Money, Technology Choice and Pattern of Exchange in Search Equilibrium

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Abstract: This paper examines the production aspect of money to bridge between the search-theoretic models and the canonical Walrasian growth models. In this paper, we argue that money can generate real effects via technology choice (high vs. low), we model explicitly the pattern of exchanges to explore through which channels money affects technology choice. We inquire (i) whether money encourages adoption of the high technology and (ii) whether the presence of trade frictions grants the high technology advantageous. While high quality goods yield greater consumption value, they incur a production time delay and a greater production cost. We allow buyers to form their best responses to accepting different types of goods. In a complete information world, we characterize the steady-state monetary equilibrium with both instantaneous and non-instantaneous production. We provide conditions under which the high technology equilibria is Pareto dominant or social welfare-enhancing, depending crucially on the quantity of money in the economy if production takes time. We examine how the introduction of money affects the technology choice by mitigating the high technology's disadvantage in production delay. We then identify a social inefficiency caused by producers' under-investment in the advanced technology in decentralized equilibrium.

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1 Introduction

Upon elaborating on the merit of division of labor and production specialization in his classic, *The Wealth of Nation*, Adam Smith presents the difficult of barter in a decentralized trading environment (trade between butcher, brewer and baker) and further illustrates the origin and use of money, emphasizing particularly on the resulting benefits from production specialization:

> "When the division of labour has been once thoroughly established, it is but a very small part of a man's wants which the produce of his own labour can supply. He supplies the far greater part of them by exchanging that surplus part of the produce of his own labour, which is over and above his own consumption, for such parts of the produce of other men's labour as he has occasion for." (Book I, Chapter IV, paragraph 1)

Smith's idea cannot be formalized in conventional neoclassical models of money that assume a transactions role for money in an environment where exchange is costless and occurs in a centralized marketplace. In this paper, we establish a search-theoretic foundation to examine how money may affect *technology choice* and *decentralized exchange patterns* in the presence of trade frictions.

Since the seminal work of Kiyotaki and Wright (1989,1991,1993), there has been a growing literature on money in search equilibrium, emphasizing that the use of a medium of exchange minimizes the time/resource costs associated with searching for exchange opportunities, hence alleviating the "double coincidence of wants" problem with barter.[1] While the study of the role of money in facilitating the trade has generated considerable insights towards understanding the origin and use of money, its roles

[1] In the prototypical search model of money, exchange is characterized by one-for-one swaps of goods and money, implying fixed prices, under which the optimal inflation issue can be studied using the arguments by Li (1995). Extensions of the Kiyotaki-Wright model with divisible goods but indivisible money to include pricing include Trejos and Wright (1995) and Shi (1995). More recent attempts to characterize pricing behavior and the distribution of cash permit divisible goods and money. For a brief survey, the reader is referred to Rupert, Schindler and Wright (2001, footnote 1) and papers cited therein.

in promoting production specialization and productivity enhancement remain largely unexplored.

The production aspect of money is especially important if one wants to bridge between the search-theoretic models and the canonical Walrasian monetary growth models where the central issue concerns the interaction between money, capital accumulation and economic advancement. In this paper, we emphasize that money can generate real effects via technology choice, which is crucial to long-run economic development. The search-theoretic framework allows us to provide a deep structure to help understand through which channels money affects technology choice with the pattern of exchange explicitly modeled.[2] We can examine (i) *whether money encourages adoption of the high technology* and (ii) *whether the presence of trade frictions grants the high technology disadvantageous.* In particular, our paper argues that due to a delay in production, trade frictions cause under-investment in high technology. Hence the introduction of money can mitigate trade frictions and improve the efficiency of technology choice.

More specifically, we consider a continuous-time search model with three groups of agents: producers, goods traders and money traders. Goods and money are indivisible, and each non-producing agent has only one unit of space to store either good or money. There are two clusters of goods: high quality and low quality, with each cluster consisting of a continuum of varieties. While high quality goods yield greater consumption values, they incur a production time delay and a greater production resource cost. At any point time, each producer must choose between the two technologies and can only produce one unit of the good of a particular type. Upon a successful production, a producer becomes a good trader with a commodity of a particular quality. The quality of goods is public information to all traders. Each buyer consumes only a subset of varieties, exclusive of those self-produced, and forms a *best response* to accepting goods of different quality within the desired subset.

The way through which money influences technology choice can be illustrated intuitively. Since the deepening of specialization entails some period for a consumer to buy the output from a producer, we have to consider inventory costs which is not

[2] Our paper is thus in sharp contrast with the ad hoc setup of money-in-the-production-function.

necessary in an autarky economy. If the use of money can save consumers' time to search for desired commodities, the time costs of inventories will be reduced. This "time-saving" effect makes the high technology's disadvantage in manufacturing costs less significant, thus creating an intensive margin in favor of the high technology. Since only producers take into account the underlying inventory costs, this time-saving effect vanishes when production becomes instantaneous and becomes more important when production takes longer time.

The main findings of the paper can be summarized as follows. First, we find the possible coexistence of two *pure* and one *mixed strategy* equilibria, where the latter is locally unstable. Second, when production is instantaneous, the mixed strategy equilibrium, if it coexists with the pure strategy high-technology equilibrium, is Pareto-dominated, and features a positive relationship between the fraction of high technology producers and the society's endowment of money. Moreover, *autarkic efficiency* is both sufficient and necessary for the high technology equilibrium to Pareto dominate the low one.[3] Third, when production takes time, the high technology equilibrium Pareto dominates the low one if, in addition to autarkic efficiency, the high technology's delay cost is not too large and the social endowment of money is sufficiently big. To money and goods traders, the introduction of money affects producers' technology choice, by *mitigating the high technology's disadvantage in production delay.* From the producers' points of view, shortened trading periods enable them to overcome high technology's disadvantage in extra manufacturing costs, but exacerbate its drawback in production delay. Fourth, by deriving the optimal quantity of money under each equilibrium, we identify a social inefficiency caused by producers' *under-investment* in the advanced technology in decentralized equilibrium. As a result, the optimal quantity of money in an equilibrium with only the high technology prevailed may be strictly less than that in an equilibrium with only the low technology.

<u>Literature Review</u>

In the money search literature, there are papers considering two types of traded goods, including Williamson and Wright (1994), Kim (1996), and Trejos (1997, 1999).

[3] Since every producer consumes his own output in an autarky economy, the technology choice must be efficient, despite the allocation of skills in the absence of trade is inefficient.

However, in these models, the low quality good is always undesirable under perfect observability, as it bears no cost to produce and generates no consumption value, compared to a high quality good yielding a strictly positive net utility gain. In order for both goods to be traded, private information about goods quality is therefore assumed. In contrast, we model more explicitly the production *process* of the two quality-differentiated goods, while assuming perfect observability of goods quality.

There are also a limited number of papers illustrating the role of money in fostering production specialization. In Shi (1997), agent can produce desired good at a higher cost than those for trade. Money enhances decentralized trade and thus creates a gain from specialization. A similar effect is considered by Reed (1998) where there is a trade-off between devoting time to trade and to maintaining production skills. Recently, Camera, Reed and Waller (2003) allow agents to choose whether to be a "jack of all trade" or a "master of one" in which money again advances individual's specialization in a decentralized trading environment. In Laing, Li and Wang (2003), a multiple-matching framework is developed where trade frictions manifest themselves in limited consumption variety and via a positive feedback between shopping and work effort decisions, money creation may have a positive effect on productive activity. In these papers, all goods are produced by an identical technology. Our paper, in contrast, goes beyond this literature by analyzing endogenous choice of two *different* types of production technologies that are associated with different production cost, production time and product quality.

The closely related work is Kim and Yao's (2001) in which the role of money is studied in an economy with divisible and heterogeneous goods. In their paper, production is instantaneous. Their focus is exclusively on the mixed strategy equilibrium, whereas the proportions of high and low technology producers are exogenously given. In contrast, our paper considers the more general case of non-instantaneous production and examines both mixed and pure strategy equilibria. Moreover, we study the welfare implications under various equilibria and with different initial social endowment of money. Furthermore, we allow money traders to determine whether they would accept either type or both types of goods and hence the proportion of producers using high/low technology is *endogenous*.

2 The Basic Model

The basic structure extends that of Kiyotaki and Wright (1993). Time is continuous. There is a continuum of infinitely-lived agents whose population is normalized to one. Following Trejos (1997) and Kim and Yao (2001), we consider the underlying production and preference structure in such a way that there is an absence of double coincidence of wants. Thus, throughout the main text of the paper, we focus exclusively on pure monetary equilibrium, with a discussion of the pure barter economy relegated to Appendix A.

Based on their activities, agents are divided into three different categories at any point in time: producers, goods traders and money traders. Both goods and money are indivisible. Each non-producing agent has only one unit of space that may be used to store either a unit of commodity or a unit of money.

There are two groups of goods: high quality (type-H) and low quality (type-L). Each group consists of a continuum of varieties whose characteristic location can be indexed on a unit circumference. At any point time, each producer can only produce one unit of the good of a particular type. Upon producing a commodity, a producer becomes a good trader instantaneously. Thus, producers can be classified as type-H or type-L, as are goods traders. The type of agents (and hence the quality of goods) is assumed to be public information to all traders.

Money is storable but cannot be consumed or produced. At the beginning of time, there are $M \in (0,1)$ units of money in the economy, so we have a measure of M money traders due to the unit-storage-space assumption. Thus, letting N_0, N_H, N_L, and N_m, respectively, be the measure of producers, type-H goods traders, type-L goods traders, and money traders,[4] population identity implies:

$$N_m + N_H + N_L + N_0 = 1 \tag{1}$$

The proportion of type-H traders to all goods traders, denoted h, and the fraction of money traders to all traders, denoted μ, can thus be expressed as:

$$h = \frac{N_H}{N_H + N_L} \tag{2}$$

[4] Due to the assumption of unit storage space and the indivisibility of money, $N_m = M$.

$$\mu = \frac{N_m}{N_m + N_H + N_L} = \frac{M}{1 - N_0}. \quad (3)$$

Traders match with each other according to a Poisson process characterized by the arrival rate parameter, β. Because that the probability for a particular pair of traders to rematch is zero in our continuum economy and that there is lack of an authority to enforce the repayment of credits or IOU's, sellers must accept money in the absence of double coincidence of wants.

