Optimal Common Value Auctions with Asymmetric Bidders

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Abstract

How do informational asymmetries between bidders affect the outcome of common value auctions? Should the seller accept bids from bidders with more precise information? If so, under what conditions? What effect do such asymmetries have on the seller's expected revenue? We analyze these questions in a simple model in which an insider competes with an outsider. Both have some information about the value of the asset for sale, but the insider's information is more precise. We derive the optimal mechanism and show that it must be biased against the insider. With an optimal mechanism, the seller's expected revenue is higher if the bidders are more asymmetrically informed. We show how the optimal mechanism can be implemented as a second-price sealed bid auction that lets the insider win only if his bid is above a hurdle price.

Keywords: Auctions, Common Value Auctions, Asymmetric Bidders, Winner's Curse

JEL codes: D44, D82

1 Introduction

Consider the following problem. A firm has gone bankrupt, and it is decided that selling its assets is the best way to proceed. The former manager (maybe the owner-manager) declares a possible interest in buying the assets. Is that good or bad news? The other potential bidders should expect the former manager to have superior information about the value of the assets. Given that winning against a better-informed competitor may mean that the winner overpaid (the so-called winner's curse), the outside bidders should bid more cautiously. Consequently, the expected sales proceeds may be lowered by the presence of the insider. On the other hand, letting the insider participate may increase competition, which may increase sales proceeds. We ask, what the optimal selling procedure is in this case: should the seller let the insider participate in an auction? If so, under what conditions? How do informational asymmetries affect the seller's expected revenue?

These questions are relevant in many other contexts. Auctions are used to offer a variety of assets to a variety of potential buyers. And some bidders will unavoidably have more reliable sources of information about the value than others, or they may simply be more experienced. For example, a local telephone company may be better informed about the potential profitability of offering cellular service in its area than an operator from a different area. Similarly, a professional car dealer will have a clearer idea about the value of a repossessed car than the average consumer. The extant literature on auctions[1] offers little guidance on how bidder asymmetry affects auction outcomes, and a seller's expected revenue.

We analyze a simple common value environment with two bidders, whose information about the value of the asset for sale is not equally precise. We call the better informed bidder the insider, and the other bidder the outsider. We derive the optimal mechanism for selling the asset and study how it can be implemented. A key variable in our analysis is the degree of bidder asymmetry: at one extreme, bidders can be symmetric and receive equally informative signals; at the other extreme, the insider may be perfectly informed about the

[1] For a recent survey, see Klemperer (1999).

asset's value, while the outsider is uninformed; intermediate cases are those in which both bidders have some private information, but the insider's is more precise.

We find that the more asymmetric bidders are, the higher the seller's expected revenue if he uses the optimal mechanism. The key to understanding this result is that the optimal mechanism must be biased against the insider, whose probability of winning the auction must be smaller than the outsider's. Given that the bidders are not symmetric, the optimal mechanism should not treat them symmetrically. Standard auctions treat bidders symmetrically and are therefore not optimal in this case.

The optimal mechanism accepts bids from the insider, since letting him participate creates competition. However, it limits this competition by being biased against the insider. This bias has two advantages. First, the insider wins the auction only if his estimate of the asset's value is high enough, i.e. when his bid is sufficiently high. This makes it possible for the seller to extract rents from the insider, in particular if his information is much better than the outsider's. Second, if the insider's bid is not sufficiently high, the outsider wins the asset. In a strongly biased mechanism, winning against the insider does not convey much information about the insider's signal, which reduces the winner's curse for the outsider.

A second result is that the optimal mechanism can easily be implemented as a modified second-price auction: the insider wins only if his bid is higher than both the outsider's bid and a hurdle price; if the insider's bid is below the hurdle price, the asset is sold at a fixed price to the outsider. The hurdle price depends on the degree of asymmetry: the better the insider's information relative to the outsider's, the higher the hurdle price. The hurdle price, thus, implements the required bias of the optimal mechanism.

Our model is most closely related to those in Bikhchandani and Riley (1991), Bulow and Klemperer (1996, 2002) and Bulow, Huang and Klemperer (1999), who also assume that the unknown value of an asset depends on the signals that bidders receive. These authors focus on properties of standard auctions: Bikhchandani and Riley (1991) focus on the properties of equilibria in ascending and second-price auctions; Bulow and Klemperer (1996) analyze a variety of models and auctions, but focus on symmetric bidders; Bulow and

Klemperer (2002) study ascending and first-price auctions and analyze properties "almost common value" auctions; Bulow, Huang and Klemperer (1999) examine how toeholds affect bidding outcomes in takeover contests. In contrast, our focus is on the study of optimal selling mechanisms in the presence of bidder asymmetry.

We are not the first to analyze common value environments in which bidders have asymmetric information. Earlier contributions differ from ours in two respects. First, earlier contributions study the properties of standard auction types,[2] while our focus is on optimal selling mechanisms. Second, earlier models typically assume either that weak bidders have no private information,[3] or that one bidder is perfectly informed and others receive only imperfect (but private) signals.[4] This makes those models more tractable, but less realistic, and an analysis of how increases in the asymmetry affect revenue or the design of the optimal mechanism is only relevant for these limit cases. We analyze a model in which both bidders receive noisy signals, and the insider's signal is more informative than the outsider's. We can vary the degree of asymmetry, leaving the expected value of the object for sale unchanged, and study how this affects the optimal mechanism, and the expected revenue it generates.

Our paper is also related to Hausch (1987), Laskowski and Slonim (1999), Kagel and Levin (1999) and Campbell and Levin (2000). The first three also consider bidders with private but differently informative signals; they place restrictions on the signals (Hausch) or bidding strategies (Laskowski and Slonim, Kagel and Levin), to solve for the optimal strategies in standard auctions. Campbell and Levin (2000) analyze several models with different information structures, in which the value of the asset for sale and the signals are binary random variables; they compute equilibrium bids and expected revenue in first-price auctions.

Other analyses of auctions with asymmetric bidders focus on settings with private values. While we believe that a common value environment is a better description of many auction situations than a model with private values, a comparison is useful. Myerson (1981) discusses

[2] See the references in footnotes 3 and 4.

[3] See e.g. Wilson (1967), Weverbergh (1979), Milgrom and Weber (1982), Engelbrecht-Wiggans et al. (1983), Hendricks and Porter (1988), Hendricks et al. (1994) or, partly, Campbell and Levin (2000).

[4] See e.g. Ortega-Reichert (1968, Ch. VII) or Kagel and Levin (1999).

an example showing that the seller can increase expected revenue by biasing the mechanism against a strong bidder; but that is not the focus of his article. Maskin and Riley (2000) focus on private value first- and second-price auctions, in which bidders are asymmetric in different ways (see also Cantillon (2000)). They find that the first-price auction generates higher expected revenue than the second-price auction; the reason for this is that the first-price auction is biased in favor of the weak bidder. Maskin and Riley do not study the properties of the optimal mechanism, however.

