## Efficient Delegation by an Informed Principal<sup>1</sup>

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Abstract: Firms organized as profit centers and the management practice of category management can reflect a form of delegation we refer to as "partial delegation." For a bargaining problem between an informed firm and an uninformed supplier, we show that a form of partial delegation by the informed firm results in a first-best equilibrium quantity. With partial delegation, the informed firm retains control of how its private information is communicated. This enables it to earn standard information rents without creating quantity distortions. Applications to spin-offs and vertical integration are discussed.

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## 1. Introduction.

Delegated authority is ubiquitous. Examples range from democratic government to representative bargaining to divisionalization in large corporations. When a subordinate has superior information, delegating decision-making authority can result in more efficient decisions but it also introduces agency costs. Much of the incentive-based literature on delegation seeks to identify organizational and informational conditions under which this tradeoff supports delegation over centralization.<sup>4</sup> In these cases, the focus is on what organizational form better serves an uninformed principal.

We seek instead to focus on the benefits of the commitment role of delegation to an informed principal in a bargaining problem. Consider the case of a firm with private valuation information bargaining with a supplier over the price and quantity of a good.<sup>5</sup> If the firm and the supplier bargain directly over the quantity to be transacted, the bargaining outcome may not yield a first best outcome (i.e. a transaction that maximizes the sum of the surpluses of the two parties in each state of nature). One well-known example leading to inefficient quantities occurs where the supplier can make a take-it-or-leave-it offer of a non-linear price schedule to the firm. The profit-maximizing price schedule for the supplier in this case will induce transaction quantities that are below first-best levels because at lower quantities the supplier pays the firm smaller information rents.

The question we examine in this paper is whether these direct bargaining inefficiencies can be eliminated if the firm delegates the authority to negotiate with the supplier to an agent. We model the agent as an independent profit center that contracts with the parent firm as well as the supplier. The delegation of decision-making thus creates a common agency game in which the firm and the supplier each offer a non-linear contract to the agent.

The delegation of decision-making to the agent can influence the interaction between the firm and the supplier in two ways. First, the firm can use its contract with the agent to eliminate the incentive for the supplier to engage in rent extraction. Specifically, by charging the agent more in high valuation states, the firm can eliminate the rent extraction incentives of the supplier such that the resulting

<sup>&</sup>lt;sup>4</sup>E.g. Baron and Besanko (1992, 1999), Melamud, Mookherjee, and Reichelstein (1992, 1995, 1997), Baliga and Sjoström (1998), Laffont and Martimort (1998), Severinov (1999), and Mookherjee and Tsumagari (2001).

<sup>&</sup>lt;sup>5</sup>In this regard, the information structure of our bargaining game is the same as in Ausubel and Deneckere (1989).

schedules induce first-best transaction levels. Second, if delegation allows the firm to move first in offering a contract to the agent, it can commit the agent to bargain more aggressively with the supplier. This benefit of delegation to the informed firm is similar to that analyzed by Schelling (1956) and Fershtman and Judd (1987) in the context of full information games. To identify the role of the first effect independent of the second, we focus instead on two games in which the firm and the supplier offer contracts simultaneously. Both games have a continuum of equilibria, which we compare to the set of incentive efficient equilibria of the initial no-delegation bargaining game. Our interest is in whether the delegation of authority to the agent can eliminate equilibria that do not have first-best quantities in asymmetric information settings, and whether delegation can lead to a Pareto improvement over the incentive efficient equilibria of the bargaining game.

We show that the answer to the first question depends critically on whether the agent is informed of the firm's private information. In the first game we consider, referred to as the partial delegation game, we assume that the agent does not know the firm's true type. In this case, the firm announces a transfer price to the agent (which is also observable to the supplier). Because the relationship between the agent and the uninformed supplier is one of full information, equilibrium quantity distortions disappear.<sup>6</sup> Yet, when one looks at the equilibrium transfers between the agent and the principals, it will be apparent that the informed principal is still earning an information rent. Thus, the partial delegation game cannot lead to a Pareto improvement relative to the bargaining game between the firm and the supplier. However, it does pare down the set of equilibria from the bargaining game in a way that is clearly beneficial to the firm when its direct bargaining power is low.

