Dollarization Traps

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Abstract

The paper analyzes dollarization in the sense of *asset substitution*, where a foreign currency competes with local assets, especially domestic capital, as a store of value, the impact of dollarization on capital accumulation and output, and why economies remain dollarized long after a successful inflation stabilization. We relate this dollarization hysteresis to a financial intermediation failure that happens during high inflation. We show that in dollarized countries, inflation stabilization policies may not have any effect on domestic capital accumulation, thus preventing such policies from stimulating growth—*i.e.* dollarized economies are vulnerable to "dollarization traps".

JEL Classification Numbers: E40, E50, F41, E41

1 Introduction

Unofficial 'dollarization' has become a pervasive phenomenon in many of the emerging market economies. Discussions of the dollarization phenomenon have often focused on either *official dollarization*, where the country's government abandons the domestic currency and replaces it with a "hard" foreign currency (such as the U.S. dollar or euro),[1] or the unofficial *currency substitution*, *i.e.* the competition between US dollars and the domestic currency as a *medium of exchange*.[2] In this paper, we focus on dollarization in the sense of *asset substitution*, where a foreign currency

[1]For a discussion of the pros and cons of full official dollarization see Berg and Borensztein (2000). Chang and Velasco (2002) examine analytically the welfare consequences of official dollarization. Barro (1999) advocates official dollarization of the whole Latin America. Edwards (2001) and Edwards and Magendzo (2003) examine empirically the effects of official dollarization for economic growth.

[2]The literature on currency substitution has been concerned with issues of real money demand, optimal money growth, inflation tax and real exchange rate in the context of endowment infinitely-lived representative agent models. Different authors adopt different money demand specifications. Calvo (1985), Canzoneri and Diba (1992) and Imrohoroglu (1996) put home and foreign currency in the utility function of the representative agent, Guidotti (1993) and Agenor and Khan (1996) use a cash-in-advance framework where individuals are required to use domestic currency to purchase domestic goods and foreign currency to purchase foreign goods. Finally, Engineer (2000) utilizes a Townsend (1980) turnpike framework.

competes with local assets, especially domestic capital, as a store of value.[3] The paper assesses the impact of dollarization on capital accumulation, a topic that has been neglected in the dollarization literature.

Is partial unofficial dollarization beneficial or harmful to growth and welfare of economic agents? The answer is not straightforward. On the one hand, dollarization enables agents to circumvent costs associated with inflation and devaluations. On the other hand, dollarization tends to reduce seigniorage revenues—and therefore may potentially lead to higher tax rates—and dollarization may impair the effectiveness of monetary policy. Perhaps more importantly, investing in foreign currency holdings could substitute for domestic capital investment.

It is well understood and documented that economies get dollarized during episodes of high inflation. However, disinflations are not necessarily followed by *de*dollarization. In particular, Argentina, Bolivia, Peru, Russia, Ukraine and Uruguay remained highly dollarized long after the inflation rate was brought down to single digits.[4] The following table provides data on two different measures of dollarization, as well as inflation, in selected countries of the former Soviet Union in 2001.

– Insert Table 1 here –

Not only various measures of dollarization remain stubbornly high years after a successful disinflation, but they also show an upward trend in all the countries listed in Table 1, according to estimates of Feige (2003).

There have been two approaches to address the dollarization hysteresis paradox. The first approach is to modify existing currency substitution models to include adjustment costs or network externalities. Oomes (2003), Cuddington and Garcia (2002), Guidotti and Rodriguez (1992) and Uribe (1997) developed models in which the cost of using a foreign currency for transactions nega-

Vegh (1995) and Sturzenegger (1997) explicitly model the production side of the economy. However, in their models labor is the only input, and so there is no substitution between productive capital and dollars. Vegh (1995) adopts a transaction technology (shopping time) approach, while Sturzenegger (1997) uses a cash-in-advance framework.

Recently several search theoretic models of money have incorporated dual (or multiple) currencies. These contributions include Matsuyama et al. (1993), Shi (1995), Head and Shi (2003) and Zhou (1997).

[3]Here, we follow Calvo (1996, Chapter 8, p. 153) who defines currency substitution as "the use of foreign currency as a means of exchange" and dollarization as "the use of foreign currency in any of its three functions: unit of account, means of exchange, *and, in particular, store of value.*" (emphasis added).

[4]Brodsky (1997), Fiege (2003), Feige and Dean (2002), Friedman and Verbetsky (2000), Guidotti and Rodriguez (1992), Havrylyshyn and Beddies (2003), Kamin and Ericsson (2003), Mongardini and Mueller (1999), Peiers and Wrase (1997), Sahay and Vegh (1996), Savastano (1996), Van Aarle and Budina (1995) describe the dollarization experience of various Latin American and East European countries.

tively depends on the share of market participants who use this currency. Once the economy gets dollarized during high inflation, there is no benefit for an individual market participant to switch back to the domestic currency as long as the others continue using dollars. The limitation of this approach is that it explains the use of a foreign currency as a medium of exchange, but not as a store of value. However, it is the store-of-value function of money that billions of dollars "under the mattress" in Latin America and Eastern Europe perform. In most of these countries retailers are obliged by law to accept the domestic currency only, which limits the use of dollars as a medium of exchange.

The second approach is to explain the hysteresis by the lack of confidence in domestic monetary assets, resulting from past inflations, devaluations and bank failures (Feige, 2003). This approach hinges on the assumption that economic agents make systematic mistakes by holding dollars "under the mattress" and forfeiting a higher return on domestic monetary instruments. The "peso problem" as a potential explanation is not consistent with very strong macroeconomic fundamentals in several of these countries (including Peru and Russia in the early 2000s).

This paper presents an alternative explanation of the dollarization hysteresis paradox. We relate it to the underdevelopment of the financial system/financial intermediation failure that happens during the period of high inflation. The link between financial underdevelopment and dollarization has been noted in several descriptive papers,[5] but it has never been modeled explicitly.

Besides addressing the dollarization hysteresis paradox, our paper overcomes several limitations of the existing literature on dollarization. There is very little research on substitution between dollar denominated assets and domestic assets other than money.[6] This is surprising, since the use of foreign currency as a store of value usually precedes the use of foreign currency as a medium of exchange (Calvo and Vegh, 1992, Heymann and Leijonhufvud, 1995). Another serious limitation of the current dollarization literature is that it neglects the *real* effects of dollarization. Specifically,

[5]Savastano (1996) argues that "the relative importance of foreign currency as an inflation hedge will be inversely related to the economy's level of financial development. An economy with a well-developed financial market is, in principle, capable of adapting rapidly to a high inflation environment by offering a rich set of fairly liquid, high-yield instruments denominated in domestic currency ('near monies') that preserve the real value of the public's portfolio."(p. 226). Chile and, especially, Brazil are examples of countries that went through periods of high inflation in the 1970s – 1990s, but avoided dollarization. These two economies arguably have the most sophisticated banking systems in the South America.

[6]Several papers, including Kareken and Wallace (1981), Chang (1994), Sibert and Liu (1998), Tandon and Wang (1999), Mourmouras and Russell (2000), analyze substitution between domestic and foreign money in the context of endowment overlapping-generations models. In these models both competing monies serve as a store of value. The focus of this literature is on the relative money demand, inflationary finance of a given budget deficit and capital controls that governments (may) impose to protect the seigniorage base.

most existing models analyze dollarization in the context of endowment economies; none of them studies the interaction between dollarization and physical capital accumulation.

This paper aims to contribute to our understanding of the consequences of dollarization when foreign currency is used as a store of value, and therefore competes with domestic capital accumulation. Specifically, we study an overlapping-generations model in which agents may hold claims to capital as well as dollars as stores of value, and they hold domestic currency to insure against liquidity shocks, like in the models with spatial separation and limited communication (Champ *et al.* (1996), Smith and Schreft (1997, 1998), and Espinosa-Vega and Yip (1999)).

In light of empirical evidence, any realistic monetary growth model should have the prediction that a rise in the steady state inflation rate is associated with a reduction (possibly non-linear) in the domestic capital stock and domestic output (see Barro (1996), Bruno (1995), Chari *et al.* (1995), Fischer (1993), Ghosh and Phillips (1998), Khan and Senhadji (2001), and Sarel (1996)). Our model has this prediction. However, as long as the marginal product of capital is diminishing, this will lead to a rise in the real return to domestic capital, which in turn results in a shift away from investment in dollars and toward capital. This "arbitrage problem" clearly contradicts the empirical evidence on dollarization in high inflation economies; *i.e.* high inflation tends to raise the demand for dollars.