We conclude that bidder asymmetry is beneficial for the seller. He should want the insider to participate in the auction. While the insider's participation may be damaging to the less well informed bidder, this need not have an adverse effect on the seller's revenue: increased bidder asymmetry increases the seller's expected revenue.

2 The Model

A seller owns an indivisible asset that can be sold to one of two bidders, $i, j \in \{1, 2\}$. All players are risk-neutral. Both bidders value the asset equally, but the value is unknown to them. Instead, each of them privately observes a signal t_i, drawn independently from the same density function f, with support $[\underline{t}, \overline{t}]$ and c.d.f. F. Denote the hazard rate by $H(t_i) = f(t_i)/(1 - F(t_i))$. The full information value of the asset is a weighted sum of the two signals:

$$v(t_1, t_2) = \psi_1 t_1 + \psi_2 t_2, \quad \text{such that} \quad \psi_1 \in [1/2, 1) \quad \text{and} \quad \psi_2 = 1 - \psi_1. \tag{1}$$

$\psi_1 < 1$ ensures that both signals are informative. Our model is similar to that introduced by Myerson (1981), and similar to the models in Bikhchandani and Riley (1991, p. 106), Bulow and Klemperer (1996, 2002) or Bulow, Huang and Klemperer (1999). These authors also assume that the true value of the asset is a function of all bidders' signals. In other words, an asset's value depends on what potential buyers are willing to pay for it; and that depends on their information. An alternative way to model common values is to assume that the true value is given, and the bidders receive noisy information about this value. Both approaches capture the idea that bidders value the asset equally (in a pure common value setup), and that bidders receive informative but imperfect signals. The model we use has the advantage that it remains tractable if bidders' signals are not equally informative.[5]

We call bidder 1 the 'insider' and bidder 2 the 'outsider', since bidder 1's signal is more informative. To see this, examine the variance of the value of the asset, conditional on bidder i's signal t_i. This conditional variance is ψ_j^2 for bidder i, and since $\psi_1 > \psi_2$, it is larger for bidder 2.

The assumptions that the weights ψ_1 and ψ_2 add up to one and that the signals t_i are i.i.d. ensure that the expected value of the asset does not depend on ψ_1 and ψ_2: it is easy to show that $E\left[v\left(t_1,t_2\right)\right] = \int_{\underline{t}}^{\bar{t}} t_i f(t_i)dt_i$, irrespective of ψ_1. This normalization allows us to examine the effect of bidder asymmetry on the seller's expected revenue, while keeping the ex-ante expected value constant. This assumption may seem restrictive, but it is without loss of generality, as we show in Section 5.

We assume that the seller's valuation of the asset is zero, and that his only goal is to maximize expected revenue. We assume that the lower bound $\underline{t}$ of the signals' support is sufficiently high, such that imposing a reserve price will turn out to be sub-optimal; a sufficient condition is that $\underline{t}H(\underline{t}) \geq \psi_1$. We also assume that the hazard rate H is increasing in the signal t_i for both bidders. This is a standard monotone hazard rate assumption, which is made for tractability reasons.

3 The Optimal Mechanism

From the revelation principle, we can restrict attention to direct mechanisms. For reported signal realizations t_1 and t_2, let $p_i(t_1,t_2)$ be the probability of giving the asset to bidder i, and let $x_i(t_1,t_2)$ be the payment that bidder i is required to make to the seller. Define the

[5] Crémer and McLean (1985) show that in the alternative model, the seller can extract all rents if bidders' signals are correlated. However, the optimal mechanism required to do so has been criticized as being unrealistic (see e.g. Klemperer (1999)), e.g. because it threatens bidders with large fines that must be enforced.

seller's expected revenue as

$$R \equiv \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{\bar{t}} \left(x_1(t_1, t_2) + x_2(t_1, t_2)\right) f(t_1) dt_1 \; f(t_2) dt_2.$$

Notice that x_i may or may not depend on p_i: a bidder may be required to make a payment even if he does not win the asset.

Define bidder i's probability of winning the asset, conditional on reported signal t_i, as

$$Q_i(t_i) \equiv \int_{\underline{t}}^{\bar{t}} p_i(t_i, t_j) f(t_j) dt_j.$$

A bidder's expected payoff depends on both the realized and the reported signal; his expected net payoff, conditional on signal t_i and announcement $\widehat{t_i}$, is defined as

$$U_i(\widehat{t_i}|t_i) \equiv \int_{\underline{t}}^{\bar{t}} \left(v(t_i, t_j) p_i(\widehat{t_i}, t_j) - x_i(\widehat{t_i}, t_j)\right) f(t_j) dt_j.$$

If bidder i truthfully reveals his signal, we have $\widehat{t_i} = t_i$; for this case, we will use the notation $V_i(t_i) \equiv U_i(t_i|t_i)$ to denote the expected payoff of a bidder with a realization t_i.

The seller solves the following optimization problem:

$$\max_{x_1, x_2 \in I\!R, p_1, p_2 \in [0,1]} \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{\bar{t}} \left(x_1(t_1, t_2) + x_2(t_1, t_2)\right) f(t_1) dt_1 \; f(t_2) dt_2 \tag{2}$$

s.t.

$$V_i(t_i) \geq 0 \quad \forall t_i, \;\; i = 1, 2 \tag{3}$$

$$V_i(t_i) \geq U_i(\widehat{t_i}|t_i) \quad \forall \widehat{t_i}, \quad \forall t_i, \;\; i = 1, 2 \tag{4}$$

$$p_1(t_1, t_2) + p_2(t_1, t_2) \leq 1 \quad \forall t_1, \; \forall t_2. \tag{5}$$

The optimal mechanism maximizes the seller's expected revenue, subject to the constraints that all parties are willing to participate (3), and they have no incentive to misrepresent their information, (4). The optimization problem (2)–(5) is quite involved. In the next lemma we show that the IC constraint (4) can be replaced by a more tractable condition, which allows us to obtain a closed form solution for the optimal mechanism.

Lemma 1 *The truthtelling constraint (4) is satisfied iff* $\frac{\partial V_i(t_i)}{\partial t_i} = \psi_i Q_i(t_i)$ *and* $\frac{\partial Q_i(t_i)}{\partial t_i} \geq 0$.

