# A Model of Gossip*

Wei Li†

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**Abstract**

This paper analyzes how the gossip process can be manipulated by biased people and the impact of such manipulation on information transmission. In this model, a single piece of information is transmitted via a chain of agents with privately known types. Each agent may be either objective or biased, with the latter type aiming to manipulate the information transmitted toward a given direction. In an indirect impact gossip model where all agents aim to influence a final decisionmaker, the biased type's equilibrium incentive to make up wrong information is independent of their position in the gossip chain. Moreover, adding just a few biased people to the population sharply decreases the amount of information transmitted. In a direct impact gossip model where every biased agent is concerned about influencing his immediate listener, gossip causes initial contamination of data, but eventually dies out as the objective people stop listening.

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†University of California, Riverside, wei.li@ucr.edu.

# 1 Introduction

A presence in virtually any social network, gossip is verbally communicated soft information that is difficult to verify during certain periods of time. Though informal, gossip and rumors can play a prominent role in financial markets, military intelligence and politics. In the finance literature, the effect of takeover rumors on stock prices is well documented (Pound and Zeckhauser 1990, Rose 1951). A political example is the sensational impeachment trial in 1917 of then Texas governor James Ferguson, which originated from some disquieting rumors; "particularly alarming was the gossip that the liquor interests had contributed substantially to his campaign fund"(Ewing 1933). The impeachment trial ended with Ferguson's suspension and eventual resignation, despite his repeated denials of the gossip and claims of innocence. These examples, among many others, demonstrate that gossip's impact may be real and powerful, even though few people really know who started a particular piece of gossip and how many people have learned it.

How can gossip be manipulated for biased purposes? What is the impact of such manipulation on information transmission? How does a person's position in a gossip chain affect his willingness to manipulate information? This paper addresses these questions. It considers a population with two different types of people, the objective ones who only pass on what they think is true information and the biased ones who derive utility from spreading biased gossip. A biased person is assumed to have reputational concerns: he does not want to be known as biased.

This paper models the gossip process in two different ways, depending on whose opinion the sender(s) of gossip tries to influence. First, when only a final decisionmaker's opinion matters in a gossip chain, an *indirect impact* gossip model is used to analyze the distortion of information and how one's position in the gossip chain affects one's incentive to lie. Second, when everyone is concerned about his *immediate* successor's opinion instead, a *direct impact* gossip model is used to analyze the persistence of a gossip chain, as the information travels farther and farther away from its source.

In the indirect impact gossip model, there are three agents $A, B, C$, located on a line. All information flows in one direction only: from $A$ to $B$ to $C$. Agent $A$ and only $A$ may receive a private signal about the true state of the world. The signal (and later the gossip) is binary: it can be either a positive signal or a negative one. Once $A$ receives the signal, he decides whether to report his signal truthfully to agent $B$. Based on what he hears from $A$, agent $B$ decides what to report to $C$ (he can report gossip that he does not hear). Agent $C$ forms an opinion about the state of the world based on what he hears from agent $B$, and forms a belief about agent $A$ and $B$'s type as well. The objective agent tries to pass on his best estimate of the true state. The biased agent tries to bias agent $C$'s opinion toward one state of the world, and also wants to pass for an objective agent in $C$'s eyes.

Three main results emerge from the analysis of the indirect impact gossip model. First, when there are few biased types in the population or when $A$'s signal is much better than the priors, an informative chain forms. The objective types pass on the gossip they hear because the gossip is more trustworthy. Therefore gossip travels through the chain of agents despite the possibility that the biased type may lie. In the informative chain, having any biased agent in the chain may cause a biased gossip to be delivered to $C$, regardless of the true signal.

Second, when there are a lot of biased type in the population or when the signal quality is not that much superior to the prior, the objective types become less trustful and tend to remain silent when they hear gossip in favor of the state preferred by the biased agents. Due to reputational concerns and having no information of his own, biased $B$ would prefer to pool with the objective $B$ in remaining silent as well. This impedes the transmission of a negative signal, and results in a completely uninformative equilibrium. The error in information transmission in the uninformative equilibrium, however, is significantly higher than that in the informative equilibrium.

Third, in an informative chain, the location of a biased agent does not affect his incentive to lie. Distance from the decisionmaker is important in $C$'s inference of agents $A, B$'s objectivity. Ex ante, agent $A$ has the information advantage of receiving the true signal while agent $B$ has a positional advantage because $C$ infers with some probability that he is simply a *messenger* of wrong gossip. biased $A, B$'s incentive to lie depends on how much blame $C$ assigns to them when a gossip turns out to be wrong and the damage a wrong gossip can impose. Surprisingly, despite the highly asymmetric positions of $A$ and $B$, in equilibrium they distort the truth with the same probability. In equilibrium, $A$'s information advantage is exactly offset by a countervailing positional *disadvantage*: even if he is honest, $B$ may still send a biased gossip to $C$, who will assign part of the blame to $A$.

In the direct impact gossip model, there are $N < \infty$ agents located on the line. Agent $A$ and only $A$ may receive a signal, and each agent may be biased or objective, just as in the indirect impact gossip model. In the direct impact gossip model, however, the biased agent $i$ cares about biasing his immediate listener $i + 1$'s view about the true state of the world, as well as being considered objective in $i + 1$'s eyes.

The main result of the direct impact gossip model is that the gossip will influence the early agents. However, a negative gossip's impact on the listener's belief gradually decreases when the sender of the gossip is located far from the source, because his information is more and more likely to be hearsay than to be the original signal. When the gossip becomes sufficiently diluted, objective agents stop listening. From this point on, all biased agents who report negative gossip will be identified as such, and therefore lose their reputation. This effect is sufficiently strong that both the objective *and* the biased agents remain silent from the same point on. Therefore communication breaks down when the chain is sufficiently long.

In both the indirect impact gossip and the direct impact model, three factors are crucial in determining how likely and how far a biased person would bias the gossip toward the direction he prefers. First, how likely he may know the true state. Second, what the objective agent in his position would do and last, the *relative* blame his listener assigns to him versus his predecessor(s). A powerful general lesson from these models is that the manipulative power of gossip thrives in a kind of limbo: there have to be *some* biased types in the population to introduce biased information, but not many, otherwise the objective agents become less trusting and the gossip channel breaks down. Moreover, there have to be *some* uninformed/less informed agents as listeners of the gossip, but not too many, otherwise the gossip dies before it influences many of them.

The current paper is close to Banerjee (1993) and Morris (2001). Banerjee (1993) first analyzed rumors as potentially useful information about some agents' action, which may reflect how profitable an investment is. However, each agent has privately known cost, and thus an agent who observes the rumor has to take an action depending on his inference about the previous agents' costs and his own cost. Banerjee shows that rumors cannot mislead everybody in the sense that a positive fraction of people who observe the rumors do not invest. His model is essentially an individual decision making problem: each agent only cares about their own return from the investment given his information. In contrast, the present paper studies a strategic game in which the biased agents attempt to bias the decisionmaker's opinion toward the state they prefer.[1]

Morris (2001) is similar to the present paper in that the informed party may be of two privately known types, one of which may want to bias the uninformed decisionmaker's action toward the one he prefers. In his model, if the unbiased informed party is sufficiently concerned about being perceived as biased, then he may not convey his true opinion at all. This paper differs from Morris (2001) in two aspects. First, the impact of the biased type (the biased party) derives not only from

[1] Informal communication in networks such as word-of-mouth communication has been analyzed by Ellison and Fudenberg (1995).

his signal, but also from source uncertainty: agent $B$ can bias the gossip without observing $A$'s signal. Second, this is not a cheap talk game. In this paper, both the cost and benefit of biased gossip are endogenous: it depends on other agent's beliefs, which in turn depend on the two types of agents' strategy.

The next section presents the indirect impact gossip model and discusses some important assumptions. Section 3 first examines informative equilibria for the parameter values such that the objective type always passes along the gossip they hear. Then I characterize equilibria when the objective type trusts the gossip less and thus may not pass on what they hear to their listener. Section 4 sets up and characterizes the equilibrium properties of the direct impact gossip model. Section 5 discusses some extensions. Section 6 concludes. Most of the proofs are collected in the Appendix.

## 2 The Indirect Impact Gossip Model: Setup

A decision based on a state variable needs to be made. The decisionmaker may hear some gossip about this variable before she makes the decision, and then the true state of the world is revealed. The state variable in question can be a takeover plan, a legislative agenda, or it can be about the integrity of a political candidate who is running for election-whether he behaved properly in some business dealings-is being speculated via the gossip process.

This paper will frequently refer to the following example. A candidate is being considered for a promotion, and his personality-whether he is collegial, ethical and will contribute to the organization in the long run-is being speculated via the gossip process. The content of the gossip affects the promotion decision, which is irrevocable by the time his true personality is learned. A candidate's personality is a somewhat vague yet important feature of him. Such information is more difficult to obtain through the formal process than one's talent, and many people may not want to become an identifiable source of critical or negative information about a person they know, which frequently results from the formal processes.

### 2.1 Agents

There are two agents $i = A, B$ who may gossip about the candidate personality. The candidate's true personality is $\eta \in \{0, -1\}$: $\eta = 0$ with probability $\pi$ and $\eta = -1$ with probability $1 - \pi$.

That is, the candidate can be either nice (0) or nasty (−1).[2] The prior is $\pi \geq \frac{1}{2}$. Agent $C$ is the decisionmaker who hears about the gossip prior to voting.

These three agents are located on a line. Agent $A, B$ can be of two types: objective ($o$) or biased ($b$). Agent $i$ is objective with probability $\theta$ and biased with probability $1-\theta$. Types are independently distributed.[3] Agent $A$ and only $A$ may receive a signal $s_A \in \{0, -1\}$. An objective agent tries to send down a gossip $g_i$ which is his best estimate of the candidate's personality, given his information $I_i$. His objective function is thus:

$$\text{Max}_{g_i \in \{0,-1\}} \quad Pr(g_i = \eta | I_i).$$

On the other hand, biased agent $A$ and $B$'s objective functions are respectively:

$$\text{Max}_{g_A} \quad EU_A = E_{g_B}[Pr(\eta = -1|g_B)|g_A] + E_{g_B,\eta}[Pr(\theta_A = o|g_B, \eta)|s_A, g_A]$$
$$\text{Max}_{g_B} \quad EU_B = Pr(\eta = -1|g_B) + E_\eta[Pr(\theta_B = o|g_B, \eta)|g_A]$$

A biased agent's objective function consists of two parts: the first half, $[Pr(\eta = -1|g_B)]$, is the posterior probability that the candidate is considered nasty in the eyes of the decisionmaker, $C$. The worse is $C$'s impression of the candidate, the less likely he will be promoted, which in turn increases a biased agent's expected utility. The next half of the objective function shows that type $b$ wants to be considered an objective type by $C$. The expression is the decisionmaker $C$'s posterior estimate of the agent's type at time $t = 3$ (see information structure in section 2.2.). This part is a reduced form for the biased type's concern for his future reputation, i.e., a person cannot exert any influence on other people's reputation if one is perceived to be biased.[4] Note that $A$ and $B$ may care about $C$'s impression for different reasons, it is only important that reputation plays a role in their gossip choice.

The objectives of type $o$ and type $b$ agent are chosen so that the biased type is the "more strategic" one and their behavior is the focus of this paper. To see this, notice that the objective type agent is no different from a statistical machine: he accounts for the possible lying of the biased type and passes on his best estimate of the state.

[2] The candidate is not a player throughout this paper.

[3] Throughout this paper, $\theta$ is assumed to be larger or equal to $\frac{1}{2}$, thus there are (weakly) more objective agents in the population.

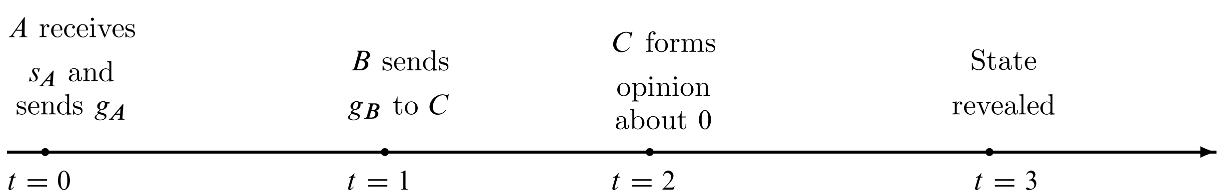
[4] Morris (2001) shows that instrumental reputation concerns can arise because the informed parties prefer to be trusted by a decisionmaker so that they can have an impact on future decisions.

