NONPARAMETRIC ESTIMATION OF DUTCH AND FIRST-PRICE, SEALED-BID AUCTION MODELS WITH ASYMMETRIC BIDDERS*

Bjarne Brendstrup
Department of Economics
University of Aarhus

Harry J. Paarsch†
Department of Economics
University of Iowa

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†Correspondence: 108 PBB; 21 East Market Street; Iowa City, Iowa 52242. Telephone: (319) 335-0936; Facsimile: (319) 335-1956; E-mail: hjp@paarsch.biz.uiowa.edu

Abstract

Within the independent private-values paradigm, we demonstrate nonparametric identification of Dutch and first-price, sealed-bid auction models when bidders are asymmetric. We also demonstrate that, in the presence of a binding reserve price, methods for estimating the distributions of valuations from data at Dutch auctions differ from those at first-price, sealed-bid auctions. The differences are illustrated by comparing the kernel-smoothed nonparametric estimators for each auction format.

1 Introduction and Motivation

During the last four decades, economists have made considerable progress in understanding the theoretical structure of strategic behavior under market mechanisms, such as auctions, when a small number of potential participants exists; see Krishna (2002) for a comprehensive presentation and evaluation of progress.

One analytic device commonly used to describe bidder motivation at auctions is a continuous random variable which represents individual-specific heterogeneity in valuations. The conceptual experiment involves each potential bidder's receiving an independent draw from a distribution of valuations. Conditional on this random variable, the bidder is assumed to act purposefully, maximizing either the expected profit or the expected utility of profit from winning the auction. Another frequently-made assumption is that the bidders are *ex ante* symmetric, their independent draws coming from the same distribution of valuations, an assumption that then allows the researcher to focus on a representative agent's decision rule when describing equilibrium behavior. However, at many real-world auctions and in many economic environments, the valuations across bidders are often better represented by draws from different distributions; *i.e.*, asymmetries are important.

Investigating equilibrium behavior in the presence of asymmetries has challenged researchers for some time. Only under the most commonly-used informational assumptions, the independent private-values paradigm (IPVP) described above, has much progress been made. In particular, under some auction mechanisms, such as the oral, descending-price (also known as *Dutch*) and the first-price, sealed-bid auction, asymmetries can induce inefficient allocations, while under other mechanisms, such as the oral, ascending-price (also known as *English*) and the second-price, sealed-bid (also known as *Vickrey*) auction, efficient allocations obtain. Moreover, when asymmetries are present, the well-known Revenue Equivalence Proposition no longer holds.

Most structural econometric research devoted to investigating equilibrium behavior at auctions has involved single-unit auctions within the symmetric IPVP. Examples include Paarsch (1992,1997); Donald and Paarsch (1993,1996,2002); Laffont, Ossard, and Vuong (1995); Guerre, Perrigne, and Vuong (2000); Haile and Tamer (2003); and Li (2003). Bajari (1997) was one of the first to investigate asymmetric auctions, analyzing low-price, sealed-bid, single-unit procurement auctions with independent cost draws from different distributions; his approach was Bayesian and thus parametric. Jofre-Bonet and Pesendorfer (2003) have investigated the effects of capacity constraints at sequential, low-price, sealed-bid procurement auctions with symmetric independent private costs, again using parametric methods, where the capacity constraints make bidders asymmetric. Building on the research of Brendstrup (2002), Brendstrup and Paarsch (2003) developed a nonparametric private-values framework within which asymmetries in valuations at multi-unit, sequential, English auctions could be investigated. In this paper, building on the pioneering research of Guerre *et al.* (2000), we develop a strategy of nonparametric identification and estimation to recover the distributions of latent valuations for different *classes* of bidders from data concerning single-unit Dutch as well as first-price, sealed-bid auctions.

