IS MODERATE-TO-HIGH INFLATION INHERENTLY UNSTABLE?*

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First draft: December 2002

This draft: June 2004

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Notes: The author would like to thank Hasan Bakshi and Pablo Burriel-Lombart for discussions that led to this research, and Athanasios Orphanides, John Roberts, Mike Woodford and especially John Driscoll for comments on previous drafts. The views expressed herein are those of the author, and do not represent those of the Federal Reserve Board or its staff.

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Abstract

The data across time and countries suggest the level and variance of inflation are highly correlated. This paper examines the effect of trend inflation on the ability of the monetary authority to ensure a determinate equilibrium and macroeconomic stability in a sticky-price model. Trend inflation increases the importance of future marginal costs for current price-setters in a staggered price-setting model. The greater importance of expectations makes it more difficult for the monetary authority to ensure stability; in fact, equilibrium determinacy cannot be achieved through reasonable specifications of nominal interest rate (Taylor) rules at moderate-to-high levels of inflation (for example, at levels around 4 percent per year). If monetary policymakers have followed these types of policy rules in the past, this result may explain why moderate-to-high inflation is associated with inflation volatility. It also suggests a revision to interpretations of the 1970s. At that time, inflation in many countries was at least moderate, which can contribute to economic instability. The results suggest that some moderate-inflation countries that have recently adopted inflation targeting may want to commit to low target inflation rates.

JEL Codes: E3, E5
Keywords: Monetary policy; equilibrium determinacy; Taylor rule; sunspot fluctuations

1. Introduction

Higher average levels of inflation are associated with larger fluctuations in inflation. The relationship between trend inflation and macroeconomic volatility is examined below in a standard new-Keynesian sticky-price model.[1] Positive rates of trend inflation raise the importance of expected future marginal costs for current price-setters in a staggered price-setting model. The greater importance of expectations makes it more difficult for the monetary authority to ensure stability. For example, much recent research has emphasized that monetary policymakers should raise real interest rates in response to an increase in inflation. In the context of a simple rule relating nominal interest rates to the output gap and inflation (e.g., Taylor (1993)), this requires that the coefficient on inflation exceed unity, a condition referred to as the Taylor principle. It is demonstrated below that the Taylor principle can be violated at fairly moderate rates of trend inflation. Equilibrium indeterminacy, and hence the possibility of sunspot fluctuations and increased macroeconomic instability, occurs for an increasingly large proportion of the range of policy settings when trend inflation rises to moderate levels (e.g., from 0 to 4 percent per year for reasonable parameter values).[2]

This result suggests that monetary policy practice and the interpretation of past events may need some reconsideration. With regard to practices, a focus on trend inflation in discussing

[1] Most recent work on sticky-price models abstracts from positive trend inflation rates (e.g., Woodford (2003)). Two exceptions are Ascari (2000) and Bakhshi et al (2002). These authors illustrate how the equilibrium in a Calvo staggered-price-setting model (Calvo (1983)) may not exist for high values of trend inflation because the infinite sum on which the currently chosen nominal price is based may not converge for high rates of inflation. This result depends on the unpalatable assumption in the Calvo model that prices can be sticky for arbitrarily long periods of time. Kiley (2002a, 2002b) discusses a number of other issues illustrating how the Calvo model fails to provide a good approximation to staggered-price setting models finite maximum lags.

policy rules is central; an overemphasis on the Taylor principle should be avoided, as macroeconomic stability requires that policymakers commit to low inflation and respond vigorously to inflation fluctuations around that trend level. A number of countries that have recently adopted inflation targeting have moderate-to-high inflation (Schmidt-Hebbel and Tapias (2002)) and this may limit the stabilization gains from their new inflation targeting framework.[3]

With respect to past experience, Clarida, Gali, and Gertler (2000) have suggested that insufficient responsiveness of nominal interest rates to expected inflation in the 1970s – i.e., violations of the Taylor principle that allowed real interest rates to fall with an increase in expected inflation – contributed to macroeconomic volatility. During that period, inflation in many countries was at least moderate. The results herein suggest that the level of inflation contributed to volatility because moderate-to-high inflation is inherently unstable when prices are rigid, regardless of how vigorously monetary authorities manipulate nominal interest rates in their efforts to stabilize inflation. This conclusion is also consistent with a substantial body of evidence showing a very strong correlation between the level and variance of inflation across countries and time (e.g., Kiley (2000)) if policymakers in that sample could be interpreted as following Taylor-type rules, as suggested in earlier work for the United States, Japan, and Germany (e.g., Clarida, Gali and Gertler (1998, 2000)).

The analysis herein will focus exclusively on the possible effects of trend inflation on

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[2] It should be emphasized that throughout this analysis the equilibria are always determinate and stable, or indeterminate. Instability herein will often refer to indeterminacy and hence the possibility that sunspot shocks increase macroeconomic volatility. This is the same notion as in Clarida, Gali, and Gertler (2000).

economic volatility, and does not consider any other costs or benefits associated with moderate inflation. However, the finding of potentially pernicious effects of moderate-to-high inflation on economic volatility adds further weight to research suggesting that low inflation is desirable due to steady-state distortions to relative prices, interactions between trend inflation and nominal tax systems, and the classical costs associated with the area under the money-demand curve stemming from positive nominal interest rates.

The next section briefly discusses the association between the level and variance of inflation. Section 3 presents a simple model illustrating the effect of trend inflation on price-setting behavior in a New-Keynesian staggered-price-setting model. The model's implications for macroeconomic volatility are examined in section 4. The final section discusses necessary further work.

2. Inflation and Its Variance

There is a long history documenting a positive relationship between the level and variance of inflation. Okun (1971) and Taylor (1981) are classic examples. Both authors demonstrate through international cross-sectional comparisons that high inflation is volatile inflation. Kiley (2000) recently reports a similar correlation across 43 countries.

For our purposes, it is important to emphasize that this finding is not driven by inclusion of very high inflation economies; rather, it is true of the G-7 economies over the past 30 years –

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[3] Benhabib, Schmitt-Grohé and Uribe (2002) and Carlstrom and Fuerst (2000) present alternative models in which forward-looking policymaking can lead to indeterminate equilibria; the latter authors emphasize the desirability of backward-looking policy while the former authors highlight an important role for nominal interest rate inertia – i.e., a role for lagged interest rates in the policy rule.

as well as during the moderate inflation conditions of the last 15 years.[4] For example, table 1 presents average inflation rates, as measured by the annual percent change in the personal consumption deflator from the national accounts, and the standard deviation of inflation for the seven economies comprising the G7 over two periods, 1974-1985 and 1986-2000. The break between time periods was chosen to roughly correspond to the period after disinflation from the higher levels of the 1970s was completed in most countries and the period of increased macroeconomic stability identified in McConnell and Perez-Quiros (2000) and OECD (2002).

Table 1: Basic statistics for consumer price inflation in the G7

	1974-1985		1985-2000	
	Average	Std. Deviation	Average	Std. Deviation
Canada	8.5	2.3	2.7	1.4
France	10.7	2.5	2.3	1.1
Germany	4.3	1.7	1.9	1.3
Italy	16.0	3.6	4.9	1.7
Japan	6.7	5.5	0.7	1.2
United Kingdom	12.1	5.7	3.9	2.1
United States	6.9	2.5	2.8	1.1

Note: Inflation is measured as the percent change in the annual average of the personal consumption deflator

In every country, the period of low inflation was also a period of more stable inflation. This relationship is also apparent across countries: Figure 1 presents the scatterplot of the 14 country/time-period pairs for average inflation and its standard deviation. The simple correlation between the level and standard deviation of inflation is 0.7 in the 14 country/time-period observations. This correlation is not driven by the high-inflation period – it remains 0.7 in the seven-country-level observations over 1985-2000.

