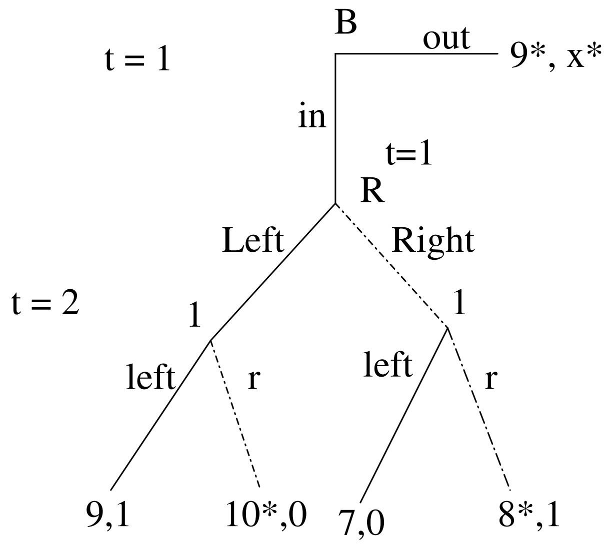

Figure 1. An extensive form game, where player B is naïve-time-inconsistent.

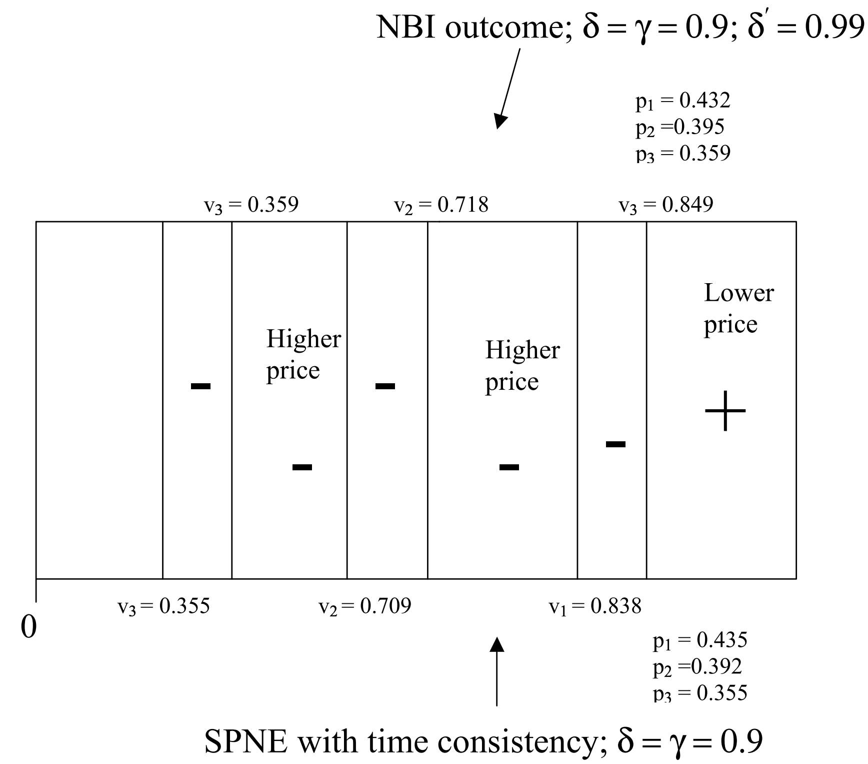

Figure 2. Comparison of consumer welfare with time consistency and with time inconsistency (under NBI).
$\delta = \gamma = 0.9; \delta' = 0.99$
Not drawn to scale

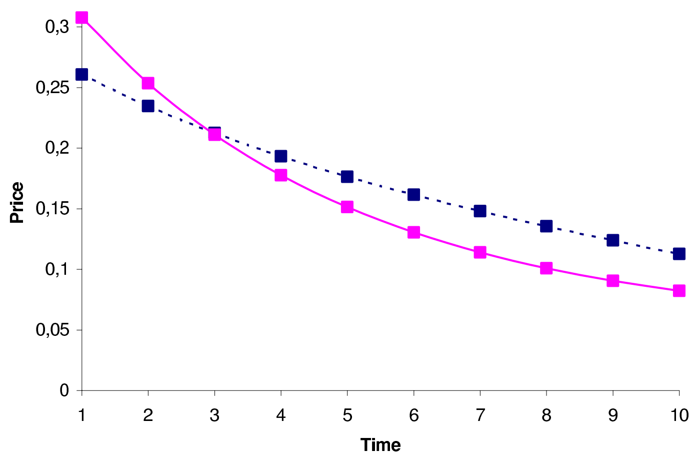

Figure 3. The path of prices with time consistent (solid path) and time inconsistent (dotted path) consumers.
$\delta = \gamma = 0.9$; $\delta' = 0.99$; $T = 10$