Proof: The proof is standard (see Myerson (1981)); it is provided in the Appendix for completeness. ■

We can now write down the seller's optimization problem as follows:

$$\max_{p,x} \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{\bar{t}} \{x_1(t_1,t_2) + x_2(t_1,t_2)\} f(t_1)dt_1 f(t_2)dt_2 \tag{6}$$

subject to

$$V_i(t_i) = V_i(\underline{t}) + \psi_i \int_{\underline{t}}^{t_i} Q_i(s_i)ds_i \quad \text{for} \quad i = 1,2 \tag{7}$$

$$Q_i'(t_i) \geq 0 \quad \text{for} \quad i = 1,2 \tag{8}$$

$$V_i(\underline{t}) \geq 0 \quad \text{for} \quad i = 1,2 \tag{9}$$

$$p_1(t_1,t_2) + p_2(t_1,t_2) \leq 1 \tag{10}$$

$$p_i(t_1,t_2) \geq 0 \quad \text{for} \quad i = 1,2. \tag{11}$$

Using Lemma 1 it is easy to see that conditions (7) and (8) are equivalent to (4). The constraints (7)–(9) together imply that (3) is satisfied for all t_i. The last two conditions are the feasibility constraints. Tedious but straightforward algebra shows that (6) can be rewritten as

$$\max_{p_i, V_i(\underline{t})} \sum_{i=1,2} \left\{ -V_i(\underline{t}) + \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{\bar{t}} \left[v(t_1,t_2) - \frac{\psi_i}{H(t_i)} \right] p_i(t_1,t_2) f(t_1)dt_1 \ f(t_2)dt_2 \right\}. \tag{12}$$

To obtain (12) from (6) we substitute for $x_i(t_1,t_2)$ from (7) and simplify. Details of the manipulations required can be found in the Appendix. The objective function (12) leads to the well-known Revenue Equivalence Theorem (see e.g. Myerson (1981) or Riley and Samuelson (1981)): In any selling mechanism with independent signals the seller's expected revenue from an incentive-compatible mechanism is completely determined by $V_i(\underline{t})$ and the probability functions p_i. The transfers x_i are determined implicitly by (7). So any incentive compatible auction that gives the same rent to bidders with the lowest signal and uses the same allocation rules p_i yields the same expected revenue.

Proposition 1 *The optimal mechanism sets* $V_1(\underline{t}) = V_2(\underline{t}) = 0$*,*

$$p_1(t_1, t_2) = \begin{cases} 1 \quad if \quad \frac{\psi_1}{H(t_1)} \leq \frac{1-\psi_1}{H(t_2)} \\ 0 \quad \text{otherwise} \end{cases} \tag{13}$$

and $p_2(t_1, t_2) = 1 - p_1(t_1, t_2)$*. The transfer payments* $x_i(t_1, t_2)$ *are implicitly given by (7).*

Proof: Clearly, there is no need to set $V_i(\underline{t}) > 0$, and setting $V_i(\underline{t}) < 0$ violates bidder i's participation constraint (9). The assumption that $\underline{t}H(\underline{t}) \geq \psi_1$ and the monotone hazard rate assumption together imply that the term in square brackets in (12) is positive for all (t_1, t_2). Thus, it is never optimal to set $p_1(t_1, t_2) = p_2(t_1, t_2) = 0$, i.e. there is no reserve price. The monotone hazard rate assumption also implies that for the solution in (13), a higher signal t_i makes it more likely that bidder i wins the object, thus satisfying (8). The allocation rule can be derived by comparing the term in square brackets in (12) for a given (t_1, t_2) and setting $p_1(t_1, t_2) = 1$ if $\left(v(t_1, t_2) - \frac{\psi_1}{H(t_1)}\right) \geq \left(v(t_1, t_2) - \frac{\psi_2}{H(t_2)}\right)$. An alternative way to derive (13) is the approach developed in Bulow and Roberts (1989) or Bulow and Klemperer (1996): the term in square brackets in (12) is sometimes referred to as a bidder's 'marginal revenue,' and the optimal mechanism allocates the asset to the bidder with the higher marginal revenue. ■

The optimal mechanism asks the bidders to announce their privately observed signals and allocates the asset based on the simple rule (13). The bidders' transfers $x_i(t_i, t_j)$ are defined implicitly, and we will discuss them in Section 4, where we also show how the optimal mechanism can easily be implemented.

It will be convenient to define the cut-off signals, $z_1(t_2)$ and $z_2(t_1)$, as follows:

$$z_1(t_2) \equiv H^{-1}\left(\frac{\psi_1}{1-\psi_1}H(t_2)\right) \tag{14}$$

$$z_2(t_1) \equiv \begin{cases} H^{-1}\left(\frac{1-\psi_1}{\psi_1}H(t_1)\right) & \text{if} \quad \frac{1-\psi_1}{\psi_1}H(t_1) \geq H(\underline{t}) \\ \underline{t} & \text{otherwise} \end{cases} \tag{15}$$

where H^{-1} is the inverse of the hazard function. Notice that z_1 is invertible and its inverse is z_2. In contrast, z_2 is only invertible if t_1 is high enough, such that $H(t_1) \geq \frac{\psi_1}{1-\psi_1}H(\underline{t})$. In this region, the inverse of z_2 is indeed z_1.

We can use these cut-off signals to rewrite the allocation rule (13) as

$$p_1(t_1, t_2) = \begin{cases} 1 & \text{if} \quad t_1 \geq z_1(t_2) \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

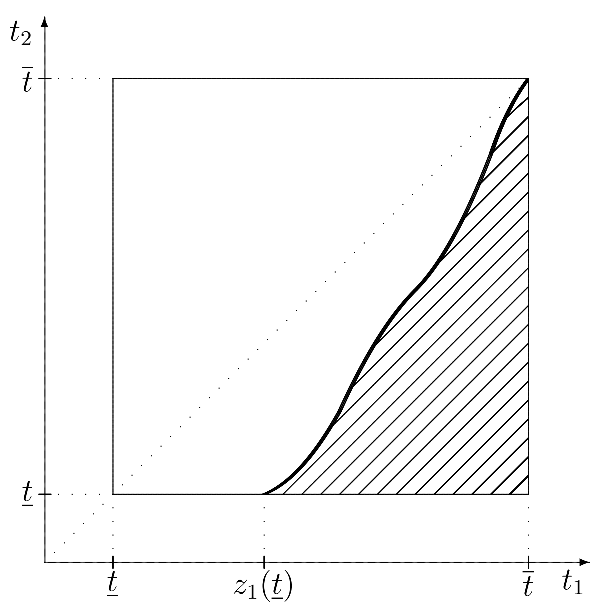

Figure 1: *The optimal allocation rule*

The optimal mechanism is biased against the insider: he wins the asset only if his signal is sufficiently higher than the outsider's. The threshold is $z_1(t_2)$, which in turn is higher than t_2 (this follows from (14), recalling that $H' > 0$). Figure 1 sketches how the optimal mechanism allocates the asset, depending on the signal realizations. The square box contains all signal combinations that are possible. The upward sloping solid curve separates signal pairs for which the insider wins (the shaded area on the lower right) from those for which the outsider wins (the remaining area, top left). For any t_1, this curve determines the threshold $z_2(t_1)$; conversely, going in the other direction, it traces $z_1(t_2)$ for all t_2. Notice that the bias is extreme for low values of t_1. If $t_1 < z_1(\underline{t})$, the insider will certainly not win the asset, irrespective of the realization of t_2.