To address the arbitrage problem the paper assumes a non-convexity in production: when the aggregate capital stock is high enough, the scale of production allows for efficient financial intermediation and a modern technology with a marginal rate of return to capital higher than in the unintermediated "primitive" technology that prevails under lower capital stocks. Our model predicts that when the inflation rate is low, the capital stock and output are high enough, and the efficient intermediated technology is used. When the inflation rises above a certain threshold, a financial intermediation failure occurs, so that the economy "jumps" to an equilibrium with a lower capital stock and the primitive technology. This feature of the model is broadly consistent with the earlier theoretical and empirical work that shows that inflation negatively affects financial intermediation activities, and that the relationship between them may be highly nonlinear.[7]

Our model has three key predictions. First, for a range of inflation rates, there exists a steady state equilibrium in which the efficient technology (and no dollars) is used, as well as a steady state equilibrium in which the inefficient technology and dollars are used. Second, in the equilib-

[7] For empirical evidence see Boyd *et al.* (2001), and for theoretical analyses see Azariadis and Smith (1996), Choi *et al.* (1996), Boyd and Smith (1998), De Gregorio and Sturzenneger (1997), and Huybens and Smith (1998,1999).

rium where the inefficient technology and dollars are used, the link between the rate of inflation and capital accumulation is severed. This implies the hysteresis in dollarization ratios, capital accumulation, and output, i.e., the central prediction of the model. It is possible for economies to become stuck—for a range of inflation rates—in low output, technology induced "development traps", where the net marginal product of capital is the same as the return from holding dollars. The only way to exit from such an equilibrium is to reduce inflation below a threshold level. Third, the "dollarization trap" steady state equilibrium occurs only when inflation is falling, i.e., during a disinflation.

These predictions are broadly consistent with the experience of many dollarized countries. In particular, Bolivia, Russia, and Ukraine experienced significant reductions in output and increased dollarization during their spells of high inflation, and remained heavily dollarized when the inflation was brought down. In these countries, the recovery of output was slow (or nonexistent) in the aftermath of stabilization. On the other hand, in Poland and Estonia, dedollarization coincided with an output boom soon after the inflation rate fell below 30 % a year.

The format of the paper is as follows. The next section describes the model. Section 3 discusses production technologies. Section 4 discusses the demand for local currency in the presence of liquidity shocks. Section 5 discusses steady-state equilibria and transitional dynamics. Section 6 considers the effects of inflation on the equilibrium. Section 7 offers concluding remarks.

2 The Model

The environment is similar to other overlapping-generations models with spatial separation and limited communication, such as Champ *et al.* (1996), Schreft and Smith (1997, 1998) and Espinosa-Vega and Yip (1999). This is a small open economy consisting of two identical locations (islands). At each date the locations are completely symmetric. In every period a continuum of agents of measure 1 is born on each island. There is no population growth. Agents live for two periods. In the first period, agents are endowed with one unit of labor which they supply inelastically on the labor market and receive the perfectly competitive market wage, w_t. They derive utility from consumption in the second period of life only. Agents born in period t maximize their expected utility function $U_t = E_t \ln c_{2,t+1}$, where the first subscript denotes the period of life and the second

- the timing of consumption.[8]

To transfer wealth between periods, agents can hold domestic currency, dollars, or invest in productive capital. We assume that there is no inflation abroad, and that purchasing power parity holds. Therefore, the gross return on dollars is always unity. We restrict our attention to equilibria in which the return on domestic currency is strictly lower than the return on dollars and capital. In other words, we abstract from deflation and liquidity traps here. However, we assume that dollars and capital are perfect substitutes for the agents. Henceforth we will refer to them as capital market assets (CMA).

At the end of each period a fixed share of young agents, $\beta < 0.5$, are relocated to another island. Agents can take only the domestic medium of exchange (domestic currency) with them. Note that our focus is on dollars as store of value and not their role as a medium of exchange. Thus, the consumption good and CMA cannot be moved across islands. An agent has to make his investment decision before he knows if he will be relocated. If he invests only in CMA, he will face starvation in the second period if he is relocated. If he holds money, he will forfeit the higher return on other assets that he could enjoy if he is not relocated.

It has been shown that in this environment competitive banks arise endogenously to provide insurance against liquidity (relocation) risk. Banks issue demand deposits and hold portfolios of currency and CMA. They use the currency to honor deposit withdrawals of agents who are relocated. The explicit treatment of the competitive bank problem is presented in section 4. It is important to note that these banks are not financial intermediaries. They are merely insurance providers.[9]

The main difference between our environment and the setup of other spatial-separation-and-limited-communication models is that we assume the existence of two productive technologies.

[8] The main results hold if we generalize the utility function to the family of constant-relative risk aversion utility function, *i.e.*, $U_t = \frac{c_{2,t+1}^{1-\sigma}-1}{1-\sigma}$, where σ, the coefficient of the relative risk aversion is greater than or equal to 1. This generalization greatly complicates the algebra but does not affect the predictions. The derivations of the main results in the generalized CRRA case are available from the authors upon request.

[9] The spatial-separation-and-limited-communication framework is just one way to derive a positive demand for real money balances such that an increase in the steady state inflation rate reduces the capital stock and output. A feasible alternative is to have a three-period OLG model, in which agents work only in the first period of life, consume only in the third period, and the domestic currency must be held for at least one period prior to the purchase of CMA. Therefore, agents hold the currency between the first and the second period of life, and invest in CMA at the end of the second period of life. This assumption is analogous to the standard cash–in–advance requirement, however in this model it is the purchases of capital market assets, rather than consumption goods, that are subject to a cash–in–advance constraint. The results that the model with this alternative setup generates are qualitatively identical to the results of the model presented in this paper. A version of the paper with this alternative setup is available from authors upon request.

One technology is more efficient, but it requires use of financial intermediation—henceforth called a "financial center"—that incurs a fixed cost to operate. The efficient technology in per capita terms is $y = Ak^{\alpha} - \phi$, where y and k are the per capita output and the capital stock, respectively, and ϕ is the cost to operate the financial center. For simplicity, we abstract from exogenous technological progress and endogenous growth.

The ability to operate the financial center is the endowment of a special agent. There is just one special agent in each generation on each island. This agent does not work when young, and maximizes his expected consumption when old. His only source of income is the profit of the financial center.

The other technology does not require a financial center, *i.e.*, the agents can use it directly, but it is less productive. The less efficient technology is $y = A\gamma k^{\alpha}$, where $\gamma < 1$. Henceforth we will refer to it as the "primitive technology." Whichever technology is used, capital depreciates fully each period.

There is a government that prints money at the end of every period and gives the seigniorage as lump-sum transfers to the young agents staying on the same island. The gross growth rate of nominal money supply is constant each period and is denoted ρ. Hence $M_t = \rho M_{t-1}$. In the steady state, the gross inflation factor equals the gross rate of the money supply growth.

The key assumption of the model is the presence of two possible production technologies, one of which requires the use of financial intermediation—a "financial center"—that is subject to a fixed cost to operate. This financial center could literally be a financial intermediation technology, a stock market, or any other technology (*e.g.* computer center, power plant, infrastructure) that is necessary to use if the efficient production technology is to be utilized.

The timing of the events is as follows. First, the young agents of generation t are born. Second, they work, earn wages, and deposit their wages with competitive banks. At the same time the old agents receive their return on capital. Third, the old agents purchase the consumption good. At the same time competitive banks make investment decisions, *i.e.* they allocate their funds between the domestic currency and CMA. If the aggregate capital stock is large enough, banks invest the CMA portion of their funds through the financial center. Otherwise, they use the primitive technology and dollars. Fourth, relocation shocks are realized. Agents hit by the shock withdraw domestic currency from banks and travel to the other island. Fifth, the government prints money and gives transfers to the agents who remain on the same island.[10] Finally, the old agents of generation $t-1$ die.

3 The Financial Center

We assume that the financial center is operated as an independent profit-maximizing entity. It is created and owned by a special agent, who is the only agent endowed with the ability to operate the financial center. Therefore the financial center is a monopoly, as long as it operates. If it operates, it accumulates all resources available for investment, and invests them in the efficient technology (this will be verified below). In other words, it attracts deposits of banks acting on behalf of individual agents and pays them just the amount they would earn on their own with the primitive technology and dollars—the "autarkic portfolio".