The second game we consider is referred to as the full delegation game. In it, the firm reveals its private information to the agent. As in the case of partial delegation, we assume that the firm and the supplier simultaneously offer contracts to the agent. The difference introduced by full delegation is that the contract offered by the firm can depend on the firm's true type while the contract offered by the supplier can depend only on a type report from the agent. We show that the full delegation game has a large set of equilibria that includes all of the incentive efficient equilibria of the bargaining game. Equilibria with quantities that are not first best arise in the full delegation game because the supplier is uncertain about the firm's costs when it offers its schedule to the firm. Thus, there will exist equilibria

<sup>&</sup>lt;sup>6</sup>Because one the effects of the transfer price is to restrict the set of profitable deals for the agent, our model of partial delegation can be interpreted as combining elements of both "incentive delegation" and "instructive delegation" as defined by Fershtman and Kalai (1997).

in which the supplier distorts the quantity schedule in order to extracts rents, and the schedule offered by the firm anticipates this rent extraction by the supplier.

We believe we see this form of delegation is two important commercial settings: corporations organized as profit centers and category management. Perhaps the best example of a corporation organized along profit-center lines is the multinational corporation. Often, a corporation doing business in a foreign country sets up a wholly-owned subsidiary to handle its foreign transactions. The subsidiary operates as a separate economic unit but it also is required to conduct business, such as trade in intermediate goods, with its parent on terms (i.e. transfer prices) usually set by the parent. Much of the literature on transfer pricing focuses on the tax benefits of this corporate structure. Our work herein suggests an efficiency benefit that may be more significant than a tax motive. Since the same basic incentives are also present in tax competition problems between two countries with a common multinational, our work is certainly not incompatible with the literature on tax-induced transfer pricing.

Our second example is the emerging management practice of category management. Large retailers like WalMart, purchase over 1500 different categories of products. Category management involves assigning a manager to each category. The manager is responsible for negotiating with outside suppliers and is compensated on the basis of her category's profits. Part of the category's costs can include costs for shelf space as well as overhead charges which are paid to the retailer.<sup>7</sup> Rather than giving the category manager discretion over how to use information such as the opportunity cost of shelf space, our results suggest that it is better for the retailer to maintain control of such information and instead embed the appropriate rent-seeking incentives into the manager's contract in a non-distortionary way.

Finally, despite the admittedly simple information structure in our model, we believe our specific results advance our understanding of two problems: corporate spin-offs and Bork's Thesis (1978). The finance literature has documented short-run excess stock price returns of 2.4% to 4.3% related to spin-off announcements.<sup>8</sup> Numerous explanations for this phenomenon exist. The one that is closest in spirit to this paper is from Aron (1991). She argues that with a multi-division firm, share price does not track the performance of any one division very closely. Spinning off a division allows owners to provide the manager with stronger incentives based on stock price. Seward and Walsh (1996), however, do not find

<sup>&</sup>lt;sup>7</sup>See Blattberg and Fox (1995-6) for details on how category management is being implemented.

<sup>&</sup>lt;sup>8</sup>See Hite and Owers (1983), Miles and Rosenfeld (1983), Schipper and Smith (1983), and Rosenfeld (1984).

any evidence that the excess returns are related to stronger managerial incentives. Our results suggest an alternative explanation. In the short run, the information of the parent about the spun-off division (e.g. Ford and Visteon) remains accurate while the parent can now extract its surplus through an arm's-length relationship. Our analysis implies that by creating this arm's-length relationship, the parent eliminates the incentives faced by suppliers that in equilibrium created quantity distortions. This is inherently a short-term advantage as over time the division's economic characteristics will likely change and its reliance on its parent for business will diminish.

Bork's Thesis is the term used by Prat and Rustichini (2002) to describe Judge Bork's argument that vertical relationships, while viewed as anti-competitive, can facilitate efficient production. Our work extends the support for this argument found in Prat and Rustichini (2002) to an environment with private information. We find this application surprising in light of the conventional wisdom that private information induces inefficiencies due to information rents.