An increase in ψ_1 increases the bias in the optimal mechanism. Notice that z_1 is increasing in ψ_1 (see (14), and recall that $H' > 0$). Thus, the higher ψ_1 is, the more the insider's signal must exceed the outsider's, in order to win. The curve in Figure 1 moves counterclockwise around $(\bar{t}, \bar{t})$, if ψ_1 increases, and the threshold $z_1(\underline{t})$ also increases; the insider's probability of winning the auction decreases. In the extreme, as $\psi_1 \to 1$, we have $z_1(\underline{t}) \to \bar{t}$, and the insider's probability of winning goes to zero. However, for smaller values of ψ_1, the insider's probability is positive. We can therefore conclude:

Corollary 1 *It is optimal to let the insider participate.*

Biasing the mechanism agains the insider increases competition between the bidders, by forcing the insider to submit high bids if he wants to win. One would expect this bias to somewhat mitigate the adverse effect that increased bidder asymmetry has on the seller's expected revenue. In fact, the optimal mechanism achieves more than that:

Proposition 2 *The seller's expected revenue is increasing in* ψ_1*.*

Proof: See the Appendix. ■

The proof consists of two steps. Starting with a given ψ_1 and the corresponding optimal mechanism, consider a small increase in ψ_1. In the first step, we construct a new mechanism that is incentive compatible and satisfies the participation constraint for both bidders. With this changed mechanism and the higher value of ψ_1, the seller's revenue is higher. In constructing the new mechanism, we do not alter the allocation rule, i.e. p_i and

Q_i remain unchanged. Because of this constraint, the changed mechanism is generally not optimal for the higher value of ψ_1; in the second step of the proof we argue that expected revenue generated by the optimal mechanism for the higher ψ_1 can only be higher.

4 The Optimal Mechanism as a Standard Auction

The discussion of the optimal mechanism in Section 3 did not include the bidders' transfers to the seller. Many different transfer schemes may be feasible for the same allocation rule. Here, we discuss one that makes the optimal mechanism resemble a somewhat modified second-price auction. This allows us to discuss the properties of the optimal mechanism and, more importantly, present a simple way to implement the optimal mechanism.

Consider bidding strategies

$$b_i^{\text{MSP}}(t_i) = \psi_i t_i + \psi_j z_j(t_i) \qquad \text{for } i \neq j,$$

a hurdle price

$$\underline{b}_1 = \psi_1 z_1(\underline{t}) + (1-\psi_1)\underline{t}$$

and a price

$$\underline{x}_2 = \psi_1 E\left[t_1 \middle| t_1 < z_1(\underline{t})\right] + (1-\psi_1)\underline{t}.$$

Proposition 3 *(Modified second-price auction) The seller can obtain the same expected revenue as with an optimal mechanism by using a second-price auction, with a hurdle price for the insider: if $b_1 < \underline{b}_1$, the outsider wins the asset and pays a fixed price $\underline{x}_2$. The bidding functions b_i^{MSP} constitute a Bayesian Nash Equilibrium for this modified second-price auction.*

Proof: See the Appendix. ∎

The mechanism described in Proposition 3 is a second-price auction, since the price that the winner pays is the second-highest bid. It is not a standard second-price auction, since the winner did not necessarily submit the highest bid: the outsider may win with a lower bid if the insider's bid is below the hurdle price $\underline{b}_1$. This bias corresponds to the bias in the optimal allocation rule described in Proposition 1, which lets the insider win only if his signal is sufficiently higher than the outsider's.

It is interesting that simply introducing a hurdle price for one bidder is sufficient to turn a standard auction into an optimal mechanism. This increases expected revenue in the same way a reserve price operates: shading bids is made less profitable, since low bids make it less likely that the asset is won, and the profit (value minus payment) enjoyed. The key difference is that a hurdle price is less costly to use than a reserve price: if the threat has to be carried out, the asset is not simply withdrawn (and destroyed, say) but instead sold to the outsider, who is willing to pay for it. A hurdle price is thus more efficient than a reserve price; however, it increases revenue only if the bidders' signals are not equally informative.

The key feature of the optimal mechanism is that it is biased against the insider: he is less likely to win the asset, and he wins only if his signal (and bid) is sufficiently high. The optimal mechanism generates higher revenue through cherry-picking: it sells the asset to the insider if and only if his signal is very high, forcing him to pay a high price. The outsider receives the asset otherwise. The outsider's willingness to pay may be low if $b_1 < \underline{b}_1$, since he can infer that the insider's signal was low. But this loss is smaller than the gain from selling to the insider: the outsider would have earned a large profit by paying an average price for a high-value asset; and given that the insider needs a high bid to win the asset, he has little scope to shade his bid and earn a large rent.

In Proposition 2 we show that the optimal mechanism generates expected revenue that is increasing in ψ_1. It is instructive to examine the modified second-price auction in the limit, as ψ_1 approaches 1.

Corollary 2 *In the limit as $\psi_1 \uparrow 1$, the modified second-price auction extracts all rents: the payoff is zero for both bidders, and the seller's revenue is equal to the expected value of the asset.*

Proof: In the limit as $\psi_1 \uparrow 1$, we have $z_1(t_2) = \bar{t}$ for all t_2. It follows that $\underline{b}_1 = \bar{t}$ and

$\underline{x}_2 = E[t_1]$, cf. the definition of $\underline{b}_1$ (recalling that $\psi_1 + \psi_2 = 1$). Bidder 2 is certain to win; he pays the expected value of the asset and, thus, gets no rents. ■

The optimal mechanism encourages competition between the bidders and yet is biased against the insider. This becomes clear if we consider the limits of ψ_1. If $\psi_1 = 1/2$, the bidders are symmetric: the cut-off signal for the insider is $z_1(\underline{t}) = \underline{t}$, and the hurdle price is $\underline{b} = \underline{t}$; the optimal mechanism can be implemented as a standard second-price auction. In contrast, in the limit as $\psi_1 \uparrow 1$, we have $z_1(\underline{t}) \uparrow \bar{t}$, i.e. the insider cannot win the asset. This is reflected in the hurdle price, which is $\underline{b} = \bar{t}$, equal to the highest possible valuation of the asset. The optimal mechanism extracts all rents since the price that the outsider pays is $\underline{x}_2 = E[t_1]$, the unconditional expected value of the asset. Thus, if $\psi_1 \uparrow 1$, the optimal mechanism mimics an exclusive sale offer to the outsider: it is as biased as possible against the insider. In contrast, if $\psi_1 = 1/2$, it mimics a standard auction and is not biased at all. For intermediate values of ψ_1, it is somewhat biased and generates higher revenue than either a standard auction or an exclusive sale offer to the outsider.

5 An Alternative Model of Bidder Asymmetry

One of the assumptions we make in the analysis so far is the normalization of the weights ψ_i, which add up to one, cf. (1). This is a convenient assumption, since we can vary the relative informativeness of the bidders' signals without changing the expected value of the asset. This in turn simplifies comparisons of different auctions, or comparisons of auctions for different bidder types, since we can focus on the level of expected revenue that is generated.