2.1 Production Technology

There are two types of technologies. The high technology can produce a unit of the high quality good at a (utility) cost of ε, while the low technology incurs a lower manufacturing cost of $\delta\varepsilon$ (with $0 < \delta < 1$) to produce one unit of the low quality good.

The two technologies also differ in the arrival rates of the respective outputs. Specifically, the production of the low technology follows a Poisson process with arrival rate of α, while that of the high technology has an arrival rate of $\eta\alpha$ (with $0 < \eta < 1$).

2.2 Preferences

Following the convention of the money-search literature, we assume that no agent would consume the good he or she produces. Moreover, each agent gains positive utility only by consuming a subset of the varieties of each type (called a consumable set), whose measure is denoted by x. Thus, x can be regarded as a taste specialization index.

Despite their taste heterogeneity, all agents have identical utility functional forms. While the consumption of the first unit of a high quality good within the consumable set yields a utility $U > 0$, any additional unit at a given point in time would not generate any extra value. Similarly, the consumption of the first unit of a low quality good within the consumable set gives an utility of θU (with $0 < \theta < 1$).[5] To ensure non-trivial technological choice, we impose:

[5] More precisely, if we index the agent by the type of goods he can produce, then utility function for agent i is $u_i(\cdot) = \Theta U I_{(i,i+x] \bmod 1}$, where Θ is the quality factor and I is the indicator function. By using $\bmod 1$, we can actually index the types of goods on a unit circle. Observe that this utility function implies that the producer cannot consume his own product.

Assumption 1: $U > \theta U > \varepsilon > \delta\varepsilon$.

That is, both types of products deliver net values to the economy. The assumption of $\theta U > \varepsilon$ guarantees the existence of mixed strategy equilibrium, as we will show later.

Moreover, we assume that each agent has a reservation value of zero, and only positive values would attract him to join in the exchange economy.

2.3 Value Functions

Denote the probability at which a money trader will accept type-i goods as π_i ($i = H, L$), while Π_i as the average probability of acceptability in the economy (which is taken as parametrically given by all individual traders). Denote the discount rate by r. Further denote V_i as the asset value of a type-i agent, where $i = 0, H, L, m$ represents producers, type-H goods traders, type-L goods traders, and money traders, respectively.

We are now well equipped to set up the Bellman equations ($i = H, L$), displayed for simplicity by assuming steady states (as in the conventional money and search literature):

$$rV_0 = \max\{\alpha(V_L - V_0 - \delta\varepsilon), \eta\alpha(V_H - V_0 - \varepsilon)\} \tag{4}$$

$$rV_i = \beta\mu x \Pi_i (V_m - V_i) \tag{5}$$

$$rV_m = \beta(1-\mu)x\left[h \max_{\pi_H}\{\pi_H(U + V_0 - V_m)\} + (1-h)\max_{\pi_L}\{\pi_L(\theta U + V_0 - V_m)\}\right]. \tag{6}$$

Equation (4) states that the flow value of a producer is the maximum incremental value, over the two technologies, from the producer state to the goods trader state net of the corresponding production cost, upon a successful arrival of the product (measured by α and $\eta\alpha$, respectively).

Recall that at a flow probability β, a goods trader of type-i can meet another trader who will be a money trader with probability μ. The chance for this money trader to like the goods trader's product is x, which will be accepted at probability Π_i. Thus, as indicated by (5), the flow value of a type-i goods trader is the incremental value from exchanging the product with money, which is the differential, $V_m - V_i$, multiplied by the flow probability, $\beta\mu x\Pi_i$.

Similarly, the flow probability for a money trader to meet a type-H goods trader whose commodity is within the consumable set is $\beta(1-\mu)xh$ and that to meet a type-L goods trader is $\beta(1-\mu)x(1-h)$. The flow value of meeting a type-i goods trader is the flow utility (U and θU, for $i = H, L$, respectively) plus the incremental value from the money trader state to the producer state ($V_0 - V_m$). A money trader may stay put (by not accepting the good, i.e., $\pi_i = 0$) or accept the trade with probability $\pi_i > 0$ (which is the best response by the money trader, possibly less than one). Thus, this flow value must be multiplied by the corresponding acceptance probability, as displayed in (6).

It is convenient to define by Δ_i ($i = H, L$) the producer's effective discount factors over the expected span of the production process and by ρ_i ($i = H, L$) the goods trader's effective discount factors for the expected waiting period for sales.

$$\Delta_H \equiv \frac{\eta\alpha}{\eta\alpha + r}; \quad \Delta_L \equiv \frac{\alpha}{\alpha + r} \tag{7}$$

$$\rho_H \equiv \frac{\beta\mu x\Pi_H}{\beta\mu x\Pi_H + r}; \quad \rho_L \equiv \frac{\beta\mu x\Pi_L}{\beta\mu x\Pi_L + r}. \tag{8}$$

Given the Poisson process, $\frac{1}{\eta\alpha}$ is the average waiting time for production and $\frac{r}{\eta\alpha}$ is the discount rate over the expected span of the production process , thus yielding the producer's effective discount factors, Δ_i. Similar explanations apply to ρ_i.

Accordingly, we can rewrite the value functions (4) and (5) in a cleaner manner,

$$V_0 = \max\{\Delta_L(V_L - \delta\varepsilon), \Delta_H(V_H - \varepsilon)\} \tag{9}$$

$$V_i = \rho_i V_m \tag{10}$$

3 Equilibria with Instantaneous Production

We begin by considering a special case with instantaneous production ($\alpha \to \infty$), which enables a complete analytic analysis of the *steady-state monetary equilibrium*. With instantaneous production, we have $N_0 = 0$, and , from (3), $\mu = M$. Moreover, (7) implies $\Delta_H = \Delta_L = 1$ and hence (9) can be rewritten as:

$$V_0 = \max\{(V_L - \delta\varepsilon), (V_H - \varepsilon)\}. \tag{11}$$

3.1 Money Trader's Best Response

To solve the equilibrium under instantaneous production, first consider the money trader. A money trader's best responses π_H and π_L are determined according to the following:

$$\pi_H \begin{cases} = 0, & \text{if } \; U + V_0 - V_m < 0 \\ \in (0,1), & \text{if } \; U + V_0 - V_m = 0 \\ = 1, & \text{if } \; U + V_0 - V_m > 0 \end{cases} \tag{12}$$

$$\pi_L \begin{cases} = 0, & \text{if } \; \theta U + V_0 - V_m < 0 \\ \in (0,1), & \text{if } \; \theta U + V_0 - V_m = 0 \\ = 1, & \text{if } \; \theta U + V_0 - V_m > 0 \end{cases} . \tag{13}$$

Thus, in the case where $U + V_0 - V_m = 0$ or $\theta U + V_0 - V_m = 0$, the corresponding best response (π_H or π_L) constitutes a mixed strategy.

In equilibrium, the individual's best response agrees with the average behavior in the economy, that is,

$$\pi_i = \Pi_i, \tag{14}$$

for $i = H, L$.

3.2 Existence

We focus on the case of nondegenerate equilibrium in which all agents participate in the exchange economy actively. Thus, a producer must have positive payoff,

$$\max\{(V_L - \delta\varepsilon), (V_H - \varepsilon)\} > 0 \tag{15}$$

Moreover, a money trader must buy at least one type of the commodities. This is valid under the following active equilibrium condition:

$$\max\{U + V_0 - V_m, \theta U + V_0 - V_m\} > 0 \tag{16}$$

The strict inequality is due to condition (15).

Since $\theta < 1$, this condition requires: $U + V_0 - V_m > 0$, and thus $\pi_H = 1$, which means the money trader will fully accept the type-H good. Based on the three different best responses towards the acceptability of the type-L good, we can have three equilibria: (A) $\pi_L^A = 0$; (B) $\pi_L^B \in (0,1)$; and (C) $\pi_L^C = 1$. We use superscript A, B, and C to denote each equilibrium whenever it is necessary. Also, we can define the effective discount factor for the purchasing period (when always accepting a good) as:

$$\rho_m = \frac{\beta(1-\mu)x}{\beta(1-\mu)x + r}. \tag{17}$$

It is not difficult to solve (V_0, V_H, V_L, V_m) from the linear equation system (6), (10) and (11), which are summarized in Table 1.1. The main task is to figure out the best responses of the agents and check the corresponding conditions on the parameters. Define $Q \equiv \frac{(\beta x + r)r\varepsilon}{\beta^2 x^2 (U-\varepsilon)}$ and consider,

Assumption 2: $Q \max\left\{\frac{\delta U - \delta\varepsilon}{\theta U - \delta\varepsilon}, 1\right\} < \frac{1}{4}$.

Assumption 3: $\dfrac{1}{\theta U - \varepsilon} + \dfrac{\theta}{1-\theta} < \dfrac{\beta x}{r}$.

We first examine the two pure strategy equilibria (A and C). In equilibrium A, no producer would choose the low technology since it yields negative flow value to producers ($h = 1$). We can show from (8) and (10) that $V_L = 0$. From (13), we know that $\pi_L = 0$, if $\theta U + V_0 - V_m < 0$. We now define:

$$M_1 \equiv \max\{1 - \frac{(\beta x + r)(\theta U - \varepsilon)}{\beta x (U - \varepsilon)}, 0\} \tag{18}$$

and $M_2 < 0.5$ such that

$$M_2(1 - M_2) \equiv \frac{(\beta x + r)r\varepsilon}{\beta^2 x^2 (U - \varepsilon)}, \tag{19}$$

which has two distinct real roots under Assumption 2. We can then establish:

Lemma 1: (Equilibrium A) *Equilibrium A exists if $S^A \equiv (0, M_1) \cap (M_2, 1 - M_2)$ is nonempty and $M \in S^A$.*

Proof: All proofs are in Appendix B.

Within the region $M \in (0, M_1)$, $\theta U + V_0 - V_m < 0$ and hence it is a money trader's best response to rejecting a trade with a type-L producer. Intuitively, in an economy swamped by too much money, money traders would buy any type of goods as soon as possible since they cannot afford the long waiting period for the second chance. This is particularly essential when the difference in the quality is not sufficiently large to make the waiting worthwhile. Since this effect due primarily to the presence of search frictions (with the quality differential accounted), it may be referred to as the *search friction effect*.

The requirement that $M \in (M_2, 1 - M_2)$ is to ensure nonnegative producer payoffs. If the amount of initial money endowment is too big, then money traders will also take the low quality goods; if the initial money endowment is too small, then there will be no producers.

