# Asymmetric Information, Stock Returns and Monetary Policy: A Theoretical and Empirical Analysis

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### Abstract

It is known that stock returns are affected by monetary policy. This paper theoretically and empirically investigates whether asymmetric information between the Federal Reserve and the public causes the relation between stock returns and monetary policy actions. The paper concludes that asymmetric information between the Federal Reserve and the public is one of the reasons of effects of monetary policy actions on stock returns. A Kalman filter algorithm is constructed to analyze the information and learning dynamics between the Federal Reserve and a representative investor. Stock prices react to monetary policy actions because monetary policy actions reveal the private information that the Fed has about future inflation and output. Investors update their expectations after observing the Fed's actions and that produces a change in stock returns. The findings of the model are empirically investigated using VAR and impulse responses verify the theoretical findings.

## 1 Introduction:

The relationship between the monetary policy and asset prices has been shown empirically and theoretically. Most of the theoretical papers focus on the effects of monetary policy on the collateral value of the firm or construct a CIA model to form a relation between monetary policy and asset returns. This paper takes a novel approach and uses the asymmetric information between the Federal Reserve and the public to produce a link between monetary policy and stock returns. Romer and Romer (2002) empirically show that the Federal Reserve

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has superior information about future inflation and output. Also, they show that commercial forecasters update their expectations about future inflation and output when they observe the actions of the Federal Reserve. Tas (2003) shows that the private information of the Federal Reserve can be used to predict future stock returns so the Federal Reserve (Fed) has valuable information that the investor would like to know. So, empirical results of Romer and Romer (2002) and Tas (2003) suggest that one of the reasons that monetary policy affects asset prices might be the information asymmetry between the Fed and the investors. Starting from those empirical findings this paper constructs the link between monetary policy and stock returns using the asymmetric information between the Fed and the public. The relationship is examined both theoretically and empirically.

In this paper, I follow a Kalman filtering algorithm as suggested by Townsend (1983) to analyze learning dynamics between Federal Reserve and a representative investor. The representative investor's expectations about future inflation and output are affected by the Federal Reserve's actions, since the forecasts of the Federal Reserve are not made public. Thus, the only way the public can access the private information that the Federal Reserve possess about future inflation and output is to observe the Federal Reserve's actions. The investors updates her expectations after observing the actions of the Fed. A simple onegood model similar to Lucas (1978) is constructed to analyze the effects of investor's learning (updating her expectations) about future inflation and output on asset prices. The MLE estimates of the coefficients of the Kalman filter are estimated by SUR and OLS than the impulse response functions are derived to analyze the dynamics of the model.

The theoretical analysis displays that the investors update their expectations after observing the Federal Reserve's actions and act according to their expectations. That change in the expectations of the investors cause the change in stock returns. Thus, the actions of the Federal Reserve effects stock returns by altering the investors' expectations. The empirical part of the paper can be divided into two parts. First, the parameters of the state-space representation of the Kalman filter is estimated using OLS and SUR. Second, the theoretical model is presented as VAR and impulse response functions are used to examine the dynamics of the model.

There are two main results of this paper. First, the asymmetric information between the Federal Reserve and the public is one of the reasons of the effects of the monetary policy on stock returns. Second, the investors (public) update their expectation about inflation and output after observing the actions of the Federal Reserve. These results are shown both theoretically and empirically.

This paper is making three major contributions to the literature. First, the paper provides theoretical and empirical evidence about the effects of the asymmetric information between the Fed and the public on the relation between stock returns and monetary policy which does not exist in the literature to our knowledge. Second, a theoretical model is constructed which verifies the empirical results of Romer and Romer (2002). The model shows that the public update their expectations after observing the Fed's actions which has been shown empirically in Romer and Romer (2002). Third, supporting arguments about the asymmetric information between the Fed and the public is presented from the financial markets.

The outline of this article as follows: Section 2 reviews the related literature and states contribution of the paper. Section 3 displays a simple Lucas type model about stock prices and the expectations of a representative investor. Section 4 presents the learning dynamics and the Kalman filter algorithm. Section 5 solves the model. Section 6 estimates the parameters of the information dynamics. Section 7 constructs the link between stock returns and the representative investors expectations. Section 8 displays the time series dynamics.Section 9 concludes.

# 2 Literature Review:

To be completed.

### 2.1 Contribution of the Paper:

The contribution of this paper can be analyzed from two perspectives, asset pricing and monetary policy. From a asset pricing point of view there are two contributions that the paper is making. First, this paper analyzes both theoretically and empirically the affects of monetary policy changes on asset prices to explain the empirical findings of Patelis (1997), Thorbecke (1997) and Tas (2003) by modeling the information asymmetry between the Fed and investor. The model and empirical findings of the paper show that the information asymmetry between the Fed and investor is one of the reasons of the effects of monetary policy actions of asset prices. Second, empirical evidence is provided that asset prices react to changes in both investor's and the Fed's expectations of inflation and output.

From a monetary policy point of view, this paper constructs a theoretical model to explain empirical findings of Romer and Romer (2002). The model concludes that investor changes his expectations after observing the Fed's actions. And the paper takes one more step and analyzes the effects of this information dynamics on asset prices.

# **3** A Simple Monetary Model:

### 3.1 Players:

### 3.1.1 Central Bank (Fed):

Central bank observes signals about current inflation  $(_{CB}\pi_t^*)$  and about current output  $(_{CB}x_t^*)$  and forms expectations about future inflation and output using its information set at time t  $(\Omega_t^{CB})$ . Then Central bank sets the federal funds target rate  $(r_t)$  using a simple forward looking rule.

$$r_t = \Psi(E\{\pi_t / \Omega_{t-1}^{CB}\}) + \gamma(E\{x_t / \Omega_{t-1}^{CB}\}) + a_{1CB}\pi_t^* + a_{2CB}x_t^* \qquad (1)$$

 $r_t =$  Federal funds target rate

 $\pi_t =$ True inflation at time t

 $x_t =$  True output at time t

 $\Omega_{t-1}^{CB} =$  Information set of the Central bank at time t-1.