## 2.2 Information Structure and Timing

At $t = 0$, agent A receives a private signal $s_A \in \{0, -1\}$ such that $Pr(s_A = \eta) = p > \pi \geq \frac{1}{2}$. At time $t = 3$, the true personality of the candidate is revealed to all agents. The indirect impact gossip game proceeds as follows:

- At $t = 0$, agent $A$ receives a signal $s_A \in \{0, -1\}$ and passes on gossip $g_A \in \{0, -1\}$ to agent $B$.
- At $t = 1$, agent $B$ passes on gossip $g_B \in \{0, -1\}$ to agent $C$, given gossip $g_A$.
- At $t = 2$, agent $C$ forms his opinion of the candidate's personality given $g_B$.
- At $t = 3$, the candidate's true personality is revealed to all, and agent $C$ forms his opinions about the objectivity of $A$ and $B$.

The following is the time line of this game. Notice that agent $C$ forms opinion about how likely the candidate is nice *prior* to the realization of state, but he forms his opinions about agent $A$ and $B$'s objectivity *after* the candidate's true personality is revealed.




## 2.3 Equilibrium

Agent $A$'s strategy is $g_A : S_A \times \Theta_A \to \Delta(\{0, -1\})$. Agent $B$'s strategy is $g_B : G_A \times \Theta_B \to \Delta(\{0, -1\})$. Agent $C$, who is not a strategic player, simply forms his opinion of how likely the candidate is nasty: $Pr(\eta = -1|g_B)$. After he observes the revealed true state, he forms his opinion on $A$, $B$'s objectivity $Pr(\theta_A = o|g_B, \eta)$ and $Pr(\theta_B = o|g_B, \eta)$ respectively.

The equilibrium concept used in this paper is Perfect Bayesian Equilibrium (PBE): each agent chooses a gossip to maximize his expected utility, given his belief about the other agent's type as well as the candidate's personality and $C$'s inferences. Every agent's belief is updated using Bayes' rule whenever possible.

Although gossip in this model is a kind of verbal message, this is not a cheap talk game: there is endogenous cost of fabricating/passing on biased gossip, even though the source of gossip is unknown to decisionmaker $C$. When biased $A$ and/or $B$ fabricate gossip, they deviate from their best estimate of the state and thus are more likely to be wrong. In expectation, they are less likely to be considered objective in the eyes of $C$.

The following simple observation helps illustrate some properties of the current model:

**Observation 1** *(1) There does not exist a babbling equilibrium in which the gossip is uncorrelated with types and signal, and the decisionmaker $C$ ignores the gossip and learns nothing;*

*(2) There does not exist a completely uninformative equilibrium in which the biased agents always sends the same gossip.*

There is no babbling equilibrium because the objective agents always send their best estimate, thus a rational $C$ should infer how likely a gossip is sent by an objective agent and use the information accordingly. Moreover, there does not exist a completely uninformative equilibrium such as Morris (2001), in which a particular message (in the current context, $g_i = -1$) is taken as a sure sign of bias and not trusted. The reason is that, suppose that there was such an equilibrium, then the biased agent will avoid any negative gossip. Therefore in equilibrium, this very gossip becomes a sign of objectivity, and will be trusted by the decisionmaker. Therefore the biased type can deviate profitably because by sending this gossip, he is considered objective with probability one *and* influence his listener(s), which is a contradiction.

# 3 The Indirect Impact Gossip Model

## 3.1 Preliminaries: Objective Agents' Behavior

Although this paper focuses on the strategic behavior of the biased agent, the biased type's behavior depends crucially on the responses of the objective type he is partially trying to emulate. The objective type's inference is influenced by how likely their predecessors are biased, the quality of $A$'s signal and how likely they believe the biased types will lie. Naturally if there is no biased agent, the gossip is accurate and they will be believed since they are the only source of information other than the priors. If there are a lot of biased people and many of them are expected to lie, however, the gossip's usefulness will be discounted heavily. Therefore this section first characterizes the objective type agents' behaviors.

Let $x$ and $y$ be the probabilities for biased $A$ and $B$ to remain silent when they hear $s_A = 0$ and $g_A = 0$ respectively.[5] Similarly, define $\alpha$ and $\beta$ as the probability that biased $A$ and $B$ state

[5] The reason that I analyze this belief now is that it will be part of the equilibrium belief in many of the cases analyzed later.

$g_A = -1$ and $g_B = -1$ upon hearing $s_A = -1$ and $g_A = -1$ respectively.[6]

**Lemma 1** *Suppose that the objective $B$ believes that: biased agents $A$ and $B$ gossip $g_A = 0, g_B = 0$ with probability $x$ and $y$ respectively when they hear $s_A = 0, g_A = 0$ and gossip $g_A = -1, g_B = -1$ when they hear $s_A = -1, g_A = -1$, then:*

*(1) objective agent $A$ always reports $g_A(o, s_A) = s_A$;*

*(2) if $\pi - p + (1-\theta)(p+\pi-1) < 0$, then objective $B$ always passes along the gossip he hears;*

*(3) if $\pi - p + (1-\theta)(p+\pi-1) \geq 0$, then there exists a $\hat{x} \equiv 1 - \frac{p-\pi}{(1-\theta)(p+\pi-1)}$ such that if $x \leq \hat{x}$, then an objective $B$ remain silent ($g_B = 0$) regardless of $g_A$. If $x > \hat{x}$, objective $B$ gossips $g_B = g_A$.*

In Lemma 1, inequality $\pi - p + (1-\theta)(p+\pi-1) < (\geq) 0$ is crucial in understanding objective $B$'s behavior. It characterizes the set of parameter values such that objective $B$ always passes on the gossip he hears. It holds if $Pr(\eta = -1 | g_A = -1) > Pr(\eta = 0 | g_A = -1)$ for all $x$. Depending on whether this inequality is true, the parameters can be divided into two cases (Case I and Case II). Case I includes all parameters such that $\pi - p + (1-\theta)(p+\pi-1) < 0$, or when the objective $B$ always passes along what he hears. This occurs when there are few biased types, or when quality of agent $A$'s private signal is much higher than everyone's prior such that even discounting for a possible biased $A$'s lying, it is still better than the prior. Case II includes parameters when $\pi - p + (1-\theta)(p+\pi-1) \geq 0$, or when the objective $B$ does not trust $g_A$ enough to pass on the gossip independently of biased $A$'s lying probability. When the information value of the gossip is not so high, then the objective $B$'s behavior depends on $A$'s mixing probability $x$. If biased $A$ lies less than a cutoff value $\hat{x}$, then the objective $B$ passes a negative gossip down to $C$. Otherwise he simply remains silent.

Figure 1 aids in understanding Case I and Case II from the primitives $p, \pi, \theta$. Notice that the when $1 > p > max\{\pi, \frac{2-\theta}{\theta}\pi - \frac{1-\theta}{\theta}\}$, the agents' behavior fall into Case I, and the region where $\pi \leq p \leq \frac{2-\theta}{\theta}\pi - \frac{1-\theta}{\theta}$, it falls into Case II. It is easy to see that the region of case I increases when $\theta$ is closer to 1: even at $\theta = 0.5$, as long as $p - \pi$ is reasonably large, the objective $B$ will pass on a negative gossip until Case II disappears at $\theta = 1$.

Given the behaviors of objective $A$ and $B$, agent $C$ needs to form an opinion of the candidate's personality $Pr(\eta = -1 | g_B)$:

[6] Therefore $x, y, \alpha, \beta$ are all probabilities that biased agents *follow* what they hear.

**Observation 2** *Given the proposed beliefs, agent $C$ believes that $Pr(\eta = -1|g_B = -1) > Pr(\eta = -1|g_B = 0)$ regardless of how much biased $A, B$ may be lying when they hear $s_A = 0$ and $g_A = 0$ respectively.*

Intuitively, there are at least as many objective people in the population as the biased ones ($\theta \geq \frac{1}{2}$). Even if biased agent $A$ and $B$ fabricate negative gossip with probability one, when $C$ hears $g_B = -1$, he correctly infers that it comes from an objective $A$ with positive probability. On the other hand, $C$ knows that $s_A = 0$ for sure if he hears $g_B = 0$. Therefore he thinks worse of the candidate after hearing negative gossip.

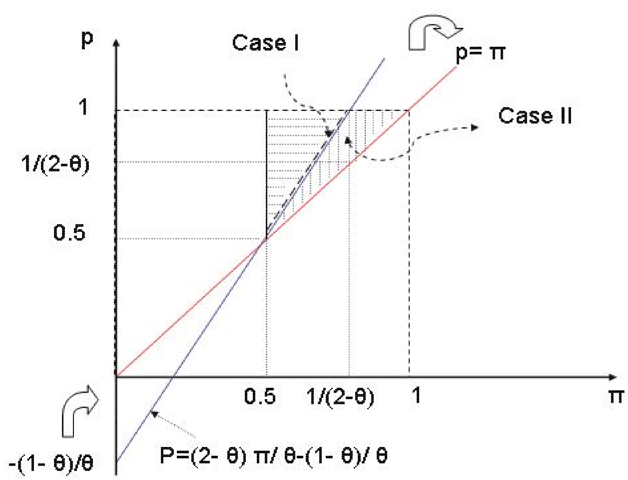



Figure 1
Case I and II: Using Primitives

### 3.2 When Objective $B$ Passes on $g_A$: Strong Negative Impact Equilibrium

This subsection analyzes equilibrium for parameters that fall into Case I, which includes three important subcases. First, it holds when $\pi = 0.5$, i.e., the prior is completely uninformative about the candidate. In this case gossip is the only available information to evaluate the candidate. Second, when $\theta$ is close enough to 1, i.e., there are few biased agents in the population. Third, it holds when $p - \pi$ is large for any given $\theta$, i.e., the quality of signal $s_A$ is much higher than people's priors.[7]

[7] The exact condition is when $p - \pi > \frac{(1-\theta)(2\pi-1)}{\theta}$.

Given the objective agent $A, B$'s behavior and how agent $C$ forms his opinion, what are the biased agents $A, B$'s strategic considerations? Recall that biased $B$ is concerned about both discrediting the candidate and posing as an objective agent in $C$'s eyes. Let $\hat{\eta}_{-1} \equiv Pr(\eta = -1 | g_B = -1), \hat{\eta}_1 \equiv Pr(\eta = -1 | g_B = 0)$. Then for any $g_A$, the difference in $B$'s expected utility for stating $g_B = -1$ and $g_B = 0$ composes of two parts:

$$
\begin{aligned}
& EU_B(g_B = -1, m, g_A) - EU_B(g_B = 0, m, g_A) \\
= \quad & \underbrace{\hat{\eta}_{-1} - \hat{\eta}_1}_{\text{damage to the candidate}} + \underbrace{E[Pr(\theta_B = o | g_B = -1, \eta) | g_A] - E[Pr(\theta_B = o | g_B = 0, \eta) | g_A]}_{\text{own reputation loss}}
\end{aligned}
$$

The first part is the net damage a negative gossip can exert on $C$'s perception of the candidate while the second part is the net loss of $B$'s perceived objectivity. As discussed in Observation 2, the first part is always positive. Therefore if the net reputation loss is small in terms of the lowered opinion in the eyes of $C$, biased $B$ will always lie.

Given $s_A$, the only source of true information, biased $A$'s consideration is shaped by a similar tradeoff to that of biased $B$'s if he states $g_A = -1$ as opposed to $g_A = 0$. A key difference here is that both $A$'s damage to the candidate and his own reputation loss will be filtered through $B$'s gossip. The following proposition characterizes equilibrium behaviors of the agents in this case:

**Proposition 1 (Strong Negative Impact Equilibrium)** *When $\pi - p + (1-\theta)(\pi + p - 1) < 0$, there exists an equilibrium such that,*

*(1.1) The objective type A passes on the true signal and the objective $B$ passes on the gossip he hears;*

*(1.2) Both biased agents A and $B$ will pass on negative gossip when they receive negative signal/gossip. i.e., $\alpha = \beta = 1$;*

*(1.3) When $\theta \in [\overline{\theta}, 1)$, biased agent A and $B$ lie completely when $s_A = 0$ and $g_A = 0$ respectively, i.e., $x = y = 0$;*

*(1.4) When $\theta < \overline{\theta}$, biased A and B lie with the same probability, i.e., $g_A(b, 0) = 0$ with probability $x \in (0, 1)$ and $g_B(b, 0) = 0$ with probability $y \in (0, 1), x = y$. Moreover, there does not exist an equilibrium in which $x \neq y$.*

*Remark 1: Honesty is never the best policy.* First, when there are few biased types, or when $\theta \approx 1$, it is very likely that both $A, B$ are objective and a wrong gossip $g_B$ is likely to be attributed to a wrong signal by nature. Therefore biased agent $A, B$'s own reputation loss of lying is almost zero and they strictly benefit in terms of damage to the candidate, hence they will report negative gossip with probability one regardless of what they hear. Second, even when $\theta$ is not very high, complete honesty cannot be part of biased agents' equilibrium strategy. Suppose the decisionmaker $C$ believes that biased $A, B$ pass on $s_A, g_A$ honestly. Then biased agents have *no* reputation loss since they behave the same way as the objective type, but the damage to the candidate, $\hat{\eta}_{-1} - \hat{\eta}_1 > 0$, is strictly positive. Therefore biased $A$ and $B$ would like to deviate and make up negative gossip with some probability.

