# Financial intermediaries, markets, and growth

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**Abstract**

This paper contributes to the literature comparing the relative performance of financial intermediaries and markets by studying an environment in which a trade-off between risk sharing and growth arises endogenously. Financial intermediaries provide insurance to households against a liquidity shock. Households can also invest directly on a financial market, if they pay a cost. In equilibrium, the ability of intermediaries to share risk is constrained by the market. Moreover, intermediaries invest less in the productive technology when they provide more risk-sharing. This creates a trade-off between risk-sharing and growth. We show the balance of intermediaries and market that maximizes welfare depend on parameter values.

*Keywords:* Financial intermediaries; Financial markets; Risk-sharing; Growth

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# 1 Introduction

This paper studies a model in which financial intermediaries (which we also call banks) provide insurance to households against a liquidity shock. The extent to which these intermediaries are able to provide risk-sharing is constrained by the fact that household can, if they pay a cost, invest directly in assets on a financial market. In equilibrium, the larger the fraction of household active on the financial market, the more investment there is in a productive technology. This creates an endogenous trade-off between risk-sharing provided by intermediaries and growth. The model allows us to compare economies that are more bank-oriented (banks hold more assets and provide more risk sharing) with economies that are more market-oriented (individuals hold more assets and banks provide less risk sharing). As an example of the former, one can think of Germany while the US is an example of the latter.[1]

In a static model, growth does not occur and consumers only care about risk sharing. Hence the trade-off has no bite and more bank-oriented systems provide higher expected utility. In a dynamic environment, however, this trade-off matters. Market-oriented economies always yield more growth and may yield more expected utility. We study the optimal balance between intermediaries and markets in steady states and show it depends on parameter values. The optimal amount of risk sharing provided by intermediaries increases with risk aversion. For risk aversion parameters sufficiently low, it is optimal that bank provide no risk-sharing at all. As risk aversion increases, the optimal amount of risk-sharing increases until banks are no longer constrained in the risk-sharing they provide. Hence, economies populated by more risk-averse consumers should be expected to be more bank-oriented and grow slower than economies populated by less risk-averse consumers. The optimal amount of risk-sharing also decreases with the fraction of patient consumers.

We build on a model by Fecht (2003) in which banks play two different roles: First, as in Diamond and Dybvig (1983), they provide insurance to consumers against preference shocks. Second, as in Diamond and Rajan (2000 and 2001), they have a better ability to monitor projects than do unsophisticated depositors and thus intermediate investment for them. We assume consumers can pay a cost to become sophisticated. Sophisticated consumers have the same monitoring ability as banks. As shown in Fecht (2003), there arises a trade-off between the ability for the bank to provide risk-sharing and the number of sophisticated depositors. We embed the static model into a dynamic overlapping generations structure, as in Ennis and

[1] Allen and Gale (1995) provide a comparison of these two types of economies.

Keister (2003). In this context there is a trade-off between the amount of risk-sharing provided by banks and growth. An increase in risk sharing implies less investment in productive assets and less growth.

There is a large literature concerned with whether a well developed financial system can promote growth, particularly in developing countries. See Levine (1997) for a review. In this context, financial intermediaries and financial markets are typically viewed as complementing each other rather than as competing. In our paper, in contrast, financial intermediaries increase risk-sharing at the cost of growth. Financial markets, on the other hand, impede the ability of intermediaries to share risk which leads to higher growth in our model.

Some papers consider specifically the effects of financial intermediaries or financial markets on growth. Levine (1991) argues financial markets may promote growth. Jappelli and Pagano (1994), in contrast, provide evidence that financial market imperfections may increase savings rate and thus growth. Bencivenga and Smith (1991) study a model in which financial intermediaries enhance growth. These papers do not consider the interaction of markets and intermediaries.

Our paper is related to a literature which compares the performance of market and intermediaries (see, for example, Bhattacharya and Padilla 1996 or Fulghieri and Rovelli 1998). Maybe closest in spirit to our paper is the work by Allen and Gale (1997). To the best of our knowledge, this paper is the only one that considers the interplay of markets and intermediaries. These author consider an environment in which a financial intermediary can provide risk-sharing to overlapping generations of households. However, a financial market constrains the ability of intermediaries to provide this risk sharing. They show a system with an intermediary and no market can provide a Pareto improvement compared to a system in which the market is active.

Our model differs from theirs in several respect. For example, we do not consider long-lived intermediaries. We assume a new generation of banks arises with each new generation of households. This implies we do not consider inter-generational risk-sharing. In our model all risk-sharing occurs within each generation. Another difference is that in their framework risk arises because of a risky productive technology. Instead, our model considers a liquidity shock as in Diamond and Dybvig (1983). Despite these differences, our results are very close to theirs, at least in our static environment. In both their and our model a bank-oriented system is preferred because it allows more risk-sharing. Further, the extend to which banks can provide risk-sharing is limited by the financial market.

However, dramatically different conclusions arise when we account for the trade-off between risk-sharing and growth in our dynamic model. Allen and Gale (1997) are unable to study the impact of risk sharing on growth because their results rely heavily on the fact that the productive asset is in fixed supply. In contrast, our setup naturally extends to a dynamic case. Hence, maybe the most important result from our paper is the fact that the implications for growth play a crucial role in the optimal choice between financial intermediaries and a market.

The remainder of the paper proceeds as follows: Section 2 describes the static environment. Section 3 embeds the static model of section 2 in an OLG framework and describes our main results. Section 4 concludes.

# 2 Static environment

The environment described in this section is very similar to Fecht (2003). The economy takes place at three dates, $t = 0, 1, 2$, and is populated by a mass 1 of households, a large number of bank which compete for the households' deposits, and a large number of entrepreneurs.

Households learn at date $t = 1$ if they are patient (with probability $q$) or impatient (with probability $1 - q$). In the former case they only derive utility from consumption at date 1, and in the later case they only derive utility from consumption at date 2. Expected utility can be written $U(c_1, c_2) = qu(c_1) + (1 - q)u(c_2)$. The function $u$ exhibits CRRA: $u(c) = \frac{c^{1-\alpha}}{1-\alpha}$, with $\alpha > 1$. Whether a households is patient or impatient is private information.

There are two production technologies in the economy: A storage technology, which returns 1 unit of good at date $t + 1$ for each unit invested are date $t$, $t = 0, 1$, and a productive technology. The productive technology is operated costlessly by entrepreneurs who are not endowed with any goods. Entrepreneurs decide at date 1 either to "behave", in which case the technology has a return of $R$ at date 2 for each unit invested at date 0, or to "shirk", in which case the date 2 return is only $\gamma R$, with $R > 1 > \gamma R > 0$. Since the focus of the paper is not on bank runs driven by pessimistic expectations by patient depositors, we assume the productive technology has negligible scrap value if liquidated at date 1. Under this assumption runs do not occur.

Competition leads entrepreneurs to promise a repayment of $R$ at date 2 for each unit invested at date 0. At date 1 a secondary market is open on which claims to the return on the productive technology can be exchanged for goods. At date 2, entrepreneurs pay out the actual return of the project to the holder of the financial claim.

Households can either become sophisticated or remain unsophisticated. Sophisticated households can monitor entrepreneurs perfectly and are able to replace a misbehaving entrepreneur without forgoing any of the expected return of the project. Thus, these households can guarantee themselves a return of $R$ at date 2 if they lend to entrepreneurs. Unsophisticated households are unable to monitor entrepreneurs. Entrepreneurs financed by such households will always shirk and their projects will return only $\gamma R$ at date 2. Households choose whether or not to become sophisticated at date 0. To become sophisticated, a household must pay a utility cost proportional to its expected utility, $(\chi - 1)\,[qu(c_1) + (1-q)u(c_2)]$, where $\chi \geq 1$.[2]

There are several way to think of this cost. It could represent the cost of learning to become a financial analyst or of getting an MBA. Alternatively, it could be the effort spend in order to monitor entrepreneurs. In either case, the cost could be measured in terms of utility, resources, or both. The size of $\chi$ could be affected by the development financial markets, or the extent to which financial instruments are standardized, among other things. In particular, we assume a benevolent government would be able to affect this cost. For example the cost could be reduced by subsidizing the schooling necessary to become sophisticated. It could be increased by imposing restrictions on who is allowed to buy and trade financial claims. Below, we will treat $\chi$ as a policy variable and think of a benevolent government that chooses the cost in order to maximizes households' welfare.

Instead of investing directly in the market, households can deposit their endowment in a bank. Banks invest the deposits they have received in storage or in financial claims on the productive technology. They can also trade in the secondary financial market at date 1. Banks can monitor entrepreneurs costlessly and thus guarantee a return of $R$ for the projects they have invested in. Further, as in Diamond and Rajan (2001), banks can credibly commit to pay this return to a third party by setting up a deposit contract. Such a contract exposes banks to runs if they attempt to renegotiate the repayments they have promised depositors.[3]

In this environment, banks potentially play two different roles. On the one hand, they

[2]Assuming a proportional cost simplifies the analysis when we study a dynamic economy. However, we expect our results to hold for more general specifications of the cost. Our results hold also for a proportional resource cost as we show below.

[3]See Diamond and Rajan (2000) for a more complete exposition of this argument.

intermediate investment for unsophisticated households and thus allow them to indirectly invest in the productive technology, as in Diamond and Rajan (2000 and 2001). On the other hand, they can provide liquidity insurance to depositors who do not know whether they will be patient or impatient, as in Diamond and Dybvig (1983).

## 2.1 Equilibrium allocation

In this section we derive the contract offered by banks. At the beginning of date 0, banks choose the deposit contract they offer households and households decide whether or not to become sophisticated simultaneously.[4] Let $d_1$ denote the payment banks promise depositors who withdraw early, and $d_2$ denote the payment banks promise depositors who withdraw late. If banks provide any insurance against the liquidity shock, then $R > d_2 \geq d_1 > 1$. Fecht (2003) shows arbitrage pins the price of claims on the productive technology in the secondary market at 1 and competitive banks will supply the claims demanded by sophisticated depositors. Consequently, all households strictly prefer to deposit their endowment in a bank as long as banks provide some liquidity insurance. Indeed, for sophisticated households depositing in the bank and withdraw at date 1 yields $d_1$ which is greater than the resale value of claims on the productive technology they could have bought. At date 1, sophisticated households choose to buy claims on the productive technology in the secondary market. For unsophisticated households, depositing in a bank is the only way to benefit from the productive technology.

To summarize, at date 1, all impatient households withdraw and consume. Sophisticated patient households withdraw from the bank and invest on the secondary market since $Rd_1 \geq d_2$, with a strict inequality if banks provide some liquidity insurance. Banks are unable to prevent sophisticated household from withdrawing their deposits since a household's type is private information.

We can now write the problem of a competitive bank. The bank tries to maximize the utility of its unsophisticated depositors subject to a resource constraint. The bank's objective function is

$$qu(d_1) + (1 - q)u(d_2) \tag{2.1}$$

[4]If banks are allowed to move first they can offer a contract under which no household has an incentive to become sophisticated. Our results also hold in this case, as the cost of becoming sophisticated still influences the contract offered by banks, but then the secondary market is inactive.

and the resource constraint is

$$[qi + (1-i)]\, d_1 + (1-q)i\frac{d_2}{R} \leq 1. \tag{2.2}$$

This constraint says the bank must have enough resources to pay $d_2$ to a fraction $1-q$ of unsophisticated depositors at date 2 and $d_1$ to all sophisticated depositors as well as a fraction $q$ of unsophisticated depositors at date 1.