However, one may wonder how restrictive an assumption this is. Specifically, changes in ψ_1 imply changes in ψ_2 in the opposite direction — if the insider becomes better informed, the outsider automatically becomes less informed. In the limit, as the insider becomes perfectly informed, and the outsider becomes totally uninformed. In this section we discuss a model in which we can vary ψ_1 and leave ψ_2 unchanged. We show that the results are unchanged. The reason is that the value of the outsider's information changes if we vary ψ_1, even if ψ_2

remains unchanged. A simultaneous change in the informativeness of both bidders' signals is *unavoidable*: if the insider's information becomes more reliable, the outsider's must become less valuable, *compared with* the insider's. And it is the *relative* informativeness of the signals that is relevant for our results.

Consider the following model, which differs from ours only in the definition of the asset's value, depending on the bidders' signals. In (1), the weights ψ_i added up to one. We now assume that ψ_2 remains constant, and we study the effects of varying ψ_1:

$$v(t_1, t_2) = \psi_1 t_1 + \psi_2 t_2, \quad \text{such that} \quad \psi_1 \geq \psi_2.$$

The insider's signal remains more informative than the outsider's, since $\psi_1 \geq \psi_2$. Adapting the definitions, results and proofs to the modified model is straightforward. When deriving the optimal mechanism, the cut-off signals can be defined as

$$\hat{z}_1(t_2) \equiv H^{-1}\left(\frac{\psi_1}{\psi_2} H(t_2)\right)$$
$$\hat{z}_2(t_1) \equiv \begin{cases} H^{-1}\left(\frac{\psi_2}{\psi_1} H(t_1)\right) & \text{if} \quad \frac{\psi_2}{\psi_1} H(t_1) \geq H(\underline{t}) \\ \underline{t} & \text{otherwise,} \end{cases}$$

and the optimal mechanism sets

$$\hat{p}_1(t_1, t_2) = \begin{cases} 1 & \text{if} \quad t_1 \geq \hat{z}_1(t_2) \\ 0 & \text{otherwise.} \end{cases}$$

The only significant changes concern the statement and proof of Proposition 2, in which we show that expected revenue is increasing in ψ_1. The result itself is unchanged: expected revenue in an optimal mechanism is increasing in ψ_1 if ψ_1 and ψ_2 do not add to one. However, this is not a meaningful result, since the expected value of the asset $(\psi_1 + \psi_2) E[t_i]$ is also strictly increasing in ψ_1. We therefore consider the ratio of expected revenue to expected value, which measures the degree of rent extraction:

Proposition 4 *Rent extraction is increasing in* ψ_1.

Proof: See the Appendix. ■

Relaxing the assumption that the weights ψ_1 and ψ_2 add up to one does not simplify the analysis, and it does not add any new insights; nor does it change any of our results. Relaxing the assumption also does not seem a very natural way to study how the asymmetry of bidders' information affects auction outcomes: the model then implies that the better informed the insider, the more valuable the asset becomes, and it is not clear why bidder asymmetry should affect the ex-ante expected value of an asset. Overall, there is no advantage from using this seemingly more general model.

6 Conclusion

We have analyzed a simple common value model in which bidders receive signals that are not equally informative. Our aim was to study the properties of optimal selling mechanisms. We found that an optimal mechanism always accepts bids from an insider, but is biased against him: the more asymmetric the bidders' information, the smaller the probability that the insider wins the asset. He wins the asset only if he has very optimistic information about its value, and is therefore willing to bid highly; otherwise, the outsider wins the asset. The more asymmetric bidders are, the more biased the optimal mechanism is. This helps to reduce the winner's curse for the outsider, thereby enhancing his willingness to pay, which benefits the seller.

A second focus of our analysis was on the implementation of the optimal mechanism. Standard auctions — sealed bid first-price or sealed bid second-price — treat all bidders in a symmetric fashion and, consequently, do not implement the optimal mechanism. However, we show that a slightly modified second-price auction is an optimal mechanism, i.e. it generates the highest possible expected revenue. The slight modification is that the insider can win only if his bid is above a hurdle price; if it is below, the asset is sold to the outsider at a pre-specified price. This is a welcome result: more often than not, optimal mechanisms are too complex to be implemented in practice.

The main task for the seller is to estimate how asymmetric the bidders are, and to use this estimate to specify a hurdle price for the stronger bidder. The seller's estimate may be imprecise, of course: she may not know with certainty how asymmetric bidders are, or how asymmetrically informed bidders feel. She will then have to determine an optimal hurdle price given noisy information about ψ_1, a problem we leave for future research.

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Appendix: Proofs

A.1 Proof of Lemma 1

$\Longrightarrow$: The following identity will be useful:

$$\begin{aligned}
U_1\left(t_1,\hat{t}_1\right) &\equiv \int_{Tj}\left[v\left(t_i,t_j\right)p_i\left(\hat{t}_i,t_j\right)-x_i\left(\hat{t}_i,t_j\right)\right]\ f_j\left(t_j\right)dt_j \\
&= \int_{Tj}\left[v\left(\hat{t}_i,t_j\right)p_i\left(\hat{t}_i,t_j\right)-x_i\left(\hat{t}_i,t_j\right)\right]\ f_j\left(t_j\right)dt_j \\
&\quad + \int_{Tj}\left[v\left(t_i,t_j\right)p_i\left(\hat{t}_i,t_j\right)-v\left(\hat{t}_i,t_j\right)p_i\left(\hat{t}_i,t_j\right)\right]\ f_j\left(t_j\right)dt_j \\
&= V_i\left(\hat{t}_i\right)+\int_{Tj}\left[v\left(t_i,t_j\right)-v\left(\hat{t}_i,t_j\right)\right]\ p_i\left(\hat{t}_i,t_j\right)f_j\left(t_j\right)dt_j \\
&= V_i\left(\hat{t}_i\right)+\psi_i\int_{Tj}\left(t_i-\hat{t}_i\right)\ p_i\left(\hat{t}_i,t_j\right)f_j\left(t_j\right)dt_j \\
&= V_i\left(\hat{t}_i\right)+\psi_i\left(t_i-\hat{t}_i\right)Q_i\left(\hat{t}_i\right). \qquad \text{(A1)}
\end{aligned}$$

(4) then implies

$$\begin{aligned}
V_i\left(t_i\right) \geq U_i\left(t_i,\hat{t}_i\right) &= V_i\left(\hat{t}_i\right)+\psi_i\ \left(t_i-\hat{t}_i\right)\ Q_i\left(\hat{t}_i\right) \\
V_i\left(\hat{t}_i\right) \geq U_i\left(\hat{t}_i,t_i\right) &= V_i\left(t_i\right)+\psi_i\ \left(\hat{t}_i-t_i\right)\ Q_i\left(t_i\right)
\end{aligned}$$

so

$$\begin{aligned}
V_i\left(t_i\right)-V_i\left(\hat{t}_i\right) &\geq \psi_i\ \left(t_i-\hat{t}_i\right)\ Q_i\left(\hat{t}_i\right) \\
V_i\left(\hat{t}_i\right)-V_i\left(t_i\right) &\geq \psi_i\ \left(\hat{t}_i-t_i\right)\ Q_i\left(t_i\right).
\end{aligned}$$