*Remark 2: free-riding among $A$ and $B$.* Biased $A$ and $B$'s incentive to lie increase together because they can free ride on the other's reputation cost. To see that $x$ increases in $y$, note that $Pr(g_A = -1|g_B = -1, \eta)$ is crucial in $C$'s inference: it assigns responsibility to $A$ and $B$ whenever $C$ hears negative gossip. This probability increases in $y$ for both states $\eta = \{0, -1\}$. In other words, if the biased $B$ lies more, the perceived probability that $A$ has initiated the negative gossip falls. Thus in equilibrium, $A$'s reputation loss in saying $-1$ falls when $B$ lies more. Similarly, in $C$'s inference, $Pr(g_A = -1|g_B = -1, \eta)$ decreases in $x$. Thus when $A$ lies more, i.e., $x$ falls, the probability that $A$ initiated the negative gossip rises, which in turn decreases the equilibrium reputation cost of biased $B$.

*Remark 3: The filtering effect.* Recall that biased $A$'s impact on the candidate and on his reputation are filtered through agent $B$. To study the filtering effect in more detail, let $\kappa \equiv [Pr(g_B = -1|g_A = -1) - Pr(g_B = -1)|g_A = 0)]$, and $\gamma \equiv Pr(\eta = 0|s_A = 0)$. Suppose that biased $A$ receives $s_A = 0$, the difference in his expected utility of reporting $g_A = -1$ and reporting $g_A = 0$ is as follows:

$$
\begin{aligned}
& EU_A(g_A = -1, b, s_A = 0) - EU_A(g_A = 0, b, s_A = 0) \\
= \; & \overbrace{[Pr(g_B = -1|g_A = -1) - Pr(g_B = -1|g_A = 0)]}^{\text{Common filtering factor}}[Pr(\eta = -1|g_B = -1) - Pr(\eta = -1|g_B = 0)] \\
+ \; & \underbrace{\Big[E_{\eta, g_B}[Pr(\theta_A = o|g_B, \eta)|s_A = 0, g_A = -1] - E_{\eta, g_B}[Pr(\theta_A = o|g_B, \eta)|s_A = 0, g_A = 0]\Big]}_{\text{net reputation loss}} \\
= \; & \kappa \qquad \overbrace{(\hat{\eta}_{-1} - \hat{\eta}_1)}^{\text{net damage to candidate}} \\
- \; & \kappa \gamma Pr(g_A = -1|g_B = -1, \eta = 0)[Pr(\theta_A = o|g_B = -1, \eta = 0) - Pr(\theta_A = o|g_B = 0, \eta = 0)] \\
- \; & \kappa (1 - \gamma) Pr(g_A = -1|g_B = -1, \eta = -1)[Pr(\theta_A = o|g_B = -1, \eta = -1) - Pr(\theta_A = o|g_B = 0, \eta = -1)]
\end{aligned}
$$

Note that both biased $A$'s damage on the candidate and his expected reputation inferred from $g_B$ by agent $C$ are filtered by a *common factor* $\kappa$. This factor measures the marginal damage a negative gossip from $A$ would exert on the candidate's reputation versus the damage a biased $B$

would exert by lying even if $A$ remains silent. Biased $A$'s expected reputation loss is filtered the same way: the probability $A$ is considered objective by $C$ when $C$ thinks $A$ has fabricated the gossip versus the probability $A$ is considered objective if $B$ passes on the null gossip.

*Remark 4: position-independent incentive to lie.* Since both $A$'s damage on the candidate and his expected reputation are filtered through the same factor, biased $A$'s incentive to lie vis-a-vis $B$'s can be analyzed with the filtering factor taken out. Thus $A$'s impact on the candidate's reputation is the same as $B$'s: $\hat{\eta}_1 - \hat{\eta}_{-1}$. Any difference in $A, B$'s mixing probabilities $x$ and $y$ is therefore driven by $A$ and $B$'s relative expected reputation losses from $C$ hearing a negative gossip rather than silence.

At first sight, it would seem that this effect could go either way. Biased $A$ has opposing incentives due to his position. On one hand, $C$ does not observe $g_A$, thus he always gives $A$ some benefit of the doubt even if $g_B$ turns out to be wrong. On the other hand, whenever $g_B \neq \eta$, $A$ is always suspected as the source where the lie originated. Similarly, biased $B$ has to weigh how likely $C$ thinks that he is mislead by $A$ versus $B$ has distorted the truth.

To understand the relationship between biased $A$ and $B$'s incentive to lie and their positions in the chain, consider the case when $p = 1$. In this case, $A$'s signal is so accurate that whenever $C$ hears a wrong gossip $g_B$, he knows that either $A$ or $B$ must have lied.

Suppose that $s_A = 0$, then there are two paths leading to $g_B = -1$, with their respective probabilities attached in the above figure. Observe from Figure 2 that with probability $(1-\theta)(1-x)$ agent $C$ infers that path 2 has occurred and $A$ has lied. In this case, agent $A$ and $B$ are considered objective by $C$ with probability 0 and $\theta$ respectively. On the other hand, with probability $(\theta + (1-\theta)x)(1-\theta)(1-y)$, agent $C$ believes that path 1 has occurred and $B$ has distorted $A$'s truthful gossip. In this case, the posterior probability that $A$ and $B$ are considered objective is $\frac{\theta}{\theta+(1-\theta)x}$ and 0 respectively. It is easy to see that in expectation, agent $A$ and $B$'s reputation are respectively $\theta(1-\theta)(1-x)$ and $\theta(1-\theta)(1-y)$, which are equal at $x = y$. Thus, there is an equilibrium in which the incentive to lie is the same regardless of the biased agent's position.

Why cannot there be an asymmetric equilibrium in which biased $A$ and $B$ mix with different probability? Recall that after the filtering effect, every biased agent derives the same relative benefit of $\hat{\eta}_{-1} - \hat{\eta}_1$ from reporting $-1$ when he hears 0. Suppose that there is an asymmetric equilibrium in which $x > y$. Then $C$ is more likely to attribute a wrong piece of gossip to $B$'s fabrication than to $A$'s. On the other hand, $B$ pays a higher reputation cost of *not* remaining silent than $A$. In other words, in expectation, $B$ gains less than $A$ by fabricating $g_B = -1$ but loses more by not reporting $g_B = 0$, thus they cannot have the same expected reputation when $x > y$.

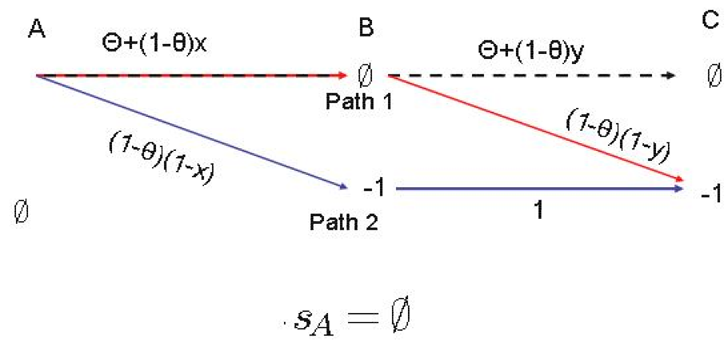



Figure 2: Filtering Effect

*Remark 5: Impact of the biased type.* The reason that this equilibrium is called strong negative impact equilibrium is that a small fraction of biased agents in either position $A$ or $B$ may lead to a biased gossip to $C$. Given that there is only one signal $s_A$ in this model, the signal is the relevant state of the world because all gossip is contingent on this signal and the agents' private types. It is simple to see that in Case I, when the signal is $s_A = 0, \theta \approx 1$, the decisionmaker $C$ may hear $g_B = -1$ with probability $\theta^2$. That is, having *any* biased agent in the chain causes a negative message to be delivered. In other words, the null signal tends to be distorted but the negative signal is perfectly communicated.

### 3.3 Case II: When Objective $B$ has More Doubts about the Gossip

One crucial part of the analysis of Case I is that the objective agent $B$ always follows the gossip, either because the fraction of biased agents in the population is small or because $A$'s signal is much more informative than everyone's prior. On the other hand, if there are more biased people in the population, and/or if $A$'s signal and therefore the gossip based on his signal become less informative, the objective agents may want to remain silent. Their doubt makes it harder for negative gossip to go around and for a gossip chain to form. The present section characterizes equilibrium behaviors in Case II, when the gossip is so diluted that objective agents become more reticent.

### 3.3.1 Different Regions of Objective Type's Response

Case II applies when $\pi - p + (1-\theta)(p+\pi-1) \geq 0$. This inequality holds in several interesting subcases: First, suppose that the fraction of the biased type $\theta$ is fixed, then case II applies when $p - \pi \in [0, \frac{1-\theta}{\theta}(2\pi - 1)]$. Note that this is true when $p, \pi$ are close, or when agent $A$'s signal is more, but not too much more accurate than the priors.

The starting point in analyzing Case II is the objective $B$'s response after hearing gossip. Recall from proposition 1.3 that objective $B$'s best response falls into two regions: first, when the objective agent $B$'s belief is that $x \leq \hat{x} \equiv 1 - \frac{p-\pi}{(1-\theta)(p+\pi-1)}$, his best response is to remain silent even if he hears a gossip against the candidate because biased $A$ fabricates negative gossip with such a high probability that the gossip cannot be trusted. Second, when $B$'s belief is such that $x > \hat{x}$, objective $B$ will pass on the gossip he hears, i.e., $g_B(o, g_A) = g_A$. The two different types of beliefs held by the objective $B$ may lead to different equilibrium behaviors. The first type of belief, i.e., $x \leq \hat{x}$ is labeled region 1, and the second type of belief falls into region 2.

The decisionmaker $C$'s updating is characterized in the following lemma:

**Lemma 2 ($C$'s Opinion in Case II)** *In Case II where* $\pi - p + (1-\theta)(\pi + p - 1) \geq 0$,

*(1) In region 1 where* $x \leq \hat{x}$, $Pr(\eta = -1 | g_B = -1) < \frac{1}{2}$;

*(2)* $Pr(\eta = -1 | g_B = -1)$ *increases in biased* $A$ *and* $B$*'s mixing probability* $x$ *and* $y$.

### 3.3.2 Uninformative Equilibrium

In Case II, the objective $B$'s belief about how much biased $A$ is lying is crucial to the agents' behavior. Define $x^*, y^*$ as the respective probability such that biased $A$ and $B$ are indifferent between fabricating negative gossip and remaining silent if they hear $s_A = 0$ and $g_A = 0$.[8] In region 1, when objective $B$ believes that biased $A$ lies with probability $x \leq \hat{x}$,

**Proposition 2 (Uninformative Equilibrium)** *If* $\pi - p + (1-\theta)(p+\pi-1) > 0$, *then*

*(2.1) there always exists an equilibrium in which objective* $A$ *states* $g_A = s_A$ *and for the biased* $A$ *to state* $g_A(b, 0) = 0$ *with probability* $x \in [0, \hat{x}]$ *and* $g_A(m, -1) = -1$. *Both the objective agent* $B$ *and the biased agent* $B$ *state* $g_B = 0$ *for any* $g_A$;

*(2.2) it cannot be an equilibrium in which* $g_A(o, s_A) = s_A, g_A(b, 0) = 0$ *with probability* $x^* \leq \hat{x}$, $g_B(o, g_A) = g_A$, $g_B(b, 0) = 0$ *with probability* $y^* \in (0, 1)$.

[8] See equation 1,2 in the Appendix.

*Remark 1*: Objective $B$'s belief is key to understand the uninformative equilibrium in this case. Notice that in the first region when the objective $B$ always remains silent, $g_B = -1$ reveals that agent $B$ is biased. Because biased $B$ can be considered objective with probability $\theta$ by simply pooling with objective $B$ and always stating $g_B = 0$, the damage to the candidate has to outweigh the loss in reputation for him to fabricate or to pass on negative information. Lemma 2.1. shows that $\hat{\eta}_{-1} - \hat{\eta}_1 < 0.5 \leq \theta$, thus for biased $B$, $EU_B(g_B = -1, b, g_A = 0) - EU_B(g_B = 0, b, g_A = 0) = \hat{\eta}_{-1} - \hat{\eta}_1 - \theta < 0$. Therefore biased $B$ will always remain silent. The reason that objective $B$'s belief is supported in equilibrium is that since the biased $A$ can exert no influence on the candidate's reputation, he may as well states $g_A(b, 0) = 0$ with probability one.