Contracts that maximize (2.1) subject to (2.2) are characterized by

$$d_1 = \frac{R}{R-(R-\Theta)(1-q)i}, \tag{2.3}$$

$$d_2 = \frac{R\Theta}{R-(R-\Theta)(1-q)i}, \tag{2.4}$$

where

$$\Theta \equiv \left[\frac{1-(1-q)i}{qi}R\right]^{\frac{1}{\alpha}}. \tag{2.5}$$

Such a contract will be an equilibrium contract only if it satisfies two incentive constraints. First, it must be the case $\gamma R d_1 \leq d_2$, otherwise unsophisticated depositors would withdraw their deposits to buy financial claims on the secondary market. This constraint is always satisfied since we assumed $1 > \gamma R$. The second constraint, which we refer to as $IC_S$, is $Rd_1 \geq d_2$. When $IC_S$ holds with equality, $\Theta = R$, and sophisticated patient depositors are indifferent between leaving their deposits in the bank and withdrawing them to invest in the secondary market. In this case, banks offer no more liquidity insurance. Define

$$\underline{i} \equiv \left[qR^{\alpha-1} + (1-q)\right]^{-1}. \tag{2.6}$$

$IC_S$ binds whenever $i \leq \underline{i}$. If this happens, the contract is given by equations (2.3) and (2.4) with $\Theta = R$.

The equilibrium mass of unsophisticated depositors, $i$, is determined by the condition that depositors must be indifferent between becoming sophisticated or remaining unsophisticated. This condition is

$$qu(d_1) + (1-q)u(d_2) = \chi\left[qu(d_1) + (1-q)u(d_1R)\right]. \tag{2.7}$$

We can use equations (2.3) and (2.4) to substitute for $d_1$ and $d_2$ in that expression. Then, using the fact that $u$ is CRRA, we can write

$$\Theta^{1-\alpha} = \chi R^{1-\alpha} + \frac{q}{1-q}(\chi - 1). \tag{2.8}$$

Using the definition of $\Theta$, we obtain the following expression for $i$

$$i = \left\{ (1-q) + \frac{q}{R} \left[ \chi \left( R^{1-\alpha} + \frac{q}{1-q} \right) - \frac{q}{1-q} \right]^{\frac{\alpha}{1-\alpha}} \right\}^{-1}. \tag{2.9}$$

It can easily be seen that an increase in $\chi$, the cost of becoming sophisticated, will lead to an increase in $i$, the fraction of unsophisticated depositors. As expected, $i = \underline{i}$ if there is no cost of becoming sophisticated, or $\chi = 1$. We can also find the cost above which no depositor becomes sophisticated, denoted by $\bar{\chi}$, by setting $i = 1$ in the above equation. We obtain

$$\bar{\chi} = \frac{(1-q)R^{\frac{1-\alpha}{\alpha}} + q}{(1-q)R^{1-\alpha} + q}. \tag{2.10}$$

If $\chi \geq \bar{\chi}$ the cost of becoming sophisticated is so high that no depositors chooses to become sophisticated.

We can derive the amount of investment in the productive technology in this economy, denoted by $K$. Part of the investment, $(1-q)i(d_2/R)$, is needed to provide consumption for unsophisticated patient depositors who withdraw at date 2. The rest, $(1-q)(1-i)d_1$ is sold to patient sophisticated depositors on the secondary market. The expression for $K$ is thus

$$K(i) = 1 - \frac{q}{1-(1-q)i(1-\frac{\Theta}{R})}. \tag{2.11}$$

It is decreasing in $i$. In particular, $K(i = \underline{i}) = 1 - q$ and

$$K(i=1) = 1 - \frac{q}{1-(1-q)(1-R^{\frac{1-\alpha}{\alpha}})}. \tag{2.12}$$

The above model gives us a way to think about financial systems being more bank-based or more market-oriented. When the cost of becoming sophisticated is high, there are few such depositors ($i$ is large) and the secondary market for financial claims is not very active. Banks are able to offer a lot of liquidity insurance but there is relatively little investment in the productive technology. Conversely, when the cost of becoming sophisticated is low, there a many such depositors ($i$ is small) and the secondary market is very active. Banks offer little liquidity insurance, or none at all, but there is more aggregate investment in the productive technology. Hence, when comparing two economies, $A$ and $B$, with a different fraction fraction of sophisticated depositors, $i_A > i_B$, we say economy $A$ is more bank oriented or, equivalently, economy $B$ is more market oriented.

## 2.2 The resource-cost case

The setup is identical except that a young household who decides to become sophisticated at the beginning of period $t$ will incur a $(1 - C)$ percent consumption loss at the end of period $t$ or the beginning of period $t + 1$, for some $C \leq 1$.[5] In this case, equation (2.7) becomes

$$qu(d_1) + (1 - q)u(d_2) = [qu(Cd_1) + (1 - q)u(Cd_1R)]\,. \tag{2.13}$$

We can use equations (2.3) and (2.4) to substitute for $d_1$ and $d_2$. Then, since $u$ is CRRA, we have

$$\Theta^{1-\alpha} = C^{1-\alpha}R^{1-\alpha} + \frac{q}{1-q}(C^{1-\alpha} - 1). \tag{2.14}$$

Using the definition of $\Theta$, we obtain the following expression for $i$

$$i = \left\{(1-q) + \frac{q}{R}\left[C^{1-\alpha}R^{1-\alpha} + \frac{q}{1-q}(C^{1-\alpha} - 1)\right]^{\frac{\alpha}{1-\alpha}}\right\}^{-1}. \tag{2.15}$$

The rest of the analysis is similar.

## 2.3 Comparison with a planner's allocation

It is interesting to compare the equilibrium allocation with the allocation chosen by a planner endowed with the technologies described above. Since bank runs do not occur in this setting, the planner does not have to be concerned with households misrepresenting their types. The planner's problem is

$$\max_{c_1,c_2} \quad qu(c_1) + (1-q)u(c_2)$$

subject to

$$qc_1 + (1-q)\frac{c_2}{R} \leq 1. \tag{2.16}$$

The planner's allocation, denoted $\{c_1^*, c_2^*\}$, is given by

$$c_1^* = \frac{1}{1 - (1 - R^{\frac{1-\alpha}{\alpha}})(1-q)}, \tag{2.17}$$

$$c_2^* = \frac{R^{\frac{1}{\alpha}}}{1 - (1 - R^{\frac{1-\alpha}{\alpha}})(1-q)}, \tag{2.18}$$

It is straightforward to see the equilibrium allocation of an economy with $i = 1$ corresponds to the planner's allocation. This occurs if the cost of becoming sophisticated is sufficiently

[5] We implicitly assume that, at date 0, when households decide to become sophisticated or not, they are able to commit to paying the resource cost when they receive $d_1$ from the bank.

high. In this static model, because capital accumulation does not matter, the expected utility of households is always decreasing as the cost of becoming sophisticated decreases. Hence, welfare is higher when banks are able to provide more risk sharing between patient and impatient depositors and the financial market is small. This result is reminiscent of Allen and Gale (1997). They study an environment in which the market constrains how much risk sharing financial intermediaries can provide. They show, in they model, having intermediaries and no financial markets is preferable to a financial market and no intermediaries. As in our static model, the intuition for their result is that more risk sharing is provided in the former case than in the latter.

A key feature of the model in Allen and Gale (1997) is that the productive asset is in fixed supply. Hence it is difficult to extend that environment to include growth. In contrast, it is straightforward to adapt our setup to a dynamic environment. The next section shows there is a real trade-off between risk-sharing and growth in a dynamic environment. Hence, the result that bank-based financial systems are always better is overturned in that context.

# 3 An OLG Environment with Growth

In this section, we embed the static model of the previous section in a two-period OLG framework along the lines of Ennis and Keister (2003). This allows us to think about how changes in the number of sophisticated households affect capital accumulation and growth.

As in Ennis and Keister (2003), but in contrast to allen and Gale (1997) or Bhattacharya and Padilla (1996), we assume a new set of banks arises with each new generation and banks maximize the expected utility of their unsophisticated depositors. In that sense, banks are not long-lived institutions in our model. Our result should extend to an environment with long-lives intermediaries as long as the amount of risk sharing that can be provided by intermediaries depends on the fraction of sophisticated depositors in the economy.

Each period is divided into two subperiods: in the first subperiod (the beginning), production occurs, factors get paid, and young households can deposit their wage income in one of a large number of perfectly competitive banks. Banks purchase existing capital from old households and decide on new investment and storage. In the second subperiod (the end), depositors observe whether they are patient or impatient and they can claim their consumption goods or shares of capital. The detail are presented below.

**The beginning of period $t$:** At the beginning of period $t$ each old household owns $K_t$ units of capital and young households are endowed with $L_t = 1$ units of time. Competitive entrepreneurs combine the capital and labor to produce a single consumption good $Y_t$ using the following production function: $Y_t = \bar{K}_t^{1-\theta} L_t{}^{1-\theta} K_t{}^{\theta}$. The assumption of perfect competition in the factor markets, and the fact that labor is supplied inelastically, implies the equilibrium real wage and real capital rental rate in units of the consumption good are given by $w_t = (1-\theta)K_t$ and $r_t = \theta$, respectively.

After the production takes place, each old household cashes in $[r_t + (1-\delta)p_t^-]K_t$ units of consumption good, consumes them, and exits the economy. Here $p_t^-$ denotes the price of capital in units of the consumption good in the beginning-of-period capital market. Note, in order for old households to be willing to rent their capital to firms before selling to the banks, it must be that $r_t \geq \delta p_t^-$. We show below this condition always holds under our parameter restrictions.

Each young household has $w_t$ units of consumption good in hand and is not sure whether she will become patient or impatient until the end of the period. These households deposit all their wage income in a perfectly competitive bank and enter a deposit contract $(d_{1t}, d_{2t})$. The bank uses part of the deposits to purchase the existing capital $(1-\delta)K_t$, at the price $p_t^-$, from old households and divides the rest of the deposits between storage and investment in new capital. As in the static model, one unit of consumption placed into the storage at the beginning of period $t$ yields one unit of consumption at the end of the period and one unit of consumption placed into the investment at the beginning of period $t$ yields $R > 1$ units of capital at the beginning of period $t+1$. Early liquidation of the investment will result in infinitesimal return. Note, only banks engage in purchasing existing capital, investing in new capital, and putting goods in storage at the beginning of the period. We impose parameter restrictions so the market for existing capital always clears.

As in the static model, young households decide whether or not to become sophisticated at the same time banks offer the deposit contract $(d_{1t}, d_{2t})$. A young household who decides to become sophisticated must exert some effort and incurs a cost of $(\chi - 1)$ percent of lifetime utility, for some $\chi \geq 1$. We consider the case of a proportional resource cost below.