In the standard theoretical analysis of Dutch and first-price, sealed-bid auctions, researchers typically note that the two auction formats are strategically equivalent. From an econometrician's perspective, however, Dutch auctions are quite different from first-price, sealed-bid auctions in two important respects: First, at Dutch auctions only the winning bid is observed, while at first-price, sealed-bid auctions all bids of participants are observed. This obvious point is well-known. A second and, to our knowledge, unnoticed point is related to the fact that, in the presence of a binding reserve price, participation is endogenous: Only those potential bidders for whom the object on sale is worthwhile choose to attend the auction, to participate. At Dutch auctions, this participation information is revealed to those bidders present at the auction. The revelation of additional information, which is absent at first-price sealed-bid auctions, makes the decision problem solved by participants at Dutch auctions different. Moreover, in the presence of asymmetries, it matters who has chosen to attend the auction. Thus, the distinction between the number of potential bidders $\mathcal{N}$ and the number of actual bidders n is particularly important when asymmetries exits. To our knowledge, this second observation has not been incorporated into the empirical literature concerned with analyzing field data from auctions; we believe this to be a potential source of mis-specification.

2 Theoretical Model

We consider the sale of a single object at auction assuming that each of the $\mathcal{N}(\geq 2)$ potential bidders is from one of J different classes where J is less than or equal to $\mathcal{N}$. To admit full generality, hereafter we set J equal to $\mathcal{N}$. A potential bidder of class j draws his individual-specific valuation independently from the cumulative distribution function $F_j(v)$ having corresponding probability density function $f_j(v)$ which is strictly continuous.

For the moment, consider an equilibrium to the first-price, sealed-bid auction game at which a bidder of class j follows the strategy β_j which is strictly increasing and differentiable in v, having inverse function ϕ_j defined to be β_j^{-1}. Given that, together, the bidders follow the strategy vector $\boldsymbol{\beta}$, which collects the strategies for each of the $\mathcal{N}$ potential bidders, the expected payoff of bidder i when his valuation is v_i and he bids b is

$$\Pi_i(b, v_i) = H_i(b)(v_i - b)$$

where $H_i(b)$ is his probability of winning, given a bid of b. However, except in the symmetric case, calculating $H_i(b)$ is difficult because it is a function of all opponents' distribution functions as well as all their inverse functions.

Our contribution is as follows: First, we note that, for bidder i who is of type j and has valuation V_i, his bid B_{ji} equals $\beta_j(V_i)$. Now, the probability density function of this bid $g_j(b)$ equals $f_j\left[\phi_j(b)\right]\phi_j'\left[\phi_j(b)\right]$ so, if bidder i has a winning bid of y, then we know that

$$\max_{h \neq i} B_{jh} < y.$$

Now, the probability of $\max_{h \neq i} B_{jh}$ is given by

$$H_i\left(y | f_1, \ldots, f_{i-1}, f_{i+1}, \ldots, f_{\mathcal{N}}, \beta_1, \ldots, \beta_{i-1}, \beta_{i+1}, \ldots, \beta_{\mathcal{N}}\right) = \prod_{h \neq i} G_h(y),$$

so the corresponding probability density function, following the notation of Balakrishnan and Rao (1998), is

$$\begin{aligned} & H_i'\left(y | f_1, \ldots, f_{i-1}, f_{i+1}, \ldots, f_{\mathcal{N}}, \beta_1, \ldots, \beta_{i-1}, \beta_{i+1}, \ldots, \beta_{\mathcal{N}}\right) \\ = & \frac{1}{[(\mathcal{N}-1)-1]!} \operatorname{Perm} \begin{bmatrix} G_1(y) & \cdots & G_i(y) & G_{i+1}(y) & \cdots & G_{\mathcal{N}}(y) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ G_1(y) & \cdots & G_i(y) & G_{i+1}(y) & \cdots & G_{\mathcal{N}}(y) \\ g_1(y) & \cdots & g_i(y) & g_{i+1}(y) & \cdots & g_{\mathcal{N}}(y) \end{bmatrix}. \end{aligned}$$

The above matrix on the right is $[(\mathcal{N}-1) \times (\mathcal{N}-1)]$ where each column represents a collection of $(\mathcal{N}-2)$ cumulative distribution functions and one probability density function for a particular bidder, excluding bidder i. Here, the symbol "Perm" outside the matrix above denotes the permanent operator. The permanent is similar to the determinant except that all the principal minors have positive sign. An example of the permanent for a (3×3) matrix is

$$\text{Perm}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a(ei + fh) + b(di + fg) + c(dh + eg).$$

Unlike the determinant, which in the transformation of random variables ensures that a probability density function integrates to one, the permanent is a counting device, like the permutation formula. It is especially useful when finding combinations from different types of distributions.