Consider this autarkic portfolio and the return on it. Two cases should be considered, corresponding to whether the return on capital is greater than or equal to the return on dollars and thus whether the autarkic portfolio consists of both capital and dollars or just capital.

3.1 Return on Capital Equals the Return on Dollars

In this case both capital and dollars are held in the autarkic portfolio, and thus the competitive banks will split their CMA portfolio such that they invest in capital up to the point where the gross marginal product of capital is unity, the same as the marginal return on dollars:

$$R^T \equiv \gamma A\alpha(k^T)^{\alpha-1} = 1 \tag{1}$$

where k^T is the per capita capital stock.[11] Solving for k^T yields:

$$k^T = (A\alpha\gamma)^{1/(1-\alpha)} \tag{2}$$

If we denote by d the total size of the autarkic portfolio of CMA, then this situation can arise only if $d > k^T$.

Hence the center pays the competitive banks a gross rate of return equal to 1, and appropriates the profit. The cost of setting up the financial center is ϕ (in per capita terms). Therefore, assuming

[10] Even though the seigniorage revenue is given to young agents, they do not consume it until they get old, as only old agents consume in this model.

[11] We denote the per capita capita capital stock as k^T, because this is the level of the capital stock in the dollarization trap (T stands for trap).

for the moment that the financial center invests only in capital, the center's profit in per capita terms is:

$$\Pi^d = A\alpha k^\alpha - k - \phi \tag{3}$$

The bank operates as long as $\Pi^d \geq 0$, and shuts down when Π^d becomes negative. Figure 1 illustrates the revenue and the expenses of the financial center. $A\alpha k^\alpha$ is the total return on capital, *i.e.*, the revenue of the center. $k+\phi$ are the expenditures of the center. Profit is non-negative when revenue is no less than the expenditures, *i.e.*, $k^* \leq k \leq k^{**}$, where k^* and k^{**} are the smaller and the larger roots of the equation $A\alpha k^\alpha - k - \phi = 0$, respectively. Finally, $k^m = (A\alpha^2)^{1/(1-\alpha)}$ is the point where profit is maximized.

We make two additional assumptions about the efficient production technology, that ensure the existence of a non-empty range of values for k, such that $\Pi^d > 0$.

Assumption 1: $k \leq k^m$, *i.e*, $A\alpha^2 k^{\alpha-1} - 1 > 0$.

Assumption 2: $A\alpha(A\alpha^2)^{\alpha/(1-\alpha)} - (A\alpha^2)^{1/(1-\alpha)} - \phi > 0$.

Assumption 1 guarantees that profit of the financial center positively depends on its scale of operation ($\frac{\partial \Pi^d}{\partial k} = A\alpha^2 k^{\alpha-1} - 1$). We need this assumption to ensure that the financial center makes nonnegative profit and operates when the capital stock is high (and when inflation rate is low, see section 4), and shuts down when the capital stock is low. The corollary of Assumption 1 is that the efficient production technology is dynamically efficient, *i.e.*, $A\alpha k^{\alpha-1} > 1$. Hence the financial center never uses dollars (dollars are always dominated in rate or return by the efficient production technology). Assumption 2 ensures that the profit of the financial center is positive at its maximum point k^m. Therefore, the equation $A\alpha k^\alpha - k - \phi = 0$ has two positive real roots, and $\Pi^d \geq 0$ for $k^* \leq k \leq k^m$.

Therefore, the financial center operates, if $d \geq k^*$, and shuts down if $d < k^*$.

3.2 Return on Capital Exceeds Return on Dollars

In this case the autarkic portfolio of CMA contains no dollars, *i.e.* $d \leq k^T$. Thus, the total return on the autarkic portfolio is $A\alpha\gamma d^\alpha > 1$. This, in turn, implies that the the financial center, if it operates, invests in capital only (since the return on capital exceeds the return on dollars). Thus, its profit is:

$$\Pi = A\alpha k^\alpha - A\alpha\gamma k^\alpha - \phi = (1-\gamma)A\alpha k^\alpha - \phi \tag{4}$$

The financial center operates if and only if $\Pi \geq 0$, or

$$d \geq \tilde{k}^* \equiv \phi^{1/\alpha}[(1-\gamma)A\alpha]^{-1/\alpha} \tag{5}$$

We can state the following result:

Lemma 1: A) If $k^T < k^*$, then $k^T < \tilde{k}^* < k^*$.

B) If $k^T > k^*$, then $k^* < \tilde{k}^* < k^T$.

C) if $k^T = k^*$, then $k^T = k^* = \tilde{k}^*$.

Proof: See appendix.

Therefore, the comparison of k^*, k^T and $\tilde{k}^*$ can yield just these three cases. Note that the financial center operates and the autarkic portfolio does not contain dollars only in cases B) and C), if $\tilde{k}^* \leq d \leq k^T$.

The findings of this subsection are summarized in Proposition 1:

Proposition 1: If $k^* > k^T$, the financial center operates if and only if $d \geq k^*$. If $k^* \leq k^T$, the financial center operates if and only if $d \geq \tilde{k}^*$.

4 Demand for Domestic Currency

Analysis of the demand for money follows Champ *et al.* (1996), Smith and Schreft (1998), and Espinosa-Vega and Yip (1999). Competitive banks arise endogenously to provide insurance against liquidity shocks. Young agents deposit their savings with the banking system. Banks issue demand deposits, and they hold a share of deposits in domestic money in order to meet liquidity needs of relocated agents. The remaining funds are invested in CMA (either directly, or through the financial center). Because of perfect competition in the banking industry, banks earn zero profit, and they set the deposit rates for agents who withdraw early (relocated agents) and for the rest of the agents, so that the expected utility of a young agent is maximized:

$$U_t = \beta \ln(c^m_{2,t+1}) + (1-\beta)\ln(c^s_{2,t+1}) \tag{6}$$

where $c^m_{2,t+1}$ is the consumption of an agent who is relocated, $c^s_{2,t+1}$ is the consumption of an agent who stays on his home island.

The maximization of (6) is conducted subject to two feasibility constraints:

$$c^s_{t+1} = \frac{w_t(1-\lambda_t)}{1-\beta} R_t + a_{t+1} \tag{7}$$

and

$$c^m_{t+1} = \frac{w_t \lambda_t}{\pi_t \beta} \tag{8}$$

where λ_t is the share of deposits held in cash, w_t is the wage income of an agent in period t, R_t is the gross rate of return on autarkic portfolio, a_{t+1} is the real value of the government transfer in period $t+1$, and $1/\pi_t$ is the gross rate of return on domestic currency. The maximization is conducted with respect to λ_t, c^m_{t+1} and c^s_{t+1}.

The maximization yields the following first-order condition:

$$\pi_t R_t c^m_{t+1} = c^s_{t+1} \tag{9}$$

Equations (7)-(9) are a linear system in λ_t, c^m_{t+1} and c^s_{t+1}. In the next section we show that λ_t is always greater than zero, but less than one.

5 Competitive Equilibrium and Steady States

This section identifies the possible equilibria. There are three possibilities for equilibria: the efficient technology is used; the primitive technology is used but agents do not hold dollars; and the primitive technology is used and agents hold both dollars and capital.

5.1 Equilibrium with the Efficient Technology

Six equations determine the competitive equilibrium of this economy when the efficient technology is used:

$$\begin{aligned}
w_t &= (1-\alpha)Ak_t^\alpha && (10)\\
R_t &= 1 \;\text{ if }\; k_{t+1} \geq k^T;\; R_t = \alpha A \gamma k_{t+1}^{\alpha-1} \;\text{ if }\; k_{t+1} < k^T && (11)\\
k_{t+1} &= (1-\lambda_t)w_t && (12)\\
a_{t+1} &= \frac{(\rho-1)\lambda_t w_t}{\pi_t(1-\beta)} && (13)\\
a_{t+1} &+ \frac{w_t(1-\lambda_t)}{1-\beta}R_t = \frac{w_t\lambda_t}{\beta}R_t && (14)\\
(1-\beta)a_{t+1} &= w_t - k_{t+1} - \frac{\lambda_t w_t}{\pi_t} && (15)
\end{aligned}$$

Equations (10) and (11) define the values of the factor prices. Equation (12) describes the law of motion of the capital stock: young agents deposit all their wage income with competitive banks, and the banks invest share $1-\lambda_t$ of deposits in capital. Equation (13) is the government budget constraint. It states that the government expenditure, $(1-\beta)a_{t+1}$, equals real seigniorage revenue, $\frac{(\rho-1)\lambda_t w_t}{\pi_t}$. Equation (14) is the first-order condition of the competitive banks' optimization problem (9). Finally, equation (15) states that a share of the wage income of young agents of generation t is acquired by the government and spent on the transfer, which equals $(1-\beta)a_{t+1}$ in per capita terms. The value of the transfer also equals the wage income, w_t, net of the value of the assets retained by the representative agent, $k_{t+1}+\frac{\lambda_t w_t}{\pi_t}$.