In the next section, we set up a simple model of principal to principal bargaining. Since our results apply to more general bargaining weights than considered in Spulber (1988), this section is of some independent interest. In Section 3, we then permit the informed principal to partially delegate bargaining responsibilities to an agent and characterize the equilibria of the resulting common agency game. In Section 4, we analyze a game with fully delegated bargaining. We offer concluding comments in Section 5.

### 2. A Model of Bargaining Between Two Principals.

Our model focuses on the strategic interaction between an informed principal and an uninformed principal who are negotiating over the terms of transaction for q units of a product. The informed principal's benefit from the quantity transacted has two components. The first is the "direct" benefit V(q,c) from the transaction, which depends on a cost parameter, c. This parameter represents the informed principal's private information and is drawn from the distribution  $F(\cdot)$  on [c,c]. The second component is the "indirect" effect from the transaction, H(q).<sup>9</sup> The gross payoff to the uninformed principal is denoted by -C(q).

The primary example we use to motivate this payoff structure as well as use for expository purposes throughout the paper is one in which the informed principal is a corporation whose production

<sup>&</sup>lt;sup>9</sup>It is not essential that  $H(\cdot)$  be independent of *c*. If  $H(\cdot,c)$  creates countervailing incentives relative to  $v(\cdot,c)$ , the ability of the informed principal to mute or magnify rents, alluded to in the introduction, could be compromised.

involves an internally produced intermediate good and an externally supplied product produced by the uninformed principal. In this case, V will be the corporation's profit from the sale of its product net of the cost of internally produced input and H is the corporation's profit from any spillovers to other products. The cost function of the external supplier is C(q). The case to be considered in this section is one of direct bargaining between the corporation and the supplier. Our model of delegated negotiation, to be considered in the next section, will introduce a subsidiary to the corporation. The corporation will still produce the internal intermediate good and provide it to the subsidiary. The subsidiary will decide how much of each intermediate good to purchase and will market the final product. For simplicity we assume that one unit of each intermediate good is needed to produce one unit of the final good.<sup>10</sup>

Assumptions: The payoff functions have the following properties:

- a) V(q,c) = v(q) cq
- b)  $v(\cdot)$  and  $H(\cdot)$  are strictly increasing, strictly concave in q, and H(0)=v(0)=0.
- c)  $C(\cdot)$  is strictly increasing, strictly convex, and C(0)=0.

Assumption (a) is adopted for convenience. All of our results hold as long as  $V(\cdot, \cdot)$  satisfies the usual single-crossing properties. Letting W(q,c) = H(q) + V(q,c) - C(q) denote the total surplus from the transaction, the first best quantity,  $q^F(c)$ , is the solution to  $W_q(q,c) = 0$ . Assumptions (b) and (c) ensure that total surplus is strictly concave, so that the first best quantity is unique. Assumption (a), and more general versions, ensure that the first-best quantities and gross payoffs will be are decreasing in c.

We represent the outcome of direct bargaining by a non-linear price schedule T(q). We refer to T(q) as a contract. It represents the transfer to be made by the corporation to the supplier. The corporation chooses the value of q to maximize profits, given T(q).<sup>11</sup> Total profit for the respective

<sup>&</sup>lt;sup>10</sup>This payoff structure is also consistent with our category management example where v is the gross profit of the category and H is the spillover profits in related categories, and with our spin-off example, again where v is the direct profit of the spin-off and H is the parent's spillover profit related to its transactions with the spun off firm.

<sup>&</sup>lt;sup>11</sup>Rochet's (1985) Taxation Principle implies that there is no loss in generality in restricting attention to non-linear price contracts. In this context, the Taxation Principle is equivalent to the Revelation Principle. We focus on non-linear contracts instead of direct contracts or mechanisms for consistency with the analysis in the next section where it will be necessary to focus on non-linear

parties are

$$\pi(q,c) = H(q) + V(q,c) - T(q) \text{ and } s(q) = T(q) - C(q).$$
(1)

Assuming that T(q) is differentiable, any strictly positive profit-maximizing quantity for the corporation, q(c), will satisfy

$$H'(q(c)) + V_q(q(c),c) - T'(q(c)) = 0 \text{ and } H'' + v_{qq} - T'' \le 0.$$
<sup>(2)</sup>

The payoff generated by the corporation's optimal quantity choice is denoted  $\Pi(c) = \pi(q(c),c)$ .