The solution of equilibrium C is quite similar to that of equilibrium A. Observe that when $\pi_L = \Pi_L = 1$, equation (5) results in $V_H^C = V_L^C$, as well as $\rho_H^C = \rho_L^C$. The producer would definitely choose the low technology to minimize his cost, which means $h = 0$. After solving the values, we find that since $U > \theta U > V_m^C - V_0^C$, for any $M \in (0, 1)$, equilibrium C exists as long as $V_0^C > 0$. Define $M_3 < 1/2$ such that

$$M_3(1 - M_3) \equiv \frac{(U - \varepsilon)\,\delta Q}{\theta U - \delta\varepsilon} \tag{20}$$

which has real root(s) under Assumption 2. Then we have:

Lemma 2: (Equilibrium C) *Equilibrium C exists if $M \in S^C \equiv (M_3, 1 - M_3)$. Moreover, $S^C \supseteq S^A$ if $0 < \delta \leq \theta < 1$.*

Equilibrium B is a bit more complicated. The money trader's mixed strategy implies $\theta U + V_0^B - V_m^B = 0$. Based on the fact that the producers are indifferent between the two technologies, we can solve the money trader's acceptability of low quality goods,

$$\pi_L^B = \Pi_L^B \equiv 1 - \frac{(1 - \delta)\varepsilon}{\rho_H(\theta U - \delta\varepsilon)}, \tag{21}$$

and the equilibrium proportion of type-H goods in the market,

$$h^B \equiv \frac{(\beta\mu x + r)(\theta U - \varepsilon)}{\beta(1 - \mu)x(1 - \theta)U}. \tag{22}$$

It is easily seen that π_L^B is increasing in μ and thus M. Moreover, h^B is increasing in μ and thus M, which implies as the amount of money increases in the economy, there are more people holding type-H goods. Defining

$$M_4 \equiv \frac{r\varepsilon}{\beta x(\theta U - \varepsilon)}, \tag{23}$$

we can obtain:

Lemma 3: (Equilibrium B) *Equilibrium B exists if* $S^B \equiv (M_4, M_1)$ *is nonempty and* $M \in S^B$. *Moreover,* $S^B \subseteq S^A$.

Under Assumptions 2 and 3, S^j $(j = A, B, C)$ is nonempty and hence with the aid of Lemmas 1-3, we can establish:

Proposition 1: (Existence and Stability) *Under Assumptions 1-3, a steady-state monetary equilibrium exists, which possesses the following properties, depending on the society's initial endowment of money* M:

(i) $\pi_L = 0$ *with* $M \in S^A$ *(equilibrium A);*

(ii) $\pi_L \in (0, 1)$ *with* $M \in S^B$ *(equilibrium B);*

(iii) $\pi_L = 1$ *with* $M \in S^C$ *(equilibrium C);*

Moreover, multiple equilibria may arise. Among the three equilibria, equilibrium A and C are locally stable, while equilibrium B is locally unstable.

Concerning the existence, Assumptions 2 and 3 ensure the nonemptiness of S^C and S^B, respectively, whereas both Assumptions together guarantee S^A is nonempty. From Lemma 3, when $M \in S^B$, the mixed strategy equilibrium B always co-exists with the pure strategy equilibrium A (as $S^B \subseteq S^A$). Moreover, when $0 < \delta \leq \theta < 1$ and $M \in S^A$, both pure strategy equilibria co-exist (as $S^A \subseteq S^C$).

We can interpret the solution intuitively with the effective discount factors defined in (8) and (17). In equilibrium A, for example, the producer bears the manufacturing cost instantaneously but should wait for both the selling and purchasing periods, so

	Equilibrium A	Equilibrium B	Equilibrium C[6]
Π_{mL}	0	π_L^B	1
h	1	h^B	0
V_0	$\dfrac{\rho_H^A \rho_m^A U - \varepsilon}{1 - \rho_H^A \rho_m^A}$	$\dfrac{\rho_H^B \theta U - \varepsilon}{1 - \rho_H^B}$, or $\dfrac{\rho_L^B \theta U - \delta\varepsilon}{1 - \rho_L^B}$	$\dfrac{\rho_L^C \rho_m^C \theta U - \delta\varepsilon}{1 - \rho_L^C \rho_m^C}$
V_H	$\dfrac{\rho_H^A \rho_m^A (U - \varepsilon)}{1 - \rho_H^A \rho_m^A}$	$\dfrac{\rho_H^B (\theta U - \varepsilon)}{1 - \rho_H^B}$	V_L^C
V_L	0	$\dfrac{\rho_L^B (\theta U - \delta\varepsilon)}{1 - \rho_L^B}$	$\dfrac{\rho_L^C \rho_m^C (\theta U - \delta\varepsilon)}{1 - \rho_H^C \rho_m^C}$
V_m	$\dfrac{\rho_m^A (U - \varepsilon)}{1 - \rho_H^A \rho_m^A}$	$\dfrac{\theta U - \varepsilon}{1 - \rho_H^B}$, or $\dfrac{\theta U - \delta\varepsilon}{1 - \rho_L^B}$	$\dfrac{\rho_m^C (\theta U - \delta\varepsilon)}{1 - \rho_L^C \rho_m^C}$
M	S^A	S^B	S^C

Table 1: Solutions for Instantaneous Production

his utility in one production cycle is $\rho_H \rho_m U - \varepsilon$. Since the effective discount factor for one production cycle is $\rho_H \rho_m$, the summation of infinite geometric series yields the solution in the first cell in Table 2.1, where other cells can be derived in an analogous fashion.

The two pure strategy equilibria are both locally stable, since small disturbance in the acceptability of the type-L goods cannot affect the producer's choice. However, equilibrium B is locally unstable. To see this we can simply disturb Π_L. If the agents believe Π_L to be a bit larger (smaller), V_L would be higher (lower). Thus the producer will prefer the low (high) technology, thereby leading to equilibrium C (A).

Equilibrium B in our model can be compared with the mixed strategy equilibrium in Kim and Yao (2001): When both types of products co-exist, the share of type-H goods (h) and the level of social welfare are increasing in the money supply (M).

3.3 Welfare Implications

Due to the assumption of instantaneous production, only the goods and money traders are considered in the commonly used equally weighted steady-state social welfare function. Observe that, $M \in (0, M_1)$ is equivalent to $V_m^A > V_m^B$, which implies

$V_H^A > V_H^B > V_L^B$, and $V_0^A > V_0^B$, *pointwise* with respect to M. Since $S^B \subseteq S^A$, for any value of $M \in S^B$, there is always an equilibrium with $\pi_L = 0$ (equilibrium A) that Pareto dominates the mixed strategy equilibrium. Since this equilibrium is locally unstable and Pareto-dominated in its existence region (see the following subsection), we put more effort towards comparing the two pure strategy equilibria, A and C.

Comparing the two pure strategy equilibria A and C, we find that both goods traders and money traders prefer (pointwise with respect to M) the technology with autarkic efficiency, i.e., that with the highest net-of-cost utility. The Pareto ranking in this case is straightforward because the producers are of measure zero. In general, it may be useful to compare the steady-state social welfare instead of Pareto rankings:

$$Z \equiv N_0 V_0 + N_L V_L + N_H V_H + N_m V_m. \tag{24}$$

We assume that social planner can set the initial amount to maximize Z. Hence we compare the maximal welfare in equilibrium A and C.

For equilibrium A and C, the social welfare levels can be computed as: $\frac{\beta x M(1-M)(U-\varepsilon)}{r}$ and $\frac{\beta x M(1-M)(\theta U-\delta\varepsilon)}{r}$, respectively. As a consequence, the *socially optimal* amount of money can be easily solved as $\min\{1/2, M_1\}$ for equilibrium A and $1/2$ for equilibrium C.[7] Since a greater amount of money renders a more severe search friction effect, it encourages the choice of low technology and makes equilibrium A not sustainable. As a result, the optimal quantity of money in equilibrium A may be strictly less than that in equilibrium C. If $M_1 > 1/2$ (which holds when θ is sufficiently small), the welfare comparison is again equivalent to autarkic efficiency. Otherwise, the social planner would choose the high technology only when it provides sufficiently more net utility than the low technology, that is,

$$\frac{U-\varepsilon}{\theta U-\delta\varepsilon} \geq \frac{1/4}{M_1(1-M_1)} > 1.$$

From (18), M_1 is decreasing in θ and independent of δ. Therefore, when the quality difference is sufficiently small, the social planner could still support the production of

[7] Since we have open intervals, M_1 is not attainable for the optimal amount of money when $M_1 \leq 1/2$. However, based on the assumption that the amount of money has a smallest unit, we can easily get around this technical problem.

type-L goods, even when the type-H goods provide more net utility. On the contrary, the production cost differential (captured by δ) does not play any role, which is a result of the take-it-or-leave-it offer to buyers whose only concern is the quality of the good. Under instantaneous production, it can do no better than the autarkic efficiency outcome, with a frictional exchange process being introduced. This conclusion would no longer be true if production itself also takes time (see Section 4 below).

Proposition 2: (Welfare and Optimal Quantity of Money) *Equilibrium B is always Pareto dominated by equilibrium A either pointwise with respect to M or in the sense of equally weighted social welfare maximization. The comparison between equilibria A and C possesses the following properties:*

(i) *under pointwise Pareto criterion, it is equivalent to the case of autarkic efficiency;*

(ii) *under social welfare maximization,*

- **a.** *it is equivalent to autarkic efficiency if $M_1 > 1/2$,*
- **b.** *the social planner is less likely to adopt the high technology than autarkic efficiency if $M_1 \leq 1/2$;*

(iii) *the socially optimal quantity of money is $\min\{1/2, M_1\}$ for equilibrium A and $1/2$ for equilibrium C.*

4 Non-instantaneous Production

When production is not instantaneous, *i.e.*, when α is finite, there is a nontrivial steady-state mass of producers, and thus $\mu > M$. This creates great algebraic complexity. Nonetheless, this exercise allows us to gain additional insights on how the introduction of money could improve technological development.

4.1 Steady-State Monetary Equilibrium

Based on the active equilibrium condition (16) we once more obtain: $\pi_H = 1$, which means money trader will fully accept the type-H goods in equilibrium. Based on the three different best responses to accepting type-L goods, we again have three equilibria: (AA): $\pi_L^{AA} = 0$; (BB): $\pi_L^{BB} \in (0,1)$; and (CC): $\pi_L^{CC} = 1$, where the labeling AA, BB, and CC correspond to A, B, and C, in the instantaneous production case.