[4] We focus on low-to-high inflation, where high is defined as 10 percent per year. The positive association between very-high inflation and its variance will not be considered, as very-high rates of inflation make a New-Keynesian sticky-price model implausible.

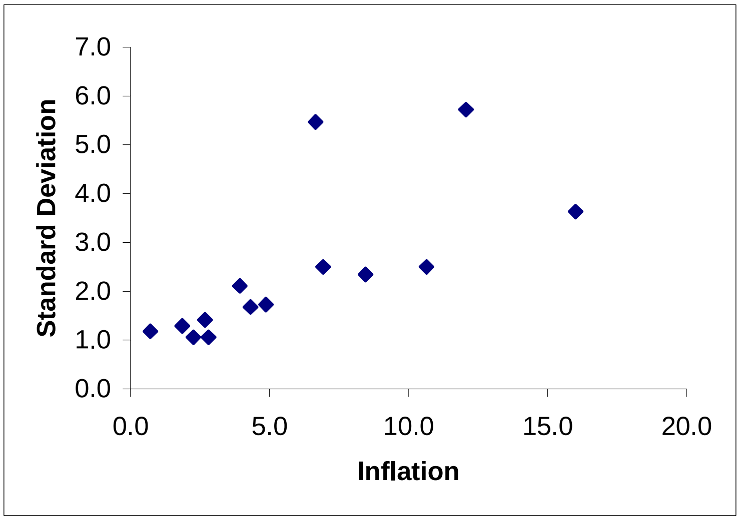

Figure 1: Scatterplot of the volatility of inflation against its level in the G7

Finally, it is important to emphasize that the positive association between the level and variance of inflation is distinct from the positive association between inflation and the dispersion of relative prices at a point in time – which is also quite strong, as documented in, for example, Fischer (1981) and Stockton (1988). These earlier studies have demonstrated that relative price dispersion and the level of inflation are positively correlated largely because relative price shocks – particularly to food and energy – increase price dispersion and inflation in the short run. This literature has also found some positive effect of trend inflation on relative price dispersion – in other words, the causality runs in both directions, albeit more strongly from relative price shocks to dispersion and inflation than vice-versa.

In contrast, the positive association between the level of inflation and its variance is not dominated by relative price shocks. Two factors suggest this interpretation: the cross-country experience partially controls for energy price shocks (e.g., common global movements in oil prices); and the correlation between the level and variance is strong between 1986 and 2000, a period of greater stability in oil prices. Two hypotheses were offered in the earlier literature for the relationship between the level of inflation and its variance. Milton Friedman's Nobel lecture (1977) suggests that high inflation causes inefficient, and hence more variable, macroeconomic policies – perhaps reflecting a diminution of political consensus; much of Friedman's conjecture concerns inflation at the high-end of the range we consider, and does not address the correlation between the rate of inflation and its variance at the moderate levels emphasized herein. Taylor (1981) suggests that accommodative monetary policies may lead to high inflation and greater variability in response to supply shocks. Our analysis will echo Taylor in emphasizing policy. But it will differ in demonstrating that moderate-to-high levels of inflation will potentially increase macroeconomic instability even when policymakers are not accommodative, because moderate-to-high inflation may both amplify the effects of intrinsic shocks and open up the possibility of self-fulfilling inflation fluctuations.

3. A Model

The model is a standard New-Keynesian description of the macroeconomy, similar to the baseline case in Clarida, Gali, and Gertler (1999).[5] The aggregate-demand side of the model – the IS curve and the monetary policy reaction function – are not affected by trend inflation and hence their derivation from microeconomic behavior will not be discussed.

Focusing first on aggregate supply, the economy consists of a large number of (symmetric) monopolistically competitive firms producing intermediate goods that are aggregated with a constant-elasticity-of-substitution aggregation function to produce the final

[5] King (2000) provides further discussion of the New-Keynesian model.

consumption good. The demand function facing firm j in period t is given by

$$D_{j,t}(\frac{X_{j,t}}{P_t}) = (\frac{X_{j,t}}{P_t})^{-\theta} Y_t, \quad \theta > 1,$$

where $X_{j,t}$ is the price charged by the firm, P_t is the aggregate price level (defined below), and Y_t is aggregate demand. For simplicity, aggregate demand and consumption are used interchangeably, and the effects of investment by firms or consumer durable purchases are ignored (following, for example, Woodford (2003)).

Firms set nominal prices for two periods. We will fix this period of price rigidity across all the levels of inflation considered below. It is reasonable to suppose that the degree of price rigidity would vary with trend inflation, as demonstrated empirically by Kiley (2000). But the observed variation across countries and time has been for large differences in trend inflation, and we will consider more modest variations in trend inflation. Future work may wish to re-examine the role of *endogenous* selection of price rigidity at different levels of trend inflation. However, previous research suggests that the results herein will not be affected to a significant extent. Fischer (1981) notes that menu cost models will imply that the degree of price stickiness will decreases with the average inflation rate, but that this decrease will generally be partial in the sense that relative price dispersion increases with trend inflation; in the model below, the increase in relative price dispersion with trend inflation is an important mechanism influencing our results. Recent research also suggests that, within calibrated dynamic-general-equilibrium models, the degree of price rigidity is not likely to be very sensitive to the average level of inflation for the differences across the G-7 since the 1970s (e.g., Klenow and Kryvtsov (2003)).

In our model, there are two classes of firms, each with mass equal to 1/2 the total; the firms differ in that they alternate the period in which they adjust their price, i.e. price-setting is staggered as in Taylor (1980). This sticky-price assumption preserves tractability and avoids some of the problems associated with the popular Calvo model.[6] The firm's profit maximization problem involves choosing the nominal price X_t that maximizes

$$\Lambda_t[X_t(\frac{X_t}{P_t})^{-\theta}Y_t - \Gamma((\frac{X_t}{P_t})^{-\theta}Y_t)] + E\left\{\Lambda_{t+1}[X_t(\frac{X_t}{P_{t+1}})^{-\theta}Y_{t+1} - \Gamma((\frac{X_t}{P_{t+1}})^{-\theta}Y_{t+1})] \,|\, t\right\},$$

where E{.|t} is the expectations operator conditional on period t information, Λ_t is the marginal utility of consumption for the firm's owners in period t (and hence the appropriate discount rate) and Γ(.) is the firm's cost function.

Manipulating the first-order condition yields an expression for the optimal price

Equation 1
$$\frac{X_t}{P_t} = \frac{\theta}{\theta-1}\frac{\Lambda_t MC_t(\frac{X_t}{P_t})^{-\theta}Y_t + E\left\{\Lambda_{t+1}\Pi_{t+1}MC_{t+1}(\frac{X_t}{P_{t+1}})^{-\theta}Y_{t+1} \,|\, t\right\}}{\Lambda_t(\frac{X_t}{P_t})^{-\theta}Y_t + E\left\{\Lambda_{t+1}\Pi_{t+1}(\frac{X_t}{P_{t+1}})^{-\theta}Y_{t+1} \,|\, t\right\}},$$

where Π_{t+1} is inflation (P_{t+1}/P_t). The real price equals a constant markup over the weighted average of marginal cost during the period for which the price is fixed, where the weights incorporate the effects of discounting and trend inflation.