**The end of period $t$:** Each young depositor realizes whether she is patient or impatient. Impatient depositors only value consumption in this subperiod when they are young while patient depositors only value consumption in the first subperiod of $t+1$ when they become old. The nature of the deposit contract is such that a depositor who claims to be impatient gets paid $d_{1t}$ in this subperiod, while a depositor who claims to be patient will get paid $d_{2t}$ in the first subperiod of $t+1$. As will be shown, the deposit contract offered by banks induces sophisticated patient depositors to misrepresent themselves as being impatient. Depositors can purchase capital from the banks at the price $p_t^+$. As was the case in the static model, banks are unable to prevent patient sophisticated depositors from withdrawing because being sophisticated is private information. Further, competition leads banks to supply the financial claims sophisticated households desire.

The price of existing capital in the first subperiod (primary) capital market under which the banks will be indifferent between purchasing existing capital and investing in new capital is given by

$$p_t^- = R^{-1}, \quad \forall t. \tag{3.1}$$

Our parameter restrictions to be specified below will ensure that this is the only equilibrium price for the existing capital in the primary market.

For convenience, we introduce the following notation:

$$X \equiv R[r_t + (1-\delta)p_t^-] = R[\theta + (1-\delta)R^{-1}] = R\theta + 1 - \delta. \tag{3.2}$$

In other words, $X$ is the return on long-term investment in the first subperiod of each period. We assume $X > 1$ and $\gamma X < 1$. Note, $X > 1$ implies $r_t \geq \delta p_t^-$, the condition for old households to strictly prefer renting their capital to firms before selling it to banks.

Given the availability of the storage technology, the equilibrium price of capital in the second subperiod (secondary) capital market must satisfy

$$p_t^+ = R^{-1}, \quad \forall t. \tag{3.3}$$

With this setup the optimal contract is essentially the same as in the previous section with $X$ replacing $R$ in the expressions below. We have, taking $i_t$ as given, the following problem

$$\begin{aligned}
\max_{d_{1t},d_{2t}} \quad & [qu(d_{1t}) + (1-q)u(d_{2t})] & \\
\text{s.t.} \quad & [qi_t + (1-i_t)]d_{1t} + (1-q)i_t\frac{d_{2t}}{X} \leq w_t & (BC) \\
& \max\{1; X\}\, d_{1t} \geq d_{2t} & (IC_S) \\
& \max\{1; X\}\, d_{1t} \leq d_{2t} & (IC_U)
\end{aligned}$$

The definitions of $\Theta_t$ and $\underline{i}$ also are very similar.

$$\Theta_t \equiv \left[\frac{1-(1-q)i_t}{qi_t}X\right]^{\frac{1}{\alpha}}, \tag{3.4}$$
$$\underline{i} \equiv [qX^{\alpha-1}+(1-q)]^{-1}. \tag{3.5}$$

Solving the maximization problem subject to the (BC) only yields:

$$d_{1t} = \frac{X(1-\theta)K_t}{X-(X-\Theta_t)(1-q)i_t}, \tag{3.6}$$
$$d_{2t} = \frac{X\Theta_t(1-\theta)K_t}{X-(X-\Theta_t)(1-q)i_t}. \tag{3.7}$$

Taking the deposit contract as given, $i_t$ is determined by

$$qu(d_{1t})+(1-q)u(d_{2t}) = \chi[qu(d_{1t})+(1-q)u(d_{1t}X)]. \tag{3.8}$$

The expression for $\bar{\chi}$ is now

$$\bar{\chi} = \frac{(1-q)R^{\frac{1-\alpha}{\alpha}}+q}{(1-q)R^{1-\alpha}+q}. \tag{3.9}$$

We consider $\chi \in [1, \bar{\chi}]$, which guarantees the endogenously determined $i_t \in [\underline{i}, 1]$. To see this, substituting (3.6) and (3.7) into (3.8) to obtain

$$i_t = \frac{X}{(1-q)X+qA}, \tag{3.10}$$

where $A$ is given by

$$A \equiv \left[\frac{q(U-1)+U(1-q)X^{1-\alpha}}{1-q}\right]^{\frac{\alpha}{1-\alpha}}. \tag{3.11}$$

For the remainder of the paper we drop the indexes for $i_t$ and $\Theta_t$ since they are time independent.

We focus on a symmetric equilibrium in which each bank holds the same portfolio. The law of motion for capital is given by

$$\begin{aligned} K_{t+1} &= (1-q)(1-i)\frac{d_{1t}}{p_t^+}+(1-q)i\frac{d_{2t}}{X}R \\ &= \frac{X-(X-\Theta)i}{X-(X-\Theta)(1-q)i}R(1-q)(1-\theta)K_t \\ &= \frac{\Theta-qX+qA}{(1-q)\Theta+qA}R(1-q)(1-\theta)K_t. \end{aligned} \tag{3.12}$$

It can be verified that the growth rate of the capital stock, defined by

$$\rho = \frac{\Theta-qX+qA}{(1-q)\Theta+qA}R(1-q)(1-\theta), \tag{3.13}$$

is strictly decreasing in $\chi$. Intuitively, a larger cost to becoming sophisticated results in less sophisticated households participating in the capital market. There is less investment in the productive technology and thus a smaller growth rate. The growth rate is greater than or equal to $1-\delta$ (implying that markets for existing capital clear) for all $\chi \in [1, \bar{\chi}]$ if and only if

$$\frac{R(1-q)(1-\theta)}{1-\delta} \geq (1-q) + qX^{\frac{\alpha-1}{\alpha}}. \tag{3.14}$$

The necessary and sufficient condition for actual growth, that is, for the growth rate to be greater than or equal to 1 (implying net investment is larger than or equal to replacement capital), for all $\chi \in [1, \bar{\chi}]$ is that

$$R(1-q)(1-\theta) \geq (1-q) + qX^{\frac{\alpha-1}{\alpha}}. \tag{3.15}$$

## 3.1 Welfare Analysis

Let $\beta \in (0,1)$ denote the social discount factor. Social welfare is equal to

$$W = \sum_{t=1}^{\infty} \beta^t [qu(d_{1t}) + (1-q)u(d_{2t})] \tag{3.16}$$

plus the utility of the initial old households given by $u([\theta + R^{-1}(1-\delta)]K_0)$, which will not affect our following analysis and thus will omitted below. Note,

$$d_{1t} = G\rho^t, \tag{3.17}$$

$$d_{2t} = \Theta G\rho^t, \tag{3.18}$$

where

$$G \equiv \frac{X(1-\theta)K_0}{X - (X-\Theta)(1-q)i}. \tag{3.19}$$

The expression for $G$ is very similar to the expression for $d_{1t}$, with $K_0$ taking the place of $K_t$. Hence, $G$ is related to the amount of investment in the storage technology. The direct effect of an increase in $G$ is to increase consumption, and thus welfare, but such an increase could reduce growth and thus, indirectly, welfare. We call $G$ the level effect. Clearly, $\Theta$ corresponds to the risk sharing effect. An increase in the value of $\Theta$ means a reduction in risk sharing. The direct effect of this is to reduce welfare. However, in equilibrium, a reduction in risk sharing is accompanied by an increase in the number of sophisticated depositors. This, indirectly, increases growth.

We are interested in the effect of a change in the cost $\chi$ on welfare. It is easy to derive the following relations:

$$\rho'(\chi) < 0, \quad \Theta'(\chi) < 0, \quad G'(\chi) > 0, \quad i'(\chi) > 0. \tag{3.20}$$

While a larger cost to becoming sophisticated tends to reduce both $d_{1t}$ and $d_{2t}$ through slowing growth, it tends to increase both $d_{1t}$ and $d_{2t}$ through increasing $G$. There is thus a tradeoff between the level of consumption households enjoy and the growth rate of the capital stock. An economy can start with a high level of consumption and grow relatively slowly or, instead, start at a lower level of consumption and grow faster. A larger cost also leads to more risk sharing and more liquidity insurance and thus tends to reduce $d_{2t}$ through decreasing $\Theta$. In this dynamic environment, there is a trade-off between growth and risk-sharing. Increasing one must decrease the other.

We think of $\chi$ as a policy variable a benevolent government can choose. The effects we just described imply a change in $\chi$ may have conflicting effect on social welfare. A given value for $\chi$ results in a given mix of markets and banks and we are interested to know which $\chi$ corresponds to an optimal structure in the sense that the resulting balance between growth and risk sharing maximizes the social welfare.

Assuming $\beta < \rho^{\alpha-1}$, we can solve for the social welfare as

$$W = \frac{\beta}{1-\alpha} \frac{G^{1-\alpha}[q + (1-q)\Theta^{1-\alpha}]}{\rho^{\alpha-1} - \beta}. \tag{3.21}$$

As expected, welfare increases with $G$, the level effect, and with $\rho$, the growth effect (recall $\alpha > 1$). An increase in $\Theta$, corresponding to a decrease in risk-sharing, affects welfare positively, which is counterintuitive. Here it is important to remember that $G$, $\rho$, and $\Theta$ are all functions of deeper parameters which are ultimately affecting welfare. An increase in $\Theta$ can be consistent with an increase in welfare if the deeper parameter responsible for the change in $\Theta$ also leads to, for example, an increase in $\rho$.

We want to find the value of $\chi$ that maximizes $W$. Such an optimum exists since $W$ is continuous on a compact domain of the cost. It is also clear that such an "optimal" cost is a function of $q, X, \theta, \delta, \alpha$, and $\beta$, but is independent of the initial capital $K_0$. An immediate implication is that a country's optimal bank-market mix is independent of its initial wealth.

We are unable to obtain analytical results for the value of $\chi$ that maximizes this expression. Instead, we look at some numerical simulations to get an idea of the trade-offs involved. Parameters for the production function are standard from the macro literature; we choose

$\theta = 0.33$, $\delta = 0.1$. The model imposes $r = \theta$. We also choose $R = 10$, so the range of equilibrium values of $i$ on our graphs below is large. Note, the inequality $rR > \delta$ is satisfied as it needs to be. Our baseline for preference parameters is $\alpha = 0.3$, $q = 0.2$, and $\beta = 0.98$. We did extensive robustness checks over the parameter space and find that our results are not sensitive to our choice of parameters.[6]

