Constitutional Rules

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Abstract

This paper proposes a normative theory of constitutional rules. We characterize the set of optimal constitutional rules under different assumptions about the degree of contractual imperfections. Our model explains why constitutions contain different *types* of rules. In particular, we derive conditions under which it is optimal, in addition to a standard decision rule (e.g., simple majority), to introduce veto rules (that block certain types of decisions) and supermajority rules (that allow the veto rule to be overruled). Our model also explains the existence of amendment rules and checks and balances.

Keywords: Constitutions, constitutional design, social contracts, majority rules, amendments, checks and balances.

JEL Classification:

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1. Introduction

A constitution is a social contract that sets out the rules that govern the way a society makes collective decisions. While these rules are complex and vary from constitution to constitution, it is possible to point some features that are shared by most constitutions. All constitutions contain rules that govern how day-to-day decisions are made, with the most common decision rule being the simple majority rule. In addition to this, many constitutions have some sort of veto rule that blocks certain changes to the status quo. This, typically happens when the changes, would violate basic political or economic rights. Such veto rules are often complemented by an additional rule that allows the veto rule itself to be overruled, but only in exceptional circumstances. This typically requires that a stricter test (e.g., the support of a supermajority) than the one implied by the decision rule must be passed.

Some examples will help clarify what we have in mind. First, the Danish constitution of 1953 states that the right to private property cannot be violated (a veto rule). Nonetheless, expropriation by the State can take place, but only if a majority of the parliament supports it and a minority of one-third of the members of the current parliament do not request that the decision must be confirmed by the parliament again *after* a new election has taken place. This effectively allows the veto rule to be circumvented, but only if a stricter test than that implied by the decision rule (a majority in the current parliament) is passed. Second, the US constitution contains many provisions designed to protect the rights of individuals (e.g., freedom of speech) and legislation that challenge these rights can be overruled by the constitutional court and thus would only pass if they are not deemed unconstitutional.

In addition, to these types of rules most constitutions also contain amendment rules that specify how the constitutions itself can be changed. The Danish constitution, for example, specifies that the constitution can only be changed after it has been confirmed in a referendum in which at least fifty per cent of those who cast their vote and forty per cent of all eligible voters support the change.

Finally, whenever constitutions have to regulate societies where clear differences exist amongst easily identifiable groups, checks and balances exist in that policy making requires that all (or some) of these groups agree. For example, in the current draft for the European Union constitution, in certain policy areas (such as foreign policy or defense) decisions cannot be made without the consent of all member states.

The purpose of this paper is to develop a normative theory that can explain why constitutions embody a range of different procedures and rules. We deliberately abstract from the agency problems that arise when decision making power is delegated and the complications that arise when agenda setting powers are allocated. We do so to focus on the implications that derive directly from the fact that societies in which individuals disagree about what should be done nonetheless need to make decisions. This seems to us to be the fundamental conflict of interest that any constitution would have to deal with, and understanding why a simple decision procedure such as the majority rule is, typically, not sufficient to enable societies to resolve this conflict adequately is of theoretical as well as of practical importance.

We consider a society with a continuum of individuals who face the task of choosing between two alternatives A and B. Alternative B – the status quo – yields the same level of welfare to all, while alternative A creates winners and losers. The magnitude of the loss depends on the precise nature of alternative A and in some case – when alternative A violates certain fundamental rights – the loss is particularly large. At the time when the the constitutional rules are laid down individuals do not yet know whether they will gain or lose from alternative A, nor do they know if adoption of alternative A will violate fundamental rights of those who lose. Thus, the constitution is designed from the original position, behind a veil of ignorance. The set of constitutional rules that maximizes expected utility depends on the details of the environment, and in particular on which aspects of the environment can be verified ex post. We make a distinction between ex ante and ex post efficiency. A constitution is ex ante efficient if from behind the veil of ignorance all individuals prefer the constitution to having the status quo for sure, while a constitution is ex post efficient if a hypothetical social planner, after having learned the nature of proposal A and distribution of winners and losers from this proposal still prefers the constitution to having the status quo for sure.

The optimal constitution always embodies *a decision rule* that specifies how large a fraction of the population must support alternative A for it to pass. In cases where it is not possible to verify *ex post* whether alternative A, if adopted, will violate the rights of those who lose or not, this is the only constitutional rule and the constitution is ex ante but not ex post efficient. In many cases, however, it is reasonable to suppose that whether A would violate some fundamental rights can be verified ex post. In such situations, the optimal constitution embodies, in addition to the decision rule, a *veto rule* that says that irrispective of the decision rule, alternative A cannot pass if it violates these (verifiable) rights. Interestingly, the veto rule is complemented in the optimal constitution by a third rule – a *supermajority rule*. This rule specifies that even if the veto rule applies, alternative A can be adopted if it commands support from a sufficiently large fraction of the population. This constitution is both ex ante and ex post efficient and there is no need for either amendment rules or for checks and balances.

These additional rules are, therefore, adapted in response to other factors. Amendment rules that allow a subset of individuals (such as those who stand to gain from proposal A) to change the original decision and/or veto rule enter the optimal constitution in situations where there exist external threats to the welfare of this subset of individuals. When this threat – say a threat of revolution or some other crisis– varies with circumstances and the original constitution cannot be made contingent on these circumstances, individuals, from behind the veil of ignorance, may want to introduce an amendment rule. This rule allows for constitutional changes to be implemented after it has become clear if the external threat is credible or not, but only if a sufficiently large number of individuals support such amendments. We show that amendment rules provide the flexibility necessary to adapt the constitution to the changing circumstances, but this flexibility comes at the cost that it must allocate decision making to a specific group of individuals (those who would benefit from the policy) rather than to someone who takes society's interest as a whole into account. Therefore, amendment rules are optimal only, from the original position's perspective, when the possibility that the threat will be carried out is high enough.

Checks and balances understood as different rules for different groups of individuals (as distinguished by wealth levels, geographical location etc.) are desirable in heterogenous societies where some groups are more likely to suffer disproportionately from changes to the status quo. Our main results here are that heterogeneity needs to be significant enough for checks and balances to be optimal but that verifiability of the policy areas in which there is heterogeneity reduces this effect.

The rest of the paper is organized as follows. In Section 2, we provide a short literature review. In Section 3 we introduce our basic collective decision problem. In Section 4 we take up the issue of optimal constitutional. In section 5 we extend the framework to take up the issue of constitutional amendment. In section 6, we study the conditions under which checks and balances should be introduced. Section 7 concludes while figures are relegated to an appendix.

2. Related Literature

In recent years, there has been a renewed interest in the fundamental questions related to constitutional design.[1] In this section, we offer a brief discuss of this literature and relate our analysis to what has gone before.

- A number of recent papers view constitutions as incomplete social contracts. Within this framework, [1] show that the optimal choice of a majority rule from behind the veil of ignorance is determined by a trade-off between two consideration. On the one hand, the desire to limit excessive ex post redistribution whereby the majority expropriates the minority suggest that the majority rule should be strict. On the other hand, it is desirable to allow enough flexibility to circumvent ex post vested

[1] The classical work in the area is [4]

interests that attempt to block socially desirable reforms. This suggests that the majority rules should be lax.[2] [2] propose a related theory of endogenous political institutions but focus on rules that contain the power of political leaders. They show that the optimal degree of "insulation" measured as the share of votes needed to block legislation (or the size of the supermajority needed to pass legislation) is determined by a trade-off between allowing the political leader enough leeway to rule and restricting the scope for misuse of power. Our approach shares with these papers the assumption that constitutional choices are made from behind the veil of ignorance, yet our goal is different. We want to understand when and why particular constitutional rules emerge. Thus, rather than analyzing how the strictness of one particular (decision) rule varies with changes in the economic environment, we are interested in the broader question of how the *set* of optimal rules itself varies with the environment.[3] Although our starting point is that constitutions are incomplete contracts in the sense that they do not necessarily provide a full state-contingent plan for all future events, we stress that appropriate responses to *certain* future events, in particular those that relate to fundamental rights, can be specified in the constitution. This is major departure from the previous work, but one, we argue, that provides valuable insights into the complexity of real world constitutions.

- Some recent papers have argued that constitutions are not written behind the veil of ignorance but by individuals who know their position in society. This literature naturally focuses on constitutional rules that are self-sustaining. [7], for example, study a situation where the decision rule used to govern future decisions is itself decided by the majority rule. They find that supermajority rules emerge in an overlapping generations framework where the young can decide on size of the supermajority that is going to be used to make decisions when they become old. Assuming that most public policies introduce immediate costs while benefits arrive later, older voters suffer more from reforms than young voters, and this provides an incentive for young voters to make the decision rule that is going to apply when they are old strict. [3] also study the endogenous choice of majority rule in a positive framework and derive conditions under which voting rules are self-sustaining. At present, we have little to say about the political economy of constitutional design, although our framework could be extended to capture some aspect of this.

- We show that checks and balances understood as different decision rules applied to different groups of individuals (e.g., as in bicameral systems) can be optimal if there

[2] This approach has been further developed by [5].

[3] [1] take steps in this direction by analyzing when it would be desirable to introduction various minority protection rules such as equal tax rates and tax limits.

is enough heterogeneity in the population. The function of checks and balances in our framework is very different from that of [8]. They focus on situations where checks and balances understood as separating decision making power between politicians can reduce policy agency problems. In our framework, checks and balances provide protection to groups of voters at risk of experiencing particularly large loses. This is so because checks and balances in our framework prevents support for alternative A to be transferred between groups.

- Finally, [6] studies a formal model of constitutional change but assumes that changes are requested and determines the optimal amendment rules as a function of the pressures from other forms of change (interpretative interventions through legislation and the courts or, at the other extreme, the possibility of a complete constitutional crisis). Our approach here is entirely different as we endogenize the possibility of change though the choice of the constitution itself.

3. Setup

We model a society where there is a status quo policy B and a possible alternative A and society must choose between them. Thus, A should be interpreted as a reform just as in [2]. There is a continuum of individuals i in this society whose utility functions are described as follows:

$$u(i,x,c)=\begin{cases} w & if \quad x=A \text{ and } i\in W \\ -y & if \quad x=A,\ i\in W^C \text{ and } c\leq\theta \\ -z & if \quad x=A,\ i\in W^C \text{ and } c>\theta \\ 0 & if \quad x=B \end{cases}$$

where $w>0$ and $z\geq y>0$. The interpretation we apply is as follows: if policy $x=B$ is chosen, we have a status quo where all individuals obtain zero utility. If $x=A$ is chosen, some individuals, those who belong to the set W (the "winners"), will get a positive utility w while all other individuals (the "losers") will get some negative utility. The exact amount of negative utility that losers get is determined by the variable c which we discuss further below.

