Endogenous Asymmetry and Entry in Sequential Multi-Unit Auctions: Identification and Estimation

Sudip Gupta
University of Wisconsin Madison
Very Preliminary and Incomplete

June 13, 2004

Abstract

This paper analyzes bidding behavior in a multi period multiple unit auction. While bidders are ex ante symmetric, the first period outcome translates the second period game to a game between asymmetric bidders. The first period outcome determines who will be a strong or a weak bidder in the second period. The possibility of future asymmetry thus affects the bidding behavior in the current symmetric environment. This leads to "excessive entry" and "overbidding' in the first period. We characterize the equilibrium in terms of the observed bid distribution and entry behavior. Using this characterization we establish the nonparametric identification of bidders' privately observed signals from bid data. We suggest a three step procdure to estimate the dynamic mixed discrete-continuous choice model.

We estimate our model and report the results. Specifically, we found that the federal government is only recovering 25% of the 'strong' buyers' willingness to pay.In the wildcat auctions, we found that the bidders are willing to pay 10 % more to take possible future informational advantage.

1 Introduction

This paper analyzes bidding behavior in a multi-period multiple unit auction. There is a single seller who has two units of the good and sells off each unit of the good sequentially via first price auctions. In the first period bidders are typically symmetric. Whoever wins the first period auction learns his valuation for the second period 'better' (in a sense to be made clear later), and becomes a strong bidder in the second period. Thus the first period outcome generates asymmetry in the second period distribution of bidders' valuations. The possibility of future asymmetry thus affects the bidding and entry behavior in the first period symmetric environment. This would lead to more entry in the first period than if we were analyzing the first period as a static game. We characterize the equilibrium in terms of the observed distribution of bids and entry behavior, which in turn can be used to recover the unobserved signals of bidders from observed variables, when bidders follow equilibrium strategies. It is important to characterize bidding behavior in such an environment. This will give important policy directions in the choice of designing auction mechanisms and reserve prices for both periods.

A well known example of this type of auctions are the oil tracts auctions[1]. In such auctions, for each geographic location, the government auctions off several oil tracts via sequential first price sealed bid auctions. Thus there are multiple goods for each locations and bidders may demand one, more or none of these goods depending on their valuations. There are two major kinds of oil and gas lease sales. A *wildcat* sale covers tracts whose geology is not well known and on which exploration involves searching for a new deposit. Firms can get pre bidding seismic informations but no on site drilling is permitted. A *drainage* sale consists of tracts in areas where a deposit has been discovered. On-site drilling is not permitted but firms owning adjacent tracts can conduct off-site drilling, which may be informative about the oil deposited in the tracts to be auctioned. Thus bidders who already have won adjacent tracts via wildcat auctions have informational advantages over other bidders for drainage tracts. This creates a clear asymmetry among bidders, separating them in two categories: those who have 'more accurate' information, we call them strong bidders, and those who do not have as accurate information as the strong bidder; we call them weak bidders. This asymmetry is generated by who has won in the previous wildcat auction. Thus bidders who bid for wildcat tracts do not only take their evaluations of the oil storage in the wildcat tracts, but they also keep in mind the informational advantages they would get in subsequent sales in that area by winning the current auction. The latter effect generates major dynamics in the bidding behavior.

We model wildcat tracts as tracts being sold in the first period of the game, and the drainage tracts are sold in the second period. In the wildcat sales bidders are typically symmetric in the sense that they receive privately observed seismic signals from the same distribution. If the signal is not above a threshold level, then bidding is not optimal. A puzzle so far has been, whether we notice some kind of overbidding as 64% of wildcat tracts turn out to be dry[2]. If the fault lay with the accuracy of the seismic surveys, one would have expected them to be carried out less and less as these are quite costly to carry out. However, that has not been seen. An alternate explanation of why people bid when they

[1] Examples also include spectrum auctions, treasury auctions etc.

[2] Descriptive statistics as reported by Hendricks and Porter ('89).

apparently should not is because bidders rationally take into account dynamic considerations. Owning an oil tract even if it has a high chance to turn out dry, gives the owner an advantage for future drainage sales. Thus, bidders rationally calculate the advantages of bidding in future drainage auction in deciding on optimal bidding strategies in a wildcat auction. Thus this dynamics lowers the threshold level of signal for bidding for wildcat tracts and overstates the expected valuation of the tract. This leads to more entry in the wildcat sales relative to the static analysis of wildcat auction. Reduced form analysis suggests that ex-post drainage tract values conditional on ex-post wildcat tract values and competition, have a significant positive effect on entry and bidding decisions for wildcat tracts.

The valuation from any tract, conditional on winning, depends on how much oil is stored in that tract minus the cost of drilling. Now for a wildcat tract, this value is likely to be same for all the bidders, since the amount of oil stored in those tracts is same for all the bidders and as they have not won any tract in the neighborhood, there are also no difference in drilling technologies. We therefore model the wildcat tracts as a common value (CV) auction. However the drilling cost is likely to be different for different bidders in drainage auctions. Since the costs associated with shifting resources from other previously won tracts is different depending on whether the bidder has won a tract in the neighborhood or not. We therefore model the drainage auctions as affiliated private value (APV) auctions: affiliated through the common component of how much oil is stored in the tract and the private component depends on the drilling cost. This is our maintained assumption throughout this paper.

Assumption : The wildcat auctions are common value auctions whereas the drainage auctions are affiliated private value auction.

Note that, this assumption is easily testable following Haile, Hong & Shum ('04). Also as will be clear later, our results will not change because of this assumption. However, the economic interpretation of the first order conditions associated with bidding will be different.

Our aim in this paper will be, to formulate a model and recover bidders' valuations for the wildcat and drainage tracts, which takes these dynamics into account. This will give important policy directions for the government to device a better auction mechanism for wildcat and drainage tracts which generates more revenue. Researchers have previously argued that drainage auctions are under priced. Our analysis will give directions for the analysis of the optimal (reserve) pricing in such an environment.

We model this as a three stage dynamic game of mixed discrete and continuous choice. The first stage is an entry game where bidders makes the discrete choice of whether to bid for the wildcat tract or not depending on the pre-bidding seismic signals received. While deciding to enter they keep the possibility of future asymmetry in mind. This future asymmetry is also taken into account while deciding how much to bid for the wildcat auction. While bidding for the drainage auction bidders know who is a strong and weak bidder. However no bidder knows their valuations from the tract perfectly. This leads to aggressive bidding by the weak bidder in equilibrium.

We show the existence of equilibrium for this dynamic game and establish uniqueness in the bidding games in the wildcat and drainage auctions.

The equilibrium bidding function is monotonic, hence we can invert it to rewrite the distribution of privately observed signals in terms of the distribution of bids and the observed entry behavior. We then use this to establish non-parametric identification of distribution of privately observed signals.

The structural parameters of our model are the distribution of unobserved signals ('pseudo values') and the 'entry parameters' (to be described later). We suggest a three step estimation procedure based on the identification results to recover the structural parameters from the data. The structural parameters of our model are the distributions of bidders' privately observed signals for wildcat and drainage auction and the sunk cost associated with submission of bids. Our procedure extends the works of Guerre, Perrigne and Vuoung (GPV) ('00) to dynamic auctions with entry.

We used data[3] on sales of wildcat tracts off the coasts of Texas and Louisiana held during the period 1954 to 1990 to estimate our model and report our results. Specifically we found that the federal government only recovers 25% of the 'strong' bidder's willingness to pay in the drainage auctions. Also in the wildcat auctions, we found that the bidders are willing to pay 10 % more to take possible future informational advantage.

2 Related Literature

Asymmetry in auctions has received increased attention recently. Maskin & Riley ('96 &'00), Athey('00), Lebrun ('99), Mcadams ('04), Reni & Zamir ('03) have characterized asymmetric first price sealed bid auctions and established monotonicity of the equilibrium under specific assumptions. Asymmetry originating from size difference (Laffont, Oscard, Vuoung('95), geographic locations (Bajari('99)), and capacity constraints (Jofre-Bonet & Pesendorfer ('03)(JBP)) among others have been studied in the literature. However except for JBP no paper has analyzed asymmetry in a dynamic context. Also to the best of our knowledge no paper has analyzed the process of asymmetry generation starting from a symmetric environment in a multiple unit framework as in our paper. This endogenous asymmetry also endogenizes entry in our model. Endogenous entry, although prevalent in procurement auctions, has received increased theoretical attention recently in Mcaffee('87), Levin & Smith ('94) among others.

[3] I am grateful to Prof. Ken Hendricks, Joris Pinkse and Rob Porter for sharing the data.

Our empirical modelling of entry is different from theirs and more related to Berry, Levinson, Pakes ('94) modelling of entry as in the standard IO literature.

The presence of possible asymmetry in drainage tract sales have been widely documented by Hendricks & Porter ('88,'89,'95) among others. However their analysis does not account for dynamics in a multiple unit structural model as in ours.

Multiple unit auction is yet to be widely studied, both in theoretical and empirical literature. Most of the analysis is confined to bidding behavior in a static and symmetric environment. A standard assumption made is that the marginal valuations of successive units are constant and independent of the allocation of other units. To the best of our knowledge no paper has analyzed the dynamics and possible future asymmetry as in our model in an uncertain environment. We have also provided explicit form of the distribution of valuation in terms of the observed bid data and equilibrium entry behavior. This makes it readily available for structural estimation of the unobserved valuations distributions and counter-factual experiments from the observed bid data, which will be relevant for devising optimum (reserve) price.

3 Model

3.1 Description and Information Structure

For simplicity of exposition, we present the model here for two bidders and two periods. However a generalization to the N bidder case in a multi-period framework is straightforward and will be sketched in the identification and estimation section.

This is a multi-stage game of incomplete information. There is a seller who has two indivisible units of the good and sells off each unit of the good in sequential first price auctions. There are two potential buyers 1 and 2. All players are risk neutral. The seller, who does not act strategically, offers the good via a first price sealed bid auction. The first period auction is called wildcat auction and the second period is called drainage auctions for reasons described earlier. Bidders receive private signals s^w and s^d about their valuations of wildcat and drainage auction respectively. It is assumed that the unknown value of oil stored in a tract of type t to a bidder i, U_i^t can be expressed as a function of all bidder's signals

$$u_i^t(s_1^t, s_2^t) = E(U_i^t | S_1^t = s_1^t, S_2^t = s_2^t)$$

where $t = \{wildcat, drainage\}$ and u_i^t is bidder $i's$ valuation and is assumed to be non-decreasing in all its arguments, strictly increasing in s_i^t and twice continuously differentiable..

This is an interdependent valuation model, as other bidder may posses information that would, if known to a particular bidder, affect the value he assigns to the tract. When bidder's valuations only depend on his signal $u_i^t(s_1^t, s_2^t) = s_i^t$, then it is called a pure private value model. On the other hand if the valuation remains the same for all bidders $u_i^t(s_1^t, s_2^t) = U$, then it is a pure common value setting. We assume that the wildcat auction is a common value (CV) auction and the drainage auction is an affiliated private value (APV) auction for reasons described earlier[4].

The distribution of signals for auction of type t, $F_t(.)$ are affiliated, in the sense defined[5] in Milgrom& Weber (MW)('82). By affiliation, high value of one signal leads to a higher value of the other signal. Note that, independence is a special of affiliation.

3.1.1 Information in the Wildcat Auction (First Period)

Let the value of oil stored in wildcat tract, drawn from F_u, be denoted by u. Bidders are typically symmetric for wildcat auctions in the sense that they privately observe conditionally independent, but affiliated signals s^w about u from the same distribution $F_W(s^w|u)$[6].

To economize on notations, let the expected common value of oil stored in an wildcat auction when bidder i has one rival, be given by,

$$U = E[u_i(s_i^w, Y_w)|s_i^w, \max_{j \neq i} s_j^w = Y_w,\ A = 2]$$

where Y_w is the maximum signal received by the rival of bidder i, and let U_0 be the same when he does not have any rival. Let u and u_0 be the values that these random variables take. Note that, since wildcat auction is a pure CV auction, due to the presence of 'winner's curse', U and U_0 need not be the same.

[4]Another approach could be not to impose any CV or APV assumption and estimate the interdependent 'values' for each auction. We can then test wheter they are CV or APV using HHS.

[5]The concept of affiliation as coined by Milgrom & Weber ('82), is what is known as Total Positivity (TP) in statistical literature. More specifically the variables $\mathbf{s} = (s_1, s_2, ..s_n)$ are said to be affiliated if for all $\mathbf{s}', \mathbf{s}'' \in S$,

$$f(\mathbf{s}' \wedge \mathbf{s}'')f(\mathbf{s}' \vee \mathbf{s}'') \geq f(\mathbf{s}')f(\mathbf{s}'')$$

where $(\mathbf{s}' \vee \mathbf{s}'') = (\max(s_1', s_1''), \max(s_2', s_2''), ..., \max(s_n', s_n''))$, and $(\mathbf{s}' \wedge \mathbf{s}'') = (\min(s_1', s_1''), \min(s_2', s_2''), ..., \min(s_n', s_n''))$ are the component wise maximum and minimum of $\mathbf{s}'$ and $\mathbf{s}''$. It is easy to verify that f is affiliated if and only if, for all $i \neq j$,

$$\frac{\partial^2}{\partial s_i \partial \mathbf{s}_j} \ln f \geq 0$$

For more details see MW.