Differentiation of  $\Pi(c)$  and (2) yields the incentive compatibility conditions that q(c) must satisfy:

$$d\Pi(c)/dc = V_c(q(c),c) = -q(c) < 0$$
(3)

and

$$\mathbf{q}'(\mathbf{c}) \le \mathbf{0}. \tag{4}$$

By integrating (3), corporation profit must equal

$$\Pi(c) = \Pi(\overline{c}) + \int_{t=c}^{c} q(t)dt.$$
<sup>(5)</sup>

In order for the corporation and the supplier to be willing to participate in the bargaining game, it must yield a non-negative payoff to each party. For the corporation, it follows from (3) that participation is guaranteed for all values of the private information if  $\Pi(\vec{c}) \ge 0$ . Similarly, participation by the supplier yields a non-negative expected return if

$$\mathscr{E}_{c}[W(q(c),c) - \Pi(c)] \ge 0.$$
(6)

Consistent with the approach used in Spulber (1988), we model the bargaining that takes place as the outcome of some (interim) incentive efficient process. Incentive efficiency, as defined by Holmström and Myerson (1983), describes the set of quantity-money transfer allocations (q(c),T(q(c))) for which there exists no other incentive compatible allocations ( $q^1(c)$ , $T^1(q^1(c))$ ) that yields higher expected profit for the supplier or higher profit for any corporation type without giving the supplier or any other

contracts.

corporation type lower profit. Myerson (1985) and Wilson (1985) show that this incentive constrained Pareto frontier can be described by maximizing the weighted welfare function

$$\hat{W} = \int_{c=\underline{c}}^{c} [\alpha(c)\pi(q(c),c) + \gamma s(q(c))]f(c)dc$$
(7)

subject to the incentive compatibility constraints, (3) and (4), and the participation constraints, (6) and  $\mathbf{II}(\vec{c}) \ge \mathbf{0}$ . The non-negative weights,  $\alpha(\cdot)$  and  $\gamma$ , are normalized so that

$$\gamma + \int_{c=\underline{c}}^{\overline{c}} \alpha(c) f(c) dc = 1.$$
(8)

Intuitively, these weights proxy the parties' relative bargaining power. In describing the incentive constrained Pareto frontier, define

$$\hat{\alpha}(c) = \mathscr{E}[\alpha(t)|t \le c] = \int_{t=c}^{c} \alpha(t)f(t)dt/F(c)$$
(9)

which is the expected weight placed on corporation profit for all types less than c. This conditional weight is important because incentive compatibility requires that if type c earns an extra dollar, all lower types must also earn an extra dollar. The conditional weight measures the cumulative effect of such a transfer.

Using (1), (7) can be rewritten as

$$\widehat{W} = \int_{c=c}^{\overline{c}} \{ \gamma W(q(c),c) + (\alpha(c) - \gamma) \Pi(c) \} f(c) dc .$$
(10)

Given (3), the proof of Proposition 1 in the Appendix shows that (6) is equivalent to

$$\mathbf{\Pi}(\overline{c}) \leq \mathbf{\Gamma}(q) \equiv \int_{c=\underline{c}}^{\overline{c}} [W(q(c),c) + W_c(q(c),c)F(c)/f(c)]f(c)dc.$$
(11)

Thus, for a given set of weights, we define the equilibrium bargaining allocation by the quantity schedule, q(c), and minimum corporation profit,  $\Pi(\vec{c})$ , that maximizes (10) subject to (3), (4), (11),  $\Pi(\vec{c}) \ge 0$ , and  $\Gamma(q) \ge 0$ .<sup>12</sup> Denote the incentive efficient quantity schedule as  $q^{B}(\cdot)$  and the corresponding

<sup>&</sup>lt;sup>12</sup>The constraint  $\Gamma(q) \ge 0$  ensures that the average surplus created by exchange is sufficient to cover the aggregate information rents earned by the corporation.

payoffs as  $\Pi^{B}(c)$  and  $s^{B}(c)$ . We will focus on welfare weights for which  $q^{B}(\cdot)$  is strictly decreasing.<sup>13</sup>