 $_{CB}\pi_t^* =$  Signal received by Central bank about time t inflation.

$$_{CB}\pi_t^* = \pi_t + \epsilon_t^{CB} \tag{2}$$

 $_{CB}x_t^* =$  Signal received by Central bank about time t output.

$$_{CB}x_t^* = x_t + \theta_t^{CB} \tag{3}$$

Inflation and output are assumed to have AR(1) processes:

$$\pi_t = \rho \pi_{t-1} + \nu_t \tag{4}$$

$$x_t = \beta x_{t-1} + \zeta_t \tag{5}$$

 $\epsilon_t^{CB}, \theta_t^{CB}, \nu_t, \zeta_t$  are jointly normally distributed , independent among themselves and over time, with mean zero and covariances  $\sigma_{\epsilon}^2, \sigma_{\theta}^2, \sigma_{\nu}^2, \sigma_{\zeta}^2$ .

### 3.1.2 Investor:

Investor observes the federal funds rate target set by the Central bank and her signals about current inflation and output. Then she forms her own expectations about future inflation and output. Then she maximizes her lifetime expected utility subject to her budget constraint.

Investor consumes only one good. The consumption good enters the economy as an endogenous endowment stream  $Y_t$ . Following Lucas (1978), the equities are modeled as claims to the endowment. The total supply of equity shares is normalized to unity. The economy is an exchange economy. So, there is no money in the economy. The Fed's only action is to determine the federal funds target rate which effects borrowing.

• Preferences of the Investor:

1.

$$U(t) = U\{C(t)\}\tag{6}$$

The investor's expected lifetime utility is given by:

$$E_t \left\{ \sum_{i=0}^{\infty} \beta^i U\{C_{t+i}\} / \Omega_t^{inv} \right\}$$
(7)

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• Budget Constraint:

### Sources of Funds:

1. Borrowing  $(b_t)$ :

It is assumed that there is no borrowing constraints, but the investor should pay the amount she borrowed the previous period (t-1) at the current period(t). The amount borrowed is subject to the interest rate which is equal to the federal funds target rate  $(r_t)$  determined by the Fed.

2. Asset  $(z_t)$ :

Asset  $(z_t)$  is a claim to next periods good  $(Y_t)$  produced by the asset.

3. Output  $(Y_t)$ :

Asset produces  $(Y_t)$  at time t. Investor can consume that good or sell it at  $p_t^c$ .

### Uses of Funds:

- 1. Consumption  $C_t$ :
- 2. New Shares  $(z_t)$ :
- 3. Debt payment:  $(1 + r_{t-1})b_{t-1}$

So, the budget constraint is:

$$p_t^c c_t + p_t^z z_{t+1} + (1 + r_{t-1}) b_{t-1} \le z_t \left( p_t^c Y_t + p_t^z \right) + b_t \tag{8}$$

### 3.1.3 Time Table:

The time table of the model is as follows:

- 1. Central bank observes its signals about current inflation and output and it sets the federal funds target rate using a forward looking Taylor rule.
- 2. Investor observes Central banks actions (change in the federal funds target rate) and updates her expectations about future inflation and output.
- 3. Investor solves her lifetime utility maximization problem subject to her budget constraint and her expectations about future inflation and future output.

# 4 Information Dynamics Between Central Bank and Investor:

As, I mentioned before it is empirically shown that Fed has superior information about future inflation and output, commercial forecasters update their forecasts after they observe monetary policy actions (Romer and Romer (2002)) and that information is useful for the investor since she can use that information to predict stock returns (Tas (2002)).

Starting from these empirically findings, I will construct a learning model using Kalman filter algorithm suggested by Townsend (1983). By using the Kalman filter I will be able to write investor's expectations about future inflation and output as a function of Central banks expectations.

### 4.1 Overview of Kalman Filter Algorithm:

### 4.1.1 The State-Space Representation of a Dynamic System:

• State Equation:

$$x_{t+1} = Fx_t + v_{t+1}$$

- $x_t = (r \ge 1)$  state vector.
- Observation Equation:

$$y_t = A'z_t + H'x_t + w_t$$

 $y_t = (n \ge 1)$  vector of variables observed at time t.

 $z_t = (k \ge 1)$  vector of exogenous or predetermined variables.

Where F, A', and H' are matrices of parameters of dimension (r x r), (n x k), and (n x r), respectively.

The (r x 1) vector  $v_t$  and the (n x 1) vector  $w_t$  are vector white noise.

#### 4.1.2 Forecast Equation of the Kalman Filter:

After many calculations and manipulations the Kalman filter gives the following forecast equation:

$$\widehat{x}_{t+1/t} = F\widehat{x}_{t/t-1} + K_t \left( y_t - A' z_t - H' \widehat{x}_{t/t-1} \right)$$
(9)

$$K_t \equiv FP_{t/t-1}H \left(H'P_{t/t-1}H + R\right)^{-1}$$

where

$$\hat{x}_{t+1/t} = E(x_{t+1}/\Omega_t) P_{t/t-1} = E\left[ \left( x_{t+1} - \hat{x}_{t+1/t} \right) \left( x_{t+1} - \hat{x}_{t+1/t} \right)' \right]$$