[6]Thus, each of the random variables s_1^w and s_2^w are affiliated with the common component u, but conditioned on the common component u they are indepedent.

The winner of the wildcat auction becomes a strong bidder in the drainage auction. The relative strongness of bidders' are determined by his location **state variable**. The location of bidder i evolves according to

$$D_i = \{ \begin{array}{l} i \text{ is strong, if } i \text{ won the wildcat auction in the same location} \\ i \text{ is weak, if } i \text{ did not win wildcat auction in the same location} \end{array}$$

The dynamics of the model is represented by the evolution of the state variable D. When bidders are bidding for wildcat tracts they are also choosing their future locations D, which affects their marginal distribution of valuations for drainage tracts. Note that, the state variable can also be a continuous distance variable, leading to a continuous type space instead of discrete one as described above.

3.1.2 Information in the Drainage Auction (Second Period)

Bidders in drainage auctions are divided into two groups: strong bidder (denoted by 1); who has won the wildcat tract in the first period, and weak bidders (denoted by 0); who did not win in the first period. They draw their private signals about the value of the drainage tract, v from different marginal distributions, $F_1(.|v)$ and $F_0(.|v)$ respectively. Thus s_1^d is the realization of the random variable s^d drawn from the distribution $F_1(.|v)$. Bidders receive affiliated signals $s_i^d \in \Re$, about the unknown value of oil stored in drainage tracts v, $v \sim F_v$, $s_i^d \sim F_i^d(s^d|v)$, and let their joint distribution be $F(v, s_i^d, s_j^d) = F_1^d(s^d|v)F_0^d(s^d|v)F_v(.)$, we assume that all the distributions are continuous.

Strong bidders evaluate the value of the tract 'better', in the sense that their signals about the value of the tract is more 'accurate' than that of the weak bidder. Thus, if the realization of v is low then strong bidder's signal s_1^d is low and vice versa. We shall use the concept of 'accuracy' as introduced in the statistics literature by Lehmann ('88) and applied recently in economics by Persico ('00) and Athey & Levin ('01). Formally

Definition 1 *Given two distributions of signals $F_1(.|v)$ and $F_0(.|v)$, we say that the realization of s_1^d drawn from $F_1(.|v)$ is more accurate than s_0^d, drawn from $F_0(.|v)$, if*

$$h_{1,v}(s^d) = F_1^{-1}[F_0(s^d|v)|v] \tag{1}$$

is non-decreasing in v, for every s^d.

Intuitively, the notion of a more accurate signal can be interpreted as the one which is more correlated with the random variable v. Note that, 'accuracy' is more general than stochastic dominance. We shall outline a procedure later on how to test the assumption of accuracy using ex post production and bid data[7].

This kind of asymmetry can be explained with the following simple example. Suppose the common value of the object is v and every bidder's valuations of the object is $s_1 = v + \epsilon_1$, and $s_0 = v + \epsilon_0$, where $\epsilon' s$ have zero mean and $var(\epsilon_1) \leq var(\epsilon_0)$. Thus the strong bidder (bidder 1) evaluates the common value of the object better than the weak bidder (bidder 0), in the sense that he receives a more concentrated signal around the true common value of the object.

To explain the above definition better, suppose v can take only two values $v_1 < v_2$. Both bidders receive signals s_1^d and s_0^d respectively about v. Bidders make inferences about v based on their signals. Now for the weak bidder any 'most powerful' test of v_1 versus v_2 has the form 'accept' v_2 if $s_0^d > \overline{s}_0^d$. The probability that weak bidder rejects the fact the value is v_2 when the true value is actually v_2 (Type I error), is given by $F_0(\overline{s}_0^d|v_2)$. That of the strong bidder would be $F_1(\overline{s}_1^d|v_2)$. The probability that the weak bidder would accept the fact that the value is v_2 when the true value is actually v_1 (Type II error), is given by $1 - F_0(\overline{s}_0^d|v_1)$. Similarly that for the strong bidder is $1 - F_1(\overline{s}_1^d|v_1)$. If we fix the same probability of Type I error for both bidders, then $F_1(\overline{s}_1^d|v_2) = F_0(\overline{s}_0^d|v_2)$, inverting this gives us the relationship in equation 1 in the above definition. The strong bidders' signal is more 'accurate' in the sense that it has lower Type II error (more 'Power'). This implies $\overline{s}_1^d \geq F_1^{-1}[F_0(\overline{s}_0^d|v_1)|v_1]$, this gives us the non-decreasing in v part of the definition[8].

3.2 Timeline of Events

The sequence of moves, for each location, are as follows:

1) Number of potential bidders[9] is common knowledge. Bidders first receive some information; s^w, about the value of oil stored in the tract, u, $s^w \sim F_W(.)$ of the wildcat tract from a seismic survey.

2) There are 2 potential bidders for any location. They simultaneously decide whether to bid for the wildcat auction or not. Bidders receive action specific shocks ϵ associated with actions of whether to enter or not, drawn independently from Logistic distribution[10]. If they decide not to bid they receive ϵ_1

[7] One simple test will be a test of correlation of the estimated valuation and exact value of oil stored. Given the expost data on oil auction we can test that the correlation is higher for strong bidder than for the weak bidder. Another test of asymmetry using 'copula' is described in Gupta ('04).

[8] For details; see Lehmann ('88, especially pp.527-528).

[9] This is not necessary. As long as the distribution of the potential bidders are known to bidders then our analysis will go through. However we abstract away from it for simplicity.

[10] The assumption of logistic distribution is not necessary. However, we adopt this as it will lead to a closed form of the choice probability of whether to enter or not (to be defined later). This will be very handy for the empirical analysis.

and if they decide to bid they receive ϵ_2 in the current period. The error ϵ is independent of the seismic survey signals and they are not errors associated with the analysis of seismic surveys. Hence they do not affect the actual amount of bids submitted. These errors are like bidders' action specific shocks, shocks received while going to bid, or following the arguments of Bajari & Hortacsu ('03) and Mckelvey & Palfrey ('95); errors made while deciding to bid. These errors could also be interpreted as the cost of conducting the seismic surveys. Let the actual number of bidders be A.

3) If they decide to bid then they further incur a sunk cost K to analyze the survey and submit a bid b_w[11].

4) Any bidder submitting a bid b_w, wins the auction with probability $\eta(b_w, B_w)$, where B_w is the maximum of the rival bid.

5) If they win the auction they learn the valuation U.

6) Next period bidders decide how much to bid for the neighboring drainage tracts :b_d. Bidders draw their signals about the valuations of the drainage tract v_i, from continuous asymmetric distributions $F_i(s^d|v)$. They win the drainage tract with a probability $\eta(b_d, B_d, D)$, where D is the state (location) variable of bidder and B_d being the maximum of the rivals' bid.

We assume that the auctioneer (government), does not act strategically and always sells the objects to the highest bidder via two independent first price auctions.

In this game bidders have three kinds of choices : a discrete choice of whether or not to bid in an wildcat tract, and a continuous choice of how much to bid for the wild cat tract and a second continuous choice of how much to bid for the drainage tract. Thus the number of bidders in an wildcat tract is endogenous. However all the potential bidders bid for a drainage tracts[12]. Thus there is no endogeneity of number of bidders in a drainage tract. Some of them may bid zero though depending on their signals received.

[11] In reality, before any wildcat sale potential bidders hire a geophysical firm to "shoot" a seismic survey for a large (50 block) area, and bear the cost (US$12 million) jointly. The interpretation of this survey across firms and they receive a signal s^w, unknown to other bidders. They typically reject half of the tracts in the 50 block area. If a firm does not reject the tract, it often purchase more data and shoot "'infill" or "cross-diagonal" lines on selected blocks by incurring additional costs (ϵ in our model) to build a better picture of the model. The cost of the information upgrade in an area is between US$1/2m to US$1 million. In addition the firm must pay for the inhouse expertise to interpret the data (K in our model).

[12] This model is extendable to allow entry in drainage auction too. Please see section 4. However, for simplicity I abstract away from entry in drainage tracts.

3.3 Value Function

The value of the wildcat tract, to the i^{th} bidder, V_i, can be written as follows:

$$V_i(s_i^w, \epsilon) = \max_{\alpha_i \in \{1,2\}} \{Q_{1;i}(s_i^w) + \epsilon_1, Q_{2;i}(s_i^w) + \epsilon_2\}, \quad i = \{1, 2\} \tag{2}$$

Equivalently,

$$V_i(s_i^w, \epsilon) = \max_{\alpha_i \in \{1,2\}} \{[Q_{1;i}(s_i^w) + \epsilon_1] I(\alpha_i = 1) + [Q_{2;i}(s_i^w) + \epsilon_2] I(\alpha_i = 2)\}$$

where $\alpha_i = 1$ represents not bidding and $\alpha_i = 2$ represents bidding, $I\{.\}$ is the indicator function, $Q_1 + \epsilon_1$ is the expected value to the bidder if he decided not to bid for the wild cat auction, and $Q_2 + \epsilon_2$ is the expected value of bidding for the wildcat tract. He receives an action specific shock ϵ_k. We assume that ϵ_k 's are independently drawn from a logistic distribution.

Integrating over the $\epsilon_k' s$, we get the smoothed value function, V_i^σ, the value function before the bidder observes his action specific shocks ϵ_k ($i.e$, before incurring the cost for more "infill"),

$$V_i^\sigma(s_i^w) = \int V_i(s_i^w, \epsilon) d\Phi(\epsilon) = \max_{\alpha_i \in \{1,2\}} \{[Q_{1;i}(s_i^w) + E(\epsilon_1)] I(\alpha_i = 1) + [Q_{2;i}(s_i^w) + E(\epsilon_2)] I(\alpha_i = 2)\}$$

Under the assumption of logistic distribution of $\epsilon_k' s$, $E(\epsilon_k) = \lambda - \ln(P(\alpha_i = k))$, $k = \{1, 2\}$, where λ is the Euler's constant.

If a bidder decides to bid $b_w(s^w)$ for the wild cat tract, he wins the tract with probability $\eta_w(b_w, B_w)$, when the maximum of his rival's bid B_w. His next period state will be D. Next period he will then bid for the drainage tract in the same location. Let the expected value from bidding for the drainage tract be $T_i(D)$.

Thus,

$$\begin{gathered} Q_{2;i}(s_i^w) = \max_{b_W \geq 0} [\{\Pr(\alpha_{-i} = 2 | s_i^w)\{(U - b_w) \times \eta_W - K \\ + \beta \textstyle\sum_{j=1}^2 \Pr(j \text{ wins} W_A) \int T_i(D) dF(s^d, D)\} \\ + \Pr(\alpha_{-i} = 1 | s_i^w)\{(U_0 - b_w) + \beta \int T_i(i \text{ is strong}) dF_1(s^d, D)\}] \end{gathered} \tag{3}$$

where

$$\Pr(\alpha_{-i} = 2 | s_i^w) = \int \Pr(\alpha_{-i} = 2 | s_{-i}^w) dF_w(s_{-i}^w | s_i^w)$$

$\Pr(\alpha_{-i} = 2 | s_{-i}^w)$ is the probability that the rival decides to bid, given rival's privately received signal s_{-i}^w. This is then integrated over s_{-i}^w given that the i^{th} bidder received a signal s_i^w; the relevant distribution function being $F_w(s_{-i}^w | s_i^w)$, which gives us the probability that bidder i assigns that his rival enters, given that the i^{th} bidder received a signal s_i^w. We shall characterize $\Pr(\alpha_i | s_i^w)$ shortly.

$\{(U - b_w) \times \eta_W - K\}$represents bidder i's expected value from bidding in the wildcat tract, and $T_i(D)$ is his expected value from bidding the drainage tract discounted by β. For notational simplicity, I have suppressed the arguments of the probability of winning; $\eta_W(b_w, B_W)$, and $\int T_i(D) dF(s^d|v, D) dF(v)$ is the ex-ante value from the drainage auction, before bidders receive their signals about the drainage tracts[13].

The terms within the first curly parentheses represents bidders' expected valuations when he has a rival. The terms within the second curly parentheses represents bidders $i's$ expected valuation when he does not have a rival. Both terms being weighted by the probability of whether he has a rival or not.

Note that using affiliation of s^w's it can be easily shown that, $\Pr(\alpha_{-i} = 2|s_i^w)$ first order stochastically dominates $\Pr(\alpha_{-i} = 2|s_i^{w\prime})$ for $s_i^w > s_i^{w\prime}$.

