Interpreting Permanent and Transitory Shocks to Output When Aggregate Demand May Not Be Neutral in the Long-run

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Abstract: I examine the statistical model of permanent and transitory shocks to output under the following structural assumptions: An aggregate supply shock that raises output will cause the price level to fall and an aggregate demand shock that initially raises output will cause the price level to rise. No assumption is made about the long-run effect of aggregate demand on output. Based on these assumptions I obtain three primary results. First, if a permanent increase in output is associated with an increase in the price level, then aggregate demand shocks must have a positive long-run effect on output. Second, output variance explained by permanent shocks will exceed the variance attributable to aggregate supply when aggregate demand shocks have a positive effect on output in the long run. Third, permanent and transitory shocks will affect price and output in qualitatively the same way as aggregate supply and aggregate demand shocks, respectively, from textbook macro theory over a range of positive and negative values for the structural parameter describing the long-run effect of aggregate demand on output. The results in this paper are used to interpret findings from empirical research and to motivate directions for further investigation.

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1. Introduction

For many years economists have been using statistical models that decompose time series into permanent and transitory components to investigate macroeconomic relationships. Much of this research has dealt with questions about aggregate real output. While initial studies employed univariate models,[1] more recent work has almost universally used time series models with multiple variables. Multivariate models are thought to be preferable because more information is used in the decomposition, multiple structural relationships can be estimated, and the permanent and transitory shocks can be orthogonal to one another.[2]

Blanchard and Quah (1989) developed one of the first multivariate models in this literature, and there have been numerous applications and extensions of their approach. An important feature of Blanchard and Quah's decomposition of output is that the permanent and transitory shocks may be able to identify the effects of aggregate supply and demand. Necessary conditions for this structure to be identified with their decomposition include: (i) the aggregate supply curve is vertical; (ii) aggregate demand shocks do not affect supply in the long run; (iii) the dynamic structure is invertible;[3] and (iv) the shocks to supply and demand are independent.

Some researchers have questioned Blanchard and Quah's bivariate approach. With only a single aggregate supply shock and a single aggregate demand shock, one concern is that their model may ignore other important structural shocks,[4] in which case their specification could be misspecified. In fact, Blanchard and Quah (1989) show in an appendix conditions under which their approach will successfully identify the effects of the aggregate supply and aggregate demand shocks even when an economy experiences multiple kinds of supply and demand shocks. Faust and Leeper (1997) elaborated on that point and extended the discussion in a number of important directions.

Another concern involves the use of unemployment rate data in the decomposition. Some economists have replaced unemployment with price data.[5] This substitution is based on the fact that textbook aggregate supply and demand theory is used to justify the statistical model and this theory is most often formulated in terms of output and the price level. To explain the behavior of unemployment requires that a labor market structure be appended to aggregate supply and demand. Another advantage of using price data in the statistical model is that theory usually predicts all kinds of supply shocks will affect output and the price level in qualitatively the same way and also that each variable will respond to the various aggregate demand shocks in qualitatively similar ways. Supply shocks typically cause output and price to move in opposite directions while demand shocks typically cause output and price to move in the same direction, at least for some portion of time after the shock occurs. On the other hand, the unemployment rate responds in a fundamentally different way to different types of supply shocks. For example, an increase in labor supply raises output and the unemployment rate whereas an increase in labor demand raises output but lowers the unemployment rate. Results from Blanchard and Quah (1989) and from Faust and Leeper (1997) show that if there is more than one type of supply shock and each type has fundamentally different effects on output and the unemployment rate, the bivariate decomposition will not identify the effects of aggregate supply and aggregate demand shocks. Consequently, I will focus on models that use price data in place of the unemployment rate.

In contrast to possible concerns about multiple kinds of structural shocks, this paper considers the possibility that aggregate demand shocks are not neutral in the long run. A substantial number of macroeconomic theories imply long-run non-neutrality, and if they are relevant, that would invalidate the primary structural assumption that justifies Blanchard and Quah's decomposition. This paper examines the effects of this alternative assumption on impulse responses and variance decompositions obtained with their statistical model.

One motivation of this paper is to determine if structural interpretations can be given to four findings from the empirical literature. First, impulse responses for postwar economies are typically consistent with textbook theory; Permanent shocks behave like aggregate supply shocks and temporary shocks behave like aggregate demand shocks. However, this result is not robust to different time periods. The second empirical finding is that a permanent increase in output is associated with an increase in the price level for most of a sample of prewar economies in Keating and Nye (1998). The term prewar is used to describe the period before World War I. These prewar responses to permanent shocks are inconsistent with the aggregate supply shock interpretation. Third, that same study finds that the amount of finite horizon output variance explained by permanent shocks tends to be larger in the pre-1914 period than in the postwar. Keating and Nye (1999) use the unemployment rate, in accord with Blanchard and Quah (1989), and obtain a similar result. Fourth, the immediate effect on output of a permanent shock exceeds the long-run effect in most estimates with pre-World War I data. This characteristic is described as short-run overshooting. While common in the prewar, short-run overshooting of output responses to permanent shocks is not observed in the postwar estimates.

I use a set of plausible structural assumptions to interpret these statistical findings. But instead of the long-run neutrality assumption, I use inequality constraints on the dynamic responses of variables to structural shocks. Specifically I assume an aggregate supply shock that raises output causes the price level to fall and an aggregate demand shock that initially raises output causes the price level to rise. The long-run effect of aggregate demand on output is not constrained. Based on these structural assumptions, the aforementioned empirical findings can be given structural interpretations.

The postwar findings suggest that textbook aggregate supply and demand theory provides a good description of economies in that period. However, the findings with prewar data that permanent output shocks cause prices and output to move in the same direction, exhibit short-run overshooting, and explain more output variance than these shocks do in the postwar, support the hypothesis that aggregate demand shocks had long-run positive output effects in the earlier sample period. Finally I show that even if impulse responses are consistent with textbook theory, as is typically the case with postwar data, aggregate demand shocks might still be having permanent effects on output. I show that permanent shocks behave like aggregate supply and transitory shocks like aggregate demand over a range of positive and negative values for the parameter describing the long-run effect of aggregate demand on output. The paper concludes by briefly summarizing some economic theories in which aggregate demand may be non-neutral in the long run, discussing whether or not each of these theories is a plausible explanation for the differences between prewar and postwar estimates and recommending potentially useful areas for future research.

2. The Structure and the Statistical Model

A statistical model provides a means of potentially discovering a structural relationship. This section will describe a statistical model and a structure in terms of the dynamic responses of variables to shocks, moving average representations (MARs) as they are called in time series analysis. The structural MAR describes the dynamic response of each variable to each structural shock. For the statistical MAR, I will use Blanchard and Quah's decomposition of output into temporary and permanent shocks. This decomposition is obtained by imposing a particular set of identification restrictions on the reduced-form parameters of a VAR. This section introduces the VAR model and characterizes each of these MARs.