We study the situation where all individuals are behind a veil of ignorance in which they do not know whether they belong in W or not and what the value of the variable c is. In this situation, they must select a mechanism (a constitution) which, once individuals are partitioned between W and W^C and once c has been selected, determines whether A or B obtains. Throughout the paper we assume that there is an independent judiciary which guarantees that the rules prescribed by the constitution will be enforced. Once the constitution has been selected, nature determines winners and losers. It first selects a value

p from a distribution F on the unit interval. Then, for each individual i independently, nature determines whether $i \in W$ or not with $p = \Pr(i \in W)$. Given that we have a continuum of individuals and given that all draws are independent, p also represents the fraction of individuals in the population who belong to W. At the same time, nature also selects the variable c from a distribution G on the interval $[a, b]$ where $a \leq \theta \leq b$. We assume that both F and G have strictly positive densities f and g respectively while $\widetilde{F}(p) = \int_0^p F(x)dx$ and $\widetilde{G}(c) = \int_a^c G(x)dx$.

Clearly, p represents the fraction of individuals who favor over A over B, while c allows us to describe different possible alternatives to the status quo. If the realization of c associated with A is such that $c \leq \theta$ then the policy is not too damaging to losers while if $c > \theta$ the consequences for them are more serious.

To summarize the timeline in our setup we have:

1. Behind the veil of ignorance, a representative individual (the constitutional designer) chooses the constitution.

2. Nature selects p and c. c is observed by everyone.

3. Given p, the sets W and W^C are determined.

4. Individuals vote for or against A. The vote result is observed by everyone.

5. The policy x is determined according to the constitution.

While we assume that c is selected and observable at the same time as p is selected, all we need is that it be observable by the time the policy is determined and so in particular we could also have assumed that W and W^C are selected before c is realized. This would follow an interpretation where A is not a single policy but a whole program of policies that all benefit a particular group of individuals where before c is realized individuals are still uncertain about exactly which policies this program will entail. This issue is discussed further when we take up constitutional change.

Also, as we discuss further below, while the verifiability of votes by a independent court is a relatively mild assumption in a democracy, the verifiability of c is a more delicate issue. At this point, all we wish to require is that if there is verifiability for any of the two variables, this also be in place by the time policy is determined.

All the assumptions above generate a setup where many important features of the collective decision problems actual societies face are absent. In particular, this is a model of direct democracy so that all the agency issues implies by the relationship between politicians and their constituencies are not dealt with here. Also, in our setup the only heterogeneity amongst policies is that generated by the random variable c so that in effect we have only two kinds of alternatives which differ by the degree of losses inflicted upon losers. This means that we do not deal with the agenda setting problem nor does the problem of Condorcet cycles arise (see [1]). Indeed, the purpose of our setup is to study what constitutional mechanisms naturally arise as solutions to even the simplest collective decision problem.

Also, our approach is normative and this for a variety of reasons. Firstly, this assumption maps nicely with the notion that constitutional designers are “founding fathers". If the individuals who design constitutions sufficiently care about the future, they will be careful not to design a constitution that makes either too easy or too difficult to select reforms because while they might be, for example, winners from today's reforms, they might be losers from tomorrows reforms. In fact, it is fairly easy to show that in infinitely repeated versions of the decision problem described here, as long as the constitutional designers care sufficiently about the future, the main results obtained in the paper still apply even if the designers know whether they are winners or losers at the present time.

Secondly, any “positive" theory of constitutional design crucially relies on the details of the bargaining process between the parties while with our normative approach we are able to generate results which do not rely on anything else other than the veil of ignorance assumption. Finally, a normative approach provides a benchmark against which actual outcomes might be measured: as we shall argue below there is ample anecdotal evidence that many of the constitutional mechanisms highlighted here can be found in actual constitutions.

4. Optimal Constitutions

4.1. Constitutions with Majority Rules

We begin our analysis by considering the following class of mechanisms:

Definition 1 *A Majority-Rule (MR) constitution is a number $m \in [0,1]$ such that*

$$x = \begin{cases} A & if \quad v \geq m \\ B & otherwise \end{cases}$$

where v is the proportion of individuals who vote in favor of policy A.

Thus, a MR constitution does not make any use of the information conveyed by the variable c: as we shall see this below this is of crucial importance. Also, since only two possibilities exist, we assume without any loss of generality that voting is sincere so that $v = p$. Given this, for the rest of the paper we use p instead of v whenever there is no risk of confusion between the two. It is important to note that this class of constitution is sole focus of much of the recent literature on constitutional rules (e.g. [1], [2], [7]).

We proceed by determining the optimal MR constitution of our decision problem. To do that, note that the optimization problem that the constitutional designer faces is

Problem 1

$$\max_{m\in[0,1]} EU(m) = \int_m^1 \left(\int_a^\theta (pw - (1-p)y)\, dG(c) + \int_\theta^b (pw - (1-p)z)\, dG(c) \right) dF(p)$$

□

The problem tells us that if A obtains (which means $p \geq m$), then the constitutional designer can expect utility w with probability p and $-y$ with probability $(1-p)$, subject to the fact that $c \leq \theta$ while she will get w with probability p and $-z$ with probability $(1-p)$ if $c > \theta$. If B obtains (which means $p < m$), then the designer gets utility zero for sure.

The solution to the problem is the following:

Proposition 1 *There is a unique optimal MR constitution*

$$m^*_{MR}(\theta, w, z, y) = \frac{z - G(\theta)(z-y)}{(w+z) - G(\theta)(z-y)}$$

Further, m^*_{MR} is increasing in z and y and decreasing in w and θ with $\lim_{z\to\infty} m^*_{MR} = 1$ and $\lim_{w\to\infty} m^*_{MR} = 0$.

Proof. Problem 1 can be rewritten, using integration by parts, as

$$\max_{m\in[0,1]} \left\{ \begin{array}{c} G(\theta)\left[(w+y)\left(1 - mF(m) - \widetilde{F}(1-m)\right) - yF(1-m)\right] \\ +(1-G(\theta))\left[(w+z)\left(1 - mF(m) - \widetilde{F}(1-m)\right) - zF(1-m)\right] \end{array} \right\}$$

The first order condition gives us $m^*_{MR}(\theta, w, z, y)$ while the second order condition is everywhere

$$-\left[G(\theta)(w+y) + (1-G(\theta))(w+z)\right] < 0$$

which means that the function is strictly concave in m and proves that m^*_{MR} is a unique maximum for problem 1. The rest of the results follows by simple differentiation and by taking limits. □

The optimal MR mechanism has some interesting properties. Firstly, as we would expect, if potential losses under A ($-y$ or $-z$) increase, it gets more difficult to pass policy while if potential gains (w) increase, it gets easier. Less intuitively, perhaps, the constitution is decreasing in θ and this because if θ increases, the probability of incurring heavy losses ($-z$) vs. smaller losses ($-y$) decreases and that makes A more appealing ex-ante. Finally, the optimal MR constitution is independent of the distribution F. Intuitively, this is because the constitution sets out conditions under which A should be chosen over B (and vice-versa) and these must be independent of the probability that such condition obtain.

Given that the purpose of this work is to go one step further than determining the optimal constitution of a certain type by determining which type of constitutions are best from a normative viewpoint, the following benchmarks will be useful:

Definition 2 *A constitution is* ex-ante efficient *if behind the veil of ignorance, society prefers the constitution to the status quo B for sure. A constitution is* ex-post efficient *if after the realization of p and c society prefers the constitution to the status quo B for sure.*

Obviously, ex-post efficiency will imply ex-ante efficiency but not vice-versa. Given these definitions, we have

Corollary 1 *m^*_{MR} is ex-ante efficient but not ex-post efficient.*

Proof. Clearly ex-ante efficiency applies here because B can be guaranteed just by setting $m = 1$ but proposition 1 shows that the unique maximum is m^*_{MR} which in turn implies that for any individual in society, $EU(m^*_{MR}) > EU(1) = 0$. On the other hand, ex-post efficiency requires that if $c \leq \theta$ then A obtains iff $pw - (1-p)y \geq 0$ while if $c > \theta$ then A obtain iff $pw - (1-p)z \geq 0$. But to guarantee the first condition would require

$$mw - (1-m)y = 0 \Leftrightarrow m = \frac{y}{y+w}$$

because if $m < \frac{y}{y+w}$, there exists a $p \in \left[m, \frac{y}{y+w}\right)$ such that A obtains while $pw - (1-p)y < 0$ and if $m > \frac{y}{y+w}$, there exists a $p \in \left(\frac{y}{y+w}, m\right]$ such that B obtains even though $pw - (1-p)y > 0$. Similarly, to guarantee the second condition would require

$$m = \frac{z}{z+w}$$

Now, it is easy to see that under our assumptions $\frac{y}{y+z} < m^*_{MR} < \frac{z}{z+w}$ so that ex-post efficiency does not obtain. $\square$

4.2. Constitutions with Supermajority Rules

We have established that MR constitutions cannot achieve ex-post efficiency which leads us to ask whether we can find an alternative class of constitutions which might satisfy this condition. The proof of corollary 1 shows that a constitution that satisfies ex-post efficiency should have the property that the majority rule be $\frac{y}{y+w}$ whenever $c \leq \theta$ and the majority rule be $\frac{z}{z+w}$ if $c > \theta$. In this section, we consider another class of constitutional rules and show that the optimal constitution within this class is ex-post efficient. We have:

Definition 3 *A Supermajority-Rule (SM) constitution is a triplet* (m, s, d) *with a majority rule* $m \in [0,1]$, *a supermajority rule* $d \in [m, 1]$ *and a veto rule* $s \in [a, b]$ *such that*

$$x = \begin{cases} A \quad if \quad ((p \geq m) \wedge (c \leq s)) \vee (p \geq d) \\ B \qquad\qquad\qquad otherwise \end{cases}$$

This class of constitutions makes full use of the information provided by the collective decision problem. The information is used by stating that if p is large enough (i.e. greater than d) then the policy is passed regardless of the losses that losers will have, but if these losses are not large, then a smaller majority (i.e. $p \geq m$) will suffice. Clearly, for any s, MR constitutions are a special case of SM constitutions where $d = m$. Note that the requirement that $d \geq m$ is imposed for consistency: if $d < m$, then m becomes irrelevant.