To solve the population distribution in the steady state, we equate the outflows and inflows from and to the population of goods and money traders to yield:

$$\Lambda\eta\alpha N_0 = \beta\mu x \Pi_H N_H \tag{25}$$

$$(1-\Lambda)\alpha N_0 = \beta\mu x \Pi_L N_L \tag{26}$$

$$\beta\mu x(\Pi_L N_L + \Pi_H N_H) = \beta(1-\mu)x[h\Pi_H + (1-h)\Pi_L]N_m \tag{27}$$

where Λ is the proportion of producers employing the high technology. From equation (26) and (25) and using $\pi_H = 1$, we can derive:

$$\Lambda = \frac{h}{h + \eta(1-h)\Pi_L} \tag{28}$$

Observe that Λ is strictly increasing in h, satisfying: $\lim_{h\to 0} \dfrac{\Lambda}{h} = \dfrac{1}{\eta}$, and $\lim_{h\to 1} \dfrac{\Lambda}{h} = 1$.

Now μ no longer equals to M. However there is a monotone increasing relationship between them, which can be seen by combining equation (27) and (25) to yield, $\Lambda\eta\alpha(1-\dfrac{M}{\mu}) = \beta\mu x h(\dfrac{M}{\mu} - M)$, or,

$$M = \frac{\mu\eta\alpha\,(\Lambda/h)}{\beta x\mu(1-\mu) + \eta\alpha\,(\Lambda/h)} \tag{29}$$

The expression could be simplified with the aid of the limiting properties under equilibrium AA or CC. As a result, the population distribution will be determined by only three endogenous variables, h, μ, and Π_L, since from (1), (2) and (3), all population masses can be expressed in terms of h, μ and M and from (28) and (29), M is a function of h, μ, and Π_L.

As before, we can solve the system using the discount rates Δ_H and Δ_L (see Table 2.2), where the equilibrium acceptability of type-L goods in equilibrium BB is:[9]

$$\pi_L^{BB} = \frac{1}{\beta\mu x}\frac{\beta\mu x\eta(\alpha+r)\theta U - \{(\beta\mu x + r)\eta\alpha + r[(\beta\mu x + r)(\eta-\delta) - \delta\eta\alpha]\}\,\varepsilon}{[(\beta\mu x + r) + \eta\,(\alpha - \beta\mu x)]\theta U + [(\beta\mu x + r)(\eta - \delta) - \delta\eta\alpha]\varepsilon}. \tag{30}$$

[9] The reader can easily check that the solution of π_L^{BB} reduces to π_L^{B} with $\alpha \to \infty$ and $\eta \to 1$.

	Equilibrium AA	Equilibrium BB	Equilibrium CC[8]
Π_L	0	π_L^{BB}	1
h	1	h^{BB}	0
V_0	$\Delta_H \dfrac{\rho_H^{AA}\rho_m^{AA}U-\varepsilon}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}$	$\Delta_H \dfrac{\rho_H^{BB}\theta U-\varepsilon}{1-\rho_H^{BB}\Delta_H}$, or $\Delta_L \dfrac{\rho_L^{BB}\theta U-\delta\varepsilon}{1-\rho_L^{BB}\Delta_L}$	$\Delta_L \dfrac{\rho_L^{CC}\rho_m^{CC}\theta U-\delta\varepsilon}{1-\rho_L^{CC}\rho_m^{CC}\Delta_L}$
V_H	$\dfrac{\rho_H^{AA}\rho_m^{AA}(U-\Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}$	$\dfrac{\rho_H^{BB}(\theta U-\Delta_H\varepsilon)}{1-\rho_H^{BB}\Delta_H}$	V_L^{CC}
V_L	0	$\dfrac{\rho_L^{BB}(\theta U-\Delta_L\delta\varepsilon)}{1-\rho_L^{BB}\Delta_L}$	$\dfrac{\rho_L^{CC}\rho_m^{CC}(\theta U-\Delta_L\delta\varepsilon)}{1-\rho_L^{CC}\rho_m^{CC}}$
V_m	$\dfrac{\rho_m^{AA}(U-\Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}$	$\dfrac{\theta U-\Delta_H\varepsilon}{1-\rho_H^{BB}\Delta_H}$, or $\dfrac{\theta U-\Delta_L\delta\varepsilon}{1-\rho_L^{BB}\Delta_L}$	$\dfrac{\rho_m^{CC}(\theta U-\Delta_L\delta\varepsilon)}{1-\rho_L^{CC}\rho_m^{CC}\Delta_L}$
μ	S^{AA}	S^{BB}	S^{CC}

Table 2: Solutions for the Case of Possitive Production Time

Accordingly, the proportion of type-H goods in the market becomes:

$$h^{BB} = \frac{r(\theta U - \Delta_H\varepsilon)}{(1-\rho_H\Delta_H)\beta(1-\mu)x(1-\theta)U}. \tag{31}$$

Note that although h^{BB} is increasing in μ, the relationship between π_L^{BB} and μ is no longer monotone.

The values in equilibria AA and CC listed in Table 2.2 can be explained intuitively. Note that the effective discount factors indicate the time costs over the respective waiting periods (production, selling, and buying). Take V_0^{AA} as an example. As the producers must wait for all the three waiting periods, the utility should be discounted by all the three factors, Δ_H, ρ_H, and ρ_m. Meanwhile, the production cost is generated at the end of the production period, so only Δ_H is attached to it. This provides the producer's value in one cycle, $\Delta_H\rho_H^{AA}\rho_m^{AA}U-\Delta_H\varepsilon$. The value is then obtained by simply dividing the one-cycle value by one minus the discount factor for a cycle, $\Delta_H\rho_H^{AA}\rho_m^{AA}$.

Repeating the same steps as in the previous section, one can derive parameter regions for μ (instead of M) to support each type of equilibrium. As shown in the Appendix, we have: $S^{AA} = (0,\mu_1)\cap(M_2, 1-M_2)$, where μ_1 solves:

$$(1-\theta)U = \frac{(\beta\mu x + r)r(U-\Delta_H\varepsilon)}{\beta^2x^2\mu(1-\mu)(1-\Delta_H)+r\beta x+r^2}; \tag{32}$$

$S^{BB} = (M_4, \mu_1)$, and, $S^{CC} = S^C$. With positive production time, $S^{AA} \supseteq S^{BB}$ still holds, and the relationship between S^{AA} and S^{CC} is the same as the discussion in the previous section. We can establish:

Proposition 3: (Existence and Stability) *Under Assumptions 1-3, a steady-state monetary equilibrium exists. Depending on the society's initial endowment of money M, it possesses the following properties:*

(i) $\pi_L = 0$ *with* $\mu \in S^{AA}$ *(equilibrium A);*

(ii) $\pi_L \in (0,1)$ *with* $\mu \in S^{BB}$ *(equilibrium B);*

(iii) $\pi_L = 1$ *with* $\mu \in S^{CC}$ *(equilibrium C);*

where multiple equilibria may arise and the stability property remains the same as in Proposition 1.

4.2 Welfare Implications

As before, we still have equilibrium AA Pareto dominates equilibrium BB. However the welfare comparison between equilibria AA and CC is a bit more sophisticated now. Let us derive the social welfare for the respective equilibria as follows:

$$Z^{AA} = \frac{\eta\alpha b}{b + \eta\alpha}\left(\frac{U - \varepsilon}{r}\right) \tag{33}$$

$$Z^{CC} = \frac{\alpha b}{b + \alpha}\left(\frac{\theta U - \delta\varepsilon}{r}\right). \tag{34}$$

where $b \equiv \beta x \mu(1-\mu)$. Obviously the optimal amount of money still satisfies $\mu = 0.5$ in each case, provided that $\mu_1 \geq 0.5$. For pointwise comparison with respect to μ, we still have the net utility terms, $U - \varepsilon$ versus $\theta U - \delta\varepsilon$ as in the instantaneous production case. However, the slow production process makes the high technology less attractive than the low technology as the multiplier on the right-hand side of (33) is less than that of (34) provided $\eta < 1$. When the net utility gain from undertaking the high technology is positive and sufficient large to overcome the disadvantage from a non-instantaneous production process, the welfare under equilibrium AA is greater than that under equilibrium CC.

Meanwhile, the autarkic values in the respective equilibria are

$$W^{AA} = \frac{U - \Delta_H \varepsilon}{1 - \Delta_H} = \frac{(\eta\alpha + r)U - \eta\alpha\varepsilon}{r} \tag{35}$$

$$W^{CC} = \frac{\theta U - \Delta_L \delta\varepsilon}{1 - \Delta_L} = \frac{(\alpha + r)\theta U - \alpha\delta\varepsilon}{r} \tag{36}$$

Again, the comparison between the two values depends crucially on the net utility gain versus the loss in a non-instantaneous production process. Formally, we define $q \equiv \dfrac{\theta U - \delta\varepsilon}{U - \varepsilon}$ and calculate two critical values for η,

$$\eta_Z = q - \frac{q\alpha(1-q)}{\alpha + b - \alpha q}; \quad \eta_W = q + \frac{r(1-\theta)U}{\alpha(U-\varepsilon)},$$

such that $Z^{AA} > Z^{CC}$ iff $\eta > \eta_Z$, and that $W^{AA} > W^{CC}$ iff $\eta > \eta_W$.

As long as the type-H goods provide more utility and the search friction effect is sufficiently small ($\mu_1 \geq 0.5$), autarkic efficiency is a sufficient (but not necessary) condition for equilibrium AA to dominate CC in social welfare sense. In other words, the monetary economy can improve technological development if search friction is negligible.

Proposition 4: (Welfare under Non-instantaneous Production) *While equilibrium AA always Pareto dominates equilibrium BB, it leads to higher welfare than equilibrium CC if $\eta > \eta_Z$. The optimal quantity of money for equilibria AA and CC are analogous to Proposition 2 after replacing M_1* with μ_1.

