Competitive Equilibria and the Core of Exchange Economies under Asymmetric Information

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Abstract

I argue in favor of a competitive screening approach for studying the question of coalition formation in exchange economies under asymmetric information. I obtain a new notion of core that refines Wilson (1978)'s coarse core. It is nonempty under the standard regularity conditions. I also justify a notion of competitive equilibrium formerly introduced by Wilson as a technical tool for proving the nonemptiness of the coarse core. Indeed, the core converges to the set of Wilson equilibria when the economy is replicated.

1 Introduction

The objective of the paper is to study the allocation of scarce resources between agents that are asymmetrically informed about the fundamentals of the economy. I assume for simplicity that the true state of the economy is verifiable when the contracts are implemented. Incentive and measurability constraints are therefore irrelevant. The main question is the following. To what extent can the agents insure themselves against risk?

The main reference for core concepts is Wilson (1978). An agreement specifies a way to split the endowment of the economy among the agents in each state. Such a function is called an allocation rule. Wilson discusses various notions of objection against given allocation rules. They differ by the amount of communication that is permitted between the agents. Two polar notions emerged: coarse objections are based on events that are common knowledge among the members of the coalition; fine objections are based on events that can be discerned by pooling the information of the members of the coalition. Obviously the fine core is included in the coarse core.

An important feature is missing from Wilson's theory. Where are the tested allocation rules coming from? A matter that was irrelevant when information

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was complete becomes crucial. This observation was made clear by Myerson (1983) in the case of an informed principal but has not been exploited so far for the core. Any offer coming from an informed agent may reveal some information to the others and influence their willingness to accept the proposal. Such considerations may greatly restrict the set of feasible agreements, see example 11 of de Clippel and Minelli (2003).

Signalling games are hard to analyze. The very complexity of the problem explains why it is difficult for the agents to reach an agreement on their own. The potential gains from trade and insurance may be substantial though. I consider that uninformed intermediaries will help the agents to coordinate in an attempt to make some profit. I study the consequences of a competition a la Bertrand between the intermediaries. In other words, I adapt the competitive screening approach initiated by Rothschild and Stiglitz (1976) in order to study coalition formation.

I obtain a new notion of core in section 3. It is nonempty if the usual regularity conditions are satisfied. It is a subset of the coarse core but bears no general relation with the fine core. I characterize in section 4 the limit of the core when the economy is replicated as in Debreu and Scarf (1963). Quite surprisingly, the limit coincides with a notion of constrained market equilibrium introduced by Wilson as a technical tool for proving the nonemptiness of the coarse core.

It is the first convergence theorem of that kind that is proved for exchange economies under asymmetric information. On the one hand, Serrano et al. (2001) show that no similar result can be obtained for the coarse or the fine core with respect to a very large class of competitive equilibrium notions, including the one introduced by Wilson. They explain why the conjectures of Kobayashi (1980, page 1647) and Yannelis (1991, remark 6.5) are wrong. On the other hand, the convergence results of Einy et al. (2001) and Forges et al. (2001) are completely different in nature from mine. In their models, the agents are symmetrically informed at the time of contracting and asymmetrically informed at the time of implementing the contracts. The main conceptual issue for defining a notion of core in this context is to impose the right restrictions on the set of feasible trades within each coalition. Einy et al. require each agent's trade to be measurable with respect to his private information and show that the associated core converges to the set of Radner (1968) equilibria. Forges et al. require each agent's trade to be incentive compatible and show that the associated core converges in some particular cases to the set of equilibria defined by Prescott and Townsend (1984). They also show by mean of an example that the result is not generally true.