If the bidder decides not to bid for the wildcat tract, he will receive his discounted expected valuation from the drainage tract where for sure he will be a weak bidder if his rival enters this period and otherwise would face the same problem and value $V_i^\sigma(s_i^{w\prime})$ again next period. Hence

$$Q_{1;i}(s_i^w) = \{\beta \Pr(\alpha_{-i} = 2|s_i^w) \times \int T_i(1 \text{ is weak}) dF_0(s^d|v) + \beta \Pr(\alpha_{-i} = 1|s_i^w) \times \int V_i^\sigma(s_i^{w\prime}) dF_w(s_i^{w\prime})\} \quad (4)$$

The expected value from the drainage tract can be written as

$$T_i(D) = \int \max_{b_D \geq 0} \{(v_i - b_D) \times \eta_D(D, .)\} dF(v|s^d, D)$$

From the bidder's perspective, the drainage tract being auctioned next period. Hence its expected value depends on the next period's state D. This in turn depends on bidders choices and outcomes in the wildcat auction this period.

We assume that the drainage auction is an affiliated private model, hence $\int v_i dF(v|s^d, D) = s_i^d(D)$.

Note that we can write the value function as a mapping into itself.

$$V_i^\sigma = \Lambda(V_i^\sigma) \quad (5)$$

Using standard arguments in the literature (see Bhattacharya & Majumdar ('89), Theorem 3.2,), it can be shown that the mapping Λ, and a unique solution to the value function exists.

[13]Note that
$F(s^d|v, D = i \text{ is strong}) = F_1(.|v)$, and $F(s^d|v, D = i \text{ is weak}) = F_0(.|v)$

3.4 Equilibrium

We solve for symmetric perfect Bayesian equilibrium in monotone strategies for each stage game. Solution method will involve backward induction. We shall first solve the drainage tract auction as a function of bidders state variable next period D. We shall substitute this solution in the value function and solve the wildcat auction which determines D.

More specifically, three stages are:

1) Solve the third stage problem of bidding in drainage auctions, as a function of state variable D,denoted by $T(D)$, which also determines part of Q_1.

2) Substitute $T(D)$ in the second stage problem to solve for how much to bid for the wildcat auction. This determines D, and part of Q_2, excluding the probability weight of rival discrete actions.

3) Compare Q_2 $+\epsilon_2$and $Q_1 + \epsilon_1$ to determine whether to bid or not for the wildcat auction in the first stage. This will determine the equilibrium probability of winning.

3.4.1 Last Stage Decision: Analysis of Drainage Tracts

The distribution of valuations of strong and weak bidders are given by $F_1(|.v)$ and $F_0(.|v)$ respectively. Note that strong bidder receive more 'accurate' signal as defined before. Let $v_1(s_1^d, s_0^d) = E[V_1|S_1^d = s_1^d, S_0^d = s_0^d]$ be the expected value to bidder 1 when he received a signal s_1^d and his rival received a signal s_0^d. $v_0(s_1^d, s_0^d)$ is defined analogously. Bidder's valuations are interdependent. i.e., $v_i(s_i^d, s_j^d)$ is non-decreasing in s_i^d, s_j^d, $i \neq j, \{i,j\} \in \{0,1\}$, Moreover we assume v_i is strictly increasing in s_i^d and $v_i(0,0) = 0$. Henceforth, we will suppress the arguments of v_i.

Let the bid by two bidders be b_1^d and b_0^d respectively. Let each bidder adopts monotone bidding strategy $b_i^d(s_i)$ with an inverse $\phi_i(b_i^d)$. Thus by bidding b, the strong bidder wins the auction with probability $\Pr(b_0^d \leq b) = G_0^d(b) = \Pr(\phi_0(b_0^d) \leq \phi_0(b)) = F_0(\phi_0(b))$, similarly the weak bidder wins the auction with probability $G_1^d(b)$. Let the supports of the distributions of signals be $[\underline{s}_i, \overline{s}_i]$ for $i \in \{0,1\}$. In general we may have $\underline{s}_0 \leq \underline{s}_1 \leq \overline{s}_1 \leq \overline{s}_0$ as described below. $\underline{s}_0$ ---- $\overbrace{s_0}$ $\downarrow$ ---- $\underline{s}_1$ ---------- $\overline{s}_1$ ------- $\overline{s}_0$

Proposition 2 *a) There exists a pure strategy equilibrium of the drainage auction. It is characterized by the following conditions*

$$\frac{F_0'}{F_0}\phi_0'(b) = \frac{1}{v_1 - b} \tag{6}$$

and

$$\frac{F_1'}{F_1}\phi_1'(b) = \frac{1}{v_0 - b} \tag{7}$$

in the common support of signals, i.e., for all $s_0, s_1 \in [\underline{s}_1, \underline{s}_1]$*, satisfying the boundary conditions,* $b_0(\overline{s}_0) = b_1(\overline{s}_1) = \overline{b}$, $b_0(\underline{s}_0) = b_1(\underline{s}_1) = \underline{b}$*, The equilibrium pair of inverse bid function is given by,* $\phi_i(b^d) = b_i^{-1}(b^d)$, $i \in \{0, 1\}$*,where* $s_i^d = \phi_i(b)$*, is the inverse bid function. When the weak bidder has a signal* $\underline{s}_0 \leq s_0 \leq \underline{s}_1$*, then one equilibrium solves the differential equation* $\frac{F_1'(\underline{s}_1)}{F_1(\underline{s}_1)}\phi_1'(b) = \frac{1}{v_0 - b_0}$*, and strong bidder submits a bid* 0*.*

Moreover we can rank the ex-ante values from the drainage auction as

$$\int T(i \text{ is strong}) dF(v|s^d, D) dF_1(s^d) > \int T(i \text{ is weak}) dF(v|s^d, D) dF_0(s^d)$$

Proof. Details are given in the appendix. Here is a sketch, first note that the utility function satisfies the single crossing property and affiliation of signals guarantees that the distribution of valuations is log-supermodular (see Athey('00)). Hence the assumptions of theorem 4.10 of Athey ('00) is satisfied and a pure strategy equilibrium exists. The exact characterization of the equilibrium in terms of the differential equations given above then can be found by taking the first order conditions with appropriate boundary conditions[14]. ■

3.4.2 First Stage Decision: Analysis of Wildcat Auction

Note that, the first stage decision of whether to bid, and how much to bid for the wildcat auction is formulated in such a way that if the seismic information does not reveal a signal above the threshold level, then not bidding is optimal. Thus number of actual bidders is endogenous in the wildcat auction.

The threshold level of signal, defined as the lowest signal at which a bidder believes the value of the tract conditional on winning (in a symmetric equilibrium) is not worth bidding, is given by,

$$\lim_{s_i^w \downarrow s^{w*}} \sup \Pr(\alpha_i = 1 | s_i^w) \to 1 \tag{8}$$

i.e., it is the supremum of the limiting signal when not bidding becomes optimal.

Below, we define the conditional choice probability (CCP) of the decision to bid for bidder i, given by,

$$\Pr(\alpha_i = 2 | s_i^w) = \int I\{\alpha_i = 2 | s_i^w\} d\Phi(\epsilon)$$

where α_i is the optimum decision of bidder i and s_i^w is the vector of signals received by bidder i and his rival.

Equivalently,

$$\begin{aligned} \Pr(\alpha_i = 2 | s_i^w) &= \Pr(Q_1 + \epsilon_{1,i} \leq Q_2 + \epsilon_{2,i} | s_i^w) \\ &= \Pr(\epsilon_{1,i} - \epsilon_{2,i} \leq Q_2 - Q_1 | s_i^w) \end{aligned}$$

[14] We assume throughout in the paper that the second order condition is satisfied.

and

$$\Pr(\alpha_i = 1|s_i^w) = 1 - \Pr(\alpha_i = 2|s_i^w)$$

Given our assumption about the unobservable ϵ_i follows a logistic distribution, we have a closed form solution for the conditional choice probability,

$$\Pr(\alpha_i = 2|s_i^w) = \frac{\exp(Q_{2;i}(s_w)/\rho)}{\sum_k \exp(Q_{k;i}(s_w)/\rho)}$$

where ρ is the smoothing parameter.

Note that, Q_2 depends on the entry choices (P_{-i}) of other bidder too; i.e. $Q_2(P_{-i})$.

All the bidders in the wildcat tracts are symmetric. Hence in equilibrium, $\Pr(\alpha_i = 2|s^w) = p^*$, for all $i = 1, 2$. $k = 1, 2$.

Hence, writing in vector notations, for each location,

$$p^*(s_i^w) = \frac{\exp(Q_{2;i}(p^*)/\rho)}{\sum_k \exp(Q_{k;i}(p^*)/\rho)} \tag{9}$$

for all $i = 1, 2$. Note that by symmetry we mean conditional symmetry, i.e., bidders with same signals follow same strategies, and strategies are monotone.

Let Y_w be the maximum signals of the rivals bid in the wildcat auction if there is a rival and zero otherwise[15], then the probability of bidder i wins the auction is the probability that his bid is higher than Y_w. Let each bidder adopts the monotone bidding strategy $b(s^w)$ with an inverse $\phi(b^w)$.

Then,

$$\begin{aligned}\Pr(i \text{ wins } W_A) = G^w_{B_w|b_i^w}(y|b_i^w) &= \Pr(b_i^w > B_w = \max b_j^w,\ for\ j \neq i| \ni\ A \text{ bidders}) \\ &= \Pr(\phi(b_i^w) > \phi(b_j^w),\ for\ j \neq i| \ni\ A \text{ bidders}) \\ &= \Pr(\alpha_{-i} = 2|s_i^w)F^w(\phi(b_{-i}^w)) + \Pr(\alpha_{-i} = 1|s_i^w)\end{aligned}$$

where the first line states that the bidder wins the auction if his bid is higher than the maximum of his actual rivals' (A) bid (B^w). The second line uses monotonicity of the bidding strategy[16] of actual bidders to express distribution of valuations signals in terms of equilibrium bid distribution of the rival bidder $(G^w_{B^w}(b_i^w))$. The third line weights these probabilities by the probability of entry of potential bidders. The first term is the probability that the rival enters, In that case bidder i wins with probability

[15] This is necessary as it will be obvious below. It may also be a reserve price in the presence of a reserve price.

[16] Note that, when we have endogeneous bidders, as in the wildcat auction, affilation (supermodularity) is not sufficient to ensure the existence of an increasing bid function. Mcaffee, Quan, and Vincent ('02) have analyzed this case and gave a sufficient condition in terms of log-supermodularity. We assume that holds here. We shall come back to it later.

$F^w(\phi(b^w_{-i}))$. The second term is the case when the rival does not enter, in that case, conditional on bidding, bidder i wins with probability one.

Let $M_{Yi|s_w} = \Pr(\alpha_{-i} = 2|s^w_i)F^w(\phi(b^w_{-i}))$, be the distribution of maximum signal of bidder's rival when his rival enters. Let the associated density function be $m_{Y_w|s_w}$. Then,

$$\begin{aligned} G^w_{B_w|b^w_i}(y|b^w_i) &= M_{Y_w|s_w}(Y_w|s^w_i) + \Pr(\alpha_{-i} = 1|s^w_i) \\ &= M_{Y_w|s_w}(\phi(B_w)|\phi(b^w_i)) + \Pr(\alpha_{-i} = 1|\phi(b^w_i)) \end{aligned} \tag{10}$$

Let g^w be the density function associated with G^w.

Lemma 3 *The equilibrium bidding rule of the wildcat bidding game can be characterized by the following equations:*

$$b_W = U - [M_{Y_w|s_w}(s^w_1|s^w_1) + \Pr(\alpha_{-i} = 1|s^w_i)] \times \frac{b'(s^w_1)}{m_{Y_w|s_w}(s^w_1|s^w_1)} + \beta \int [T(1\ is\ strong) - T(1\ is\ weak)]dF(s^d|v, X')dF(v) \tag{11}$$

with the terminal condition, $b_W(s^{w^*}) = 0$.

and writing in terms of the distribution of bids,

$$b_W = U - \frac{G^w_{B_w|s_w}(b^w_1|b^w_1)}{g^w_{B_w|s_w}(b^w_1|b^w_1)} + \beta \int [T(1\ is\ strong) - T(1\ is\ weak)]dF(.|v, X')dF(v) \tag{12}$$

Proof. *In the appendix.* ■

The first order condition states that bid equals expected valuation plus a mark down and plus a markup. The markdown accounts for the level of competition in the wildcat sale. The mark up accounts for the discounted incremental effect on the future discounted profits if bidder 1 wins the contract instead of another firm.