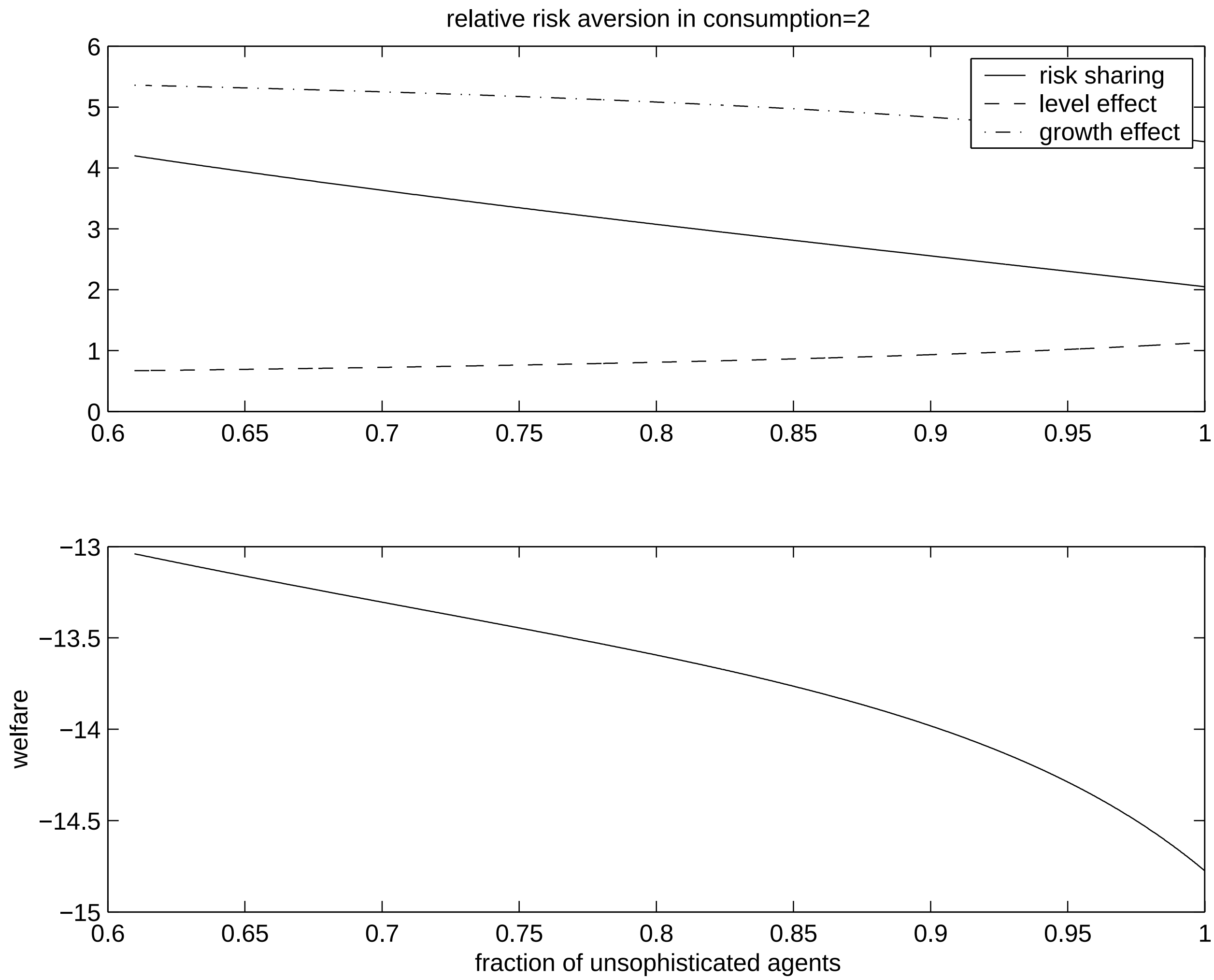


Figure 1. The case with a utility cost

Our first numerical exercise concerns the effect of risk sharing on the optimal trade-off between financial intermediaries and the market. We use the baseline parameters for all variables except for the coefficient $\alpha$ which we let vary. In each figure, we provide two graphs. The top graph shows the evolution of $\Theta$, $G$, and $\rho$ for different values of $i$. Here, $i$ is determined

[6]We use Matlab to compute the analytical solutions to the model. The code is available from the authors upon request.

endogenously as $\chi$ varies between 1 and $\bar{\chi}$.[7] The bottom graph shows the evolution of welfare for different values of $i$.

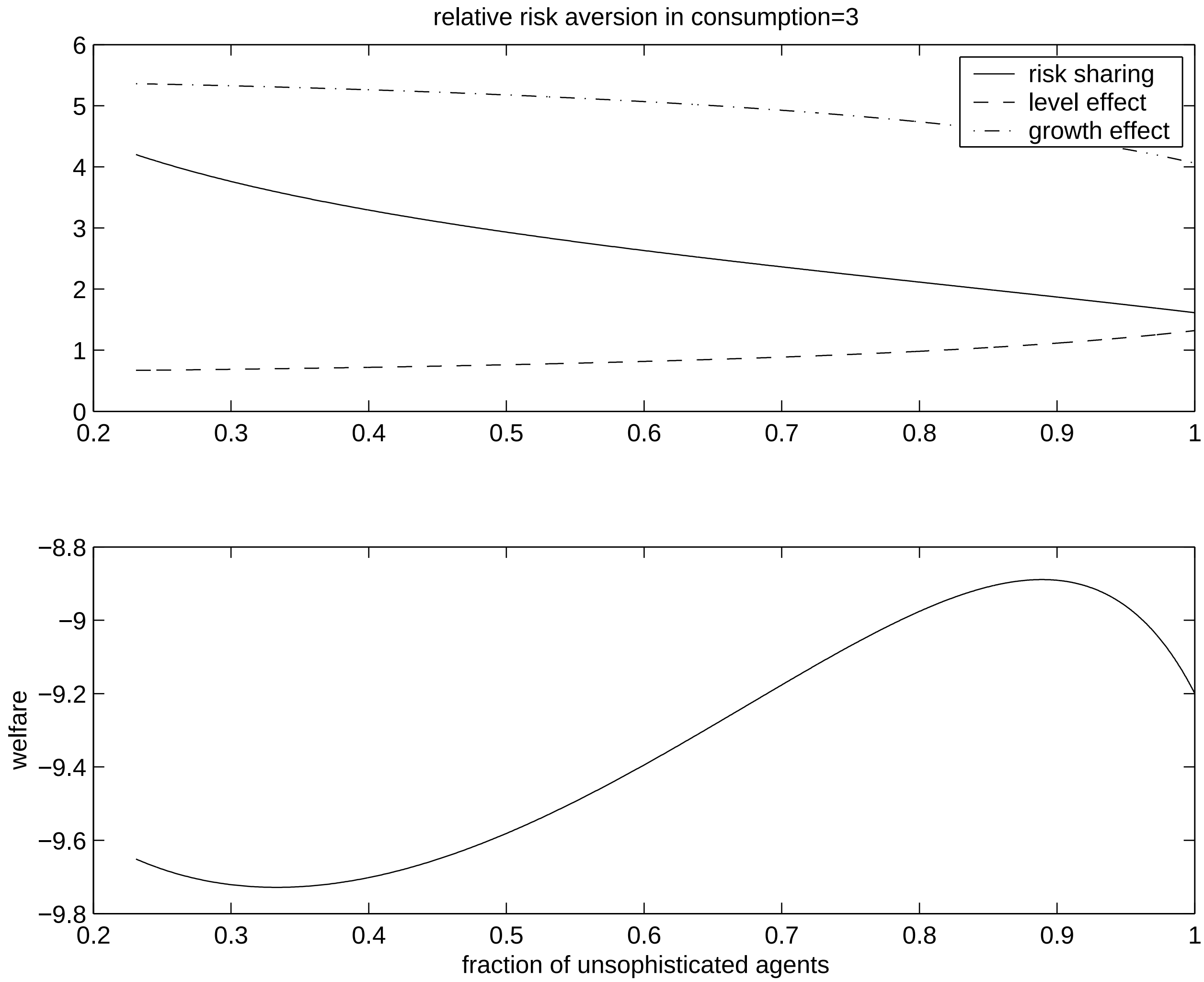


Figure 2. The case with a utility cost

As can be seen in Figures 1, 2, and 3, the maximum amount of welfare is reached for a higher level of the cost $\chi$ as the value of $\alpha$ increases. When the coefficient of risk aversion is low ($\alpha = 2$), as in Figure 1, welfare is maximized when the cost of becoming sophisticated is zero and bank offer no risk sharing. For a higher coefficient or risk aversion ($\alpha = 3$), as in figure 2, the optimal cost $\chi$ belongs to the interval $(1, \bar{\chi})$. It is optimal for banks to offer some risk sharing, but less than in the static case. Finally, for an even higher coefficient of risk aversion ($\alpha = 5$), as in Figure 3, the optimal cost is high enough that no household becomes

[7]There is a bijective mapping between $\chi$ and $i$.

sophisticated. In this case banks are not constrained in the amount of risk sharing they can provide but growth is slow.

The graphs representing $\Theta$, $G$, and $\rho$ are very similar in each case. As expected, the growth effect decreases with $i$ as there is less investment in the productive technology. An increase in $i$ also means a decrease in $\Theta$ which corresponds to an increase in risk sharing as the difference between $d_{1t}$ and $d_{2t}$ decreases. Finally, an increase in $i$ is accompanied by an increase in the level effect $G$.

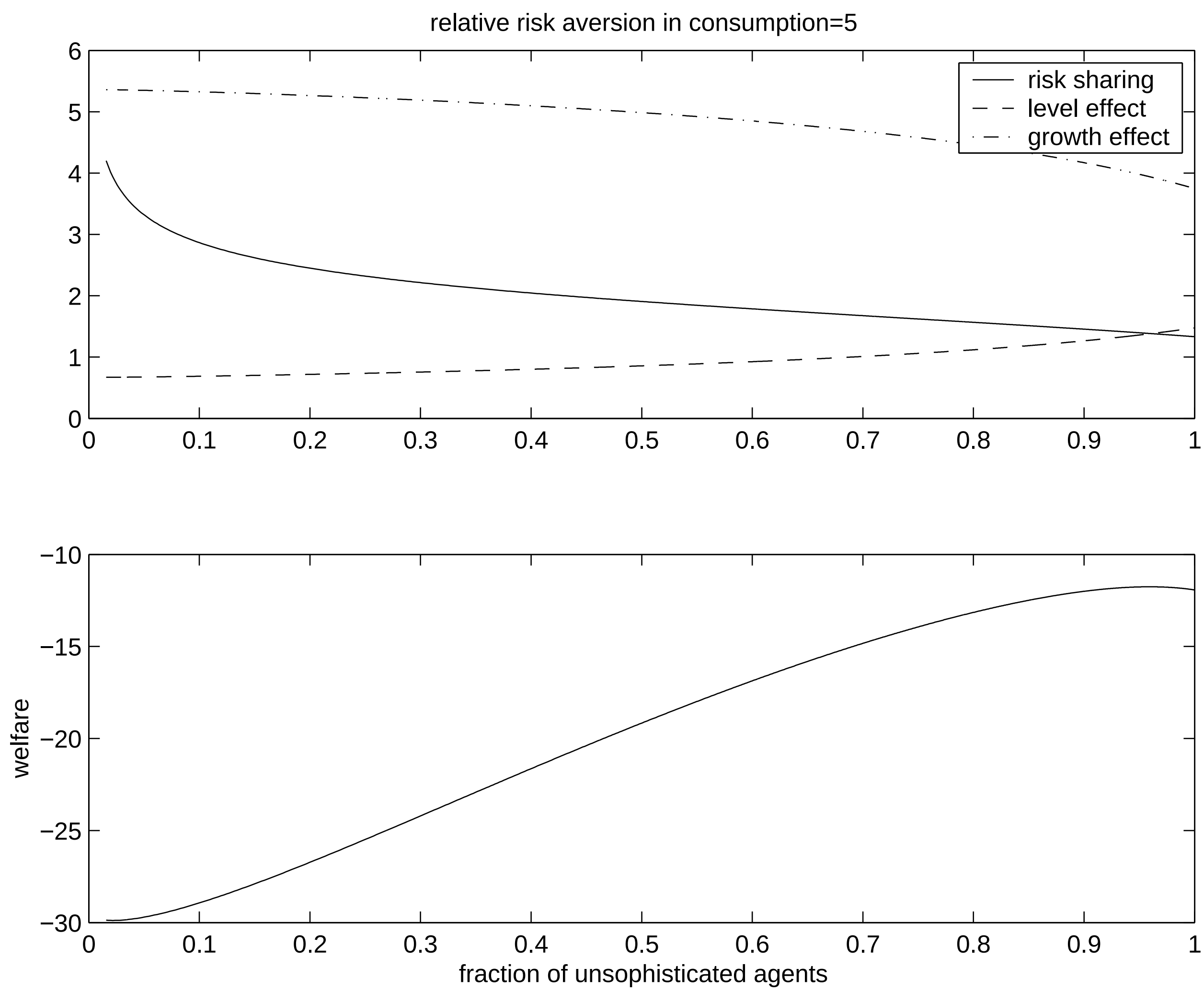


Figure 3. The case with a utility cost

By looking a the individual effects, it is possible to get an idea of how the overall welfare changes. Comparing Figures 1, 2, and 3 the main difference, for small values of $i$ (corresponding to small values of $\chi$) is in the risk sharing effect. The increase in the amount of risk sharing provided by banks, as $i$ increases from low values, is much faster in Figure 1 than in

Figure 2, and in Figure 2 than in Figure 3. Comparing the same figures, the main differences for large value of $i$ (corresponding to large values of $\chi$) are in the growth and the level effect.