## 4.2 State-Space Representation of The Dynamic Model of Information Asymmetry:

### 4.2.1 Analysis of the Expectations of the Central Bank :

• Central Banks Expectation About Future Inflation:

$${}_{CB}\pi^*_t = \pi_t + \epsilon^{CB}_t$$

$$\pi_t = \rho \pi_{t-1} + \nu_t$$

The equations above form a state-space representation. So, let  $x_t = \pi_t$ ,  $y_t = {}_{CB}\pi^*_{t+1}$ , A = 0,  $\theta = \sigma^2_v$ ,  $R = \sigma^2_\varepsilon$ ,  $F = \rho$ , H = 1. So, using equation (9) we can write that:

$$E\left[\pi_{t+1}/\Omega_t^{CB}\right] = \rho E\left[\pi_t/\Omega_{t-1}^{CB}\right] + K_t \left({}_{CB}\pi_{t+1}^* - E\left[\pi_t/\Omega_{t-1}^{CB}\right]\right)$$
(10)  
$$K_t \equiv \rho P_{t/t-1} \left(P_{t/t-1} + \sigma_{\varepsilon}^2\right)^{-1}$$

Using the proposition 13.1 at Hamilton (1994), one can say that  $P_{t/t-1}$  converges to some constant P. So the time dimension (t) of  $K_t$  drops out.

Call  $P = \Sigma$ . So,

$$K_t \equiv \rho \Sigma \left( \Sigma + \sigma_{\varepsilon}^2 \right)$$

So, equation (10) can be written as the following:

$$M_{t+1}^{CB} = \rho M_t^{CB} + K \left( {}_{CB} \pi_{t+1}^* - M_t^{CB} \right)$$
(11)

 $M_{t+1}^{CB} = E\left(\pi_{t+1}/\Omega_t^{CB}\right)$ Equation (11) can be written as:

$$M_{t+1}^{CB} = \alpha_0 M_t^{CB} + \alpha_{1CB} \pi_{t+1}^*$$
 (12)

where  $\alpha_0 = \rho - K$  and  $\alpha_1 = K$ .

Same manipulations can be done for  $N_{t+1}^{CB} = E\left(x_{t+1}/\Omega_t^{CB}\right)$ , investors expectations of future output at time t. The equation for  $N_{t+1}^{CB}$  is:

$$N_{t+1}^{CB} = \beta_0 N_t^{CB} + \beta_{1CB} x_{t+1}^* \tag{13}$$

# 4.2.2 Analysis of the Information Dynamics Between The Fed and Investor:

Using equation (11) and (12), the information dynamics between central bank and the investor can be written as a state-space representation after some trivial computations and manipulations as the following:

• The state equation:

$$\begin{bmatrix} M_{t+1}^{CB} \\ N_{t+1}^{CB} \\ \pi_{t+1} \\ \pi_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha_0 & 0 & \alpha_1 \rho & 0 \\ 0 & \beta_0 & 0 & \beta_1 \beta \\ 0 & 0 & \rho & 0 \\ 0 & 0 & 0 & \beta \end{bmatrix} \begin{bmatrix} M_t^{CB} \\ N_t^{CB} \\ \pi_t \\ \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} \alpha_1 \nu_{t+1} + \alpha_1 \epsilon_{t+1}^{CB} \\ \beta_1 \zeta_{t+1} + \beta_1 \theta_{t+1} \\ \nu_{t+1} \\ \zeta_{t+1} \end{bmatrix}$$
$$x_{t+1} = Fx_t + V_{t+1}$$
(14)

$$\begin{split} M^{CB}_{t+1} &= E\left(\pi_{t+1}/\Omega^{CB}_t\right) \\ N^{CB}_{t+1} &= E\left(x_{t+1}/\Omega^{CB}_t\right) \end{split}$$

• The observation equation:

$$[r_{t}] = \begin{bmatrix} \psi & \gamma & a_{1} & a_{2} \end{bmatrix} \begin{bmatrix} M_{t}^{CB} \\ N_{t}^{CB} \\ \pi_{t} \\ x_{t} \end{bmatrix} + \begin{bmatrix} \alpha_{1} \epsilon_{t+1}^{CB} + \alpha_{1} \theta_{t}^{CB} \end{bmatrix}$$
$$y_{t} = H' x_{t} + W_{t}$$
(15)

Investor is assumed to receive no signal about inflation and output. As stated in Romer and Romer (2000):

"the Federal Reserve appears to possess information about the future state of the economy that is not known to the market participants. Our estimates suggest that if they had access to the Federal Reserve's forecast of inflation, commercial forecasters would find it nearly optimal to discard their forecasts and adopt the Federal Reserve's."

So, following Romer and Romer the assumption that investor does not receive any signal about inflation and output is reasonable.<sup>1</sup>

So, equations (14) and (15) represent a system which the Kalman filtering algorithm can be applied directly. If we make use of the Kalman filter and apply equation (9) to equations (14) and (15) we get,

<sup>&</sup>lt;sup>1</sup>Assuming that investor receives signals does not change the results, but makes the analysis more complicated.

$$E\left(\left[\begin{array}{c}M_{t+1}^{CB}\\N_{t+1}^{CB}\\\pi_{t+1}\\x_{t+1}\end{array}\right]/\Omega_{t}^{inv}\right) = FE\left(\left[\begin{array}{c}M_{t}^{CB}\\N_{t}^{CB}\\\pi_{t}\\x_{t}\end{array}\right]/\Omega_{t-1}^{inv}\right) + K_{t}\left(\left[r_{t}\right] - H'E\left(\left[\begin{array}{c}M_{t}^{CB}\\N_{t}^{CB}\\\pi_{t}\\x_{t}\end{array}\right]/\Omega_{t-1}^{inv}\right)\right)\right)$$

$$(16)$$

If we substitute equation (15) into (16) and rearrange, we get the following result about investor's expectations about next term inflation and output:

$$E\left(\left[\begin{array}{c}M_{t+1}^{CB}\\N_{t+1}^{CB}\\x_{t+1}\\x_{t+1}\end{array}\right]/\Omega_{t}^{inv}\right) = (F - K_{t}H')E\left(\left[\begin{array}{c}M_{t}^{CB}\\N_{t}^{CB}\\\pi_{t}\\x_{t}\end{array}\right]/\Omega_{t-1}^{inv}\right)$$
(17)
$$+K_{t}\left(H'\left[\begin{array}{c}M_{t}^{CB}\\N_{t}^{CB}\\\pi_{t}\\x_{t}\end{array}\right] + \left[\alpha_{1}\epsilon_{t+1}^{CB} + \alpha_{1}\theta_{t}^{CB}\right]\right)$$
$$K_{t} = FP_{t/t-1}H(H'P_{t/t-1}H + R)^{-1}$$
(18)

$$P_{t/t-1} \equiv E\left[ (x_{t+1} - E(x_{t+1}/\Omega_t) (x_{t+1} - E(x_{t+1}/\Omega_t)') \right]$$
(19)

 $E\left(\pi_{t+1}/\Omega_t^{inv}\right) =$  Investors expectations about future inflation  $E\left(x_{t+1}/\Omega_t^{inv}\right) =$  Investors expectations about future output

Equation (16) shows that investors expectations about future inflation and output are affected by Central Bank's expectations. The following equations about investor's expectations of future inflation and output can be derived from equation (16):

$$E\left(\pi_{t+1}/\Omega_{t}^{inv}\right) = A(3,1)E\left(M_{t}^{CB}/\Omega_{t-1}^{inv}\right) + A(3,2)E\left(N_{t}^{CB}/\Omega_{t-1}^{inv}\right) + A(3,3)E\left(\pi_{t}/\Omega_{t-1}^{inv}\right) + A(3,4)E\left(x_{t}/\Omega_{t-1}^{inv}\right) + B\left(3,1\right)M_{t}^{CB} + B\left(3,2\right)N_{t}^{CB} + B\left(3,3\right)\pi_{t} + B\left(3,3\right)x_{t} + B\left(3,3\right)x_{t} + K(3,1)(\alpha_{1}\epsilon_{t+1}^{CB} + \alpha_{1}\theta_{t}^{CB})$$
(20)

$$E\left(x_{t+1}/\Omega_{t}^{inv}\right) = A(4,1)E\left(M_{t}^{CB}/\Omega_{t-1}^{inv}\right) + A(4,2)E\left(N_{t}^{CB}/\Omega_{t-1}^{inv}\right) + A(4,3)E\left(\pi_{t}/\Omega_{t-1}^{inv}\right) + A(4,4)E\left(x_{t}/\Omega_{t-1}^{inv}\right) + B\left(4,1\right)M_{t}^{CB} + B\left(4,2\right)N_{t}^{CB} + B\left(4,3\right)\pi_{t} + B\left(4,3\right)x_{t} + K(4,1)(\alpha_{1}\epsilon_{t+1}^{CB} + \alpha_{1}\theta_{t}^{CB})$$
(21)

$$\begin{split} A &= (F - K_t H') \\ B &= K_t H' \end{split}$$

By proposition 13.1 at Hamilton (1994) it can be shown that  $P_{t/t-1}$  converges to some constant *P*.So the time dimension of  $K_t$  drops out and  $K_t$  converges to some constant *K*.

The main implications of equations (19) and (20) are the relationship between investor's expectations and the Fed's expectations about future inflation and output. By using Kalman filter algorithm we showed that there is a hierarchical information system between the Fed and investor's through the federal funds target rate. Same hierarchical information dynamics can be solved using Bayesian updating algorithm but Kalman filter is preferred for analytic and econometric purposes. The Bayesian method makes use of the whole history (back to t = 0) and the dimensions of the state vector increases in dimension with the length of history- there are more and more innovations. So, Kalman filter algorithm is preferred because contemporary mean beliefs may be updated from past mean beliefs and the contemporary observation  $y_t$  alone - mean beliefs capture all that is necessary for forecasting from the infinite history.

# 5 Investor's Maximization Problem:

As mentioned before the economy is a one good Lucas (1978) economy with borrowing. The representative investor maximizes her lifetime expected utility :

$$MAX_{\{C_t, b_t, z_{t+1}\}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i U\{C_{t+i}\} / \Omega_t^{inv} \right\}$$

subject to her budget constraint:

$$p_t^c c_t + p_t^z z_{t+1} + (1 + r_{t-1}) b_{t-1} \le z_t \left( p_t^c Y_t + p_t^z \right) + b_t$$

### 5.1 Solution:

Bellman Equation:

$$v\left\{z_{t}, Y_{t}, b_{t-1}\right\} = \max_{z_{t+1}, b_{t}} \left[ U\left\{\frac{1}{p_{t}^{c}}\left[z_{t}\left(p_{t}^{c}Y_{t}+p_{t}^{z}\right)+b_{t}-p_{t}^{z}z_{t+1}\right]-\left(1+r_{t-1}\right)b_{t-1}\right\}\right] + \beta E_{t}v\left\{z_{t+1}, Y_{t+1}, b_{t}\right\}$$

The first-order conditions of the maximization problem above:  $z_{t+1}$ :

$$\frac{p_t^z}{p_t^c} U'(C_t) = \beta E_t v_1 \{ z_{t+1}, Y_{t+1}, b_t \}$$

 $b_t$ :

$$\frac{1}{p_t^c} U'(C_t) = -\beta E_t v_3 \{ z_{t+1}, Y_{t+1}, b_t \}$$

Envelope Conditions:

$$v_{1} \{z_{t}, Y_{t}, b_{t-1}\} = \frac{1}{p_{t}^{c}} (p_{t}^{c} Y_{t} + p_{t}^{z}) U'(C_{t})$$
$$v_{3} \{z_{t}, Y_{t}, b_{t-1}\} = -(1+r_{t}) \frac{1}{p_{t}^{c}} U'(C_{t})$$

