Individual Euler Equations Rather Than Household Euler Equations

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Abstract

This paper focuses on the identification and estimation of the intertemporal and intratemporal elasticities of substitution for the wife and separately for the husband using individual Euler equations. To that end, the household is represented as a group of agents making joint decisions. By means of this framework, individual Euler equations are derived and used to identify and estimate the parameters of interest. The main advantage of this approach is that the key parameters can be identified for all household members and not only for the household as a whole. To implement this approach it is essential to deal with an important issue: individual Euler equations depend on individual consumption which is not observable. In this paper it is shown that individual Euler equations are identified when only data on household consumption, individual labor supply and individual wages are observed. The identification

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strategy is then used to estimate the elasticities of substitution using the Consumer Expenditure Survey.

1 Introduction

An extensive literature in public finance uses intertemporal models to study consumption, savings and labor supply and to evaluate alternative policies. To that end it is important to know which parameters govern individual behavior and to have reliable estimates of them. In dynamic models the two key parameters are the intertemporal and intratemporal elasticities of substitution. The focus of this paper is on the identification and estimation of these elasticities for the wife and separately for the husband using individual Euler equations.

In the Health and Retirement Study, allowing for only four categories of risk preferences, more than 50 percent of couples report that the wife's risk preferences differ from the husband's. It is well documented that wives are expected to live several years longer than their husbands.[1] Life-time variations in costs and opportunities - due to children, unemployment of the spouse and business cycle changes - differ dramatically between household members.[2] Consequently, to evaluate alternative policies on savings, consumption and life-cycle labor supply it is essential to model household intertemporal decisions as the joint decisions of its members. This approach is feasible only if reliable estimates of the intertemporal and intratemporal elasticities of substitution are available for the wife and separately for the husband.

As discussed in Browning, Hansen and Heckman (1999), the estimation of those two parameters is almost exclusively based on Euler equations. One of the major challenges in the estimation of Euler equations is the lack of consumption data at the individual level. The traditional solution to this problem is to assume that the household behaves as a single agent. Under this assumption, it is possible to assign a unique utility function to the whole household and derive household Euler equations, which depend only on household consumption. The results obtained with this approach are extremely important to understand

[1] See Lundberg et al (forthcoming) for a discussion on this issue.

[2] For a discussion see Heckman (1978), Killingsworth (1979) and Heckman and MaCurdy (1980).

consumption, savings and labor supply dynamics.[3] However, two shortcomings characterize this approach. First, in Mazzocco (2002) it is shown that a household can be represented using a unique utility function if and only if the conditions for Gorman aggregation are satisfied. In a static framework, Thomas (1990), Browning et al. (1994), Lundberg et al. (1997), Browning and Chiappori (1998) and Chiappori et al. (2002) find strong evidence against this assumption. In an intertemporal environment, the standard approach is rejected in Lundberg et al. (forthcoming) and Mazzocco (2002). Second, the unitary approach generates estimates for the intertemporal and intratemporal elasticities for the household as a whole, but not for individual members.

In this paper, the household is modelled as a group of agents making joint decisions and individual Euler equations are used to identify and estimate intertemporal and intratemporal elasticities. This framework has two main advantages. First, it is not affected by an aggregation problem. Second, elasticities of substitution can be estimated separately for the wife and the husband. To implement this approach it is essential to deal with an important issue: individual Euler equations depend on individual consumption which is not observable. This paper focuses on the identification and estimation of individual Euler equations when only data on household consumption, individual labor supply and wages are available, i.e. the information available in the Consumer Expenditure Survey (CEX). In particular, the paper deals with the following two issues:

a) Suppose that husband and wife are characterized by individual preferences and cooperate. Moreover, suppose that only total consumption, individual labor supplies, wages and interest rates are observed. It is shown that individual Euler equations are identified given the limited amount of information. Specifically, if both agents work, Euler equations of husband and wife can be identified up to a constant. Consequently, the intertemporal and intratemporal elasticities of substitution can be determined for both the wife and the husband. If only one agent works, Euler equations for the spouse supplying labor can be identified up to a constant. For the spouse not working, only the consumption Euler equation can be identified.

b) Using the suggested identification strategy, individual Euler equations are then esti-

[3] For a survey on Euler equations see Browning and Lusardi (1996).

mated by using the panel structure of the CEX. Specifically, the identification method is first evaluated using the sample of households with only one member and no children. Since for this subset of households individual consumption is equivalent to household consumption, the intertemporal model can be estimated by using both the standard method and the identification strategy. The results indicates that the identification method is a reliable tool to estimate individual Euler equations. Afterward, the identification method is employed to estimate the individual Euler equations of couples. It is found that the intertemporal elasticity of substitution of women is lower than the intertemporal elasticity of substitution of men. Moreover, the difference is statistically significant. Finally, the individual Euler equations are estimated separately for single females and males with no children. The results suggest that the intertemporal elasticity of substitution of single females and males is larger than the corresponding elasticity for married females and males.

Euler equations have been estimated for the past 20 years, as reported in the survey by Browning and Lusardi (1996). The identification and estimation approach that I propose is new, as I consider Euler equations for each household member and not for the entire household. I employ an intertemporal framework in which each spouse is represented by individual preferences, therefore generalizing the static collective model developed by Chiappori (1988, 1992). Chiappori (1988, 1992) shows that in a static framework individual preferences can be identified under some separability restrictions. Blundell, Chiappori, Magnac and Meghir (2001) extend Chiappori's results to allow for households in which only one spouse works. While this project is concerned with the identification of preferences, the focus is on household intertemporal optimization. Specifically, the goal is to identify and estimate individual Euler equations and, as a byproduct, the intertemporal elasticities of substitution for each household member. Lundberg, Startz and Stillman (forthcoming) use a three-period collective model with limited commitment and no uncertainty to explore the retirement-consumption puzzle. Lundberg and Pollack (2001) use a non-stationary multi-stage game to analyze theoretically the location decision of a married couple. They show that marital decisions involving the future are in general not efficient.

The paper is organized as follows. In section 2, the standard approach is discussed individual. In section 3, the intertemporal collective model is introduced and individual Euler equations are derived. Section 4 outlines the identification procedure. Section 5 discusses the empirical implementation. Section 6 presents some preliminary results. Section 8 concludes the paper.

2 The Standard Approach

Consider a household composed of 2 members living for $\mathcal{T}$ periods. In each period $t \in \{0, ..., \mathcal{T}\}$ and state of the world $\omega \in \Omega$, member i consumes a private consumption good in quantity $c^i(t, \omega)$ and supplies labor in quantity $h^i(t, \omega)$.[4] Denote with $l^i = T - h^i$ leisure of member i, where T is the time available to each spouse in each period. At each (t, ω), member i is endowed with an exogenous stochastic income, $y^i(t, \omega)$. For any given (t, ω), the household can either consume or save in a risk-free asset. Let $b(t, \omega)$ and $R(t)$ denote respectively the amount of wealth invested in the risk-free asset at (t, ω) and the gross return on the risk-free asset.[5] Let $Y(t, \omega) = \sum_{i=1}^{2} y^i(t, \omega)$ and $C(t, \omega) = \sum_{i=1}^{2} c^i(t, \omega)$. The utility functions are assumed to be twice continuously differentiable.