Notice that the results of social welfare comparison are essentially driven by the values of goods and money traders. Provided that the two technologies provide the same values to producers in autarky, the sellers and buyers in the monetary exchange economy would prefer the high one (pointwise with respect to μ), since

$$\frac{1 - \Delta_H}{1 - \rho_H^{AA}\rho_m^{AA}\Delta_H} > \frac{1 - \Delta_L}{1 - \rho_L^{CC}\rho_m^{CC}\Delta_L}$$

However, in terms of Pareto criteria, we must also examine the welfare of producers, whose relative gain from employing the high technology can be written as:

$$\begin{aligned} V_0^{AA} - V_0^{CC} &= \left(\frac{1 - \Delta_H}{1 - \rho_H^{AA}\rho_m^{AA}\Delta_H} W^{AA} - \frac{1 - \Delta_L}{1 - \rho_L^{CC}\rho_m^{CC}\Delta_L} W^{CC}\right) - (1-\theta)U \\ &= \left[\frac{1 - \rho_L^{CC}\rho_m^{CC}\Delta_L}{1 - \rho_H^{AA}\rho_m^{AA}\Delta_H}\frac{U - \Delta_H\varepsilon}{\theta U - \Delta_L\delta\varepsilon} - 1\right]\frac{\theta U - \Delta_L\delta\varepsilon}{1 - \rho_L^{CC}\rho_m^{CC}\Delta_L} - (1-\theta)U. \end{aligned}$$

The term in the square bracket is similar to the value comparison for goods and money traders, but the last term may upset such a comparison if θ is sufficiently lower than one. This last term can be viewed as the difference in inventory costs per unit of goods, which driven by the time-consuming trading period in the monetary economy with search friction. Thus, even when the high technology provides a higher autarkic value, the producers may still prefer the low technology when frictional exchanges are taken into account.

Another interesting finding is that the gains from employing the high technology need not be maximized at the welfare-optimizing quantity of money. In particular, we can identify a *time-saving effect* from $\frac{1}{1-\rho_L^{CC}\rho_m^{CC}\Delta_L}$, which is increasing in $\mu(1-\mu)$. In fact, it is the only effect in the case of instantaneous production, since $\Delta_H = \Delta_L = 1$. When production takes time, there also exists a *mitigation effect*, which is decreasing in $\mu(1-\mu)$ as long as it takes more time to produce the type-H goods ($\Delta_H < \Delta_L$).[10] Intuitively, a longer waiting period to trade would mitigate the disadvantage of the high technology in production time to a greater extent. When the expected trading period approaches to its minimum, 0.5, the mitigation effect may be strong enough to dominate the time-saving effects under some parameter values. Figure 2 illustrates a numerical example, in which the sign of producers' gain depends on the amount of money and the mitigation effect dominates the time saving effect near the optimal amount of money.

5 Conclusion

An interesting message our model has delivered is that the use of money affects only producers' technology choices (in favor of the high technology) in the instantaneous production model, but its effect is pervasive if production takes time. Moreover, we identify a social inefficiency caused by producers' under-investment in the advanced technology in decentralized equilibrium. Furthermore, in the case of mixed strategy equilibrium, the share of high-technology output is increasing in the quantity of money.

[10] This effect is via the term, $\frac{1-\rho_L^{CC}\rho_m^{CC}\Delta_L}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H} = \frac{\Delta_L}{\Delta_H} + \frac{1}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}(1-\frac{\Delta_L}{\Delta_H})$.

The implication of our model could go beyond the technology choice issue. Should we regard the high technology as a production plan of high volume, and the low technology as one with low volume, it becomes a binary output quantity model, where the utilities, manufacturing costs and production times are all increasing in the scale of production. This may shed light on the possibility of multiple equilibria in the multiple consumption units or divisible goods setup. For instance, in a simple case with constant return and cost to scale, the highest possible volume of output is best in the sense of social welfare. The optimal volume of output will be determined by the relevant set of parameters (similar to S^A), which depends on the quantity of money in the economy.

In this paper, we assume perfect observability throughout. To another extreme, if buyers cannot detect the quality of the commodities trade at all, then V_H always equals V_L and producers will always choose the cost-saving technology without investing in the high technology. In the case of partial observability, we expect similar results as Trejos (1997). If the high technology has adequate relative efficiency over the low, then the buyers would prefer type-H goods whenever they are able to identify its quality. It is therefore straightforward to conclude that the presence of private information will not eliminate the positive role of money in production efficiency as long as partial observability is preserved.

Appendix A: Technology Choice in a Pure Barter Economy

In this appendix, we investigate the technology choice issue in a scenario of a pure barter economy. On the basis of the notation we employ in Section II, we can set up the related values functions:

$$rV_0 = \max\{\alpha(V_L - V_0 - \delta\varepsilon), \eta\alpha(V_H - V_0 - \varepsilon)\}, \tag{A1}$$

$$rV_H = \beta x^2[h\Pi_{HH} \max_{\pi_{HH}}\{\pi_{HH}(U + V_0 - V_H)\} + (1-h)\Pi_{LH} \max_{\pi_{HL}}\{\pi_{HL}(\theta U + V_0 - V_H)\}], \tag{A2}$$

$$rV_L = \beta x^2[h\Pi_{HL} \max_{\pi_{LH}}\{\pi_{LH}(U+V_0-V_L)\}+(1-h)\Pi_{LL} \max_{\pi_{LL}}\{\pi_{LL}(\theta U+V_0-V_L)\}], \tag{A3}$$

where $\pi_{i,j}$ indicates the probability for i-type goods trader to accept j-type commodities. The equilibrium population equations are

$$\Lambda\eta\alpha N_0 = \beta x^2[h\Pi_{HH}\pi^*_{HH} + (1-h)\Pi_{LH}\pi^*_{HL}]N_H, \tag{A4}$$

$$(1-\Lambda)\alpha N_0 = \beta x^2[h\Pi_{HL}\pi^*_{LH} + (1-h)\Pi_{LL}\pi^*_{LL}]N_L. \tag{A5}$$

The active equilibrium condition similar to condition (16) yields

$$\Pi_{HH} = \pi^*_{HH} = \Pi_{LH} = \pi^*_{LH} = 1. \tag{A6}$$

As a result, we can rewrite equation (A2) and (A3) as

$$rV_H = \beta x^2[h(U + V_0 - V_H) + (1-h)\pi^*_{HL}(\theta U + V_0 - V_H)], \tag{A7}$$

$$rV_L = \beta x^2[h\Pi_{HL}(U + V_0 - V_L) + (1-h)\Pi_{LL}\pi^*_{LL}(\theta U + V_0 - V_L)], \tag{A8}$$

and solve Λ as a function of h

$$\Lambda = \frac{h[h + (1-h)\pi^*_{HL}]}{h[h + (1-h)\pi^*_{HL}] + (1-h)\eta[h\Pi_{HL} + (1-h)\Pi_{LL}\pi^*_{LL}]}. \tag{A9}$$

In the instantaneous production case, $V_0 = \max\{(V_L - \delta\varepsilon), (V_H - \varepsilon)\}$. Observe that $\theta U + V_0 - V_H \geq \theta U - \varepsilon > 0$ under Assumption 1. Therefore $\Pi_{HL} = \pi^*_{HL} = 1$. Similarly $\theta U + V_0 - V_L \geq \theta U - \delta\varepsilon > 0$, and $\Pi_{LL} = \pi^*_{LL} = 1$. From (A7) and (A8), we can find that $V_L = V_H$, which means only the low technology would be chosen, since $V_L - \delta\varepsilon > V_H - \varepsilon$.

When we have non-instantaneous production, it is a bit more complicated. If $V_H \leq V_L$, the producers will choose only the low technology, which requires less production cost and shorter production time. From equation (A7) and (A8) as well as $h = 0$, we can find that

$$V_H = \frac{\beta x^2 \pi^*_{HL}(\theta U + V_0)}{r + \beta x^2 \pi^*_{HL}}$$

and

$$V_L = \frac{\beta x^2 \Pi_{LL} \pi^*_{LL}(\theta U + V_0)}{r + \beta x^{22} \Pi_{LL} \pi^*_{LL}}$$

Hence $\pi^*_{HL} \leq \Pi_{LL}\pi^*_{LL}$. Meanwhile, $(\theta U + V_0 - V_H) \geq (\theta U + V_0 - V_L)$ implies that $\pi^*_{HL} \geq \pi^*_{LL} \geq \Pi_{LL}\pi^*_{LL}$. Since $\pi^*_{HL} = \pi^*_{LL} = 0$ leads to $V_L = 0$ and $V_0 < 0$, we must have $\pi^*_{HL} = \pi^*_{LL} = 1$, which is discussed in Case 1.

If $V_H > V_L$, we have $\theta U + V_0 - V_H < \theta U + V_0 - V_L$, and thus $\pi^*_{HL} \leq \pi^*_{LL}$. Note that we cannot have both mixed strategies at the same time. Therefore, we have only four cases to discuss: (1) $\pi^*_{HL} = \pi^*_{LL} = 1$; (2) $0 < \pi^*_{HL} < \pi^*_{LL} = 1$; (3) $0 = \pi^*_{HL} \leq \pi^*_{LL} < 1$; and (4) $0 = \pi^*_{HL} < \pi^*_{LL} = 1$.