2.1 The VAR

In general, the VAR representation exists and is unique, and can be written as:

$$\beta(L)\Delta X_t = e_t \tag{1}$$

where $X_t = (Y_t, P_t)'$ is the vector of variables, e_t is the vector of residuals, Δ = 1-L is the first difference operator and $\beta(L)=I-\beta_1 L-\beta_2 L^2 ...-\beta_\kappa L^\kappa$ represents the coefficients in the VAR with the identity matrix and each β_j for j=1,2, ... ,κ a 2×2 matrix and κ the number of lags in the VAR. Deterministic features such as constants, deterministic trends or dummy variables that might be essential for conducting a valid empirical analysis have been omitted without loss of generality. The first difference of Y will ensure there are permanent shocks to output in a linear model with constant coefficients.

This specification is different than Blanchard and Quah because the second variable is ΔP, with P the logarithm of the price level, instead of the unemployment rate. Hence, ΔP is (approximately) equal to the rate of inflation. If the choice of second variable does not alter the identification restrictions for the statistical model of output or the theoretical assumptions, then the results derived in this paper for output are independent of that choice.

2.2 The Structural Moving Average Representation

Assume the economic structure has the following MAR:

$$\Delta X_t = \theta(L)\varepsilon_t \tag{2}$$

where $\varepsilon_t = (\varepsilon_t^S, \varepsilon_t^D)'$ is a vector of shocks to aggregate supply and aggregate demand, respectively, and $\theta(L) = \theta_0 + \theta_1 L + \theta_2 L^2 + ... = \sum_{j=0}^{\infty} \theta_j L^j$ specifies the dynamic responses of ΔY and ΔP to these structural shocks with $\theta_j = \begin{bmatrix} \theta_j^{YS} & \theta_j^{YD} \\ \theta_j^{PS} & \theta_j^{PD} \end{bmatrix}$ for all j. If we assume supply and demand shocks are independent, a standard assumption in the structural VAR literature, then it is convenient to normalize these shocks to have variances equal to one: $E\varepsilon_t \varepsilon_t' = I$.

Appendix A shows how recursive substitutions are used to transform equation (2), the system for

ΔX, into the system in terms of X:

$$X_t = X_0 + \theta_0\varepsilon_t + (\theta_0 + \theta_1)\varepsilon_{t-1} + (\theta_0 + \theta_1 + \theta_2)\varepsilon_{t-2} + \ldots \tag{3}$$

from which responses of X to structural shocks are obtained:

$$\frac{\partial X_t}{\partial \varepsilon_{t-k}} = \sum_{j=0}^{k} \theta_j \equiv \Phi_k \ . \tag{4}$$

The last equality defines Φ_k as the sum of the first k parameter matrices in $\theta(L)$, a definition that will be convenient in later analysis. Note that Φ_k is a 2×2 matrix:

(5)

where $\Phi_k^{vi} = \sum_{j=0}^{k} \theta_j^{vi}$ for v=Y,P and i=S,D. While economists want to estimate these structural responses, the paper is concerned with conditions under which the statistical model may be unable to obtain consistent estimates of the dynamic structure.

The long-run responses of variables to shocks are obtained by letting k go to infinity in (4):

$$\Phi_k = \begin{bmatrix} \Phi_k^{YS} & \Phi_k^{YD} \\ \Phi_k^{PS} & \Phi_k^{PD} \end{bmatrix} \tag{6}$$

where the last equality comes from setting L=1 in $\theta(L)$. The $\theta(1)$ matrix represents the long-run multipliers for structural shocks,[6] and it can be written as:

$$\theta(1) = \begin{bmatrix} \Theta_{YS} & \Theta_{YD} \\ \Theta_{PS} & \Theta_{PD} \end{bmatrix}. \tag{7}$$

2.3 The Statistical Model's Moving Average Representation

Let the MAR for the statistical model be written as:

$$\Delta X_t = C(L)\mu_t \tag{8}$$

where $\mu_t = (\mu_t^P, \mu_t^T)'$ is the vector of permanent and transitory shocks, respectively, and

$C(L) = C_0 + C_1 L + C_2 L^2 + ... = \sum_{j=0}^{\infty} C_j L^j$ are the impulse responses of ΔX to these shocks with

C_j a 2×2 matrix for all non-negative integer values of j. In all applications of the bivariate framework, the permanent and transitory shocks are assumed independent, and therefore the variance of each shock in the statistical model can be normalized to one for convenience: $E\mu_t\mu_t' = I$.

Equation (8) is in first differences, but it can be transformed into a relationship in terms of X using recursive substitution, following the same procedure that was used with (2) to generate (3):

$$X_t = X_0 + C_0\mu_t + (C_0 + C_1)\mu_{t-1} + (C_0 + C_1 + C_2)\mu_{t-2} + ... \quad . \tag{9}$$

Hence, the impulse responses of Y and P to permanent and transitory shocks are obtained:

$$\frac{\partial X_t}{\partial \mu_{t-k}} = \sum_{j=0}^{k} C_j \ . \tag{10}$$

Letting k go to infinity in (10) yields the long-run effects of these shocks on the variables:

$$\lim_{k\to\infty}\left(\frac{\partial X_t}{\partial \mu_{t-k}}\right) = \sum_{j=0}^{\infty} C_j = C(1) , \tag{11}$$

where the last equality comes from evaluating C(L) at L=1. C(1) represents the sum of coefficients in C(L), and it can be written as:

$$C(1) = \begin{bmatrix} C_{YP} & 0 \\ C_{PP} & C_{PT} \end{bmatrix} \tag{12}$$

where C_{vi} is the long-run response of price or output, v∈(Y,P), to a permanent or transitory shock, i∈(P,T). C(1) is made lower triangular by setting C_{YT}=0, the restriction that forces temporary shocks to not have a permanent effect on output, as seen from equation (11):

$$\lim_{k\to\infty}\left(\frac{\partial Y_t}{\partial \mu_{t-k}^T}\right) = C_{YT} = 0 \ .$$

3 Relationships between the Statistical and Structural MARs

The statistical MAR will not be the same as the structural MAR unless the identification restrictions in the statistical model are valid structural restrictions. An easy way to see how the two MARs are related is to map each of them into the VAR. A VAR is a system of equations in which each variable is a function of lagged endogenous variables and a serially uncorrelated error. The statistical decomposition is transformed into the VAR representation by multiplying equation (8) by $C_0C(L)^{-1}$. The structure is transformed into the VAR by multiplying equation (2) by $\theta_0\theta(L)^{-1}$. These mappings determine how VAR residuals:

$$e_t = C_0\mu_t = \theta_0\varepsilon_t \tag{13}$$

and VAR coefficients:

$$\beta(L) = C_0C(L)^{-1} = \theta_0\theta(L)^{-1} \tag{14}$$

are functions of parameters from both the statistical model and the structure. I use these equations to describe a well-known method of calculating coefficients in the statistical decomposition and, more importantly, to characterize the way that coefficients from the statistical model are related to structural parameters when $\Theta_{YD}\neq 0$.