Proposition 4 *The equilibrium of the entire game is characterized by the equations 6, 7 ,9and 11*

Proof. (Sketch) Existence of the equilibrium is a complicated issue. Here is a brief sketch of the arguments which are yet to be formalized. The game has three stages (two periods). The first period consists of two stages and the second period one. The first stage decision is whether to enter the wildcat auction or not. Second stage is how much to bid for wildcat and the third stage is how much to bid for the drainage tracts. The proof of existence of equilibrium will involve backwards induction. The third stage is a game of affiliated private value auctions with asymmetric buyers, asymmetry arising from the second stage outcome. The second stage is a bidding game of pure common value models with symmetric bidders, in the sense that they receive the signals from the same distribution. The first stage is a game of entry with symmetric potential bidders. We shall first solve the last stage game The existence of equilibrium in the stage game, follows from the analysis Jackson,Simon, Swinkels and Zane ('02), Maskin & Riley ('96,'00) and Athey ('00). We shall plug the equilibrium values of $T(D)$, in the second stage problem and solve for the bidding strategies for bidder who has decided to enter. This is common value environment with independent types and the existence of equilibrium in this stage game again follows from theorem 2 of Maskin & Riley('00b) given opponents' entry behavior. The existence of equilibrium of the entire game now involves solving for the equilibrium entry probabilities P. Note that given the second and third stage equilibrium decisions of bidders, and the assumptions of *iid* extreme value distribution of $\epsilon' s$, the equilibrium entry probability is a continuous function of rival's expected behavior (see equation 9) . Hence Brower's fixed point theorem applies and an equilibrium exists. ■

Proposition 5 *There will be excessive entry in the wildcat auctions if there is a drainage auction in the following period, compared to if there were no drainage auction in the second period. i.e., the threshold level of signal is lower in the former case.*

Proof. In the Appendix. ■

4 Identification

4.1 Identification of Valuations in the Drainage Auction

In this section we establish the identification of the distribution of signals from the observed bidding behavior of bidders for the drainage and wildcat auctions respectively. Note that, for simplicity of exposition, in our theoretical analysis we have so far assumed that there are only two bidders. For the identification and estimation of the OCS auction data we generalize the first order conditions to more than two bidders below. We assume that there are two types of bidders in the drainage auctions. Let there be n_{1d} strong bidders (type 1) and n_{0d} weak bidders (type 0). Note that n_{1d} and n_{0d} are endogenous. They are determined by who has won in the wildcat auctions. However, before bidding for the drainage auctions, bidders (and the econometrician) can observe n_{1d} and n_{0d}.

Let

$$v_{1i}(s_{1i}^d, y_{1i}^d, y_{0i}^d) = E[V_{1i}^d | S_{1i}^d = s_{1i}^d, Y_1^d = \max_{j \in strong, j \neq i,} s_{1j}^d = y_{1i}^d, Y_0^d = \max_{j \in Weak} s_{0j}^d = y_{0i}^d]$$

be the expected value to bidder i of type 1 when he received a signal s_{1i}^d and the maximum signal of his rival of type 1 received a signal y_{1i}^d, and that of his rival of type 0 be y_{0i}^d. $v_{0i}(s_{0i}^d, y_{1i}^d, y_{0i}^d)$ is defined analogously. Note that bidders are asymmetric across the groups but symmetric within each group. Hence $v_{1i}(s^d_{1i}, y^d_{1i}, y^d_{0i}) = v_1(s^d_{1i}, y^d_{1i}, y^d_{0i})$ for all $i \in$Type 1, and $v_{0i}(s^d_{0i}, y^d_{1i}, y^d_{0i}) = v_0(s^d_{0i}, y^d_{1i}, y^d_{0i})$,for all $i \in$Type 0. We assume $v_k(s^d_{ik}, y^d_{ki}, y^d_{ji})$ is non-decreasing in $s^d_{ik}, y^d_{ki}, y^d_{ji}$, for all i, and $j \neq k, \{j,k\} \in \{0,1\}$, Moreover we assume v_k is strictly increasing in s^d_{ik} and $v_k(0,0,0) = 0$. Henceforth, for notational simplicity, we will suppress the arguments of v_k.

Each of 'strong' bidders receives a private signal[17] s^d_{1i} about his valuation v^d_1 and chooses b^d_{1i} to maximize $E[(v_{1i} - b^d_{1i})I(B^d_{-i} \leq b^d_{1i})|s_{1i}]$, where $B^d_{-i} = \max\{b^d_{1i}(s^*_{1i}), b^d_{0i}(s^*_{0i})\}$, $y^*_{1i} = \max_{j\neq i,j} s^d_{1j}$ and $y^*_{0i} = \max_{j\neq i,j} s^d_{0j}$. and $b^d_{1i}(.)$ and $b^d_{0i}(.)$ are the equilibrium strategies of 'strong' and 'weak' bidders respectively. We restrict our attention to symmetric, strictly increasing and differentiable equilibrium strategies. By 'symmetry' we mean symmetry within each sub-group of strong and weak bidders. Since we have modeled drainage auctions as an APV model, we have $E(v^d_{1i}|s^d_{1i}) = s^d_{1i}$., hence the problem of a representative 'strong' bidder is thus

$$
\begin{aligned}
&max_{b^d_{1i}}(v^d_{1i} - b^d_{1i})\Pr(y^*_{1i} \leq \phi^d_1(.) \; and \; y^*_{0i} \leq \phi^d_0(.)|s^d_{1i}) \\
&= max_{b^d_{1i}}(v^d_{1i} - b^d_{1i})F_{y^*_1,y_0|s_{1i}}(y^*_{1i} \leq \phi^d_1(.),\, y^*_{0i} \leq \phi^d_0(.)|s^d_{1i})
\end{aligned}
$$

where $\phi^d_1(.)$ and $\phi^d_0(.)$ are the inverse of the equilibrium strategy $b^d_{ji}(.), j = \{0,1\}$.

We derive the equilibrium conditions below for the common support of signals,

Differentiating with respect to b^d_{1i}, we get the first order differential equation

$$
\begin{aligned}
-F_{y^*_1,y_0|s_{1i}}(\phi^d_1(b^d_{1i}), \phi^d_0(b^d_0)|s^d_{1i}) + (v^d_{1i} - b^d_{1i})[\tfrac{\partial F_{y^*_1,y_0|s_{1i}}(\phi^d_1(b^d_{1i}),\phi^d_0(b^d_0)|s^d_{1i})}{\partial y^*_1} \times \tfrac{1}{b^{d\prime}_1(\phi_1(b_{1i}))} \\
+\tfrac{\partial F_{y^*_1,y_0|s_{1i}}(\phi^d_1(b^d_{1i}),\phi^d_0(b^d_0)|s^d_{1i})}{\partial y_0} \times \tfrac{1}{b^{d\prime}_0(\phi_0(b_{1i}))}] = 0
\end{aligned}
$$

for all $s_{1i} \in [\underline{s}^d_1, \overline{s}^d_1]$, where $b^d_{1i} = b^d_{1i}(s^d_{1i})$, with the boundary condition $b^d_1(\underline{s}^d_1) = \underline{s}^d_1$.

Similarly for the weak bidders we have the first order condition,

$$
\begin{aligned}
-F_{y_1,y^*_0|s_0{}_i}(\phi^d_1(b^d_{1i}), \phi^d_0(b^d_0)|s^d_{0i}) + (v^d_{0i} - b^d_{0i})[\tfrac{\partial F_{y_1,y^*_0|s_{1i}}(\phi^d_1(b^d_{1i}),\phi^d_0(b^d_0)|s^d_{1i})}{\partial y_1} \times \tfrac{1}{b^{d\prime}_1(\phi_1(b_{0i}))} \\
+\tfrac{\partial F_{y_1,y^*_0|s_{0i}}(\phi^d_1(b^d_{1i}),\phi^d_0(b^d_0)|s^d_{0i})}{\partial y^*_0} \times \tfrac{1}{b^{d\prime}_0(\phi_0(b_{0i}))}] = 0
\end{aligned}
$$

To establish identification of distribution of private signals we need to uniquely express the distribution of observed bids in terms of the distribution of signals assuming that the bidders follow equilibrium strategies.

Now, observe that the conditional distribution of bids are given by

[17]Note that, the assumption of APV would imply that $v_k(s^d_{ik}, y^d_{ki}, y^d_{ji}) = s^d_{ik}$. The signals are still affiliated and hence so are s^d_{ik}, y^d_{ki} and y^d_{ji}.

$$
\begin{aligned}
G_{B_1^{d*},B_0^d|b_1^d}(X,X|x) &= \Pr(B_1^{d*} \leq X, B_0 \leq X | b_1^d = x) \\
&= \Pr(y_1^{d*} \leq \phi_1^d, y_0^d \leq \phi_0^d(X) | s_1^d = \phi_1^d(x)) \\
&= F_{y_1^*,y_0|s_{1i}}(\phi_1^d(X), \phi_0^d(X) | \phi_{1i}^d(x))
\end{aligned}
$$

Differentiating we get,

$$
\begin{aligned}
\frac{dG_{B_1^{d*},B_0^d|b_1^d}(X,X|x)}{dX} &= \frac{\partial F_{y_1^*,y_0|s_{1i}}(\phi_1^d(X),\phi_0^d(X)|\phi_{1i}^d(x))}{\partial y_1^{d*}} \times \frac{1}{b_1^{d\prime}(\phi_1^d(X))} \\
&+ \frac{\partial F_{y_1^*,y_0|s_{1i}}(\phi_1^d(X),\phi_0^d(X)|\phi_{1i}^d(x))}{\partial y_0^d} \times \frac{1}{b_0^{d\prime}(\phi_0^d(X))}
\end{aligned}
$$

Using the above we can rewrite the first order conditions for the strong bidders as

$$
v_1^d = b_1^d + \frac{G_{B_1^{d*},B_0^d|b_1^d}(b_1^d, b_1^d | b_1^d)}{dG_{B_1^{d*},B_0^d|b_1^d}(b_1^d, b_1^d | b_1^d)/dX} = \xi_1^d(b_1^d, G) \tag{13}
$$

Similarly for the weak bidders, we have

$$
v_0^d = b_0^d + \frac{G_{B_1^d,B_0^{d*d}|b_1^d}(b_0^d, b_0^d | b_0^d)}{dG_{B_1^d,B_0^{d*d}|b_1^d}(b_0^d, b_0^d | b_0^d)/dX} = \xi_0^d(b_0^d, G) \tag{14}
$$

The following lemma establishes the identification of distribution of private signals from bid distribution.

Lemma 6 *a)The affiliated distribution of privately observed signals for the drainage auction are non-parametrically identified from the observed bids.*

b) The expected value of winning the drainage auctions, $T(i\ \textit{is strong})$ and $T(i\ \textit{is weak})$ are identified from the observed bids.

Proof. In the Appendix. ■

4.2 Identification of Valuations in the Wildcat Auctions

The first order conditions associated with the Bayesian equilibrium strategies for the bidders who have already entered to bid in the wildcat auction is given by

$$
b_W = U - \frac{G^w_{B_w|s_w}(b_1^w | b_1^w)}{g^w_{B_w|s_w}(b_1^w | b_1^w)} + \beta \int [T(1\ \textit{is strong}) - T(1\ \textit{is weak})] dF(.|v, D) dF(v) \tag{15}
$$

Lemma 7 *Given β, the distribution of U from the common value model for wildcat auction is identified for the bidders who submitted bids, from the observed distribution of bids and the data on actual and potential number of bidders*[18].

Proof. In the Appendix. ■

5 Estimation Strategy

The structural parameters of interest are F_w, F_1^d, F_0^d, K, and U_0. The estimation of our model will proceed in three stages. In the first stage we will estimate F_1^d, F_0^d non-parametrically, in the second stage we will estimate F^w non-parametrically, in the third stage we will estimate the 'entry parameters' K and U_0.

First Step :In the first step we shall estimate the bid distributions from the drainage auctions for a particular location and recover the 'pseudo' values using the equations 24 and 14. Note that the right hand side of both these equations are represented in terms of the distributions of observed bids. We therefore need to estimate $\frac{G_{B_1^{d*},B_0^d|b_1^d}(C,C|c)}{dG_{B_1^{d*},B_0^d|b_1^d}(C,C|c)/dC}$ and $\frac{G_{B_1^d,B_0^{d*d}|b_1^d}(b_1^d,b_1^d|b_1^d)}{dG_{B_1^d,B_0^{d*}|b_1^d}(b_0^d,b_0^d|b_0^d)/dC}$. The standard procedure for estimation as developed in GPV('00) or LPV('02) does not apply here as if both n_1^d and n_0^d are strictly positive then the terms above involves a trivariate distribution and a total derivative, see Campo, Perrigne and Vuoung ('03) for more details.

The ratio in 24 can be interpreted as

$$\frac{\Pr(B^{d*} \leq b_1, B_0^d \leq b_1, b_1 = b_1)}{\Pr(B^{d*} = b_1, B_0^d \leq b_1, b_1 = b_1) + \Pr(B^{d*} \leq b_1, B_0^d = b_1, b_1 = b_1)}$$

Let L be the number of auctions and $K(.)$ be a kernel.