To see that the above equation collapses to the probability density function of the highest order statistic when the G_js are identical, recall that $H'(y)$, the probability density function of Y, the highest order statistic from $(\mathcal{N} - 1)$ independently and identically distributed draws from $G(b)$ is

$$H'(y) = (\mathcal{N} - 1)G(y)^{\mathcal{N}-2}g(y).$$

With some loss of generality, consider an example when $\mathcal{N}$ is four and focus on bidder 1. Thus, $(\mathcal{N} - 1)$ is three and

$$\begin{aligned} H_1'(y) &= \frac{1}{(3-1)!}\text{Perm}\begin{bmatrix} G_2(y) & G_3(y) & G_4(y) \\ G_2(y) & G_3(y) & G_4(y) \\ g_2(y) & g_3(y) & g_4(y) \end{bmatrix} \\ &= \frac{1}{2}\left\{ \begin{array}{c} G_2(y)\left[G_3(y)g_4(y) + G_4(y)g_3(y)\right] \\ +G_2(y)\left[G_3(y)g_4(y) + G_4(y)g_3(y)\right] \\ +g_2(y)\left[G_3(y)G_4(y) + G_4(y)G_3(y)\right] \end{array} \right\} \end{aligned}$$

which one can show, by direct substitution, is

$$H'(y) = 3G(y)^2 g(y)$$

when G_j equals $G(y)$ for $j = 2, 3, 4$. The purpose of introducing the equation above and the example is to illustrate that $H_i'(y)$, the probability density function of the highest competing bid at the auction, is a "mixture" of the probability density function, the f_js, where the "mixing" weights vary with y. Thus, the model is nonparametrically unidentified when only data on the number of potential bidders $\mathcal{N}$ and the winning bid are observed; see Athey and Haile (2002) who apply the research of Meilijson (1981) as well as Prakasa Rao (1992). Thus, we assume that the researcher can determine the identity of each potential bidder and, if necessary, assign collections of potential bidders to particular classes.

Having obtained the expected payoff for bidder i, given that his opponents follow strategies $\boldsymbol{\beta}_{-i}$, we can derive the necessary first-order condition of his decision problem

$$\max_b H_i(b)(v_i - b)$$

which is

$$H_i'(b)(v_i - b) = H_i(b).$$

A first-order condition like the one above must hold for each bidder; the solution to this system of differential equations, one for each bidder, together with the relevant boundary conditions, typically stated in terms of the reserve price r as well as the upper bound of support $\overline{v}$, constitutes an equilibrium to this first-price, sealed-bid auction game; see Maskin and Riley (2000) as well as Krishna (2002). Note that the conditions we have assumed on the distribution functions imply that we can invoke a theorem of Lebrun (1999) to assert that an equilibrium exists.

As an illustration, consider the following:

Example 1 *Let $\mathcal{N}$ be two and assume that bidder 1's valuation is drawn from F_1, while bidder 2's valuation is drawn from F_2. Denote the strategy functions by β_1 and β_2 and their inverse functions by ϕ_1 and ϕ_2, respectively. Under these assumptions, $H_1(b)$ reduces to $F_2[\phi_2(b)]$ and $H_2(b)$ reduces to $F_1[\phi_1(b)]$, so the necessary first-order conditions are*

$$\begin{aligned} F_2[\phi_2(b)] &= F_2'[\phi_2(b)]\,\phi_2'(b)\,[\phi_1(b) - b] \\ F_1[\phi_1(b)] &= F_1'[\phi_1(b)]\,\phi_1'(b)\,[\phi_2(b) - b] \end{aligned}$$

which, when rearranged, yield

$$\begin{aligned} \frac{F_2[\phi_2(b)]}{F_2'[\phi_2(b)]}\frac{1}{[\phi_1(b) - b]} &= \phi_2'(b) \\ \frac{F_1[\phi_1(b)]}{F_1'[\phi_1(b)]}\frac{1}{[\phi_2(b) - b]} &= \phi_1'(b)\,. \end{aligned}$$