Crucially for SM constitutions to be applicable, we must assume that c be observable and verifiable by the courts. We will discuss this point further below. To find the optimal SM constitution we need to solve

Problem 2

$$\max_{m, d \geq m, s} EU(m, s, d)$$

$$= \begin{cases} \int_m^d \left(\int_a^s (pw - (1-p)y)\, dG(c) \right) dF(p) \\ + \int_d^1 \left(\int_a^\theta (pw - (1-p)y)\, dG(c) + \int_\theta^b (pw - (1-p)z)\, dG(c) \right) dF(p) & if \quad s \leq \theta \\ \int_m^d \left(\int_a^\theta (pw - (1-p)y)\, dG(c) + \int_\theta^s (pw - (1-p)z)\, dG(c) \right) dF(p) \\ + \int_d^1 \left(\int_a^\theta (pw - (1-p)y)\, dG(c) + \int_\theta^b (pw - (1-p)z)\, dG(c) \right) dF(p) & if \quad s > \theta \end{cases}$$

□

The solution is given by

Proposition 2 *There is a unique optimal SM constitution*

$$(m^*_{SM}, s^*_{SM}, d^*_{SM}) = \left(\frac{y}{y+w}, \theta, \frac{z}{z+w} \right)$$

Proof. Consider the case in which $s \leq \theta$ first. Problem 2 can be rewritten as

$$\max_{m,d \geq m,s} \begin{bmatrix} \int_m^1 \left(\int_a^s (pw-(1-p)y)\, dG(c)\right) dF(p) \\ + \int_d^1 \left(\int_s^\theta (pw-(1-p)y)\, dG(c) + \int_\theta^b (pw-(1-p)z)\, dG(c)\right) dF(p) \end{bmatrix}$$

which, using integration by parts, becomes

$$\max_{m,d\geq m \in [0,1], s\in[a,b]} \begin{bmatrix} G(s) \left[(w+y)\left(1-mF(m)-\widetilde{F}(1-m)\right)-yF(1-m)\right] \\ +G(\theta-s)\left[(w+y)\left(1-dF(d)-\widetilde{F}(1-d)\right)-yF(1-d)\right] \\ +(1-G(\theta))\left[(w+z)\left(1-dF(d)-\widetilde{F}(1-d)\right)-zF(1-d)\right] \end{bmatrix}.$$

It is easy to see that the problem as a unique maximum for m at $\frac{y}{y+w}$. Consider now d. The FOC is

$$G(\theta-s)\left[y-d(w+y)\right]+(1-G(\theta))\left[z-d(w+z)\right]=0$$

which gives a unique solution

$$d(s)=\frac{G(\theta-s)\,y+(1-G(\theta))z}{G(\theta-s)(w+y)+(1-G(\theta))(w+z)}$$

while the SOC

$$-G(\theta-s)(w+y)-(1-G(\theta))(w+z)<0$$

shows that this solution is also unique. Consider now the first derivative with respect to s evaluated at the optimal m,d:

$$g(s)\begin{bmatrix} \left[(w+y)\left(1-\frac{y}{y+w}F(\frac{y}{y+w})-\widetilde{F}\left(1-\frac{y}{y+w}\right)\right)-yF\left(1-\frac{y}{y+w}\right)\right] \\ -\left[(w+y)\left(1-d(s)\,F(d(s))-\widetilde{F}(1-d(s))\right)-yF(1-d(s))\right] \end{bmatrix}$$

which is equal to

$$g(s)\left\{\int_{\frac{y}{y+w}}^{d(s)}(pw-(1-p)y)\,dF(p)\right\}$$

Now, $d(s)>\frac{y}{y+w}$ for any s and $(pw-(1-p)y)>0$ for any $p>\frac{y}{y+w}$ which means that the derivative in question is strictly positive. Thus the optimal $s=\theta$ and

$$d(\theta)=\frac{z}{w+z}$$

which proves our result for the case in which $s\leq\theta$. For the other case, an analogous proof applies giving us the desired result □

From the proposition above, the following is immediate.

Corollary 2 *The optimal SM constitution is ex-post efficient. Also*

$$EU\left(m_{SM}^*, s_{SM}^*, d_{SM}^*\right) > EU(m_{MR}^*)$$

Proof. The first statement follows immediately by noting that with the optimal SM constitution, A will obtain iff $p \geq \frac{y}{y+w}$ whenever $c \leq \theta$ while it will obtain iff $p \geq \frac{z}{z+w}$ whenever $c > \theta$. The second statement follows immediately by noting that $(m^*_{SM}, s^*_{SM}, d^*_{SM})$ is a unique solution to the maximization problem 2 while m^*_{MR} is equivalent to (m^*_{MR}, s, m^*_{MR}) which is sub-optimal in the problem 2 □

Figure 1 below shows expected utilities for the optimal SM and MR constitutions as a function of z.

Figure 1 here

As the figure clearly shows, expected utility with a majoritarian constitution, expected utility tends to zero as z increases. This is because m^*_{MR} tends to one as z goes to infinity. So with a majoritarian constitution, a large losses for A whenever $c > \theta$ make it very difficult to pass policy even when losses for the case in which $c \leq \theta$ are relatively small. On the other hand, supermajoritarian constitutions can distinguish between the two kinds of policies and only make it difficult to pass policies for which $c > \theta$. In fact is it immediate to see that

$$\lim_{z \to \infty} EU\left(m^*_{SM}, s^*_{SM}, d^*_{SM}\right) = EU\left(m^*_{SM}, s^*_{SM}, 1\right)$$

4.3. Discussion

The constitution presented above, rely on the observability and verifiability of the random variables p and c. In other words, if these mechanism are to be used, an independent court system has to be able to ascertain the realizations of p and c. The observability and verifiability of these variables is necessary because otherwise losers would always have an incentive to claim that p is smaller or c is greater than they actually whenever A should obtain while winners would have the incentive to do the opposite. A natural question that arises then, is to what extent observability and verifiability are possible in reality. Democratic societies have developed, through elections and referenda, a fairly reliable way of determining whether a certain majority of society is in favor of certain policies or not. In the case of elections, different electoral rules are more or less capable of representing society's true preferences, but the choice of less reliable electoral rules (such as majoritarian rules) can be ascribed to reasons outside of the simple setup of this paper: for example, it is often claimed that majoritarian electoral rules can generate more executive stability in parliamentary systems because of Duverger's law.[4] Thus, at a fundamental level, the assumption that p is verifiable in a democracy, is quite mild.

[4]According to Duverger's law, first-past-the-post systems, tend to generate two-party legislatures.

With respect to c, however, things are quite different. In our setup, one interpretation is that c controls individuals' utilities in case A is passed so that verifiability would require that these individuals (the losers in our setup) be able to provide verifiable information about their utility. In many cases, this is not a realistic assumption so that only constitutions that rely on p, such as the constitution m^*_{MR} can be designed. Indeed much of the most recent literature (e.g. [1], [2], [7]) has focused on constitutions as majority rules for this reason.

While we agree with the basic premise that it is indeed impossible to design constitutions where majority rules are designed in accordance with the utilities of the winners and losers for each policy, we believe that there are, for example, policy areas where the cost of losing is always especially significant. Thus, we argue that to the extent that some policy areas are clearly defined (and thus observable and verifiable), special majority rules that pertain to them can be specified in the constitution.

As an illustration, consider the following simple extension of our setup. Imagine that alternative policies $A(c)$ differ by the dis-utility that they give to losers while winners always get w and B always give zero utility to everyone. So

$$u\left(i \in W^C, A, c\right) = \begin{cases} -y_1 & if \quad c \leq \theta_1 \\ -y_2 & \quad \theta_1 < c \leq \theta_2 \\ -y_3 & \quad \theta_2 < c \end{cases}$$

where $a < \theta_1 < \theta_2 < b$ and $y_1 < y_2 < y_3$. Suppose further that it is possible to verify whether $c > \theta_2$ or not but that it isn't possible to verify whether $\theta_1 < c \leq \theta_2$ or $c \leq \theta_1$. For example, if $c \leq \theta_1$, then we have some fiscal policy, if $\theta_1 < c \leq \theta_2$ we have some different fiscal policy, while if $c > \theta_2$, the policy limits some fundamental freedoms. While it may be very difficult to determine ex-ante which of the fiscal policies will generate losses $-y_1$ and which $-y_2$, it is clearly much easier to assume that more losses will occur to any individual who loses some fundamental liberty. Thus, an optimal constitutional response to this problem would be to have a constitution

$$(m, s, d) = \left(\frac{G(\theta_2)\, y_2 - G(\theta_1)\,(y_2 - y_1)}{G(\theta_2)\,(w + y_2) - G(\theta_1)\,(y_2 - y_1)}, \theta_2, \frac{y_3}{y_3 + w}\right)$$

where a majority rule that takes the fiscal policies into account coexists with a supermajority rule for dealing with fundamental freedoms.[5]

Actual constitutions seem to map well with our argument: while there is a majority of policy areas for which it is difficult to identify ex-ante which will provide smaller or greater losses, there are policy areas for which gains and losses are easily identifiable as being greater or smaller than usual and for which special decision rules are devised. The

[5]The proof of this result follows immediately from the proofs of propositions 1 and 2.

advantages of such constitutions are clear: if in some society some policy areas are specifically delicate and these policy areas are easily to identify and verifiable, supermajoritarian constitutions allow for the creation of special policy making rules for those areas without necessarily making all policy-making decisions difficult.

[insert some examples from real constitutions here].

5. Optimal Constitutional Change

In this section we slightly modify our previous setup to allow for the possibility that a crisis arises if the probability that a high-loss policy can be implemented is large enough. As in the previous section, we investigate possible constitutions in this setting and we show that if the probability that a crisis obtains is large enough, constitutions with constitutional amendments can be optimal.