In every period t, k_t is predetermined. The system (10)-(15) can be solved for $R_t, k_{t+1}, w_t, a_{t+1}, \pi_t$ and λ_t. Lemma 2 establishes the existence and uniqueness of the solution.

Lemma 2. The system (10)-(15) has a unique solution such that $\lambda_t \in (0,1)$. *Proof: See Appendix.*

Taking into account that $\pi_t = \rho$ in the steady state and eliminating w_t and a_{t+1}, the system (11)-(16) can be rewritten for the steady state in the following way:

$$R\frac{\lambda}{\beta} = \frac{(1-\lambda)R}{1-\beta} + \frac{\left(1-\frac{1}{\rho}\right)\lambda}{1-\beta} \tag{16}$$

$$k = (1-\lambda)(1-\alpha)Ak^{\alpha} \tag{17}$$

$$R = 1 \text{ if } k \geq k^T;\ R = \alpha A\gamma k^{\alpha-1} \text{ if } k < k^T \tag{18}$$

The following proposition establishes the main comparative statics result.

Proposition 2: If the financial center operates, an increase in the steady-state money supply growth rate raises the share of bank assets held in currency, and reduces the per capita capital stock.

Proof: See Appendix.

The negative relationship between the steady-state inflation rate and the capital stock established in the Proposition 2 is often termed the reversed Mundell-Tobin effect.

Lemma 3 below establishes that the dynamical equilibrium analyzed above is stable, *i.e.*, it converges to the steady state.

Lemma 3. The steady state equilibrium described by the system (11)-(16) is stable.

Proof: See Appendix.

For $k^* > k^T$, the condition $d \geq k^*$ is equivalent to $k \geq k^*$ (since dollars are not held). This allows us to calculate values of ρ compatible with the efficient-technology steady-state equilibrium. Specifically, it follows from $k \geq k^*$, Proposition 2 and (16)-(18) that the equilibrium exists if and only if:

$$\rho \in [1, \rho^*], \tag{19}$$

where:

$$\rho^* = \frac{1}{1 - \frac{1}{\beta} + \frac{1}{\lambda^*}} \tag{20}$$

and

$$\lambda^* = 1 - \frac{(k^*)^{1-\alpha}}{(1-\alpha)A} \tag{21}$$

Equation (20) is obtained by solving (16) for ρ taking into account that $R = 1$ if $k \geq k^T$. Equation (21) is obtained by solving (17) for λ for $k = k^*$.

If $k^* \leq k^T$, the condition $d \geq \tilde{k}^*$ is equivalent to $k \geq \tilde{k}^*$ (since dollars are not held). This allows us to calculate values of ρ compatible with the efficient-technology steady-state equilibrium. It follows from $k \geq \tilde{k}^*$, Proposition 2 and (16)-(18) that the equilibrium exists if and only if:

$$\rho \in [1, \tilde{\rho}^*], \tag{22}$$

where

$$\tilde{\rho}^* = \frac{1}{1 - \tilde{R}^* \left(\frac{1}{\beta} - \frac{1}{\tilde{\lambda}^*}\right)} \tag{23}$$

$$\tilde{\lambda}^* = 1 - \frac{(\tilde{k}^*)^{1-\alpha}}{(1-\alpha)A} \tag{24}$$

$$\tilde{R}^* = \alpha\gamma A \left(\tilde{k}^*\right)^{\alpha-1} \tag{25}$$

Equation (23) is obtained by solving (16) for ρ taking into account that $R = \alpha\gamma A k^{\alpha-1}$ if $k \leq k^T$. Equation (24) is obtained by solving (17) for λ for $k = \tilde{k}^*$.

Note that whether k^* is greater than or less than k^T is a parametric restriction and thus only one of the sets $[1, \rho^*]$ and $[1, \tilde{\rho}^*]$ is relevant for any given parameterization.

5.2 Steady-State Equilibrium with the Primitive Technology and No Dollars

The primitive technology is used if and only if the financial center cannot make a profit. From Proposition 1 this happens whenever $d < k^*$ if $k^* > k^T$, and whenever $d < \tilde{k}^*$ if $k^* \leq k^T$. Dollars are not present in the autarkic portfolio, if and only if $d \leq k^T$. Hence, a steady state equilibrium with no dollars occurs if $d \equiv k < k^T$ in case A ($k^* > k^T$), and if $d \equiv k < \tilde{k}^*$ in cases B-C ($k^* \leq k^T$).

Six equations determine the competitive equilibrium of this economy when the inefficient technology is used:

$$
\begin{aligned}
w_t &= (1-\alpha)A\gamma k_t^{\alpha} && (26)\\
R_t &= \alpha\gamma A k_{t+1}^{\alpha-1} && (27)\\
k_{t+1} &= (1-\lambda_t)w_t && (28)\\
a_{t+1} &= \frac{(\rho-1)\lambda_t w_t}{\pi_t(1-\beta)} && (29)\\
a_{t+1} &+ \frac{w_t(1-\lambda_t)}{1-\beta}R_t = \frac{w_t\lambda_t}{\beta}R_t && (30)\\
(1-\beta)a_{t+1} &= w_t - k_{t+1} - \frac{\lambda_t w_t}{\pi_t} && (31)
\end{aligned}
$$

The system (26)-(31) is almost identical to the system (10)-(15). The two differences are in the factor prices. Given that the inefficient technology is used, the wage income expression has the coefficient γ in it. Furthermore, since by assumption dollars are not used, the return on CMA is greater than or equal to unity, and it coincides with the return on capital using the inefficient technology.

In every period t, k_t is predetermined. The system (26)-(31) can be solved for $R_t, k_{t+1}, w_t, a_{t+1}, \pi_t$ and λ_t. Lemma 4 establishes the existence and uniqueness of the solution.

Lemma 4. The system (26)-(31) has a unique solution such that $\lambda_t \in (0,1)$.

The proof mirrors the proof of Lemma 2 and is omitted.

Taking into account that $\pi_t = \rho$ in the steady state and eliminating w_t and a_{t+1}, the system (26)-(31) can be rewritten for the steady state in the following way:

$$
\begin{aligned}
R\frac{\lambda}{\beta} &= \frac{(1-\lambda)R}{1-\beta} + \frac{\left(1-\frac{1}{\rho}\right)\lambda}{1-\beta} && (32)\\
k &= (1-\lambda)(1-\alpha)A\gamma k^{\alpha} && (33)\\
R &= \alpha A\gamma k^{\alpha-1} && (34)
\end{aligned}
$$

The following proposition establishes the main comparative statics result.

Proposition 3: If $k < k^T$ and the financial center does not operate, an increase in the steady-state money supply growth rate raises the share of bank assets held in currency, and reduces the per capita capital stock.

The proof mirrors the proof of Proposition 2 and is omitted.

Lemma 5. The steady state equilibrium described by the system (26)-(31) is stable.

The proof mirrors the proof Lemma 3 and is omitted.

For $k^* > k^T$, the condition $d \equiv k < k^T$ allows us to calculate values of ρ compatible with the primitive-technology steady-state equilibrium in case A. It follows from $k < k^T$, equations (32)-(34) and Proposition 3, that the equilibrium exists if and only if:

$$\rho > \rho_1 \tag{35}$$

where:

$$\rho_1 = \frac{1}{1 - \left(\frac{1}{\beta} - \frac{1}{\lambda^T}\right)} \tag{36}$$

$$\lambda^T = 1 - \frac{(k^T)^{1-\alpha}}{(1-\alpha)A} \tag{37}$$

Equation (36) is obtained by solving (32) for ρ for $R = 1$ (at $k = k^T, R = 1$). Equation (37) is obtained by solving (33) for λ for $k = k^T$.