The solution will depend on the relative aggregate welfare weights,  $\hat{\alpha}(\vec{c})$  and  $\gamma$ . When  $\hat{\alpha}(\vec{c}) \leq \gamma$ , the key welfare trade-off balances the direct welfare losses from trading an inefficient quantity with the cost to the supplier of paying the corporation an information rent that is increasing in q. If  $\lambda$  denotes the multiplier on  $\Gamma(q)$ , the incentive efficient quantity exchanged is defined by

$$W_q(q(c),c) = \left(1 - \frac{\hat{\alpha}(c)}{\gamma + \lambda}\right) \frac{F(c)}{f(c)}.$$
(12)

In addition, when  $\hat{\alpha}(\overline{c}) < \gamma$ ,  $\Pi(\overline{c}) = 0$ .

When  $\hat{\alpha}(\vec{c}) > \gamma$ , the key trade-off balances the direct welfare losses from trading an inefficient quantity with the cost of distributing the information rents among different types of the corporation. Now the incentive efficient quantity exchanged is defined by

$$W_{q}(q(c),c) = \left(1 - \frac{\hat{\alpha}(c)}{\hat{\alpha}(\bar{c}) + \lambda}\right) \frac{F(c)}{f(c)}$$
(13)

and  $\Pi(\overline{c})$  is set at its maximal level as defined by (11).

**Proposition 1a.** Assume the welfare weights imply  $\alpha(\overline{c}) \leq \gamma$ . Then the incentive efficient allocation is fully separating (i.e.  $(q^B)'(\cdot) < 0$ ) and satisfies (12) if, and only if,

$$\left(\left(1-\frac{\hat{\alpha}(c)}{\gamma+\lambda^*}\right)\frac{F(c)}{f(c)}\right)'+1\geq 0$$
(14)

where  $\lambda^*$  is the multiplier associated with  $\Gamma(q) \ge 0$ . Moreover,  $\Pi(c) = 0$  if  $\alpha(c) < \gamma$  and  $\Pi(c) \in [0, \Gamma(q^B)]$  if  $\alpha(c) = \gamma$ .

**Proposition 1b.** Assume the welfare weights imply  $\hat{\alpha}(c) > \gamma$ . Then the incentive efficient allocation is

<sup>&</sup>lt;sup>13</sup>When the solutions that follow imply  $q'(\cdot) > 0$  for some *c*, an "ironing" procedure implies that the incentive efficient quantity schedule is constant on some open sets of *c*. We do not address these cases as the added complication does not contribute materially to our discussion.

fully separating (i.e.  $(q^B)'(\cdot) < 0$ ) and satisfies (13) if, and only if,

$$\left(\left(1-\frac{\hat{\alpha}(c)}{\hat{\alpha}(\bar{c})+\lambda^{**}}\right)\frac{F(c)}{f(c)}\right)'+1\geq0$$
(15)

where  $\lambda^{**}$  is the multiplier associated with  $\Gamma(q) \ge 0$ . Moreover,  $\Pi(c) = \Pi(q^B)$ .

In principal agent models that endow the uninformed principal with all the bargaining power, i.e.  $\gamma=1$ , the usual increasing inverse hazard rate assumption,  $(F/f)^{1/2} = 0$ , is sufficient for conditions (14) and (15) to hold. With distributed bargaining power, extreme welfare weights can imply pooling in the quantity schedule. For instance, (14) is equivalent to

$$\alpha(c) \leq 1 + \hat{\alpha}(c) + (\gamma + \lambda^* - \hat{\alpha}(c))(F(c)/f(c))'.$$
<sup>(16)</sup>

Thus, the characterizations in Propositions 1a and 1b only apply if  $\alpha(\cdot)$  is sufficiently uniform.

Several examples help illustrate important features of the incentive efficient quantity schedules. The first example concerns the possibility of a bargaining equilibrium which generates the first best quantity schedule,  $q^{F}(c)$ .