Our original setup is modified as follows: after the constitutional decision is made but before policies are implemented, nature selects a state of nature $\omega \in \{\overline{\omega}, \underline{\omega}\}$ with $\gamma = \Pr(\omega = \overline{\omega})$. We interpret the state of nature $\underline{\omega}$ as meaning that there is no possibility of a crisis, in which case everything is as in the previous section, while if $\overline{\omega}$ obtains, there is a possibility of a crisis. Specifically, we assume that if $\overline{\omega}$ obtains then there exists a value ψ with $\theta \leq \psi \leq b$ such that if with the constitution that is used to determine policy, there is positive probability that a policy A obtains with $c > \psi$, then losers will choose to have a constitutional crisis. If this occurs, the crisis is successful and the outcome is B with probability $(1 - \alpha)$ and is not successful and the outcome is decided by the constitutional rules with probability α. We will discuss this setup further below.

The timeline, then, is as follows:

1. Constitution is chosen behind veil of ignorance
2. Nature selects ω, which is observed by everyone and p.
3. Given p, the sets W and W^C are determined.
4. If $\omega = \overline{\omega}$ and the current constitution gives a positive probability to a policy with $c > \psi$ to obtain, then we have a crisis: nature determines if it is successful. If it is, then B obtains and the process ends. If it isn't successful, then we go to step 4.
5. Nature selects c which is observed by everyone.
6. Individuals vote for or against A. The result of the vote is observed by everyone.
7. The policy x is determined according to the constitution.[6]

[6] As mentioned before, the fact that here c is selected after nature determines the sets W and W^C would also work in the framework of section 3, so that there is no inconsistency between the two setups.

The timing here is more crucial than in the previous setup. While the collective decision problem we describe is literally a one-shot problem, real constitutions have to deal with many policy decisions on a daily basis. What this timeline represents is, in reduced form, a dynamic setup where once the constitution is chosen, several policy decision are made that constitute a specific program so that what we term policies A and B should be interpreted as a bundle of decisions which may differ in their cost to losers (that is, may differ because of their value c) but generate the same set of winners and losers. Thus, the interpretation is that if $c > \theta$, the program will include policies which are particularly costly to losers. The assumption that c is revealed after W and W^C are determined only means that the exact details of this program are unknown at the time in which people know whether the program will favor them or not. In this context, our interpretation of ω is that it captures the notion that losers may or may not be in a position to resist the enactment of a program which can potentially be disastrous for them. Implicit in this interpretation is the argument that in any society the relative strength of different sides in not necessary captured just by p (how many individuals are on each side) but may be distorted in one way or another. For example, we could have that while a certain side has the numbers to enact a certain program of policies, the other side has particularly strong interest groups or even a credible threat of recourse to violence that allows them to threaten a crisis. If one such case occurs, i.e. if $\omega = \overline{\omega}$, then we capture the extent to which this is a real threat through the exogenous parameters ψ and α. The first measures how losers trade-off the risk of getting a really bad policy as opposed to benefits of a crisis while the second is an inverse measure of the probability of success of a crisis.[7]

The main point of the whole setup, however, is that ω will generally not be verifiable. While each side's relative size can be measured by p through mechanisms such referenda or elections, what makes for the actual strength of each side and its ability to threaten each other will be perhaps easy to recognize but cannot be easily described and cannot be therefore be part of a constitution. *[include examples such as the vulnerability of French governments to strikes here]*

5.1. Optimal Constitutions when ω is verifiable

Situations of instability, which is what ω captures here, while easily observable, will mostly be unverifiable. However, in this subsection, we study the optimal constitution when ω is verifiable to provide a benchmark for the analysis of the unverifiable case below. We also

[7] In the model, we assume that p and ω are drawn independently by nature and we also impose no connection between α and ψ. These assumptions, given the discussion above, are not particularly realistic: for example, it would be reasonable to assume that p and ω be negatively correlated because sheers numbers *are* important in democracies. However, this would complicate the model excessively and would not change our results in a significant way.

assume that c is verifiable. In order to proceed we need to slightly generalize our definition of SM constitutions:

Definition 4 *A Generalized Supermajority-Rule (GSM) constitution is a quadruple* (m, s, d, t) *with a majority rule* $m \in [0,1]$, *a supermajority rule* $d \in [m, 1]$ *and veto rules* $s \in [a, b]$, $t \in [s, b]$ *such that*

$$x = \begin{cases} A & if \quad ((p \geq m) \wedge (c \leq s)) \vee ((p \geq d) \wedge (c \leq t)) \\ B & \qquad\qquad otherwise \end{cases}$$

Thus, we slightly changed the definition to allow for an additional veto rule even when $p \geq d$. This generalization was redundant in our previous setup since whenever the chances of a crisis are zero, it is always optimal to set $t = b$. Verifiability implies that a $\omega-$dependent constitution can be designed for each of the possible realization of ω so that the problem faced behind the veil of ignorance is

Problem 3 If $\omega = \underline{\omega}$ then the problem is[8]

$$\max_{m,s,d,t} EU(m, s, d, t)$$

If $\omega = \overline{\omega}$ then the problem is

$$\max_{m,s,d,t} \begin{cases} EU(m, s, d, t) & if \quad (s \wedge t) \leq \psi \\ \alpha EU(m, s, d, t) & if \quad (s \vee t) > \psi \end{cases}$$

□

To save space we will now use the notation:

$$\Delta = G(\theta) \left[(w + y) \widetilde{F} \left(\frac{w}{w + y} \right) - (w + z) \widetilde{F} \left(\frac{w}{w + z} \right) \right]$$

and

$$\Lambda = w - (w + z) \widetilde{F} \left(\frac{w}{w + z} \right)$$

The following proposition follows immediately:

Proposition 3 *The unique optimal GSM constitution when* ω *is verifiable is*

$$(m_V^*, s_V^*, d_V^*, t_V^*)(\omega) = \begin{cases} \left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, b \right) & if \qquad \omega = \underline{\omega} \\ \left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, b \right) & if \quad (\omega = \overline{\omega}) \wedge (\alpha \geq \overline{\alpha}) \\ \left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, \psi \right) & if \quad (\omega = \overline{\omega}) \wedge (\alpha < \overline{\alpha}) \end{cases}$$

where

$$\overline{\alpha} = \frac{G(\psi) \Lambda - \Delta}{\Lambda - \Delta}$$

[8] $EU(m, s, d, t)$ is too cumbersome to write extensively here but it is a natural extension of the expression described in problem 2.

Proof. The statement with regard to the case $\omega = \underline{\omega}$ is straightforward as it follows the pattern developed in the proof of proposition 2. Consider now the case $\omega = \overline{\omega}$. The nature of the problem implies that either a constitution guarantees no crisis or a successful crisis will happen with probability $(1-\alpha)$. Thus the optimal constitution that guarantees no crisis is one where the problem is exactly the same as problem 2 *but* ψ replaces b. This gives us $\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, \psi\right)$ as a unique solution. On the other hand, the best constitution that will trigger a successful crisis with probability $(1-\alpha)$ is again, uniquely, the standard constitution because any other constitution that triggers a crisis will have the resulting expected utility from A multiplied by α. So we have that we either choose to receive $EU\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, \psi\right)$ or $\alpha EU\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, b\right)$. Now then, $\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, b\right)$ will be chosen iff

$$\alpha \geq \frac{EU\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, \psi\right)}{EU\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, b\right)} = \frac{G(\psi)\Lambda - \Delta}{\Lambda - \Delta}$$

which is what we were looking for □

This is a clear benchmark for us because it tells us what the optimal constitution is when the flexibility to adjust the constitution for different realizations of ω exists and this from the perspective of the original position which implies that this is the decision that a social planner will make. In particular, the cut-off point $\overline{\alpha}$ indicates the point at which, conditional on $\overline{\omega}$ obtaining, it is optimal for the constitutional designer to switch from one possibility to the other. If ω in not verifiable, then clearly this trade-off cannot be made conditional on $\overline{\omega}$ obtaining.

5.2. Optimal Constitutions when ω is not verifiable

The clear consequence of unverifiability of ω is that a ω-contingent constitution is not available any more. This means that constitutional designer actually face the following problem:

Problem 4 Let $EU(m,s,d,t)$ the expected utility for a GSM constitution. Then the problem is

$$\max_{m,s,d,t} \begin{cases} EU(m,s,d,t) & if \quad (s \wedge t) \leq \psi \\ (\alpha\gamma + (1-\gamma)) EU(m,s,d,t) & if \quad (s \vee t) > \psi \end{cases}$$

□

A simple comparison of problem 3 with problem 4 already shows intuitively that the difference between the two problems is not just that the optimal constitution cannot be made contingent on ω but also that conditional on $\overline{\omega}$ obtaining, the trade-off between choosing a "safe" constitution with $t = \psi$ and a "risky" one with $t = b$ is different. Indeed we have:

Proposition 4 *The unique optimal GSM constitution when ω is not verifiable is*

$$(m^*_{UV}, s^*_{UV}, d^*_{UV}, t^*_{UV}) = \begin{cases} \left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, b\right) & if \quad (\alpha \geq \overline{\overline{\alpha}}) \\ \left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, \psi\right) & if \quad (\alpha < \overline{\overline{\alpha}}) \end{cases}$$

where

$$\overline{\overline{\alpha}} = 1 - \frac{(1 - G(\psi))\Lambda}{\gamma(\Lambda - \Delta)} < \overline{\alpha}$$

Proof. Just as we argued in the proof of proposition 3, the unique optimal constitution that guarantees no crisis is $\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, \psi\right)$ while the unique optimal constitution that allows for the probability of a successful crisis is $\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, b\right)$. Thus , from the original position, we have that we either choose the former option which gives us $EU\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, \psi\right)$ or the latter which gives us $EU\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, b\right)$ if either $\underline{\omega}$ obtains (which occurs with probability $(1-\gamma)$) or if $\overline{\omega}$ obtains but the crisis is not successful (which happens with probability $\alpha\gamma$). Now then, $\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, b\right)$ will be chosen iff

$$\alpha\gamma + (1-\gamma) \geq \frac{EU\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, \psi\right)}{EU\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, b\right)} \Leftrightarrow \alpha \geq 1 - \frac{(1 - G(\psi))\Lambda}{\gamma(\Lambda - \Delta)}$$

Now to show that $\overline{\overline{\alpha}} < \overline{\alpha}$ we first notice that

$$\overline{\alpha} - \overline{\overline{\alpha}} = \frac{\Lambda(1-\gamma)(1 - G(\psi))}{\gamma(\Lambda - \Delta)}$$

where we need to study the sign of Λ and $\Lambda - \Delta$. Consider Λ first. Note that

$$\Lambda > 0 \Leftrightarrow \frac{w}{w+z} > \widetilde{F}\left(\frac{w}{w+z}\right)$$

which is always true. Since

$$\Lambda - \Delta = EU\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, b\right)$$

and we know the latter is ex-ante efficient, then $\Lambda - \Delta$ is also strictly positive which completes the proof $\square$