For $k^* \leq k^T$, the condition $d \equiv k < \tilde{k}^*$ allows us to calculate values of ρ compatible with the primitive-technology steady-state equilibrium in cases B-C. It follows from $k < \tilde{k}^*$, equations (32)-(34) and proposition 3, that the equilibrium exists if and only if:

$$\rho > \tilde{\rho}_2^* \tag{38}$$

where:

$$\tilde{\rho}_2^* = \frac{1}{1 - \tilde{R}^* \left(\frac{1}{\beta} - \frac{1}{\tilde{\lambda}_2^*}\right)} \tag{39}$$

$$\tilde{\lambda}_2^* = 1 - \frac{(\tilde{k}^*)^{1-\alpha}}{(1-\alpha)A\gamma} \tag{40}$$

and

$$\tilde{R}^* = \alpha\gamma A \left(\tilde{k}^*\right)^{\alpha-1} \tag{25}$$

Equation (39) is obtained by solving (32) for ρ. Equation (40) is obtained by solving (33) for λ.

5.3 Steady State Equilibrium with Dollars

A steady state equilibrium with dollars can exist only if $k^T < k^*$. Otherwise, the financial center starts operating at a capital stock below k^T. Taking into account that $k_t = k^T$ and $R_t = 1$, the dynamics of the model are described by the following equations:

$$\begin{aligned}
w_t &= (1-\alpha)A\gamma\left(k^T\right)^{\alpha} && (42)\\
k^T + \delta_t &= (1-\lambda_t)w_t && (43)\\
a_{t+1} &= \frac{(\rho-1)\lambda_t w_t}{\pi_t(1-\beta)} && (44)\\
a_{t+1} &+ \frac{w_t(1-\lambda_t)}{1-\beta} = \frac{w_t\lambda_t}{\beta} && (45)\\
(1-\beta)a_{t+1} &= w_t - (k^T + \delta_t) - \frac{\lambda_t w_t}{\pi_t} && (46)
\end{aligned}$$

where δ_t is the per capita stock of dollars. Equation (43) is the asset accumulation equation when dollars are held. The right-hand side is the real value of labor income invested in CMA, and the left-hand side is holdings of capital and dollars. The system (42)-(46) can be solved for $a_{t+1}, w_t, \delta_t, \lambda_t$ and π_t.

Lemma 6. The system (42)-(46) has a unique solution, such that $\lambda_t \in (0,1)$.

Proof: See Appendix.

The following system describes the steady-state equilibrium of the model

$$\begin{aligned}
\frac{1-\lambda}{1-\beta} + \frac{\left(1-\frac{1}{\rho}\right)\lambda}{1-\beta} &= \frac{\lambda}{\beta} && (47)\\
(1-\lambda)(1-\alpha)A\gamma\left(k^T\right)^{\alpha} &= k^T + \delta && (48)
\end{aligned}$$

Equations (47)-(48) are obtained from (42)-(46) by substituting out w_t and a_{t+1} and taking into account that $\pi_t = \rho$ in the steady state.

Proposition 4 presents the main comparative statics result of this section.

Proposition 4. A disinflation reduces the share of deposits held in cash and increases the dollar holdings.

Proof: See Appendix.

A disinflation (a reduction in ρ) reduces λ, the share of the wage income held in currency, and therefore, increases the right-hand side of (48). The capital stock is pinned down by the arbitrage condition, and is therefore independent of the inflation rate. Hence the disinflation raises the dollar holdings δ.

The following lemma shows that the model does not have any transitional dynamics in this "dollarization trap" equilibrium.

Lemma 7. For a given rate of the money supply growth, the share of bank deposits held in domestic currency, λ, the capital stock, k, the inflation rate, π, and the dollar holdings, δ, are all time invariant and equal to their steady-state values.

Proof: See Appendix.

Next we calculate the values of ρ compatible with this dollarization trap equilibrium. The lowest possible money growth rate—denoted as ρ_2—makes the right-hand side of (48) equal to k^*, *i.e.*, saving is sufficiently high that $d = k^*$ which induces a switch back to the efficient technology. Algebraically,

$$\rho_2 = \frac{1}{\frac{1}{\lambda_2} - \frac{1}{\beta} + 1} \tag{49}$$

and

$$\lambda_2 = 1 - \frac{k^*}{(1-\alpha)\gamma A(k^T)^\alpha} \tag{50}$$

Here equation (49) is obtained from equation (47) solved for ρ, and equation (50) is obtained from (48) solved for λ under the assumption that the right-hand side of (48) is equal to k^*.

Similarly, the highest possible money growth rate—denoted as ρ_1—consistent with this equilibrium makes the right-hand side of (48) equal to k^T (zero demand for dollars). Algebraically,

$$\rho_1 = \frac{1}{\frac{1}{\lambda_1} - \frac{1}{\beta} + 1} \tag{51}$$

and

$$\lambda_1 = 1 - \frac{k^T}{(1-\alpha)\gamma A(k^T)^\alpha} = 1 - \frac{\left(k^T\right)^{1-\alpha}}{(1-\alpha)\gamma} \tag{52}$$

Here equation (51) is obtained from equation (47) solved for ρ, and equation (52) is obtained from (48) solved for λ under the assumption that the right-hand side of (48) is equal to k^T.

$\rho_1 > \rho_2$, because the right-hand side of (48) is monotonically decreasing in ρ. Hence it attains the lower value, k^T, at a higher level of the steady state inflation, than the higher value, k^*.

5.4 Summary

Two different cases emerge depending on whether k^T is greater than, less than, or equal to, k^*.

Case A. $k^* > k^T$. All three equilibria are feasible, and there may could be multiple equilibria for a range of the money supply growth rates. If $\rho \in (1, \rho_2)$, only the efficient-technology equilibrium exists. If $\rho \in [\rho_2, \rho_1)$ both the efficient-technology steady state and the steady state with the inefficient technology and dollars are possible. Lemma 8 in the Appendix proves that $\rho_1 < \rho^*$. Thus, for $\rho \in [\rho_1, \rho^*)$ there exist two steady equilibria: the efficient-technology equilibrium and the inefficient-technology-no-dollars equilibrium. Finally, for $\rho \geq \rho^*$ the only equilibrium is with the inefficient technology and no dollars. Thus, the range of the gross inflation factor values $[\rho_2, \rho^*]$ is compatible with two different steady-state equilibria. Figure 3 illustrates this case.

Case B-C. $k^T \geq k^*$. The inefficient-technology-and-dollars equilibrium is not feasible. If $\rho \leq \tilde{\rho}^*$, the efficient-technology equilibrium exists. If $\rho \geq \tilde{\rho}_2$, the primitive-technology-no-dollars equilibrium exists. Therefore the range of the gross inflation factor values $[\tilde{\rho}_2, \rho^*]$ is compatible with two different steady-state equilibria, one of which uses the efficient technology and on which uses the inefficient technology (and no dollars). Figure 4 illustrates this case.

6 Dynamics of Inflation and Disinflation

6.1 Qualitative Features of a Dollarization Trap

A dollarization trap arises only if $k^* > k^T$. In this case, assume that inflation is low, but rising. The economy starts at point A and gradually moves to point B (Figure 3). Along the way the efficient technology is used, but the capital stock is falling due to higher inflation. Any temporary deviation from the steady state equilibrium dies out, because steady states are stable.

If the inflation rate exceeds ρ^*, a bifurcation takes place. The financial center shuts down, and agents have to use the primitive technology instead. The economy "jumps" from point B to point C. Along the trajectory D-F, there is still a negative relationship between the inflation rate and the capital stock. Therefore, a disinflation raises the capital stock above k^*, the level of capital stock at C. However, if the inflation rate falls below ρ_1, the investment and output recovery halts. The economy is stuck in a trap, where the level of the capital stock is pinned down by return on capital.

Disinflation translates not into a larger capital investment, but into larger dollar holdings. Hence our model is consistent with the empirical evidence that falling inflation sometimes coexists with a *rising* dollarization. Only when the inflation rate falls to ρ_2, another bifurcation takes place, the financial center resumes operations, and the capital stock "jumps" from k^T to k_2.

It is important to note that because of the multiplicity and stability of equilibria, the level of the capital stock (and output) during disinflation is lower than the level during rising inflation at the same inflation rate. Moreover, as the disinflation progresses (as the economy moves from D to E), the gap rises and reaches its maximum when the inflation rate falls to ρ_2.