Case 1: $\pi^*_{HL} = \pi^*_{LL} = 1$. It implies $V_H = V_L$, and the producers only choose the low technology ($h = 0$). The solutions are provided in Table 2.3 with

$$\rho_b = \frac{\beta x^2}{\beta x^2 + r}. \tag{A10}$$

The required condition is

$$\theta U + V_0 - V_L > 0$$

Case 2: $0 < \pi^*_{HL} < \pi^*_{LL} = 1$. The immediate implication is

$$\theta U + V_0 - V_H = 0 \tag{A11}$$

.Based on equation (A11), we can rewrite the value functions as

$$V_H = \frac{\beta x^2 h(1-\theta)U}{r}, \tag{A12}$$

$$V_0 = \frac{\beta x^2 h(1-\theta)U}{r} - \theta U, \tag{A13}$$

$$V_L = \frac{\beta x^2[h\Pi_{HL} + (1-h)] + r\Pi_{HL}}{\beta x^2[h\Pi_{HL} + (1-h)] + r} V_H. \tag{A14}$$

Observe that $V_0 \geq 0$ implies $h > 0$ and consequently $rV_0 = \eta\alpha(V_H - V_0 - \varepsilon) = \eta\alpha(\theta U - \varepsilon)$ in the case of positive production time.[11] We can combine it with equation (A13) to obtain the proportion of type-H goods

$$h_b = \frac{(\eta\alpha + r)\theta U - \eta\alpha\varepsilon}{\beta x^2 (1-\theta) U}. \tag{A15}$$

If $h = h_b < 1$, we can substitute (A15) into the expressions of V_H and V_0

$$V_H = \frac{(\eta\alpha + r)\theta U - \eta\alpha\varepsilon}{r} = \frac{\theta U - \Delta_H \varepsilon}{1 - \Delta_H} \tag{A16}$$

and

$$V_0 = \frac{\eta\alpha(\theta U - \varepsilon)}{r} = \Delta_H \frac{\theta U - \varepsilon}{1 - \Delta_H}. \tag{A17}$$

In order to make the producers indifferent between the two technologies, we need

$$V_L - V_0 - \delta\varepsilon = \eta(V_H - V_0 - \varepsilon). \tag{A18}$$

With the help of equations (A11), (A14) (A16), and (A17) we can convert equation (A18) into

$$\frac{1 - \Pi_{HL}}{\beta x^2 [h \Pi_{HL} + (1-h)] + r} = \frac{(1-\eta)\theta U + (\eta - \delta)\varepsilon}{(\eta\alpha + r)\theta U - \eta\alpha\varepsilon}.$$

and solve the cross-type acceptability, denoted as π_b. Note that $\pi_b < 1$ as long as $\theta U > \delta\varepsilon$. Actually this equilibrium is unstable if we disturb the acceptability Π_{HL} slightly away from its equilibrium level.

The other subcase is that $h_b = 1$. We must have some particular cost-utility ratio to satisfy equation (A15). Moreover, we need $\Pi_{HL} < \pi_b$ to discourage the producers from choosing the low technology. As a consequence, this equilibrium does not hold generically.

Case 3: $0 = \pi^*_{HL} \leq \pi^*_{LL} < 1$. If $\pi^*_{LL} > 0$, we have $V_L = 0$ and $\theta U + V_0 - V_L = 0$, which implies $V_0 < 0$. Similarly for $\pi^*_{LL} = 0$, we also have $V_0 < 0$ from the requirement of $V_L = 0$ and $\theta U + V_0 - V_L < 0$. None of them is plausible.

Case 4: $0 = \pi^*_{HL} < \pi^*_{LL} = 1$. It demands $V_L < \theta U + V_0 < V_H$. While the cross-type acceptability is zero, we may have separating equilibrium with

$$V_H = \frac{h\beta x^2 (U - \Delta_H \varepsilon)}{h\beta x^2 (1 - \Delta_H) + r},$$

[11] In a pure barter economy with instantaneous production, we have $V_0 = V_H - \varepsilon$, and hence we need $\theta U = \varepsilon$. It implies that this mixed equilibrium may not hold generically in the instantaneous production case.

	Equilibrium A[b]	Equilibrium B[b]	Equilibrium C[b]
π^*_{HL}	0	π_b	1
π^*_{LL}	1	1	1
h	0, or h_s, or 1	h_b	0
V_0	$\max\{\Delta_H(V_H-\varepsilon),\Delta_L(V_L-\delta\varepsilon)\}$	$\Delta_H\dfrac{\theta U-\varepsilon}{1-\Delta_H}$	$\Delta_L\dfrac{\rho_b\theta U-\delta\varepsilon}{1-\rho_b\Delta_L}$
V_H	$\dfrac{h\beta x^2(U-\Delta_H\varepsilon)}{h\beta x^2(1-\Delta_H)+r}$	$\dfrac{\theta U-\Delta_H\varepsilon}{1-\Delta_H}$	$\dfrac{\rho_b(\theta U-\Delta_L\delta\varepsilon)}{1-\rho_b\Delta_L}$
V_L	$\dfrac{(1-h)\beta x^2(U-\Delta_H\varepsilon)}{(1-h)\beta x^2(1-\Delta_H)+r}$	$\dfrac{\alpha+r}{r}\eta(\theta U-\varepsilon)+\delta\varepsilon$	$\dfrac{\rho_b(\theta U-\Delta_L\delta\varepsilon)}{1-\rho_b\Delta_L}$

Table 3: Solutions for Pure Barter Economy

$$V_L=\frac{(1-h)\beta x^2(U-\Delta_H\varepsilon)}{(1-h)\beta x^2(1-\Delta_H)+r},$$

and

$$V_0=\max\{\Delta_H(V_H-\varepsilon),\Delta_L(V_L-\delta\varepsilon)\}$$

Since an increase in h leads to bigger V_H and smaller V_L, the function $f(h)=\Delta_H(V_H-\varepsilon)-\Delta_L(V_L-\delta\varepsilon)$ is strictly increasing in h. Moreover, it is easy to find that $f(1)>0$, and $f(0)<0$. Consequently, there exists a unique $h_s\in(0,1)$, such that $f(h_s)=0$.

Now the producer's choice depends on the current level of h. If $h>h_s$, only the high quality goods will be produced. If $h<h_s$, we have the pure strategy equilibrium with only low technology. It means that the pure barter economy with zero cross-type acceptability would stick to the *old* technology. This is the typical *trap effect*.

Proposition A1 (pure barter economy) *In pure barter economy with instantaneous production, Assumption 1 implies that producers would choose the low technology only. In the case of non-instantaneous production, the producers would choose low technology provided perfect cross-type acceptability, and stick to the old technology in the case of zero cross-type acceptability. The mixed equilibrium is unstable in an economy with positive production time, and non-existent in the instantaneous production case.*

Appendix B: Proofs

In this appendix, we provide detailed mathematical derivations of some fundamental relationships and propositions presented in the main text.

Proof of Lemma 1:

In equilibrium A, we need $\pi_L = 0$, and hence $\theta U + V_0 - V_m < 0$. Using the solutions provided in Table 2.1, we can obtain

$$\theta U + \frac{\rho_H^A \rho_m^A U - \varepsilon}{1 - \rho_H^A \rho_m^A} - \frac{\rho_m^A (U - \varepsilon)}{1 - \rho_H^A \rho_m^A} < 0$$

or

$$\theta U - \varepsilon + \frac{\rho_H^A \rho_m^A (U - \varepsilon)}{1 - \rho_H^A \rho_m^A} - \frac{\rho_m^A (U - \varepsilon)}{1 - \rho_H^A \rho_m^A} < 0.$$

Therefore,

$$\frac{\theta U - \varepsilon}{U - \varepsilon} < \frac{\rho_m^A (1 - \rho_H^A)}{1 - \rho_H^A \rho_m^A}$$

Employing the definition of (8) and (17), we can multiply $(\beta \mu x + r)[\beta(1-\mu)x + r]$ to both the numerator and the denominator. Now we have

$$\frac{\theta U - \varepsilon}{U - \varepsilon} < \frac{\beta(1-\mu)x}{\beta x + r}$$

or

$$M < M_1 \equiv 1 - \frac{(\beta x + r)(\theta U - \varepsilon)}{\beta x (U - \varepsilon)} \tag{B1}$$

where we use the equilibrium result that $\mu = M$.

In addition, we also need the producer's value to be positive, *i.e.*

$$\frac{\rho_H^A \rho_m^A U - \varepsilon}{1 - \rho_H^A \rho_m^A} > 0.$$

Hence

$$\frac{\varepsilon}{U} < \rho_H^A \rho_m^A = \frac{\beta^2 x^2 \mu (1-\mu)}{\beta^2 x^2 \mu (1-\mu) + (\beta x + r) r}$$

or

$$\mu(1-\mu) > Q \equiv \frac{(\beta x + r) r \varepsilon}{\beta^2 x^2 (U - \varepsilon)}. \tag{B2}$$

Observe that the quadratic equation given by the equality in (B2) has two real roots within the interval $(0, 1)$, if Assumption 2 holds. To differentiate the two roots, we define the smaller root to be M_2. As a result, condition (B2) can be written as $M_2 < M < 1 - M_2$ in equilibrium.

In conclusion, the existence region for equilibrium A is given by $M < M_1$ and $M_2 < M < 1 - M_2$.

Proof of Lemma 2:

The derivation of the existence region is analogous to that of condition (B2). We only have to replace U and ε with θU and $\delta\varepsilon$ respectively. In addition, if $0 < \delta \le \theta < 1$ and Assumption 1 holds,

$$\frac{(\beta x + r)r\varepsilon}{\beta^2 x^2 (U - \varepsilon)} = \frac{(\beta x + r)r\delta\varepsilon}{\beta^2 x^2 (\delta U - \delta\varepsilon)} \ge \frac{(\beta x + r)r\delta\varepsilon}{\beta^2 x^2 (\theta U - \delta\varepsilon)}.$$

As a result, $S^A \subseteq S^C$.

Derivation of h^B and π^B*:*

Since $\theta U + V_0^B - V_m^B = 0$, we can rewrite the money holder's value (6) as

$$rV_m = \beta(1-\mu)xh(1-\theta)U.$$

Based on the solutions listed in Table 2.1, we have

$$h^B = \frac{r}{\beta(1-\mu)x(1-\theta)U}\frac{\theta U - \varepsilon}{1-\rho_H^B} = \frac{(\beta\mu x + r)(\theta U - \varepsilon)}{\beta(1-\mu)x(1-\theta)U}$$

While the producers are indifference between the two technologies, the two solutions of V_0^B listed in Table 2.1 should be the same, *i.e.*

$$\frac{\rho_H^B \theta U - \varepsilon}{1-\rho_H^B} = \frac{\rho_L^B \theta U - \delta\varepsilon}{1-\rho_L^B} = \frac{\rho_L^B(\theta U - \delta\varepsilon)}{1-\rho_L^B} - \delta\varepsilon.$$

Note that

$$\frac{\rho_L^B}{1-\rho_L^B} = \frac{\beta\mu x \Pi_L}{r} = \frac{\rho_H^B}{1-\rho_H^B}\Pi_L.$$

Therefore

$$\frac{\rho_H^B \theta U - \varepsilon}{1-\rho_H^B} = \frac{\rho_H^B(\theta U - \delta\varepsilon)}{1-\rho_H^B}\Pi_L - \delta\varepsilon$$

$$\pi^B = \Pi_L = \frac{\rho_H^B \theta U - \varepsilon + (1-\rho_H^B)\delta\varepsilon}{\rho_H^B(\theta U - \delta\varepsilon)} = 1 - \frac{(1-\delta)\varepsilon}{\rho_H^B(U - \delta\varepsilon)}$$

Proof of Lemma 3:

The conditions for existence come from the requirement of $V_0 > 0$, and $h^B, \pi^B \in (0,1)$, where h^B and π^B are given by equation (22) and (21), respectively. Assumption

1 implies that $h^B > 0$, while the condition $h^B < 1$ is equivalent to $\mu = M < M_1$. The latter comes from the fact that

$$\begin{aligned}(\beta x M_1 + r)(\theta U - \varepsilon) &= \left[\beta x - \frac{(\beta x + r)(\theta U - \varepsilon)}{U - \varepsilon} + r\right](\theta U - \varepsilon) \\ &= (\beta x + r)\frac{(1-\theta)U}{U - \varepsilon}(\theta U - \varepsilon) \\ &= \beta x(1 - M_1)(1-\theta)U\end{aligned}$$

and that h^B is increasing in μ.