2 The Model

The model is the same as in Wilson (1978). Let N be the finite set of agents. Let L be the finite set of goods. The future state of the economy is uncertain. Let Ω be the finite set of possible states. Let π be the common prior that describes the relative probability of those states. I assume without loss of generality that $\pi(\omega) > 0$ for each $\omega \in \Omega$. The agents may have some private information. The information of agent i is summarized by a partition $\mathcal{P}_i$ of the set Ω. For each $\omega \in \Omega$, let $P_i(\omega)$ be the element of the partition that contains ω. The interpretation goes as follows. When the future state of the economy is ω, agent i knows and only knows that it will be an element of $P_i(\omega)$. His beliefs are derived from π by Bayesian updating. *Events* are subsets of Ω. The probability $\pi(\omega|\mathcal{E})$ of any state ω given an event $\mathcal{E}$ equals 0 if $\omega \notin \mathcal{E}$ and equals $\pi(\omega)/\pi(\mathcal{E})$ if $\omega \in \mathcal{E}$. The true state of the economy is common knowledge among the agents at some future date. It determines their preferences and endowments. Let $e_i : \Omega \to \mathbb{R}^L_+$ be the function that specifies the initial endowment of agent i and let $u_i : \mathbb{R}^L_+ \times \Omega \to \mathbb{R}$ be the function that specifies his preferences. The agents maximize their expected utilities when facing some uncertainty. The utility function of each agent is strongly increasing, continuous and concave in each state of the economy. Decisions are taken today about the way to redistribute the endowments when the state will be common knowledge. An *allocation rule* is a function $a : \Omega \to \mathbb{R}^{L\times N}_+$. It is *feasible* if $\sum_{i\in N} a_i(\omega) \leq \sum_{i\in N} e_i(\omega)$ for each $\omega \in \Omega$. There are typically opportunities for insurance, even if the agents are asymmetrically informed. The expected utility of agent i for some allocation rule a conditional on some event $\mathcal{E}$ is $E(u_i(a_i)|\mathcal{E}) = \sum_{\omega\in\Omega} \pi(\omega|\mathcal{E})u_i(a_i(\omega), \omega)$. The model boils down to a classical exchange economy when there is only one possible state. It coincides with the traditional model of exchange economies under (symmetric) uncertainty when $\mathcal{P}_i = \{\Omega\}$ for each $i \in N$.

3 The Core

Uninformed intermediaries simultaneously offer contracts to the agents. I assume for simplicity that each intermediary proposes exactly one contract to each agent. A *contract* for agent i is a function $c_i : \Omega \to \mathbb{R}^L$. The offer of an intermediary to the agents may be summarized by a function $c : \Omega \to \mathbb{R}^{L\times N}$. The agents then simultaneously choose the contracts they prefer among those offered. I assume for simplicity that each agent signs for at most one contract. Time goes by, uncertainty is resolved and agreed-upon contracts are realized. Suppose an agent i signed for a contract c_i offered by an intermediary j. Let $l \in L$ and let $\omega \in \Omega$ be the state of the economy. Then agent i receives (resp. pays) $c^l_i(\omega)$ units of good l from (resp. to) intermediary j when $c^l_i(\omega)$ is positive (resp. negative). Of course, an agent cannot pay more than what he owns. The preferences of the intermediaries are assumed to be continuous and to satisfy the following two properties: 1. bankruptcy is impossible meaning that it leads to a very low payoff, for instance a heavy jail sentence; 2. the preferences are strongly increasing in the sense that having more of any good in any state of the economy is better ceteris paribus. Each profile of strategies generates an allocation rule. An allocation rule belongs to the *core* if it is supported by some subgame perfect equilibrium. The core is characterized by the following notion of blocking. An allocation rule a is *blocked* by an allocation rule a' if

$S(a, a', \omega) := \{i \in N | E(u_i(a'_i)|P_i(\omega)) > E(u_i(a_i)|P_i(\omega))\} \neq \emptyset$ for some $\omega \in \Omega$ and $\sum_{i \in S(a,a',\omega)} a'_i(\omega) \leq \sum_{i \in S(a,a',\omega)} e_i(\omega)$ for each $\omega \in \Omega$.

Theorem 1 *An allocation rule belongs to the core if and only if it is feasible and cannot be blocked by any alternative allocation rule.*

The intuition is clear. The intermediaries exactly break even for each good in each state of the economy at equilibrium. Hence no intermediary should be able to enter the market, to propose an alternative contract, to break even for each good in each state of the economy and to keep some good in some state of the economy, taking into account which agents are going to choose the alternative proposal. The property stated in the proposition is a convenient reformulation of this simple idea in terms of allocation rules. Perez-Castrillo (1994) obtains a result in that vain for cooperative games with transferable utility under complete information. It can be shown that the theorem remains valid even if the intermediaries and the agents are respectively allowed to offer and to sign many contracts.