This helps explain the shape of welfare as a function of $i$. For low values of $i$, an increase in the coefficient of risk aversion increases the effect on risk sharing. This means the effect on welfare from an increase in $\chi$ gradually changes from being negative to becoming positive. The main driving force of the changes for higher values of $i$ are the changes in the growth and the level effect. These go in opposite direction and it is hard to see from the graphs why the growth effect becomes relatively less important as the coefficient of risk aversion increases. Nevertheless, for a high enough value of this coefficient, welfare is maximized if no household becomes sophisticated.

To summarize the results from our numerical exercises, we can say that if two economies $A$ and $B$ are populated by households who have coefficients of risk aversion $\alpha_A$ and $\alpha_B$, respectively, where $\alpha_A > \alpha_B$, then households in economy $A$ prefer a more bank oriented system than households in economy $B$. As a consequence, economy $A$ will have a lower level of capital than economy $B$. When $\alpha$ is sufficiently small, the optimal system is such that banks provide no risk sharing. Intuitively, if consumers are not very risk averse they do not value risk sharing very much and an increase in risk sharing cannot compensate for a decrease in the level of consumption that accompanies a reduction of the capital stock. Conversely, if households are sufficiently risk averse the optimal system is such that banks are not constrained in the amount of risk sharing they provide.

In the appendix we report the result of another experiment where we change the value of $q$, keeping all other parameters as in our baseline case. In Figures 7, 8, and 9, we see that if $q$ is sufficiently small ($q = 0.1$), welfare is maximized in a bank-only system. As $q$ increases ($q = 0.2$), the maximum welfare is reached with a mix of banks and market, where banks play a smaller and smaller role. Finally, for high values of $q$ ($q = 0.3$), a market-only system maximizes welfare. The intuition for this result is straightforward. If the probability of becoming impatient is small, households put more weight on the consumption they get when they are sophisticated, which increases when there is less risk sharing.

We also did some experiments changing $\beta$ while keeping other parameters constant. Perhaps surprisingly, changes in $\beta$ have very little effect on the value of $\chi$ that maximizes social welfare. We do not report graphs for this case.

### 3.2 The resource-cost case

We now consider the case of a resource cost. All relations up to (3.7) hold as before. Taking the deposit contract as given, the equation for determining $i_t$ is now given by

$$qu(d_{1t}) + (1-q)u(d_{2t}) = qu(Cd_{1t}) + (1-q)u(Cd_{1t}R). \tag{3.22}$$

Let $\underline{C}$ denote the cost which leads to $i = 1$. Then,

$$\underline{C} = \left[ \frac{(1-q)R^{-\frac{(\alpha-1)^2}{\alpha}} + qR^{\frac{\alpha-1}{\alpha}}}{(1-q) + qR^{\frac{\alpha-1}{\alpha}}} \right]^{\frac{1}{\alpha-1}}. \tag{3.23}$$

We consider $C \in [\underline{C}, 1]$, which guarantees the endogenously determined $i_t \in [\underline{i}, 1]$. To see this, substitute (3.6) and (3.7) into (3.22) to obtain

$$i_t = \frac{R}{(1-q)R + qB}, \tag{3.24}$$

which is constant over time, where

$$B \equiv \left[ \frac{q(C^{1-\alpha} - 1) + C^{1-\alpha}(1-q)R^{1-\alpha}}{1-q} \right]^{\frac{\alpha}{1-\alpha}}. \tag{3.25}$$

It can then be verified that as $C$ varies from 1 to $\underline{C}$, $i_t$ varies from $\underline{i}$ to 1. Note that since the corresponding $\Theta_t > 1$ and $\gamma R < 1$, the solution in (3.6) and (3.7) satisfies ($IC_U$). The solution also satisfies ($IC_S$) since $R \geq \Theta_t$. Note also that since $i_t \leq 1$, we have $\Theta_t \geq R^{1/\alpha}$. We again drop the indexes for $i_t$ and $\Theta_t$ since they are time independent.

Since $B$ is increasing in $C$, $i$ is decreasing in the cost of becoming sophisticated. In words, the smaller $C$, the larger the fraction of households who choose to become sophisticated.

The analysis so far is homomorphic to the case with a utility cost, with the underlying linkage $C^{1-\alpha} = \chi$. The implication for capital accumulation is, however, slightly different here. We shall again focus on a symmetric equilibrium in which each bank holds the same portfolio. The law of motion for capital in one region is now given by

$$\begin{aligned} K_{t+1} &= (1-q)(1-i)C\frac{d_{1t}}{p_t^+} + (1-q)i\frac{d_{2t}}{R}X \\ &= \frac{CR - (CR - \Theta)i}{R - (R-\Theta)(1-q)i} X(1-q)(1-\theta)K_t \\ &= \frac{\Theta - qCR + qCB}{(1-q)\Theta + qB} X(1-q)(1-\theta)K_t. \end{aligned} \tag{3.26}$$

Note, unlike in the case with a utility cost there are here two opposite effects of a resource cost on the growth rate. The smaller the cost of becoming sophisticated, the more households want to become sophisticated. This tends to help investment and growth on the one hand. On the other hand, as more households become sophisticated, they use resources to pay the cost. It can be shown the positive effect always dominates the negative effect. In consequence, the growth rate, defined by

$$\rho = \frac{\Theta - qCR + qCB}{(1-q)\Theta + qB} X(1-q)(1-\theta), \tag{3.27}$$

is strictly increasing in $C$. It is then easy to show the growth rate is greater than or equal to $1-\delta$ for all $C \in [\underline{C}, 1]$ if and only if (3.14) holds, and it is greater than or equal to 1 for all $C \in [\underline{C}, 1]$ if and only if (3.15) holds.

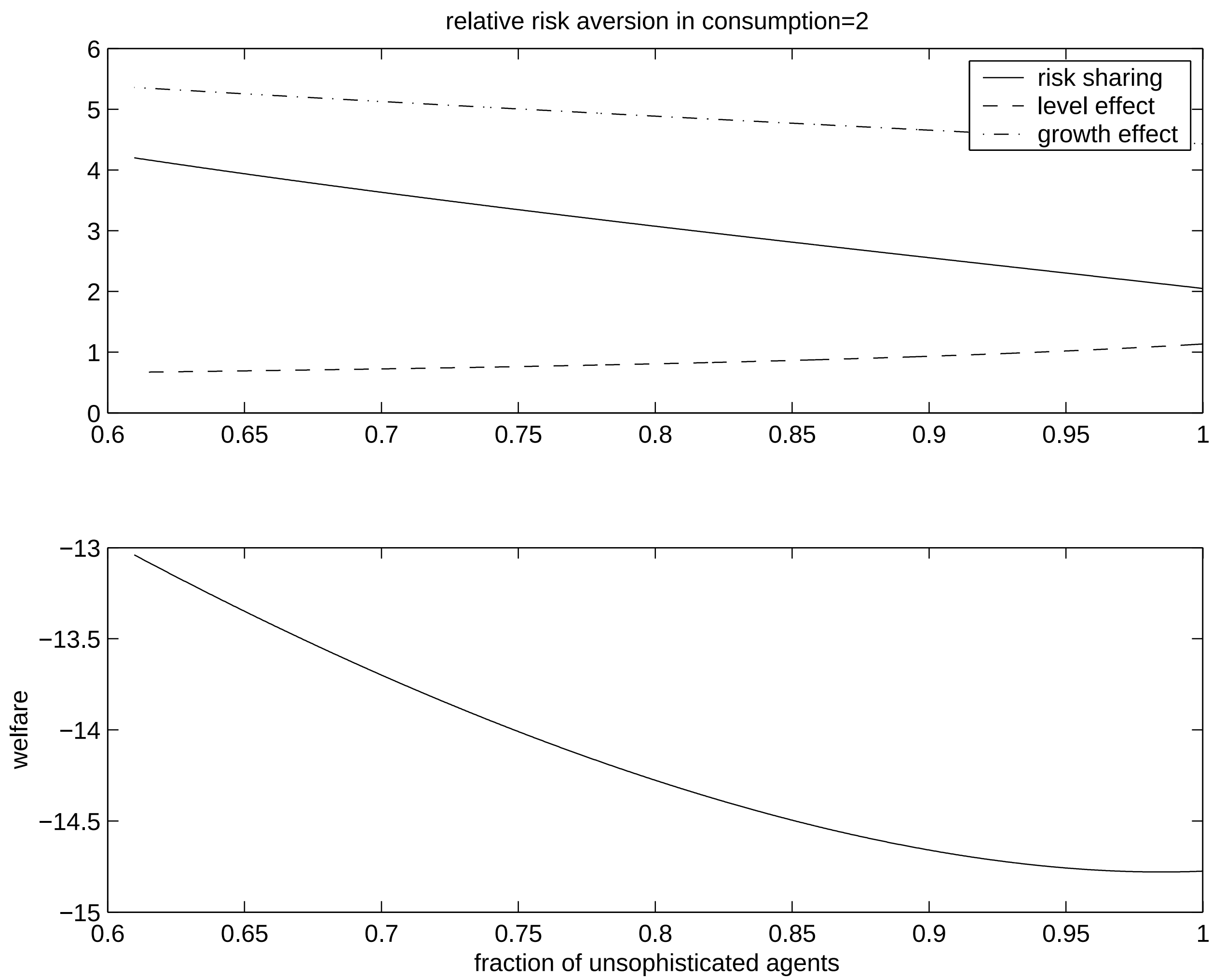


Figure 4. The case with a resource cost

Thus, regardless of how the cost is modelled, a general lesson is that a smaller cost leads to more sophisticated households and a more market-oriented economy. While this results in less risk sharing and less liquidity insurance, it promotes more economic growth. What mix of banks and markets is optimal depends on what mix of growth and risk sharing is optimal from a welfare point of view. We turn now to examining this issue.

The expression for welfare in this case is similar to the utility-cost case. It is easy to derive the following relations.

$$\rho'(C) > 0, \quad \Theta'(C) > 0, \quad (G)'(C) < 0, \quad i'(C) < 0. \tag{3.28}$$

We run a similar set of numerical experiments for the resource-cost case as we did for the utility-cost case. The parameters for our baseline experiments are the same except for $\alpha$. We let $\alpha = 2.85$. This value is chosen because, as can be seen on Figure 5, in this case welfare is the same in an economy where banks provide no risk sharing as it is in an economy where banks are unconstrained in the amount of risk sharing they provide.