Meanwhile, $V_0 > 0$ iff

$$\rho_H^B = \frac{\beta\mu x}{\beta\mu x + r} > \frac{\varepsilon}{\theta U}$$

or

$$\mu > M_4 \equiv \frac{r\varepsilon}{\beta x(\theta U - \varepsilon)}. \tag{B3}$$

Observe that condition (B3), along with Assumption 1, implies that

$$\pi^B > 1 - \frac{(1-\delta)\theta U}{\theta U - \delta\varepsilon} = \frac{\delta(\theta U - \varepsilon)}{\theta U - \delta\varepsilon} > 0,$$

while Assumption 1 also implies that $\pi^B < 1$.

Now consider the relationship between S^B and S^A. We know that S^B is non-empty, iff $M_4 < M_1$.Observe that, with $Q \equiv \frac{(\beta x + r)r\varepsilon}{\beta^2 x^2(U-\varepsilon)}$, we have

$$M_4(1 - M_1) = \frac{r\varepsilon}{\beta x(\theta U - \varepsilon)}\frac{(\beta x + r)(\theta U - \varepsilon)}{\beta x(U - \varepsilon)} = Q. \tag{B4}$$

Hence $M_1(1 - M_1) > M_4(1 - M_1) = Q$, and $M_4(1 - M_4) > M_4(1 - M_1) = Q$. By Lemma 1, $M_1 \in S^A$, and $M_4 \in S^A$. Consequently, $S^B = [M_4, M_1) \subseteq S^A$.

Proof of Proposition 1:

Since the stability is proved in the body text, only remaining work is to show that all the existence regions are non-empty under Assumption 1-3. Given Assumption 2, we know that $\frac{1}{2} \in (M_2, 1 - M_2)$, and $\frac{1}{2} \in S^C$. Now we need to establish $M_2 < M_1$. One sufficient condition is that $Q < M_1(1 - M_1)$, which boils down to

$$(U - \varepsilon)r\varepsilon < (\theta U - \varepsilon)[\beta x(1-\theta)U - r(\theta U - \varepsilon)],$$

or

$$\frac{1}{\theta U - \varepsilon} + \frac{\theta}{1-\theta} < \frac{\beta x}{r}.$$

Note that $Q < M_1(1-M_1)$ and equation (B4) imply $M_4 < M_1$. As a result, Assumption 1-3 guarantee that S^B is nonempty.

Derivation of the social welfare in the instantaneous production case:

In equilibrium A, the social welfare

$$\begin{aligned} Z^A &= MV_m^A + (1-M)V_H^A \\ &= M\frac{\rho_m^A(U-\varepsilon)}{1-\rho_H^A\rho_m^A} + (1-M)\frac{\rho_H^A\rho_m^A(U-\varepsilon)}{1-\rho_H^A\rho_m^A} \\ &= \frac{U-\varepsilon}{1-\rho_H^A\rho_m^A}\rho_m^A[\rho_H^A + (1-\rho_H^A)M] \\ &= \frac{U-\varepsilon}{(\beta x + r)r}\beta(1-\mu)x(\beta\mu x + rM) \\ &= \frac{\beta x M(1-M)(U-\varepsilon)}{r}, \end{aligned}$$

where the last equality employs the equilibrium result that $\mu = M$. Analogously, we can derive

$$Z^B = \frac{\beta x M(1-M)(\theta U - \delta\varepsilon)}{r}.$$

Proof of Proposition 2:

For each $M \in S^B$, $M < M_1$ and $\rho_H^A = \rho_H^B$. We have

$$\frac{V_m^A}{V_m^B} = \frac{\rho_m^A(1-\rho_H^B)}{1-\rho_H^A\rho_m^A}\frac{U-\varepsilon}{\theta U-\varepsilon} = \frac{\beta(1-\mu)xr}{(\beta x + r)r}\frac{U-\varepsilon}{\theta U - \varepsilon} > 1$$

and hence $V_H^A = \rho_H^A V_m^A > \rho_H^B V_m^B = V_H^B$. While the producers are indifferent between the two technologies, $V_H^B - \varepsilon = V_L^B - \delta\varepsilon$. Consequently $V_H^B > V_L^B$. So the goods trader's value in equilibrium A is always higher than that in equilibrium B. To the producers, we also have $V_0^A = V_H^A - \varepsilon > V_H^B - \varepsilon = V_0^B$. With the knowledge that $S^B \subseteq S^A$, we can conclude that equilibrium A Pareto dominates equilibrium B either for same M or at the optimal amount of money. The other parts are straightforward.

Derivation of h^{BB} and π^{BB}:

Since $\theta U + V_0^B - V_m^B = 0$, we can rewrite the money holder's value (6) as

$$rV_m = \beta(1-\mu)xh(1-\theta)U.$$

Based on the solution listed in Table 2.2, we have

$$h^{BB} = \frac{r}{\beta(1-\mu)x(1-\theta)U}\frac{\theta U - \Delta_H\varepsilon}{1-\rho_H^{BB}\Delta_H}$$

While the producers are indifference between the two technologies, two solutions for V_0^{BB} listed in Table 2.1 should be the same. Since $\theta U + V_0^{BB} - V_m^{BB} = 0$, we can also equate two solutions for money holder's value

$$\frac{\theta U - \Delta_H\varepsilon}{1-\rho_H^{BB}\Delta_H} = \frac{\theta U - \Delta_L\delta\varepsilon}{1-\rho_L^{BB}\Delta_L}$$

Therefore

$$\rho_L^{BB}\Delta_L = \rho_H^{BB}\Delta_H + \frac{\Delta_H\varepsilon - \Delta_L\delta\varepsilon}{\theta U - \Delta_H\varepsilon}(1-\rho_H^{BB}\Delta_H)$$

$$\frac{\rho_H^{BB}}{1-\rho_H^{BB}}\Pi_L = \frac{\rho_L^{BB}\Delta_L}{\Delta_L - \rho_L^{BB}\Delta_L} = \frac{(\theta U - \Delta_H\varepsilon)\rho_H^{BB}\Delta_H - (\Delta_H\varepsilon - \Delta_L\delta\varepsilon)(1-\rho_H^{BB}\Delta_H)}{(\theta U - \Delta_H\varepsilon)(\Delta_L - \rho_H^{BB}\Delta_H) + (\Delta_H\varepsilon - \Delta_L\delta\varepsilon)(1-\rho_H^{BB}\Delta_H)}$$

$$\begin{aligned}
\pi^{BB} &= \Pi_L = \frac{r}{\beta\mu x}\frac{(\theta U - \Delta_H\varepsilon)\rho_H^{BB}\Delta_H - (1-\rho_H^{BB}\Delta_H)(\Delta_H - \Delta_L\delta)\varepsilon}{(\theta U - \Delta_H\varepsilon)(\Delta_L - \rho_H^{BB}\Delta_H) + (1-\rho_H^{BB}\Delta_H)(\Delta_H - \Delta_L\delta)\varepsilon} \\
&= \frac{r}{\beta\mu x}\frac{\rho_H^{BB}\Delta_H\theta U - (\Delta_H - \Delta_L\delta + \rho_H^{BB}\Delta_H\Delta_L\delta)\varepsilon}{(\Delta_L - \rho_H^{BB}\Delta_H)\theta U + (\Delta_H - \Delta_L\delta + \rho_H^{BB}\Delta_H\Delta_L\delta - \Delta_H\Delta_L)\varepsilon}
\end{aligned}$$

After substituting the expressions of the effective discount factors, we can obtain the result given in the main text. Note that when $\Delta_H = \Delta_L = 1$,

$$\begin{aligned}
\pi^{BB} &= \frac{r}{\beta\mu x}\frac{\rho_H^{BB}\theta U - (1-\delta+\rho_H^{BB}\delta)\varepsilon}{(1-\rho_H^{BB})\theta U - (1-\rho_H^{BB})\delta\varepsilon} \\
&= \frac{r}{\beta\mu x}\frac{(\theta U - \delta\varepsilon)\rho_H^{BB} - (1-\delta)\varepsilon}{(\theta U - \delta\varepsilon)(1-\rho_H^{BB})} \\
&= \frac{(\theta U - \varepsilon)\rho_H^{BB} - (1-\delta)\varepsilon}{(\theta U - \delta\varepsilon)\rho_H^{BB}} = \pi^B
\end{aligned}$$

Proof of Proposition 3:

By comparing the solution for producer's values (V_0) in Table 2.1 and 2.2, we can find that the condition for $V_0 > 0$ would not change in the non-instantaneous production case. However, in Equilibrium AA, the condition $\theta U + V_0 - V_m < 0$ leads to

$$\theta U + \Delta_H\frac{\rho_H^{AA}\rho_m^{AA}U - \varepsilon}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H} - \frac{\rho_m^{AA}(U - \Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H} < 0$$

or

$$\frac{(1-\rho_m^{AA})(U-\Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H} < (1-\theta)U$$

Note that the left-hand side is strictly increasing in μ, since

$$\begin{aligned}
\frac{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}{1-\rho_m^{AA}} &= 1+\frac{\rho_m^{AA}-\rho_H^{AA}\rho_m^{AA}\Delta_H}{1-\rho_m^{AA}} = 1+\frac{\rho_m^{AA}(1-\rho_H^{AA}\Delta_H)}{1-\rho_m^{AA}} \\
&= 1+\frac{\rho_m^{AA}(1-\Delta_H)}{1-\rho_m^{AA}}+\frac{\rho_m^{AA}(1-\rho_H^{AA})\Delta_H}{1-\rho_m^{AA}} \\
&= 1+\frac{\beta(1-\mu)x}{r}(1-\Delta_H)+\frac{\beta(1-\mu)x}{\beta\mu x+r}\Delta_H.
\end{aligned}$$