Proof: I consider the case where only two intermediaries are competing. The argument can easily be adapted to show that the theorem remains valid for any number of intermediaries greater or equal to two.
$\Rightarrow$) Let $c = (c_i)_{i \in N}$ be a profile of contracts that is supported by some subgame perfect equilibrium of the game. It is not necessarily directly offered by one of the two intermediaries. It is the outcome of the game following the equilibrium contract offers and selections.
a) I prove that $\sum_{i \in N} c_i(\omega) = 0$ for each $\omega \in \Omega$. Of course, $\sum_{i \in N} c_i(\omega) \leq 0$ for each $\omega \in \Omega$ as otherwise one of the two intermediaries bankrupts for some good in some state of the economy and therefore prefers to offer the null contract. Suppose now that $\sum_{i \in N} c_i^l(\omega) < 0$ for some $\omega \in \Omega$ and some $l \in L$. Then, one may construct for each $\epsilon > 0$ a profile of contracts d such that the continuation payoff in good l for the intermediary proposing d is at least 0 when $\sum_{i \in N} c_i^l(\omega) = 0$ and equals $-\sum_{i \in N} c_i^l(\omega) - \epsilon$ when $\sum_{i \in N} c_i^l(\omega) < 0$, for each $\omega \in \Omega$. For sure one of the two intermediaries strictly prefers to get the aggregate equilibrium payoff rather than his own equilibrium payoff. By continuity of the preferences, proposing d is a profitable deviation for at least one of the two intermediaries for each $\epsilon > 0$ small enough. I now sketch how d may be defined. The idea is to equally distribute to the agents in each state of the economy ϵ additional units of each good in excess supply. Formally, $d_i^l(\omega) := c_i^l(\omega) + \epsilon/n$ for each $(i, l, \omega) \in N \times L \times \Omega$ such that $\sum_{j \in N} c_j^l(\omega) < 0$ and $d_i^l(\omega) := c_i^l(\omega)$ for each other triple (i, l, ω). The payoff in each good l of an intermediary offering such a deviating contract indeed equals $-\sum_{i \in N} c_i^l(\omega) - \epsilon$ when $\sum_{i \in N} c_i^l(\omega) < 0$, for each $\omega \in \Omega$. Nevertheless, he could possibly bankrupt in states ω such that $\sum_{i \in N} c_i^l(\omega) = 0$ for each $l \in L$. I slightly modify the definition of d at those states by making transfers between the agents. I focus on states ω such that $\sum_{i \in S(c+e,d+e,\omega)} d_i^l(\omega) > 0$ for some $l \in L$. Let $i \in S(c+e, d+e, \omega)$ be such that $d_i^l(\omega) > 0$. A small amount of good l is transferred from agent i to the other agents. Hence every agent signs the deviating contract should it

be proposed and should the future state of the economy be ω. The deviating intermediary now exactly breaks even in that state as well. There could be new states where the intermediary bankrupts. Fortunately, applying the procedure recursively (using the fact that Ω is finite), I find a contract d such that $\sum_{i\in S(c+e,d+e,\omega)} d_i^l \leq 0$ for each $(l,\omega) \in L \times \Omega$. Notice though that this non positive number is not necessarily the continuation payoff of the deviating intermediary as some agents not in $S(c+e,d+e,\omega)$ could also sign his contract should the future states be ω. Indeed, c is not necessarily available anymore and indifferent agents could sign the alternative contract as well. Therefore I modify one more time the definition of d by imposing that agent i receives nothing in ω if $i \notin S(c+e,d+e,\omega)$. The above modifications do not affect the definition of d at states ω such that $S(c+e,d+e,\omega) = N$. So, the payoff in each good l of an intermediary offering such a deviating contract remains equal to $-\sum_{i\in N} c_i^l(\omega) - \epsilon$ when $\sum_{i\in N} c_i^l(\omega) < 0$, for each $\omega \in \Omega$.
b) Let a be the allocation rule associated to c, i.e. $a = c + e$. It follows from the previous paragraph that a is feasible. Suppose now that it is blocked by some allocation rule a'. Let c' be the profile of contracts defined as follows: $c_i'(\omega) := (1-\epsilon)a_i'(\omega) - e_i(\omega)$ when $i \in S(a,a',\omega)$ and $c_i'(\omega) := 0$ when $i \notin S(a,a',\omega)$, for each $\omega \in \Omega$ ($\epsilon > 0$ is very small). The intermediary proposing c' breaks even in each state of the economy and keeps some strictly positive amount of some goods in some states of the economy. Hence c' is a profitable deviation for both intermediaries, as it follows from the previous paragraph that they both exactly break even at equilibrium in each good and in each state of the economy.
$\Leftarrow$) Let a be a feasible allocation rule that cannot be blocked by any allocation rule. I consider the following strategies. Both intermediaries propose $c = a - e$. All the agents go to the first intermediary. If the first intermediary proposes something different from c, then each agent chooses to stay with him if and only if he strictly prefers his proposal to c. Otherwise the agents go to the second intermediary. If the second intermediary proposes something different from c, then each agent chooses to follow him if and only if he strictly prefers his proposal to c. Otherwise the agents stay with the first intermediary. These strategies are clearly part of a subgame perfect equilibrium and the associated outcome is a. ♣