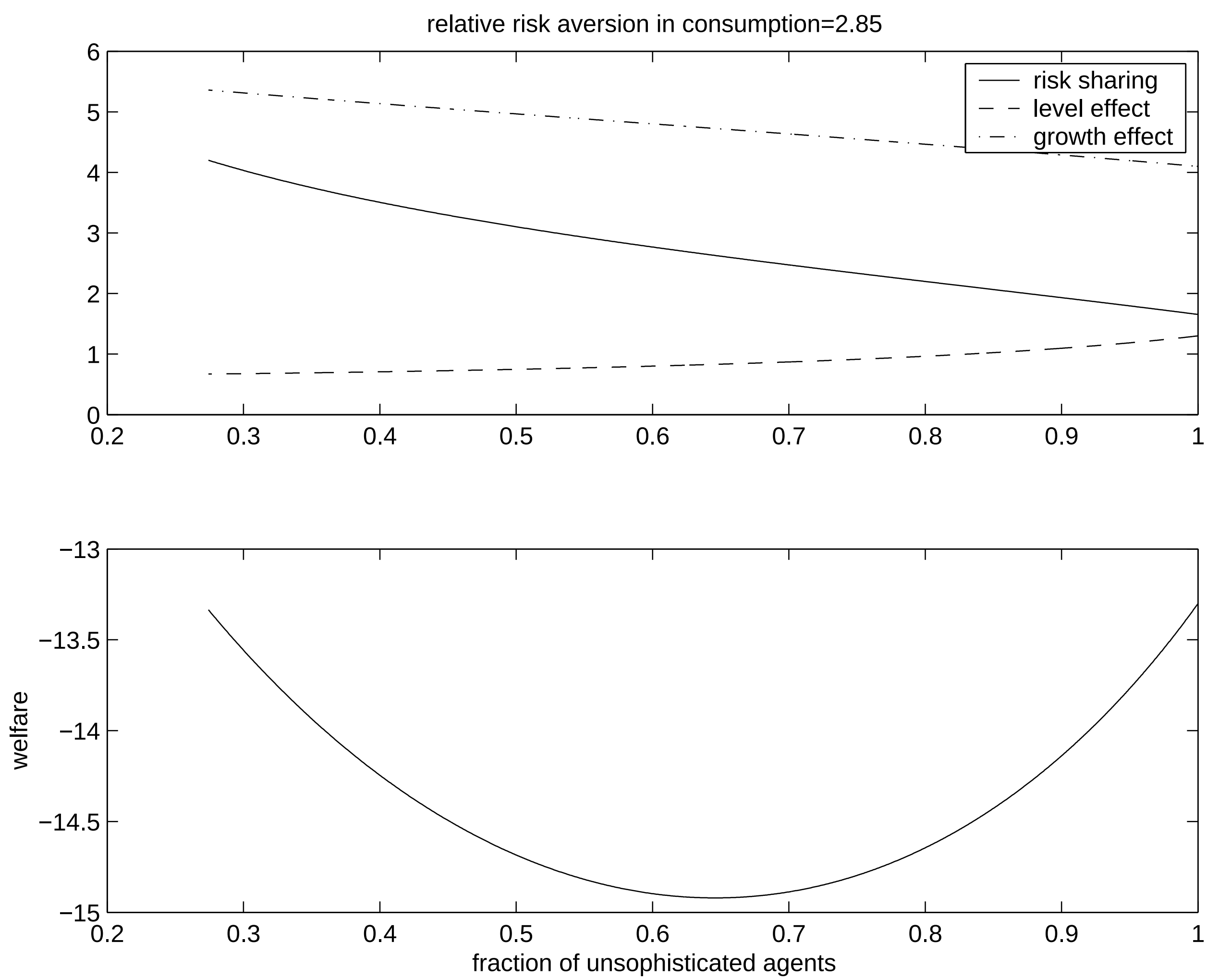


Figure 5. The case with a resource cost

Figures 4, 5, and 6, graph welfare, as well as the three effects that determine it, for different values of the risk-aversion coefficient (in these graphs, $\alpha = 2, 2.85$, and 4, respectively). The graphs confirm the general story told in the utility-cost case. When risk aversion increase, there is a shift from a market-oriented to a bank-oriented system. Interestingly, with a resource cost we were unable to find cases where the optimal cost corresponds to $i \in (\underline{i}, 1)$. In words, welfare is maximized either when banks provide no risk-sharing, or when they are unconstrained in how much risk-sharing they can provide.

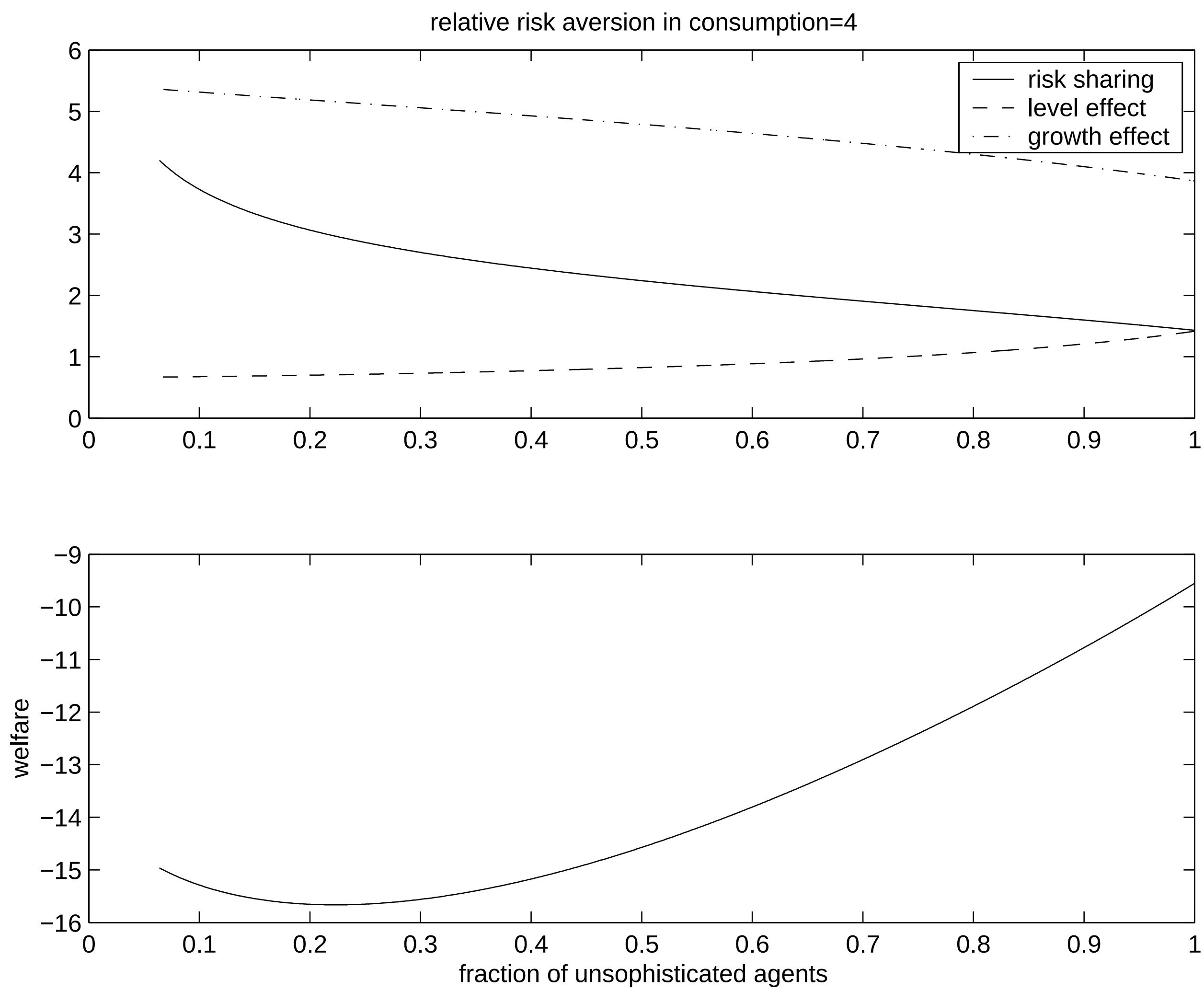


Figure 6. The case with a resource cost

As noted above, one important difference between the utility-cost and the resource-cost case is that in the latter the cost paid to become sophisticated reduces the capital stock and thus the growth rate of the economy. This effect helps explain why having a mix of banks and markets is never optimal in the resource-cost case.

Figures 10, 11, and 12, in the appendix, show welfare for different values of $q$, the fraction of impatient depositors in the economy. As was the case for the utility cost, an increase in $q$ leads to a shift from a market-dominated system to a bank-dominated system in the resource-cost case. The intuition for this result is the same for both type of costs. Finally, we considered different values of $\beta$. Again, changes in the value of $\beta$ have very little impact on the value of $\chi$ that maximizes social welfare. We do not report graphs for this case.

# 4 Conclusion

This paper contributes to the literature comparing the relative performance of financial intermediaries and markets by studying an environment in which a trade-off between risk sharing and growth arises endogenously. We consider a model in which financial intermediaries provide insurance to households against a liquidity shock, as in Diamond and Dybvig (1983). Households can also invest directly on a financial market, if they pay a cost. In equilibrium, we show the ability of intermediaries to provide risk-sharing is constrained by the market. The more households invest directly in the market, the less risk-sharing intermediaries can provide. Moreover, intermediaries invest less in the productive technology when they provide more risk sharing. This creates a trade-off between risk-sharing and growth.

We are able to show economies that are more market-oriented always enjoy higher growth, although not necessarily higher welfare. We are unable to obtain analytical solutions for welfare so we provide some numerical examples. In particular, we are interested in the optimal balance between intermediaries and markets (or equivalently between risk-sharing and growth) in different economies. We find, everything else being equal, economies in which households are more risk averse should be more bank-oriented. The intuition is that if households care less about risk, they value the increase in the growth rate of the economy more than the loss in risk sharing. These results are robust to changes in the value of the parameters in our numerical simulations.

It is interesting to contrast our paper with the work by Allen and Gale (1997). These author study an environment in which a financial intermediary provides insurance to households and show a market constrains the ability of the intermediary to share risk. This result is very similar to what we obtain in our static model and one conclusion one might draw is that financial intermediaries a preferable to markets because of their ability to provide risk-sharing. This result, however, is overturned when we consider a dynamic setting and take into account the fact that there might be a trade-off between risk-sharing and growth. This, we think, is the most interesting finding of our paper.