Rearranging we have

$$\psi_i\ \left(t_i-\hat{t}_i\right)\ Q_i\left(t_i\right) \geq V_i\left(t_i\right)-V_i\left(\hat{t}_i\right) \geq \psi_i\ \left(t_i-\hat{t}_i\right)\ Q_i\left(\hat{t}_i\right).$$

W.l.o.g., let $t_i > \hat{t}_i$. Divide by $\left[t_i - \hat{t}_i\right]$,

$$\psi_i \, Q_i \left(t_i\right) \geq \frac{V_i \left(t_i\right) - V_i \left(\hat{t}_i\right)}{\left[t_i - \hat{t}_i\right]} \geq \psi_i \, Q_i \left(\hat{t}_i\right)$$

i.e. Q_i must be weakly increasing in t_i. Taking the limit we have the result:

$$\psi_i \, Q_i \left(t_i\right) \geq \frac{\partial V_i \left(t_i\right)}{\partial t_i} \geq \psi_i \, Q_i \left(t_i\right)$$

$$\frac{\partial V_i \left(t_i\right)}{\partial t_i} = \psi_i \, Q_i \left(t_i\right).$$

$\Longleftarrow$: Show that if

$$\frac{\partial V_i \left(t_i\right)}{\partial t_i} = \psi_i \, Q_i \left(t_i\right)$$

and $Q_i' \left(t_i\right) \geq 0$ then

$$V_i \left(t_i\right) \geq U_i \left(t_i, \hat{t}_i\right) \quad \forall t_i, \hat{t}_i.$$

From $\frac{\partial V_i(t_i)}{\partial t_i} = \psi_i \, Q_i \left(t_i\right)$ we get,

$$V_i \left(t_i\right) = V_i \left(\hat{t}_i\right) + \int_{\hat{t}_i}^{t_i} \frac{\partial V_i \left(t\right)}{\partial t} dt = V_i \left(\hat{t}_i\right) + \int_{\hat{t}_i}^{t_i} \psi_i \, Q_i \left(s_i\right) \, ds_i. \tag{A2}$$

From (A1) above we have,

$$U_i \left(t_i, \hat{t}_i\right) = V_i \left(\hat{t}_i\right) + \psi_i \left(t_i - \hat{t}_i\right) Q_i \left(\hat{t}_i\right).$$

Substituting for $V_i \left(\hat{t}_i\right)$ in (A2),

$$V_i \left(t_i\right) = U_i \left(t_i, \hat{t}_i\right) - \psi_i \left(t_i - \hat{t}_i\right) Q_i \left(\hat{t}_i\right) + \int_{\hat{t}_i}^{t_i} \psi_i \, Q_i \left(t\right) \, dt. \tag{A3}$$

If $t_i > \hat{t}_i$, then since Q is weakly increasing, we can substitute for the lower bound on $Q_i \left(s_i\right)$

to obtain the following inequality:

$$\begin{aligned} V_i(t_i) &\geq U_i\left(t_i, \hat{t}_i\right) - \psi_i\left(t_i - \hat{t}_i\right) Q_i\left(\hat{t}_i\right) + \int_{\hat{t}_i}^{t_i} \psi_i\, Q_i\left(\hat{t}_i\right)\ dt \\ &= U_i\left(t_i, \hat{t}_i\right) - \psi_i\left(t_i - \hat{t}_i\right) Q_i\left(\hat{t}_i\right) + \psi_i\, Q_i\left(\hat{t}_i\right)\left(t_i - \hat{t}_i\right) \\ &= U_i\left(t_i, \hat{t}_i\right). \end{aligned}$$

If $t_i < \hat{t}_i$, then rewrite (A3) as

$$V_i(t_i) = U_i\left(t_i, \hat{t}_i\right) - \psi_i\ \left(t_i - \hat{t}_i\right)\ Q_i\left(\hat{t}_i\right) - \int_{t_i}^{\hat{t}_i} \psi_i\, Q_i(t)\ dt$$

and we have (since Q is weakly increasing)

$$\begin{aligned} V_i(t_i) &\geq U_i\left(t_i, \hat{t}_i\right) - \psi_i\ \left(t_i - \hat{t}_i\right)\ Q_i\left(\hat{t}_i\right) - \psi_i\ \int_{t_i}^{\hat{t}_i} Q_i(t_i)\, dt \\ &= U_i\left(t_i, \hat{t}_i\right) - \psi_i\ \left(t_i - \hat{t}_i\right)\ Q_i\left(\hat{t}_i\right) - \psi_i\ \left(\hat{t}_i - t_i\right)\ Q_i\left(\hat{t}_i\right) \\ &= U_i\left(t_i, \hat{t}_i\right) \end{aligned}$$

i.e. (4) is satisfied. ∎

A.2 Derivation of (12) from (6)

Rewrite (6) as

$$\max_{p_i, x_i} \sum_{i=1,2} \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{\bar{t}} v(t_1, t_2) p_i(t_1, t_2) f(t_1) dt_1 f(t_2) dt_2 \tag{A4}$$

$$+ \sum_{i=1,2} \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{\bar{t}} \{x_i(t_1, t_2) - v(t_1, t_2) p_i(t_1, t_2)\} f(t_1) dt_1 f(t_2) dt_2.$$

The summands in the last term of (A4) can be rewritten (for $i \neq j$) as:

$$\begin{aligned}
&\int_{\underline{t}}^{\bar{t}} \left(\int_{\underline{t}}^{\bar{t}} \{x_i(t_1, t_2) - v(t_1, t_2) p_i(t_1, t_2)\} f(t_j) dt_j \right) f(t_i) dt_i \\
&= - \int_{\underline{t}}^{\bar{t}} V_i(t_i) f(t_i) dt_i \\
&= - \int_{\underline{t}}^{\bar{t}} V_i(\underline{t}) f(t_i) dt_i - \int_{\underline{t}}^{\bar{t}} \left(\psi_i \int_{\underline{t}}^{t_i} Q_i(s_i) ds_i \right) f(t_i) dt_i \\
&= -V_i(\underline{t}) - \psi_i \int_{\underline{t}}^{\bar{t}} \left(\int_{s_i}^{\bar{t}} f(t_i) dt_i \right) Q_i(s_i) ds_i \\
&= -V_i(\underline{t}) - \psi_i \int_{\underline{t}}^{\bar{t}} (1 - F(s_i)) \left(\int_{\underline{t}}^{\bar{t}} p_i(s_i, t_j) f(t_j) dt_j \right) ds_i \\
&= -V_i(\underline{t}) - \psi_i \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{\bar{t}} (1 - F(t_i))\, p_i(t_i, t_j) f(t_j) dt_j dt_i \\
&= -V_i(\underline{t}) - \psi_i \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{\bar{t}} \left[\frac{1 - F(t_i)}{f(t_i)} p_i(t_i, t_j) \right] f(t_j) dt_j f(t_i) dt_i.
\end{aligned}$$