Let S be a coalition. An event $\mathcal{E} \subseteq \Omega$ is *common knowledge* among the members of S if it can be written as a union of elements of $\mathcal{P}_i$ for each $i \in S$. An allocation rule a is *feasible for* S if $\sum_{i\in S} a_i(\omega) \leq \sum_{i\in S} e_i(\omega)$ for each $\omega \in \Omega$. Coalition S has a *coarse objection* against an allocation rule a if there exist an allocation rule a' feasible for S and an event $\mathcal{E}$ that is common knowledge among the members of S such that $E(u_i(a_i')|P_i(\omega)) > E(u_i(a_i)|P_i(\omega))$ for each $i \in S$ and each $\omega \in \mathcal{E}$. The *coarse core* is the set of feasible allocation rules against which no coalition has a coarse objection (see Wilson, 1978).

Theorem 2 *The core is a subset of the coarse core.*

Proof: Let a be an allocation rule and let $(S, a', \mathcal{E})$ be a coarse objection against a. Then a is blocked by the allocation rule a'' where $a_i''(\omega) := (1-\epsilon)a_i'(\omega)$ if

$(i, \omega) \in S \times \mathcal{E}$ and $a_i''(\omega) := e_i(\omega)$ otherwise ($\epsilon > 0$ is very small). ♣

Let a be an allocation rule. A coalition S has a *fine objection* against a if there exist an event $\mathcal{E}$ and an allocation rule a' feasible for S such that the two following properties are true at each $\omega \in \mathcal{E}$: 1) $\bigcap_{i \in S} P_i(\omega) \subseteq \mathcal{E}$; 2) $E(u_i(a_i')|\mathcal{E} \cap P_i(\omega)) > E(u_i(a_i)|\mathcal{E} \cap P_i(\omega))$ for each $i \in S$. The *fine core* is the set of feasible allocation rules against which no coalition has a fine objection (see Wilson, 1978). The fine core is a subset of the coarse core. In fact, the fine core is the smallest conceivable core according to Wilson, as fine objections allow for any kind of information sharing. This is wrong once we try to understand how the agreements emerge. Example 11 in de Clippel and Minelli (2003) illustrates this point when the tentative agreements are proposed by the agents themselves. I adapt the example in order to show that some fine core allocations may be blocked.

Example 1 I consider a sunspot economy with asymmetric information. There are two agents, two goods and two equiprobable states for the economy. Agent 1 knows the future state while agent 2 does not: $\mathcal{P}_1 = \{\{\omega_1\}, \{\omega_2\}\}$ and $\mathcal{P}_2 = \{\{\omega_1, \omega_2\}\}$. The endowments are defined as follows: $e_1(\omega) = (0, 100)$ and $e_2(\omega) = (1, 100)$ for each $\omega \in \Omega$. The utility functions are defined as follows: $u_1(x, \omega) = 200x^1 + x^2 - 100$ and $u_2(x, \omega) = x^1 + x^2 - 101$ for each $x \in \mathbb{R}^2_+$ and each $\omega \in \{\omega_1, \omega_2\}$. It is mutually beneficial to exchange good 1. Good 2 is money. Let a be the feasible allocation rule defined as follows: $a_1(\omega_1) := (1, 198)$, $a_1(\omega_2) := (1, 0)$, $a_2(\omega_1) := (0, 2)$ and $a_2(\omega_2) := (0, 200)$. It belongs to the fine core but not to the core. Indeed, it favors too much agent 1 in ω_1 and hence is blocked by the allocation rule a' defined as follows: $a_1'(\omega_1) := (0, 100)$, $a_1'(\omega_2) := (1, 98)$, $a_2'(\omega_1) := (1, 100)$ and $a_2'(\omega_2) := (0, 102)$.