The aggregate price level P_t is given by the standard equation

[6] Besides the problems alluded to in footnote 1, Kiley (2002a, 2002b) demonstrates that the Calvo model provides a poor approximation to staggering (which, given this is usually the motivation for adopting the simpler Calvo setup, is important) and has a curious relative price distribution that greatly distorts welfare losses to inflation (as the relative price distribution is the key in computing such losses, e.g. Woodford (2003)).

Equation 2 $P_t = [\frac{1}{2} X_t^{1-\theta} + \frac{1}{2} X_{t-1}^{1-\theta}]^{\frac{1}{1-\theta}}$.

In order to complete the specification of the firm's problem, expressions for marginal cost and the discount rate are necessary. Assuming that household's preferences are separable in consumption and leisure, insurance markets are complete and preferences are of the constant-relative-risk-aversion (CRRA) form with risk aversion equal to σ implies that the discount rate for period t+j (Λ_{t+j}) is $\beta^{j-t} Y_{t+j}^{-\sigma}$. If the disutility from labor supply takes the typical power function form, household j's decision regarding its hours supply ($H_{j,t}$) is governed by the intratemporal optimality condition

$$\frac{W_t}{P_t} = H_{j,t}^{\phi} Y_t^{\sigma},$$

where W_t/P_t is the real wage and $1/\phi$ is the labor supply elasticity. Finally, household j is assumed to be attached to firm j (i.e., labor markets are sector-specific). The production function of firm j is given by

$$Y_{j,t} = H_{j,t}^{a},$$

implying that total costs for firm j in period t is

$$\Gamma(Y_{j,t}) = \frac{W_t}{P_t} H_{j,t} = \frac{W_t}{P_t} Y_{j,t}^{\frac{1}{a}}.$$

Differentiating and then substituting the expression for the real wage above yields marginal cost for a firm charging nominal price X_t in period k

Equation 3 $MC_{t+k} = \frac{1}{a}(\frac{X_t}{P_{t+k}})^{-\omega\theta} Y_{t+k}^{\omega+\sigma}, \quad \omega = \frac{\phi}{a} + \frac{1}{a} - 1.$

The parameter ω represents the elasticity of marginal cost with respect to the firm's own output and will be an important parameter below.

Inserting equation 3 into equation 1 along with the expression for the discount factor yields the solution for the optimal price chosen by a firm in period t

Equation 4 $\frac{X_t}{P_t} = [\frac{\theta}{\theta-1}\frac{1}{a}\frac{Y_t^{1+\omega} + E\left\{\beta\Pi_{t+1}^{1+\theta+\omega\theta}Y_{t+1}^{1+\omega} \mid t\right\}}{Y_t^{1-\sigma} + E\left\{\beta\Pi_{t+1}^{\theta}Y_{t+1}^{1-\sigma} \mid t\right\}}]^{\frac{1}{1+\omega\theta}}$

Log-linearizing equations 4 and 2 around the steady state values of relative prices, output, and inflation ($P_{t+1}/P_t = \Pi$) yields (with lower-case letters denoting log-deviations from steady-state levels)

$$x_t - p_t = d_1 y_t + E\left\{d_2 y_{t+1} + d_3 \pi_{t+1} \mid t\right\},$$

$$d_1 = \frac{1}{1+\omega\theta}[\frac{1+\omega}{1+\beta\Pi^{1+\theta+\omega\theta}} - \frac{1-\sigma}{1+\beta\Pi^{\theta}}],$$

$$d_2 = \frac{1}{1+\omega\theta}[\frac{(1+\omega)\beta\Pi^{1+\theta+\omega\theta}}{1+\beta\Pi^{1+\theta+\omega\theta}} - \frac{(1-\sigma)\beta\Pi^{\theta}}{1+\beta\Pi^{\theta}}],$$

$$d_3 = \frac{1}{1+\omega\theta}[\frac{(1+\theta+\omega\theta)\beta\Pi^{1+\theta+\omega\theta}}{1+\beta\Pi^{1+\theta+\omega\theta}} - \frac{\theta\beta\Pi^{\theta}}{1+\beta\Pi^{\theta}}].$$

Equation 5 $\pi_t = \Pi^{1-\theta}[x_t - p_t] + x_{t-1} - p_{t-1}.$

These expressions look cumbersome, but should be familiar for the case where trend inflation equals zero (Π = 1) and there is no discounting (β = 1); in that case, these equations simplify to

$$x_t - p_t = \frac{1}{2}\frac{\omega+\sigma}{1+\omega\theta}[y_t + E\{y_{t+1} \mid t\}] + \frac{1}{2}E\{\pi_{t+1} \mid t\}$$

$$\pi_t = x_t - p_t + x_{t-1} - p_{t-1},$$

which are equivalent to the staggered price-setting specification in Taylor (1980), Chari, Kehoe, and McGrattan (2000), and Kiley (2002a, 2002b). However, in the present case trend inflation raises the importance of future output and inflation in decisions regarding the current price (i.e., d_2 and d_3 are increasing in trend inflation). This occurs because firms realize that demand for their product will be higher in the future, after inflation has eroded the real value of their nominal prices; hence, firms place a larger weight on future developments in setting current prices when inflation is higher. Note that these responses are related to the increase in relative price dispersion that accompanies higher inflation in the staggered-price setting model, consistent with earlier empirical work and the emphasis in recent work (e.g., Woodford (2003)) on the importance of this channel in staggered price setting models for aggregate welfare.

For the analysis of equilibrium determinacy, the set of stochastic disturbances affecting the economy can be ignored. But the analysis of volatility will require some set of exogenous disturbances. Both for simplicity and in line with earlier work (Clarida, Gali, and Gertler (1999)), a cost-push shock is appended to the log-linearized equation for relative prices, yielding

Equation 6 $$x_t - p_t = d_1 y_t + E\{d_2 y_{t+1} + d_3 \pi_{t+1} \mid t\} + u_t,$$

where u is an i.i.d. disturbance term.

The remainder of the model follows the new-Keynesian literature. The IS equation links the deviation of current output from its steady-state or potential value (y) to the real interest rate deviation (the nominal rate i minus future inflation) and future output

Equation 7 $y_t = E\{y_{t+1} \mid t\} - \frac{1}{\sigma}[i_t - E\{\pi_{t+1} \mid t\}]$.

Micro-foundations for this equation can be found in the consumption Euler equation (equation 7 is a log-linearized version of such an equation). As noted above, output replaces consumption for simplicity (reflecting its dominance in aggregate demand). A more thorough discussion can be found in Woodford (2003).

The aggregate demand side of the model is closed with a specification of monetary policy, which follows a forward-looking (with respect to inflation) Taylor rule

Equation 8 $i(t) = \gamma_\pi E\{\pi_{t+1} \mid t\} + \gamma_y y_t$.