Denote $\mu_1=\mu_1(\Delta_H)$ as the solution for

$$(1-\theta)U=\frac{(1-\rho_m^{AA})(U-\Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}=\frac{(\beta\mu x+r)r(U-\Delta_H\varepsilon)}{\beta^2x^2\mu(1-\mu)(1-\Delta_H)+r\beta x+r^2}. \tag{B5}$$

Hence we need $\mu<\mu_1$ to guarantee $\theta U+V_0-V_m<0$. By Assumption 1, $\theta U>\varepsilon$. Hence $\mu_1<1$. When $\Delta_H=1$,

$$1-\frac{\theta U-\varepsilon}{U-\varepsilon}=\frac{(1-\theta)U}{U-\varepsilon}=\frac{\beta\mu x+r}{\beta x+r}=1-\frac{\beta(1-\mu)x}{\beta x+r}.$$

Hence $\mu_1(1)=M_1$. Moreover,

$$\begin{aligned}
&\frac{(1-\rho_m^{AA})(U-\Delta_H\varepsilon)}{1-\rho_H^{AA}\rho_m^{AA}\Delta_H}-\frac{(1-\rho_m^{AA})\varepsilon}{\rho_H^{AA}\rho_m^{AA}} \\
=\;& (1-\rho_m^{AA})\frac{\rho_H^{AA}\rho_m^{AA}(U-\Delta_H\varepsilon)-(1-\rho_H^{AA}\rho_m^{AA}\Delta_H)\varepsilon}{(1-\rho_H^{AA}\rho_m^{AA}\Delta_H)\rho_H^{AA}\rho_m^{AA}} \\
=\;& (1-\rho_m^{AA})\frac{\rho_m^{AA}\rho_H^{AA}U-\varepsilon}{(1-\rho_H^{AA}\rho_m^{AA}\Delta_H)\rho_H^{AA}\rho_m^{AA}}\geq 0
\end{aligned}$$

as long as $V_0^{AA}>0$. It means the right-hand side of (B5) is just a constant plus a term that is increasing in Δ_H. Recall that this term is also strictly increasing in μ. Therefore the implicit function $\mu_1(\Delta_H)$ given by (B5) is decreasing in Δ_H, and $\mu_1(\Delta_H)\geq\mu_1(1)=M_1$ in non-instantaneous production case, where $\Delta_H<1$.

Consequently, Assumptions 1-3 also implies that all the existence regions are non-empty in the case of non-instantaneous production.

Derivation of the social welfare in the non-instantaneous production case:

Consider equilibrium AA with $h = 1$ first. From equation (25)-(29), along with the population identity $N_m + N_H + N_L + N_0 = 1$ and $N_m = M$ in equilibrium, we can solve

$$N_0 = \frac{\mu - M}{\mu}, \text{ and } N_H = \frac{M(1-\mu)}{\mu}.$$

Based on the equation (9), (10) and the solutions listed in Table 2.2, we have

$$\begin{aligned}
Z^{AA} &= \frac{\mu - M}{\mu} V_0^{AA} + \frac{M(1-\mu)}{\mu} V_H^{AA} + M V_m^{AA} \\
&= \frac{\mu - M}{\mu} \Delta_H (\rho_H^{AA} V_m^{AA} - \varepsilon) + \frac{M(1-\mu)}{\mu} \rho_H^{AA} V_m^{AA} + M V_m^{AA} \\
&= V_m^{AA} \left[\frac{\mu - M}{\mu} \Delta_H \rho_H^{AA} + \frac{M(1-\mu)}{\mu} \frac{\beta\mu x}{\beta\mu x + r} + M \right] - \frac{\mu - M}{\mu} \Delta_H \varepsilon \\
&= \frac{\rho_m^{AA}(U - \Delta_H \varepsilon)}{1 - \rho_H^{AA} \rho_m^{AA} \Delta_H} \left[\frac{\mu - M}{\mu} \Delta_H \rho_H^{AA} + M \frac{\beta x + r}{\beta\mu x + r} \right] - \frac{\mu - M}{\mu} \Delta_H \varepsilon \\
&= \frac{\mu - M}{\mu} \frac{\rho_m^{AA}(U - \Delta_H \varepsilon)}{1 - \rho_H^{AA} \rho_m^{AA} \Delta_H} \left[\Delta_H \rho_H^{AA} + \frac{M\mu}{\mu - M} \frac{\beta x + r}{\beta\mu x + r} \right] - \frac{\mu - M}{\mu} \Delta_H \varepsilon
\end{aligned}$$

Recall that, when $h = 1$, $M = \dfrac{\mu\eta\alpha}{\beta x \mu(1-\mu) + \eta\alpha}$, and,

$$\frac{\mu - M}{\mu} = \frac{\beta x \mu(1-\mu)}{\beta x \mu(1-\mu) + \eta\alpha}$$

$$\frac{M\mu}{\mu - M} = \frac{\eta\alpha}{\beta x(1-\mu)}$$

As a consequence,

$$\begin{aligned}
\frac{\mu}{\mu - M} Z^{AA} &= \frac{\rho_m^{AA}(U - \Delta_H \varepsilon)}{1 - \rho_H^{AA} \rho_m^{AA} \Delta_H} \left[\Delta_H \rho_H^{AA} + \frac{\eta\alpha}{\beta x(1-\mu)} \frac{\beta x + r}{\beta\mu x + r} \right] - \Delta_H \varepsilon \\
&= \frac{(U - \Delta_H \varepsilon)\eta\alpha \left[\beta\mu x \beta x(1-\mu) + (\eta\alpha + r)(\beta x + r) \right]}{\beta^2 x^2 \mu(1-\mu) r + (\eta\alpha + r)(r\beta x + r^2)} - \Delta_H \varepsilon \\
&= \frac{(U - \Delta_H \varepsilon)\eta\alpha}{r} - \Delta_H \varepsilon \\
&= \frac{\eta\alpha U - \Delta_H \varepsilon(\eta\alpha + r)}{r} \\
&= \frac{\eta\alpha(U - \varepsilon)}{r}
\end{aligned}$$

and

$$Z^{AA} = \frac{\mu - M}{\mu} \frac{\eta\alpha(U - \varepsilon)}{r} = \frac{\beta x \mu(1-\mu)}{\beta x \mu(1-\mu) + \eta\alpha} \frac{\eta\alpha(U - \varepsilon)}{r}.$$

We can compute Z^{BB} analogously.

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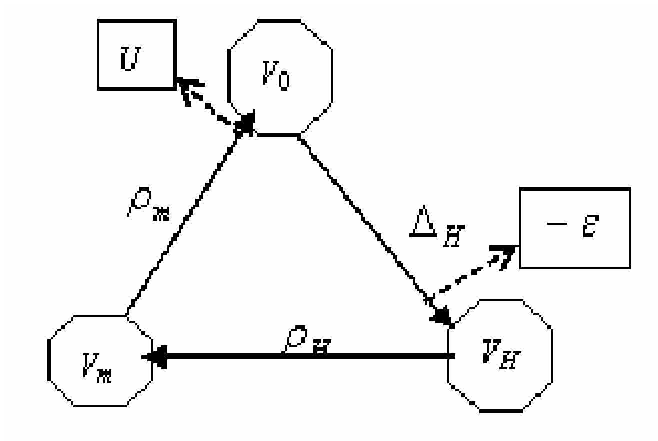

Figure 1: Steady-State Inflows and Outflows

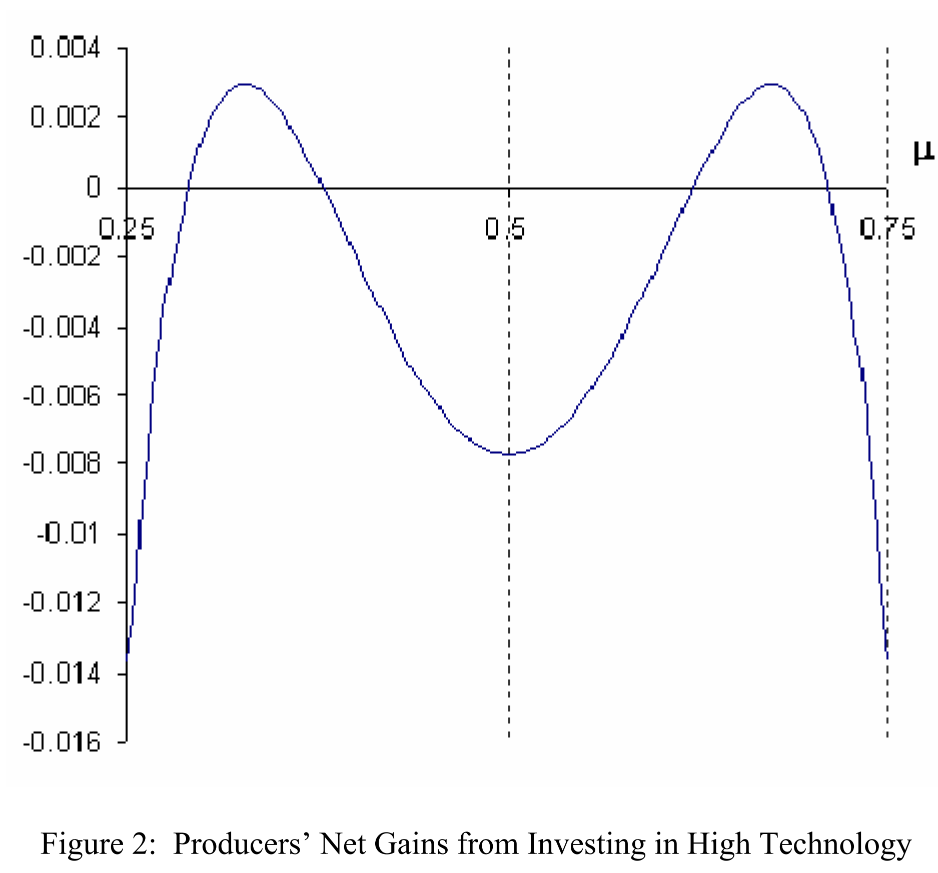

Figure 2: Producers' Net Gains from Investing in High Technology