The main obstacle in modelling household intertemporal behavior is that consumption is only measured at the household level. The standard solution to this problem is to assume that the household behaves as a single agent. Under this assumption a single utility function can be assigned to the entire household. Following the literature on consumption, it is assumed that preferences are defined over a composite consumption good. To allow for non-separability between consumption and leisure, preferences depend also on leisure. Specifically, let $V(C, l^1, l^1)$ be the household utility function. Then the intertemporal allocation is the solution of the following problem:

$$\max_{\left\{C_t, b_t, h_t^i\right\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,2}} E_0\left[\sum_{t=0}^{\mathcal{T}} \beta^t V\left(C_t, T - h_t^1, T - h_t^2\right)\right] \tag{1}$$

$$s.t.\ C_t + b_t \leq \sum_{i=1}^{2}\left(y_t^i + w_t^i h_t^i\right) + R_t b_{t-1} \quad \forall (t, \omega)$$

$$b_T \geq 0 \quad \forall \omega.$$

[4]Public goods are not modelled in this paper. This important issue is left for future research.

[5]All the results of the paper apply if a risky asset is introduced in the model.

Using a standard argument, the following household Euler equations can be derived,

$$V_C\left(C_t, T-h_t^1, T-h_t^2\right) = \beta E_t\left[V_C\left(C_{t+1}, T-h_{t+1}^1, T-h_{t+1}^2\right) R_{t+1}\right], \tag{2}$$

$$V_{l^i}\left(C_t, T-h_t^1, T-h_t^2\right) = \beta E_t\left[V_{l^i}\left(C_{t+1}, T-h_{t+1}^1, T-h_{t+1}^2\right) \frac{R_{t+1} w_t^i}{w_{t+1}^i}\right] \quad i=1,2, \tag{3}$$

where V_C and V_{l^i} are the marginal utilities of household consumption and member i's leisure.[6] Household Euler equations can be used to test the validity of the intertemporal model and to estimate intertemporal elasticities of substitution.

This approach is characterized by two shortcomings. First, by means of this approach only intertemporal elasticity of substitution for the whole household can be computed. Several policy questions require reliable estimates of individual intertemporal elasticities, i.e. one for each spouse. Second, this framework ignores that households are composed of several agents, possibly with different preferences.

3 A Collective Approach

Suppose that the households that we observe in the data satisfy the following two conditions: (i) the two spouses cooperate, i.e. any decision is on the Pareto frontier;[7] (ii) each member is characterized by individual preferences. In particular, suppose that individual preferences are intertemporally separable, depend on a composite private consumption good and leisure and that member i's preferences can be represented by means of the utility function $v^i(c^i, T-h^i)$. To emphasize the main contribution of this paper, the individual within-period utility function will be decomposed in the monotone function Ψ^i, which describes the individual preferences for intertemporal allocations, and the function u^i, which characterizes the

[6] It is also possible to derive cross Euler equations, i.e. Euler equations that relate consumption today with leisure tomorrow and vice versa.

[7] The idea that household members cooperate is well established in the literature, see for instance Becker (1973, 1974, 1991) and Chiappori (1992). Additionally, the general assumption of efficiency has the advantage of imposing no restriction on which point of the Pareto frontier will be chosen. Attanasio and Mazzocco (2002), Aura (2002), Lundberg, Startz and Stillman (forthcoming) and Mazzocco (2002) analyze the effect of limited commitment on household intertemporal behavior.

preferences for intra-period allocations, i.e.

$$v^i(c^i, T-h^i) = \Psi^i \left[u^i(c^i, T-h^i)\right].$$

Throughout the paper it will be assumed that individual utility functions are twice continuously differentiable.

The allocation of resources can then be characterized as the solution of the following Pareto problem:

$$\max_{\left\{c_t^i, b_t, h_t^i\right\}_{t\in\mathbf{T},\omega\in\Omega}^{i=1,2}} \mu^1(\Theta) E_0\left[\sum_{t=0}^{\mathcal{T}} \beta_1^t \Psi^1\left[u^1(c_t^1, T-h_t^1)\right]\right] + \mu^2(\Theta) E_0\left[\sum_{t=0}^{\mathcal{T}} \beta_2^t \Psi^2\left[u^2(c_t^2, T-h_t^2)\right]\right] \tag{4}$$

$$\sum_{i=1}^{2} c_t^i + b_t \leq \sum_{i=1}^{2}\left(y_t^i + w_t^i h_t^i\right) + R_t b_{t-1} \quad \forall (t,\omega)$$

$$b_T \geq 0 \quad \forall \omega, \in \Omega$$

for some pair of Pareto weights $(\mu^1(\Theta), \mu^2(\Theta))$, where Θ is the set of variables affecting the decision power of individual members.[8]

Even if preferences are heterogeneous, it is always possible to construct the representative agent corresponding to the household solving the following problem for a given level of individual leisure:

$$\bar{V}\left(C, \left\{T-h^i\right\}, \left\{\mu^i(\Theta)\right\}\right) = \max_{\left\{c^i\right\}_{i=1,2}} \mu_1(\Theta)\Psi^1\left[u^1\left(c^1, T-h^1\right)\right] + \mu_2(\Theta)\Psi^2\left[u^2\left(c^2, T-h^2\right)\right]$$

$$s.t. \sum_{i=1}^{2} c^i = C$$

However, the household aggregator $\bar{V}$ will generally depend on the distribution factors, i.e. on all the variables affecting the decision power. In Mazzocco (2002), it is shown that household preferences do not depend on the distribution factors if and only if the conditions for Gorman aggregation are satisfied. Using the aggregator $\bar{V}$, household Euler equations can be written in the form,

$$\bar{V}_C\left(C_t, \left\{T-h_t^i\right\}, \left\{\mu^i(\Theta)\right\}\right) = \beta E_t\left[\bar{V}_C\left(C_{t+1}, \left\{T-h_{t+1}^i\right\}, \left\{\mu^i(\Theta)\right\}\right) R_{t+1}\right], \tag{5}$$

$$\bar{V}_{l^i}\left(C_t, \left\{T-h_t^i\right\}, \left\{\mu^i(\Theta)\right\}\right) = \beta E_t\left[\bar{V}_{l^i}\left(C_{t+1}, \left\{T-h_{t+1}^i\right\}, \left\{\mu^i(\Theta)\right\}\right)\frac{R_{t+1} w_t^i}{w_{t+1}^i}\right], \tag{6}$$

[8]To be precise, the weights μ are a function of the Pareto weights and of the altruism parameters.

where $\bar{V}_C$ and $\bar{V}_{l^i}$ are the partial derivatives of $\bar{V}$ with respect to C and l^i. Consequently, household Euler equations depend on all variables affecting decision power unless Gorman aggregation applies. In Mazzocco (2002), it is tested whether household Euler equations depend on the distribution factors. The test is based on the following argument. If the standard model (1) is a complete characterization of household intertemporal optimization, the household Euler equations (2) and (3) should be satisfied for all families independently of the number of decision-makers in the household. If the collective formulation (4) is correct, household Euler equations should be satisfied for families with one decision-maker, but rejected for families with several decision-makers. Using the PSID and the CEX, after controlling for self selection, I find that Euler equations are strongly rejected for couples, but cannot be rejected for singles. This seems to indicate that it is important to find an alternative solution to the lack of individual data on consumption.