6.2 Dollarization Trap and Financial Development

What factors affect the likelihood of a dollarization trap? The trap arises only if $k^* > k^T$, therefore all the factors that reduce k^* relative to k^T makes the trap less likely. In particular, lemma 9 below asserts that a reduction in ϕ lowers k^*, but it does not affect k^T.

Lemma 9: $\frac{\partial k^*}{\partial \phi} > 0$.

Proof: See Appendix.

In figure 1, a reduction in ϕ shifts the expenditure line of the financial center, $k + \phi$, down, and hence the point of intersection of this line with the revenue curve $A\alpha k^\alpha$ shifts to the left. This finding is very intuitive. ϕ is inversely related to the level of financial development. Therefore, a more developed financial system makes the dollarization trap less likely. When ϕ approaches 0, k^* approaches 0 as well. The dollarization trap becomes impossible.

7 Conclusion

This paper studies the link between inflation, the demand for foreign currency as a store of value, and capital accumulation. Our principal aim in the paper is to identify circumstances which can explain key empirical facts in dollarized countries. These empirical facts are that inflation is a main cause of dollarization, that dollarization coincides with adverse real economic performance, and that both the level of dollarization and the performance of the real economy are very slow to reverse following inflation stabilization.

The key assumption in the paper is that the efficiency of the production technology depends positively on the level of capital accumulation. This assumption appears in slightly different form in various endogenous growth models. In our model, this assumption is critical for explaining dollarization hysteresis. The reason is that, to be consistent with the stylized facts in dollarized economies, a sufficiently high inflation rate must reduce the marginal product of capital, decrease capital accumulation, and induce a higher demand for dollars. This is impossible if there is a single neoclassical production technology.

There are three main implications of the model. First, there exist steady states with a relatively high capital stock, low dollarization and low inflation, as well as steady states with a relatively lower capital stock, higher inflation, and substantial dollarization. Second, for a range of intermediate inflation rates, there co-exist a steady state equilibrium in which the efficient technology (and no dollars) is used and a steady state equilibrium in which the inefficient technology and dollars are used. Third, in the equilibrium where the inefficient technology and dollars are used, the link between the rate of inflation and capital accumulation is severed. This implies that hysteresis in dollarization ratios, capital accumulation, and output is a central prediction of the model. It is possible for economies to become stuck—for a range of inflation rates—in low growth, technology induced "development traps", where the net marginal product of capital is the same as the return from holding dollars. The only way to exit from such an equilibrium is to reduce inflation below a threshold level.

Appendix

Proof of Lemma 1.

It is easy to verify that $\Pi^d > \Pi$ if and only if $k < k^T$, $\Pi^d < \Pi$ if and only if $k > k^T$, and $\Pi^d = \Pi$ if and only if $k = k^T$.

First, consider the case when both k^* and $\tilde{k}^*$ are smaller than k^T. We know that $\Pi^d > \Pi$ for $k < k^T$. Therefore, $\Pi^d(\tilde{k}^*) > \Pi(\tilde{k}^*) = 0$. Given that $\Pi^d(k)$ is increasing in k, the root of the equation $\Pi^d(k) = 0$, defined as k^*, is smaller than $\tilde{k}^*$. Hence $k^* < \tilde{k}^* < k^T$.

Second, consider the case when both k^* and $\tilde{k}^*$ are greater than k^T. We know that $\Pi > \Pi^d$ for $k > k^T$. Therefore, $\Pi^d(\tilde{k}^*) < \Pi(\tilde{k}^*) = 0$. Given that $\Pi^d(k)$ is increasing in k, the root of the equation $\Pi^d(k) = 0$, defined as k^*, is greater than $\tilde{k}^*$. Hence $k^T < \tilde{k}^* < k^*$.

Third, we show the impossibility of the case $k^* < k^T < \tilde{k}^*$. If this inequality were true, then $\Pi^d(\tilde{k}^*) < 0$ (because $\Pi^d(k) < \Pi(k)$ for $k > k^T$ and $\tilde{k}^*$ is defined as the root of $\Pi(k) = 0$). Therefore, $k^* > \tilde{k}^*$. This is a contradiction. The impossibility of the case $\tilde{k}^* < k^T < k^*$ can be shown in a similar way.

Fourth, we show that if $\tilde{k}^* = k^*$, then $\tilde{k}^* = k^* = k^T$. If $k = \tilde{k}^* = k^*$, then $\Pi(k) = \Pi^d(k) = 0$. Solving $\Pi(k) = \Pi^d(k)$ for k yields $k = k^T$.

Finally, we show that if k^T equals either k^*, or $\tilde{k}^*$, then it equals the other critical value as well. Suppose that $k^T = k^*$ (the case when $k^T = \tilde{k}^*$ can be proved in a similar way). Then, $\Pi^d(k^T) = 0$. However, we already know that for $k = k^T$, $\Pi^d(k) = \Pi(k)$. Hence, $\Pi(k^T) = 0$. Therefore, k^T is the root of $\Pi(k) = 0$, and $k^T = \tilde{k}^*$. Q.E.D.

Proof of Lemma 2.

We will first show that that $\pi_t = \rho$. In words, the inflation rate equals the money growth rate not only in the steady state equilibrium, but also in a neighborhood of it.

Combining (13) and (15), we get:

$$\frac{\lambda_t w_t(\rho - 1)}{\pi_t} = w_t - k_{t+1} - \frac{\lambda_t w_t}{\pi_t} \tag{53}$$

Equation (53) can be rewritten as:

$$\frac{\rho \lambda_t w_t}{\pi_t} = w_t - k_{t+1} \tag{54}$$

Combining (54) with (12) yields:

$$\frac{\rho}{\pi_t}\lambda_t w_t = w_t - (1-\lambda_t)w_t = \lambda w_t, \tag{55}$$

which simplifies to

$$\pi_t = \rho \tag{56}$$

Hence we can replace equation (13) with

$$a_{t+1} = \frac{(1-\frac{1}{\rho})\lambda_t w_t}{1-\beta} \tag{57}$$

By substituting out w_t and a_{t+1} the system (10)-(15) can be written as:

$$\frac{R_t\lambda}{\beta} = \frac{1-\lambda_t}{1-\beta}R_t + \frac{(1-\frac{1}{\rho})\lambda_t}{1-\beta} \tag{58}$$

$$k_{t+1} = (1-\lambda_t)(1-\alpha)Ak_t^\alpha \tag{59}$$

$$R_t = 1 \quad \text{if} \quad k_{t+1} \geq k^T; \ = \alpha A\gamma k^{\alpha-1} \text{ if } \quad k_{t+1} < k^T \tag{60}$$

We next show the existence and uniqueness of a triplet λ_t, k_{t+1}, R_t that satisfies the system (58)-(60). Equation (59) describes a continuous negative relationship between k_{t+1} and λ_t, such that k_{t+1} attains its maximum value if $\lambda_t = 0$, and $k_{t+1} = 0$ if $\lambda_t = 1$. Equation (58) can be rewritten as:

$$\lambda_t = \frac{R_t}{\frac{R_t(1-\beta)}{\beta} + R_t - (1-\frac{1}{\rho})} \tag{61}$$

Differentiating (61) with respect to R_t, we get:

$$\frac{\partial\lambda_t}{\partial R_t} = \frac{-(1-\frac{1}{\rho})}{\left(\frac{R_t(1-\beta}{\beta} + R_t - (1-\frac{1}{\rho})\right)^2} < 0 \tag{62}$$

Given the relationship between k_{t+1} and R_t,

$$\frac{\partial\lambda_t}{\partial k_{t+1}} = \frac{\partial\lambda_t}{\partial R_t}\frac{\partial R_t}{\partial k_{t+1}} \geq 0 \tag{63}$$

Therefore the system (58)-(60) can be shown graphically using the $\lambda_t - k_{t+1}$ plane. Equation (59) is represented as a downward-sloping curve in Figure 2. Given the result (63), equations (58) and (60) are represented as an upward-sloping curve. Hence the two curves intersect in at most one point. Furthermore, using (61), we can show that

$$\lim_{k_{t+1}\to 0} \lambda_t = \beta < 1 \tag{64}$$

Hence, by continuity, these two curves intersect in exactly one point, and $\lambda_t < 1$. Q.E.D.

Proof of Proposition 2.