Note that the term $G_{B_1^{d*},B_0^d,b_1^d}(x,y,z)$ can be estimated as

$$G_{B_1^{d*},B_0^d,b_1^d}(x,y,z) = \frac{1}{h_{g_1}L}\sum_{l=1}^{L}\frac{1}{n_1^d}\sum_{i=1}^{n_1^d}1(B_{1il}^{d*} \leq x)1(B_{0l}^d \leq y)K_G(\frac{z - b_{1il}}{h_{g_1}})$$

Similarly $G_{B_1^d,B_0^{d*},b_1^d}(b_1^d,b_1^d|b_1^d)$ can be estimated as,

[18] Note that U depends on both signals (s^w) and the actual number of bidders (A). Unless more structure is imposed the distribution of signals are not identified from the distribution of U. see Laffont & Vuoung ('96) and Li, Perrigne & Vuoung ('00) for one such approach.

$$G_{B_1^d,B_0^{d*},b_0^d}(x,y,z)=\frac{1}{h_{g_0}L}\sum_{l=1}^{L}\frac{1}{n_0^d}\sum_{i=1}^{n_0^d}1(B_{1l}^d\le x)1(B_{0l}^{d*}\le y)K(\frac{z-b_{0il}}{h_{g_0}})$$

The denominators of 24 can be estimated as,

$$\widehat{D}_{11}(x,y,z)=\frac{1}{h_{g_1}^2L}\sum_{l=1}^{L}\frac{1}{n_1^d}\sum_{i=1}^{n_1^d}K(\frac{x-B_{1il}^{d*}}{h_{g_1}})1(B_{0l}^d\le y)K(\frac{z-b_{1il}}{h_{g_1}})$$

$$\widehat{D}_{12}(x,y,z)=\frac{1}{h_{g_1}^2L}\sum_{l=1}^{L}\frac{1}{n_1^d}\sum_{i=1}^{n_1^d}1(B_{1il}^{d*}\le x)K(\frac{y-B_{0il}^d}{h_{g_1}})K(\frac{z-b_{1il}}{h_{g_1}})$$

Similarly for 14,we have

$$\widehat{D}_{01}(x,y,z)=\frac{1}{h_{g_{01}}^2L}\sum_{l=1}^{L}\frac{1}{n_0^d}\sum_{i=1}^{n_0^d}K(\frac{x-B_{1il}^d}{h_{g_1}})1(B_{0l}^{d*}\le y)K(\frac{z-b_{0il}}{h_{g_1}})$$

$$\widehat{D}_{02}(x,y,z)=\frac{1}{h_{g_0}^2L}\sum_{l=1}^{L}\frac{1}{n_0^d}\sum_{i=1}^{n_0^d}1(B_{1il}^d\le x)K(\frac{y-B_{0il}^{d*}}{h_{g0}})K(\frac{z-b_{0il}}{h_{g_0}})$$

Therefore the private values from the drainage auctions are estimated as,

$$\widehat{v}_1^d=b_1^d+\frac{\widehat{G}_{B_1^{d*},B_0^d,b_1^d}(b_1^d,b_1^d,b_1^d)}{\widehat{D}_{11}(b_1^d,b_1^d,b_1^d)+\widehat{D}_{12}(b_1^d,b_1^d,b_1^d)}=\widehat{\xi}_1^d(b_1^d,G) \tag{16}$$

$$\widehat{v}_0^d=b_0^d+\frac{\widehat{G}_{B^d,B_0^{d*},b_0^d}(b_0^d,b_0^d,b_0^d)}{\widehat{D}_{01}(b_0^d,b_0^d,b_0^d)+\widehat{D}_{02}(b_0^d,b_0^d,b_0^d)}=\widehat{\xi}_0^d(b_0^d,G) \tag{17}$$

We plug these pseudo values into the objective function to get back the equilibrium value function for the strong and weak bidders respectively. More specifically, we get

$$T_1(1\text{ is strong})=\int\max_{b_D\ge 0}\{(v_1-b_1)\times\eta_D(X',.)\}dF(s_1^d)$$

Now,

$$(v_1-b)\times\eta_D(D,.)=\frac{G_{B_1^{d*},B_0^d|b_1^d}(X,X|x)}{dG_{B_1^{d*},B_0^d|b_1^d}(X,X|x)/dX}\times G_{B_1^{d*},B_0^d|b_1^d}(X,X|x)$$

$\Rightarrow$

$$T_1(1\ is\ strong)=\int\frac{G_{B_1^{d*},B_0^d|b_1^d}(X,X|x)}{dG_{B_1^{d*},B_0^d|b_1^d}(X,X|x)/dX}\times G_{B_1^{d*},B_0^d|b_1^d}(X,X|x)f_1(s_1^d)ds_1^d$$

Now,

$$f_1\frac{\partial b^{-1}(b)}{\partial b}=g_1(b)$$

and

$$\frac{\partial b^{-1}(b)}{\partial b} = \frac{1}{\frac{\partial b(c)}{\partial c}}$$

By substituting, we get

$$T_1(1\ is\ strong) = \int \frac{G_{B_1^{d*}, B_0^d|b_1^d}(X, X|x)}{dG_{B_1^{d*}, B_0^d|b_1^d}(X, X|x)/dX} \times G_{B_1^{d*}, B_0^d|b_1^d}(X, X|x) g_1(b_1) db_1$$

Similarly,

$$T_0(1\ is\ weak) = \int \frac{G_{B_1^d, B_0^{d*d}|b_1^d}(b_1^d, b_1^d|b_1^d)}{dG_{B_1^d, B_0^{d*}|b_1^d}(b_0^d, b_0^d|b_0^d)/dX} \times G_{B_1^d, B_0^{d*d}|b_1^d}(b_1^d, b_1^d|b_1^d) g_0(b_0) db_0$$

The bid distributions are estimated using nonparametric density estimation. The integration is evaluated numerically.

Second Stage

In the second stage we first non parametrically estimate the bid distributions for each wildcat sales G^w and g^w respectively. We then plug these and T_1 and T_0 in the following first order equation characterizing the equilibrium, to get the 'pseudo' values,

$$b_W = U - \frac{G^w_{B_w|s_w}(b_1^w|b_1^w)}{g^w_{B_w|s_w}(b_1^w|b_1^w)} + \beta \int [T(1\ is\ strong) - T(1\ is\ weak)] dF(.|v, D) dF(v) \tag{18}$$

Third Stage

The estimated 'pseudo; values from the drainage and wildcat auctions help us calculate the choice specific value functions

$$Q_{2;i}(s_i^w) = \max_{b_W \geq 0} [\{\Pr(\alpha_{-i} = 2|s_i^w)\{(U - b_w) \times \eta_W - K + \beta \sum_{j=1}^{2} \Pr(j \text{ wins} W_A) \int T_i(D) dF(s^d|v, D) dF(v)\}$$
$$+ \Pr(\alpha_{-i} = 1|s_i^w)\{(U_0 - b_w) + \beta \int T_i(i \text{ is strong}) dF_1(s^d|v) dF(v)\}] \tag{19}$$

and

$$Q_{1;i}(s_i^w) = \{\beta \Pr(\alpha_{-i} = 2|s_i^w) \times \int T_i(1 \text{ is weak}) dF_0(s^d|v) dF(v) + \beta \Pr(\alpha_{-i} = 1|s_i^w) \times \int V_i^\sigma(s_i^{w\prime}) dF_w(s_i^{w\prime})\} \tag{20}$$

Now the parameters to be estimated are K and U_0 respectively.

We will exploit the discrete choice of entry to estimate the sunk cost parameter K and U_0. We shall use the choice probabilities of entry

$$p^*(s_i^w) = \frac{\exp(Q_{2;i}(p^*)/\rho)}{\sum_{k=1}^{2} \exp(Q_{k;i}(p^*)/\rho)} \tag{21}$$

to form the likelihood function

$$L(K, U_0|.) = \prod_{l=1}^{L} p_l^*(s_i^w)$$

where L is the number of wildcat auctions. Maximization of these likelihood would give us an estimate of the parameters[19] (K, U_0).

Note that given the estimates from the first two steps the right hand side of 21, is known except the discrete decision of the opponents of type i. We can follow two steps here, first note that the 21 is a continuous mapping from opponents decisions and hence a fixed point exists. We solve this fixed point by solving for the p_j, for all potential entrant j in a wildcat location by Nelder-Meade or Newton method by simultaneous solution of j equations and then form the likelihood. Thus it is a nested procedure, where the fixed point equations is solved inside the nests and the likelihood function is maximized.

In another procedure we may avoid computing the fixed point calculations. From the data if we can estimate the nonparametric choice probabilities of the opponents p_{-i} then we can plug in those estimates and maximize the likelihood. This procedure is feasible, since we have data on entry behaviors of all potential entrants and many other observed heterogeneities also. This procedure is computationally more attractive.

6 Practical Issues

The observed distribution of bids is in general highly skewed with a large number of observations in the lower end. We therefore apply log- transformation to the distribution of bids. The logarithm transformation to the distribution of bids translates equations 16 and 17 to

$$v_1^d = \exp(c_1^d)(1 + \frac{G_{C_1^{d*}, C_0^d|c_1^d}(c_1^d, c_1^d|c_1^d)}{dG_{C_1^{d*}, C_0^d|c_1^d}(c_1^d, c_1^d|c_1^d)/dX}) - 1 = \tau_1^d(c_1^d) \tag{22}$$

$$v_0^d = \exp(c_0^d)(1 + \frac{G_{C_1^d, C_0^{d*d}|c_1^d}(c_0^d, c_0^d|c_0^d)}{dG_{C_1^d, C_0^{d*d}|c_1^d}(c_0^d, c_0^d|c_0^d)/dX}) - 1 = \tau_0^d(c_0^d) \tag{23}$$

[19] We assume that the expected value of the tract if there are no rival bidder is constant across auctions, i.e., U is constant across L.

where $c = \log(1+b)$, $G_{C_1^{d*}, C_0^d | c_1^d}(c_1^d, c_1^d | c_1^d)$ is the conditional density of $(C_1^{d*}, C_0^d) = (\max_{i \neq 1} \log(1 + b_{1i}^d), \max_i \log(1+b_{0i}))$ given $\log(1+b_1)$, c_1 being chosen arbitrarily among n_1 values, and $dG_{C_1^{d*}, C_0^d | c_1^d}(c_1^d, c_1^d | c_1^d)/dX$ is the appropriate total derivative.

Since Kernel density estimators are not well estimated close to the boundaries of their support ('boundary effect'), we use trimming as used in GPV and LPV in our simulation. Specifically, for each n_i,

$$\begin{aligned} \widehat{v}_{il}^d &= \tau_i(c_{il}) \text{ if } \quad h_i \leq c_{il} \leq b_{\max} - h_i \\ &= \infty \qquad\qquad \text{otherwise} \end{aligned} \tag{24}$$

for $i = \{0, 1\}$, $l = 1, 2, ...L$.

Similar log-transformations and trimming is applied to data on wildcat bids too.

The marginal densities of $\widehat{v}_i^d$ is estimated by

$$\widehat{f}(\widehat{v}_{il}^{d*}) = \frac{1}{h_{g_i}^2 n_i^d L} \sum_{l=1}^{L} K\Big(\frac{x - \widehat{v}_{il}^{d*}}{h_{g_1}}\Big)$$

6.1 Choice of Bandwidths and Kernels

Uniform consistency of the estimation of the 'pseudo values' requires compact kernels and specific bandwidth rates (see GPV). We used triweight Kernels as has been used in the literature (see GPV, LPV). The choice of bandwidths requires more attention. We used the bandwidths $h = c(nL)^{-1/5}$, and $c = 2.978 \times 1.06\widehat{\sigma}_d$, where $\widehat{\sigma}$ is the standard deviation of bids and h uses the same formula and its values will be different in different applications depending on the value of n,L and c. for that data set.

7 Monte Carlo Simulation

7.1 Simulation of Drainage Auction

The analytical solution of asymmetric private value model is available only for a very special case. We adopt the solution presented in Krishna ('02) to generate distribution of bids and recover the private values from there. We perform this simulation for two bidders. We assume that strong bidder's valuation is distributed uniformly in $[0, w_1]$ and that of weak bidder be distributed uniformly $[0, w_2]$. Note that, here the strong bidder's valuation stochastically dominates that of the weak bidder, which is a special case of our assumption of strongness. Krishna has shown that in this case the closed form solution for the bidding rule is,

$$b_i^d(s_i) = \frac{1}{k_i s_i}(1 - \sqrt{1 - k_i s_i^2})$$

where $k_i = \frac{1}{w_i^2} - \frac{1}{w_j^2}$, $i = \{1, 2\}$.

We set $w_1 = \frac{4}{3}$ and $w_2 = \frac{4}{5}$, and generate L =100 draws from the respective uniform distributions to generate the bid data using the above equation. We then estimate the 'pseudo values' by the methods described above for the drainage auction. We used a triweight kernel and bandwidths used in Campo, Perrigne & Vuoung ('03). We present the estimated distribution of 'pseudo values' in the graph. More specifically, we used the following bandwidth $h_1 = c_1(n_1 L)^{-1/5}$, and $h_2 = c_2(n_0 L)^{-1/6}$, $c_1 = 2.978 \times 1.06\widehat{\sigma}_{d_1}$, $c_2 = 2.978 \times 1.06\widehat{\sigma}_{d_2}$.