# 5 Appendix

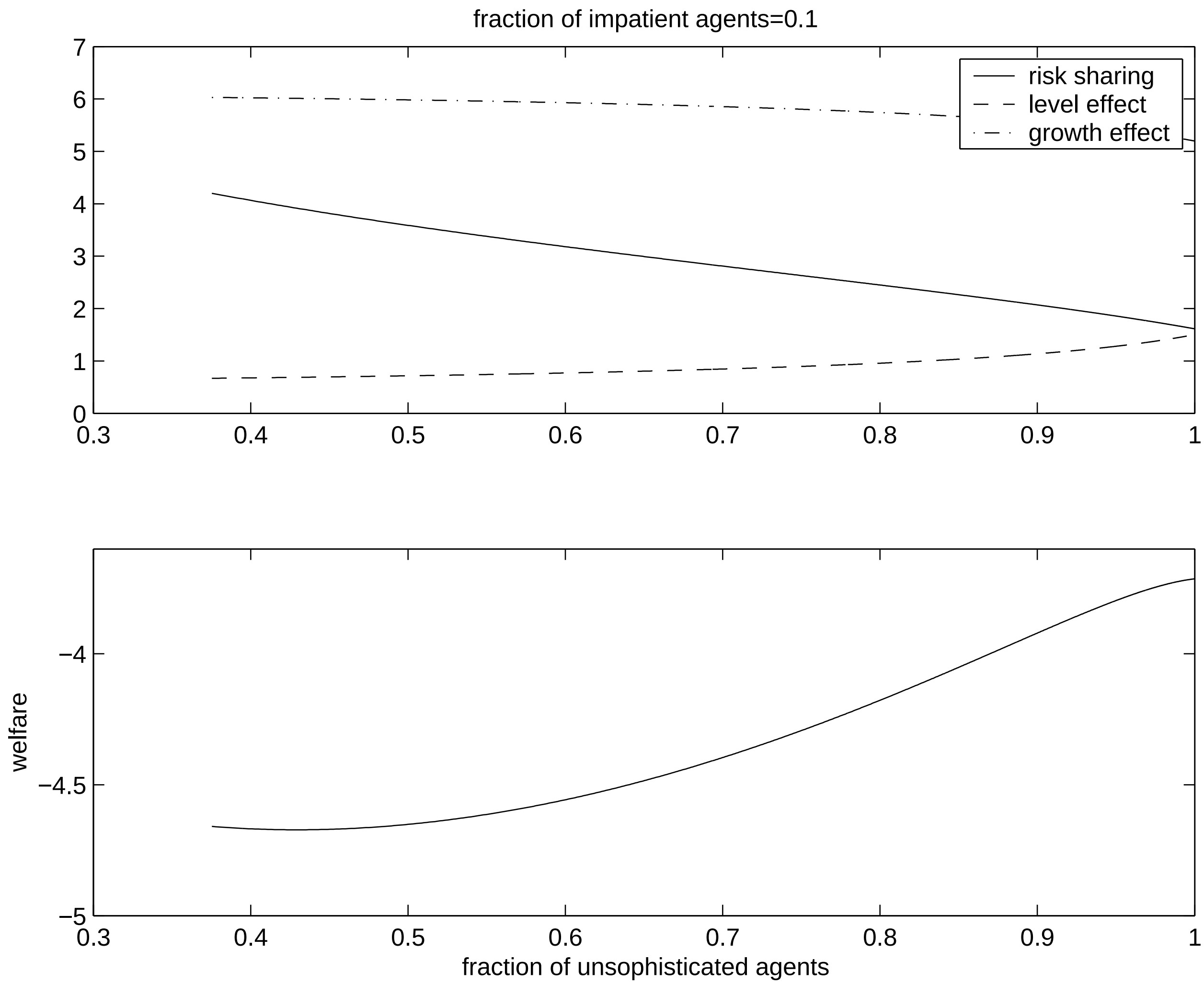


Figure 7. The case with a utility cost

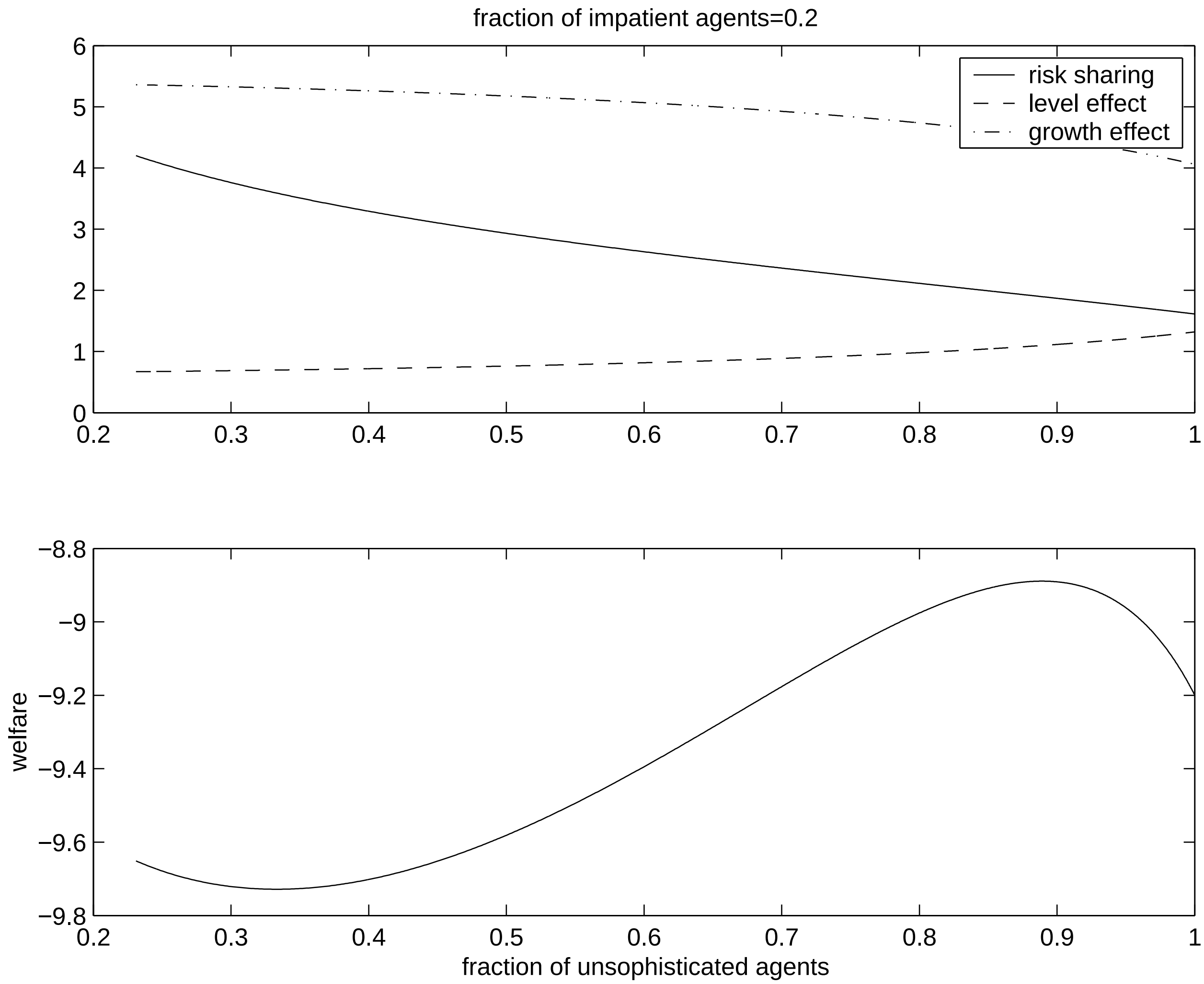


Figure 8. The case with a utility cost

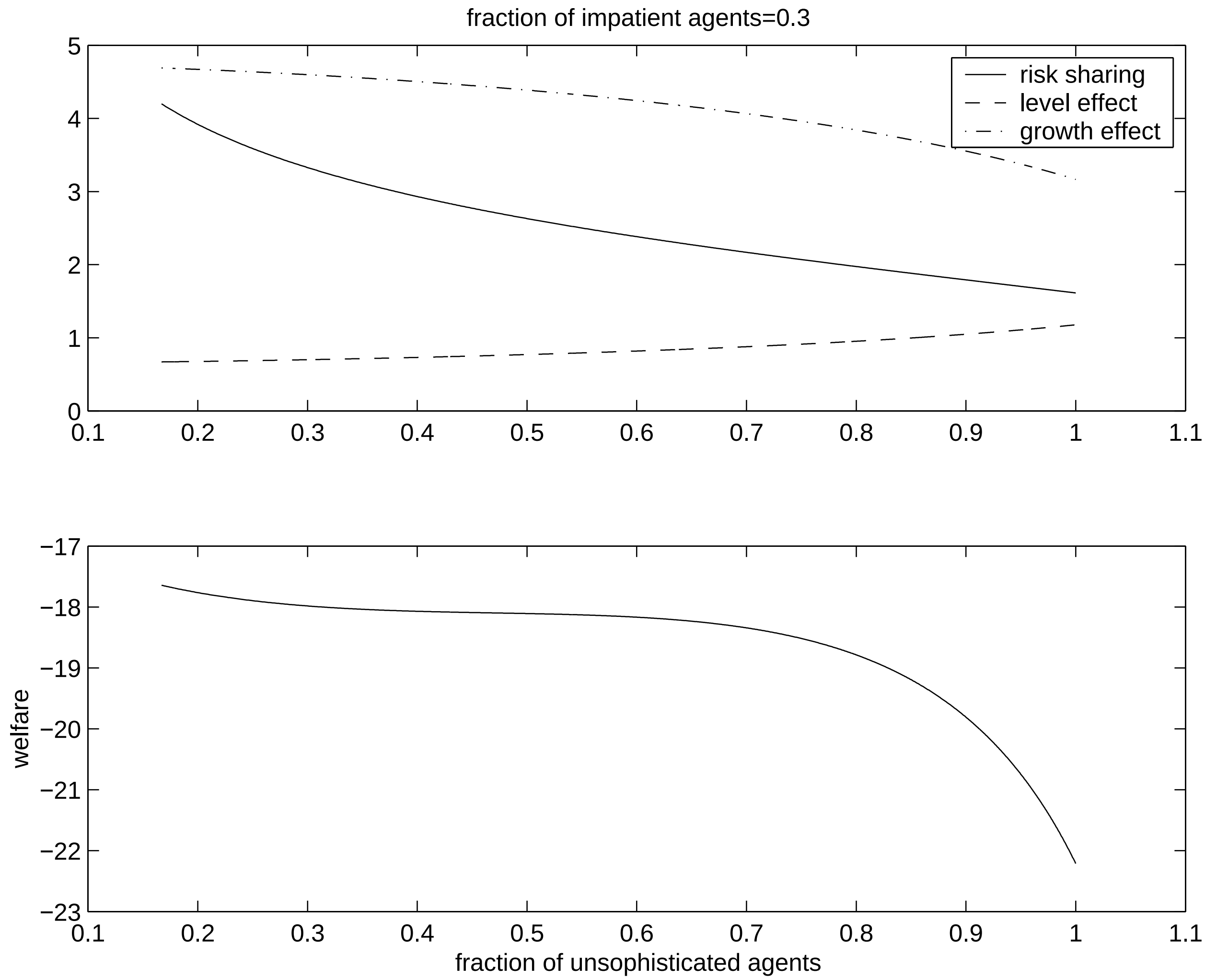


Figure 9. The case with a utility cost

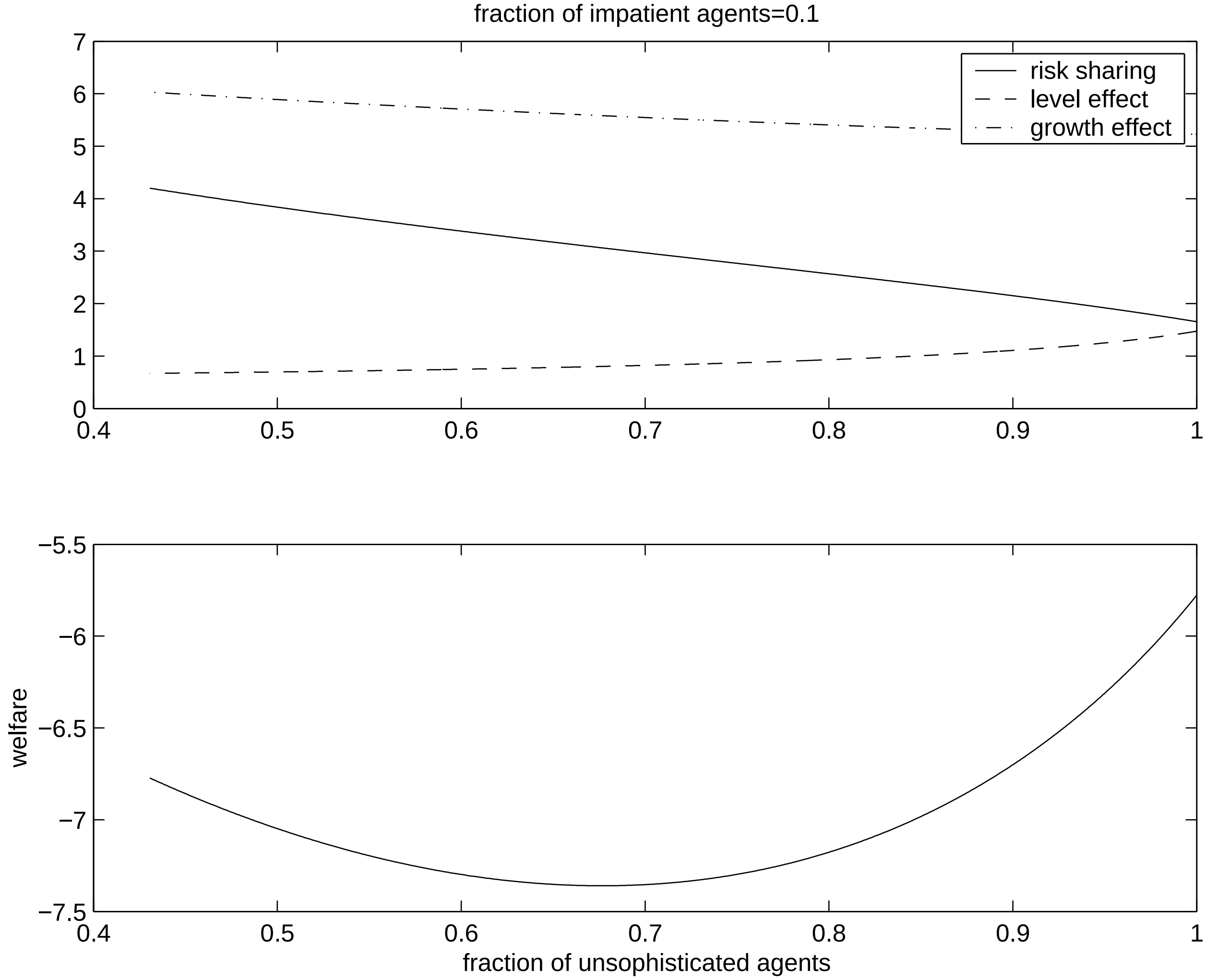