*Remark 2: Inefficiency of the uninformative equilibrium.* Since both biased and objective $B$ remain silent in the uninformative equilibrium, no information reaches $C$. Therefore $C$ must rely on his priors to judge the candidate and the socially useful information that the candidate is nasty $s_A = -1$ is thus lost and $C$ cannot learn anything from the gossip channel.

### 3.3.3 Informative Equilibrium

If objective agent $B$ believes that biased $A$ reports $g_A = 0$ with probability $x > \hat{x}$, then he would pass on a negative gossip. Thus the biased $B$ would say $g_B(m, 0) = 0$ with probability $y^* < 1$. Can there be an informative equilibrium in which the objective $B$ believes biased $A$ is relatively honest and therefore passes on negative gossip *and* the biased $A$ in equilibrium lies with small probability?

**Proposition 3 (Informative Equilibrium)** *Consider Case II where $\pi - p + (1-\theta)(p + \pi - 1) > 0$. Assume that biased $A$'s mixing probability $x^* > \hat{x}$, then in equilibrium, objective $A$ always reports $g_A = s_A$; biased $A$ reports $g_A(b, 0) = 0$ with probability $x^*$ and reports $g_A(b, -1) = -1$ with probability one. Objective $B$ reports $g_B = g_A$ and biased $B$ reports $g_B(b, 0) = 0$ with probability $y^*$ and passes on the negative gossip with probability one.*

*Remark: multiple equilibria.* The above proposition states that there may be an informative equilibrium if biased $A$ 's mixing probability $x^* > \hat{x}$. Thus when the objective listener $B$ thinks biased $A$ lies a lot, he remains silent and no news gets passed on to $C$; when he thinks the biased $A$ does not fabricate negative gossip so much, he passes on the gossip and $C$ can hear both types of gossip with positive probability.

In contrast to the uninformative equilibrium, however, the informative equilibrium may not always exist because $\hat{x}$ may be very large for certain parameter values. An intuitive example that an informative equilibrium does not exist can be seen from Figure 1: when $p = \pi = 0.5$, then the bound is $\hat{x} = 1$, that is, objective $B$ only passes on negative gossip if the biased $A$ passes on a null signal honestly. However, in equilibrium, the biased $A$ will always have some incentive to lie ($x = 1$ cannot be part of the equilibrium profile). Thus in this case, objective $B$ always remains silent and the equilibrium is the uninformative one.

### 3.4 Position-Independent Incentives with More Than Three Agents

The above analysis focus on the three agents model. This section shows that the key conclusions extend to longer chains where every biased agent cares about a final decisionmaker's opinion.

Consider the same setup as in Section 2, except that there are $K$ agents who may pass on the gossip. For simplification, let $p = 1$, i.e., agent $A$ receives a perfect signal. Agent $K$ is the decisionmaker. All objective agents pass on their best estimates of the state, while every biased agent is concerned about agent $K$'s impression of themselves and $K$'s belief that the candidate is nasty. Then the following proposition characterizes equilibrium in this case:

**Proposition 4** *Suppose that* $\theta^K > \frac{2\pi-1}{p+\pi-1}$*, then in the indirect impact gossip game with* $K$ *agents,*

*(4.1) there exists an informative equilibrium in which all objective agents pass on their signal/gossip truthfully;*

*(4.2) all biased agents from* $A$ *up to agent* $K-1$ *reports* $g_i(b,1) = 1$ *with the same probability* $x \in [0,1)$*, and reports* $g_i(b,-1) = -1$*.*

*(4.3) If* $\theta^K \leq \frac{2\pi-1}{p+\pi-1}$*, there always exists an uninformative equilibrium in which every biased agent before agent* $i < K$ *lies completely and everyone after* $i$ *remain silent.*

**Proof:** For claim (1) and (3), see the Appendix.

For claim (2), suppose that each biased agent $i$ reports $g_i(b,0) = 0$ with probability $x_i$. Agent $K$ may receive gossip $g_{k-1} \in \{0,-1\}$. Then there are two cases:

Case I: when the gossip $g_{k-1} = \eta$, then all agents on the chain are considered objective with probability $\theta$ in the case of $\eta = -1$ and $\frac{\theta}{\theta+(1-\theta)x_i}$ in the case of $\eta = 1$. Therefore if all $x_i = x$, the agents receive identical reputation.

Case II: suppose that gossip $g_{k-1} \neq \eta$, then consider agent $i$ and $i+1$'s expected reputation when they hear a null gossip but report a negative gossip. In the opinion of the decisionmaker $K$, if $x_i = x_{i+1}$, then:

$$Pr(\theta_i = o | g_{k-1} = -1, \eta = 0) - Pr(\theta_{i+1} = o | g_{k-1} = -1, \eta = 0)$$

$$= \quad Pr(g_{i-1} = -1)\theta + Pr(g_{i-1} = 0, g_i = -1) * 0 + \frac{\theta}{\theta + (1-\theta)x_i} Pr(g_i = 0)$$
$$- \quad \Big[ Pr(g_{i-1} = -1)\theta + Pr(g_{i-1} = -1, g_i = -1)\theta + Pr(g_i = 0, g_{i+1} = -1) * 0 + \frac{\theta}{\theta + (1-\theta)x_{i+1}} Pr(g_{i+1} = 0)$$
$$= \quad -\theta(1-\theta)(1-x_i)Pr(g_{i-1} = 0) + \theta(1-\theta)(1-x_{i+1})Pr(g_{i-1} = 0) = 0$$

Hence biased agent $i$ and $i+1$ receive the same reputation payoff when they report a negative gossip. Similarly, they receive the same payoff when they report the null signal truthfully. Therefore they have the same incentive to lie and mixes at probability $x$. ‖

*Remark 1: possible length of a chain with informative equilibrium.* Condition $\theta^K > \frac{2\pi-1}{p+\pi-1}$ is a generalized version of inequality $\pi - 1 + (1-\theta)(p + \pi - 1) < 0$, which separates Case I from Case II in the three agents model. When it is satisfied, every objective agent in the chain would pass on a negative gossip, even if *every* biased agent before him lies with probability one.

This is a sufficient condition for the existence of an informative equilibrium, and the violation of this condition is sufficient for the existence of a completely uninformative equilibrium.[9] Notice that the length of such a "guaranteed" informative chain is quite short even when the fraction of objective type is high in the population. For instance, consider the case when $p = 1, \pi = 0.6$, then the length $K$ is determined by $\theta^K > \frac{1}{3}$. For $\theta = 0.95$, the maximum length of the informative chain is only 21 people; for $\theta = 0.8$, the maximum length of the informative chain is only 4. In other words, even when people are very likely to be objective and the signal is perfect, the information contained in the signal may not reach the decisionmaker for a chain longer than $K$.

*Remark 2: position-independent incentive to lie.* Recall that in three agent model biased $A$ and $B$ have the same incentive to lie when they hear negative signal/gossip, in equilibrium $x = y$. This insight is more general than the three agent setting: the key factor is that each pair of neighbors have the same incentive to lie. Therefore in every informative equilibrium the biased agent's incentive to lie is the same regardless of their position.

### 3.5 Efficiency of the Indirect Impact Gossip Model

The above sections settled the equilibrium behaviors in the indirect impact gossip process. Recall that in Case I, the negative signal goes through while the positive signal is heavily diluted if there is biased agent in the chain. In contrast, in the uninformative equilibrium in Case II, no information gets through because biased agent $B$ worries about losing all his reputation. One

[9] Similar to the possibility of multiple equilibria in the three agents model, for a range of parameter values, there may be both informative and uninformative equilibria.

important question is the efficiency of the indirect impact gossip model in terms of information transmission. The measure of Mean Absolute Error (MAE), defined to be the ex ante likelihood a true signal goes through the chain and reaches agent $C$, is employed to take into account signal distortion in both directions:

$$\begin{aligned} MAE &\equiv E_{s_A, g_B}\left[|Pr(s_A = 0|g_B) - I_{s_A=0}|\right] \\ &= \sum_i \sum_j \left[|Pr(s_A = 0|g_B = j) - I_{s_A=0}|\right] Pr(g_B = j \& s_A = i) \end{aligned}$$

The measure MAE calculates the difference between how likely agent $C$ thinks signal $s_A = 0$ was received and whether the true signal received is $s_A = 0$. For example, when $\theta = 0$, $MAE = 0$: since there is only objective type agent who reports their best estimate of the state, the signal is conveyed to agent $C$ with probability one and there is no error. The following proposition characterizes the error introduced by different fraction of biased type, holding $p, \pi$ fixed:

**Proposition 5 (Information Transmission Error due to the biased Agents)** *Given any $p, \pi$, there exists $\theta^*$ such that:*

*(5.1) Whenever $\theta \geq \theta^*$, th equilibrium falls into Case I, where the objective agent $B$ always passes on what he hears, MAE is increasing and concave in $\epsilon \equiv 1 - \theta$.*

*(5.2) At $\theta < \theta^*$, MAE features a discontinuous jump up. Moreover, MAE in uninformative equilibrium is higher than that of Case I equilibrium.*

This proposition is established using equilibrium properties of Case I and Case II. For example, when $\theta$ is sufficiently close to zero, Proposition 1 shows that biased agent $A$ and $B$ both lie completely, i.e., $x = y = 0$. Then $MAE = \frac{2}{\frac{1}{[p\pi+(1-p)(1-\pi)](1-\theta^2)} + \frac{1}{p(1-\pi)+\pi(1-p)}}$, which is increasing and concave in $\epsilon$.

The error jumps upward due to the change in equilibrium behavior of the objective agents, namely the equilibrium may become the uninformative one. Note that the error may increase as a result of no signal coming through. Intuitively, as long as there is some information leaked through the gossip process, the decisionmaker $C$ should have a better idea about the candidate with the leaked information than without.[10] In the extreme case of Case II, when objective agent entertains too much doubt to pass on the negative gossip, agent $C$ has the same information as his prior. The error with the gossip process is weakly smaller than the error with no information.

[10] It is also due to the fact that the message is binary, so that biased type cannot distort the gossip infinitely. Moreover, it becomes more complicated if $C$ makes decision based on the value of information.

Figure 4 is generated fixing the value of $\pi = 0.7, p = 0.95$ and allows $\theta$ to vary to illustrate the above theorems. Note that if $\theta > 0.62$, the parameter values fall into that of Case I when the objective $B$ always passes on what hears. If $\theta < 0.62$, then objective $B$'s response depends on the lying probability of biased $A, B$.

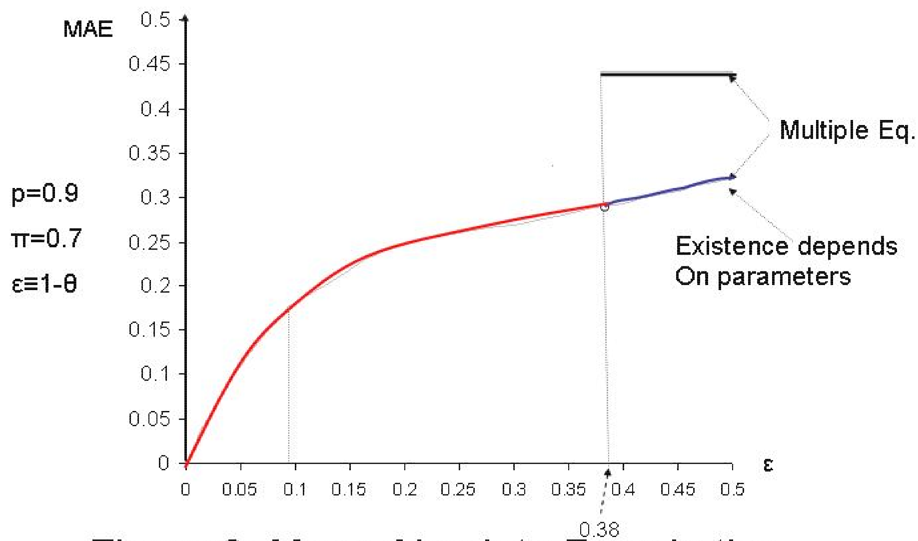



Figure 3: Mean Absolute Error in the Indirect-Impact Model

One way to interpret this proposition is that in regions with a large population and relatively distant interpersonal relationships, there may be less reasons for gossip; however, each biased gossip has a very large marginal impact on the reputation of the target of the gossip. On the other hand, in small, close-knit communities, there may be more reason for biased type to exist, but each of them will have smaller impact on the margin. In certain environments with a very high fraction of biased type, the impact on objective people's belief falls drastically because normal people have little trust for the hearsay.