Given equation (13) and the identity covariance matrices assumption for the shocks in each MAR, the covariance matrix for residuals:

$$\Sigma_e = C_0C_0' = \theta_0\theta_0' \ , \tag{15}$$

is a function of short-run parameters from the structure and also a function of short-run coefficients from the statistical decomposition. A relationship between the statistical decomposition, the structure and $\beta(1)$, the matrix describing the sum of VAR coefficients, is obtained by setting L=1 in equation (14):

$$\beta(1) = C_0 C(1)^{-1} = \theta_0 \theta(1)^{-1} . \tag{16}$$

The first identity in equation (16) yields: $C_0 = \beta(1)C(1)$. Insert this expression into the first identity in equation (15) and simplify:

$$C(1)C(1)' = \beta(1)^{-1} \Sigma_e \beta(1)'^{-1} . \tag{17}$$

This equation illustrate a popular method for estimating parameters in the statistical decomposition. Given that C(1) is a triangular matrix, the C(1) parameters can be obtained by Cholesky decomposition of the right-hand side of equation (17). Insert the estimate of C(1) into the first equality from equation (16), solve for C_0, insert the value for C_0 into the first equality in equation (14) and solve for C(L), the dynamic responses of variables to permanent and transitory shocks.

Now we investigate how the statistical decomposition is related to structure. Notice that the last identify in (14) can be manipulated to yield: $C_0 = \theta_0\theta(1)^{-1}C(1)$. Insert this equation into the second equality of (15) and simply, to obtain:

$$C(1)C(1)' = \theta(1)\theta(1)' . \tag{18}$$

The standard assumption from textbook theory is that aggregate demand shocks are long-run neutral which is given by $\Theta_{YD} = 0$ in the structural model. This condition, along with the assumptions that the structure is invertible and that the structural shocks are orthogonal to one another, implies that the permanent and transitory shocks identify the dynamic effects of supply and demand, respectively. The $\theta(1)$ matrix is lower triangular when $\Theta_{YD} = 0$, and because the lower triangular factor of a symmetric matrix is unique,[7] equation (18) yields $C(1) = \theta(1)$. Hence, C(1) identifies the long-run effects of aggregate demand and aggregate supply on Y and P. This last identity is combined with equation (14) which maps these two representations into the VAR coefficients to show that $C(L)=\theta(L)$. As one might expect, the statistical model identifies the complete dynamic responses of each variable to each structural shock when the identification restrictions are valid.

This paper is concerned with the more general case in which $\Theta_{YD} \neq 0$ is a possibility. Using the second identities from equations (14) and (16), it is easy to derive the relationship between the statistical model's impulse responses and the structural responses:

$$C(L) = \theta(L)\theta(1)^{-1}C(1) \tag{19}$$

or equivalently:

$$C_j = \theta_j\theta(1)^{-1}C(1) \qquad \text{for all j.} \tag{20}$$

Using the definition of θ(1) from (7) and the definition of C(1) from (12) in equation (18), it is straightforward to calculate the relationship between the long-run coefficients from the statistical model and long-run structural parameters:

$$C_{YP} = \left(\Theta_{YS}^2 + \Theta_{YD}^2\right)^{1/2}, \quad C_{PP} = \frac{\Theta_{YS}\Theta_{PS} + \Theta_{YD}\Theta_{PD}}{C_{YP}} \quad \text{and} \quad C_{PT} = \frac{\Theta_{YS}\Theta_{PD} - \Theta_{YD}\Theta_{PS}}{C_{YP}}.$$

Using these three identities, the following result is easily obtained:

$$\theta(1)^{-1}C(1) = \frac{\begin{bmatrix}\Theta_{YS} & -\Theta_{YD} \\ \Theta_{YD} & \Theta_{YS}\end{bmatrix}}{C_{YP}} . \tag{21}$$

Insert (20) into (10), the equation describing responses from the statistical model, and then use the definition of structural responses from (4) to obtain:

$$\frac{\partial X_t}{\partial \mu_{t-k}} = \sum_{j=0}^{k} C_j = \sum_{j=0}^{k} \theta_j\theta(1)^{-1}C(1) = \Phi_k\theta(1)^{-1}C(1) . \tag{22}$$

Equation (22) characterizes the relationship between the statistical model's impulse response function and the structure's dynamics. Substituting (21) and (5) into (22) yields:

$$\frac{\partial X_t}{\partial \mu_{t-k}} = \frac{\begin{bmatrix} \Phi_k^{YS} & \Phi_k^{YD} \\ \Phi_k^{PS} & \Phi_k^{PD} \end{bmatrix} \begin{bmatrix} \Theta_{YS} & -\Theta_{YD} \\ \Theta_{YD} & \Theta_{YS} \end{bmatrix}}{C_{YP}} \tag{23}$$

Equation (23) shows how the impulse responses for the statistical model are a function of the structural parameters. If $\Theta_{YD} \neq 0$, then the statistical model's coefficients are nonlinear functions of structural parameters, not consistent estimates of the structure. This result suggests that when the statistical model uses inappropriate identification restrictions, it will be uninformative about the structure of the economy. However, if other assumptions about the economic structure are available, this misspecified statistical model might still be able to infer important facts about the underlying economic structure. Equation (23) will be used to answer questions that motivate this paper.

4. Structural Assumptions

If we are unable or unwilling to take a stand on some features of the structure, it is impossible, in general, to give structural interpretations to empirical models.[8] More to the point, economists are unable to infer how the impulse responses to permanent or transitory shocks are related to the structure without assumptions of some kind concerning how the economy operates. Blanchard and Quah, along with many others, have taken the position that aggregate demand is long-run neutral to interpret their statistical model, but whether or not this structural hypothesis is correct, Blanchard and Quah's decomposition is able to identify a statistical model with permanent and transitory shocks. If we are unwilling to assume that aggregate demand is long-run neutral with respect to real output, then we will need alternative structural assumptions to give an economic interpretation to the statistical model.

Fortunately other assumptions are available. Economic theory often places bounds on the qualitative responses of variables to structural shocks.[9] For example, most theories predict that a beneficial aggregate supply shock will raise output[10] and have a negative effect on the price level:

$$\text{A1:} \qquad \frac{\partial Y_t}{\partial \varepsilon^S_{t-k}} = \Phi^{YS}_k > 0 \quad \text{for all k;}$$

$$\text{A2:} \qquad \frac{\partial P_t}{\partial \varepsilon^S_{t-k}} = \Phi^{PS}_k < 0 \quad \text{for all k.}$$

These assumptions are weak enough to allow for the possibility that supply shocks also shift the aggregate demand curve. If supply shocks cause both curves to shift in the same direction, then assumption A2 requires that the demand curve not shift by as much as supply. An example of an aggregate supply factor that could shift both curves in the same direction is a permanent increase in productivity. Aggregate demand would also shift to the right because a permanent increase in productivity increases investment demand by raising the expected future marginal product of capital.