Independently of the number of household members, under the assumption of efficiency, individual Euler equations should always be satisfied.[9] In particular, individual consumption and leisure should satisfy the following intertemporal optimality conditions:[10]

$$\Psi_t^{i\prime} u_c^i\left(c_t^i, T-h_t^i\right) = \beta_i E_t\left[\Psi_{t+1}^{i\prime} u_c^i\left(c_{t+1}^i, T-h_{t+1}^i\right) R_{t+1}\right], \tag{7}$$

$$\Psi_t^{i\prime} u_l^i\left(c_t^i, T-h_t^i\right) = \beta_i E_t\left[\Psi_{t+1}^{i\prime} u_l^i\left(c_{t+1}^i, T-h_{t+1}^i\right) \frac{R_{t+1} w_t^i}{w_{t+1}^i}\right], \tag{8}$$

Consequently, if individual consumption and individual labor supply were observed, it would be possible to test the intertemporal model of household behavior, and more important to estimate individual elasticities of substitutions. Unfortunately, consumption is only measured at the household level. The remaining sections discuss the identification and estimation of individual Euler equations if total consumption, individual labor supplies, wages and interest rates are observed, but individual consumption is not.

[9] In this paper I abstract from the important issue of liquidity constraints.

[10] Individual Euler equations relating consumption today with leisure tomorrow and vice versa can also be derived.

4 M-consumption Functions and Identification of Individual Euler Equations

Consider a household characterized by an arbitrary pair of individual utility functions $\Psi^1\left[u^1(c^1, T-h^1)\right]$ and $\Psi^2\left[u^2(c^2, T-h^2)\right]$ which depend on a private composite good and leisure. To be able to test the model and identify the intertemporal elasticities it is important to answer the following question. Which variables do we observe? Micro datasets contain at best information on total household consumption, individual labor supplies and wages. With the exception of clothing no survey contains data on individual consumption. However, according to the theory, there should be a precise relationship between individual consumption on one side and labor supply and individual wages on the other. In this section, this link between unobservable and observable variables is used to show that individual Euler equations can be identified.

Assumption 1 *In each period, at least one member is working and can choose freely leisure.*

This assumption is crucial for the identification method. The first part of the assumption should not be controversial since in the CEX about 98 percent of households contain at least one member supplying a positive amount of labor. The second part of the assumption is more controversial. In the CEX, for both husbands and wives there is a lot of concentration around 40 hours of labor supply per week. However, there is much more variation among wives. If in each period at least one of the two members is working, the marginal rate of substitution between leisure and individual consumption must be equal to the wage rate divided by the price of consumption. Without loss of generality suppose that in period t member 1 satisfies assumption 1. Then the first order conditions of the Intertemporal Collective Model (4) imply,

$$\frac{u_l^1\left(c_t^1, T-h_t^1\right)}{u_c^1\left(c_t^1, T-h_t^1\right)} = q^1\left(c_t^1, h_t^1\right) = w_t^1. \tag{9}$$

If the function $q^1\left(c,h\right)$ is invertible, it is possible to determine individual consumption as a function of individual labor supply and wage rate, i.e. as a function of observable variables:

$$c_t^1 = g^1\left(w_t^1, h_t^1\right). \tag{10}$$

The function $g^1(w_t^1, h_t^1)$ corresponds to the m-consumption function introduced by Browning (1999). The following proposition establishes the condition under which $q^1(c, h)$ is invertible.

Proposition 1 *The m-consumption function $g^1(w^1, h^1)$ is well-defined if*

$$u_{lc}^1\left(c^1, T-h^1\right) u_c^1\left(c^1, T-h^1\right) - u_{cc}^1\left(c^1, T-h^1\right) u_l^1\left(c^1, T-h^1\right) \neq 0 \tag{11}$$

for any c^1 and h^1 that satisfy (9) for some feasible w^1.

Proof. For any c^1, h^1, w^1 satisfying (9) define,

$$d^1\left(c^1, h^1, w^1\right) = q^1\left(c_t^1, h_t^1\right) - w_t^1 = 0.$$

By the implicit function theorem, $g^1(w^1, h^1)$ is well-defined if $\dfrac{\partial d^1}{\partial c^1} \neq 0$. Which implies condition (11). ■

Consequently, even if individual consumption is not observed, it is possible to derive a function that relates it to observable variables. Total household consumption C is also observed and this information has not been used so far. Since by assumption households are composed of 2 members, member 2's consumption can be calculated as the difference between total consumption and consumption of member 1,[11]

$$c_t^2 = C_t - c_t^1 = C_t - g^1\left(w_t^1, h_t^1\right).$$

By means of these results, individual Euler equations of member can be characterized as functions of observed variables. In particular, substituting the m-consumption functions (10) for c_t^1 and c_{t+1}^1 in the marginal utilities of members 1 and 2, the following transformed marginal utilities can be derived:

$$v^1\left(w^1, h^1\right) = u_c^1\left(g^1\left(w^1, h^1\right), T-h^1\right), \tag{12}$$

$$f^1\left(w^1, h^1\right) = u_l^1\left(g^1\left(w^1, h^1\right), T-h^1\right), \tag{13}$$

$$v^2\left(C, w^1, h^1, h^2\right) = u_c^2\left(C - g^1\left(w^1, h^1\right), T-h^2\right), \tag{14}$$

$$f^2\left(C, w^1, h^1, h^2\right) = u_l^2\left(C - g^1\left(w^1, h^1\right), T-h^2\right). \tag{15}$$

[11]Since many couples have children it will be important in future research to extend the model to include them.