## 4 The Direct Impact Gossip Model

The previous model features a gossip chain with indirect impact, where every biased agent is concerned about the view of the final decisionmaker. In other situations, there may not be a clearly identified decisionmaker. Instead, everyone is concerned about how people close to them think of their objectivity. What happens to the impact of gossip as it passes through a large number of people who are only concerned about their immediate neighbor's opinion?

## 4.1 Setup

### 4.1.1 Agents and Information Structure

In the direct impact gossip model there are $N$ agents, where $N$ is finite. Each agent may be type $o$ or type $b$ with probability $\theta$ and $1-\theta$ respectively. Types are independently distributed. Agent $A$ and only agent $A$ may receive a private signal about the candidate; the signal is accurate with probability $p$.

The information structure and timing are similar to the indirect impact gossip model, except that now each biased agent cares about influencing the next agent's opinion of the candidate and the next agent's impression of them. The reason for this change in biased agents' objective function is that in a very long chain of people, it may be less plausible for every agent to care about the last person's opinion instead of those close in the chain.

Formally, type $o$ agent $i$ still passes on the gossip that is the best estimate of the state: $g_i = 0$ iff $Pr(\eta = 0|I_i) \geq Pr(\eta = -1|I_i)$. Type $b$ agent $i$'s objective is to maximize:

$$EU_i = Pr(\eta = -1|g_i) + E_\eta[Pr(\theta_i = o|g_i, \eta)|g_{i-1}]$$

over $g_i$. As before, the first part is agent $i+1$'s view of the candidate given the gossip $g_i$. The second part is agent $i+1$'s view of how objective $i$ is, after the true state realizes.

### 4.1.2 Timing

Agent $A$ may hear a signal $s_A \in \{0, -1\}$, and he passes on a gossip $g_A \in \{0, -1\}$ down to agent $B$, who may then pass a gossip $g_B$ on to the next agent. The game ends at agent $N$, who was the last one to hear gossip $g_{N-1} \in \{0, -1\}$.

### 4.1.3 A Word on Equilibrium

Agent $A$'s strategy is $g_A : S_A \times \Theta_A \to \Delta(0, -1)$ and all other agents $i$'s strategy is $g_i : G_{i-1} \times \Theta_i \to \Delta(0, -1)$. The equilibrium concept used here is still Perfect Bayesian Equilibrium (PBE). This model is a finite (in both the time horizon and the action space of each player) extensive form game with perfect recall that satisfies all conditions in Selten (1975) and Kreps and Wilson (1982). Thus, there exists a sequential equilibrium by Theorem 5 of Selten (1975) and Proposition 1 of Kreps and

Wilson (1982).[11] Therefore all the following analysis will assume equilibrium existence and only derive the properties of equilibrium.

## 4.2 Equilibrium Analysis

This section proceeds to characterize the properties of the equilibrium via a sequence of observations and lemmas. The first observation of the direct impact gossip game is that everyone remains truthful cannot be part of an equilibrium:

**Observation 3** *Contamination of Information*

*For $N > 1$, there does not exist an equilibrium such that all biased agents pass on the gossip they hear truthfully with probability one.*

Suppose there exists such an equilibrium. Then all objective agents will report $g_i(o, g_{i-1}) = g_{i-1}$ because the gossip reflects true information $s_A$. Then at least one biased agent will have an incentive to state $g_i(b, g_{i-1} = 0) = -1$ with probability one. In this putative equilibrium, the posterior probability that he is objective is $\theta$ regardless of his gossip, yet his impact on the next agent is strictly positive: $Pr(\eta = -1|g_{N-1} = -1) - Pr(\eta = -1|g_{N-1} = 0) = Pr(\eta = -1|s_1 = -1) - Pr(\eta = -1|s_1 = 0) > 0$. Therefore some biased agent will deviate.

A similar observation shows that the biased agents distort the information if it is highly informative and trustworthy. The probability of having a biased agent in a chain of length $N$ is $1 - \theta^N \to 1$ when $N$ is very large. Hence in a direct impact gossip, the gossip would almost surely have been contaminated.

**Observation 4** *As long as the objective agents are sending both messages with positive probability, the biased agents will do so as well.*

In this game, at least the very first objective agent will follow his signal truthfully and therefore both $g_A = 0$ and $g_A = -1$ can be heard with positive probability. Suppose that the biased agent $i$ only sends $g_i(b, g_{i-1}) = 0$, then $g_i = -1$ shows one is objective *and* lowers the listener's view of the candidate. Thus by deviating, the biased type increases both components of his expected utility. Recall that the objective type in the indirect impact gossip model becomes less trusting and may remain silent unless $\theta$ is very high and/or the quality of signal is high, similarly:

[11] Fudenberg and Tirole (1991) shows that the concepts of sequential equilibrium and PBE coincide in multi-stage game of incomplete information when the players' types are independent and each player has at most two possible types.

**Lemma 3** *(1) In equilibrium, if an objective agent $i$ remains silent regardless of what he hears, i.e., $g_i(o, g_{i-1}) = 0$, then all objective agents $N \geq j > i$ will report $g_j(o, g_{j-1}) = 0$.*

*(2) In equilibrium, if an objective agent $i$ reports $g_i(o, g_{i-1}) = g_{i-1}$, a biased agent $i$ will report $g_i(b, -1) = -1$ and $g_i(b, 0) = 0$ with probability $x_i > 0$.*

This first part of this lemma establishes the fact that if one objective agent stops listening, every objective one after him will follow suit because they have the same preferences of only passing on what they believe to be reliable information. Since the quality of information can only become worse, once gossip becomes too diluted to be trusted by one, it becomes untrustworthy to all.

The second part of the lemma shows that, whenever the objective agents are still listening, the biased agent will have an incentive to distort his gossip with positive probability. Intuitively, the fact that objective agent $i$ passes along the gossip means that in his inference, the candidate is more likely to be nasty when he hears a negative gossip. Therefore when $g_{i-1} = 0$, biased $i$ can fabricate a negative gossip with a small probability: the informativeness of a negative gossip from him will still be of some value to his listener and thus can bias their view of the candidate toward $\eta = -1$.

Lemma 3 establishes that the biased type agents always want to bias the gossip as long as the objective agents are listening. Their impact dissipates very quickly, however, once the objective agents stops listening and passing on negative gossip. A natural question is whether there is such a point when all objective agents stop listening. If the gossip chain is long, the answer is yes.

**Lemma 4** *For $N$ sufficiently large, there exist agents $k$ and $k+1$ such that all objective agent $j \leq k$ reports $g_j(o, g_{j-1}) = g_{j-1}$ but the objective agent $k+1$ always remains silent, i.e., $g_{k+1}(o, g_k) = 0$.*

Note that in order to have a cutoff point in the gossip chain such that all objective agents after $k$ remain silent, it is not sufficient to establish that all biased agents up to $k$ lie with positive probability. Suppose that each successive biased agent lies with smaller and smaller probability, then it is possible that objective agent can still believe that the candidate is more nasty upon hearing a negative gossip, albeit with smaller and smaller probability. Formally, it may be the case that agent $i+1$'s posterior probability about the candidate is $Pr(\eta = -1 | g_i = -1) - Pr(\eta = 0 | g_i = -1)$ is decreasing but always greater than zero. In other words, the posterior probability that the candidate is slightly more nasty may decrease but does not fall below $\frac{1}{2}$, which implies that the objective type will forever pass on the gossip they hear. This cannot be part of an equilibrium: suppose so, then it must be that each biased agent lies with smaller and smaller probability. Thus the damage on the candidate is bounded from below by a positive number but the reputation loss approaches zero, which is a contradiction.

Next it is simple to see that the biased agent will stop spreading gossip soon because their impact on the next objective type becomes smaller and smaller and they have to pay a reputation cost of $\theta$: by spreading a negative gossip, they reveal that they are biased for sure. Moreover, the next lemma shows that the biased agents will become silent *immediately*:

**Lemma 5** *All biased agents $i > k$ always remain silent.*

Intuitively, since it is impossible for any agent after $k$ to hear the negative gossip from an *objective* source, the information value of any negative gossip is at most $Pr(\eta = -1|g_k = -1) \leq \frac{1}{2}$, which is smaller than the reputation cost of $\theta$ a biased agent gains by being silent. Finally, the key insights of the direct impact gossip model are summarized below:

**Proposition 6** *In an equilibrium of the direct impact gossip game with sufficiently large $N$:*

*There exists an agent $k$ such that all objective agents up to $k$ pass along $g_{i-1}$ truthfully, i.e., $g_i(o, g_{i-1}) = g_{i-1}$. All biased agents $i \leq k$ report $g_i(b, -1) = -1$, and $g_i(b, 0) = 0$ with probability $x_i < 1$. Both objective and biased agents from $k + 1$ to $N$ always remain silent.*

**Proof**: immediate from Lemma 3-5. ‖

Hence the direct impact gossip model shows that gossip may have an impact on the people who hear it early. Gradually, its negative impact shows up in another direction: people stop listening and the channel completely breaks down. This is a caution against the use of informal communication channels: even though the gossip may exert negative impact in a small group of people, as suggested in the indirect impact gossip model, the gossip eventually become worthless.

## 5 Extensions and Discussions of the Linear Gossip Model

The linear gossip models studied in the previous sections (both the indirect impact gossip and the direct impact gossip models) are based on a number of special assumptions. This section discusses some key modeling assumptions used in the previous analysis and how relaxing them may or may not change the basic insight of the linear model.

## 5.1 Multiple Sources of Gossip

In this model, $A$ and only $A$ may obtain a signal about the candidate and thus gossip becomes the only potential source of information. What if there are multiple sources of information? Consider the three agent indirect impact model as in Section 3. Suppose that agent $B$ may obtain an independent signal of equal quality to that of $A$'s, i.e., $Pr(s_B = \eta) = p$. Then the following is true:

**Result 1** *Suppose that agent $B$ receives a private signal $s_B \in \{0, -1\}$, $Pr(\eta = s_B) = p$, then in equilibrium, then*

*(1) objective $A$ reports $g_A = s_A$ and objective $B$ reports $g_B = s_B$ regardless of $g_A$;*

*(2) biased $B$ reports $g_B(b, g_A = 0, s_B = 0) = 0$ with probability $y_1$ and $g_B(b, g_A = 0, s_B = -1) = 0$ with probability $y_2$. Moreover, $y_1 \leq y^*, y_2 \geq y^*$.*

Note that the objective type agent does not use the information contained in the gossip because his signal is weakly more accurate than the gossip, the biased $B$ is more likely to be held accountable for wrong gossips. This effect may further prevent the information to reach decisionmaker $C$.

More generally, manipulative gossip thrives in the limbo: there have to be some biased types in the population, but not many, otherwise the objective agent become less trusting (Section 3). Moreover, there have to be uninformed/less informed agents as listeners of the gossip (Section 5), but not too many, otherwise the gossip dies before it influences any of them (Section 4).

## 5.2 Gossip in Network

Both the indirect impact gossip and direct impact gossip model assume a linear structure. In reality, however, gossip may spread in complicated networks: people may hear similar gossip about the same subject from different sources even though the gossip was originated from the same source. For example, in the long chain model, what would happen if agent $i$ has a small chance of hearing a piece of gossip from someone other than his immediate predecessor?

Suppose there are only three agents. Agent $C$ hears gossip $g_A$ with probability $\rho \approx 0$, then it can be shown that biased $B$ has smaller incentive to fabricate negative gossip than in the previous case: he risks losing his reputation completely if $C$ learns $g_A$. However, this effect may increase biased $A$'s incentive to lie: suppose he remains silent, his damage on the candidate becomes much smaller. First, biased $B$ is less willing to lie for the fear of being caught. Second, $g_A = 0$ may be learned by $C$ and becomes a permanent source of positive information for the candidate. Thus the biased $A$ may want to lie more than when $C$ cannot hear what he says.

Another way to analyze gossip in social network is to consider a circular model of gossip. Agents are located in circles, someone may receive a new signal about the candidate's personality with a very small probability. Then there is no such strong distinction of agents with information and without, and a steady state of circular gossip process can be analyzed.