Iterate the envelope conditions forward and insert them into the first order conditions:

$$\frac{p_{\tilde{t}}^{z}}{p_{t}^{c}}U'(C_{t}) = \beta E_{t} \left\{ \frac{1}{p_{t+1}^{c}} \left( p_{t+1}^{c} Y_{t+1} + p_{t+1}^{z} \right) U'(C_{t+1}) \right\}$$
(22)

$$\frac{1}{p_t^c} U'(C_t) = \beta E_t \left\{ (1 + r_{t+1}) \frac{1}{p_{t+1}^c} U'(C_{t+1}) \right\}$$
(23)

Rearrange (21) and (22):

$$p_t^z = \beta E_t \left\{ \frac{p_t^c}{p_{t+1}^c} \left( p_{t+1}^c Y_{t+1} + p_{t+1}^z \right) \frac{U'(C_{t+1})}{U'(C_t)} \right\}$$
(24)

$$p_t^c = \beta E_t \left\{ (1 + r_{t+1}) \frac{1}{p_{t+1}^c} \frac{U'(C_{t+1})}{U'(C_t)} \right\}^{-1}$$
(25)

 $\frac{U'(C_{t+1})}{U'(C_t)}$  is the stochastic discount factor. If we denote  $m = \frac{U'(C_{t+1})}{U'(C_t)}$ . Then equation (23) which is the asset pricing equation becomes:.

$$p_t^z = \beta p_t^c E_t \left\{ Y_{t+1} m \right\} + \beta E_t \left\{ \frac{1}{\pi_{t+1}^c} p_{t+1}^z r m \right\}$$

$$\pi_{t+1}^c = \frac{p_{t+1}^c}{p_t^c}$$
(t+1) inflation rate.

 $E_t \{Y_{t+1}m\} = \sigma_{Y_{t+1},m} + E_t \{E_t \{Y_{t+1}\}m\} - E_t \{E_t \{Y_{t+1}\} E_t \{m\}\} + E_t \{Y_{t+1}E_t \{m\}\} + E_t \{M\} + E_t \{$ 

 $\sigma_{Y_{t+1,m}}$  = Covariance of  $Y_{t+1}$  and m.

Since, the expectations of the investor are affected by expectations of the central bank, the asset price is affected by the expectations of the central bank.

# 6 Estimation of The Parameters of the Information Dynamics:

The previous sections explain how Kalman filter can be applied to analyze the information dynamics between the Central Bank and the investor and how this information dynamics affect the stock returns. To be able to apply the methods developed in the previous sections we need the parameters of the state-space representation F, H' and K. In this section the parameters will be estimated using Zellner's SUR method and OLS.

### 6.1 Data:

1. The expectations of the Central bank and investor about future inflation and output  $(M_t^{CB}, N_t^{CB}, M_t^{inv}, N_t^{inv})$ :

a. FED'S FORECASTS: Fed's forecasts are from the Greenbooks of the Federal Reserve Board of Governors which are available at Federal Reserve Bank of Philadelphia's research web page. The quarterly greenbook data is available for 1969:1 to 1995:4. Greenbook forecasts of GDP deflator are used for inflation and forecasts of GDP are used for output.

b. COMMERCIAL FORECASTS: Commercial forecasts are from Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters. SPF forecasts are available quarterly from 1968:4 to 2002:3. Data for the period of 1968:1 to 1995:4 is used. The mean of the GDP deflator forecasts and mean of the nominal GDP forecasts are used as investor's expectations.

- Inflation: is the rate of change in Consumer Price Index taken from CRSP U.S. Treasury and Inflation.
- 3. Output: GDP taken from the FRED (Federal Reserve Bank of St. Louis).
- 4. Federal Funds Rate: Federal funds rate is taken from FRED (Fedfunds). The original data is monthly so to make the data quarterly the Federal funds rates are summed over three month intervals.

### 6.2 Estimation of The Parameters using SUR and OLS:

We need to estimate parameters in the state equation and the observation equation. The state equation looks like a VAR subject to general exclusion restrictions. As mentioned in Hamilton (1994) a VAR subject to general exclusion restrictions can be viewed as a system of seemingly unrelated regressions. The state equation is estimated using a Feasible GLS methodology and an iterated FGLS methodology which gives the maximum likelihood estimates. The observation equation is estimated using OLS.

#### 6.2.1 The State Equation:

The state equation is the following as mentioned before:

$M_{t+1}^{CB}$		$\alpha_0$	0	$\alpha_1 \rho$	0		$M_t^{CB}$	$\alpha_1 \nu_{t+1} + \alpha_1 \epsilon_{t+1}^{CB}$
$N_{t+1}^{CB}$	_	0	$\beta_0$	0	$\beta_1\beta$		$N_t^{CB}$	$\beta_1 \zeta_{t+1} + \beta_1 \theta_{t+1}$
$\pi_{t+1}$	_	0	0	$\rho$	0		$\pi_t$	$^+$ $\nu_{t+1}$
$x_{t+1}$		0	0	0	$\beta$		$x_t$	$\zeta_{t+1}$
				$x_{t+1}$	$=Fx_{i}$	t +	$-V_{t+1}$	

This equation looks like a VAR subject to general exclusion restrictions. As mentioned in Hamilton (1994) a VAR subject to general exclusion restrictions can be viewed as a system of seemingly unrelated regressions. The state equation is estimated using a Feasible GLS methodology and an iterated FGLS methodology which gives the maximum likelihood estimates.