Applying a similar argument to Ψ^1 and Ψ^2, the following transformed monotone functions can be obtained:

$$\chi^1\left(w^1,h^1\right) = \Psi^{1\prime}\left[u^1\left(g^1\left(w^1,h^1\right),T-h^1\right)\right], \tag{16}$$

$$\chi^2\left(C,w^1,h^1,h^2\right) = \Psi^{2\prime}\left[u_c^2\left(C-g^1\left(w^1,h^1\right),T-h^2\right)\right]. \tag{17}$$

The individual consumption Euler equations can then be written as functions of observables by using $\upsilon^1,\upsilon^2,\chi^1$ and χ^2. Up to this point the main restrictions are contained in assumption 1. However, to derive the individual Euler equations using $\upsilon^1,\upsilon^2,\chi^1$ and χ^2 the previous argument must be used for period t and period $t+1$. Consequently, assumption 1 must be replace by the following more restrictive assumption.

Assumption 2 *At least one member is working and can choose freely leisure at t and $t+1$.*

Under this assumption the individual Euler equations can be rewritten in the following form:

$$\begin{aligned}\chi^1\left(w_t^1,h_t^1\right)\upsilon^1\left(w_t^1,h_t^1\right) &= \beta_1 E_t\left[\chi^1\left(w_{t+1}^1,h_{t+1}^1\right)\upsilon^1\left(w_{t+1}^1,h_{t+1}^1\right)R_{t+1}\right], \\ \chi^2\left(C_t,w_t^1,h_t^1,h_t^2\right)\upsilon^2\left(C_t,w_t^1,h_t^1,h_t^2\right) &= \beta_2 E_t\left[\chi^2\left(C_t,w_t^1,h_t^1,h_t^2\right)\upsilon^2\left(C_{t+1},w_{t+1}^1,h_{t+1}^1,h_{t+1}^2\right)R_{t+1}\right].\end{aligned}$$

Given that total household consumption, individual labor supplies and wage rates are observed, the functions $\upsilon^1,\upsilon^2,\chi^1$ and χ^2 can be identified non-parametrically or parametrically by using standard methods developed in the past 20 years for the estimation of Euler equations. Note that to avoid selection biases, only the consumption Euler equations have been employed so far.

The functions f^1,f^2 and μ can then be recovered using the intra-period optimality conditions. Specifically, since the function υ^1 is known, f^1 can be identified using the following equation:

$$\frac{u_l^1\left(g^1\left(w^1,h^1\right),T-h^1\right)}{u_c^1\left(g^1\left(w^1,h^1\right),T-h^1\right)} = \frac{f^1\left(w^1,h^1\right)}{\upsilon^1\left(w^1,h^1\right)} = w^1 \tag{18}$$

Since the functions υ^1 and υ^2 are known, μ can be identified using the following equation:

$$\frac{u_c^1\left(g^1\left(w^1,h^1\right),T-h^1\right)}{u_c^2\left(C-g^1\left(w^1,h^1\right),T-h^2\right)} = \frac{\upsilon^1\left(w^1,h^1\right)}{\upsilon^2\left(C,w^1,h^1,h^2\right)} = \mu \tag{19}$$

Finally, under the assumption that member 2 supplies a positive amount of labor and can choose freely leisure, the function f^2 can be recovered using the following equation:

$$\frac{u_l^2\left(C-g^1\left(w^1,h^1\right),T-h^2\right)}{u_c^2\left(C-g^1\left(w^1,h^1\right),T-h^2\right)} = \frac{f^2\left(C,w^1,h^1,h^2\right)}{\upsilon^2\left(C,w^1,h^1,h^2\right)} = w^2 \tag{20}$$

Note that the assumption that agent 2 works at time t and can choose freely leisure is only used for the identification of f^2.

However, we are not interested in $v^1, v^2, f^1, f^2\chi^1$ and χ^2, but rather in u_c^1, u_l^1, u_c^2, $u_l^2, \Psi^{1\prime}$ and $\Psi^{2\prime}$ and in the corresponding individual elasticities of substitution. To identify u_c^i, u_l^i from v^i, f^i, initially suppose that husband and wife both work. Then variations in labor supply and wages are observed for both spouses and v^i and f^i. From relations (14) and (15) we can deduce that,

$$v_{h^2}^2 = -u_{cl}^2, \quad f_w^2 = -u_{lc}^2 g_w^1, \quad f_{h^1}^2 = -u_{lc}^2 g_h^1.$$

Given that v^2 and f^2 are known functions, their derivatives are known as well and g_w^1 and g_h^1 can be identified. In particular,

$$g_w^1 = \frac{f_w^2}{v_{h^2}^2}, \quad g_h^1 = \frac{f_{h^1}^2}{v_{h^2}^2} \tag{21}$$

where the result follows from $u_{lc}^2 = u_{cl}^2$. Hence (21) provides a partial differential system, which can be integrated to give $g\left(w^1, h^1\right)$ up to the constant of integration. From relations (14) and (15) we can also deduce,

$$v_w^2 = -u_{cc}^2 g_w^1, \quad f_{h^2}^2 = -u_{ll}^2,$$

which imply that u_{cc}^2, u_{ll}^2 and u_{cl}^2 can be identified as follows,

$$u_{cc}^2 = -\frac{v_w^2 v_{h^2}^2}{f_w^2}, \quad u_{ll}^2 = -f_{h^2}^2, \quad u_{cl}^2 = -v_{h^2}^2. \tag{22}$$

The system can be solved to derive u_c^2 and u_l^2 up to a constant. From equations (12) and (13), we obtain,

$$v_w^1 = u_{cc}^1 g_w^1, \quad f_w^1 = u_{lc}^1 g_w^1, \quad f_{h^1}^1 = u_{lc}^1 g_h^1 - u_{ll}^1,$$

which imply,

$$u_{cc}^1 = \frac{v_w^1 v_{h^2}^2}{f_w^2}, \quad u_{ll}^1 = \frac{f_w^1 f_{h^1}^2}{f_w^2} - f_{h^1}^1, \quad u_{cl}^1 = \frac{f_w^1 v_{h^2}^2}{f_w^2}. \tag{23}$$

Hence u_c^1 and u_l^1 can be identified up to a constant.

It remains to show that $\Psi^{i\prime}$ can be recovered for $i = 1, 2$. The following equations can be derived from (16):

$$\chi_w^1 = \Psi^{1\prime\prime} v^1 g_w$$

Since $\chi_w^1, \chi_l^1, v^1, f^1, g_w$ and g_l are known, the two equations can be solved for χ_w^1 and χ_l^1 to obtain,

$$\Psi^{1\prime\prime} = \frac{\chi_w^1}{v^1 g_w},$$

which implies that $\Psi^{1\prime}$ is identified up to a constant.

By means of equation (17) it can be shown that

$$\chi_C^2 = \Psi^{2\prime\prime} v^2.$$

Consequently,

$$\Psi^{2\prime\prime} = \frac{\chi_C^2}{v^2},$$

and $\Psi^{2\prime}$ can be identified up to a constant.

Finally, all the constants of integration can be identified with the exception of the constant for $g(w^1, h^1)$, by noting that

$$u_c^i = v^i, \quad u_l^i = f^i, \quad \Psi^{i\prime} = \chi^i, \qquad for \quad i = 1, 2$$

and that v^i, f^i and χ^i are known functions for $i = 1, 2$.