This specification is relatively standard. A substantial body of earlier work (e.g., Clarida, Gali, and Gertler (2000), Bullard and Mitra (2002), and Woodford (2003)) has demonstrated that equilibrium determinacy, and hence macroeconomic stability, can be achieved in this framework if the real interest rate increases with inflation ($\gamma_\pi > 1$). This property has been labeled the Taylor principle, following the influential work of Taylor (1993, 1999). Two considerations drive our emphasis on forward-looking behavior. First, Clarida, Gali, and Gertler (1998, 2000) and Orphanides (2002) both argue that monetary policy behavior in the United States, Germany and Japan has been well-described by this type of behavior. In addition, central banks that have adopted inflation targeting have placed increased focus on inflation expectations and have characterized their behavior in this regard as consistent with an equation like 8 (for example, see the summary of twenty countries' experience with inflation targeting in Schmidt-Hebbel and Tapias (2002)). Of course, decisions in actual practice invariably include factors not included in the model. Section 3 will examine the sensitivity of the results to alternative specifications of the monetary-policy rule.

The model can be compactly expressed as a second-order expectational difference equation

Equation 9 $AE\{z_{t+1} \mid t\} + Bz_t + Cz_{t-1} = Du_t$ **,**

where z_t is a 4x1 vector containing the relative price set at t, inflation, output, and the nominal interest rate. A, B, C, and D are matrices containing structural coefficients. Equation 9 has a unique rational-expectations solution in which fluctuations are driven solely by the cost-push shock (u) when the number of roots of the matrix polynomial on the left-hand side that lie inside the unit circle equal the number of predetermined endogenous variables (one in this case, reflecting the lagged relative price in the inflation equation). When more than one of these roots lie inside the unit circle, rational-expectations solutions in which sunspots – non-fundamental shocks – can drive fluctuations are also possible; this multiplicity is termed indeterminacy herein.[7] Such indeterminacy is undesirable as non-fundamental shocks could increase the variability of the economy, a notion pursued, for example, in Clarida, Gali, and Gertler (2000).

In the next section we explore the importance of trend inflation for determinacy, and then discuss volatility within the determinate region of the parameter space. It is instructive to consider equation 6 once again, the expression for price setting (suppressing the shock):

$$x_t - p_t = d_1 y_t + d_2 y_{t+1} + d_3 \pi_{t+1}.$$

When inflation equals zero, the parameter d_3 is always less than one-half – the fraction of current price setters. In that case, expectations are not overly important. In contrast, positive trend inflation increases the importance of expectations (both parameters d_2 and d_3), which should make the economy more susceptible to sunspot fluctuations.

The intuition is most clear in a simple difference equation. Consider the univariate difference equation for a variable z that is hit by an autocorrelated stochastic disturbance u,

$$z_t = \lambda E\{z_{t+1} \mid t\} + u_t, \quad u_t = \rho u_{t-1} + e_t, \quad e_t \sim i.i.d.$$

Focusing first on the case where there is a determinate, stationary equilibrium ($|\rho| < 1$, $|\lambda| < 1$), the solution to this equation is

$$z_t = \frac{1}{1-\lambda\rho} u_t.$$

The variance of z is increasing in the importance of forward-looking behavior (the parameter λ) if the shock is positively autocorrelated. While our model (equation 9) is not subject to persistent cost-push shocks by assumption, there are lagged endogenous variables which could generate similar implications through endogenous persistence.

Now consider the possibility of self-fulfilling expectations. When the weight on expectations is large (λ greater than one), indeterminacy and the possibility of sunspot fluctuations arise. In this case, the volatility of the variable z could be higher due to non-fundamental shocks. We will see in the next section that this intuition regarding the importance of the weight on expectations carries over into our more complex model.[8]

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[7] Farmer (1993) provides a good introduction to indeterminacy and the possibility of sunspot equilibria in rational expectations models of the sort discussed in this research.

4. Results on Equilibrium Determinacy and Volatility

A. Indeterminacy

We focus first on determinacy of equilibrium and return later to volatility under policy settings consistent with a unique equilibrium. Our results are derived from an extensive set of numerical exercises, in which the model is solved using the AIM algorithm originally developed by Anderson and Moore (for a recent presentation, see Anderson (2000)). We first assign a baseline set of parameter values to the model. Since the sticky price model assumes that nominal prices are fixed for two periods, a period is assumed to correspond to one-half year. Table 2 presents values for most of the parameters. The discount factor (β) is set at 0.96 per year, implying a real interest rate of approximately 4 percent. The coefficient of relative risk aversion (σ) is set at ¼; while this value is quite low, Woodford (2003) justifies a low value by noting that the inverse of this parameter governs the interest sensitivity of aggregate demand and that this sensitivity is substantially higher than the intertemporal elasticity of substitution of consumption once investment in business capital and consumer durables – both absent from the model – are considered. The elasticity of output with respect to labor input (a) is set at 2/3, approximately the value in US data. The baseline setting for the markup of prices over marginal cost ($1/(\theta-1)$) is 10 percent (Woodford (2003) typically uses a similar value, and other values of this parameter are discussed below). The labor supply elasticity ($1/\phi$) equals 1 in the baseline calibration. This value for the labor supply elasticity lies above traditional estimates (MaCurdy (1981), Altonji (1986), and Abowd and Card (1989)) but below the common assumption in dynamic general equilibrium models that labor supply is perfectly elastic (an assumption associated with indivisible labor

[8] King and Wolman (2004) provide a different example in which greater forward-looking behavior raises the

supply). This baseline value is near the recent estimates of Mulligan (1998), and alternatives are considered below.

Table 2: Baseline parameter values

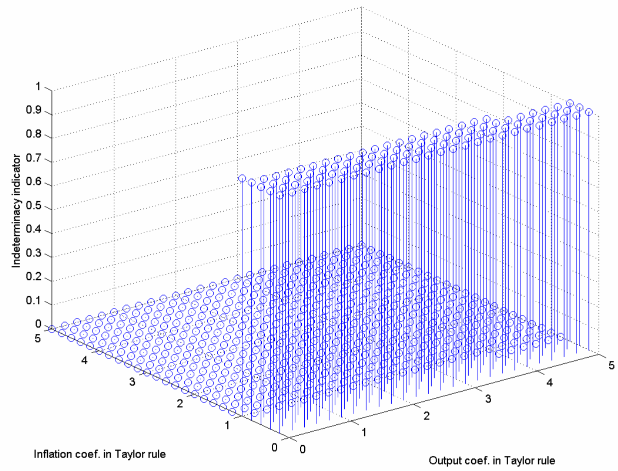
Parameter	Description	Value
β	Discount factor	0.96
σ	Coef. of rel. risk avers.	¼
a	Elast. Of Y w.r.t. H	2/3
$1/(\theta-1)$	Markup (at zero infl.)	.10
$1/\phi$	Labor supply elast.	1