We plotted the estimated bid distribution function in figure 1 and compared it with the actual distribution.

Insert Figure 1 About Here

8 Data

We apply our model to data[20] on sales of wildcat and drainage tracts off the coasts of Texas and Louisiana held during the period 1954 to 1990. The government sells off these tracts to the highest bidder via a sealed bid auction and charges his bid. Bidder for these tracts are oil companies. For each tract the data set contains the date of sale; acreage; location (Latitude and Longitude); the identity of all bidders and the amounts they bid; whether the government accepted the high bid; the number, date and depth of any wells that were drilled; and monthly production of oil,condensate, natural gas and other hydrocarbons through 1991. We also have information on drilling costs of wildcat and production wells obtained from annual surveys by the American Petroleum Institute. Typically an wildcat tract consists of 5000 to 5760 acres and covers on an average 0.0463 degrees of longitude and 0.0405 degrees of latitude. There are generally eight drainage tracts surrounding an wildcat tract and each one covers around 2500 acres. The strong and weak bidders are identified using the latitude and longitude information. Specifically, a strong bidder for a drainage tract is a bidder who has owned the nearest wildcat tract in the neighborhood before the drainage tract sold.

I present below a descriptive statistics of the tracts offered

$Table 1$

[20] I am grateful to Prof. Ken Hendricks and Prof. Joris Pinkse for sharing the data.

Period	# of Tracts	Tracts Receiving Bids	Bids Per Tract	Sold	Total Winning Bid
1954-1960	950	454	2.94	419	621
1961-1967	1460	841	2.95	801	1317
1968-1974	2041	1269	4.04	1103	12855
1975-1982	6811	2753	2.59	2383	26591
1983-1990	136952	8011	1.38	7582	14394

Selected Statistics as reported by (HP'89) on wildcat and drainage tracts are given below[21].

Table 2

	Wildcat	Drainage
Number of Tracts	1056	144
Number of Tracts Drilled	748	124
Number of Productive Tracts	385	86
Average Winning Bid	2.67	5.76
Average Net Profits	1.22	4.63
Average Tract Value	5.27	13.51
Average Number of Bidders	3.46	2.73

Table 3

	Wins by Neighbor Firms	Wins by Non-Neighbor Firms
No. of Tracts	59	55
No. of Tracts Drilled	47	51
No. of Productive Tracts	36	31
Average Winning Bid	6.04	4.87
Average Gross Profits	12.75	4.45
Average Net Profits	6.71	-0.42

This suggests the following major points supporting our hypothesis:

[21]We are presenting descriptive staistics for data used by HP('89). Our final data set will be different for two main reasons. First, because our data set will cover more observations as we have observation for post 1970 too. Second, we did not model joint bidding in our model. During a part of the sell period, joint bidding was allowed. We will have todrop those observations. However the basic features of the data are expected to remain same.

1) **Strong bidder has informational advantage:** Both social rents and net profits are much higher on tracts won by a strong bidder. Discounted social value as measured by ex post revenue minus drilling costs was on an average US \$12.75 million for tracts won by strong bidders and US \$4.45 million for tracts won by weak bidders.

Net profit measured as ex post revenue minus the drilling costs minus the bid was on an average US\$6.71 million for tracts won by strong bidders and only US\$-0.42 million for tracts won by weak bidders. This suggests that information however noisy has some role to play in deciding how much to bid for the drainage tracts, and considering the fact that most of the drainage tracts were won by strong bidders, it suggests evidence that the drainage tracts were under priced and reserve prices should be increased.

2) Higher gross profit per acre from drainage tracts than wildcat tracts: Average gross profit per acre measured as ex post revenue minus cost for wildcat tracts was US\$ 793.7 million and that for drainage tracts was US\$4863.8 million.

3) Number of Bidders is endogenous: All firms submit bids in less than half of the tracts offered for sale. Thus pre-bidding seismic signals may play a role in determining whether to bid or not.

4) 'Excessive Entry' in wildcat tracts: Out of the wildcat tracts sold oil was found only in 36 % of them whereas in more than 60% of the cases oil was found in drainage tracts.

8.1 Reduced Form Prediction

In this section we explore if the bidders were taking the potential profitability of the drainage tracts also into account while deciding to bid for the wildcat tracts. We only observe the decision to bid and the amount of bid for the wildcat tracts. However we have ex-post informations on tracts' value of oil, drilling cost, acerage of the drainage tracts and whether the tracts were dry or not. Although these ex-post information are also unavailable to the firm while entering and bidding for the wildcat tracts, we use them as a reasonable proxy about bidder's information level.

In the theoretical model we have identified that there will be excessive entry in the wildcat auctions and bidders will also bid higher in the wildcat auction depending on the informational advantage he will foresee as a strong bidder over the weak bidder. We take ex-post gross profit of the drainage tracts as a proxy for the profitability of the drainage tracts.

8.1.1 Entry in Wildcat

In this section we report evidence of the presence of effects of drainage tracts on the decision to bid for the wildcat tracts controlling for the competition and profitability of wildcat tracts. In the following table we report OLS regression results for entering the wildcat auction regressed on the ex-post value of drainage tracts (dv) conditional on the ex-post values of wildcat tracts (wv).

OLS Regression of Entry Decision

Dependent Variable:	No of Bidders in Wildcat	$p-$values
wv	3.030e-05	0.1638
wv^2	-2.670e-10	0.0561
Dv	3.529e-05	0.1067
Dv^2	-1.746e-10	0.0580
Constant	2.885e+00	8.61e-16

The estimated elasticity on the basis of the median level is reported below.

Elasticity Calculation for Entry Based on Median Value

Dependent Variable:	Elasticity
wv	0.038
dv	0.036

Thus conditional on the wildcat values the entering decision is significantly affected by the value of the drainage tracts. The elasticity is almost same for wildcat and drainage values.

Poisson Regression; Incidence rates on Entry

Dependent Variable:	No of Bidder in Wildcat	$p-$values
$w\pi$	1.001	0.008
$w\pi^2$	1	0.002
$D\pi$	1.002	0.001
$D\pi^2$	1	0.001

Thus,increase in the drainage tract values increases the number of bidders by 0.2%.

8.1.2 Bidding in Wildcat Tracts

Conditional on entry the bidding decision in wildcat tracts is regressed on ex-post wildcat and drainage values.

OLS Regression of BiddingDecision

Dependent Variable:	Log Bid Wildcat	$p-$ values
wv	1.252e-05	0.17
wv^2	-5.093e-11	0.38
Dv	9.353e-06	0.31
Dv^2	-6.993e-11	0.07
wn	2.984e-01	1.39e-14
Constant	1.354e+01	$<$ 2e-16

The estimated elasticity on bidding decision based on the above regression is reported below.

Elasticity Calculation Based on Median Value

Dependent Variable:	Elasticity
wv	0.02
dv	0.01
wn	0.59

Thus all the reduced form variables have expected signs. Although the estimated effects are low, but in general bids are in millions of dollars hence their absolute effects are not small.

9 Structural Estimation Results

**(Very Preliminary and Incomplete)

In this section we present the structural estimation results for only a subset of the data[22]. Main structural elements of our model is the pre-bidding expected values of the drainage tracts to the strong

[22]Specifically, I report here a specific set of drainage tracts auctioned in the period 1970-1979 where only two bidders were present and whose corresponding wildcat auctions also had two bidders. There is no specific reason but simplicity for this. More results for the entire relevant data set and the estimated sunk cost parameter from the entry game will follow.

and weak bidders, pre-bidding expected value of the wildcat tracts, the sunk cost parameter K and the value of the wildcat tract if no bidder bids U_0.

In figures 2 and 4 we represent the estimated functions of $\widehat{\xi}_1$ and $\widehat{\xi}_0$ which is the inverse of the equilibrium strategy as given by equations (16) and (17). Both these functions are increasing suggesting that the underlying valuation distribution being affiliated private values may not be rejected by the data. The estimated density of $\widehat{v}_1$ and $\widehat{v}_0$ are depicted in figures 3 and 5. The mean, median and variance of the strong and weak bidders' estimated valuations are reported in the table (all values are in US1982\$ in millions) below.

Table 4

	Strong Bidder	Weak Bidder
Mean	2.75	1.71
Median	2.57	1.71
Variance	2.02	0.39

Insert Figure $2-5$ About Here

It appears that the density of the weak bidder has less mean median and variance than the strong bidders. This suggests that weak bidders are less likely to draw high private values than the strong bidders. This is in corroboration of the special case of asymmetry of our basic model where the strong bidder has higher valuation than the weak bidder. However a more formal test of this of this is necessary[23].

In figures 6 we depict $\widehat{U}$ as estimated by equation (18) which also takes into account the possible dynamics. The estimated density of $\widehat{U}$ is represented in figure 7.

Insert Figure $6-7$ About Here

The informational rent as measured by $\frac{\widehat{v}_1-b_1}{\widehat{v}_1}$ for the strong bidder in the drainage auction and similar calculations for others in wildcat and drainage auctions are given in the following table.

Table 5

[23] A newly developed test of asymmetry is a test based on Copula by Gupta ('04) for first price auctions. In that test, we have modelled asymmetry of valuations as a particular form of 'coupledness' of the joint density in terms of their respective marginals. The association is then represented by a particular 'copula' parameter. Given the ex-post values of the oils in these tracts, we can test whether the 'association' parameter of the strong bidder's valuations with ex-post oil values is statistically higher than that of the weak bidder.

Summary Statistics of Informational Rent

Auction Type	Mean	Median	3rd Quantile
Wildcat Auction (no Dynamics)	0.65	0.7	0.81
Wildcat Auction (with Dynamics)	0.54	0.59	0.7
Drainage Auction: Strong Bidder	0.71	0.77	0.87
Drainage Auction: Weak Bidder	0.63	0.62	0.73

Thus on an average the winner's informational rent is 75% for strong bidders and 61% for weak bidders in the drainage auctions. Thus on an average the government is capturing only 39 % from the weak bidders willingness to pay and 25% from the strong bidder's willingness to pay. These preliminary numbers also suggests that the weak bidder sheds his valuation less than the strong bidder, i.e., they bid more aggressively, as argued in the theoretical analysis. These numbers are consistent but slightly different and higher to the analysis of Camp, Perrigne Vuong('03) where they analyzed asymmetry originating due to the presence of joint or solo bidders. As evident from the theoretical analysis, our model of asymmetry is different from theirs. The informational rent in the wildcat auctions are quite high too. In the wildcat auctions, we found that the bidders are willing to pay 10 % more to take possible future informational advantage. This is possibly because of the presence of the possible dynamics.

INCOMPLETE.

10 Conclusion and Ongoing Work

In this paper we have formulated a dynamic auction model where asymmetry of bidders is endogenous. The seller sells multiple units of similar goods via a sequence of first price auctions in two periods. The first period winner becomes a strong bidder in the second period bidding game in the sense that he learns his valuations better. For our application of the OCS oil tract auctions, this asymmetry is characterized by the location state variable of the bidders in the second period. If the bidder wins the first period (wildcat tract auction) then he wins an oil tract closer to the tract being sold next period (drainage tract auction).This is more informative as the strong bidder can conduct off-site drilling to gather more information about the tract to be sold than the weak bidder. The possibility of future asymmetry affects bidders' bidding behavior in the first period. The first period is characterized by an entry stage and a bidding stage. In the entry stage bidders incur a sunk cost of seismic survey to enter to bid . The possibility of future asymmetry affects both bidders' entry and bidding behavior in the first stage and leads to ‘excessive’ entry and ‘overbidding’ relative to the static auction game. We have characterized the equilibrium and established nonparametric identification of the ‘pseudo values’ in both stages. We suggested a three stage procedure to semi-parametrically estimate the model. We used data on oil tract auctions off the coast of Lousiana and Texas to estimate our model. Preliminary descriptive statistics and reduced form analysis lends support to the possibility of excessive entry and overbidding as an equilibrium behavior. We report our structural estimates of our model. We found that the government was recovering on an average only 25% strong bidder willingness to pay from the drainage auctions. In the wildcat auctions, we found that the bidders are willing to pay 10 % more to take possible future informational advantage.