Below, we shall argue that, when a binding reserve price r exists, it can be misleading to analyze Dutch and first-price, sealed-bid auctions using the same extensive-form game because at Dutch auctions participants typically have additional information in the form of the actual number of bidders present at the auction; bidders whose valuations are below the reserve price have no reason to be present at the auction. Thus, endogenous participation, when present, is observed at Dutch auctions and becomes a predetermined variable. Actual participation is typically (and reasonably so) assumed unobserved at first-price, sealed-bid auctions, so the presence of a reserve price only affects the boundary condition. At Dutch auctions, on the other hand, the information set is different from that at first-price, sealed-bid auctions. To illustrate this consider the following:

Example 2 *Let $\mathcal{N}$ be two and assume that $F_i(v)$ equals v on $[0,1]$ for $i = 1,2$ where the reserve price r is strictly positive. Now, the equilibrium strategy at a first-price, sealed-bid auction for v which weakly exceeds r is*

$$\begin{aligned} \beta(v) &= \mathcal{E}\left[\max(Y,r)|v > Y\right] \\ &= r\frac{r}{v} + \frac{1}{v}\int_r^v y dy \\ &= \frac{1}{2}\left(\frac{r^2+v^2}{v}\right) \end{aligned}$$

At a Dutch auction, the same bidder will be able to observe whether his opponent is absent (i.e., had a draw below the reserve price), so it is easy to see that, when his opponent is absent, his optimal strategy, conditional on that information, is to bid the reserve price. Hence, when only one bidder attends the auction, he bids the reserve price

$$\beta(v) = r.$$

The above example illustrates that Dutch and first-price, sealed-bid auctions are not strategically equivalent in the presence of a binding reserve price when actual participation is known at Dutch auctions, a common feature at these auctions. In the following two sections, we develop a structural-econometric framework within which to identify and to estimate the primitives of the economic model, specifically the underlying distributions of latent valuations, in the two informational settings.

3 First-Price, Sealed-Bid Auctions

3.1 Nonparametric Identification

In this section, we demonstrate that the underlying distributions of latent valuations are nonparametrically identified from the bids tendered at first-price, sealed-bid auctions. Our strategy relies on the fact that the observed bids contain information concerning both the corresponding valuation and the equilibrium strategy. To see this, note from the first-order condition that

$$v_i = b_i + \frac{H_i(b_i)}{H_i'(b_i)}. \tag{1}$$

Thus, we can use the twin hypotheses of purposeful behavior and equilibrium, to identify bidder i's private value v_i as function of his equilibrium bid and the distribution of the highest bid of his opponents. Note that, when it is possible to obtain the distribution of the highest of the opponent's bids, no reason exists to solve for the equilibrium strategies. Moreover, because all the bids of participants and their identities are recorded, it is possible to estimate the H_i function from observed data. The next proposition claims that under our assumptions the underlying distributions are nonparametrically identified from observables.

Proposition 3 *F_i is identified nonparametrically on $[r, \overline{v}]$ from the observed bids, the number of potential bidders, and the identities of the bidders.*

Proof. See the proof of Proposition 2 in the next section. ■

3.2 Nonparametric Estimation

We now describe how to estimate the distributions of the underlying valuations. The basic idea, which is due to Guerre *et al.* (2000), is simple, yet elegant and powerful. Note that, while H_i and H_i' are unknown, they can be estimated from the observed bids as well as the bidders identities. Having estimated H_i and H_i', we can use (1) to form an estimate of bidder i's valuation using the framework of Guerre *et al.* (2000).

As a description of the analysis would be, almost word-for-word, as in Guerre *et al.* (2000), we direct the reader to their paper for specific details.

Here, we simply outline enough of the approach to make clear our use of their techniques, so that later we can illustrate how the methods of Guerre *et al.* (2000) differ from those we propose to analyze data from asymmetric Dutch auctions when the reserve price binds.