Note: Discount factor expressed at an annual rate

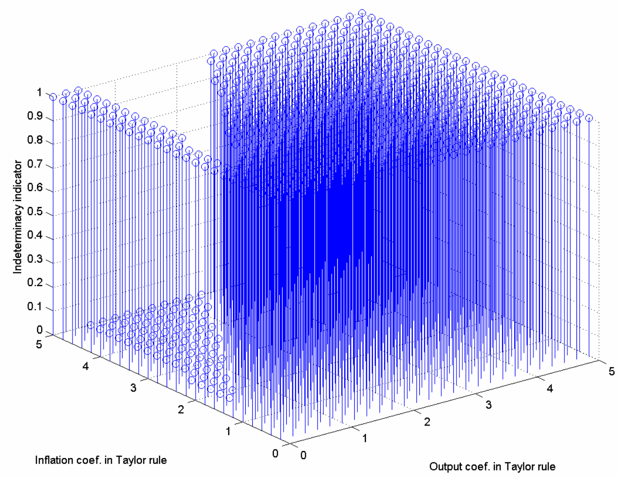
The remaining parameter values are the trend inflation rate (Π) and the coefficients in the Taylor rule. Our experiments consider the possibility of indeterminacy for different values of these parameters. For each set of parameters, the indeterminacy of equilibrium is examined numerically, and the results for a range of inflation rates and policy settings are summarized in figure 2. The three panels of figure 2 correspond to trend inflation rates of 0 percent, 4 percent, and 8 percent, respectively. (The value for inflation refers to the steady-state percent change in the price level *at an annual rate*). Each panel presents whether indeterminacy arises for coefficients in the Taylor rule ranging from 0 to 5 for both output and the one-period-ahead inflation forecast; indeterminacy is indicated by a value of one for the indicator. As shown in panel A, indeterminacy, and hence the possibility of sunspot fluctuations, arises with trend inflation equal to zero when the Taylor principle is violated (γ_π not greater than 1), except for cases with a strong output response, in which case the Taylor principle is slightly relaxed.

potential for multiple equilibria in a New-Keynesian model.

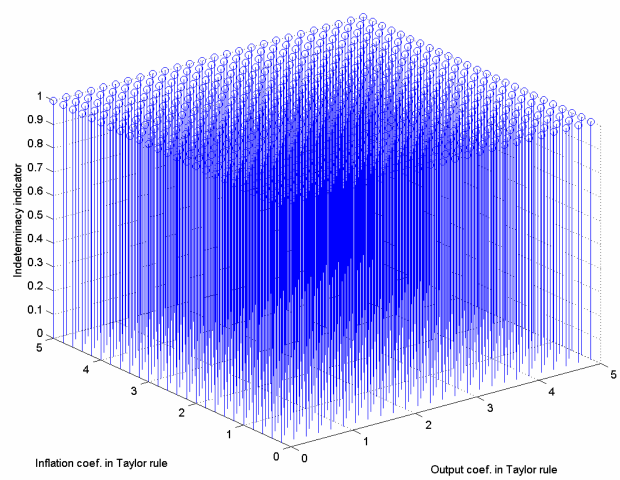
Figure 2: Equilibrium Determinacy for Different Trend Inflation Rates (Π)



A. Π = 1.00



B. Π = 1.04



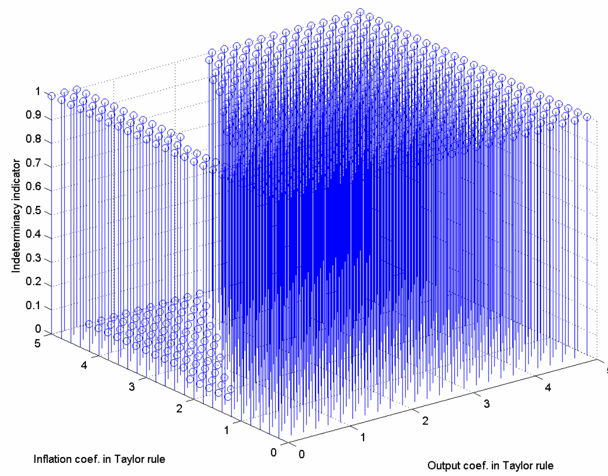
C. Π = 1.08

Panels C and D illustrate how this condition changes with trend inflation. For trend inflation of 4 percent, indeterminacy is possible for a much wider range of the parameter space: the output response (γ_y) must be slightly positive, but not too large; and the inflation response (γ_π) must be greater than one, and by a significant amount for larger values of the output response. When trend inflation is 8 percent, no set of responses in the rule yields determinacy.

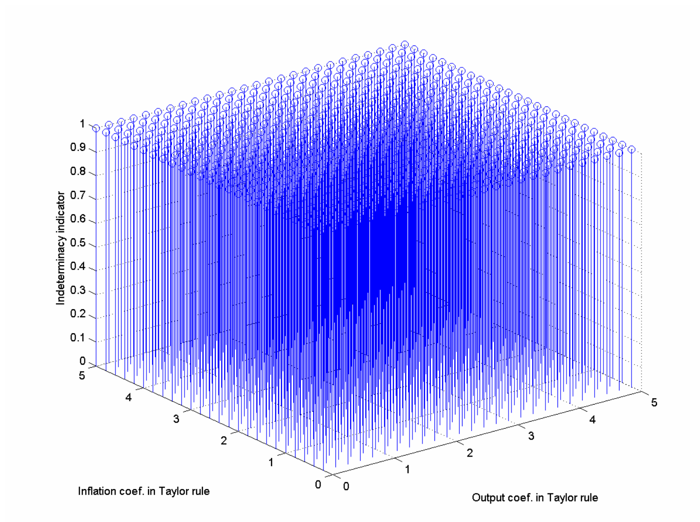
As mentioned earlier, higher trend inflation increases the role of expectations in the dynamic system, contributing to the possibility of sunspot fluctuations. Other parameters are important as well – in particular, the labor supply elasticity and the elasticity of demand for a firm's product. Firms know that demand will be higher when inflation erodes their real price – i.e., in the future. The strength of this effect is driven by the demand elasticity; and low labor supply elasticities imply that marginal cost is more sensitive to high demand, which implies that future profits are eroded to a greater extent by price rigidity with high inflation and lower labor supply elasticities. Therefore, the demand elasticity and labor supply elasticity play important roles in determining the importance of inflation for forward-looking behavior.

Figure 3 presents the indeterminacy regions for different values of the labor supply elasticity, holding all other parameters at their baseline values and fixing trend inflation at 4 percent ($\Pi = 1.04$). Panel A. reproduces the baseline results. Panel B shows that a low labor supply elasticity (¼, similar to estimates cited above) makes indeterminacy likely, while a high labor supply elasticity (infinity) makes indeterminacy less of a problem for policymakers. Again, higher weight on expectations, in this case through a lower labor supply elasticity, makes indeterminacy a larger potential problem.

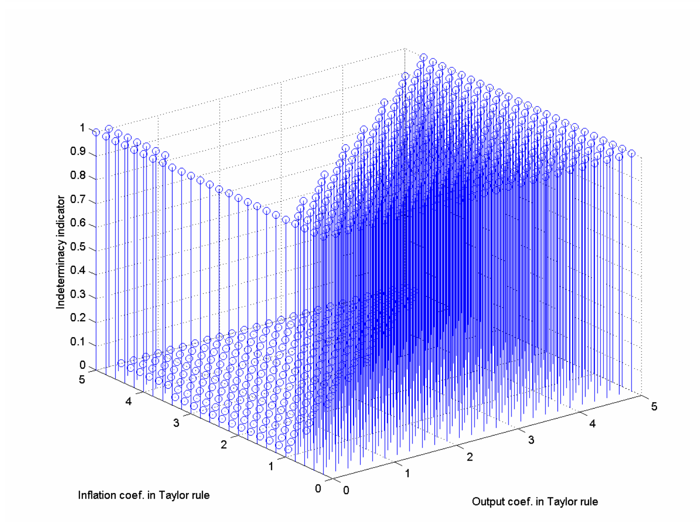
Figure 3: Equilibrium Determinacy for Different Labor Supply Elasticities (1/ϕ)