There is some debate in the literature about the long-run effects of aggregate demand on output. Since the long-run aggregate supply curve is vertical in virtually all modern macroeconomic theories, that means a shift in the aggregate demand curve will not have a long-run effect on output unless it has a permanent effect on some factor in the aggregate supply curve. There are a number of different theories that predict aggregate demand may be non-neutral in the long run. Some theories predict an increase in aggregate demand will cause output to rise in the long run while other theories find the opposite effect. A list of prominent examples from the literature includes:

1. Non-Superneutrality: A permanent increase in the growth rate of money may raise or lower output in the long-run depending on particular features of the structure;
2. Long-Run Fiscal Policy Effects: An increase in government spending may crowd-out or crowd-in investment in the long-run, affecting the stock of capital and consequently long-run aggregate supply, and changes in marginal tax rates may have supply-side effects;

3. Hysteresis: The natural rate of unemployment may depend on past levels of the unemployment rate, and so aggregate demand may affect the natural rate and as a result the long-run level of output;
4. Coordination Failures: Coordination problems may yield multiple equilibria, allowing aggregate demand to potentially affect the long-run equilibrium position attained by the economy;
5. Destabilizing Price Flexibility: Rather than bring about general equilibrium, price flexibility may destabilize the economy causing aggregate demand to have persistent effects on output.

Irrespective of the ultimate consequence for output, I assume that a positive aggregate demand shock raises output for at least K periods after the shock occurs:

$$\text{A3:} \qquad \frac{\partial Y_t}{\partial \varepsilon^D_{t-k}} = \Phi^{YD}_k > 0 \quad \text{for } k=0,1...,K \quad \text{with} \quad 0< K < \infty.$$

This assumption allows for the possibility that after K periods output may fall below its pre-shock level in response to a beneficial aggregate demand shock. There are at least two ways that this might occur. The first is if aggregate demand has a negative long-run effect on output. Obviously, with $\Theta_{YD}<0$, the response of output to a positive aggregate demand shock must eventually become negative. A second way would be if output exhibits damped cycles around its steady state. When an economy experiences this sort of behavior, output may fall below its initial level as the economy dynamically adjusts. Such dynamics are more likely to cause a decline in output if aggregate demand is neutral in the long-run, but negative output responses may occur even when demand has a positive long-run output effect. The likelihood of this effect is a function of the cyclical amplitude relative to the long-run effect on output.

I also assume that aggregate demand shocks which initially cause output to rise will have a positive effect on the price level:[11]

$$\text{A4:} \qquad \frac{\partial P_t}{\partial \varepsilon^D_{t-k}} = \Phi^{PD}_k > 0 \quad \text{for all k.}$$

If a positive aggregate demand shock also shifts the long-run aggregate supply curve to the left, $\Theta_{YD}<0$, then A4 will hold because the movement of each curve raises the price level. On the other hand, if a positive aggregate demand shock shifts long-run aggregate supply to the right, $\Theta_{YD}>0$, then the supply curve must not shift by as much as demand does for A4 to hold.

Aggregate demand neutrality may be thought of as a reasonable working hypothesis, but it is inconsistent with many different macroeconomic theories. And while assumptions A1 through A4 may not hold for every conceivable structure, these assumptions are consistent with most economic theories. Furthermore, these assumptions do not rule out the possibility that aggregate demand shocks are neutral in the long run because no assumption is made about Θ_{YD}.

5. Results

Using permanent-transitory shock decompositions with pre-World War I data for 10 countries that have relatively long time series, Keating and Nye (1998) find that a permanent increase in output is associated with an increase in the price level for 8 countries. In 5 of these cases, the effect is statistically significant.[12] This evidence strongly rejects the textbook structure which underlies Blanchard and Quah's (1989) decomposition, at least for the prewar sample, because if permanent shocks are supply shocks they should cause price and output to move in opposite directions. Is a rejection of the structural hypothesis all that can be inferred or does this empirical evidence tell us something more about the structure of prewar economies? Proposition 1 provides an answer to this question.

Proposition 1: Given assumptions A1, A2 and A4, if a permanent increase in output is associated with an increase in the price level, then aggregate demand must have a positive effect on output in the long run.

Proof of Proposition 1:

From equation (23), the response of price to a permanent increase in output is

$$\frac{\partial P_t}{\partial \mu^P_{t-k}} = \frac{\Phi^{PS}_k \Theta_{YS} + \Phi^{PD}_k \Theta_{YD}}{C_{YP}} \ .$$

The condition on Θ_{YD} that makes this response positive is:

$$\Theta_{YD} > \frac{-\Phi^{PS}_k \Theta_{YS}}{\Phi^{PD}_k}$$

and the structural assumptions guarantee that the right side is positive. Q.E.D.

Keating and Nye (1998) speculated that their evidence with pre-1914 data might support theories in which aggregate demand shocks have positive long-run effects on output, and Proposition 1 provides formal theoretical support for this interpretation. Since the price level rises at all points on the impulse responses for 8 countries in the pre-1914 sample, Θ_{YD} must be large enough to satisfy the inequality for all k in these economies. These price responses reject the structural model used to justify the identification restrictions, but they also imply, under arguably more plausible structural assumptions, that aggregate demand had a positive long-run effect on output in a number of prewar economies.

Another finding in Keating and Nye (1998) is that impulse responses are fundamentally different across the two time periods. The immediate effect on output of a permanent shock in prewar data is larger than the long-run effect for 7 of the 10 countries used in the study.[13] Short-run overshooting responses are not observed in postwar data from any of these countries. Based on economic theory and properties of the statistical model, I argue that the only plausible structural explanation for this overshooting is that aggregate demand shocks had permanent positive effects on output. The response of output to a permanent shock is taken directly from equation (23):

$$\frac{\partial Y_t}{\partial \mu^P_{t-k}} = \frac{\Phi^{YS}_k \Theta_{YS} + \Phi^{YD}_k \Theta_{YD}}{C_{YP}} \quad . \qquad (24)$$

If Θ_{YD}=0 then the response of output to a permanent shock is identical to the response of output to supply. I am not familiar with any economic theory in which short-run overshooting characterizes the response of output to a permanent supply shock. Theory typically shows a gradual output adjustment to a permanent supply shock, with the possibility of cyclical dynamics as the economy approaches a steady state. Thus Θ_{YD}=0 is unable to explain short-run overshooting.

Now consider the Θ_{YD}<0 case. This assumption along with A1, A3 and equation (24) implies:

$$\frac{\partial Y_t}{\partial \mu^P_{t-k}} < \Phi^{YS}_k \text{ for small k,}$$

because when Θ_{YD} is negative, the coefficient on Φ^{YS}_k in (24) is positive and less than one and the second term is negative. Furthermore, for any non-zero Θ_{YD}:

$$\lim_{k\to\infty}\left(\frac{\partial Y_t}{\partial \mu^P_{t-k}}\right) = C_{YP} = (\Theta^2_{YS} + \Theta^2_{YD})^{1/2} > \Theta_{YS} \quad .$$

The implication is that when Θ_{YD} is negative, permanent shocks have a smaller short-run effect and a larger long-run effect on output than aggregate supply shocks. Therefore, a permanent shock to output will not exhibit short-run overshooting if aggregate demand has a negative long-run effect on output, given that the response of output to aggregate supply can not experience short-run overshooting.