The following two propositions summarize the results of this section.

Proposition 2 *Let u^1 and u^2 be von Neumann-Morgenstern utility functions. Assume that both agents work and that either u^1 or u^2 satisfies the invertibility condition (11). Then u_c^i, u_l^i and $\Psi^{i\prime}$ can be identified for $i = 1, 2$ up to the constant associated with $g(w^1, h^1)$.*

Consider a household in which only one spouse works. Without loss of generality suppose that agent 1 supplies labor. The function $g^1(w^1, h^1)$ is still well-defined and the approach outlined for households in which both members are employed can be implemented setting $h^2 = 0$. Since no variation in member 2's labor supply is observed, u_l^2 cannot be identified. The following theorem summarizes the result.[12]

Proposition 3 *Let u^1 and u^2 be von Neumann-Morgenstern utility functions. Assume that only agent 1 works and that u^1 satisfies the invertibility condition (11). Then u_c^i and $\Psi^{i\prime}$ can be identified for $i = 1, 2$ up to the constant associated with $g(w^1, h^1)$. Moreover, u_l^1 can also be identified up to the same constant.*

[12] A formal proof of this result is not included because it is mostly a replication of the argument used for households in which both members work. The proof is available on request.

Three remarks are in order. First, the identification of the individual elasticities of substitution is unaffected by the fact that individual Euler equations are identified up to the additive constant corresponding to $g(w^1, h^1)$. Second, the suggested identification strategy requires the following five standard assumptions: (i) in each period, at least one agent works; (ii) altruism is additive; (iii) m-consumption functions are well defined (iv) utility functions are twice continuously differentiable. No additional assumption on the functional form of u^1 and u^2 is required for the identification strategy to work. Moreover, no assumption on the exogeneity of the labor force participation decision is needed. Third, the method proposed in this section generates a set of overidentifying restrictions, which can be employed to test the model.

5 Empirical Implementation

The identification strategy is implemented assuming a specific parametric formulation for individual preferences. Specifically, suppose that the one-period utility function can be written in the form,[13]

$$\Psi^i \left[u^i \left(c^i, T - h^i\right)\right] = \frac{\left[\left(a_i + c^i\right)^{\sigma_i} \left(b_i + T - h^i\right)^{1-\sigma_i}\right]^{1-\rho_i}}{\sigma_i \left(1 - \rho_i\right)},$$

where $0 < \sigma_i < 1$, $\rho_i > 0$. For this specification of preferences, the intra-period condition (9) becomes,

$$q^i \left(c^i, h^i\right) = \frac{1 - \sigma_i}{\sigma_i} \frac{a_i + c^i}{b_i + T - h^i} = w^i.$$

Consequently, the m-consumption function for agent 1 can be written in the form,

$$c^1 = g^1 \left(w^1, h^1\right) = \frac{\sigma_1}{1 - \sigma_1} w^1 \left(b_1 + T - h^1\right) - a_1.$$

The functions v^1, f^1, v^2 and f^2 can now be computed and the following transformed consumption Euler equations can be derived,

$$1 = \beta_1 E_t \left[\left(\frac{w^1_{t+1}}{w^1_t}\right)^{\gamma_1 - 1} \left(\frac{b_1 + T - h^1_{t+1}}{b_1 + T - h^1_t}\right)^{-\rho_1} R_{t+1}\right], \tag{24}$$

[13] The utility function is divided by σ_i to normalize the multiplicative constant of the marginal utility of consumption to 1.

$$1 = \beta_2 E_t \left[\left(\frac{C_{t+1} - \phi_1 w^1_{t+1} \left(T - h^1_{t+1}\right) + a_1 + a_2}{C_t - \phi_1 w^1_t \left(T - h^1_t\right) + a_1 + a_2} \right)^{\gamma_2 - 1} \left(\frac{b_2 + T - h^2_{t+1}}{b_2 + T - h^2_t} \right)^{\theta_2} R_{t+1} \right], \qquad (25)$$

where $\gamma_i = \sigma_i (1 - \rho_i)$, $\theta_i = (1 - \sigma_i)(1 - \rho_i)$, $\phi_i = \dfrac{\sigma_i}{1 - \sigma_i}$. The first equation represents the consumption Euler equation of member 1, whereas the second equation is the consumption Euler equations of member 2. The coefficients γ_i, θ_i and ϕ_1 will be estimated using the Generalized Method of Moments (GMM).

5.1 The Data

To implement the identification procedure, the dataset must have the following two characteristics. First, information on total household consumption, individual labor supply, wages and interest rates must be available. Second, the dataset should have a panel structure to determine consumption, labor supply and wage dynamics for each household. The CEX survey satisfies these requirements. Since 1980, the CEX survey has been collecting data on household consumption, labor supply, wages and demographics. The survey is a rotating panel organized by the Bureau of Labor Statistics (BLS). Each quarter about 4500 households, representative of the US population, are interviewed: 80% are reinterviewed the following quarter, while the remaining 20% are replaced by a new randomly selected group. Each household is interviewed at most for four quarters and detailed information are elicited in regard to expenditures for each of the three months preceding the interview, and in regard to labor supply and demographics for the quarter preceding the interview.[14]

The data used in the estimation cover the period 1982-1998. The first two years are dropped, since the data were collected with a different methodology. As in Meghir and Weber (1996) and Attanasio and Mazzocco (2002), the rotating feature of the panel is used, i.e. household level data for the four quarters available are employed. Consequently, I drop all households that are not in the survey for all four interviews. I exclude from the sample rural households, household in which the head is younger than 21 and older than 65, household in which the head is self-employed and households with incomplete income responses.

[14] Each household is interviewed for five quarters, but the first interview is used to make contact and no information is publicly available.

The estimation of individual Euler equations will be performed separately for the subsample of singles with no children and the subsample of couples with family size equal to two and no children. Singles with no children are useful to determine the performance of the identification method relative to the standard method, since for this subgroup individual consumption is identical to household aggregate consumption and therefore observable. The subsample of couples is restricted to married or cohabiting couples with no children because the identification method applies to households with two members and no public good. Households experiencing a change in marital status are dropped from the sample.

To implement the identification procedure, at least one household member must be employed for two consecutive periods and able to choose freely labor supply. Since wives display greater variation in labor supply conditional on supplying labor, in the estimation wives will be used to derive individual consumption as a function of individual labor supply and wages. Consequently, I exclude all households in which the wife is not working in consecutive periods. It is important to note that in the CEX individual labor data are collected during the first and last interviews unless a member of the household reports changing his or her employment. In the second and third interviews the labor data are set equal to the data reported in the first interview. To cope with this limitation of the data, for each household the first and last quarters of the survey period are defined as the two consecutive periods used in the identification and estimation of individual Euler equations. The wife claims to be employed in the first and last interview in about 76 percent of the households in the sample.