We are currently extending our model in many directions. First, we are allowing entry in the second period (drainage auction) too. Since these tracts are sold sequentially and each wildcat tract is surrounded on an average by eight drainage tracts, which need not be sold on the same date, this seems a natural extension. Second, we are considering modelling the possibility of entering in different wildcat tracts (first period) sold on the same date. Since many tracts are sold on the same date, bidders do participate in selective tracts depending on the seismic surveys. Third, we have characterized what should be optimal reserve price the government should charge for wildcat tracts given bidders dynamic behavior. The distributions of ‘pseudo values’ estimated in this paper is an building block to that reserve price. This reserve price is characterized and semiparametrically estimated by a methodology similar to Li, Perrignme and Vuoung (03). Fourth, we are investigating the optimal reserve price the government should charge in the second period (drainage auctions). This is a complicated issue as this reserve price should depend on the first period bids and any observable production. But bidders while bidding in the first period will figure that out and underbid. This would lead to the so called ‘Ratchet Effect’. Fifth, this is even more ambitious. The government is currently selling the objects in two sequential first price auctions. This need not be the optimal mechanism. A first price followed by a second price, or some other mechanism may be ‘better’ for the government. This is a major theoretical issue unexplored so far.

11 Appendix

11.1 Proofs of the Theoretical Model

Proposition 8 *There exists a pure strategy equilibrium of the drainage auction. It is characterized by the following conditions*

$$\frac{F_0'}{F_0}\phi_0'(b) = \frac{1}{v_1 - b} \tag{25}$$

and

$$\frac{F_1'}{F_1}\phi_1'(b) = \frac{1}{v_0 - b} \tag{26}$$

satisfying the boundary conditions,$F_i(\phi_i(b^*)) = 1,\ i \in \{0,1\}$*, satisfying the boundary conditions,* $b_0(\overline{s}_0) = b_1(\overline{s}_1) = \overline{b}$, $b_0(\underline{s}_0) = b_1(\underline{s}_1) = \underline{b}$*, The equilibrium pair of inverse bid function is given by,* $\phi_i(b^d) = b_i^{-1}(b^d),\ i \in \{0,1\}$*,where* $s_i^d = \phi_i(b)$*, is the inverse bid function. When the weak bidder has a signal* $\underline{s}_0 \leq s_0 \leq \underline{s}_1$*, then one equilibrium solves the differential equation* $\frac{F_1'(\underline{s}_1)}{F_1(\underline{s}_1)}\phi_1'(b) = \frac{1}{v_0 - b_0}$*, and strong bidder submits a bid* 0.

Moreover, we can rank the ex-ante values from the drainage auction as

$$\int T(i \text{ is strong}) dF_1(.|v)dF(v) > \int T(i \text{ is weak}) dF_0(.|v)dF(v)$$

Proof. First note that the utility function has the single crossing property and affiliation of signals guarantees that the distribution of valuations is log-supermodular (see Athey('00)). Hence the assumptions of theorem 4.10 of Athey ('00) is satisfied and a pure strategy equilibrium exists.

To establish the conditions stated above, note that, bidder i solves the following problem,

$$T_i(D) = \max_{b_i^d \geq 0}\{(v_i - b_i^d) \times F_j(s_j < \phi_j(b)),\ i,j = \{0,1\},\ i \neq j$$

Taking logarithm and differentiating with respect to b_i^d, we get the first order conditions,

$$\frac{F_0'}{F_0}\phi_0'(b) = \frac{1}{v_1 - b}$$

and

$$\frac{F_1'}{F_1}\phi_1'(b) = \frac{1}{v_0 - b}$$

satisfying the relevant boundary conditions as described below.

For $\overline{s}_0 \geq \overline{s}_1 \Rightarrow b_0(\overline{s}_0) = b_1(\overline{s}_1) = \overline{b}$, since otherwise, if say $b_0(\overline{s}_0) \geq b_1(\overline{s}_1)$, then weak bidder wins for sure when his valuation is $\overline{s}_0$ and pays more than he needs to, he could increase his payoff by bidding slightly less than $b_0(\overline{s}_0)$.The boundary condition at the lower end point is more tricky. It is straightforward to show that though $b_1(\underline{s}_1) = \underline{b}_1$, and $b_0(\underline{s}_0) = 0$. However note that for all $s_0 \in (\underline{s}_0, \underline{s}_1)$, weak bidder loses for sure, and he is indifferent in submitting any bid or 0. However then the strong bidder would be better off in submitting $\underline{b}_1 = 0$ too. But then the weak bidder for all his signals $s_0 \in (\underline{s}_0, \underline{s}_1)$ can submit a bid slightly above zero and win the auction. Therefore the weak bidder has to submit a higher than zero bid . We conjecture that the weak bidder submitting a bid b_0, solving the differential equation $\frac{F_1'(\underline{s}_1)}{F_1(\underline{s}_1)}\phi_1'(b) = \frac{1}{v_0 - b_0}$ for all $s_0 \in (\underline{s}_0, \underline{s}_1)$, is a Bayes-Nash equilibrium, where $\phi_i(.)$ is the inverse bid function for bidder i. For all $s_0, s_1 \in [\underline{s}_1, \underline{s}_1]$, the equilibrium is characterized by the standard differential equation as characterized below. Let the supports of these distributions be $[\underline{b}_i^d, \overline{b}_i^d]$, respectively for $i \in \{0, 1\}$.

Since the equilibrium bid distributions[24] are, $G_i(b) = F_i(\phi_i(b))$, $i = \{0, 1\}$, hence $G_i' = F_i'(\phi_i(b))\phi_i'(b)$, where $'$ represents the derivative.

Hence, rewriting the first order conditions, we get everything in terms of the bid distribution functions,

$$\frac{G_0'}{G_0} = \frac{1}{v_1 - b} \tag{27}$$

and

$$\frac{G_1'}{G_1} = \frac{1}{v_0 - b} \tag{28}$$

Hence the expected value of valuations conditional on winning by the strong bidder is

$$T(i \text{ is strong}) = G_0 \times \frac{G_0}{G_0'} \tag{29}$$

Similarly,

$$T(i \text{ is weak}) = G_1 \times \frac{G_1}{G_1'} \tag{30}$$

Note that this no longer holds true for our case as Persico's problem was covert information acquisition and our case is overt information acquisition. In our case the weak bidder knows that he is weak and adjusts his bid accordingly (aggressively). The intuition of the proof is that we need to show that in equilibrium $\int T(i\ is\ strong)dF_1(.|v)dF(v) > \int T(i\ is\ weak)dF_0(.|v)dF(v)$

$\Rightarrow$ after they have received their signals for the drainage auctions (s_1^d, and s_0), then in equilibrium

$$(v_1 - b_1)F_0(.|v)dF_1(.|v)dF(v) > \int (v_0 - b_0)F_1(.|v)dF(v)$$

we have to integrate this over all the possible signals (s_1^d, and s_0), they may get, i.e., over $F_1(s_1|.)$ and $F_0(.|v)$, since they are evaluating this during the wildcat bidding entering stage. (think of equilibrium dominance here)

[24] The bid distributions exists by the existence of monotonic strategies.

Now since the weak bidder must bid higher than the strong bidder to have the same probability of winning. Let us fix this probability to the winning probability of the weak bidder (say p_0) then multiply $(v_1 - b_1)$ with this winning probability of weak bidder, since the weak bidder is bidding higher then

$(v_1 - b_1) \times p_0 \geq (v_0 - b_0) \times p_0$, now since the strong bidder is doing better even now when he is not playing the equilibrium strategy then it must be the case the strong bidder is doing better in equilibrium by incentive compatibility.

Hence the proof of the proposition. ■

Proof of Lemma 3

Now when bidder 1 decides to bids for the wildcat auction, in the simple case of two bidders he is solving the following

$$\begin{gathered}\max_{b_W \geq 0}[\Pr(\alpha_{-i} = 2|s_i^w)\{(U - b_W) \times F^w(\phi(b_j^w) \leq s_i^w \ for \ j \neq i)) - K + \epsilon_2 \\ +\beta[T(1 \ is \ strong)]F^w(\phi(b_j^w) \leq s_i^w \ for \ j \neq i)) \\ +\beta[T(1 \ is \ weak)]F^w(\phi(b_j^w) \leq s_i^w \ for \ j \neq i)\} + (U_0 - b_w)\Pr(\alpha_{-i} = 1|s_i^w)]\end{gathered}$$

where $T(X', 1 \text{ is strong})$, $T(X', 1 \text{ is weak})$ are as given above.

Equivalently,

$$\begin{gathered}\max_{b_W \geq 0}[(U - b_W) \times \Pr(\alpha_{-i} = 2|s_i^w) \times F^w(\phi(b_j^w) \leq s_i^w \ for \ j \neq i)) - K + \epsilon_2 \\ +\beta[T(1 \ is \ strong)]\Pr(\alpha_{-i} = 2|s_i^w) \times F^w(\phi(b_j^w) \leq s_i^w \ for \ j \neq i)) \\ +\beta[T(1 \ is \ weak)]\Pr(\alpha_{-i} = 2|s_i^w) \times F^w(\phi(b_j^w) \leq s_i^w \ for \ j \neq i) + (U_0 - b)\Pr(\alpha_{-i} = 1|s_i^w)]\end{gathered}$$

Equivalently, using the definitions of $M(.)$, we get,

$$\begin{gathered}\max_{b_w \geq 0}\{(U - b_W)M_{Y_w|s_1}(y|\phi(b_1^w)) - K + \epsilon_2 + \beta M_{Y_w|s_w}(y|\phi(b_1^w))[\textstyle\int T(1 \ is \ strong)dF_1(.|v)dF(v)] \\ +\beta[1 - M_{Y_w|s_w}(y|\phi(b_1^w))][\textstyle\int T(1 \ is \ weak)dF_0(.|v)dF(v)]\} + (U_0 - b)\Pr(\alpha_{-i} = 1|s_i^w)\end{gathered}$$

The first order condition implies,

$$\begin{gathered}\{(U - b_W)\tfrac{m_{Y_w|s_{wi}}(s_1^w|s_1^w)}{b'(s_w)} - M_{Y_{wi}|s_i}(s_1^w|s_1^w) + \beta\tfrac{m_{Y_w|s_{wi}}(s_1^w|s_1^w)}{b'(s_w)}[\textstyle\int T(1 \ is \ strong)dF_1(.|v)dF(v)] \\ -\beta[\tfrac{m_{Y_w|s_w}(s_1^w|s_1^w)}{b'(s_w)}][\textstyle\int T(1 \ is \ weak)dF_0(.|v)dF(v)]\} - \Pr(\alpha_{-i} = 1|s_i^w) = 0\end{gathered} \tag{31}$$

with the terminal condition, $b_W(s^{w^*}) = 0$.

Equivalently,

$$(U-b_W)\frac{m_{Y_w|s_w}(s_1^w|s_1^w)}{b'(s_1^w)} - M_{Y_w|s_{wi}}(s_1^w|s_1^w) + \beta[\frac{m_{Y_w|s_w}(s_1^w|s_1^w)}{b'(s_1^w)}]\int[T(1\ is\ strong) - T(1\ is\ weak)]dF_1(.|v,D)dF(v)$$
$$= \{(U-b_W) + \beta\int[T(1\ is\ strong) - T(1\ is\ weak)]dF(.|v,D)dF(v)\}\frac{m_{Y_w|s_w}(s_1^w|s_1^w)}{b'(s_1^w)}$$
$$-M_{w|s_i}(s_1^w|s_1^w) - \Pr(\alpha_{-i}=1|s_i^w) = 0 \tag{32}$$

Equivalently,

$$\{(U-b_W) + \beta\int[T(1\ is\ strong) - T(1\ is\ weak)]dF(.|v,D)dF(v)\}\frac{m_{Y_w|s_w}(s_1^w|s_1^w)}{b'_w(s_1^w)}$$
$$-M_{Y_w|s_w}(s_1^w|s_1^w) - \Pr(\alpha_{-i}=1|s_i^w) = 0$$

$\Rightarrow$

$$b_W = U - \{M_{Y_w|s_w}(s_1^w|s_1^w) + \Pr(\alpha_{-i}=1|s_i^w)\}\times\frac{b'(s_1^w)}{m_{Y_w|s_w}(s_1^w|s_1^w)} + \beta\int[T(1\ is\ strong) - T(1\ is\ weak)]dF(.|v,D)dF(v) \tag{33}$$

Note that, under the assumption of monotonic bidding strategies, we expressed the bid distribution function,

$$G^w_{B_w|b_i^w}(y|b_i^w) = M_{Y_w|s_w}(\phi(B_w)|\phi(b_i^w)) + \Pr(\alpha_{-i}=1|\phi(b_i^w))$$

with the density function,

$$g^w_{B_w|b_i^w} = \frac{\Pr(\alpha_{-i}=2|s_1^w)F^{w'}(s_1^w|s_1^w)}{b'(s_1^w)}$$
$$= \frac{m(.)}{b'(s_1^w)}$$

Substituting in the first order conditions, we get,

$$b_W = U - \frac{G^w_{B_w|b_i^w}(b_1^w|b_1^w)}{g^w_{B_w|b_i^w}(b_1^w|b_1^w)} + \beta\int[T(1\ is\ strong) - T(1\ is\ weak)]dF(.|v,D)dF(v) \tag{34}$$

Proposition 9 *There will be excessive entry in the wildcat auctions if there is a drainage auction in the following period, compared to if there were no drainage auction in the second period. i.e., the threshold level of signal is lower in the former case.*

Proof. (Sketch) The relative values of $Q_1+\epsilon_1$ and $Q_2+\epsilon_2$ will determine whether the bidder will decide to enter or not. Assumption of extreme value distribution of ϵ will imply a closed form solution for the choice probabilities as showed above. The threshold level of signal (s^{w^*}), below which not bidding is optimal can be written as

$$\lim \sup_{s_i^w \downarrow s^{w*}} \Pr(\alpha_i = 1|s_i^w) \to 1 \tag{35}$$

i.e., it is the supremum of that limiting signal when not bidding becomes optimal.