A. $1/\phi = 1.00$



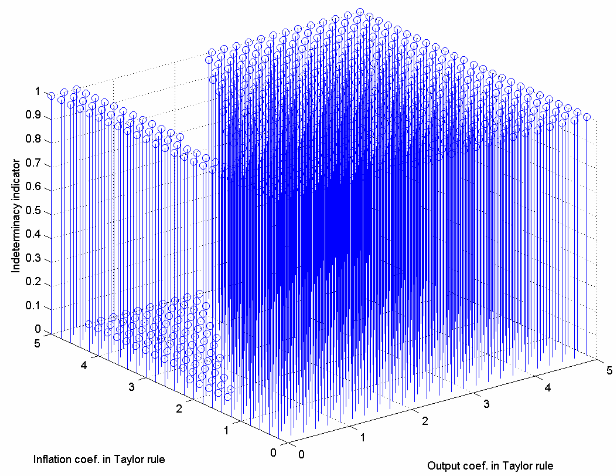
B. $1/\phi = 1/4$



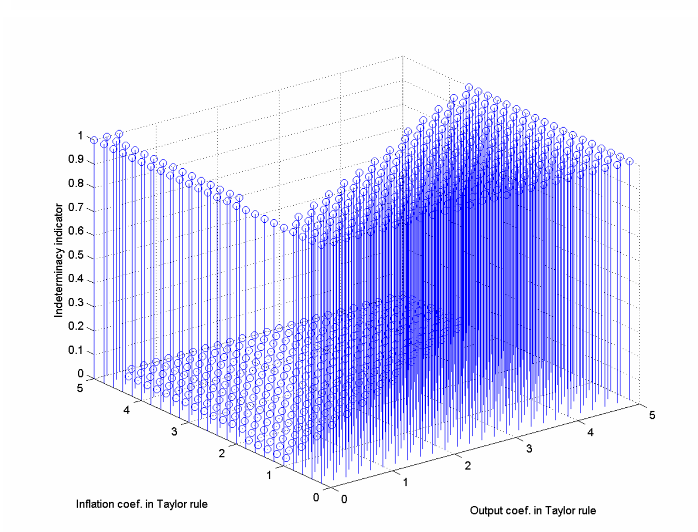
C. $1/\phi = \infty$

Figure 4 presents the results on indeterminacy for the baseline demand elasticity and a lower elasticity (consistent with a markup of 20 percent), again with trend inflation of 4 percent. A lower elasticity (higher markup) lowers the importance of future demand in price setting, and this shrinks the region of policy settings over which indeterminacy is a concern.

Figure 4: Equilibrium Determinacy for Different Demand Elasticities (θ)



A. $\theta = 11$



B. $\theta = 6$

In summary, sunspot fluctuations are a possible concern when trend inflation is moderate, at least under the type of forward-looking Taylor-rule that has been discussed as a reasonable characterization of behavior for some central banks. Even values of trend inflation of 4 percent per year substantially shrink the range of policy settings that deliver equilibrium determinacy. At trend inflation of 8 percent, policy settings within the forward-looking Taylor rule framework cannot ensure determinacy for reasonable values of other parameters. Alternative policy rules are discussed in subsection C below.

B. Volatility in the Determinate Region

The previous section demonstrated that the increasing importance of forward-looking behavior with trend inflation in sticky-price models raises the potential problem of indeterminacy, which could increase macroeconomic volatility through the possibility of sunspot fluctuations. Even within the parameter space consistent with a determinate equilibrium, greater forward looking behavior could contribute to increased volatility in response to fundamental shocks, as highlighted in our simple example at the end of section 3.

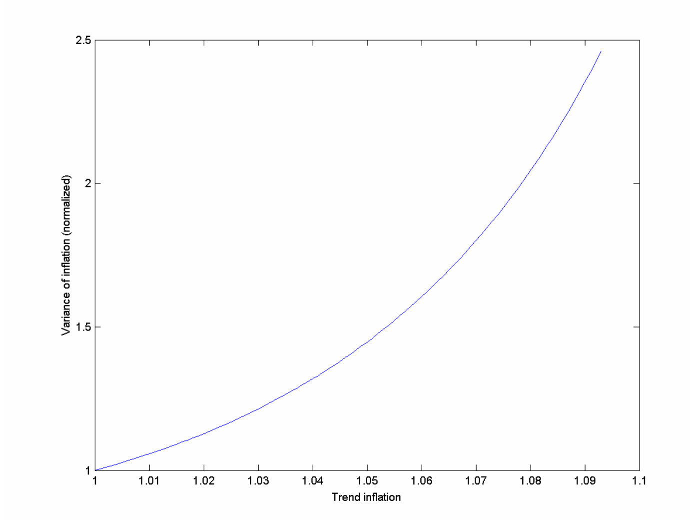
The implications of trend inflation for the volatility of inflation and output in our model are examined numerically using a markup of 20 percent, an infinite labor supply elasticity, and traditional values for the coefficients in the Taylor rule ($\gamma_\pi = 1.5$, $\gamma_y = 0.5$). Other parameters equal their values in Table 2. (The high markup and high labor supply elasticity were chosen to allow a substantial range for inflation within which the equilibrium is determinate; the qualitative results do not depend on the specific values chosen for these parameters). Figure 5 presents the variances of inflation and output for different trend inflation rates, normalized so that each equals 1 at a zero percent inflation rate. (In other words, the values within the panel represent the variances relative to those that prevail at zero inflation given that an i.i.d. cost-push shock is the only disturbance hitting the economy. The variances of inflation and output across panels A and

B are no comparable because the normalization is made within each panel and does not preserve the relative inflation/output variances; given the simplicity of the model, such a comparison is arguably uninteresting).

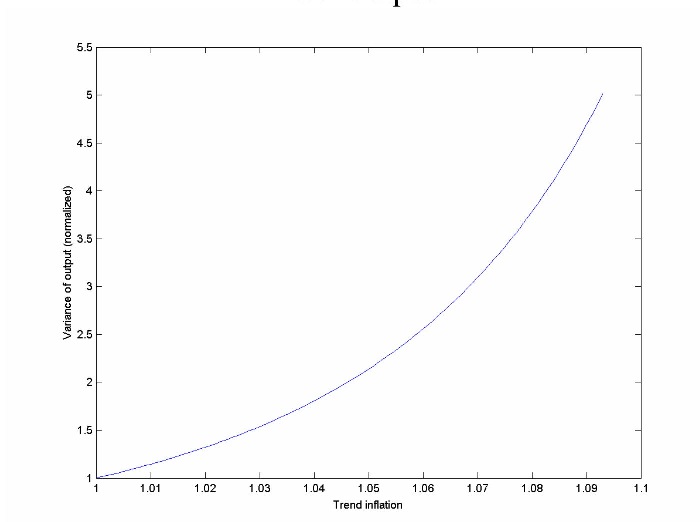
The results illustrate than the intuition from the simple example considered earlier in this subsection carries over to the sticky-price model. Both output and inflation volatility rise with trend inflation when the monetary policy settings are held fixed. (Note that the variances are not presented up to trend inflation of 10 percent, as equilibrium determinacy fails prior to that level of inflation). This suggests that moderate trend inflation may contribute to macroeconomic instability both through its effect on the transmission of fundamental shocks and through the possibility of sunspot-induced volatility.