This leaves Θ_{YD}>0 as the only possible structural explanation for short-run overshooting. In general, the response of output to a permanent shock is a linear combination of output responses to aggregate supply and demand, and when Θ_{YD} is positive, the coefficient on Φ^{YD}_k in equation (24) is positive. Macroeconomic theories often predict an aggregate demand shock will have its peak effect on output within a year, an effect that is qualitatively similar to the short-run overshooting observed in models with annual prewar data. If aggregate demand has a long-run positive output effect and this effect is sufficiently large, the response of output to a permanent shock could inherit short-run over-shooting behavior from the dynamic response of output to aggregate demand.

Most of the research with permanent and transitory decompositions has used postwar data. Empirical results from this sample period are typically consistent with the aggregate supply interpretation of permanent shocks and the aggregate demand interpretation of transitory shocks. Therefore, economists often reach the conclusion that the textbook aggregate demand and supply structure provides a good description of postwar economies.

Another interesting finding is that the amount of variance explained by permanent shocks to output tends to be larger in the pre-1914 period than in the post-World War II period. This finding is obtained by Keating and Nye (1999) who followed Blanchard and Quah and used the unemployment rate and also by Keating and Nye (1998) who used inflation in place of the unemployment rate. This relationship can only occur at finite horizons because as the forecast horizon goes to infinity permanent shocks explain 100% of the variance of output by construction. While there may be a variety of significant differences between prewar and postwar economies, it would be interesting to determine whether a significant difference in the long-run output effect of aggregate demand, by itself, could explain observed differences in variance decomposition across these two periods. The following proposition addresses this question:

Proposition 2. If the aggregate supply and demand structure applies to two economies, demand shocks to Economy A are long-run neutral, demand shocks to Economy B may have a long-run effect on output, and this is the only difference between these two economies, then the fraction of finite horizon output variance associated with permanent shocks is larger for Economy B if and only if aggregate demand has a long-run positive effect on output in Economy B.

Proof of Proposition 2: See Appendix B.

Let Economy A be a postwar economy for which empirical results are usually consistent with textbook theory. Let Economy B be a pre-1914 economy. Output variance explained by permanent shocks tends to be larger in the earlier period for the sample of countries studied by Keating and Nye (1998,1999). If neutrality holds in the postwar economies and the only significant difference between these two sample periods is that aggregate demand may be non-neutral in the prewar, then Proposition 2 tells us that a positive long-run effect of aggregate demand on output in the prewar period by itself can explain why permanent shocks account for more output variance in this earlier sample period. Hence, Proposition 2 provides further support for the hypothesis that some economies in the late 19th and early 20th centuries experienced a permanent increase in output from aggregate demand shocks.

As is clear from the appendix, proving Proposition 2 is equivalent to showing that permanent shocks will explain more output variance than the amount attributable to supply, if and only if the aggregate demand shocks have a long-run positive effect on output. It turns out that this proof requires the assumption that a permanent shock always has a positive effect on output. This assumption is strongly supported by empirical results in Keating and Nye (1998,1999) and is used to rule out extremely negative values of Θ_{YD}.

A criticism of the permanent-transitory decomposition literature is that few studies have provided formal tests of the identification restrictions. While researchers using postwar data have often found that their statistical models are consistent with textbook theory, failure to reject a theory, of course, does not mean that a theory is necessarily correct. In most cases, consistency with a theory is based on qualitative properties of impulse response functions. An interesting question is: How are the qualitative features of a statistical model's impulse responses affected by different values for the structural parameters?

Proposition 3: Impulse responses to permanent and transitory shocks are consistent with the qualitative effects of textbook aggregate supply and aggregate demand shocks, respectively, over a range of values for Θ_{YD}.

Proof: Calculate bounds on Θ_{YD} such that the statistical model finds that the shocks that permanently increase output cause price to fall and temporary shocks that initially increase output cause price to rise. The following discussion presents and interprets these bounds.

Based on equation (23), it is easy to show that the temporary shock causes output and the price level to rise, respectively, when

$$\frac{\Phi_k^{YD}\Theta_{YS}}{\Phi_k^{YS}} > \Theta_{YD} > \frac{\Phi_k^{PD}\Theta_{YS}}{\Phi_k^{PS}}.$$

Assumptions A1 through A4 imply that if Θ_{YD} is greater than the negative value on the right for all k and less than the positive number on the left for small values of k (small meaning non-negative k that are less than K), then the statistical model will yield impulse responses to temporary shocks that appear like the aggregate demand shocks from textbook macro theory. If we ever observe output and price responses to temporary shocks moving in opposite directions for small k, that would imply Θ_{YD} falls outside the range of values given above. Unfortunately, observing this sort of response would not tell us whether the long-run effect of demand is positive or negative.[14]

Now examine the responses to a permanent shock. Based on equation (23), a permanent output shock has a negative effect on price when:

$$\frac{-\Phi_k^{PS}\Theta_{YS}}{\Phi_k^{PD}} > \Theta_{YD}.$$

Given A1, A2 and A4, this expression sets a positive upper bound on Θ_{YD} such that the permanent shock to output will cause a drop in the price level. Proposition 1 addressed the eight pre-World War I economies for which this inequality was violated.

Next determine conditions under which the permanent shock will always have a positive effect on output. This response is positive at long horizons, by construction, for any value of Θ_{YD}. Consider first the case when Φ_k^{YD} is positive. Under this assumption the permanent shock to output is positive after k periods when

$$\Theta_{YD} > \frac{-\Phi_k^{YS}\Theta_{YS}}{\Phi_k^{YD}} ,$$

implying that Θ_{YD} is greater than this negative number, given A1, A3 and the currently maintained assumption that Φ_k^{YD} is positive.

What if Φ_k^{YD} is negative for certain values of k? Then the permanent shock will have a positive effect on Y after k periods when:

$$\Theta_{YD} < \frac{-\Phi_k^{YS}\Theta_{YS}}{\Phi_k^{YD}} .$$

There are two important differences between this inequality and the previous one. First, the inequality is reversed because of division by Φ_k^{YD}, which is now assumed negative, and the second difference is that the right-hand side of the inequality is now a positive number. This new condition sets a positive upper bound on Θ_{YD}. Thus we have established a range of positive and negative values for Θ_{YD} that permit the permanent shock to have a positive effect on output for all k.