Substituting into (A4), the objective function can be rewritten as

$$\max_{p_i, V_i(\underline{t})} \sum_{i=1,2} \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{\bar{t}} v(t_1, t_2) p_i(t_1, t_2) f(t_1) dt_1 f(t_2) dt_2$$

$$+ \sum_{i=1,2} \left(-V_i(\underline{t}) - \psi_i \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{\bar{t}} \left[\frac{1 - F(t_i)}{f(t_i)} p_i(t_i, t_j) \right] f(t_j) dt_j f(t_i) dt_i \right)$$

which can be rearranged to yield (12).

A.3 Proof of Proposition 2

Consider the optimal mechanism for a given ψ_1. The seller's expected revenue is given by

$$
\begin{aligned}
&E\left[v\left(t_1,t_2\right)\right]-\int_{\underline{t}}^{\bar{t}} V_1\left(t_1\right) f\left(t_1\right) dt_1-\int_{\underline{t}}^{\bar{t}} V_2\left(t_2\right) f\left(t_2\right) dt_2 \\
&=E\left[v\left(t_1,t_2\right)\right]-\psi_1 \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{t_1} Q_1\left(s_1\right) ds_1\ f\left(t_1\right) dt_1-\left(1-\psi_1\right) \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{t_2} Q_2\left(s_2\right) ds_2\ f\left(t_2\right) dt_2 .
\end{aligned}
$$

For ε satisfying $0<\varepsilon<1-\psi_1$, this is strictly less than

$$
\begin{aligned}
E\left[v\left(t_1,t_2\right)\right]-\psi_1 \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{t_1} Q_1\left(s_1\right) ds_1\ f\left(t_1\right) dt_1-\left(1-\psi_1\right) \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{t_2} Q_2\left(s_2\right) ds_2\ f\left(t_2\right) dt_2 \\
+\varepsilon \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{t}\left(Q_2(s)-Q_1(s)\right) ds\ f(t)\, dt,
\end{aligned}
$$

since for a given signal s, the insider's probability of winning $Q_1(s)$ is smaller than the outsider's, $Q_2(s)$. We can rewrite this as

$$
E\left[v\left(t_1,t_2\right)\right]-\left(\psi_1+\varepsilon\right) \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{t_1} Q_1\left(s_1\right) ds_1 f\left(t_1\right) dt_1-\left(1-\psi_1-\varepsilon\right) \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{t_2} Q_2\left(s_2\right) ds_2 f\left(t_2\right) dt_2 .
$$

This term describes the seller's expected revenue if the true parameter is $\psi_1+\varepsilon$, but he uses the allocation rule that would be optimal if the true parameter was $\psi_1<\psi_1+\varepsilon$ (while maintaining incentive compatibility: the transfers $x_i\left(t_1,t_2\right)$ are implicitly determined by (7)). Thus, expected revenue can be increased if ψ_1 increases, without changing the allocation rule. By switching to the optimal allocation rule, the seller may additionally increase expected revenue. ■

A.4 Proof of Proposition 3

We first show that the strategies form an equilibrium. Denote by β_i^{MSP} the inverse of b_i^{MSP}. The equilibrium payoffs are

$$V_1^{\text{MSP}}(t_1) = \begin{cases} 0 & \text{if} \quad t_1 < z_1(\underline{t}) \\ \int_{\underline{t}}^{\beta_2^{\text{MSP}}(b_1(t_1))} (\psi_1 t_1 + (1-\psi_1)t_2 - b_2(t_2)) f(t_2)\, dt_2 & \text{if} \quad t_1 \geq z_1(\underline{t}) \end{cases}$$
$$= \begin{cases} 0 & \text{if} \quad t_1 < z_1(\underline{t}) \\ \psi_1 \int_{\underline{t}}^{\beta_2^{\text{MSP}}(b_1(t_1))} (t_1 - z_1(t_2)) f(t_2)\, dt_2 & \text{if} \quad t_1 \geq z_1(\underline{t}) \end{cases}$$

$$\begin{aligned} V_2^{\text{MSP}}(t_2) &= \int_{\underline{t}}^{z_1(\underline{t})} (\psi_1 t_1 + (1-\psi_1)t_2 - b_1(t_1)) f(t_1)\, dt_1 \\ &\quad + \int_{z_1(\underline{t})}^{\beta_1^{\text{MSP}}(b_2(t_2))} (\psi_1 t_1 + (1-\psi_1)t_2 - b_1(t_1)) f(t_1)\, dt_1 \\ &= \int_{\underline{t}}^{z_1(\underline{t})} (\psi_1 t_1 + (1-\psi_1)t_2 - \psi_1 t_1 - (1-\psi_1)\underline{t}) f(t_1)\, dt_1 \\ &\quad + \int_{z_1(\underline{t})}^{\beta_1^{\text{MSP}}(b_2(t_2))} (\psi_1 t_1 + (1-\psi_1)t_2 - \psi_1 t_1 - (1-\psi_1)z_2(t_1)) f(t_1)\, dt_1 \\ &= (1-\psi_1) \int_{\underline{t}}^{z_1(\underline{t})} (t_2 - \underline{t}) f(t_1)\, dt_1 + (1-\psi_1) \int_{z_1(\underline{t})}^{\beta_1^{\text{MSP}}(b_2(t_2))} (t_2 - z_2(t_1)) f(t_1)\, dt_1. \end{aligned}$$

Both are nonnegative. Now consider different possible deviations:

1. Deviations from $b_1^{\text{MSP}}(t_1)$ to $b_1^{\text{MSP}}(t_1')$ for $t_1, t_1' \geq z_1(\underline{t})$.

   (a) Deviations $b_1^+ > b_1^{\text{MSP}}(t_1)$. In many cases, this will not affect the insider's payoff. The only changes arise if he now wins the auction but would have lost it by bidding $b_1^{\text{MSP}}(t_1)$. That happens if

   $$t_2 \in \left(\beta_2^{\text{MSP}}\left(b_1^{\text{MSP}}(t_1)\right), \beta_2^{\text{MSP}}\left(b_1^+\right)\right].$$