### 5.3 Mechanism Design: Formal versus Informal Communication Channels

Why do informal channels of information transmission exist? One reason may be that informal process such as gossip serves as a substitute to the formal processes due to the lack of accountability generated by its source uncertainty. Controversial information that may lead to lawsuits or sensitive information that requires deniability typical in the political arena may never be communicated through formal channels. Moreover, people may prefer silence to vouching for something they are uncertain about. Another reason may be that, from a psychological perspective, people gossip because they like to feel important, knowledgeable and it is an important part of our social life (Allport and Postman 1946-1947). The first reason and some questions associated with it, e.g., when should the decisionmaker insist on formal process and when they should also rely on informal processes, are interesting research questions even though they are not explicitly addressed in the current paper.

One particular question is to compare the efficiency associated with the formal process and informal process such as gossip: how likely it is for the true signal to be communicated? Note that the information distortion is not necessarily a verdict against the informal process: the accountability associated with the formal process can affect the content and the total amount of information transmitted as well. If some information may be used against a clearly identifiable agent, the agent may only pass on pieces of information that he is most confident about or the least controversial ones. Therefore important, decision-relevant information may be lost if there are only formal channels for information flows.

### 5.4 When the Objective Agents Attempt to Influence the Next Agent's Beliefs

In the current model, the objective agents behave as automata: they only impart their best estimate of the state of the world. However, it seems also plausible that the objective type tries to convince their listener of what they perceive as the true state of the world. Consider the equilibrium in Case I when the negative information gets transmitted perfectly while the positive one $s_A = 0$ does not. Then an objective agent may be concerned that his silence will be interpreted as a too positive signal of the candidate's quality whereas he simply thinks that the candidate is more likely to be nice than nasty. Thus he may become less willing to state $g_i = 0$, which in equilibrium further increases the value of a null gossip. These effects reinforce each other and full unraveling may occur. Such strategic considerations by the objective type are explored in Kőszegi and Li (**?**).

## 6 Conclusion

Information flows through both formal and informal channels. Information economics typically focuses on environments where information flows through the formal channels, where the source of information and the transmission details are known. In many other environments, however, information goes through relatively informal channels where the source and who have learned it are unclear. This paper focuses on gossip, a special form of such informal channels.

Can useful information be communicated through the gossip channel? Does gossip have real influence? How does one's position in a gossip chain affect his incentive to lie? This model is a first attempt at answering these questions. In a indirect impact gossip model, when the source of gossip is much more accurate than people's priors and there are few biased people in the population, gossip can influence people's perception about the object of gossip. Moreover, even though in equilibrium gossip may be biased, ex ante, useful information still leaks through, and the errors in decision-making becomes smaller. In the direct impact gossip model, however, the objective people will eventually entertain sufficient doubt of the gossip they hear because of the gradual, but sure contamination of the information. There is a point when objective people stop passing on gossip and this information channel breaks down.

## Appendix

**A.1. Proof of Lemma 1**:

(1) By assumption, $p \geq \pi \geq \frac{1}{2}$. Simple algebra can show that:

$$Pr(\eta = 0 | s_A = 0) = \frac{\pi p}{\pi p + (1-\pi)(1-p)} \geq Pr(\eta = -1 | s_A = 0)$$
$$Pr(\eta = -1 | s_A = -1) = \frac{(1-\pi) p}{\pi(1-p) + (1-\pi) p} \geq Pr(\eta = 0 | s_A = -1)$$

Therefore the objective $A$ should report the signal he receives.

(2) For agent $B$, given the proposed beliefs, the posterior probability that the candidate is nice is:

$$\begin{aligned}
Pr(\eta = 0|g_A = -1) &= \frac{Pr(g_A = -1|\eta = 0)Pr(\eta = 0)}{Pr(g_A = -1|\eta = 0)Pr(\eta = 0) + Pr(g_A = -1|\eta = -1)Pr(\eta = -1)} \\
&= \frac{[1 - p + p(1 - x)(1 - \theta)]\pi}{[1 - p + p(1 - x)(1 - \theta)]\pi + [p + (1 - p)(1 - x)(1 - \theta)](1 - \pi)} \\
Pr(\eta = 0|g_A = 0) &= Pr(\eta = 0|s_A = 0) \geq Pr(\eta = -1|s_A = 0)
\end{aligned}$$

Simple algebra can show that $Pr(\eta = -1|g_A = -1) > Pr(\eta = 0|g_A = -1)$ if $\pi - p + (1 - x)(1 - \theta)(p + \pi - 1) < 0$. At $x = 0$, the above inequality simplifies into $\pi - p + (1 - \theta)(p + \pi - 1) < 0$. Thus if this inequality holds, $Pr(\eta = -1|g_A = -1) > Pr(\eta = 0|g_A = -1) \forall x$ and the objective $B$ should pass on the gossip $g_A$.

(3) If $\pi - p + (1 - \theta)(p + \pi - 1) \geq 0$, then $Pr(\eta = -1|g_A = -1) = Pr(\eta = 0|g_A = -1)$ at $x \leq \hat{x} = 1 - \frac{p - \pi}{(1-\theta)(p+\pi-1)}$. The third part of the lemma is true by continuity. ‖

**A.2. Proof of Proposition 1:**

(1.1) Immediate from part 2) of Lemma 1.

The proofs for claims (1.2)-(1.4) are quite long, thus two lemmas and their proofs are presented first to simplify the exposition:

**Lemma 6** *Given the proposed strategy, biased B's continuation strategy satisfies:*

*(1) If $g_B(b, 0) = 0$ with probability $y \in [0, 1)$, then $\beta = 1$;*

*(2) If $\theta \in [\overline{\theta}, 1)$, then $y = 0$;*

*(3) If $\theta$ is sufficiently close to $\frac{1}{2}$, then $y \in (0, 1)$.*

**Proof**:

(1) The difference between biased $B$'s expected payoff given $g_A$ is:

$$\begin{aligned}
& EU_B(g_B = -1, b, g_A) - EU_B(g_B = 0, b, g_A) \\
= \; & \hat{\eta}_{-1} - \hat{\eta}_1 + E[Pr(\theta_B = o|g_B = -1, \eta)|g_A] - E[Pr(\theta_B = o|g_B = 0, \eta)|g_A]
\end{aligned}$$

The first part of the above equation, $\hat{\eta}_{-1} - \hat{\eta}_1$, does not depend on the gossip $B$ hears because it is $C$'s impression of the candidate before learning the true state. Thus we only need to calculate the difference in $B$'s own reputation by reporting $g_B = -1$ versus $g_B = 0$ after receiving gossip $g_A = 0$ and $g_A = -1$ respectively. Define $\omega_1 \equiv Pr(g_A = -1|g_B = -1, \eta = 0), \omega_2 \equiv Pr(g_A = -1|g_B = -1, \eta = -1)$:

$$\begin{aligned}
& E[Pr(\theta_B = o|g_B = -1, \eta)|g_A = 0] - E[Pr(\theta_B = o|g_B = 0, \eta)|g_A = 0] \\
= \; & \left[\gamma\omega_1\theta + (1 - \gamma)\omega_2\theta\right] - \frac{\theta}{\theta + (1 - \theta)y} \\
& E[Pr(\theta_B = o|g_B = -1, \eta)|g_A = -1] - E[Pr(\theta_B = o|g_B = 0, \eta)|g_A = -1] \\
= \; & \left[\mu\omega_1\theta + (1 - \mu)\omega_2\theta\right] - \frac{\theta}{\theta + (1 - \theta)y}
\end{aligned}$$

Thus,

$$\begin{aligned}
& E[Pr(\theta_B = o|g_B = -1, \eta)|g_A = 0] - E[Pr(\theta_B = o|g_B = 0, \eta)|g_A = 0] \\
- \; & \left[E[Pr(\theta_B = o|g_B = -1, \eta)|g_A = -1] - E[Pr(\theta_B = o|g_B = 0, \eta)|g_A = -1]\right] \\
= \; & (\gamma - \mu)\theta\left[\omega_1 - \omega_2\right]
\end{aligned}$$

Since $\mu \equiv \frac{\pi(1-p)}{p+\pi-2p\pi}$, $\gamma \equiv \frac{p\pi}{p\pi+(1-p)(1-\pi)}$, it is easy to see that $\mu < \gamma$. Simple algebra can show that $\omega_1 < \omega_2$. Therefore the difference in expected reputation $E[Pr(\theta_B = o|g_B = -1, \eta)|g_A] - E[Pr(\theta_B = o|g_B = 0, \eta)|g_A]$

is higher for $g_A = -1$. If $B$ is indifferent between $g_B = -1$ and $g_B = 0$ after $g_A = 0$, then he strictly prefers to report $g_B = -1$ after $g_A = -1$.

(2) From the text, we know that $y = 1$, i.e., complete honesty, can never be part of an equilibrium strategy. Assume $\theta \approx 1$ and we only need to consider the case when $B$ hears $g_A = 0$ (from part (1) of this lemma, as long as $B$ is mixing when $g_A = 0$, he reports $g_B = -1$ when $g_A = -1$).

$$\begin{aligned} & \hat{\eta}_{-1} - \hat{\eta}_1 \\ = & \frac{(1-p)(1-\theta)[(1-x)+(\theta+(1-\theta)x)(1-y)+p](1-\pi)}{[p\pi+(1-p)(1-\pi)](1-\theta)[1-x+(\theta+(1-\theta)x)(1-y)]+p(1-\pi)+\pi(1-p)} - \frac{(1-p)(1-\pi)}{p\pi+(1-p)(1-\pi)} \end{aligned}$$

From Lemma 2, the above expression increases in both $x$ and $y$. By continuity, $\hat{\eta}_{-1} - \hat{\eta}_1 > 0$ at any $x > 0, y > 0$ if it is positive at $x = 0, y = 0$. At $x = 0, y = 0$:

$$\hat{\eta}_{-1} - \hat{\eta}_1 = \frac{1}{1 + \frac{p\pi[(1+\theta)+(1-p)]}{(1-p)(1-\pi)[(1+\theta)+p]}} - \frac{1}{1 + \frac{p\pi}{(1-p)(1-\pi)}} > 0$$

Moreover, when $\theta \approx 1$,

$$\begin{aligned} & E[Pr(\theta_B = o|g_B = -1, \eta)|g_A = 0] - E[Pr(\theta_B = o|g_B = 0, \eta)|g_A = 0] \\ = & \left[\gamma\omega_1\theta + (1-\gamma)\omega_2\theta\right] - \frac{\theta}{\theta + (1-\theta)y} \\ \approx & \gamma\theta + (1-\gamma)\theta - 1 \approx 0 \end{aligned}$$

Together, the above equations show that when $\theta$ is sufficiently close to 1, the damage on the candidate is strictly positive while biased $B$'s expected reputation loss is approximately 0. Thus $EU_B(g_B = -1, b, g_A = 0) - EU_B(g_B = 0, b, g_A = 0) > 0$ and biased $B$ always report $g_B = -1$ regardless of $g_A$.

(3) When $\theta \approx \frac{1}{2}$, from part (1) of this Lemma, we only need to consider the case when $B$ hears $g_A = 0$.

$$\begin{aligned} \Delta_B & \equiv EU_B(g_B = -1, b, g_A = 0) - EU_B(g_B = 0, b, g_A = 0) \\ & = \hat{\eta}_{-1} - \hat{\eta}_1 + E[Pr(\theta_B = o|g_B = -1, \eta)|g_A = 0] - E[Pr(\theta_B = o|g_B = 0, \eta)|g_A = 0] \\ & = \hat{\eta}_{-1} - \hat{\eta}_1 + \gamma\omega_1\theta + (1-\gamma)\omega_2\theta - \frac{\theta}{\theta + (1-\theta)y} \end{aligned}$$

First, from part (2), at $y = 1$ for any given $x$, $\Delta_B > 0$ and biased $B$ would always lie with some probability. Since both $\hat{\eta}_{-1} - \hat{\eta}_1$ and the difference in expected reputation increase in $y$, we need to check the behavior of biased $B$ at $y = 0$ for any given $x$:

$$\begin{aligned} \Delta_B & = \hat{\eta}_{-1} - \hat{\eta}_1 + E[Pr(\theta_B = o|g_B = -1, \eta)|g_A = 0] - E[Pr(\theta_B = o|g_B = 0, \eta)|g_A = 0] \\ & < \frac{\pi(1-\pi)(2p-1)}{1 - \theta[\theta(1+x) - 1][p\pi + (1-p)(1-\pi)]} - 1 + \theta \\ & \Leftrightarrow \frac{\pi(1-\pi)(2p-1) - (1-\theta) + \theta(1-\theta)[\theta(1+x) - 1][p\pi + (1-p)(1-\pi)]}{1 - \theta[\theta(1+x) - 1][p\pi + (1-p)(1-\pi)]} < 0 \end{aligned}$$