Based on the large number of empirical studies that assume neutrality of aggregate demand, this assumption would seem to have a significant amount of credibility in the economics profession. It is no surprise that empirical findings consistent with the textbook model are interpreted as support for this theory. But in proving Proposition 3, we have determined a range of values for Θ_{YD} that permit the responses to permanent and transitory shocks to appear consistent with the effects of aggregate supply and demand shocks, respectively. While Θ_{YD}=0 is included in this range of values, the qualitative properties of impulse responses do not serve as a reliable basis for judging that Θ_{YD} is essentially equal to zero. And if Θ_{YD} is not very close to zero, the impulse responses from the statistical decomposition of output may differ substantially from the dynamic effects of structural shocks.

6. Discussion and Conclusions

This paper attempts to provide structural interpretations for a number of empirical results. One important conclusion is that aggregate demand had a permanent positive effect on output in prewar economies. This conclusion derives from the tendency for the prewar period's permanent output shocks to move prices and output in the same direction, obtain output responses that in the short run overshoot the long-run response, and explain more output variance than the permanent shocks in the postwar.

An important area for future research is to investigate possible structures that may have caused aggregate demand to be non-neutral in pre-World War I economies. While quite a few potential explanations exist, a number of them seem unlikely based on observable differences between economies of these two periods. For example, non-neutral long-run effects from permanent changes in the growth rate of money are not evident in the prewar period. For non-superneutrality to have been a significant factor, there would need to have been persistent or permanent changes in the growth rate of the money supply. Such changes would be expected to show up as unit root behavior in inflation, money growth and other nominal variable growth rates, even if another kind of non-stationarity provides a better description of the data generating process. In fact, a unit root for inflation can be rejected in data from the pre-1914 sample period. It is more difficult to reject this hypothesis for inflation in the postwar period, and so long-run non-superneutrality stands a better chance of being a factor in postwar economies.[15]

Crowding-out or crowding-in effects from fiscal spending and permanent output effects from tax rate policy are also potential explanations of permanent output effects from aggregate demand. But once again these possibilities are more likely a factor in postwar economies. Tax rates and government's share of output were both very low in the prewar period. Except for times of war, large-scale government involvement in the macroeconomy did not occur until after the Great Depression.

Hysteresis theories[16] of the labor market provide another mechanism through which aggregate demand may have long-run effects on the level of output. For example, if recessions cause a permanent loss in the stock of human capital, then the marginal product of labor will decline. This would cause a permanent decline in the demand for labor which could increase the natural unemployment rate. Hence, an adverse shock to aggregate demand would not only induce a recession, but also would cause a reduction in full-employment output if hysteresis is a factor. And positive aggregate demand shocks would cause the opposite effect. Consequently, the unemployment rate would likely exhibit permanent changes resembling unit root behavior. While tests with postwar unemployment rate data have some difficulty rejecting a unit root, a unit root is easily rejected for unemployment rates from the pre-1914 sample period. This weakens the case for the hysteresis explanation of permanent output effects from aggregate demand in prewar economies.

To summarize, hysteresis, long-run non-superneutrality of money and permanent output effects from fiscal policy do not appear to be important factors in prewar economies. Each of these effects has a better chance of being relevant during the postwar, but it is this period for which impulse responses are most often consistent with standard textbook macro theory.

Coordination failure theories provide a more plausible explanation of permanent output effects from aggregate demand in pre-World War I economies. A coordination failure may occur when economic decisions have strategic complementarities or spillover effects.[17] Simple examples of this effect are when the liquidity of a financial market depends on the number of other market participants or when the utility of a communications device (a telegraph, a telephone, the Internet, etc.) depends on the number of other users of that same technology. Coordination failure economies often have multiple equilibria. Consequently, a positive aggregate demand shock may push the economy to a new equilibrium that has a higher level of economic activity, and a negative shock could do the opposite. The observation that transactions costs were falling throughout the nineteenth and twentieth centuries is consistent with this theoretical explanation.[18] Transactions costs for businesses and individuals were lowered by expansion of commercial banking and financial intermediation services, advances in transportation and improvements in communications technology. It is plausible that a reduction in transactions costs over time may have transformed economies from coordination failure structures in the 19th century to modern structures for which textbook macro theory is a good description.

Destabilizing price flexibility is another plausible explanation for permanent output effects from aggregate demand. While macroeconomic theory typically tells us that more rapid price adjustment causes aggregate demand to have smaller output effects, there are some theories in which falling prices may actually push the economy away from full employment. For example, Fisher (1933) describes how deflation and debt may produce an adverse aggregate outcome. Keynes (1936), Tobin (1975) and DeLong and Summers (1986) have emphasized how falling prices might raise real interest rates and reduce spending,[19] an effect that may be particularly significant when nominal interest rates are close to the zero bound. Consistent with these ideas is the evidence (discussed briefly in footnote 11) that prices were more flexible in the pre-1914 period than in the postwar and the fact that many economies experienced periods of substantial deflation during the prewar.[20]

While there is some evidence consistent with destabilizing price flexibility and some that is consistent with coordination failures, this in no way proves that either theory is correct. Formal tests with prewar data are required for these hypotheses or any others that might be offered to explain the empirical results. Determining why aggregate demand is not neutral in many pre-World War I economies is important in its own right. And it is also possible that the same structural mechanisms may have caused aggregate demand shocks to have permanent output effects in modern economies. While the postwar results are largely consistent with textbook theory, Proposition 3 shows that this evidence does not necessarily mean the mechanisms causing prewar non-neutrality of aggregate demand have become irrelevant. But if these prewar effects continue to be important, their influence is now relatively smaller given that price levels almost never rise following a permanent increase in output in the postwar. Finding such mechanisms are still relevant would enable us to develop a better understanding of the structure of modern economies.

Of course, there is no guarantee the mechanisms that affected pre-1914 economies are still important. Fundamentally different structural mechanisms that invalidate long-run neutrality of aggregate demand may be at work in the postwar. An implication of Proposition 3 is that economists should test the hypothesis that aggregate demand is neutral, rather than assert neutrality as is commonly done with structural VARs identified by long-run restrictions. Some tests of long-run neutrality propositions have been performed, but more tests are required along with new testing methods or improvements to currently available procedures.[21] If the tests generally fails to reject neutrality, then the vast empirical research employing neutrality assumptions can be given structural interpretations. However, wide-spread rejection of neutrality would call for new empirical work on a wide array of macroeconomic questions.

The basic ideas in this paper can be applied to a bivariate permanent and transitory shock decomposition for any variable.[22] The structural assumptions will, of course, depend on the variable that is being decomposed into these two types of shocks as well as the other variable in the model.

New insights might be obtained by extending this bivariate analysis to a setting with more than two shocks. With this extension we could investigate the structural implications of models that identify multiple permanent shocks to output and multiple temporary shocks, for example.[23] Taking the analysis from this paper to such models is not, however, a simple extension. One complication will be matrix algebra that is certainly more tedious. A more difficult problem would be if the number of inequalities increases to the point where it becomes difficult or impossible to obtain unambiguous results. Nevertheless, because of the many papers that have identified multiple permanent and/or multiple transitory shocks, this extension may well be worth pursuing

This paper provides a deeper understanding of the relationship between permanent-transitory shock decompositions of output and economic structure. In general, when the key identification assumption is not a valid structural restriction, this decomposition will obtain inconsistent estimates of the structural responses. Nevertheless, if plausible structural assumptions are available, the permanent-transitory shock decomposition may still be used to infer important facts about the economy.