### 6.2.2 The Observation Equation:

The state equation is the following:  $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ 

$$[r_t] = \begin{bmatrix} \psi & \gamma & a_1 & a_2 \end{bmatrix} \begin{bmatrix} M_t^{CB} \\ N_t^{CB} \\ \pi_t \\ x_t \end{bmatrix} + \begin{bmatrix} \alpha_1 \epsilon_{t+1}^{CB} + \alpha_1 \theta_t^{CB} \end{bmatrix}$$
$$y_t = H' x_t + W_t$$

This equation is estimated using OLS. Since, we sum fedfunds rate to have quarterly data, we have serial correlation. To deal with this problem we estimated Newey-West standard errors.

The derivation of the Kalman filter requires the mean squared error (MSE) associated with the updated projection  $P_{t/t-1}$ . Which is equal to,

$$P_{t/t-1} = E\left[\left(x_{t+1} - \hat{x}_{t+1/t}\right)\left(x_{t+1} - \hat{x}_{t+1/t}\right)'\right].$$
  
To calculate  $P_{t/t-1}$  we will use the following equation,

$$P_{t+1/t} = F \left[ P_{t/t-1} - P_{t/t-1} H \left( H' P_{t/t-1} H + R \right)^{-1} H' P_{t/t-1} \right] F' + Q \quad (26)$$

$$E(V_t V_\tau') = \begin{cases} Q & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases}$$
(27)

 $V_t$  is the error term of the state equation.

Application of equation (24) requires the knowledge of  $P_{1/0}$ . Following Hamilton (1994), we calculate  $P_{1/0}$  using the following equation:

$$vec(P_{1/0}) = [I_{r^2} - (F \otimes F)]^{-1} vec(Q)$$
 (28)

Using the maximum likelihood estimates of the state and observation equations shown at tables 1-4 and using the solutions to the kalman filter algorithm as displayed in table 5, the expectations of the investor can be written as the following:

$$E\left(\left[\begin{array}{c}M_{t+1}^{CB}\\N_{t+1}^{CB}\\x_{t+1}^{T}\\x_{t+1}\end{array}\right]/\Omega_{t}^{inv}\right) = \left[\begin{array}{ccccc}0.2762 & -0.0457 & 0.5898 & 0.0445\\-0.1540 & 0.7677 & -0.0811 & 0.1379\\-0.1851 & -0.0184 & 0.7805 & 0.0179\\-0.1075 & -0.0107 & -0.0566 & 0.8434\end{array}\right] E\left(\left[\begin{array}{c}M_{t}^{CB}\\N_{t}^{CB}\\x_{t}\\x_{t}\end{array}\right]/\Omega_{t-1}^{inv}\right) + \left[\begin{array}{c}0.4598 & 0.0457 & 0.2422 & -0.0445\\0.1540 & 0.0153 & 0.0811 & -0.0149\\0.1851 & 0.0184 & 0.0975 & -0.0179\\0.1075 & 0.0107 & 0.0566 & -0.0104\end{array}\right] \left[\begin{array}{c}M_{t}^{CB}\\N_{t}^{CB}\\x_{t}\\x_{t}\end{array}\right] + \left[\begin{array}{c}0.1167\\0.0391\\0.0470\\0.0273\end{array}\right] \left[\alpha_{1}\epsilon_{t+1}^{CB} + \alpha_{1}\theta_{t}^{CB}\right]$$
(29)

# 7 Asset Prices and Investors' Expectations:

As it can be seen from equation (24) the asset price at time t depends on investors expectactions of next period inflation and next period output. For simplification, to be able to analyze the time series dynamics of our model, we will assume that there is a linear relationship between time t asset price and expected next period inflation and expected next period output. We assume the following equation using equation (24):

$$p_t^z = \beta_1^p E\left(\pi_{t+1}/\Omega_t^{inv}\right) + \beta_2^p E\left(x_{t+1}/\Omega_t^{inv}\right)$$
(30)

where

 $p_t^z =$ Asset price at time t

 $E\left(\pi_{t+1}/\Omega_t^{inv}\right)$  = Investors expectations about future inflation at time t $E\left(x_{t+1}/\Omega_t^{inv}\right)$  = Investors expectations about future output at time t

Table 6 displays the OLS results of equation (30) estimated using price indices of equally and value weighted portfolios and 10 different size portfolios.

# 8 Time-Series Dynamics:

In summary, the economic dynamics can be displayed as the following using equations 4,5,29,30:

 $\begin{vmatrix} p_{t-1}^{\tilde{r}} \\ E\left(\pi_{t+1}/\Omega_{t}^{inv}\right) \\ E\left(x_{t+1}/\Omega_{t}^{inv}\right) \\ E\left(M_{t+1}/\Omega_{t}^{inv}\right) \\ E\left(N_{t+1}/\Omega_{t}^{inv}\right) \\ M_{t+1}^{CB} \\ N_{t+1}^{CB} \\ n_{t+1}^{T} \\ \pi_{t+1} \\ x_{t+1} \end{vmatrix} =$ 

ſ	0	$\beta_1^p$	$eta^p_2$	0	0	0	0	0	0 -
	0	0.7805	0.0179	-0.1851	-0.0184	0.1851	0.0184	0.0975	-0.0179
	0	-0.0566	0.8434	-0.1075	-0.0107	0.1075	0.0107	0.0566	-0.0104
	0	0.5898	0.0445	0.2762	-0.0457	0.4598	0.0457	0.2422	-0.0445
	0	-0.0811	0.1379	-0.1540	0.7677	0.1540	0.0153	0.7805	0.0179
	0	0	0	0	0	0.736	0	0.832	0
	0	0	0	0	0	0	0.783	0	0.123
	0	0	0	0	0	0	0	0.878	0
	0	0	0	0	0	0	0	0	0.833
	- E E E	$\begin{array}{c} p_{t-2}^{z} \\ (\pi_{t}/\Omega_{t-1}^{inv}) \\ (x_{t}/\Omega_{t-1}^{inv}) \\ (M_{t}/\Omega_{t-1}^{inv}) \\ (M_{t}/\Omega_{t-1}^{inv}) \\ (N_{t}/\Omega_{t-1}^{inv}) \\ M_{t}^{CB} \\ N_{t}^{CB} \\ \pi_{t} \\ x_{t} \end{array}$	+ eri	ror term					