Proposition 4 highlights the nature of the problem created by unverifiability: whenever $\alpha < \overline{\overline{\alpha}}$, $\left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, \psi\right)$ will be chosen, regardless of the realization of ω. Now, since $\alpha < \overline{\alpha}$ as well, there is no loss in case $\overline{\omega}$ obtains, (as in both cases $t = \psi$) but if $\underline{\omega}$ obtains, then there is a loss because the UV constitution still imposes $t^*_{UV} = \psi$ whereas the impossibility of a crisis makes it optimal to set $t = b$ which is what happens with t_V^*. Whenever $\alpha \in \left[\overline{\overline{\alpha}}, \overline{\alpha}\right)$ we have the opposite problem: there is no loss whenever $\underline{\omega}$ obtains (as both constitutions set optimally $t = b$) but there is a loss if $\overline{\omega}$ obtains because the UV constitution imposes $t_{UV}^* = b$ even though the probability of a successful crisis is now relatively high and $t_V^* = \psi$. As we've mentioned before, this occurs because it is impossible for the constitutional designer to replicate the verifiable case as it doesn't know for sure what the realization of ω is. This analysis leads us to ask whether we can find constitutions of a different nature that can help bridge the gap imposed by unverifiability. We argue below that constitutions that allow for constitutional amendment can provide a (partial) answer.

5.3. Optimal Constitutions with Amendment Rules

The main problem highlighted by the analysis above is that unverifiability leads to a lack of flexibility in the constitution which cannot be adjusted depending on the realization of ω. One way out of the problem would be to allow the constitution to be renegotiated between all individuals after ω is observed. This is not possible in our setup, however, because individuals already know whether they are winners or losers and have contrasting incentives in the constitution to be designed so that unanimity cannot be achieved. The best we can do is to give one of the sides a monopoly decision over a new constitution, *provided* this side represents enough individuals. In other words, we propose an amendment rule. Note that this adds a few steps between step 3 and step 4 in our timeline as follows:

3a. Winners propose a (possibly) new constitution to the original constitution chosen in period 1.

3b. If the proposal differs from the original constitutions, individuals vote for or against the proposed change.

3c. The amendment rule in the old constitution determines whether the new constitution is implemented or not.

Formally, a constitution with these features is then

Definition 5 *An Amendable (AM) constitution is a triplet (m, s, r) with a majority rule $m \in [0, 1]$, a veto rule $s \in [a, b]$, and an amendment rule $r \in [m, 1]$ such that if*

$$p \geq r \quad then \quad x = \begin{cases} A & if \quad ((p \geq \overline{m}) \wedge (c \leq \overline{s})) \\ B & \qquad otherwise \end{cases}$$

while if

$$p < r \quad then \quad x = \begin{cases} A & if \quad ((p \geq m) \wedge (c \leq s)) \\ B & \qquad otherwise \end{cases}$$

where $\overline{m}$ and $\overline{s}$ are chosen by a representative winner.

The definition highlights two extremely important things: the first is that this is not a ω-dependent mechanism. This is crucial because, as we shall see in detail below, AM constitutions are in general *not* outcome-equivalent to any GSM constitution unless ω is verifiable. This then provides a normative justification for amendment rules in the realistic case in which ω is unverifiable, provided AM constitutions outperform GSM constitutions. As we shall see, this turn out to be indeed the case. Also note that AM constitutions take the notion of constitutional amendment seriously, in the sense that the amendment rule does not have policy implications directly but *only* through an actual change in the constitutional rules. This is precisely why GSM constitutions and AM constitutions generate, in general, different outcomes.

The problem for this particular case can be written as

Problem 5 Let

$$(\overline{m}, \overline{s}) = \arg\max_{\overline{m} \leq r, \overline{s}} \begin{cases} G(\overline{s})\, w & if \quad \overline{s} \leq \psi \\ \alpha G(\overline{s})\, w & if \quad \overline{s} > \psi \end{cases}$$

Also, let $EU(m, s, 1, b)$ the expected utility for a GSM constitution. Then the problem is

$$\max_{m,s,r} \left[(1-r)\, EU(\overline{m}, \overline{s}, 1, b) + rEU(m, s, 1, b)\right]$$

□

The problem says that if $p \geq r$, then winners have a free hand in determining a new majority and veto rule. We assume that, knowing that $p \geq r$ whenever they do get to choose, they will never choose a $\overline{m} > r$ thus risking to lose A. Given that, they will choose a veto rule $\overline{s}$ where the only effective constraint is that for $\overline{s} > \psi$ a crisis might be triggered. At time 1., given this behavior, a designer in the original position then has to choose (m, s, r).

Given this, we have:

Proposition 5 *The unique optimal AM constitution is*

$$(m^*_{AM}, s^*_{AM}, r^*_{AM}) = \left(\frac{y}{y+w}, \theta, \frac{z}{z+w}\right)$$

with[9]

$$\overline{m}^* \leq r^*_{AM}$$

and

$$\overline{s}^* = \begin{cases} b & if \quad (\omega = \underline{\omega}) \vee ((\omega = \overline{\omega}) \wedge (\alpha \geq \widetilde{\alpha})) \\ \psi & if \quad (\omega = \overline{\omega}) \wedge (\alpha < \widetilde{\alpha}) \end{cases}$$

[9] While the AM optimal constitution is unique, all values of $\overline{m} \leq r^*_{AM}$ are optimal for reasons we have already discussed.

where

$$\widetilde{\alpha} = G(\psi)$$

Proof. Consider the optimal behavior for winners conditional on $p \geq r$. Given the chosen r they will set any $\overline{m}$ that satisfies the constraint $\overline{m} \leq r$ and set $\overline{s} = \psi$ if $\omega = \overline{\omega}$ (which they can observe) and $\alpha G(b)\, w = \alpha w < G(\psi)\, w$ and $\overline{s} = b$ in all other cases. This is because if $\omega = \underline{\omega}$ there can be no crisis and the probability of getting w is maximized by setting $\overline{s} = b$. If $\omega = \overline{\omega}$ setting $\overline{s} = b$ will still be the optimal way to maximize the chance of getting w except that the then a crisis is triggered and so payoffs only obtain with probability α. The best way to avoid a crisis is then to set $\overline{s} = \psi$ but that of course only gets w with probability $G(\psi)$. Hence the optimal choice $\overline{s}^*$.

Given this, at time 1, the problem is equivalent to maximizing the following problem.

If $\omega = \underline{\omega}$ then

$$\max_{m,s,r} EU(m, s, r, b)$$

If $\omega = \overline{\omega}$ then the problem is

$$\max_{m,s,r} EU(m, s, r, b)$$

when $\alpha \geq \widetilde{\alpha}$ and

$$\max_{m,s,r} EU(m, s, r, \psi)$$

otherwise. But these maximization problems are all identical to problem 2 with the exception of the last one which just differs by the fact that ψ replaces b in the problem. Thus, applying the proof in proposition 2, we have the desired result $\square$

It is easy to see that the optimal AM constitution is equivalent to the ω-dependent GSM constitution

$$(m_{EQ}, s_{EQ}, d_{EQ}, t_{EQ})(\omega) = \begin{cases} \left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, b\right) & if \quad \omega = \underline{\omega} \\ \left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, b\right) & if \quad (\omega = \overline{\omega}) \wedge (\alpha \geq \widetilde{\alpha}) \\ \left(\frac{y}{y+w}, \theta, \frac{z}{z+w}, \psi\right) & if \quad (\omega = \overline{\omega}) \wedge (\alpha < \widetilde{\alpha}) \end{cases}$$

which differs from the optimal ω-dependent GSM constitution only because the cut-off value $\overline{\alpha}$ is replaced by $\widetilde{\alpha}$. The following corollary then allows us to provide some comparisons:

Corollary 3 *We have that*

$$\widetilde{\alpha} - \overline{\alpha} = \frac{\Delta(1 - G(\psi))}{\Lambda - \Delta} > 0$$

Further

$$\frac{\partial\left(\widetilde{\alpha}-\overline{\alpha}\right)}{\partial\psi},\frac{\partial\left(\overline{\alpha}-\overline{\overline{\alpha}}\right)}{\partial\psi},\frac{\partial\left(\widetilde{\alpha}-\overline{\alpha}\right)}{\partial y},\frac{\partial\left(\overline{\alpha}-\overline{\overline{\alpha}}\right)}{\partial y},\frac{\partial\left(\overline{\alpha}-\overline{\overline{\alpha}}\right)}{\partial\gamma} < 0$$

$$\frac{\partial\left(\widetilde{\alpha}-\overline{\alpha}\right)}{\partial\theta},\frac{\partial\left(\overline{\alpha}-\overline{\overline{\alpha}}\right)}{\partial\theta},\frac{\partial\left(\widetilde{\alpha}-\overline{\alpha}\right)}{\partial z},\frac{\partial\left(\overline{\alpha}-\overline{\overline{\alpha}}\right)}{\partial z} > 0$$

Proof. The first statement follows immediately by showing that $\Delta > 0$. The sign of Δ is then determined by the sign of the expression

$$(w+y)\,\widetilde{F}\left(\frac{w}{w+y}\right)-(w+z)\,\widetilde{F}\left(\frac{w}{w+z}\right)$$

which is non-negative iff

$$\frac{\frac{w}{w+y}}{\frac{w}{w+z}} \leq \frac{\widetilde{F}\left(\frac{w}{w+y}\right)}{\widetilde{F}\left(\frac{w}{w+z}\right)}$$

To prove that this is always true, we need to check under which conditions the following relation holds between two values $x_1 < x_2$ in the support of F :

$$\frac{x_2}{x_1} > \frac{\widetilde{F}\left(x_2\right)}{\widetilde{F}\left(x_1\right)}$$