Appendix A: Recursive Substitution

Equation (2) can be written as:

$$X_t = X_{t-1} + \theta(L)\varepsilon_t .$$

This relationship holds for any time period, and so:

$$X_{t-1} = X_{t-2} + \theta(L)\varepsilon_{t-1} .$$

Substituting this equation for X_{t-1} into the first equation yields:

$$X_t = X_{t-2} + \theta(L)\varepsilon_t + \theta(L)\varepsilon_{t-1}$$

Then a similar substitution is made for X_{t-2}, followed by X_{t-3}, etc., to obtain:

$$X_t = X_0 + \sum_{k=0}^{t-1} \theta(L)\varepsilon_{t-k} \quad .$$

This equation can be written as:

$$\begin{aligned} X_t = X_0 &+ (\theta_0\varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \ldots) + (\theta_0\varepsilon_{t-1} + \theta_1\varepsilon_{t-2} + \theta_2\varepsilon_{t-3} + \ldots) + \\ &+ (\theta_0\varepsilon_{t-2} + \theta_1\varepsilon_{t-3} + \theta_2\varepsilon_{t-4} + \ldots) + \ldots \end{aligned}$$

Matching up the coefficients on each ε yields equations (3) and (4).

And precisely the same method is used with equation (8) to obtain equations (9) and (10).

Appendix B: Proof of Proposition 2.

The three structural assumptions in this proposition are:

I. For Economy A, the textbook aggregate supply and demand model describes the structure, hence the permanent-transitory shock decomposition identifies structural effects.

II. For Economy B, aggregate demand shocks have permanent effects on output, hence the permanent-transitory shock decomposition is unable to identify effects of aggregate supply and demand.

III. Both economies have identical short-run and intermediate-run structures.

Calculate the k-step forecast error for each economy:

For Economy A, equation (3) gives the permanent-transitory shock decomposition, and therefore the k-step forecast error is:

$$X_t - E_{t-k}X_t = \sum_{j=0}^{k-1}\Phi_j\varepsilon_{t-j} = \sum_{j=0}^{k-1}\begin{bmatrix}\Phi_j^{YS} & \Phi_j^{YD}\\ \Phi_j^{PS} & \Phi_j^{PD}\end{bmatrix}\begin{bmatrix}\varepsilon_{t-j}^{S}\\ \varepsilon_{t-j}^{D}\end{bmatrix}. \tag{i}$$

For economy B, equation (9) yields the decomposition because aggregate demand has a permanent effect on output. Hence, the k-step forecast error for this economy is:

$$X_t - E_{t-k}X_t = \sum_{j=0}^{k-1}\Phi_j\theta(1)C(1)^{-1}\mu_t = \frac{\displaystyle\sum_{j=0}^{k-1}\begin{bmatrix}\Phi_j^{YS} & \Phi_j^{YD}\\ \Phi_j^{PS} & \Phi_j^{PD}\end{bmatrix}\begin{bmatrix}\Theta_{YS} & -\Theta_{YD}\\ \Theta_{YD} & \Theta_{YS}\end{bmatrix}\begin{bmatrix}\mu_{t-j}^{P}\\ \mu_{t-j}^{T}\end{bmatrix}}{\left(\Theta_{YS}^2+\Theta_{YD}^2\right)^{1/2}} \tag{ii}$$

Although Φ_j can't be the same for all j because the two economies are different in the long run, these structural parameters are the same for each economy for some range of finite k because of Assumption III. Recall that output is the first element in X. Then, the k-step forecast variance for output associated with permanent shocks for Economy A is obtained from equation (i):

$$\frac{\displaystyle\sum_{j=0}^{k-1}(\Phi_j^{YS})^2}{\displaystyle\sum_{j=0}^{k-1}(\Phi_j^{YS})^2+(\Phi_j^{YD})^2}\ , \tag{iii}$$

and for Economy B is obtained from equation (ii):

$$\frac{\displaystyle\sum_{j=0}^{k-1}\left(\Phi_j^{YS}\Theta_{YS}\right)^2+\left(\Phi_j^{YD}\Theta_{YD}\right)^2+2\Phi_j^{YS}\Theta_{YS}\Phi_j^{YD}\Theta_{YD}}{\displaystyle\sum_{j=0}^{k-1}\left(\left(\Phi_j^{YS}\right)^2+\left(\Phi_j^{YD}\right)^2\right)\left(\Theta_{YS}^2+\Theta_{YD}^2\right)}\ . \tag{iv}$$

The question is what conditions on Θ_{YD} will guarantee that permanent shocks explain a larger fraction of output variance in Economy B? Multiplying by positive denominators and collecting terms, we can show that expression (iv) is greater than expression (iii) when:

$$2\Theta_{YS}\Theta_{YD}\sum_{j=0}^{k-1}\Phi_j^{YS}\Phi_j^{YD} + \Theta_{YD}^2\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right] \; > \; 0$$

It is convenient to divide by Θ_{YS} squared, and factor the expression as follows:

$$\left(\frac{\Theta_{YD}}{\Theta_{YS}}\right)\left\{2\sum_{j=0}^{k-1}\Phi_j^{YS}\Phi_j^{YD} + \left(\frac{\Theta_{YD}}{\Theta_{YS}}\right)\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right]\right\} \; > \; 0 \; . \qquad \text{(v)}$$

From assumptions A1 and A3 we know that $\sum_{j=0}^{k-1}\Phi_j^{YS}\Phi_j^{YD}$ is positive. However, $\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right]$ may be zero, negative or positive depending on the relative importance of supply and demand shocks for output during the first k periods. Consider each of these three cases.

When $\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right]$ is:

Case 1: zero, $\dfrac{\Theta_{YD}}{\Theta_{YS}} > 0$ satisfies (v);

Case 2: negative, $0 < \dfrac{\Theta_{YD}}{\Theta_{YS}} < \dfrac{-2\sum_{j=0}^{k-1}\Phi_j^{YS}\Phi_j^{YD}}{\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right]} > 0$; satisfies (v);

positive, (v) is satisfied by either:

Case 3a: $\dfrac{\Theta_{YD}}{\Theta_{YS}} > 0$; or

Case 3b: $\dfrac{\Theta_{YD}}{\Theta_{YS}} < \dfrac{-2\sum_{j=0}^{k-1}\Phi_j^{YS}\Phi_j^{YD}}{\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2 - \left(\Phi_j^{YS}\right)^2\right]} < 0.$

For Cases 1, 2 and 3a, $\dfrac{\Theta_{YD}}{\Theta_{YS}}$ is positive, and therefore Θ_{YD} is positive. However, Case 3b yields a negative value for this parameter. If we can show that these negative Θ_{YD} values are irrelevant, that will complete a proof that Θ_{YD} is positive.