The CEX dataset contains monthly data on consumption. However, only quarterly variables are available for labor data. Consequently, in the estimation quarterly variables are employed. Total consumption is computed as the sum of food at home, food out, tobacco, alcohol, other nondurable goods and services such as heating fuel, public and private transportation, personal care and semidurable goods which include clothing and shoes. In particular, from the definition of total consumption I exclude consumer durables, housing, education and health expenditure. Total consumption is deflated using the Consumer Price Indices published by the BLS. Specifically, the price index for the composite good is calculated as a weighted average of individual price indices, with weights equal to the expenditure share for the particular consumption good. The gross hourly wage rate is computed using three variables: the amount of the last gross pay; the time period of the last gross pay covered; the number of hours usually worked per week in the corresponding period. Since the wage rate is not directly observed, the measure used in this paper might be affected by endogeneity. In particular, the amount of the last gross pay is likely to be affected by the number of hours of work in a given period. However, this criticism applies to any work unless wage rates are directly observed. Moreover, even in this case the wage rate will depend on hours of work through the investment in human capital. To calculate the after tax wage rate, federal effective tax rates are generated using the NBER's TAXSIM model. Finally, the real after tax wage rate is determined using the individual price indices. Total time available to each household member in a quarter is set equal to 1040, which is equivalent to 80 hours per week. Quarterly individual labor supply is obtained multiplying by 13 the number of hours usually worked per week in the corresponding period. The interest rate is the quarterly average of the 3-month Municipal bond rate preceding the interview to avoid taxation issues. The real interest rate is calculated by using the household price indices. The real gross interest rate used in the estimations is computed compounding the real quarterly gross interest rate for the three quarters that separate the first interview from the last interview.

To estimate the model by GMM, a set of valid instruments is required. Under the assumption of rational expectations, any lagged information is a suitable instrument.[15] However, only two consecutive observations are available for each household and these two observations are required to compute the consumption, leisure and wage growth. Consequently, no lagged variable at the individual level is left to construct the instrument set. To address this problem, the set of instrument is constructed using lagged cohort variables, where cohort variables are computed using the year of birth of the household head. Summary statistics for the main variables are reported in table 1.

[15] The existence of measurement errors may introduced unexpected dependence between the Euler equation error term and lagged information. For this reason all instruments will be calculated as the first or higher lag of current variables.

Table 1: Summary Statistics

Independent Variable	Mean for Singles	Mean for Couples
Real Consumption per Quarter	1577.7	2545.1
Husband's Labor Supply per Week	42.7	44.4
Wife's Labor Supply per Week	-	32.1
Condit. Wife's Labor Supply per Week	-	38.8
Husband's After Tax Wage per Hour	8.1	9.2
Wife's Before Tax Wage per Hour	-	6.6
Real After Tax Interest Rate	7.02	7.09
Number of Observations	9753	5192
Number of Families	2438	1298

5.2 Econometric Issues

To compare the results obtained in this paper with the previous literature, the individual parameters will be estimated using only the consumption Euler equations of both members. The identification method requires that at least one household member supplies labor in period t as well as period $t+1$. In the CEX, about 98 percent of married males between the ages of 21 and 65 with all four interviews supply labor during the entire sample period, whereas only 75 percent of married females with all four interviews supply labor during the entire sample period. However, there is much more variation in the labor supply of married women. Therefore, the wife will be used to derive individual consumption as a function of individual leisure and wage rate.

The Euler equations will be estimated using the Generalized Method of Moments (GMM), because this approach is general enough to estimate the non-linear as well as the linear version of the model. However, the GMM is not free of problems. As for any estimator, the GMM estimator is consistent only if measurement errors are not an issue. If the GMM is used to estimate non-linear equations, the measurement error problem is exacerbated. Using a Montecarlo simulation Carroll (2001) finds that measurement errors should bias the estimates of intertemporal substitution downward. To verify the impact of measurement errors on the estimation, a linearized version of the model is also estimated. The GMM estimation has also an important advantage: it does not require the log-linearization of the

Euler equations. Carroll (2001) and Ludvigson and Paxon (2001) find that the approximation method may introduce a substantial bias in the estimation of the preference parameter. On the other hand, Attanasio and Low (2000) show that using long panels it is possible to estimate consistently log-linearized Euler equations.

A well known result is that, to estimate consistently Euler equations, a relatively large number of time periods is needed, but not necessarily on the same household. The sample used in this project covers 17 years. It is therefore likely that the aggregate shocks will average out.

In the estimation I control for heterogeneity only by allowing for seasonal dummies. Specifically, the utility function of each household member is multiplied by a heterogeneity term, $\exp\left(\sum_{j=1}^{m} \xi_j z^j\right)$, where z is a vector of seasonal dummies. However, the criteria employed to construct the two subsamples used in the estimation should reduce the importance of heterogeneity.

The subsample of couples is restricted to households in which the wife is working in the first and last interview. From a theoretical point of view this criterium to construct the sample should not introduction selection bias, since the individual consumption Euler equations are satisfied for each household member independently of labor supply choices. Empirically, the selection bias may affect the estimates if labor supply status is correlated with the expectation errors conditional on a given stage of the business cycle and the sample period is not long enough to cover the entire business cycle. However, the period 1982-1998 used in this paper should be long enough to address this problem.

Finally, this paper abstracts from the important issue of liquidity constraints. If household members are restricted in their ability to borrow, Euler equations are replaced by inequalities as shown in Zeldes (1989).

6 Results

The identification procedure developed in the previous sections relies on the theoretical structure of the model. It is therefore important to evaluate the performance of the identification method. To that end the identification method is first employed to estimate the individual Euler equations of households with only one member. In this case total and individual consumption are identical and both observable. It is therefore possible to estimate individual Euler equations using both the standard method and the identification procedure. Specifically, in the identification method, the Euler equation (24) is estimated, i.e.

$$E_t\left[\left(\frac{w_{t+1}}{w_t}\right)^{\gamma-1}\left(\frac{b+T-h_{t+1}}{b+T-h_t}\right)^{-\rho}\beta R_{t+1}\right]=1.$$

where $\gamma=\sigma(1-\rho)$. In the standard method, the individual Euler equations (7) and (8) are employed, which under the assumptions on preferences can be written in the form,

$$E_t\left[\left(\frac{a+C_{t+1}}{a+C_t}\right)^{\gamma-1}\left(\frac{b+T-h_{t+1}}{b+T-h_t}\right)^{\theta}\beta R_{t+1}\right]=1,$$

$$E_t\left[\left(\frac{a+C_{t+1}}{a+C_t}\right)^{\gamma}\left(\frac{b+T-h_{t+1}}{b+T-h_t}\right)^{\theta-1}\frac{w_t}{w_{t+1}}\beta R_{t+1}\right]=1,$$

where $\gamma=\sigma(1-\rho)$ and $\theta=(1-\sigma)(1-\rho)$.