Following the display above the dynamics of this system can be written as a first-order VAR :

$$\mathbf{z}_t = \mathbf{A}\mathbf{z}_{t-1} + \mathbf{w}_t \tag{31}$$

It is a well known fact that an AR(1) equation like equation 31 can be written as a infinite MA process:

$$\begin{aligned} \mathbf{z}_t &= \mathbf{A}(\mathbf{L})\mathbf{z}_t + \mathbf{w}_t \\ & [I - \mathbf{A}(\mathbf{L})] \, \mathbf{z}_t = \mathbf{w}_t \\ \mathbf{z}_t &= [I - \mathbf{A}(\mathbf{L})]^{-1} \, \mathbf{w}_t \\ &= \sum_{i=0}^{\infty} (\mathbf{A}(\mathbf{L}))^i \, \mathbf{w}_t \\ &= \sum_{i=0}^{\infty} (\mathbf{A})^i \, \mathbf{w}_{t-i} \\ &= \mathbf{w}_t + \mathbf{A} \mathbf{w}_{t-1} + (\mathbf{A})^2 \, \mathbf{w}_{t-2} \dots \end{aligned}$$

So, we can derive the impulse response functions which gives us the consequences of a one-unit increase in one of the variables on another variable. Since,

$$\frac{\partial \mathbf{z}_{t+s}}{\partial \mathbf{w}_t} = \left(\mathbf{A}\right)^i \tag{32}$$

These nice properties of the impulse response functions make them an ideal methodology the analyze the results of the Kalman Filter algorithm and analyze the dynamics of our system.

### 8.1 Impulse Response Functions:

### 8.1.1 Response of Asset Prices:

The impulse response functions are displayed in figures 1 and 2. We calculated  $\beta_1^p$  and  $\beta_2^p$  for value and equally-weighted portfolios and 10 different size portfolios using OLS. Table 6 displays the results of OLS regressions. Size10 is the index of largest size portfolio and size1 is the index of smallest portfolio.

Figure 1 displays the impulse response functions of value and equally-weighted portfolios. Dotted line is response of value-weighted and straight line is equally-weighted portfolio index. The impulse response functions reveal that when investors expectations about inflation  $(E(\pi_{t+1}/\Omega_t^{inv}))$  changes by one unit the asset price (portfolio index) declines. The maximum effect is at the fifth period after fifth period the affect of shock to investor's inflation expectations about output  $(E(x_{t+1}/\Omega_t^{inv}))$  changes by one unit the asset price (portfolio index)) changes by one unit the asset price (portfolio index)) changes by one unit the asset price (portfolio index)) changes by one unit the asset price (portfolio index) increases. The effect declines after the first period and dies off after 30 periods. Asset price responds negatively to a shock to investors expectations of central bank's expectation of next term inflation and output  $(E(M_t/\Omega_{t-1}^{inv}), E(N_t/\Omega_{t-1}^{inv}))$ . Asset price responds positively to a shock to central bank's expectation of next term inflation and output  $(M_t^{CB}, N_t^{CB})$ . Asset price responds positively to a shock to central bank's expectation of next term inflation  $(\pi_t)$  and negatively to a shock to next term output  $(x_t)$ .

Figure 2 displays the impulse response functions of largest and smallest size portfolios. Dotted line is response of largest and straight line is smallest portfolio index. One of the main results of figure 2 is that smallest size portfolio responds more to shocks than largest size portfolio. The impulse response functions reveal that when investors expectations about inflation  $(E(\pi_{t+1}/\Omega_t^{inv}))$ changes by one unit the asset price (portfolio index) declines. The maximum effect is at the fifth period after fifth period the affect of shock to investor's inflation expectation starts to decline and dies off after 20th period. When investors expectations about output  $(E(x_{t+1}/\Omega_t^{inv}))$  changes by one unit the asset price (portfolio index) increases. The effect declines after the first period and dies off after 30 periods. Asset price responds negatively to a shock to investors expectations of central bank's expectation of next term inflation and output  $\left(E\left(M_t/\Omega_{t-1}^{inv}\right), E\left(N_t/\Omega_{t-1}^{inv}\right)\right)$ . Asset price responds positively to a shock to central bank's expectation of next term inflation and  $\operatorname{output}(M_t^{CB}, N_t^{CB})$ . Asset price responds positively to a shock to next term inflation( $\pi_t$ ) and negatively to a shock to next term output  $(x_t)$ .

# 8.1.2 Response of Investor's Expectations to Changes in Central Bank's Expectations:

Using the impulse response functions, we can also analyze the how investor's expectations react to changes in the Fed's expectations. Romer and Romer (2002) empirically show that commercial forecasters modify their forecasts in response to monetary-policy actions. In this paper, we construct a model including asset prices and hierarchical information structure that explains this behavior and we also support their findings with our empirical findings.

Figure 3 displays the impulse responses of investor's expectations to shocks to the Fed's expectations. It can be seen from the graphs that investor's expectations especially respond to the changes in the Fed's inflation expectation.