The key to ruling out the negative values is to note that permanent output shocks always have a positive effect on output in the estimates. This positive output response places a lower bound on Θ_{YD}, a negative number that is not as negative as the values of Θ_{YD} in Case 3b. Hence, the negative values of Θ_{YD} that satisfy (v) are so negative that a permanent shock would cause output to fall at some point in the impulse response, a condition that is ruled out by the evidence.

The response of output to a permanent shock is given by equation (24), and if the first k responses are positive this equation implies:

$$\frac{\Theta_{YD}}{\Theta_{YS}} > \frac{-\Phi_j^{YS}}{\Phi_j^{YD}} \quad \text{for j=0,1,...,k-1.}$$

Let $\rho_j = \dfrac{\Phi_j^{YS}}{\Phi_j^{YD}}$. Each ρ_j is positive because of assumptions A1 and A3. Define ρ_* as the minimum over all ρ_j for j = 0, 1, 2,..., k-1. Based on the previous inequality we can see that if $\dfrac{\Theta_{YD}}{\Theta_{YS}}$ were smaller than $-\rho_*$, some portion of output's response to a permanent shock would be negative. Since output does not fall in response to a permanent shock, $-\rho_*$ sets a lower bound for $\dfrac{\Theta_{YD}}{\Theta_{YS}}$. The negative values in Case 3b are ruled out if:

$$\frac{-2\sum_{j=0}^{k-1}\Phi_j^{YS}\Phi_j^{YD}}{\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2-\left(\Phi_j^{YS}\right)^2\right]} < -\rho_* . \tag{vi}$$

To show that the inequality in (vi) holds, eliminate Φ_j^{YS} using the definition of ρ_j, and remembering $\sum_{j=0}^{k-1}\left[\left(\Phi_j^{YD}\right)^2-\left(\Phi_j^{YS}\right)^2\right]$ is positive for this analysis, manipulate (vi) into the following inequality:

$$\sum_{j=0}^{k-1}(2\rho_j-\rho_*+\rho_*\rho_j^2)\left(\Phi_j^{YD}\right)^2 \quad > \ 0.$$

Since ρ_j is positive for all j and $\rho_j \geq \rho_*$ for all j, this inequality is unquestionably true. Therefore (vi) holds, ruling out Case 3b and completing the proof of Proposition 2.

Notes

1. See Campbell and Mankiw (1987), Clark (1987), Cochrane (1988) and Watson (1986), for example.

2. Quah (1992) presents theoretical results showing potential advantages of a multivariate approach.

3. Lippi and Reichlin (1993) develop methods for handling non-invertible structures. Blanchard and Quah (1993) discuss relevancy and implications of non-invertibility in their reply.

4. Shapiro and Watson (1988), King Plosser, Stock and Watson (1991), Gamber and Joutz (1993) and Amed, Ickes, Wang and Yoo (1993) are early examples of models with more than two shocks.

5. Bordo (1993), Bayoumi and Eichengreen (1994), Karras (1994) and Keating and Nye (1998) use inflation instead of the unemployment rate.

6. While it is true that $\Phi_\infty=\theta(1)$, it is useful to distinguish between finite horizon effects, Φ_k for finite values of k, and long-run effects, $\theta(1)$, in the analysis to follow.

7. See Hamilton (1994, p.91).

8. This point is related to the fact that correlation does not always imply causation.

9. Faust (1998) and Uhlig (1999) employ sign restrictions to estimate structures. While Waggoner and Zha (2003) discuss these two papers in light of problems that may arise from inappropriately normalizing equations in a multivariate system, they also point out that recursive models are not subject to such problems. But I only use sign restrictions for interpretation of the statistical model. The permanent-transitory shock decomposition is recursive, and is therefore not subject to these normalization concerns.

10. Basu, Fernald and Kimball (1999) is a rare exception that theoretically shows output initially falling after a technological improvement. I am, however, unaware of any empirical evidence for such effects.

11. I could have made the even weaker assumption that this price response is non-negative. Permitting $\Phi_k^{PD} = 0$ allows us to interpret results under the assumption that prices are sticky for some time following an aggregate demand shock. When this response is zero, there is only one new insight: A permanent increase in output will unambiguously lower the price level. This is interesting because there is some evidence that prices are sticky in the postwar and in nearly all postwar estimates price falls with a permanent increase in output. Furthermore, in all but two of the prewar estimates price rises with a permanent increase in output. This finding rejects $\Phi_k^{PD} = 0$, which is consistent with the view of some economists that price adjustment was comparatively fast in the prewar. See the discussion in Calomiris and Hubbard (1989) and their references to differences in the speed of price adjustment.

12. Section 4 in their paper discusses how problems with the quality and consistency of pre-1914 data are unable to explain this unusual finding.

13. Short-run overshooting in prewar samples is found for the US, UK, Sweden, Japan, Italy, Germany and France. Such behavior is not observed for Canada, Denmark and Norway

14. Keating and Nye (1998) find that temporary shocks cause price and output to move in opposite directions for half of the countries in the prewar sample. This is one more finding consistent with aggregate demand not being long-run neutral in that period. Keating and Nye propose another potential explanation: The possibility that there were two types of supply shocks, one type which has only a temporary output effect and the other which can have a permanent effect.

15. Bullard and Keating (1995), Bae and Ratti (2000), Crosby and Otto (2000) and Rapach (2003) address superneutrality propositions using bivariate models in which inflation is decomposed into permanent and transitory shocks. Ahmed and Rogers (2000) also use permanent inflation shocks to address long-run superneutrality, using a model with cointegration and more than two variables.

16. See Blanchard and Summers (1986) for an interesting exposition and Ball (1999) who argues that hysteresis provides a reasonable explanation for postwar variation in unemployment rates.

17. Cooper and John (1988) is seminal work in this area and Cooper (1999) provides an excellent discussion of the literature on macroeconomic coordination failures.

18. See Wallis and North (1986). I thank John Nye for pointing out this paper to me.

19. Tobin (1993) reviews important contributions to research on destabilizing price flexibility.

20. The postwar sample ended in 1994, and since then Japan has had to contend with periods of persistent deflation. More recent Japanese data might be useful for testing the destabilizing price stability hypothesis in a postwar economy.

21. Fisher and Seater (1993) and King and Watson (1997) have developed different methods for testing neutrality propositions. But neither of these approaches is easily extended to models with more than two endogenous variables.

22. Other bivariate decompositions in the literature include output-per-hour by Gali (1999), stock prices by Cochrane (1994) and inflation by authors cited in footnote 17, for example. This list is not exhaustive because the number of papers using bivariate permanent-transitory shock decompositions is immense.

23. Gonzalo and Ng (2001) provide a general method for identifying multiple shocks of both types.

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Appendix C: Unpublished Impulse Responses from Keating and Nye (1998)