To evaluate the effect of measurement errors and non-linearities on the identification method, three different versions of the model are estimated. First, a linearized version of the individual Euler equations is estimated. In particular, the intercepts a and b are set equal to zero and the individual Euler equations (24), (7) and (8) are linearized using standard methods. The results for this version of the model are reported in tables 2 and 3 for two different sets of instruments. The identification method performs well in the sense that under standard assumptions the estimates obtained using the identification method are not statistically different from the estimates obtained using the standard method. The estimates for the coefficient ρ are between 0.32 and 0.38 depending on the set of instruments. The estimates for σ are between 0.81 and 0.89. The estimates of β are between 0.957 and 0.948. The standard model is also estimated under the assumption that consumption and labor supply are separable. In this case the coefficient ρ is almost twice as large relative to the estimates obtained without the assumption of separability. This indicates that models with separability tend to bias downward the estimates of the intertemporal elasticity of substitution.

Second, the non-linear version of the model with a and b equal to zero is estimated. The results are reported in tables 4 and 5. The estimates of ρ are slightly larger but the differences are not statistically significant under standard assumptions. The coefficient estimates of σ are slightly smaller but once again the difference is negligible. The main problem is represented by the estimation of the β, since the identification method tends to overestimate the discount factor.

Third, the non-linear model in which a and b are functions of demographics is estimated. Tables 6 and 7 report the results obtained by allowing a and b to be functions of age and two education dummies for two different sets of instruments. The identification method performs well with the exception of the estimation of β. The estimates of ρ and σ are similar to the estimates obtained using the first two versions of the model. As for the non-linear model with no intercepts, the estimates of β are lower using the standard method and the differences are statistically significant.

The estimates for couples are obtained using equation (24) for the wife and (25) for the husband, i.e.

$$E_t\left[\left(\frac{w_{t+1}^1}{w_t^1}\right)^{\gamma_1-1}\left(\frac{b_1+T-h_{t+1}^1}{b_1+T-h_t^1}\right)^{-\rho_1}\beta_1 R_{t+1}\right]=1, \tag{26}$$

$$E_t\left[\left(\frac{C_{t+1}-\phi_1 w_{t+1}^1\left(T-h_{t+1}^1\right)+a_1+a_2}{C_t-\phi_1 w_t^1\left(T-h_t^1\right)+a_1+a_2}\right)^{\gamma_2-1}\left(\frac{b_2+T-h_{t+1}^2}{b_2+T-h_t^2}\right)^{\theta_2}\beta_2 R_{t+1}\right]=1, \tag{27}$$

The coefficients are estimated using two different sets of instruments. The sum of the consumption intercepts, a_1+a_2, is function of individual ages, b_1 is a function of two education dummies for agent 1 and b_2 is a function of two education dummies for agent 2. The results are reported in tables 8 and 9. The wife's ρ is estimated to be between 1.61 and 1.85, whereas the husband's ρ is estimated to be between 1.26 and 1.55. Moreover, the difference between the wife's estimated ρ and the husband's estimated ρ is statistically significant. The wife's σ is around 0.8 and statistically significant. The husband's σ is around 0.5 but it is not precisely measured. Moreover the difference is not statistically significant. The quarterly discount factors are estimated to be similar and around 0.95. The standard unitary model with separability between consumption and leisure is also estimated. As for singles the estimate of ρ is larger and equal 2.08 and the estimate of the quarterly discount factor is much smaller and equal to 0.9.

Tables 10 and 11 contains the coefficient estimates for the non-linear model when the sample of singles is divided in single females and single males. The estimates of ρ for single females is around 0.55, whereas the estimates for single males is around 0.35. For single females σ is not precisely estimated using the identification method. Consequently, the difference between the estimates obtained using the standard and the identification method is large relative to the results obtained using different samples. The estimate of σ for single males is around 0.75.

7 Conclusions

In this paper, the identification and estimation of individual Euler equations is analyzed. It is shown that individual Euler equations can be identified parametrically and non-parametrically observing only data on household total consumption, individual labor supply and wages, i.e. with the limited information available in the CEX. Moreover, assuming a specific utility function for each household member, individual Euler equations are estimated separately for singles, couples, single females and single males.

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A Tables with Results

Table 2: Singles, Linear Approximation: Identification Method vs Standard Methods vs Standard Method with Separability. First Set of Instruments.

Parameters	Identification	Standard	Standard with Separability
ρ	0.36*	0.35**	0.68*
	[0.19]	[0.15]	[0.39]
σ	0.86**	0.87**	-
	[0.36]	[0.18]	
β	0.957**	0.948**	0.860**
	[0.01]	[0.006]	[0.13]
J-Statistics	16.8	51.5	19.9
$P > \chi^2$	0.82	0.38	0.83
number of observations			2438

Asymptotic standard errors in brackets. All models are estimated with GMM, by using the following instruments: second and third lags of after tax income growth, gross pay growth, labor supply growth all calculated at the cohort level; the second to fourth lags of real wage growth, price index growth, all calculated at the cohort level; the second to fifth lags of real consumption growth, leisure growth, real gross interest rate growth, all calculated at the cohort level; age. The instruments for last column: second to fourth lags of income growth, price index, marginal tax growth, real gross interest rate growth, all calculate at the cohort level; second to fifth lags of real consumption, real wage growth, labor supply growth, all calculate at the cohort level; age.

Table 3: Singles, Linear Approximation: Identification Method vs Standard Methods vs Standard Method with Separability. Second Set of Instruments.

Parameters	Identification	Standard
ρ	0.38**	0.32**
	[0.19]	[0.16]
σ	0.89**	0.81**
	[0.38]	[0.11]
β	0.957**	0.948**
	[0.005]	[0.003]
J-Statistics	16.9	52.8
$P > \chi^2$	0.85	0.40
number of observations		2438

Asymptotic standard errors in brackets. All models are estimated with GMM, by using the following instruments: second and third lags of after tax income growth, gross pay growth, all calculated at the cohort level; the second to fourth lags of real wage growth, price index growth, labor supply growth, all calculated at the cohort level; the second to fifth lags of real consumption growth, leisure growth, real gross interest rate growth, all calculated at the cohort level; age.

Table 4: Singles, Non-linear Model with no Intercept: Identification Method vs Standard Methods. Second Set of Instruments.

Parameters	Identification	Standard
ρ	0.44**	0.45**
	[0.22]	[0.18]
σ	0.74**	0.60**
	[0.36]	[0.15]
β	0.935**	0.899**
	[0.01]	[0.01]
J-Statistics	14.2	30.9
$P > \chi^2$	0.92	0.98
number of observations		2438

Asymptotic standard errors in brackets. All models are estimated with GMM, by using the following instruments: second and third lags of after tax income growth, price index growth, all calculated at the cohort level; second to fourth lags of labor supply growth, real gross interest rate growth, all calculated at the cohort level; the second to fifth lags of real consumption growth, real wage growth, leisure growth, gross pay growth, all calculated at the cohort level.