Figure 5: The Variances of Inflation and Output at Different Values of Trend Inflation

A. Inflation



B. Output



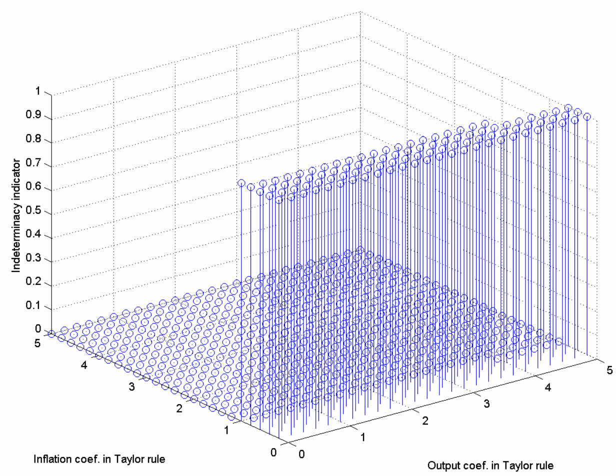
C. Robustness to Alternative Policy Rules

The analysis has held fixed the policy rule. An alternative to a forward-looking Taylor rule is the Taylor rule with contemporaneous inflation,

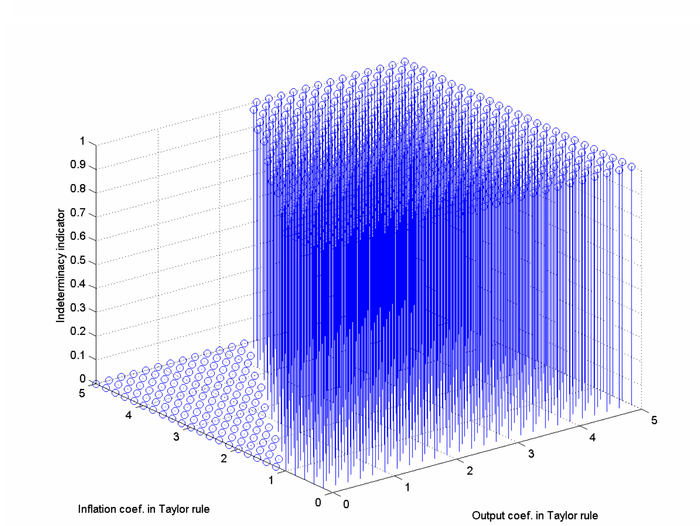
$$i(t) = \gamma_\pi \pi_t + \gamma_y y_t .$$

Figure 6 presents the indeterminacy regions for trend inflation rates of zero and 4 percent at the baseline parameter values in Table 2 using this alternative Taylor rule. While the indeterminacy region is slightly different, the picture is much the same as that with the forward-looking rule: trend inflation substantially increases the parameter space over which indeterminacy and sunspot fluctuations are a possible concern.

Figure 6: Equilibrium Determinacy under a Contemporaneous Taylor Rule



A. $\Pi = 1.00$



B. $\Pi = 1.04$

This examination of a slightly different form of the Taylor rule is only a very tiny fraction of the very general set of rules that could be considered. An analysis of optimal rules under discretion and commitment by Ascari and Ropele (2004) has been performed for a Calvo sticky-price model since the initial drafts of this paper circulated. These authors illustrate that optimal rules are subject to the same effects as documented herein for the Taylor rule. While subsequent research may discover policy rules that perform well in the presence of trend inflation, the results herein remain important given the central role that Taylor-type rules have played in central bank practice according to a number of researchers (e.g., Clarida, Gali, and Gertler (1998, 2000) and Orphanides (2002)).

D. Empirical Relevance

The analysis has illustrated that the impact of trend inflation on determinacy and volatility may be apparent at quite moderate levels of inflation, suggesting that equilibrium indeterminacy and the instability possible from sunspot fluctuations may be a serious concern for moderate trend inflation. This effect may be relevant for some historical episodes, in light of the different average rates of inflation witnessed in the G-7 since the 1970s..

It is clear from table 1 that average inflation rates in the earlier period were well within the range that can lead to equilibrium indeterminacy or affect macroeconomic volatility from fundamental shocks. This may suggest that the conclusion of Clarida, Gali, and Gertler (2000) – that the failure of policymakers to increase real interest rates in response to increases in expected inflation was a source of aggregate instability in the 1970s – should be re-interpreted: In fact, moderate-to-high inflation is inherently unstable under the Taylor-type rules currently in vogue. Hence, it may have been the high level of inflation, not the policy actions attempting to stabilize fluctuations around that high level, which contributed to macroeconomic volatility in the 1970s.

This result may also be relevant in light of the work of Orphanides (2002), which finds that the Federal Reserve may have been following a forward-looking interest rate rule in the 1970s that satisfied the Taylor principle, but relied too heavily on output gap estimates that suggested the economy was operating substantially below potential and hence pursued a policy that was excessively loose. To the extent such actions can be characterized as a medium-term shift in the inflation target (a reasonable description, as a persistently large and negative output gap estimate inserted into a Taylor rule can be expressed algebraically as a rule with the correct output gap estimate and a new, higher inflation target by simple substitution into equation 8), Orphanides' conclusions, in conjunction with the result herein that medium-to-high inflation targets can generate instability, are consistent with the notion that monetary policy contributed to macroeconomic volatility in the 1970s, as argued for different reasons by Clarida, Gali, and Gertler (2000).

Finally, it is important to remember that herein we have focused on instability under monetary rules that have been suggested as summaries of the behavior of most inflation-targeting central banks (Schmidt-Hebbel and Tapias (2002)). A substantial number of such inflation targeters, particularly in developing countries, pursue targets that are moderate-to-high. Lower target inflation rates may contribute to macroeconomic stability.

5. Conclusion

Evidence across time and countries suggests that moderate-to-high inflation tends to be less stable, but it has not been clear from previous work whether this is an intrinsic feature of such regimes. The analysis herein suggests that moderate-to-high inflation is inherently unstable in New-Keynesian models with staggered-price setting. This finding may explain the evidence across time and countries. Further, it suggests a re-interpretation of the 1970s. Clarida, Gali, and Gertler (2000) and Orphanides (2002) have agreed that monetary policy in the United States was well characterized by a Taylor-type rule at that time, but disagree with respect to whether the Taylor principle was followed and therefore whether volatility was increased by sunspot fluctuations. It has been shown herein that high rates of inflation contribute to the possibility of sunspot fluctuations and, within the model examined, amplify the effect of cost-push shocks on macroeconomic volatility. Given this, the vigorous responses of nominal interest rates to expected inflation called for by Clarida, Gali, and Gertler (2000) may not have been sufficient to deliver significantly increased stability in the 1970s until a commitment to low inflation had been put in place. This result is also relevant for central banks in countries with moderate-to-high inflation that have recently adopted (or are considering for the future) inflation targeting: Stability under such regimes is only possible with low trend inflation rates in the new-Keynesian model.