There is no general inclusion relation between the core and the fine core. Here is an example where the fine core is a subset of the core.

Example 2 I adapt example 2 of Wilson (1978). There are three agents, one good (money) and two equiprobable state for the economy. Agent 3 knows the future state while agents 1 and 2 do not: $\mathcal{P}_1 = \mathcal{P}_2 = \{\omega_1, \omega_2\}$ and $\mathcal{P}_3 = \{\{\omega_1\}, \{\omega_2\}\}$. The endowments are defined as follows: $e(\omega_1) = (100, 0, 0)$ and $e(\omega_2) = (0, 100, 0)$. The utility functions are defined as follows: $u_i(x, \omega) = \sqrt{x}$ for each $i \in \{1, 2, 3\}$, each $x \in \mathbb{R}_+$ and each $\omega \in \{\omega_1, \omega_2\}$. The full-insurance allocation rule giving 50 dollars to each of the two first agents in each state of the economy belongs to both the core and the coarse core but not to the fine core. If the agents can communicate, then agent 3 will meet agent 1 when the state is ω_1, convince him that the future state is favorable to him and agree with him to implement a different allocation, for instance $(75, 0, 25)$.

Theorem 3 *Suppose that each agent is endowed with a strictly positive amount of each good in each state of the economy. Then, the core is not empty.*

4 Convergence

An allocation rule a is a *Wilson equilibrium* if it is feasible and there exists a price system $p : \Omega \to \mathbb{R}^L_+$ such that $E(u_i(a_i)|P_i(\omega)) \geq E(u_i(a'_i)|P_i(\omega))$ for each $a'_i \in \mathbb{R}^{L\times\Omega}_+$ for which $\sum_{\omega' \in P_i(\omega)} p(\omega').a'_i(\omega') \leq \sum_{\omega' \in P_i(\omega)} p(\omega').e_i(\omega')$, for each $\omega \in \Omega$ and each $i \in N$ (see footnote 6 in Wilson, 1978). It is a natural generalization of the Arrow-Debreu equilibrium in markets with contingent commodities to economies with asymmetric information when inside trading is prohibited. Indeed, in a world with contingent commodities, the uninformed 'invisible hand' specifies a price for each commodity in each state of the economy in order to clear all the markets. The agents do not learn anything by observing the price vector as it does not depend on the future state of the economy. They maximize their expected utilities under the additional constraint that they may not sell contingent commodities associated to states that they know are not going to occur. I further analyze example 2 of Wilson (1978) in order to illustrate the concept.

Example 3 There are three agents, one good (money) and three equiprobable states for the economy. The following table specifies the information and the endowments of the agents.

$Agent(i)$	$\mathcal{P}_i$	$e_i(\omega_1)$	$e_i(\omega_2)$	$e_i(\omega_3)$
1	$\{\{\omega_1\}, \{\omega_2, \omega_3\}\}$	5	1	3
2	$\{\{\omega_2\}, \{\omega_1, \omega_3\}\}$	3	5	1
3	$\{\{\omega_3\}, \{\omega_1, \omega_2\}\}$	1	3	5

The following table specifies an allocation rule a that is a Wilson equilibrium for the price vector $(1, 1, 1)$. It also specifies the associated net-trades.

$Agent(i)$	$a_i(\omega_1)$	$a_i(\omega_1)$	$a_i(\omega_1)$	$z_i(\omega_1)$	$z_i(\omega_2)$	$z_i(\omega_3)$
1	5	2	2	0	1	-1
2	2	5	2	-1	0	1
3	2	2	5	1	-1	0

The equilibrium concept allows for some insurance between the two agents that are not fully informed. This would be impossible if the prices of the three contingent commodities were varying with the future state of the economy, much as in rational expectations equilibria. Markets have to clear ex-post with Wilson's concept. If for instance the future state is ω_1, there is excess supply (resp. demand) of money in state 2 (resp. 3), as agent 1 is prohibited to buy or sell these contingent commodities. This is irrelevant as these claims will not have to be satisfied. What matters on the other hand is the fact that the demand for money from agent 3 is met by the supply of money by agent 2 in state 1. Varying the states, there are three equilibrium equations to be satisfied in total, not nine. The market is not fully decentralized in that sense. I think instead of an uninformed trading organization proposing the prices in order to match demand with supply ex-post.