Totally differentiating (16) yields:

$$\frac{1-\beta}{\beta}(\lambda dR + Rd\lambda) = (1-\lambda)dR - Rd\lambda + d\lambda - \frac{1}{\rho}d\lambda + \frac{\lambda}{\rho^2}d\rho \tag{65}$$

Equations (17) and (18) imply a negative relationship between k and λ and non-positive relationship between k and R. Hence,

$$dR = \phi d\lambda \tag{66}$$

where $\phi \geq 0$. Substituting (66) into (65) yields:

$$\left[\frac{(1-\beta)\lambda}{\beta} - (1-\lambda)\right]\phi d\lambda + \left[\frac{1-\beta}{\beta}R + (R-1) + \frac{1}{\rho}\right]d\lambda = \frac{\lambda}{\rho^2}d\rho \tag{67}$$

$\frac{(1-\beta)\lambda}{\beta} - (1-\lambda) > 0$, because from equation (16)

$$\frac{(1-\beta)\lambda}{\beta}R - (1-\lambda)R = \left(1 - \frac{1}{\rho}\right)\lambda > 0$$

Therefore,

$$\frac{\partial\lambda}{\partial\rho} = \frac{\lambda}{\left[\left(\frac{(1-\beta)\lambda}{\beta} - (1-\lambda)\right)\phi + \left(\frac{1-\beta}{\beta}R + (R-1) + \frac{1}{\rho}\right)\right]\rho^2} > 0 \tag{68}$$

Taking into account (17), (68) implies that

$$\frac{\partial k}{\partial\rho} < 0 \tag{69}$$

Q.E.D.

Proof of Lemma 3.

Taking into account that $\pi_t = \rho$, and substituting out w_t and a_{t+1}, the system (10)-(15) can be written as:

$$\frac{R_t\lambda_t}{\beta} = \frac{1-\lambda_t}{1-\beta}R_t + \frac{\left(1-\frac{1}{\rho}\right)\lambda_t}{1-\beta} \quad (70)$$

$$k_{t+1} = (1-\lambda_t)(1-\alpha)Ak_t^{\alpha} \quad (71)$$

$$R_t = 1 \text{ if } \ k_{t+1} \geq k^T; \ \ \alpha A\gamma k_{t+1}^{\alpha-1} \text{ b if } \ k_{t+1} < k^T \quad (72)$$

Solving (70) for λ_t in terms of R_t, one obtains:

$$\lambda_t = \frac{R_t}{\frac{R_t}{\beta} + \frac{1}{\rho} - 1} \quad (73)$$

which implies that:

$$\frac{\partial \lambda_t}{\partial R_t} = \frac{\frac{R_t}{\beta} + \frac{1}{\rho} - 1 - R_t\frac{1}{\beta}}{\left(\frac{R_t}{\beta} + \frac{1}{\rho} - 1\right)^2} = \frac{-1+\frac{1}{\rho}}{\left(\frac{R_t}{\beta} + \frac{1}{\rho} - 1\right)^2} < 0 \quad (74)$$

Totally differentiating equation (71) in a neighborhood of the steady state, we get:

$$dk_{t+1} = (1-\lambda)A(1-\alpha)\alpha k^{\alpha-1}dk_t - \frac{\partial \lambda_t}{\partial R_t}\frac{\partial R_t}{\partial k_{t+1}}A(1-\alpha)k^{\alpha}dk_{t+1} \quad (75)$$

which implies

$$\frac{\partial k_{t+1}}{\partial k_t} = \frac{(1-\lambda)A(1-\alpha)\alpha k^{\alpha-1}}{1+\frac{\partial \lambda_t}{\partial R_t}\frac{\partial R_t}{\partial k_{t+1}}A(1-\alpha)k^{\alpha}} = \frac{\alpha}{1+\frac{\partial \lambda_t}{\partial R_t}\frac{\partial R_t}{\partial k_{t+1}}A(1-\alpha)k^{\alpha}} \quad (76)$$

The last transformation relies on the fact that in the steady state:

$$k = (1-\lambda)A(1-\alpha)k^{\alpha-1}$$

The denominator of (76) is no less than unity, because both $\frac{\partial \lambda_t}{\partial R_t}$ and $\frac{\partial R_t}{\partial k_{t+1}}$ are non-positive.

Therefore, $\frac{\partial k_{t+1}}{\partial k_t} < 1$, and the dynamical system converges to the steady state equilibrium. Q.E.D.

Proof of Lemma 6.

First of all, it is straightforward to combine equations (43), (44) and (45) to show that $\pi_t = \rho$ (the procedure is exactly the same as in the proof of Lemma 2). Hence we can replace equations (44) and (46) with

$$a_{t+1} = \left(1-\frac{1}{\rho}\right)\frac{\lambda_t w_t}{1-\beta} \quad (77)$$

Furthermore equation (42) uniquely determines w_t. If we substitute (44) into (45), we get an equation with just one unknown, λ_t. Solving for λ_t yields:

$$\lambda_t = \frac{1}{\frac{1}{\beta} + \frac{1}{\rho} - 1} \tag{78}$$

Finally, substituting (78) into (43) gives us the value for δ_t. Q.E.D.

Proof of Proposition 4.

Equation (78) shows that λ_t is time invariant in a neighborhood of the steady state for a given value of ρ. Furthermore, it shows that λ is positively related to ρ. A reduction in λ following a disinflation, increases the right-hand side of (48), and therefore, raises the dollar holdings δ. Q.E.D.

Proof of Lemma 7.

It follows from equation (78) that λ_t is time invariant in a neighborhood of the steady state for a given value of ρ. Therefore, the model does not have any transitional dynamics. The return on dollars determines the level of the capital stock. Hence $k_{t+1} = k^T$ at every point in the neighborhood of the steady state with dollars. Equation (48) determines the value of δ_t for $k_t = k^T$ and $\lambda_t = \lambda(\rho)$. Q.E.D.

Proof of Lemma 8.

We will prove the lemma by contradiction. Let's assume that $\rho_1 > \rho^*$. In that case,

$$1 - \lambda^* > 1 - \lambda_1 \tag{79}$$

By definition of ρ^*,

$$[1 - \lambda^*]A(1 - \alpha)(k^*)^{\alpha} = k^*, \tag{80}$$

or

$$[1 - \lambda^*]A(1 - \alpha)(k^*)^{\alpha - 1} = 1 \tag{81}$$

By definition of ρ_1,

$$[1 - \lambda_1]A\gamma(1 - \alpha)(k^T)^{\alpha} = k^T \tag{82}$$

We know that $k^T = (A\alpha\gamma)^{1/(1-\alpha)}$. Substituting into (82), we get:

$$1 - \lambda_1 = \frac{\alpha}{1 - \alpha} \tag{83}$$

From (79), (81), and (83), it follows that

$$A(1-\alpha)(k^*)^{\alpha-1} = \frac{1}{1-\lambda^*} < \frac{1}{1-\lambda_1} = \frac{1-\alpha}{\alpha} \tag{84}$$

Hence,

$$A\alpha(k^*)^{\alpha-1} < 1 \tag{85}$$

This result contradicts the assumption of dynamic efficiency of the efficient technology. Q.E.D.

Proof of Lemma 9.

By definition of k^*,

$$A\alpha(k^*)^{\alpha} = k^* + \phi \tag{86}$$

Totally differentiating equation (86), we get:

$$A\alpha^2(k^*)^{\alpha-1}dk^* = dk^* + d\phi \tag{87}$$

Therefore,

$$\frac{\partial k^*}{\partial \phi} = \frac{1}{A\alpha^2(k^*)^{\alpha-1} - 1} > 0 \tag{88}$$

The last equation holds by Assumption 1. Q.E.D.