The analysis herein was kept as simple as possible to convey the main ideas. Extensions to more fully-specified general equilibrium models may be useful, especially to consider the impact of investment and different assumptions regarding labor and product markets on the sensitivity of firm's price-setting behavior to future conditions. It is also important to note that the analysis herein assumed that firms' prices were rigid for two periods. Some recent work has assumed that firms index their nominal prices to inflation (e.g., Christiano, Eichenbaum, and Evans (2001)). The main results of this paper do not generalize to that case, as the effect of inflation on relative prices is key to firm's increased sensitivity to future conditions. We do not view such indexation as a plausible characterization of price-setting in the developed countries considered for several reasons (the first two were offered by Ascari and Ropele (2004)). Survey evidence on price rigidity points to fixed nominal prices without indexation. Gray (1976) demonstrated that full indexation is unlikely to be optimal. And empirical findings such as those in Christiano, Eichenbaum, and Evans (2001) – which suggest a role for indexation across long samples of US history – should be interpreted carefully, as their econometric work clearly mixes data from at least two different periods of trend inflation, as suggested above, while their theoretical model is a linearized approximation of a model around a single rate of trend inflation. A transition between different steady states would lead to a role of lagged inflation in a time-series decomposition of inflation because the break between periods generates unit-root like behavior in inflation, even when indexation is absent (e.g., Erceg and Levin (2003)).

Finally, there is no evidence that the range of inflation experienced in the United States over the postwar period has been associated with differences in price rigidity sufficient to affect the analysis herein, indicating that the mechanisms identified in this study may be operative in similar economies. But the data suggest that nominal price rigidities are less important at much higher rates of inflation (e.g., Kiley (2000)). Very high inflation clearly lowers the relevance of sticky prices for economic fluctuations.

References

Abowd, John M. and David Card (1989) On the Covariance Structure of Earnings and Hours Changes. *Econometrica*, 57(2), March, pages 411-445.

Altonji, Joseph G. (1986) Intertemporal Substitution in Labor Supply: Evidence from Micro Data. *Journal of Political Economy* 94(3), Part 2, June, pages S176-S215.

Anderson, Gary (2000) The Anderson-Moore Algorithm. Federal Reserve Board Occasional Staff Study 4, http://www.federalreserve.gov/pubs/oss/oss4/aimindex.html.

Ascari, Guido (2000) Staggered Price and Trend Inflation: Some Nuisances. Mimeo.

Ascari, Guido and Tiziano Ropele (2004) The Optimal Stability-Oriented Monetary Policy: Optimal Monetary Policy Under Low Trend Inflation. Mimeo.

Bakhshi, Hasan et al. (2002) Endogenous Price Stickiness, Trend Inflation, and the New-Keynesian Phillips Curve. Mimeo.

Benhabib, Jess, Stephanie Schmitt-Grohé and Martin Uribe (2002) Designing Monetary Policy: Backward-Looking Interest-Rate Rules and Interest Rate Smoothing. Mimeo, Rutgers University, October.

Bullard, James and Kaushik Mitra (2002) Learning About Monetary Policy Rules. *Journal of Monetary Economics*, 49(6), September, pages 1105-1129.

Calvo, Guillermo A. (1983) Staggered Prices in a Utility Maximizing Framework. *Journal of Monetary Economics* 12, pages 383-398.

Carlstrom, Charles T. and Timothy J. Fuerst (2000) Forward-Looking Versus Backward-Looking Taylor Rules. Federal Reserve Bank of Cleveland Working Paper 0099.

Chari, V. V., Patrick J. Kehoe and Ellen R. McGrattan (2000) Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?, *Econometrica*, 68(5), September, pages 1151-1179.

Christiano, Lawrence J., Martin Eichenbaum and Charles Evans (2001) Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. NBER Working Paper No. 8403, July.

Clarida, Richard, Jordi Gali and Mark Gertler (1998) Monetary Policy Rules in Practice: Some International Evidence. *European Economic Review* 42, pages 1033-106.

Clarida, Richard, Jordi Gali and Mark Gertler (1999) The Science of Monetary Policy. *Journal of Economic Literature* 37(4), December, pages 1661-1707.

Clarida, Richard, Jordi Gali and Mark Gertler (2000) Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. *Quarterly Journal of Economics* 115(1), February, pages 147-80.

Erceg, Christopher and Andrew Levin (2003) Imperfect Credibility and Inflation Persistence. *Journal of Monetary Economics*, 50(4), May, pages 915-44.

Farmer, Roger (1993) *The Macroeconomics of Self-Fulfilling Prophecies*. The MIT Press, Cambridge, MA.

Fischer, Stanley (1981) Relative Shocks, Relative Price Variability, and Inflation. *Brookings Papers on Economic Activity*, 2, 381-441.

Friedman, Milton (1977) Nobel Lecture: Inflation and Unemployment. *Journal of Political Economy*, 1977, v. 85, iss. 3, pp. 451-72.

Gray, Joanna A. (1976) Wage Indexation: A Macroeconomic Approach. *Journal of Monetary Economics* 2, pages 221—235.

Kiley, Michael T. (2000) Price Stickiness and Business Cycle Persistence. *Journal of Money, Credit, and Banking* 32(1), February, pages 28-53

Kiley, Michael T. (2002a) Partial Adjustment and Staggered Price Setting. *Journal of Money, Credit, and Banking* 34(2), May, pages 283-298

Kiley, Michael T. (2002b) The Lead of Output over Inflation in Sticky Price Models. *Economics Bulletin*, 5(5), August, pages 1-7.

King, Robert G. (2000), The New IS-LM Model: Language, Logic and Limits. Federal Reserve Bank of Richmond *Economic Quarterly* 86(3), pages 45-103.

King, Robert G. and Alexander Wolman (2004) Monetary Discretion, Pricing Complementarity and Dynamic Multiple Equilibria, Federal Reserve Board International Finance Discussion Papers, Number 2004-802.

Klenow, Peter J., and Oleksiy Kryvtsov (2003) State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation? Federal Reserve Bank of Minneapolis, Mimeo

MaCurdy, Thomas E. (1981) An Empirical Model of Supply in a Life Cycle Setting. *Journal of Political Economy* 88(6), December, pages 1059-85.

McConnell, Margaret and Gabriel Perez-Quiros (2000), Output Fluctuations in the United States: What Has Changed Since the Early 1980s? *American Economic Review* 90(5).

Mulligan, Casey B. (1998) Substitution over Time: Another Look at Life Cycle Labor Supply. *NBER Macroeconomics Annual 1998*, Vol. 13.

Okun, Arthur (1971) The Mirage of Steady Inflation. *Brookings Papers on Economic Activity*, 2, 485-98.

Organisation for Economic Cooperation and Development (OECD) (2002) *Economic Outlook* Vol. 71.

Orphanides, Athanasios (2002) Monetary Policy Rules and the Great Inflation. Federal Reserve Board Finance and Economics Series Discussion Papers No. 2002-8.

Schmidt-Hebbel, Klaus and Matias Tapias (2002) Monetary Policy Implementation and Results in Twenty Inflation Targeting Countries. Central Bank of Chile Working Papers No. 166.

Stockton, David J. (1988) Relative Price Dispersion, Aggregate Price Movement, and the Natural Rate of Unemployment. *Economic Inquiry*, January 1988, v. 26, iss. 1, pp. 1-22.

Taylor, John B. (1980) Aggregate Dynamics and Staggered Contracts. *Journal of Political Economy*. 88, pages 1-23.

Taylor, John B. (1981) On the Relationship Between the Variability of Inflation and the Average Inflation Rate. *Carnegie Rochester Conference Series on Public Policy*, 15, 57-85.

Taylor, John B. (1993) Discretion versus Policy Rules in Practice. *Carnegie-Rochester-Conference-Series-on-Public-Policy* 39, December, pages 195-214.

Taylor, John B. (ed.) (1999) *Monetary Policy Rules.* NBER Conference Report series. Chicago and London: University of Chicago Press.

Woodford, Michael (2003) *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton, NJ.