Table 5: Singles, Non-linear Model with no Intercept: Identification Method vs Standard Methods. Third Set of Instruments.

Parameters	Identification	Standard
ρ	0.46^{**}	0.52^{**}
	[0.22]	[0.18]
σ	0.79^{**}	0.66^{**}
	[0.39]	[0.19]
β	0.935^{**}	0.897^{**}
	[0.01]	[0.01]
J-Statistics	14.5	37.6
$P > \chi^2$	0.94	0.92
number of observations		2438

Asymptotic standard errors in brackets. All models are estimated with GMM, by using the following instruments: second and third lags of after tax income growth, price index growth, all calculated at the cohort level; second to fourth lags of labor supply growth, all calculated at the cohort level; the second to fifth lags of real consumption growth, real wage growth, leisure growth, gross pay growth, real gross interest rate growth, all calculated at the cohort level.

Table 6: Singles, Non-linear Model with Intercepts Functions of Age and Education Dummies: Identification Method vs Standard Methods. First Set of Instruments.

Parameters	Identification	Standard
ρ	0.42^{*}	0.44^{**}
	[0.24]	[0.20]
σ	0.76^{*}	0.65^{**}
	[0.42]	[0.19]
β	0.941^{**}	0.923^{**}
	[0.01]	[0.01]
J-Statistics	13.8	29.2
$P > \chi^2$	0.84	0.97
number of observations		2438

Asymptotic standard errors in brackets. All models are estimated with GMM, by using the following instruments: second and third lags of after tax income growth, price index growth, all calculated at the cohort level; second to fourth lags of labor supply growth, real gross interest rate growth, all calculated at the cohort level; the second to fifth lags of real consumption growth, real wage growth, leisure growth, gross pay growth, all calculated at the cohort level. The intercepts are linear functions of two education dummies and age.

Table 7: Singles, Non-linear Model with Intercepts Functions of Age and Education Dummies: Identification Method vs Standard Methods. First Set of Instruments.

Parameters	Identification	Standard
ρ	0.45^{*}	0.49^{**}
	[0.24]	[0.21]
σ	0.81^{*}	0.70^{**}
	[0.44]	[0.23]
β	0.940^{**}	0.921^{**}
	[0.01]	[0.01]
J-Statistics	14.1	36.9
$P > \chi^2$	0.87	0.88
number of observations		2438

Asymptotic standard errors in brackets. All models are estimated with GMM, by using the following instruments: second and third lags of after tax income growth, price index growth, all calculated at the cohort level; second to fourth lags of labor supply growth, all calculated at the cohort level; the second to fifth lags of real consumption growth, real wage growth, leisure growth, gross pay growth, real gross interest rate growth, all calculated at the cohort level. The intercepts are linear functions of two education dummies.

Table 8: Couples, Non-linear Model with Intercepts: Identification Method vs Standard Methods.

Parameters	Wife	Husband	Parameter Difference	Unitary Model with Separability
ρ	1.61**	1.26**	0.35**	2.08**
	[0.26]	[0.31]	[0.16]	[0.45]
σ	0.79**	0.41	0.38	-
	[0.25]	[0.76]	[0.89]	
β	0.868**	0.862**	0.006	0.716**
	[0.03]	[0.04]	[0.07]	[0.07]
ϕ	10.75	-	-	-
	[12.35]			
J-Statistics			43.5	21.6
$P > \chi^2$			0.62	0.76
number of observations				1298

Asymptotic standard errors in brackets. All models are estimated with GMM, by using the following instruments: second and third lags of husband's and wife's log gross pay, real gross interest rate growth, all calculated at the cohort level; second to fourth lags of husband's and wife's labor supply, all calculated at the cohort level; the second to fifth lags of real consumption growth, after tax income growth, marginal tax rate, real consumption, all calculated at the cohort level; wife's and husband's age. The sum of the consumption intercepts are linear functions of wife's and husband's age. The husband's leisure intercept is a linear function of two husband's education dummies. The wife's leisure intercept is a linear function of two wife's education dummies.

Table 9: Couples, Non-linear Model with Intercepts: Identification Method vs Standard Methods.

Parameters	Wife	Husband	Parameter Difference
ρ	1.85**	1.55**	0.30**
	[0.26]	[0.30]	[0.15]
σ	0.83**	0.58	0.25
	[0.15]	[0.97]	[1.09]
β	0.841**	0.832**	0.009
	[0.04]	[0.05]	[0.08]
ϕ	14.36	-	-
	[12.51]		
J-Statistics			43.1
$P > \chi^2$			0.55
number of observations			1298

Asymptotic standard errors in brackets. All models are estimated with GMM, by using the following instruments: second and third lags of husband's and wife's log gross pay, real gross interest rate growth, all calculated at the cohort level; second to fourth lags of husband's and wife's labor supply, all calculated at the cohort level; the second to fifth lags of real consumption growth, after tax income growth, real consumption, all calculated at the cohort level; wife's and husband's age. The sum of the consumption intercepts are linear functions of wife's and husband's age. The husband's leisure intercept is a linear function of two husband's education dummies. The wife's leisure intercept is a linear function of two wife's education dummies.

Table 10: Single Females, Non-linear Model, no Intercepts: Identification Method vs Standard Methods.

Parameters	Identification	Standard
ρ	0.50*	0.58**
	[0.26]	[0.27]
σ	0.81*	0.48**
	[0.47]	[0.19]
β	0.927**	0.894**
	[0.02]	[0.01]
J-Statistics	6.2	21.7
$P > \chi^2$	0.99	0.98
number of observations		1146

Asymptotic standard errors in brackets. All models are estimated with GMM, by using the following instruments: second and third lags of gross pay growth, labor supply growth, real gross interest rate growth, all calculated at the cohort level; second to fourth lags of real consumption, all calculated at the cohort level; the second to fifth lags of real consumption growth, leisure growth, all calculated at the cohort level.

Table 11: Single Males, Non-linear Model, no Intercepts: Identification Method vs Standard Methods.

Parameters	Identification	Standard
ρ	0.35**	0.33**
	[0.16]	[0.16]
σ	0.84**	0.67**
	[0.40]	[0.14]
β	0.949**	0.913**
	[0.01]	[0.01]
J-Statistics	13.2	31.5
$P > \chi^2$	0.83	0.86
number of observations		1146

Asymptotic standard errors in brackets. All models are estimated with GMM, by using the following instruments: second and third lags of real consumption price index growth,all calculated at the cohort level; second to fourth lags of real consumption growth, labor supply growth, all calculated at the cohort level; the second to fifth lags of leisure growth, gross pay growth, real interest rate growth, all calculated at the cohort level.