In the first step, we construct a sample of pseudo private-values from (1) using the nonparametric estimates of H_i and H_i', while in the second step, we use this pseudo sample to estimate nonparametrically the probability density function of bidder i's latent valuation. One estimator that can be used is the following: In the first step, we estimate the distributions H_i and H_i' from the observed bids using, for example,

$$\begin{aligned}\widehat{H}_i(b) &= \frac{1}{T}\sum_{t=1}^{T}\mathbf{1}\left(\max_{j\neq i} B_{jt} \leq b\right)\\ \widehat{H}_i'(b) &= \frac{1}{Th}\sum_{t=1}^{T} k\left(\frac{b-\max_{j\neq i} B_{jt}}{h}\right)\end{aligned}$$

where k is a kernel function and h is the bandwidth. From these estimates, we construct the pseudo values

$$\widehat{V}_{it} = B_{it} + \frac{\widehat{H}_i(B_{it})}{\widehat{H}_i'(B_{it})}.$$

We then use these pseudo values to estimate the probability density function of the latent valuations

$$\widetilde{f}_i\,(v) = \frac{1}{Th}\sum_{t=1}^{T} k\left(\frac{v-\widehat{V}_{it}}{h}\right)$$

or, to avoid boundary problems, we use the empirical distribution function

$$\widetilde{F}_i(v) = \frac{1}{T}\sum_{t=1}^{T}\mathbf{1}\left(\widehat{V}_{it} \leq v\right).$$

The details of this approach can be found in Guerre *et al.* (2000), and the generalizations of their basic framework (*e.g.*, to admit the truncation induced by a binding reserve price) can be applied to this model as well.

4 Dutch Auctions

4.1 Nonparametric Identification

In this section, we demonstrate that the distributions of underlying valuations are identified from observables at Dutch auctions. To understand the results, it is important to note that, on the one hand, the bids now contain information concerning actual competition but, on the other hand, less information exists because only winning bids are observed. The practical implication of these two facts is that the methods proposed for first-price, sealed-bid auctions are no longer relevant.

In what follows, we assume that, in the presence of a binding reserve price, more than one potential bidder attends the auction. From the first-order condition, we know that

$$v_i = y_i + \frac{H_i\left(y_i|P\right)}{H_i'\left(y_i|P\right)} \tag{2}$$

where y_i is the winning bid and the H_i functions are now conditioned on the realized competition which involves P, the set of bidders whose valuations have weakly exceeded the reserve price. With this observation, we claim that the F_is are nonparametrically identified from observables.

Proposition 4 *F_i is nonparametrically identified on $[r, \overline{v}]$ from the observed winning bid, the number and identities of the potential bidders, the number and identities of actual bidders, and the identity of the winner.*

Proof. Denote the set of participating bidders by P. Define the random variable Y to be the maximum of $(B_1, \ldots, B_n)$ where n is the numbers of participating bidders. Let I be the index of the winner; *i.e.*, Y equals B_j means I equals j. We observe the distribution of bids for winners, given

P. Denote the population cumulative distribution function of the winning bid Y at an auction won by bidder i by $W_i(y|P)$. Our task is to identify $\{F_i\}_{i=1}^n$, from $\{W_i(y|P)\}_{i=1}^n$. To do so, we need to find the distribution of the bids $G_i(y|P)$ so we can apply the approach of Guerre *et al.* (2000), The proof is similar to one by Prakasa Rao (1992) and is presented here because it gives the reader some understanding concerning the estimation procedure presented below.

Now $W_i(y|P)$ is the union of two disjoint events: B_i being the maximum among $(B_1, ..., B_n)$ and B_i being less than or equal to y. Thus,

$$
\begin{aligned}
W_i(y|P) &= \Pr(Y \leq y|I = j) \\
&= \int_{-\infty}^{y} \prod_{j \neq i} G_j(t|P)\, dG_i(t|P) \\
&= \int_{-\infty}^{y} \frac{\prod_{j=1}^{n} G_j(t|P)}{G_i(t|P)} dG_i(t|P) \\
&= \int_{-\infty}^{y} \frac{\Pr(y \leq t)}{G_i(t|P)} dG_i(t|P) \\
&= \int_{-\infty}^{y} \frac{\sum_{j=1}^{n} W_j(t|P)}{G_i(t|P)} dG_i(t|P) \\
&= \int_{-\infty}^{y} \sum_{j=1}^{n} W_j(t|P)\, d\log G_i(t|P).
\end{aligned}
$$