   The value in the integral in V_1^{MSP} is decreasing in t_2, so it is sufficient to show that

$$t_1 = z_1(\beta_2^{\text{MSP}}\left(b_1^{\text{MSP}}\left(t_1\right)\right)),$$

since then the payoff is negative for all higher signals t_2, those for which the insider now wins. Transform that equation,

$$\begin{aligned} z_2\left(t_1\right) &= z_2\left(z_1(\beta_2^{\text{MSP}}\left(b_1^{\text{MSP}}\left(t_1\right)\right))\right) \\ z_2\left(t_1\right) &= \beta_2^{\text{MSP}}\left(b_1^{\text{MSP}}\left(t_1\right)\right) \\ b_2^{\text{MSP}}\left(z_2\left(t_1\right)\right) &= b_1^{\text{MSP}}\left(t_1\right) \end{aligned}$$

Substitute on the left-hand side,

$$b_2^{\text{MSP}}\left(z_2\left(t_1\right)\right) = (1-\psi_1)z_2\left(t_1\right) + \psi_1 z_1(z_2\left(t_1\right))$$

and since z_2 is the inverse of z_1, we can rewrite it as

$$\begin{aligned} b_2^{\text{MSP}}\left(z_2\left(t_1\right)\right) &= (1-\psi_1)z_2\left(t_1\right) + \psi_1 t_1 \\ &= b_1^{\text{MSP}}(t_1). \end{aligned}$$

So the extra payoffs are indeed negative.

(b) Deviations $b_1^- < b_1^{\text{MSP}}\left(t_1\right)$. In many cases, this will not affect the insider's payoff. The only changes arise if he now loses the auction but would have won it by bidding $b_1^{\text{MSP}}\left(t_1\right)$. That happens if

$$t_2 \in \left(\beta_2^{\text{MSP}}\left(b_1^-\right), \beta_2^{\text{MSP}}\left(b_1^{\text{MSP}}\left(t_1\right)\right)\right].$$

We know from above that the payoff with $t_2 = \beta_2^{\text{MSP}}\left(b_1^{\text{MSP}}\left(t_1\right)\right)$ is zero, and positive for smaller t_2. So the forgone payoffs are positive.

2. Deviations from $b_1^{\text{MSP}}(t_1)$ to $b_1^{\text{MSP}}(t_1')$ for $t_1, t_1' < z_1(\underline{t})$. One way or another, the insider does not win, so this cannot affect his payoff.

3. Deviations from $b_1^{\text{MSP}}(t_1)$, for $t_1 \geq z_1(\underline{t})$ to $b_1^{\text{MSP}}(t_1')$, for $t_1' < z_1(\underline{t})$. With the equilibrium bid, the insider's payoff would be positive; with the deviation, he loses the auction, losing a positive payoff.

4. Deviations from $b_1^{\text{MSP}}(t_1)$, for $t_1 < z_1(\underline{t})$ to $b_1^{\text{MSP}}(t_1')$, for $t_1' \geq z_1(\underline{t})$. Instead of losing for sure, the insider now may win the auction. This is the case if $t_2 \leq \beta_2^{\text{MSP}}(b_1(t_1'))$. The insider's payoff if he wins (by deviating to $b_1^{\text{MSP}}(t_1')$) is $t_1 - z_1(\beta_2^{\text{MSP}}(b_1(t_1')))$. This is negative: by definition we have $t_1 < z_1(\underline{t})$, and we must have $\beta_2^{\text{MSP}}(b_1(t_1')) \geq \underline{t}$.

5. Deviations from $b_2^{\text{MSP}}(t_2)$. Similar to part 1 and therefore omitted.

For any signal pair (t_1, t_2), the allocation generated by the modified second-price auction is identical to that generated by the optimal mechanism (cf. Proposition 1). Both bidders expect zero revenue if their signal is $\underline{t}$, so from the Revenue equivalence theorem (cf. (12)), the two mechanisms generate the same expected revenue. ■

A.5 Proof of Proposition 4

The proof consists of two steps. The first step is similar to the proof of Proposition 2, the only change being that we divide all terms by the expected value of the asset for the respective levels of ψ_1. In other words, we consider rent extraction, and not the nominal level of expected revenue. The result of this first step is that an increase in ψ_1 by some $\varepsilon > 0$ and a simultaneous decrease in ψ_2 by the same ε increases rent extraction (and expected revenue). In the second step, we show that increasing both ψ_1 and ψ_2 by the same factor $\gamma > 1$ does not change the degree of rent extraction. Notice that multiplying both ψ_i by a factor γ leaves the cut-off values $\hat{z}_1(t_2)$ and $\hat{z}_2(t_1)$ unchanged, and therefore the $Q_i(t_i)$ are also unchanged. Let the superscripts ψ and $\gamma\psi$ refer to values of the respective variables for values ψ_i and $\psi_i \gamma$.

$$\begin{aligned}
&\frac{R^{\psi}}{E\left[v^{\psi}\left(t_1,t_2\right)\right]} \\
&= \frac{E\left[v^{\psi}\left(t_1,t_2\right)\right] - \int_{\underline{t}}^{\bar{t}} V_1^{\psi}\left(t_1\right) f\left(t_1\right) dt_1 - \int_{\underline{t}}^{\bar{t}} V_2^{\psi}\left(t_2\right) f\left(t_2\right) dt_2}{E\left[v^{\psi}\left(t_1,t_2\right)\right]} \\
&= \frac{\left(\psi_1+\psi_2\right) E\left[t\right] - \psi_1 \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{t_1} Q_1^{\psi}\left(s_1\right) ds_1 f\left(t_1\right) dt_1 - \psi_2 \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{t_2} Q_2^{\psi} ds_2 f\left(t_2\right) dt_2}{\left(\psi_1+\psi_2\right) E\left[t\right]} \\
&= \frac{\left(\gamma\psi_1+\gamma\psi_2\right) E\left[t\right] - \gamma\psi_1 \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{t_1} Q_1^{\gamma\psi}\left(s_1\right) ds_1 f\left(t_1\right) dt_1 - \gamma\psi_2 \int_{\underline{t}}^{\bar{t}} \int_{\underline{t}}^{t_2} Q_2^{\gamma\psi} ds_2 f\left(t_2\right) dt_2}{\left(\gamma\psi_1+\gamma\psi_2\right) E\left[t\right]} \\
&= \frac{E\left[v^{\gamma\psi}\left(t_1,t_2\right)\right] - \int_{\underline{t}}^{\bar{t}} V_1^{\gamma\psi}\left(t_1\right) f\left(t_1\right) dt_1 - \int_{\underline{t}}^{\bar{t}} V_2^{\gamma\psi}\left(t_2\right) f\left(t_2\right) dt_2}{E\left[v^{\gamma\psi}\left(t_1,t_2\right)\right]} \\
&= \frac{R^{\gamma\psi}}{E\left[v^{\gamma\psi}\left(t_1,t_2\right)\right]}
\end{aligned}$$

With a suitable choice of γ, for a given ε,

$$\gamma = \frac{\psi_2}{\psi_2 - \varepsilon}$$

the two steps show that increasing ψ_1 and leaving ψ_2 unchanged increases rent extraction. ■