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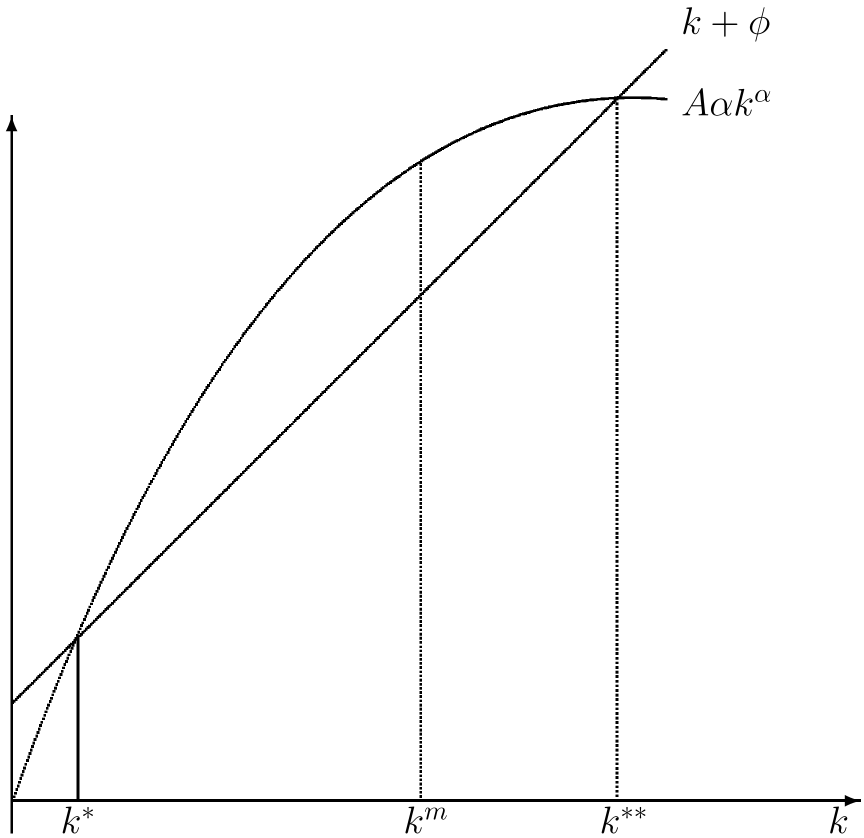
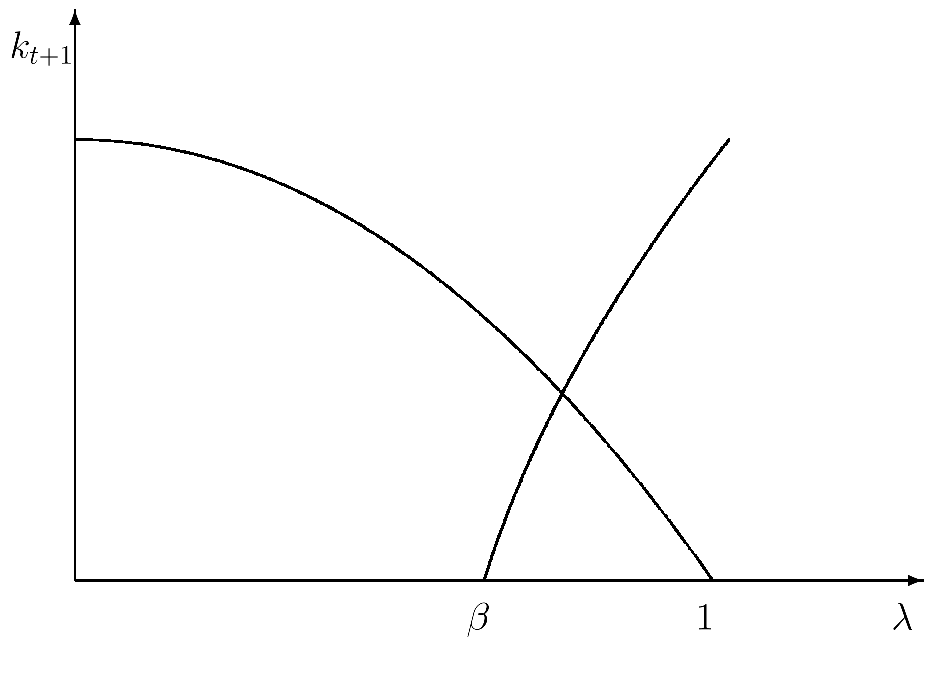
Figure 1



Fugure 2



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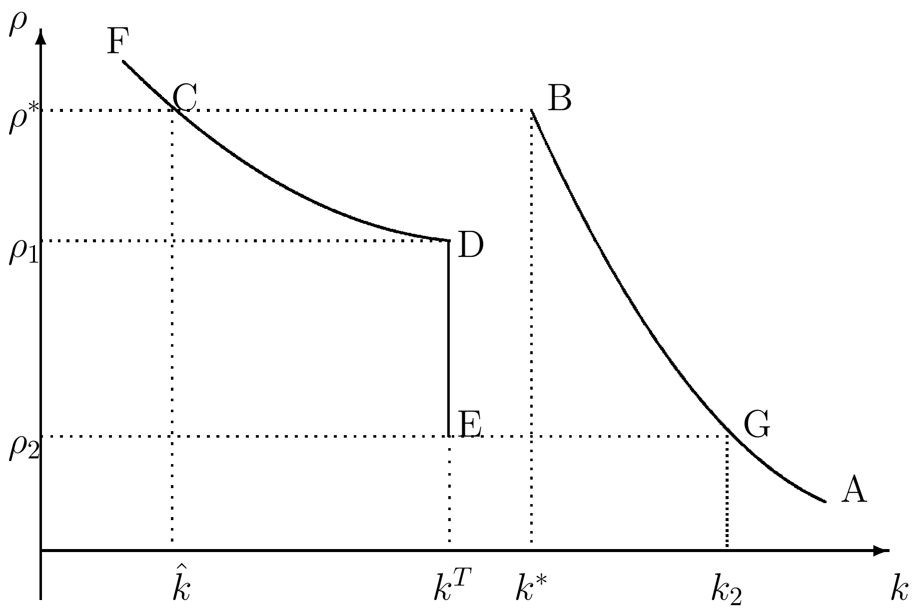

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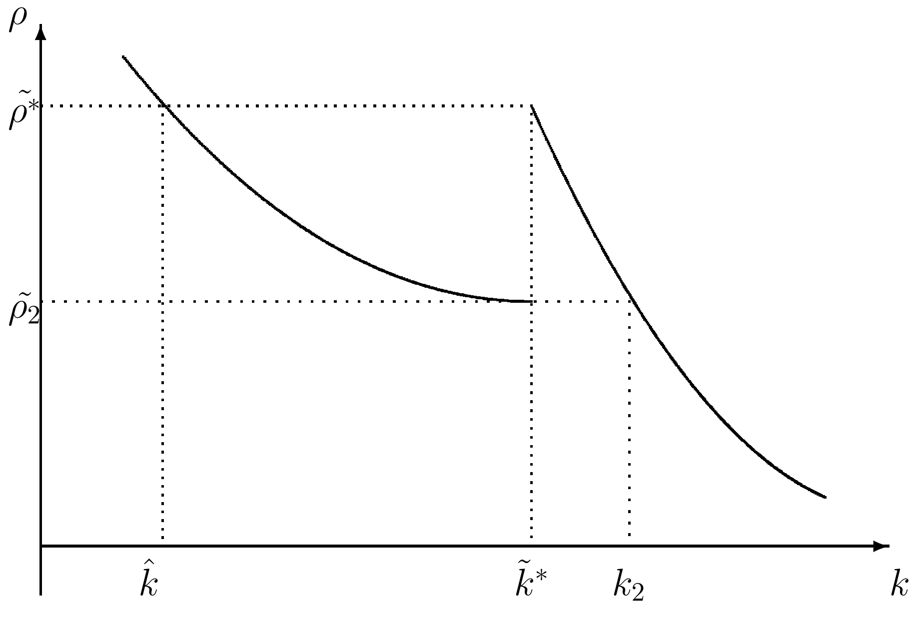

Table 1. Dollarization in Selected FSU Countries in 2001.

Country	CDI	DI	Annual Inflation
Latvia	67.5	30.8	2.5 %
Lithuania	34.7	32.9	1.3 %
Russia	73.5	24.5	21.6 %
Ukraine	37.8	19.4	6.1 %
Armenia	68.1	50.3	3.4 %
Azerbaijan	82.3	48.8	1.5 %
Georgia	80.2	44.9	4.7 %
Kazakhstan	89.7	47.5	8.4 %
Kyrgyz Rep.	52.3	25.2	7.0 %
Turkmenistan	54.4	39.9	11.4 %

CDI - Comprehensive Dollarization Index = (foreign currency + foreign currency bank deposits) / (M2 + foreign currency in circulation)

DI - Dollarization Index =

= Foreign currency bank deposits / M2.

Source: Feige (CES, 2003 and Havrylyshyn and Beddies, CES, 2003).