Therefore,

$$
dW_i(y|P) = \sum_{j=1}^{n} W_j(y|P)\, d\log G_i(y|P)
$$

or, equivalently,

$$
\begin{aligned}
G_i(y|P) &= \exp\left(\int_{-\infty}^{y} \left[\sum_{j=1}^{n} W_j(t|P) \right]^{-1} dW_i(t|P) \right) \\
&= \left[\sum_{j=1}^{n} W_j(t|P) \right]^{\alpha_i}
\end{aligned}
$$

where α_i is the probability that i wins the auction. As i was arbitrary, we have identified the distributions of the bids. Now, from the distribution of bids, we can find $H_i\left(y|P\right)$ via

$$H_i\left(y|P\right) = \prod_{j \neq i} G_j\left(y|P\right)$$

and the probability density function from

$$H_i'\left(y|P\right) = \frac{d\left[\prod_{j \neq i} G_j\left(y|P\right)\right]}{dy}.$$

The remainder of the argument follows Guerre *et al.* (2000). ■

4.2 Nonparametric Estimation

In this section, we demonstrate how to estimate the distributions of latent valuations. The idea is to use consistent estimators of $W_j\left(t|P\right)$ and $W_j'\left(t|P\right)$ as well as α_i to define the estimator

$$\widehat{G}_i(y|P) = \left[\sum_{j=1}^{n} \widehat{W}_j(t|P)\right]^{\frac{T_i}{T}}$$

and then to form estimators of $H_i\left(y|P\right)$ and $H_i'\left(y|P\right)$ via

$$\widehat{H}_i\left(y|P\right) = \prod_{j \neq i} \widehat{G}_j\left(y|P\right)$$

and

$$\widehat{H}_i'\left(y|P\right) = \frac{d\left[\prod_{j \neq i} \widehat{G}_j\left(y|P\right)\right]}{dy}.$$

At this point, we can use the estimation strategy developed for first-price, sealed-bid auctions; *i.e.*, form the pseudo values

$$\widehat{V}_{it} = Y_{it} + \frac{\widehat{H}_i(Y_{it}|P)}{\widehat{H}_i'(Y_{it}|P)}$$

and then use these pseudo values to estimate the truncated probability density function of the latent valuations

$$f_i^*(v) = \frac{f_i(v)}{[1 - F_i(r)]}$$

via $\widetilde{f_i^*}(v)$ an estimate of $f_i^*(v)$ based on the pseudo values. The main result of this section is given in the next theorem:

Theorem 5 *Given that*

$$\widehat{W}_i'(y|P) \overset{a.s.}{\to} W_i'(y|P)$$
$$\widehat{W}_i(y|P) \overset{a.s.}{\to} W_i(y|P)$$

$\widetilde{f_i^*}(v)$ *is a consistent estimator of* $\frac{f_i(v)}{[1-F_i(r)]}$.

Proof. We seek to show that $\widetilde{f_i^*}(v)$ is a consistent estimator of $\frac{f_i(v)}{[1-F_i(r)]}$. To do this, we invoke results from Guerre *et al.* (2000). However, we first need to show that $\widehat{H}_i(y|P)$ and $\widehat{H}_i'(y|P)$ are consistent estimates of $H_i(y|P)$ and $H_i'(y|P)$. Consider

$$\widehat{G}_i(y|P) = \left[\sum_{j=1}^{n} \widehat{W}_j(t|P)\right]^{\frac{T_i}{T}}.$$

To show that this is a consistent estimator of $G_i(y|P)$, we note that, by the continuous-mapping theorem, the term on the right-hand side of the equal sign below

$$\left|\widehat{G}_i(y|P) - G_i(y|P)\right| = \left|\left[\sum_{j=1}^{n} \widehat{W}_j(t|P)\right]^{\frac{T_i}{T}} - \left[\sum_{j=1}^{n} W_j(t|P)\right]^{\frac{T_i}{T}}\right|$$