When $\theta \approx \frac{1}{2}$, $\Delta_B < 0$. By continuity of $\Delta_B$ in $y$, there exists a cutoff $\overline{\theta}$ such that for all $\theta > \overline{\theta}$, biased $B$ reports $g_B = -1$ and for all $\theta \le \overline{\theta}$, biased $B$ mixes with probability $y \in (0, 1)$. ‖

**Lemma 7** *Define $\overline{\theta}$ implicitly as the solution to $(1-\theta)+\theta(1-\theta)^2[p\pi+(1-p)(1-\pi)]-\pi(1-\pi)(2p-1) = 0$, then given the proposed strategy, biased $A$'s continuation strategy satisfies:*

*(1) If $g_A(b, 0) = 0$ with probability $x \in [0, 1)$, then $\alpha = 1$;*

*(2) If $\theta \in [\overline{\theta}, 1)$, then $x = 0$;*

*(3) If $\theta$ is sufficiently close to $\frac{1}{2}$, then $x \in (0, 1)$.*

**Proof**:

(1) Similar to the proof for Lemma 7, the biased $A$'s expected utilities given $s_A = 1$ are the following:

$$\begin{aligned}
& EU_A(g_A = -1, b, s_A = 0) - EU_A(g_A = 0, b, s_A = 0) \\
= & [Pr(g_B = -1|g_A = -1) - Pr(g_B = -1)|g_A = 0)][Pr(\eta = -1|g_B = -1) - Pr(\eta = -1|g_B = 0)] \\
+ & E_\eta[Pr(\theta_A = o|g_B = -1, \eta)][Pr(g_B = -1|g_A = -1) - Pr(g_B = -1|g_A = 0)] \\
+ & E_\eta[Pr(\theta_A = o|g_B = 0, \eta)]Pr(g_B = -1|g_A = 0) - Pr(g_B = 0|g_A = 0)] \\
= & \kappa\Bigg[(\hat{\eta}_{-1} - \hat{\eta}_1) - \Big[\gamma\omega_1[Pr(\theta_A = o|g_B = -1, \eta = 0) - Pr(\theta_A = o|g_B = 0, \eta = 0)] \\
+ & (1-\gamma)\omega_2[Pr(\theta_A = o|g_B = -1, \eta = -1) - Pr(\theta_A = o|g_B = 0, \eta = -1)]\Big]\Bigg]
\end{aligned}$$

To begin with, the common filtering factor $\kappa$ does not affect the relative strength of $\hat{\eta}_{-1} - \hat{\eta}_1$ and $A$'s expected reputation. Therefore similar to $B$, biased $A$ faces the tradeoff of $\hat{\eta}_{-1} - \hat{\eta}_1$ versus the difference in his expected reputation after stating $-1$ and 1.

Recall from part (1) of Lemma 7 that $\mu < \gamma$ and $\omega_1 < \omega_2$. Moreover, $\frac{\theta(1-p)}{1-p+p(1-\theta)(1-x)} < \frac{\theta p}{p+(1-p)(1-\theta)(1-x)}$, therefore:

$$\begin{aligned}
\Delta_A \equiv & EU_A(g_A = -1, b, s_A = 0) - EU_A(g_A = 0, b, s_A = 0) \\
- & [EU_A(g_A = -1, b, s_A = -1) - EU_A(g_A = 0, b, s_A = -1)] \\
= & (\gamma - \mu)\big[\omega_1[\frac{\theta(1-p)}{1-p+p(1-\theta)(1-x)} - \frac{\theta}{\theta+(1-\theta)x}] - \omega_2[\frac{\theta p}{p+(1-p)(1-\theta)(1-x)} - \frac{\theta}{\theta+(1-\theta)x}]\big] < 0
\end{aligned}$$

Therefore the difference in expected reputation $E[Pr(\theta_A = o|g_B = -1, \eta)|s_A, g_A] - E[Pr(\theta_A = o|g_B = 0, \eta)|s_A, g_A]$ is higher for $g_A = -1$. If $A$ is indifferent between $g_A = -1$ and $g_A = 0$ after $s_A = 0$, then he strictly prefers reporting $g_A = -1$ after $s_A = -1$.

(2) If $\theta \in [\overline{\theta}, 1)$

From the text, we know that $x = 1$, i.e., complete honesty, can never be part of an equilibrium strategy. Assume $\theta \approx 1$ and we only need to consider the case when $A$ hears $s_A = 0$ (from part (1) of this lemma, as long as $A$ is mixing when $s_A = 0$, he reports $g_A = -1$ when $s_A = -1$).

From part (2) of Lemma 7,

$$\hat{\eta}_{-1} - \hat{\eta}_1 = \frac{1}{1 + \frac{p\pi[(1+\theta)+(1-p)]}{(1-p)(1-\pi)[(1+\theta)+p]}} - \frac{1}{1 + \frac{p\pi}{(1-p)(1-\pi)}} > 0$$

$A$'s expected reputation loss approaches $\theta - 1 \approx 0$ when $\theta$ is close to 1. Thus $EU_A(g_A = -1, b, s_A = 0) - EU_A(g_A = 0, b, s_A = 0) > 0$ for $\theta$ sufficiently large.

(3) If $\theta$ is sufficiently close to $\frac{1}{2}$, since both $\hat{\eta}_{-1} - \hat{\eta}_1$ and the difference in expected reputation increase in $x$, we need to check the behavior of biased $A$ at $x = 0$ for any given $y$:

$$\begin{aligned}
\Delta_A = & \hat{\eta}_{-1} - \hat{\eta}_1 + E[Pr(\theta_B = o|g_B = -1, \eta)|g_A = 0] - E[Pr(\theta_B = o|g_B = 0, \eta)|g_A = 0] \\
< & \frac{\pi(1-\pi)(2p-1)}{1 - \theta[\theta(1+x) - 1][p\pi + (1-p)(1-\pi)} - 1 + \theta \\
\Leftrightarrow & \frac{\pi(1-\pi)(2p-1) - (1-\theta) + \theta(1-\theta)[\theta(1+x) - 1][p\pi + (1-p)(1-\pi)]}{1 - \theta[\theta(1+x) - 1][p\pi + (1-p)(1-\pi)]}
\end{aligned}$$

When $\theta \approx \frac{1}{2}$, $\Delta_A < 0$. By continuity of $\Delta_A$ in $x$, there exists a cutoff $\overline{\theta}$ such that for all $\theta > \overline{\theta}$ biased $A$ will always report $g_A = -1$ and for all $\theta \leq \overline{\theta}$, biased $A$ will mix with probability $x \in (0, 1)$. ‖

Given Lemma 7 and Lemma 8, the proof of the rest of Proposition 1 is immediate:

(1.2) By Part (1) of Lemma 7 and Lemma 8.

(1.3) By Part (2) of Lemma 7 and Lemma 8.

(1.4) Biased $A$ and $B$ lie with positive probability $x$ and $y$ when $\theta < \overline{\theta}$ is true by Part (3) of Lemma 7 and Lemma 8. Consider the expected reputation of biased $A$ and $B$ if they report the negative gossip versus remaining silent when $x = y$:

$$\begin{aligned} & \Delta_A - \Delta_B = \gamma\omega_1[Pr(\theta_A = o|g_B = -1, \eta = 0) - Pr(\theta_A = o|g_B = 0, \eta = 0)] \\ + \quad & (1-\gamma)\omega_2[Pr(\theta_A = o|g_B = -1, \eta = -1) - Pr(\theta_A = o|g_B = 0, \eta = -1)] \\ - \quad & [\gamma\omega_1\theta + (1-\gamma)\omega_2\theta - \frac{\theta}{\theta + (1-\theta)y}] = 0 \end{aligned}$$

Therefore when biased $A$ and $B$ both lie, they lie with the same probability. ‖

**A.3. Proof of Lemma 2:**

1) From part 3) of Lemma 1, if $\pi - p + (1-\theta)(\pi + p - 1) \geq 0$, then for $x \leq \hat{x}$, objective $B$'s posterior $Pr(\eta = -1|g_A = -1) \leq Pr(\eta = 0|g_A = -1)$, hence objective $B$ reports $g_B = 0$. Therefore $g_B = -1$ can only come from biased $B$ and it is simple to see that $Pr(\eta = -1|g_B = -1) \leq Pr(\eta = -1|g_A = -1) \leq \frac{1}{2}$.

2) Using Bayes' rule, it is simple to see that:

$$Pr(\eta = -1|g_B = -1) \quad = \quad \frac{(1-p)(1-\theta)[(1-x) + (\theta + x(1-\theta))(1-y) + p](1-\pi)}{[p\pi + (1-p)(1-\pi)](1-\theta)[1 - x + (\theta + (1-\theta)x)(1-y)] + p(1-\pi) + \pi(1-p)}$$

Moreover, it is obvious that $\frac{\partial}{\partial y} Pr(\eta = -1|g_B = -1) > 0$. The sign of its derivative with respect to $x$ is:

$$\begin{aligned} & sign\left[\frac{\partial}{\partial x} Pr(\eta = -1|g_B = -1)\right] \\ = \quad & sign\big[[1 - x + (\theta + (1-\theta)x)(1-y)] + p] - [1 - x + (\theta + (1-\theta)x)(1-y)] + 1 - p]\big] \\ = \quad & sign(2p-1) \geq 0 \end{aligned}$$

Therefore the posterior estimate of the candidate $\hat{\eta}_{-1}$ strictly increases in $x$ and $y$. ‖

**A.4. Proof of Proposition 2:**

(2.1) From Lemma 2, objective $B$ believes that $Pr(\eta = -1|g_A = -1) \leq Pr(\eta = 0|g_A = -1)$ if biased $A$ reports $g_A(b, 0) = 0$ with probability $x \in [0, \hat{x}]$. Therefore objective $B$ will report $g_B(o, g_A) = 1$.

For the biased $B$, his expected reputation of reporting $g_B = -1$ versus $g_B = 0$ is:

$$\begin{aligned} & EU_B(g_B = -1, b, g_A) - EU_B(g_B = 0, b, g_A) \\ = \quad & Pr(\eta = -1|g_B = -1) - Pr(\eta = -1|g_B = 0) - \theta \\ = \quad & Pr(\eta = -1|g_B = -1) - (1-\theta) - \theta < 0 \end{aligned}$$

(2.2) The cutoff $x^*, y^*$ are implicitly defined by biased $A$ and $B$'s mixing constraints respectively:

$$\begin{aligned} & \Delta_A = 0 \\ \Leftrightarrow \quad & \frac{(1-p)(1-\theta)[(1-x) + (\theta + (1-\theta)x)(1-y) + p](1-\pi)}{[p\pi + (1-p)(1-\pi)](1-\theta)[1 - x + (\theta + (1-\theta)x)(1-y)] + p(1-\pi) + \pi(1-p)} - \frac{(1-p)(1-\pi)}{p\pi + (1-p)(1-\pi)} \\ = \quad & \frac{\pi(1-\pi)(2p-1)}{1 - \theta[\theta(1+x) - 1][p\pi + (1-p)(1-\pi)} - 1 + \theta \\ \Leftrightarrow \quad & \frac{\pi(1-\pi)(2p-1) - (1-\theta) + \theta(1-\theta)[\theta(1+x) - 1][p\pi + (1-p)(1-\pi)]}{1 - \theta[\theta(1+x) - 1][p\pi + (1-p)(1-\pi)]} \end{aligned} \tag{1}$$

$$\begin{aligned} & \Delta_B = 0 \\ \Leftrightarrow \quad & \frac{(1-p)(1-\theta)[(1-x) + (\theta + (1-\theta)x)(1-y) + p](1-\pi)}{[p\pi + (1-p)(1-\pi)](1-\theta)[1 - x + (\theta + (1-\theta)x)(1-y)] + p(1-\pi) + \pi(1-p)} - \frac{(1-p)(1-\pi)}{p\pi + (1-p)(1-\pi)} \\ = \quad & \frac{\pi(1-\pi)(2p-1)}{1 - \theta[\theta(1+x) - 1][p\pi + (1-p)(1-\pi)} - 1 + \theta \\ \Leftrightarrow \quad & \frac{\pi(1-\pi)(2p-1) - (1-\theta) + \theta(1-\theta)[\theta(1+x) - 1][p\pi + (1-p)(1-\pi)]}{1 - \theta[\theta(1+x) - 1][p\pi + (1-p)(1-\pi)]} \end{aligned} \tag{2}$$

From above, if the mixing probability $x^* \leq \hat{x}$, then the objective $B$ will not pass on the negative gossip. Thus the biased $B$ will remain silent by above, which means no information can reach $C$. ‖

### A.5. Proof of Proposition 3:

If biased $A$ is indifferent between the two gossips at the mixing probability $x^* > \hat{x}$, then by Proposition 2, objective $B$ will pass on the negative gossip to $C$. Thus biased $B$ will lie with probability $y^*$ and this is an equilibrium. ‖

### A.6. Proof of Proposition 4:

**Proof:** Claim (4.2) is proved in the text. ‖

### A.7. Proof of Proposition 5:

(5.1) When $\theta < \theta^*$, Proposition 1 shows that biased agent $A$ and $B$ both lie completely, i.e., $x = y = 0$, the error introduced by the biased agents becomes:

$$MAE_1 = \frac{2}{\frac{1}{[p\pi+(1-p)(1-\pi)](1-\theta^2)} + \frac{1}{p(1-\pi)+\pi(1-p)}}$$

$$= \frac{2}{\frac{1}{[p\pi+(1-p)(1-\pi)]\epsilon(2-\epsilon)} + \frac{1}{p(1-\pi)+\pi(1-p)}}$$

Given that $\epsilon \leq \frac{1}{2}$, It is easy to see $\frac{\partial}{\partial\epsilon}MAE_1 > 0$, and $\frac{\partial^2}{\partial\epsilon^2}MAE_1 < 0$, thus it is increasing and concave in $\epsilon$.