Figure 10. The case with a resource cost

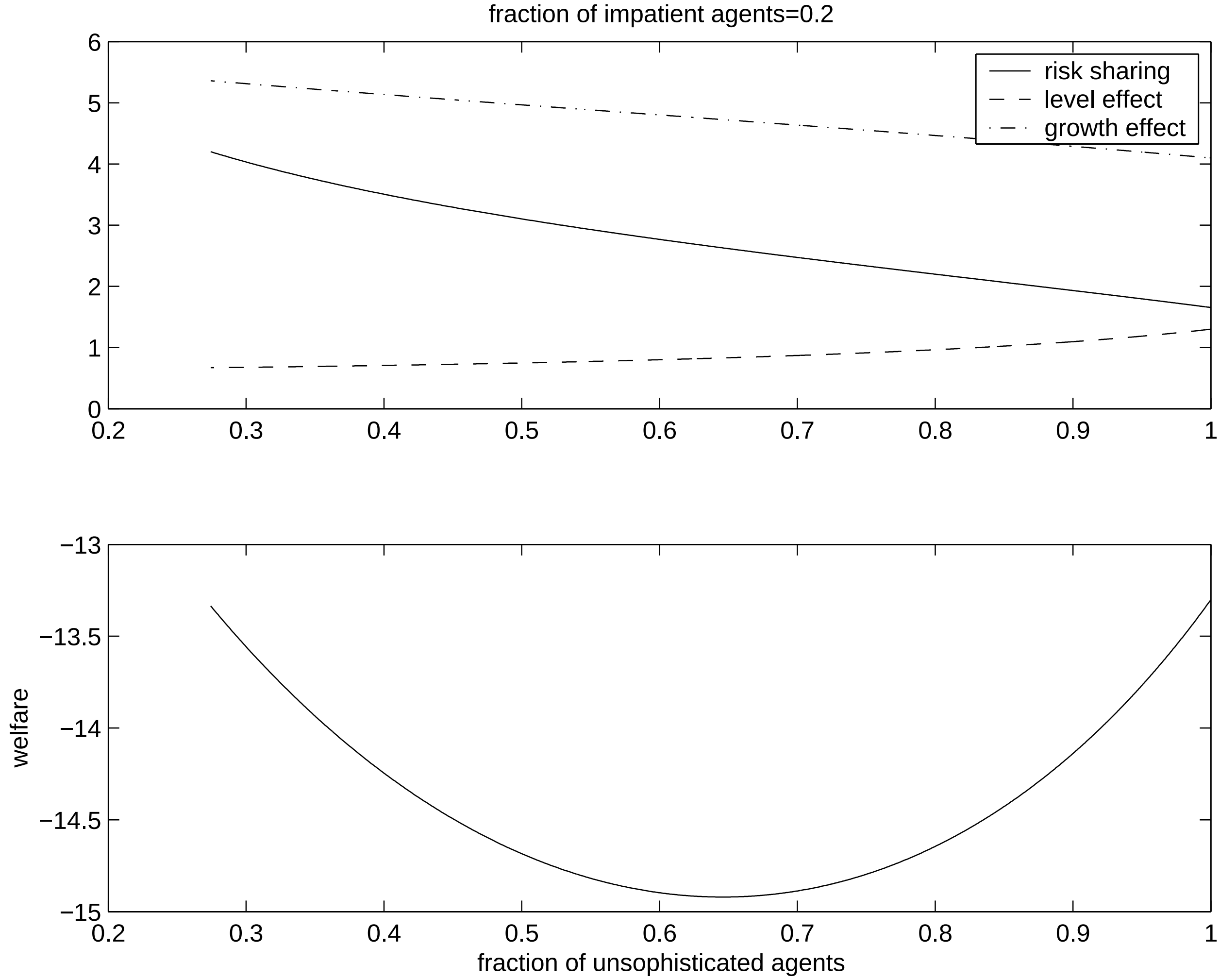


Figure 11. The case with a resource cost

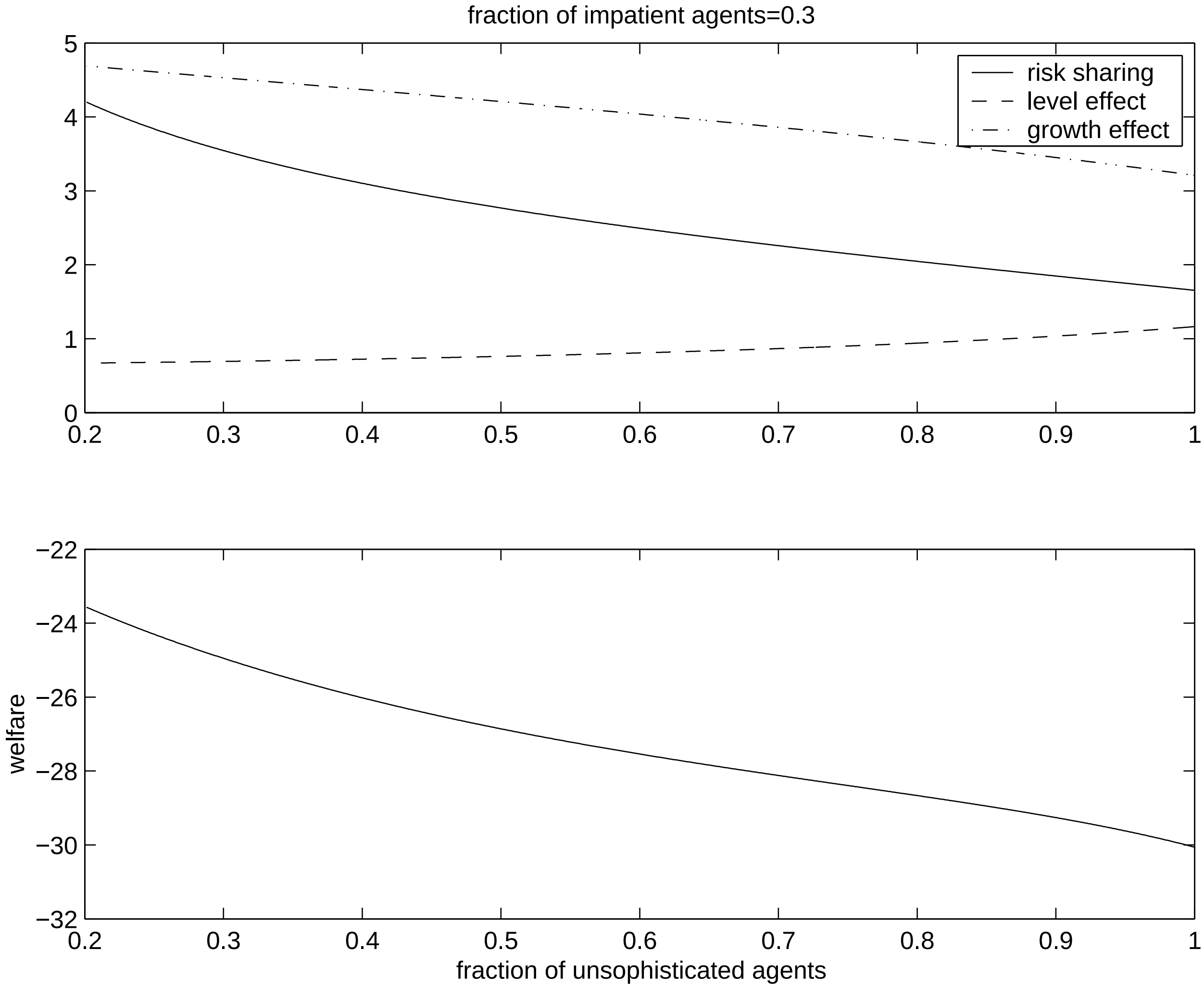


Figure 12. The case with a resource cost

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