goes to zero, so $\widehat{G}_i(y|P)$ is consistent estimator of $G_i(y|P)$. To see that $\widehat{G}_i'(y|P)$ is a consistent estimate of $G_i'(y|P)$, we again apply the continuous-mapping theorem. Consistency of $\widehat{H}_i(y|P)$ and $\widehat{H}_i'(y|P)$ then follows from the continuous-mapping theorem. The remainder of the proof of the theorem follows from Guerre *et al.* (2000), so we do not repeat it here. Two estimators that satisfy the conditions are the empirical distribution function and a kernel estimator of the densities where an appropriate choice of the kernel and bandwidth has been made. Thus,

$$\begin{aligned}\widehat{W}_i(y|P) &= \frac{1}{T}\sum_{t=1}^{T_i} \mathbf{1}\left(Y_t \leq y\right) \\ \widehat{W}_i'(y|P) &= \frac{1}{Th}\sum_{t=1}^{T_i} k\left(\frac{y-Y_t}{h}\right).\end{aligned}$$

■

5 Finite-Sample Properties

In this section, we present some experimental evidence documenting the finite-sample behavior of the estimator for Dutch auctions without a reserve price. We begin by noting the well-known fact that, when the bidders are asymmetric, it is only possible to find closed-form solutions to the equilibrium bid strategies in a few special case. Because we are interested in the finite-sample properties of our estimator, we do not want any errors, which might arise due to numerical approximations of the equilibrium bidding strategies, to influence the results concerning the estimator. Therefore, we use a previously-solved example from Maskin and Riley (2000). In this example, bidder 1 draws his valuation from a uniform on $\left[0, \frac{4}{3}\right]$, while bidder 2 draws from $\left[0, \frac{4}{5}\right]$. In the notation of Krishna (2002), the equilibrium bidding strategies then have the following closed-form solutions:

$$\beta_1(v) = \frac{1}{v}\left(\sqrt{1+v^2}-1\right)$$

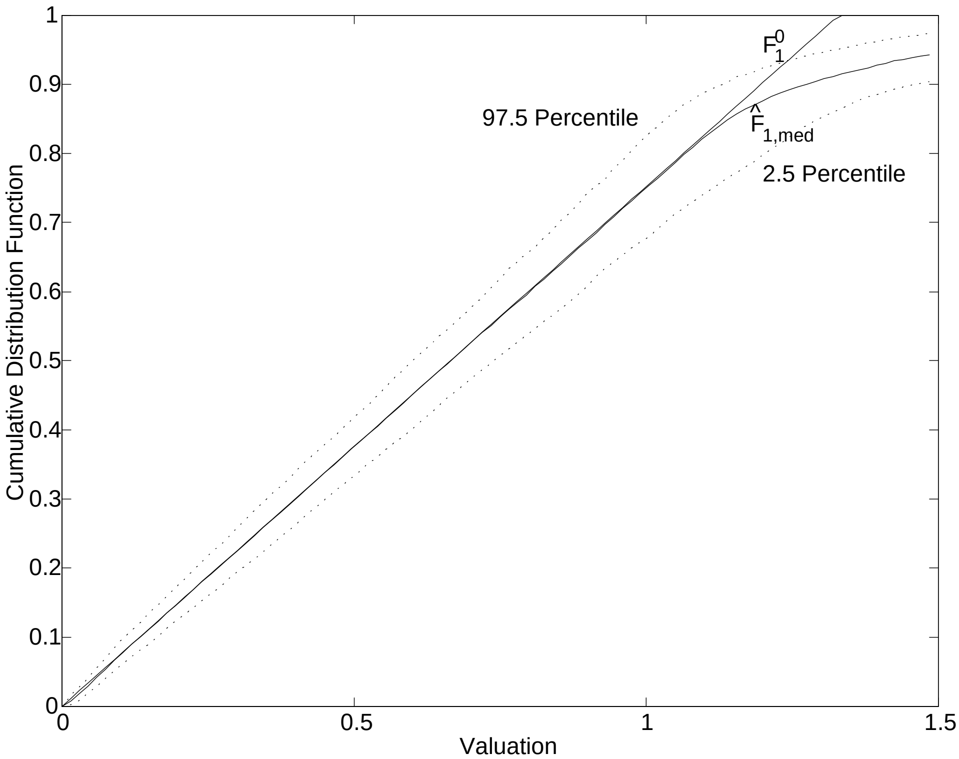
and

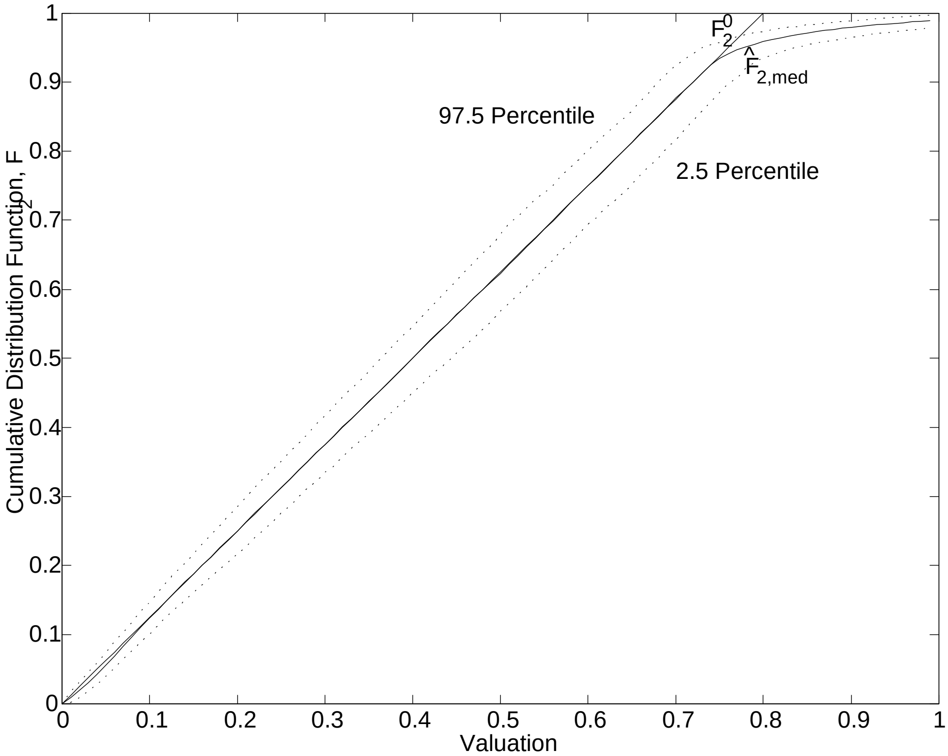
$$\beta_2(v) = \frac{1}{v}\left(1 - \sqrt{1 - v^2}\right).$$

Given this setup, we conducted a Monte Carlo experiment involving samples of 250 observations which we then replicated 5,000 times. We employed the estimator described above using the biweight kernel

$$k(x) = \begin{cases} \frac{15}{16}\left(1 - x^2\right)^2 & \text{for } |x| < 1; \\ 0 & \text{otherwise;} \end{cases}$$

where the bandwidth was 0.0826; $(0.25 \times T^{-\frac{1}{5}})$ where T is the sample size, in this case 250. The results are depicted in the graphs below where the median $\widehat{F}_{i,med}$, the 97.5 and 2.5 percentiles are reported as well as the truth F_i^0. The first figure is for type 1 bidders, while the second is for type 2 bidders.





These results are quite promising. In particular, the median of the estimators and F_i^0 are virtually the same. Of some concern is the poor behavior at the upper bound of support. This finding is, however, consistent with the sort of behavior that Guerre *et al.* (2000) encountered.

6 Conclusion

We have illustrated how the pioneering work of Guerre *et al.* (2000) can be extended to identify and to estimate nonparametrically the distributions of latent valuations using data from single-unit, first-price, sealed-bid auctions when potentially *all* bidders are asymmetric; *i.e.*, have valuations which are draws from different distributions. We have also demonstrated that, in the presence of a binding reserve price, strategic behavior at Dutch auctions is different from that at first-price, sealed-bid auctions because actual participation is typically observed at Dutch auctions and reasonably assumed unobserved at first-price, sealed-bid auctions. The additional information in the identities of those present at Dutch auctions changes the econometric analysis substantially. We have illustrated the differences by comparing the kernel-smoothed nonparametric estimators for each auction format. A small-scale Monte Carlo experiment confirms that the finite-sample behavior of the estimator is, by and large, good.

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