Let $\Omega = \widetilde{F}\left(x_1\right)$ and assume that $F\left(x_2\right) = F(x_1)$. Then

$$\widetilde{F}\left(x_2\right) = \Omega + F(x_1)\left(x_2 - x_1\right)$$

which implies

$$\frac{x_2}{x_1} > \frac{\widetilde{F}\left(x_2\right)}{\widetilde{F}\left(x_1\right)} \Leftrightarrow \Omega > x_1 F(x_1)$$

which is impossible. If $F\left(x_2\right) > F(x_1)$, then the situation above is modified only by the fact that

$$\widetilde{F}\left(x_2\right) > \Omega + F(x_1)\left(x_2 - x_1\right)$$

which means that the impossibility described above is further reinforced. So, Δ is always positive. The comparative statics results obtain by differentiating the relevant expressions *[Note: we have the actual values of these derivatives, so that we also know their relative size: should we add this? Also, comparative statics with respect to w are to do].* $\square$

The most important result of this subsection is that even AM constitutions cannot fully replicate the optimal ω-dependent GSM constitution and this is simply because the former has the same flexibility of the latter but that comes at the cost of allowing just winners, not a social planner, to modify the constitution. Equally interesting is the fact that the bias goes in the opposite direction than that generated by the optimal UV constitution. The bias can be explained by the fact that different forces are at work. With UV constitutions, the decision to set $t = b$ comes at too low a level of α because there is uncertainty as to whether $\overline{\omega}$ obtains or not. In other words, if the decision to switch from $t = \psi$ to $t = b$ was done at $\overline{\alpha}$ instead of $\overline{\overline{\alpha}}$, the alignment with the V constitution conditional on $\overline{\omega}$ obtaining would be more than negatively compensated by the fact that in case $\underline{\omega}$ obtains, the constitution is suboptimal for a larger range of values of α. With AM constitutions, instead, this problem is completely resolved but as just mentioned another problem comes in in that while the constitution is flexible to the different realizations of ω, that flexibility puts the decision in the hands of winners, not social planners. Winners are biased in the sense that they are more keen to obtain A than individuals in the original position are. This means that they take the threat of a successful crisis more seriously and are therefore more reluctant to choose $\overline{s} = b$ for a given value of α than individuals in the original position are to set $t = b$ in a V constitution. *[add comparative statics discussion].*

5.4. Discussion

In this section we extended the model of the previous section to allow for the possibility that unverifiable events occur between the time the constitution is chosen and policy decisions have to be made. What we wished to represent is the notion that in reality, A is a program supported by a group of individuals which consists of a bundle of alternatives to the status quo. In this context, c represents the event in which the program will have policies that are particularly negative for the opponents of the program. The unverifiable events that matter here are that, in contrast to the previous section, there is a possibility that losers from the enactment of the program have the power to stop the constitutional process.

Thus, we investigated what the optimal response to this problem might be. We have shown that a fundamental trade-off exists between the benefit of having enough flexibility to respond to changes in ω and the identity of the designer of the constitution. Specifically, UV constitution are chosen in the original position so that they are representative of society's choices as a whole but do not have the flexibility to respond to changes in ω. The alternative is to have constitutions with amendment rules. These do provide the required flexibility because they can be changed in response to changes in ω but this comes at the cost of having to let one specific side, the winners, make the changes so that the new constitution is different from the constitution that would be decided in the original position (i.e. a constitution V).

Note however, that if α was known ex-ante, i.e. at time 1., a constitutional designer could solve the unverifiability problem entirely by choosing a UV constitution if $\alpha \geq \overline{\alpha}$ and an AM constitution if $\alpha < \overline{\alpha}$ because UV constitutions only create problems only if

$\alpha < \overline{\alpha}$ while AM constitutions only create problems if $\alpha \in [\overline{\alpha}, \widetilde{\alpha})$.[10] This gives us some sort of “comparative statics" for constitutional structures that map well with our intuition: if α is high the possibility of a successful crisis is low which means that the cost of the rigidity that comes with UV constitutions is zero because whatever the realization of ω the optimal constitution is the same. On the other hand, with AM constitutions, the cost of the disalignment of preferences between winners and individuals in the original position might still be there even if α is relatively high (i.e. if $\alpha \in [\overline{\alpha}, \widetilde{\alpha})$) because winners are more concerned about crises than individuals in the original position. If α is low the costs of rigidity that come with UV constitutions are high because the difference between the case $\omega = \overline{\omega}$ and $\omega = \underline{\omega}$ is now significant. This means that now flexibility is more important and AM constitutions perform better. The resulting message of all this is that constitutions should be more flexible if constitutional designers expect future instability to be particularly high.

One more point is worth mentioning: the analysis of this section crucially hinges on the notion that c is verifiable, which, we argued in the previous section, cannot always be taken for granted. If c is not verifiable, we've shown that only MR constitutions are possible. But then, the choice of an amendment rules becomes meaningless: any rule r that allowed a majority rule m to be changed, would be have to be equal to m itself and therefore redundant. To see that note that if $r < m$, then m is redundant because whenever $p \geq r$ then m can be changed by winners to r thus making r the effective majority rule. On the other hand, with a MR constitution, A obtains whenever $p \geq m$ so that any rule $r > m$ would be redundant: the only reason for winners to use r would be to reduce m in order to make A easier but if $p \geq r$ is satisfied, then $p > m$ as well and so that there is never a need for r.

What this means is that constitutional amendments, as mechanisms to modify other constitutional rules, make no sense if these other rules only depend on the realization of p because constitutional amendments are mechanisms that depend on p themselves. To make sense of constitutional amendment you need to have constitutions which include rules which depend on the realization of other variables (which have to be observable and verifiable). In our simple setup, c is such a variable and many more may be described by more complex settings, but the main point is that by describing constitutions as mechanisms that depend only on realization of p (as much of the recent literature does), a meaningful theory of constitutional change cannot be provided.

[We might want to have one more point related to this is the confusion in much of the literature between supermajority and amendment rules (e.g. Boudroux and Prichard) which may be explained by the fact that for $\alpha = 1$ (i.e. effectively the setup in section 4),

[10] If $\alpha \geq \widetilde{\alpha}$ then V, UV and AM constitutions are the same so the choice of constitution between UV and AM is in any case optimal and therefore irrelevant.

AM and SM constitutions are equivalent]

6. Optimal Constitutions with Heterogeneity

The recent literature in political economics has put a significant emphasis on the importance of checks and balances as an instrument to reduce the agency problem between politicians and citizens (see especially [8] for a fundamental contribution in this regard). In this section we again slightly modify our original setup to study another sort of checks and balances which do not arise from the agency problem mentioned above but from possible heterogeneities between individuals that arise at a interim stage, after the constitution has been chosen but before p and c are determined. To clarify things, note that we can conceivably have heterogeneity between individuals at the ex-ante stage, at an interim stage, and at the ex-post stage. In sections 3 to 5, we dealt with a setup where we had ex-post heterogeneity, in the sense that heterogeneity between individuals only came about when nature determined whether individuals were winners or losers.

Here, instead, we deal with the case in which *before* nature determines winners and losers, but *after* a constitution is chosen, nature makes a first selection of individuals in two categories. Specifically, we will look at two different possibilities:

i. Nature determines that any eventual loser, conditional on $c > \theta$ obtaining, will get losses $-z_1$ with some probability and losses $-z_2$ with complementary probability.

ii. Nature determines that for any individual there is a probability that the distribution function for p is $F_1(p)$ and a complementary probability that the distribution function for p is $F_2(p)$.

The interpretation for case i. is that from the original position perspective individuals do not know whether they will be winners or losers but they know that, conditional on $c > \theta$ obtaining, some losers will have losses $-z_1$ and the rest losses $-z_2$. The interpretation for case ii. is that from the original position perspective, individuals know that their probability of being winners or losers is determines can be determined by one of two different distributions. The corresponding notions of ex-ante heterogeneity are:

iii. Before the constitution is chosen, it is common knowledge that conditional on $c > \theta$ obtaining, some individuals will get losses $-z_1$ if they turn out to be losers while the other individuals will get losses $-z_2$ in the corresponding case

iv. Before the constitution is chosen, it is common knowledge that the distribution function for p is $F_1(p)$ for some individuals and that it is $F_2(p)$ for the others.

Ideally, we would like to be able to study cases iii. and iv. but if we did, we could not use the original position device. Thus, we study cases i. and ii., where instead of knowing for sure in which of the groups they belong to before the constitution is chosen, individuals only know that they belong to either of them with some probability so that we still have ex-ante homogeneity. This still allows us to draw parallels with cases iii. and iv. For example, in the analysis below, we will show that if there is enough heterogeneity, checks and balances constitutions will be preferable to simple constitutions because they provide protection for the most disadvantaged group and in the original position individuals want to make sure the constitution does not penalize them too much in case they belong to the disadvantaged group. This is analogous to saying that in cases iii. or iv., members of disadvantaged groups accept to participate in the constitution for a whole society only if the constitution provides them enough checks and balances. We will reconsider these parallels at the end of the section.

Given this setup, we assume that simple constitutions and constitutions with checks and balances are available. With the former, the constitutional rules apply to the whole of society while with the latter group-specific rules are available. As with the previous sections, the notion of verifiability is crucial. If the heterogeneity was not verifiable, then only simple constitutions would be available. We assume that heterogeneity is captured by the random variable h: for each individual nature independently selects a state of nature $h \in \left\{\overline{h}, \underline{h}\right\}$ with $\Pr\left(h = \overline{h}\right) = q$ and given the discussion above, we assume that as soon as h becomes observable, it is also verifiable.

With these assumptions, we have a timeline identical to our original timeline except for the fact that h is also selected and before p is determined:

1. Constitution is chosen behind veil of ignorance
2. Nature selects h which is observable by everyone
3. Nature selects c and p where the former is observable by everyone.
4. Given p, the sets W and W^C are determined.
5. Individuals vote for or against A. The result of the vote is observed by everyone.
6. The policy x is determined according to the constitution.

In the rest of the section, we will study simple vs. checks and balances constitutions in this setup for 3 different scenarios:

I. The particular form of case i. in which

$$u\left(i \in W^C, A, \overline{c} > \theta, h\right) = \begin{cases} -z_1 & if \quad h = \overline{h} \\ -z_2 & if \quad h = \underline{h} \end{cases}$$

$\Pr\left(h = \overline{h}\right) = \frac{1}{2}$ for both c verifiable and unverifiable.