Theorem 4 *Suppose that each agent is endowed with a strictly positive amount of each good in each state of the economy. Then the set of Wilson equilibria is not empty.*

Theorem 5 *The set of Wilson equilibria is a subset of the core.*

I replicate the agents of the economy described in section 2 as in Serrano et al. (2001). Let $\bar{k}$ be the number of replicas. Each agent of the original economy now appears $\bar{k}$ times. Hence, there are $\bar{k}N$ agents in the replicated economy. Let $i \in N$ and let $k \in \{1, \dots, \bar{k}\}$. Copy k of agent i is denoted $i.k$. Agent $i.k$ in the replicated economy has the same endowment, the same information and the same utility function as agent i in the original economy.

Theorem 6 *Suppose that each agent's utility function is strictly concave in each state of the economy and that each agent is endowed with a strictly positive amount of each good in each state of the economy. Then, the core shrinks to the set of Wilson equilibria as the number of replicas $\bar{k}$ tends to infinity.*

Serrano et al. (2001) argue on a simple example that no similar result can be obtained for the coarse or the fine core. I now explain why their argument does not apply to the core.

Example 4 Consider a sunspot economy with two agents, two goods and two equiprobable states for the economy. Agent 1 knows the future state while agent 2 does not: $\mathcal{P}_1 = \{\{\omega_1\}, \{\omega_2\}\}$ and $\mathcal{P}_2 = \{\{\omega_1, \omega_2\}\}$. The endowments are defined as follows: $e_1(\omega) = (24, 0)$ and $e_2(\omega) = (0, 24)$ for each $\omega \in \Omega$. The utility functions are defined as follows: $u_1(x, \omega) = u_2(x, \omega) = \sqrt{x^1 x^2}$ for each $x \in \mathbb{R}^2_+$ and each $\omega \in \{\omega_1, \omega_2\}$. Let a be the feasible allocation rule defined as follows: $a_1(\omega_1) := (15, 15)$, $a_1(\omega_2) := (8, 8)$, $a_2(\omega_1) := (9, 9)$ and $a_2(\omega_2) := (16, 16)$. Serrano et al. show that the $\bar{k}$-replication of a belongs to the coarse core but is not a Wilson equilibrium of the $\bar{k}$-replicated economy, for each $\bar{k} \in \mathbb{N}$. It is already blocked in the second replica. The second replication of a is given by:

a	1.1	1.2	2.1	2.2
ω_1	$(15, 15)$	$(15, 15)$	$(9, 9)$	$(9, 9)$
ω_2	$(8, 8)$	$(8, 8)$	$(16, 16)$	$(16, 16)$

It is blocked by the following allocation rule:

a'	1.1	1.2	2.1	2.2
ω_1	$(31/2, 31/2)$	$(0, 0)$	$(17/2, 17/2)$	$(0, 0)$
ω_2	$(10, 13/2)$	$(10, 13/2)$	$(28, 11)$	$(0, 0)$

The coalitions are forming as follows: $\{1.1, 2.1\}$ if the future state is ω_1 and $\{1.1, 1.2, 2.1\}$ if the future state is ω_2.

5 Proofs

I construct a fictitious classical exchange economy. Agents are couples $(i, \mathcal{E})$ where $i \in N$ and $\mathcal{E} \in \mathcal{P}_i$. Let $\mathcal{N}$ be the set of fictitious agents. There are $L \times \Omega$ (contingent) goods. The consumption set of agent $(i, \mathcal{E})$ is $C(i, \mathcal{E}) = \mathbb{R}_+^{L \times \mathcal{E}}$. His endowment is $proj_{C(i,\mathcal{E})}(e_i)$. His utility for a bundle $x \in C(i, \mathcal{E})$ is $U_{(i,\mathcal{E})}(x) = \sum_{\omega \in \Omega} \pi(\omega|\mathcal{E}) u_i(x(\omega), \omega)$. An *allocation* $\mathfrak{a}$ is a list of bundles, one for each fictitious agent, i.e. $\mathfrak{a} \in \times_{(i,\mathcal{E}) \in \mathcal{N}} C(i, \mathcal{E})$. The concepts of core and competitive allocations are defined as usual, see for instance Debreu and Scarf (1963).