Now,

$$\Pr(\alpha_i = 1|s_i^w) = \frac{\exp(Q_{1;i}(s_w)/\rho)}{\sum_k \exp(Q_{k;i}(s_w)/\rho)} = \frac{1}{1 + \exp(Q_{2;i}(s_w) - Q_{1;i}(s_w))/\rho}$$

Let

$$Z_D(s^w) = \{Q_{2;i}(s^w) - Q_{1;i}(s^w)|\textit{there is a drainage auction}\}$$

and

$$Z_{ND}(s^w) = \{Q_{2;i}(s^w) - Q_{1;i}(s^w)|\textit{there is } \text{no } \textit{drainage auction}\}$$

then after we substitute the equilibrium values of $Q_{2;i}(s^w)$ and $Q_{1;i}(s^w)$ using equations 27, 28, 9and 15, it can be shown that,

$$Z_D(s^w) - Z_{ND}(s^w) = 2\beta \times \Pr(\alpha_{-i} = 1|s^w) \times \int T(1 \text{ is weak}) dF_0(.|v)dF(v) \geq 0$$

Since every thing is monotonic in s^w, comparison of the above expressions would help us conclude,

$$\begin{aligned} &\{s^{w^*}|\lim\sup_{s_i^w \downarrow s^{w*}} \Pr(\alpha_i = 2|s_i^w) \to 1, \text{ there is a drainage auction}\} \\ &\leq \{s^{w^*}|\lim\sup_{s_i^w \downarrow s^{w*}} \Pr(\alpha_i = 2|s_i^w) \to 1, \text{ there is no drainage auction}\} \end{aligned}$$

■

11.2 Identification Proofs

Lemma 10 *a)The affiliated distribution of privately observed signals for the drainage auction are non-parametrically identified from the observed bids.*

b) The expected value of winning the drainage auctions, $T($i is strong$)$ and $T($i is weak$)$ are identified from the observed bids.

Proof. a) The proof is very similar to the proof of Campo, Perigne & Vuoung ('02). Let the joint distribution of bids from the asymmetric drainage auction be $G(.)$ with the support $[\underline{b}, \overline{b}]^n$. Let there be two distributions of private values $F_d(.)$ and $\widetilde{F}_d(.)$ leading to the same joint distribution of bids. Let $b_1^d(., F)$ and $b_0^d(., F)$ and $\widetilde{b}_1^d(., F)$, $\widetilde{b}_0^d(., F)$ be the strictly increasing Bayesian equilibrium strategies corresponding to $F_d(.)$ and $\widetilde{F}_d(.)$ respectively. Therefore they satisfy the first order differential equations. Hence

$$F(s_1^d, s_0^d) = \Pr(\xi_1^d(b_1^d, G) \leq s_1^d, \xi_0^d(b_0^d, G) \leq s_0^d) = G(\xi_1^{d^{-1}}(s_1^d, G), \xi_0^{d^{-1}}(s_0^d, G))$$

$$\widetilde{F}(s_1^d, s_0^d) = \Pr(\xi_1^d(b_1^d, G) \leq s_1^d, \xi_0^d(b_0^d, G) \leq s_0^d) = G(\xi_1^{d^{-1}}(s_1^d, G), \xi_0^{d^{-1}}(s_0^d, G))$$

Hence $F(s_1^d, s_0^d) = \widetilde{F}(s_1^d, s_0^d)$ on their common support $[\underline{s}_1^d, \overline{s}_0^d]^n \doteq [\xi_0^d(\underline{b}^d, G), \xi_1^d(\overline{b}^d, G)]^n$. Hence the asymmetric APV model of the drainage auction is identified.

b) It is a standard result that if the distribution is uniquely identified and has a well defined expectation, then it has a unique expectation. Since $T(s^d)$ is an expectation with respect to the random variable s^d, hence it is unique and so is the expectation of the difference. ■

Lemma 11 *Given β, the distribution of U from the common value model for wildcat auction is identified for the bidders who submitted bids, from the observed distribution of bids and the data on actual and potential number of bidders*[25].

Proof. First note that by lemma (b) on the identification of the drainage auction, the third term is identified from data on drainage auctions.

Now recall that for the two bidder case, we had,

$$\begin{aligned} G^w_{B_w|b_i^w}(y|b_i^w) &= M_{Y_w|s_w}(Y_w|s_i^w) + \Pr(\alpha_{-i} = 1|s_i^w) \\ &= M_{Y_w|s_w}(\phi(B_w)|\phi(b_i^w)) + \Pr(\alpha_{-i} = 1|\phi(b_i^w)) \\ &= \Pr(\alpha_{-i} = 2|s_i^w)F^w(\phi(b_{-i}^w)) + \Pr(\alpha_{-i} = 1|\phi(b_i^w)) \end{aligned} \tag{36}$$

A straightforward generalization of the above for the N bidder case is

$$G^w_{B_w|b_i^w}(y|b_i^w) = \sum_{j \neq i} \Pr(\alpha_j = 2|s_i^w)F^w(\phi(b_{-i}^w)) + \Pr(\alpha_j = 1|\phi(b_i^w), \text{ for all } \ j)$$

Since $G^w_{B_w|b_i^w}(y|b_i^w)$ and $\Pr(\alpha_j = 2|s_i^w)$ and $\Pr(\alpha_j = 1|s_i^w)$ are observable from the observed data on bids and entry behavior, $F^w(\phi(b_{-i}^w))$ is identified. Hence the expected common value component U is identified using the first order condition. ■

[25] Note that U depends on both signals (s^w) and the actual number of bidders (A). Unless more structure is imposed the distribution of signals are not identified from the distribution of U. see Laffont & Vuoung ('96) and Li, Perrigne & Vuoung ('00) for one such approach.

11.3 Bootstrap

For the sunk cost K and common value element when no bidder is present U_0 we use the spatial block bootstrap procedure described below to compute the standard error.

1. Select B wildcat tracts randomly.

2.Add all tracts in the neighborhood tracts sold on or before that tract in the sample to calculate the potential bidders.

3.For each wildcat tracts selected, select all the drainage tracts associated with the wildcat tracts.

4. Perform the estimation procedure described before in three stages and estimate all the parameters.

5. Compute all statistics for the bootstrap sample.

6. Repeat steps $2-5$ B times.

Note that the block bootstrap is necessary to accommodate the spatial dependence of wildcat and drainage tracts inherent in the model.

References

[1] Arnold, B.C., N. Balakrishnan, and H.N. Nagaraja (1992) "*A First Course in Order Statistics*" New York: John Wiley & Sons.

[2] Athey, S. (2000) "Single Crossing Properties and the Existence of Pure Strategy Equilibria in Games of Incomplete Information" *Econometrica*, 69, 861-890.

[3] Athey, S, (2000) " Monotone Comparative Statics under Uncertainty" Working Paper, Stanford University.

[4] Athey, S. and P. Haile (2002) "Identification of Standard Auction Models" *Econometrica*, 70, 2107-2140.

[5] Athey, S. and Levine J. (2001) "The value of Information in Monotone Decision Problems" Working paper, Stanford University.

[6] Bajari, P. and Hortacsu, A. (2002) "Auction Models when Bidders Make Small Mistakes: Evidence from Theory and Estimation" working Paper.

[7] Bhattacharya, R.N. and M. Majumdar (1989) "Controlled Semi-Markov Models- The Discounted Case" *Journal of Statistical Planning and Inference*, 21, 365-381.

[8] Campo, S.., Perrigne, I. and Vuong, Q. (2003) "Asymmetry in First Price Auctions with Affiliated Private Values" forthcoming *Journal of Applied Econometrics.*

[9] Guerre, E., Perrigne, I., and Vuong,Q. (2000) "Optimal Nonparametric Estimation of First Price Auctions", *Econometrica*, 68,525-574.

[10] Gupta, S. (2004) "A Copula Based Semi-Parametric Estimation and Testing of First Price Auctions" Working Paper.

[11] Gupta, S. (2002) "Stochastic Approximation and Explorations of Discrete Choice Dynamic Programming Problems" Working Paper.

[12] Haile,P., Hong,H. and Shum,M. (2002) "Non-parametric Tests for Common Values in First-Price Sealed Bid Auctions" Working Paper, Unoversity of Wisconsin- Madison.

[13] Hendricks K. and R. Porter (1988) "An Empirical Study of an Auction with Asymmetric Information" *American Economic Review*, pp.865-883.

[14] Hendricks K, R. Porter and B. Boudreau (1987) " Information, Returns, and Bidding Behavior in OCS Auctions: 1954-1969" *Journal of Industrial Economics*, pp 517-542.

[15] Hendricks,K., Pinkse,J., and Porter, R.(2002) "Empirical Implication of Equilibrium Bidding in First-Price, Symmetric, Common Value Auctions" working Paper

[16] Hopkins, Ed.(2002) " Two Competing Models of How People Learn in Games" *Econometrica,* 70. pp2141-2166.

[17] Jofre-Bonet, Maria and M. Pesendorfer (2003) "Estimation of a Dynamic Auction Game" forthcoming *Econometrica.*

[18] Krishna, K. (1993) " Auctions with Endogeneous Valuations: The Persistence of Monopoly Revisited", *American Economic Review*, 83, 147-160.

[19] Laffont J.J., H. Oscard and Q. Vuoung (1995) " Econometrics of First Price Auctions", *Econometrica*, 63, 958-980

[20] Lebrun, B. (1999) "First Price Auctions in the Asymmetric N Bidder Case" *International Economic Review*, 40, 125-142.

[21] Lehmann E.L. (1988) "Comparing Location Experiments" *The Annals of Statistics, 16,* pp.521-533.

[22] Li, T., Perrigne, I. and Vuong, Q. (2002) " Structural Estimation of the Affiliated Private Values Auction Model" *Rand Journal of Economics,*33, forthcoming.

[23] Li, T., Perrigne,I. and Vuong, Q. (2000) "Conditionally Independent Private Information in OCS Wildcat Auctions" *Journal of Econometrics*, 98, 129-161.

[24] Maskin, E & J. Tirole (1988) "A Theory of Dynamic Oligopoly" I & II, *Econometrica, 56.* pp. 549-600.

[25] Maskin, E & J. Tirole (1998) "Markov Perfect Equilibrium" mimeo, Harvard University.

[26] Maskin, E. and Riley, J. (1996) "Uniqueness of Equilibrium in Asymmetric Auctions" *working paper,*

[27] Maskin, E. and Riley, J. (2000a) "Asymmetric Auctions" *Review of Economic Studies*, 67, 413-438.

[28] Maskin, E. and Riley, J. (2000b) "Equilibrium in Sealed High Bid Auctions" *Review of Economic Studies*, 67, 439-454.

[29] McAdams David (2003) " Characterizing Equilibria in Asymmetric First Price Auction" Working paper, MIT.

[30] Mcaffe,P. Quan,D. Vincent, D. (2002) "How to Set Minimum Acceptable Bids, with an Application to Real Estate Auctions", *Journal of Industrial Economics,* Dec. pp 391.

[31] Mcaffe,P.and Vincent, D. (1992) "Updating the Reserve Price in Common-Value Auctions", *American Economic Review,* May. pp 512-518.

[32] Mcaffe,P. McMillan (1987) "Auction with Entry"

[33] McKelvey and Palfrey (1995) "Quantal Response Equilibrium for Normal form Games" *Games and Economic Behavior*, 10(1) pp.6-38.

[34] Milgrom & Weber (1982) "A Theory of Auctions and Competitive Bidding", *Econometrica, 50(5), pp 1089-1122.*

[35] Milgrom & Weber (1986) "A Theory of Distributional Strategies for Games with Incomplete Information" *Mathematics of Operations Research* 10: 619-631.

[36] Persico N. (2000) "Information Acquisition in Auctions" *Econometrica, 68,* pp.135-148.

[37] Porter R. (1995) " The Role of Information in US Oil and Gas Lease Auction" *Econometrica, 63,* pp.1-27.

[38] Rao, B.L.S.P. (1992) *"Identifiability in Stochastic Models: Characterization of Probability Distributions"* Academic Press, New York.

[39] Reny P. Shmuel Zamir (2002) "On the existence of Pure Strategy Monotone Equilibria in Asymmetric First Price Auction" Working Paper, Unversity of Chicago.

[40] Seim, Katja (2003) " An Empirical Model of Firm Entry with Endogeneous Product- Type Choices" working paper, Stanford University.