(5.2) When $\theta \in (\theta^*, \underline{\theta}]$, Proposition 1 shows that both biased $A$ and $B$ will lie with some probability. First, the MAE associated becomes:

$$MAE_2 = \frac{2}{\frac{1}{[p\pi+(1-p)(1-\pi)](1-\theta)[1-x+(\theta+(1-\theta)x)(1-y)]} + \frac{1}{p(1-\pi)+\pi(1-p)}}$$

$$= \frac{2}{\frac{1}{[p\pi+(1-p)(1-\pi)]\epsilon[1-x+(1-\epsilon+\epsilon x))(1-y)]} + \frac{1}{p(1-\pi)+\pi(1-p)}}$$

First, compare the denominator of the two MAE, since $1-\theta^2 > (1-\theta)[1-x+(\theta+(1-\theta)x)(1-y)]$, it is easy to see that $MAE_1 < MAE_2$. Next, for given $x, y$, $\frac{\partial}{\partial\epsilon}MAE_2 > 0$.

(5.2) When $\theta > \underline{\theta}$, then in the case of uninformative equilibrium, we can see that the error is:

$$\overline{MAE} = 2[p\pi + (1-p)(1-\pi)][p(1-\pi) + \pi(1-p)]$$

which is higher than the informative case. ‖

### A.8. Proof of Lemma 3:

(1) The objective agent $i$ may hear $g_{i-1} \in \{0, -1\}$. If he reports $g_i = 0$, then $Pr(\eta = 0|g_{i-1}) \geq Pr(\eta = -1|g_{i-1})$. The objective agent $i+1$ will thus infer from $g_i \in \{0, -1\}$ that:

$$Pr(\eta = 0|g_i) - Pr(\eta = -1|g_i) = \sum_l [Pr(\eta = 0|g_{i-1} = l) - Pr(\eta = -1|g_{i-1} = l)]Pr(g_{i-1} = l|g_i) \geq 0$$

Hence objective $i+1$ reports $g_{i+1}(o, g_i) = 0$. Similarly, all objective agents after $i+1$ will remain silent as well.

(2) Step 1: let the reduced form belief of agent $i$ is:

$$Pr(\eta = 0|g_{i-1} = -1) \equiv q_1, Pr(\eta = 0|g_{i-1} = 0) \equiv q_2$$

Suppose that objective $i$ pass along what he hears in equilibrium, then it must be the case that:

$$Pr(\eta = 0|g_{i-1} = -1) < Pr(\eta = -1|g_{i-1} = -1) \Leftrightarrow q_1 < \frac{1}{2}$$
$$Pr(\eta = 0|g_{i-1} = 0) < Pr(\eta = -1|g_{i-1} = 0) \Leftrightarrow q_2 \geq \frac{1}{2}$$

Step 2: biased $i$'s strategy depends on the following two inequalities:

$$\begin{aligned}
\Delta_1 &\equiv EU_i(\theta_i = b, g_i = -1, g_{i-1} = 0) - EU_i(\theta_i = b, g_i = 0, g_{i-1} = 0) \\
&= \hat{\eta}_{-1} - \hat{\eta}_1 + E[Pr(\theta_i = o|g_i = -1, \eta)|g_{i-1} = 0] - E[Pr(\theta_i = o|g_i = 0, \eta)|g_{i-1} = 0] \quad (3)\\
\Delta_2 &\equiv EU_i(\theta_i = b, g_i = -1, g_{i-1} = -1) - EU_i(\theta_i = b, g_i = 0, g_{i-1} = -1) \\
&= \hat{\eta}_{-1} - \hat{\eta}_1 + E[Pr(\theta_i = o|g_i = -1, \eta)|g_{i-1} = -1] - E[Pr(\theta_i = o|g_i = 0, \eta)|g_{i-1} = -1] \quad (4)
\end{aligned}$$

Suppose that $g_i(b, 0) = 0$ with probability $x_i$ and $g_i(b, -1) = 0$ with probability $\alpha_i$, consider the following strategy profiles for the biased $i$:

To begin with, $x_i = 1, \alpha_i = 0$ cannot be part of an equilibrium strategy profile. Suppose that it is, then:

$$\begin{aligned}
& \Delta_1 - \Delta_2 = 0 \\
= \; & \hat{\eta}_{-1} - \hat{\eta}_1 + \theta - \theta \\
= \; & Pr(\eta = -1|g_{i-1} = -1)[Pr(g_{i-1} = -1|g_i = -1) - Pr(g_{i-1} = -1|g_i = 0)] \\
+ \; & Pr(\eta = -1|g_{i-1} = -1)[Pr(g_{i-1} = 0|g_i = -1) - Pr(g_{i-1} = 0|g_i = 0)] \\
= \; & (1 - q_1)[1 - 0] + (1 - q_2)(-1) = q_2 - q_1 > 0
\end{aligned}$$

Define the following posterior probabilities:

$$\begin{aligned}
\mu_1 &\equiv Pr(g_{i-1} = -1|g_i = -1, \eta = 0) \\
&= \frac{[\theta + (1-\theta)\alpha_i]Pr(g_{i-1} = -1)}{[\theta + (1-\theta)\alpha_i]Pr(g_{i-1} = -1) + (1-\theta)(1-x_i)Pr(g_{i-1} = 0)} \\
\mu_2 &\equiv Pr(g_{i-1} = 0|g_i = 0, \eta = 0) \\
&= \frac{(1-\theta)(1-\alpha_i)Pr(g_{i-1} = -1)}{(1-\theta)(1-\alpha_i)Pr(g_{i-1} = -1) + [\theta + (1-\theta)x_i]Pr(g_{i-1} = 0)]} \\
\mu_3 &\equiv Pr(g_{i-1} = -1|g_i = -1, \eta = -1) \\
&= \frac{[\theta + (1-\theta)\alpha_i]Pr(g_{i-1} = -1)}{[\theta + (1-\theta)\alpha_i]Pr(g_{i-1} = -1) + (1-\theta)(1-x_i)Pr(g_{i-1} = 0)} \\
\mu_4 &\equiv Pr(g_{i-1} = 0|g_i = 0, \eta = -1) \\
&= \frac{(1-\theta)(1-\alpha_i)Pr(g_{i-1} = -1)}{(1-\theta)(1-\alpha_i)Pr(g_{i-1} = -1) + [\theta + (1-\theta)x_i]Pr(g_{i-1} = 0)]}
\end{aligned}$$

The difference between biased $i$'s expected utility of reporting $g_i = -1$ versus $g_i = 0$ given $g_{i-1} = 0$ and $g_{i-1} = -1$ is:

$$\begin{aligned}
& \Delta_1 - \Delta_2 \\
= \; & E_\eta[Pr(\theta_i = o|g_i = -1, \eta)|g_{i-1} = 0] - E_\eta[Pr(\theta_i = o|g_i = 0, \eta)|g_{i-1} = 0] \\
- \; & [E_\eta[Pr(\theta_i = o|g_i = -1, \eta)|g_{i-1} = -1] - E_\eta[Pr(\theta_i = o|g_i = 0, \eta)|g_{i-1} = -1]] \\
= \; & [Pr(\eta = 0|g_{i-1} = 0) - Pr(\eta = 0|g_{i-1} = -1)][Pr(\theta_i = o|g_i = -1, \eta = 0) \\
- \; & Pr(\theta_i = o|g_i = 0, \eta = 0) - [Pr(\theta_i = o|g_i = -1, \eta = -1) - Pr(\theta_i = o|g_i = 0, \eta = -1)]] \\
= \; & (q_2 - q_1)[Pr(\theta_i = o|g_{i-1} = -1, g_i = -1, \eta = 0)\mu_1 - Pr(\theta_i = o|g_{i-1} = 0, g_i = 0, \eta = 0)\mu_2 \\
- \; & Pr(\theta_i = o|g_{i-1} = -1, g_i = -1, \eta = -1)\mu_3 + Pr(\theta_i = o|g_{i-1} = 0, g_i = 0, \eta = -1)\mu_4] \quad (5)
\end{aligned}$$

Next, $x_i = 1, \alpha_i < 1$ cannot be part of equilibrium strategy profile. Suppose it is, then equation (5) must be (weakly) positive. Observe that:

$$-(q_2 - q_1)\theta[\frac{q_1(\theta + (1-\theta)\alpha_i)}{q_1(\theta + (1-\theta)\alpha_i) + (1-q_1)(1-\theta)(1-\alpha_i)} - \frac{(1-q_1)(\theta + (1-\theta)\alpha_i)}{(1-q_1)(\theta + (1-\theta)\alpha_i) + q_1(1-\theta)(1-\alpha_i)}] < 0$$

Hence equation (5) cannot be weakly positive, contradiction.

Third, $x_i < 1, \alpha_i < 1$ cannot be part of equilibrium profile because $\Delta_1 = \Delta_2 = 0$ is impossible.

Therefore the biased $i$ must state $x_i < 1, \alpha_i = 1$, it follows that $\Delta_1 = 0$.

STEP 3: From $\Delta_1 = 0$ at $x_i > 0$,

$$\Delta_1 = \hat{\eta}_{-1} - \hat{\eta}_1 + E[Pr(\theta_i = o|g_i = -1, \eta)|g_{i-1} = 0] - E[Pr(\theta_i = o|g_i = 0, \eta)|g_{i-1} = 0]$$
$$\Delta_2 = \hat{\eta}_{-1} - \hat{\eta}_1 + E[Pr(\theta_i = o|g_i = -1, \eta)|g_{i-1} = -1] - E[Pr(\theta_i = o|g_i = 0, \eta)|g_{i-1} = -1]$$

Hence $1 - x_i > \psi$. ‖

**A.9. Proof of Lemma 4**:

From Lemma 4, suppose that in equilibrium, all objective agent $i$ up to $k$ pass on what they hear truthfully and biased $i$ reports $g_i(b, 0) = 0$ with probability $x_i < 1$, then the following must be true:

$$\begin{aligned} & Pr(\eta = 0|g_i = -1) - Pr(\eta = 0|g_{i-1} = -1) \\ = \; & [1 - Pr(\eta = -1|g_i = -1)] - q_1 \\ = \; & [1 - (1-q_1)\mu_1 - (1-q_2)(1-\mu_1)] - q_1 \\ = \; & q_2 - (q_2 - q_1)\mu_1 - q_1 = (q_2 - q_1)(1-\mu_1) > 0 \end{aligned}$$

Thus objective agent $i+1$ believes that $Pr(\eta = 0|g_i = -1)$ is larger than that of the objective agent $i$. Hence if $N$ is large enough, there must exist a $k$ such that $Pr(\eta = 0|g_{k-1} = -1) - Pr(\eta = 0|g_{k-2} = -1) \approx 0$ and, for objective agent $k+1$, $Pr(\eta = 0|g_k = -1) - Pr(\eta = 0|g_{k-1} = -1) > 0$. Therefore objective agent $k+1$ will remain silent. ‖

**A.10. Proof of Lemma 5**:

Consider biased agent $k+1$'s expected utility after reporting $g_{k+1} = -1$ and $g_{k+1} = 0$:

$$EU_{k+1}(g_{k+1} = -1, b, g_k) = [Pr(\eta = -1|g_{k+1} = -1) + 0 \leq Pr(\eta = -1|g_k = -1) \leq \frac{1}{2}$$
$$EU_{k+1}(g_{k+1} = 1, b, g_k) = [Pr(\eta = -1|g_{k+1} = 0) + \theta = (1 - \pi + \theta) > \frac{1}{2}$$

Hence the biased $k+1$ is strictly better off by remaining silent. Similarly, all biased agent after him will also remain silent. ‖

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