## **9** Conclusion:

This paper constructs and solves a model of hierarchical information between the central bank and investor to analyze the effects of the information asymmetry between the central bank and investor on asset prices. The parameters of the state-space representation are estimated by using SUR and OLS. Then, using those parameter estimates a Kalman filtering algorithm is applied . The Kalman filter results and the linear relationship between the asset prices and expected inflation and expected output are displayed in a VAR format. Using the VAR , impulse response functions are drawn which gives us how asset prices respond to different shocks to the model. Our main result is asset prices respond to changes in the investor's inflation and output expectations. We also found that investor's expectations are affected by the Fed's expectations. Finally, we conclude that the information asymmetry between the Fed and the investor's in one of the main reasons of the affects of changes in monetary policy on asset prices. This argument is supported both empirically and theoretically in the paper.

Table 1: Maximum Likelihood estimation of the Parameters of the State Space Representation: The SUR estimation of the State Equation Results of Feasible GLS

	defqtr1	gdpqtr1	cpiretper	gdpchange
defqtrllag1	0.736			
	(18.59)**			
cpiretlag1	0.832		0.878	
	(6.51)**		(20.12)**	
gdpqtrllag1		0.783		
		(11.96)**		
gdpchangelag1		0.123		0.833
		(1.11)		(16.61)**

Observations 108 108 108 108

Absolute value of z statistics in parentheses

\* significant at 5%; \*\* significant at 1%

Table 1: (Cont.)					
Results of I	terated Feasi	ble GLS			
	defqtr1	gdpqtr1	cpiretper	gdpchange	
defqtr1lag1	0.702				
	(17.92)**				
cpiretlag1	0.932		0.877		
	(7.33)**		(20.10)**		
gdpqtr1lag1		0.764			
		(11.75)**			
gdpchangelag	1	0.138		0.830	
		(1.24)		(16.55)**	
Observations	108 1	08 108 10	8		
Absolute value of z statistics in parentheses					
* significant at 5%; ** significant at 1%					

Table 2: Maximum Likelihood estimation of the Parameters of the State Space Representation: The OLS estimation of the Observation Equation (Newey-West Standard Errors)

	fedfundsqtr
defqtr1lag1	3.940
	(6.38)**
gdpqtr1lag1	0.392
	(0.83)
cpiretper	2.075
	(1.82)
gdpchange	-0.381
	(0.41)

Observations 109

t statistics in parentheses

\* significant at 5%; \*\* significant at 1%

Table 3: The Error terms of the state and observation equation to complete the state space representation (Calculated using Feasible GLS SUR results) Q matrix (The variance of the errors of the state equation) symmetric c[4,4]

	ql	q2	q3	q4
ql	.70230522			
q2	.06332782	3.0798733		
qЗ	.17096363	.17111468	.55979938	
q4	.23580346	.82817182	.40359586	3.9391462

R matrix (The variance of the errors of the observation equation) symmetric onur[1,1]

R

R 62.121805

Table 4 : The Mean Squared Error (P)matrix for the Kalman Filter Algorithm The Mean Squared Error (P) converges , the values that P matrix converged are displayed here. (Calculated using Feasible GLS SUR results)

2.6154	0.2266	1.1147	1.2074
0.8104	10.9746	0.1283	5.7008
1.0946	0.3495	1.2419	0.9071
1.1970	5.8163	0.9071	12.5648

Table 5: The Matrixes that are calculated for solution of the Kalman Filter Algorithm: (Calculated using Feasible GLS SUR results)

### $K = F^*P^*H^*inv(H'^*P^*H+R)$

K =

0.1167

0.0391

0.0470

0.0273

### (F-K\*H')

F-(K\*H')

ans =

0.2762	-0.0457	0.5898	0.0445
-0.1540	0.7677	-0.0811	0.1379
-0.1851	-0.0184	0.7805	0.0179
-0.1075	-0.0107	-0.0566	0.8434

### K\*H′

К\*Н'

### ans =

0.4598	0.0457	0.2422	-0.0445
0.1540	0.0153	0.0811	-0.0149
0.1851	0.0184	0.0975	-0.0179
0.1075	0.0107	0.0566	-0.0104

Table 6 : The Regressions (OLS) Results of the Different Stock Portfolios on Expected Inflation and Expected Output:

Portfolio Return	Spfdeflg (1 Quarter	Spfgdp1g (1 Quarter
	Ahead SPF Inflation	Ahead SPF Output
	Forecast)	Forecast)
Value-Weighted	0.743	79.048
	(0.20)	(6.07)**
Equally-Weighted	0.703	55.787
	(0.25)	(5.70)**
Size 10	0.653	48.371
	(0.26)	(5.63)**
Size 9	0.701	60.620
	(0.23)	(5.74)**
Size 8	0.870	70.600
	(0.24)	(5.73)**
Size 7	0.682	67.549
	(0.21)	(6.16)**
Size 6	0.951	86.076
	(0.22)	(5.79)**
Size 5	0.876	84.482
	(0.21)	(5.78)**
Size 4	0.760	89.700
	(0.18)	(6.09)**
	0.055	0.4 51.6
Size 3	0.657	84.516
	(0.17)	(6.29)**
	0.400	07 100
Size 2	0.498	97.100
	(U.12)	(0.30)^^
		122 504
Size i	(0.029)	133.304
	(U.12)	$(0.70)^{1}$

Observations 132 Absolute value of t statistics in parentheses \* significant at 5%; \*\* significant at 1%

# Figure 1:

Impulse Response Functions of Indices of Value and Equally Weighted Portfolios: Dotted line is response of value-weighted and straight line is equally-weighted portfolio index.















# Figure 2:

Impulse Response Functions of Indices of Largest and Smallest Size Portfolios: Dotted line is response of largest and straight line is smallest portfolio index.









# Figure 3:

Impulse Response Functions of Investor's Inflation and Output Expectations: Dotted line is response to a shock to the Fed's Inflation Expectation and straight line response to a shock to the Fed's Output Expectation.