II. The same case but $\Pr\left(h = \overline{h}\right) = q \neq \frac{1}{2}$ and c is verifiable

III. Case ii. in which

$$\begin{aligned} \text{if } h &= \overline{h} \text{ then } F_i(p) = F_1(p) \\ \text{if } h &= \underline{h} \text{ then } F_i(p) = F_2(p) \end{aligned}$$

for some specific distributions $F_1(p)$ and $F_2(p)$ and c is verifiable.

Note that as with p, given that there is a continuum of individuals and that draws for h are independent, q represents the fraction of individuals in society for which $h = \overline{h}$.

6.1. Heterogeneity in Costs with Symmetric Groups

We now consider the first of our scenarios as described above. We assume w.l.o.g. that $z_1 < z_2$ so that individuals for which $h = \underline{h}$ will be disadvantaged compared to individuals for whom $h = \overline{h}$. The verifiability of the variable h allows us to consider the random variable p_1, the proportion of winners amongst the advantaged and p_2, the proportion of winners amongst the disadvantaged.

One important change we also have to make is that in order to provide solutions in closed form, we need to make specific assumptions on the distributions $F(p)$ and $G(c)$. In particular we assume they are both uniforms on $[0, 1]$ and $[a, b]$ respectively. Finally, just to keep the algebra simple, we let $w = y$. We start with the case in which c is not verifiable:

Definition 6 *A Simple Majority-Rule (SMR) constitution is a number $m \in [0, 1]$ such that*

$$x = \begin{cases} A & if \quad p_1 + p_2 \geq 2m \\ B & otherwise \end{cases}$$

A Majority-rule constitution with Checks and Balances (CBMR) is a pair $(m_1, m_2) \in [0, 1]^2$ such that

$$x = \begin{cases} A & if \quad (p_1 \geq m_1) \wedge (p_2 \geq m_2) \\ B & otherwise \end{cases}$$

The difference between the two constitutions is clear enough: with a SMR constitution, all that matters is that on average society be in favor (or not) of A while with checks and balances, we need that A achieve a majority in both groups. Given these definitions then, our problems become:

Problem 6 Define the following objects:

$$\begin{aligned} u_i &= p_i w - (1-p_i)w \\ v_i &= p_i w - (1-p_i)z_i \\ E_i(p_i, z_i) &= \int_a^\theta \frac{u_i}{b-a}dc + \int_\theta^b \frac{v_i}{b-a}dc \end{aligned}$$

Then for a SMR constitution the problem becomes

$$\max_m EU(m) = \begin{cases} \frac{1}{2}\left[\int_{2m-1}^1 \int_{2m-p_2}^1 E_1(p_1,z_1)dp_1dp_2 + \int_{2m-1}^1 \int_{2m-p_1}^1 E_2(p_2,z_2)dp_2dp_1\right] & if \quad m \geq \frac{1}{2} \\ \left[\begin{array}{l} \frac{1}{2}\left[\int_{2m}^1 \int_0^1 E_1(p_1,z_1)dp_1dp_2 + \int_0^{2m} \int_{2m-p_2}^1 E_1(p_1,z_1)dp_1dp_2\right] \\ +\frac{1}{2}\left[\int_{2m}^1 \int_0^1 E_2(p_2,z_2)dp_2dp_1 + \int_0^{2m} \int_{2m-p_1}^1 E_2(p_2,z_2)dp_2dp_1\right] \end{array}\right] & if \quad m < \frac{1}{2} \end{cases}$$

While for a CBMR constitution the problem becomes

$$\max_{m_1,m_2} EU(m_1,m_2) = \frac{1}{2}\left[\int_{m_2}^1 \int_{m_1}^1 E_1(p_1,z_1)dp_1dp_2 + \int_{m_1}^1 \int_{m_2}^1 E_2(p_2,z_2)dp_2dp_1\right]$$

□

The problem above leads immediately to the following:

Proposition 6 *There is a unique optimal SMR constitution*

$$m^*_{SMR} = \frac{(z_1+z_2)(b-\theta) + 2w(\theta-a)}{(z_1+z_2)(b-\theta) + 2w(b+\theta-2a)}$$

with $m^*_{SMR} \in \left(\frac{1}{2}, 1\right)$ *for* $z_1 + z_2 > 2w$, $m^*_\theta < 0$ *and* $m^*_{z_1}, m^*_{z_2}, m^*_b, m^*_a > 0$. *There is also a unique optimal CBMR constitution*

$$m^*_{CBMR} = (m^*_1, m^*_2) = \left(\frac{(3\theta-2a-b)w + (b-\theta)3z_1}{3((b+\theta-2a)w + (b-\theta)z_1)}, \frac{(3\theta-2a-b)w + (b-\theta)3z_2}{3((b+\theta-2a)w + (b-\theta)z_2)}\right)$$

with $m^*_j \in \left(\frac{1}{3}, 1\right)$ *for* $z_j > w$, *increasing in* z_j, b, a *and decreasing in* θ. *Finally,* $m^*_1 < m^*_2$.

Proof. The result follows from the usual maximization procedure which shows that the objective functions are strictly concave functions of m, m_1 and m_2 respectively while the comparative statics are straightforward □

For the purposes of comparison between the two cases, we then get:

Corollary 4 *A CBMR constitution is preferable to a SMR constitution iff*

$$z_2 > \overline{z}_2 = \frac{(b+\theta-2a)w + 2(b-\theta)z_1}{b-\theta} > z_1$$

where $\overline{z}_2$ is increasing in z_1 and θ while it is decreasing in a and b.

Proof. The result follows from the analyzing the expression

$$EU\left(m^*_{CBMR}\right) - EU\left(m^*_{SMR}\right) = Q[A + Bz_2 + Cz_2^2]$$

where

$$Q = \left[\begin{array}{c} \frac{-8(a-b)^2 w^3}{27[((b+\theta-2a)w+(b-\theta)z_1)((b+\theta-2a)w+(b-\theta)z_2)]} \\ \times \frac{1}{[(2w(b+\theta-2a)+(z_1+z_2)(b-\theta))^2]} \end{array}\right] < 0$$

$$A = \left[\begin{array}{c} w^2(4a(a-b-\theta)+(b+\theta)^2) - 2z_1(b-\theta)^2 \\ +wz_1(b^2-\theta^2-2a(b-\theta)) \end{array}\right]$$

$$B = (b^2-\theta^2-2a(b-a))w + 5z_1(b-\theta)^2$$

$$C = -2(b-\theta)^2 < 0$$

This is a quadratic equation, so it has at most two real roots. The candidate roots are

$$z_2^1 = \frac{(b+\theta-2a)w + 2(b-\theta)z_1}{b-\theta}$$

$$z_2^2 = \frac{-(b+\theta-2a)w + (b-\theta)z_1}{2\,(b-\theta)}.$$

We note that $z_2^2 < z_1 < z_2^1$ and that $C < 0$. This implies that for $z_2 > z_2^1$, we get that $EU\left(m^*_{CBMR}\right) - EU\left(m^*_{SMR}\right) > 0$ and for $z_2 \in [z_1, z_2^1]$ we get $EU\left(m^*_{CBMR}\right) - EU\left(m^*_{SMR}\right) \leq 0$. The comparative statics are immediate □

Thus we get that checks and balances are only optimal if there is enough heterogeneity in the system, in the sense that it is not sufficient that z_2 be larger than z_1 but the difference needs to be significant enough. To get some intuition for this result, suppose that $z_1 = z_2 = z$ and consider the probability that policy A obtains with the optimal CBMR constitution:

$$\left[\int_{m_1}^{1}\left(\int_{m_2}^{1} dp_1\right) dp_2\right]_{m_1=m_1^*, m_2=m_2^*} = \frac{16}{9}\left(2aw - bw - bz - w\theta + z\theta\right)^{-2}\left(b-a\right)^2 w^2$$

whereas the same probability with the optimal SMR constitution is (given that $m^*_{SMR} > \frac{1}{2}$) is

$$\left[\int_{2m-1}^{1}\left(\int_{2m-p_2}^{1} dp_1\right) dp_2\right]_{m=m^*_{SMR}} = 2\left(2aw - bw - bz - w\theta + z\theta\right)^{-2}\left(b-a\right)^2 w^2$$

where the first is clearly smaller than the latter. Since for the homogeneous case the optimal SMR constitution is the optimal MR constitution, CBMR must be suboptimal. Intuitively, this is because checks and balances are redundant in the homogeneous case: the optimal choice of m already makes sure that A is chosen whenever it is optimal to do so from the original position's perspective. If from this point of homogeneity, we just add a negligible amount of heterogeneity, this will still not be enough to compensate.

A similar result to the one presented above applies to the case in which c is verifiable. Clearly, since the heterogeneity only applies to the cases in which $c > \theta$, we will contrast simple supermajoritarian constitutions with constitutions where checks and balances only apply to the supermajority rule. More precisely,

Definition 7 *A Simple Supermajority rule (SSM) constitution is a triplet* (m, s, d) *with* $m \in [0,1], s \in [a,b], d \in [m,1]$ *such that*

$$x = \begin{cases} A \quad if \quad ((p_1 + p_2 \geq 2m) \wedge (c \leq s)) \vee (p_1 + p_2 \geq 2d) \\ B \qquad\qquad\qquad\qquad otherwise \end{cases}$$

A Supermajority-rule constitution with Checks and Balances (CBSM) is a quadruplet (m, s, d_1, d_2) *with* $m \in [0,1], s \in [a,b], (d_1, d_1) \in [m,1]^2$ *such that*

$$x = \begin{cases} A \quad if \quad ((p_1 + p_2 \geq 2m) \wedge (c \leq s)) \vee ((p_1 \geq d_1) \wedge (p_2 \geq d_2)) \\ B \qquad\qquad\qquad\qquad otherwise \end{cases}$$

Since the problem to solve is extremely similar to problem 6 above, we will dispense with that and immediately provide the corresponding, result:

Proposition 7 *There is a unique optimal SSM constitution*

$$(m^*_{SSM}, s^*_{SSM}, d^*_{SSM}) = \left(\frac{1}{2}, \theta, \frac{z_1 + z_2}{2w + z_1 + z_2}\right)$$

while there is also a unique optimal CBSM constitution

$$(m^*_{CBSM}, s^*_{CBSM}, d^*_1, d^*_2) = \left(\frac{1}{2}, \theta, \frac{3z_1 - w}{3z_1 + 3w}, \frac{3z_2 - w}{3z_2 + 3w}\right)$$

where the latter generates higher expected utility than the former whenever

$$z_2 > \overline{\overline{\overline{z}}}_2 = w + 2z_1$$

Proof. The maximization problem is again tedious but straightforward. Taking the expected utility difference $EU(m^*_{CBSM}, s^*_{CBSM}, d^*_1, d^*_2) - EU(m^*_{SSM}, s^*_{SSM}, d^*_{SSM})$ we get

$$\frac{8}{27}\frac{(b-\theta)(w - z_1 + 2z_2)(z_2 - 2z_1 - w)w^3}{(b-a)(2w + z_1 + z_2)^2(w + z_2)(w + z_1)}$$

where everything is positive except for the terms

$$(w - z_1 + 2z_2)(z_2 - 2z_1 - w)$$

These are zero if

$$2z_1^2 - wz_2 - 5z_1z_2 - w^2 - wz_1 + 2z_2^2 = 0$$

We have two roots in z_2: $\frac{1}{2}z_1 - \frac{1}{2}w$ and $w + 2z_1$ where the solution is the second because it is bigger than z_1 and the result is proven $\square$

Just as in the context where c was not verifiable, here too we have that a sufficiently high degree of heterogeneity is necessary before checks and balances become the optimal constitutional structure. However

Corollary 5 $\overline{z}_2 - \overline{\overline{z}}_2 > 0$ *and it is increasing in* w*, the difference* $\theta - a$ *and decreasing in the difference* $b - \theta$*.*

Proof. The difference $\overline{z}_2 - \overline{\overline{z}}_2$ is

$$2(b-\theta)^{-1}(\theta - a)\,w > 0$$

$\square$

What this result essentially says is that the distortion that checks and balances provide in the homogeneous case is smaller when c is verifiable. In other words, if $z_2 \in (\overline{\overline{z}}_2, \overline{z}_2)$ and if c is unverifiable, checks and balances would not be recommended because while they might be useful *conditionally* on $c > \theta$, they are negative enough if $c \leq \theta$ to make the overall case negative. In the same situation, when c is verifiable, there can be checks and balances because they can be limited just to policies with $c > \theta$. Clearly, as the set $[a, \theta]$ expands (or $[\theta, b]$ shrinks), the distinction gets more pronounced.

6.2. Heterogeneity in Costs with Asymmetric Groups

This is similar to the case above but we only look at the c verifiable case and we consider a generic q [TO DO]

6.3. Heterogeneity in the Probability of Being a Winner

This is case III above [TO DO]

6.4. Discussion

[INCOMPLETE] We have considered heterogeneity in various forms. In the first subsection, we looked at the case in which heterogeneity takes the form that some individuals will be eventually disadvantaged in terms of the losses they get if particularly some policies are passed. We have shown that from the original position's perspective, if this heterogeneity is particularly significant, checks and balances might be called for. This is because with them each group is effectively granted veto power over the relevant policies (if c is verifiable) or over policy making as a whole (if c is not verifiable). Given this, when the representative individual decides which constitutional form to use will trade-off the fact that if she happens to be in the disadvantaged group she will benefit from checks and balances with the fact that if she isn't, there is a veto power that stops policy from being enacted even if for society at large there is an ample majority in favor..

For example, with checks and balances we could have that if the majority required to pass a certain policy is $1/4$ for both groups, then if $p_2 = \frac{1}{5}$ the policy won't pass even when $p_1 = \frac{9}{10}$ so that $\frac{1}{2}(p_1 + p_2) = \frac{11}{20} > \frac{1}{2}$. Also we have shown that the trade-off is less severe when c is verifiable on those issues where there is heterogeneity because policy areas where there is no heterogeneity would not be affected.

Another important point is that the interim-heterogeneity framework we've used can be of guidance in thinking about the more realistic framework where heterogeneity occurs ex-ante, before constitutional decisions are taken. In our framework, constitutional designers might choose to use checks and balances because they fear they might end up in the disadvantaged group. In a framework where this is determined before constitutions are chosen, members of the disadvantaged group might be able to impose checks and balances as a condition for joining the constitution itself, or as a condition for avoiding stability in the system. *[Examples of the first case from EU, of other cases from different ethnic groups in countries such as Belgium. Raise the question: why there are few C&B for certain clearly identifiable and verifiable groups in society (e.g. women, minorities), how do they differ from the case of ethnic groups in Belgium?]*

7. Conclusions

[INCOMPLETE] In this paper we have developed a framework for analyzing optimal constitutional design from a normative perspective. Contrary to much of the recent literature on the issue, we have not focused on exogenously given constitutional mechanisms (e.g. majority rules) and studied how they change with changes in specific parameters in the environment, but we've studied what constitutional mechanisms are optimal responses to the features of the environment. In particular, our results imply that majority rules are optimal mechanisms only to the extent that different policy dimensions are not verifiable. We argued that verifiability is instead possible in a significant number of cases and that this explains the frequent existence of more sophisticated mechanisms which includes veto rules and supermajority rules.

We further extended the framework by showing that the issue of (un)verifiability plays a crucial role in explaining constitutional amendment rules. More specifically we showed that if the risks of instability to the system are significant enough, then amendment rules are capable of providing the flexibility needed to deal with them while if these risks are not significantly high then that flexibility comes as a liability because it allocates the monopoly of constitutional decision to a specific part of society. We also argued that the issue of constitutional amendment cannot be dealt with within the context of simple majoritarian constitutions.

Finally, we looked at the issue of how constitutional structures can respond to heterogeneity amongst members of society and we have characterized under what conditions it is optimal to have checks and balances in the sense of providing disadvantaged groups veto powers over policy. We have shown that checks and balances are costly mechanisms that should only be used when the heterogeneity is sufficiently significant. In addition, we have shown how being able to verify the policy issues where heterogeneity occurs alleviates the problem. *[Add discussion of two missing sub-cases]*

The collective decision problem we have described in this paper is extremely simple. As mentioned above, the most obvious omissions is that we do not look at the agency issues involved in the relationship between citizens and politicians and we do not have an endogenous mechanism for developing policy proposal. A natural next step would then be to incorporate some of these features in a more sophisticated framework to see how the optimal constitutions would evolve in response to these features[11].

A second potentially very interesting area for future research would be an analysis of the issues from a positive perspective. As mentioned before this is not a simple task because there is a danger that many results would not be robust the details of the constitutional bargaining stage. Nevertheless, this is a necessary step if we wish to further understand the nature of the basic foundations of democracies.

[11] [2] take a first step in this direction by making the policy maker a politician who also cares about private benefits and can choose policies that provide him with private benefits. The optimal majority rules thus responds to the politician's incentives.

References

[1] Aghion, P. and P. Bolton (2003) “Incomplete Social Contracts". *Journal of the European Economic Association* 1(1), 38-67.

[2] Aghion, P., A. Alesina and F. Trebbi (2002) “Endogenous Political Institutions". NBER working paper no. 9006.

[3] Barbera, S and M. Jackson (2001) “Choosing how to Choose: Self-Stable Majority Rules", mimeo California Institute of Technology.

[4] Buchanan, J.M. and G. Tullock (1965) *The Calculus of Consent: Logical Foundations of Constitutional Democracy*. Ann Arbor: University of Michigan Press.

[5] Gersbach, H. (2002) “Democratic Mechanisms: Double Majority Rules and Flexible Agenda Costs". Mimeo University of Heidelberg, Germany.

[6] Giovannoni, F. (2003) “Amendment Rules in Constitutions", *Public Choice* 115, 37-61

[7] Messner, M. and M.K. Polborn (2004) “Voting on Majority Rules". *Review of Economic Studies* 71, 115-132.

[8] Persson, T, G. Roland and G. Tabellini (1997) “Separation of Powers and Political Accountability". *Quarterly Journal of Economics* 112, 1163-1202.

Appendix A: Figures

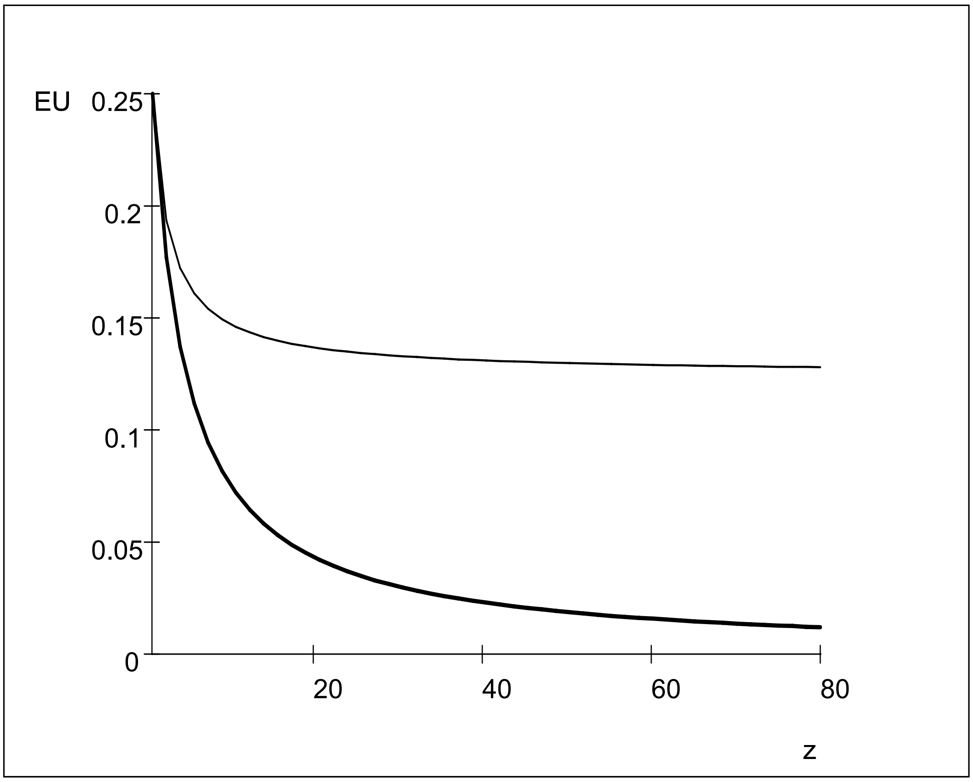

Figure 1. The thick line is $EU(m^*_{MR})$ while the thin line is $EU\left(m^*_{SM}, s^*_{SM}, d^*_{SM}\right)$ as a function of z, for $w = y = 1, a = 0, \theta = 1, b = 2$ and F, G uniform distributions.