There is a natural way to map allocations of the fictitious economy with allocation rules of the original economy. Let $\mathfrak{a} \in \times_{(i,\mathcal{E}) \in \mathcal{N}} C(i, \mathcal{E})$. Then $a(\mathfrak{a})_i(\omega) = \mathfrak{a}_{(i, P_i(\omega))}(\omega)$, for each $(i, \omega) \in N \times \Omega$. Let $a \in \mathbb{R}_+^{L \times N \times \Omega}$. Then $\mathfrak{a}(a)_{(i,\mathcal{E})}(\omega) = a_i(\omega)$ if $\mathcal{E} = P_i(\omega)$ and $\mathfrak{a}(a)_{(i,\mathcal{E})}(\omega) = 0$ if $\mathcal{E} \neq P_i(\omega)$, for each $(i, \mathcal{E}) \in \mathcal{N}$ and each $\omega \in \Omega$. If an allocation $\mathfrak{a}$ belongs to the core of the fictitious economy, then the allocation rule $a(\mathfrak{a})$ belongs to the core of the original economy. Reciprocally, if an allocation rule a belongs to the core of the original economy, then the allocation $\mathfrak{a}(a)$ belongs to the core of the fictitious economy. Similarly, if an allocation $\mathfrak{a}$ is competitive in the fictitious economy, then the allocation rule $a(\mathfrak{a})$ is a Wilson equilibrium in the original economy. If an allocation rule a is a Wilson equilibrium in the original economy, then the allocation $\mathfrak{a}(a)$ is competitive in the fictitious economy. In addition, the fictitious economy associated to the replicated economy described in section 4 coincides with the replication a la Debreu and Scarf of the fictitious economy associated to the original economy. Hence theorems 3, 4, 5 and 6 are corollaries of the results stated in Debreu and Scarf (1963).

The trick that consists in constructing a fictitious economy is not new. Wilson himself considers fictitious agents in order to prove the non-emptiness of the coarse core. Kobayashi (1980) highlights the link between a variant of Wilson's equilibria and the competitive equilibria of the fictitious economy in a framework with only one good. Similar arguments also appear in the literature on sunspot economies with symmetric information and restricted market participation (see Balasko et al., 1995). In that framework, Goenka and Shell (1997, theorem 7.3) indicate that the core of the fictitious economy converges to the set of sunspot equilibria (see also the last paragraph of section 4.1 in Serrano et al., 2001). As already mentioned, this directly follows from Debreu and Scarf (1963). What is more remarkable is the fact that the cores of the original and of the fictitious economies coincide. I don't know whether there is anything more than a formal relation between sunspot economies with restricted market participation and economies under asymmetric information. In any case, the former would constitute only a small subclass of the latter.

6 Conclusion

Theories focusing only on the stability of given allocation rules are not fully satisfactory when the information is asymmetrically distributed among the agents. Indeed, one has to explain in addition how the agreements emerge. The idea of uninformed intermediaries helping the agents to coordinate in an attempt to make some profit is simple, intuitively appealing and powerful. The agents do not have to trust the information revealed by other agents. There is no need to wonder what they learn from the offers as the intermediaries are uninformed. The agents choose whether to sign a contract independently of what the other agents do. The intermediaries do not need high analytical abilities either. They simply anticipate the choice of the agents as in any subgame perfect equilibrium. The intermediaries bear no risk as they break even in each state of the economy. Yet, the set of possible outcomes is relatively small as it is included in the coarse core and may even exclude some fine core allocations. Objections were bound to arise from coalitions in Wilson's theory. I endogenously determine the coalitions that are forming by comparing different allocation rules. The insights gained for the core help to better understand competitive behavior thanks to the convergence result.

Many open questions remain to be addressed. I focus on three of them. First the theory should be extended to situations where the information is not verifiable at the time of implementing the agreements. Myerson (1995, 2003) considers entrepreneurs devising alternative market organizations and mediators helping the agents to coordinate. It would be nice if his core concepts could be characterized as the set of equilibrium outcomes associated to some explicit competitive screening game. One should then try to obtain convergence results. Second communication and information transmission could be discussed by adapting the framework. The intermediaries may be partially informed and learning additional information by observing the choices of the agents. The agents themselves could learn some information before choosing a contract if they observe the choice of other agents. Third one may study how the results are affected by allowing the intermediaries to withdraw their offers. It is true that they break even on the equilibrium path. They also break even when proposing an objection. Nevertheless, the active intermediaries facing an objection could possibly bankrupt and be willing to withdraw their offer. This in turn would change the profitability of the deviation. Similar considerations were studied for the model of Rothschild and Stiglitz (1976) (see for instance Wilson (1977)).

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