*Example 1:* By setting  $\alpha(c) = \gamma = .5$ , the necessary conditions in (12) yield  $q^{B}(\cdot) = q^{F}(\cdot)$  for sufficiently large total welfare that  $\lambda^{*} = 0$ . To verify that  $\Gamma(q^{F}) \ge 0$ , note that  $d[W(q^{F}(c),c) - \Pi(c)]/dc = 0$  from (5) and the definition of the first best quantities. It then follows that  $\Gamma(q^{F}) = W(q^{F}(c),c)$ , so  $q^{F}(\cdot)$  will be a solution to (12) in this case as long as  $W(q^{F}(c),c) \ge 0$  Combining this observation with Proposition 1a yields the following result:

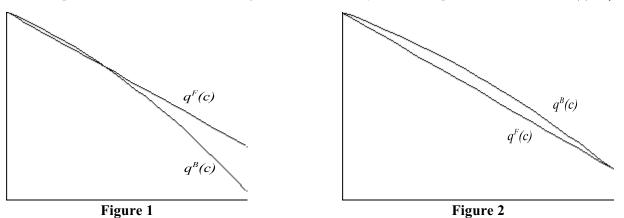
*Corollary 2:* There is a class of incentive efficient equilibria in which  $q^{B}(\cdot) = q^{F}(\cdot)$ . For these equilibria, the payoffs to the players are  $\Pi^{B}(c) = \Pi^{B}(c) + \int_{t=c}^{c} q^{F}(t)dt$  and  $\mathscr{E}_{c}[s^{B}(c)] = W(q(c),c) - \Pi^{B}(c)$ , where  $\Pi^{B}(c) \in [0, W(q^{F}(c),c)]$ .

In addition to the first best quantity schedule, it is also possible to generate incentive efficient quantity schedules which have outputs that may exceed or fall short of the first best quantities. The following examples illustrate the variety of possible outcomes.

*Example 2.* Assume  $\alpha(c) = 0$  and  $\gamma = 1$ . This is the standard principal-agent formulation. With an increasing inverse hazard rate,  $q^{B}(\cdot)$  solves  $W_{q} = F/f$ . In this case the supplier distorts the quantity

schedule downward in order to reduce the size of information rents earned by the corporation. As a result, the corporation's payoff will be strictly less than in any of the equilibria associated with the first best quantities for  $c \in [\underline{c}, \overline{c}]$ .

*Example 3.* Assume  $\alpha(c) = .75 - .7c$  for  $c \in [0,1]$  and assume F(c) = c. Thus,  $\gamma = .6$  and  $\hat{\alpha}(c) = .75 - .35c$  so Proposition 1a applies. Figure 1 illustrates  $q^{B}(\cdot)$  and  $q^{F}(\cdot)$ . Note that  $q^{B}(c) < q^{F}(c)$ only when c > 3/7. Unlike typical incomplete information problems, the optimal quantity can be inefficiently high. The reason for this can best be understood in the context of the next example. *Example 4.* Assume  $\alpha(c) = .75 - .4c$  for  $c \in [0,1]$  and assume F(c) = c. Thus,  $\gamma = .45$ ,  $\hat{\alpha}(c) = .75 - .2c$ , and Proposition 1b applies. Figure 2 shows that  $q^{B}(c) \ge q^{F}(c)$  for all c. Moreover, both the lowest and highest types trade a first-best quantity. On average, gains to the corporation are valued more than gains to the supplier. This suggests that, again on average, all the surplus should accrue to the corporation. Since, corporation profits are weighted differentially, the incentive exists for intermediate types to trade a higher quantity. Exceeding the first-best level increases profits for lower types at the expense of higher types. As (11) reveals the unweighted benefit is  $-W_c \cdot F$  or  $q \cdot F$ . For all but the lowest type, the marginal welfare benefit of trading more than the first-best level,  $\hat{\alpha}(c)$ , is offset by the marginal cost of overall surplus reduction realized by all types,  $\hat{\alpha}(\vec{c})$ , due to trading a second-best quantity. For the lowest and highest possible type, these marginal welfare effects exactly offset, implying a first-best quantity. From (14) it is clear that key to generating this excess quantity distortion is a decreasing welfare weight function as it implies that the benefits of the increased trade should accrue to higher welfare types. The last example will show that one can also generate inefficiently low trade quantities even when  $\hat{\alpha}(c) > \gamma$ .



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*Example 5.* Assume  $\alpha(c) = 1.4c$  for  $c \in [0,1]$  and assume F(c) = c. Thus,  $\gamma = .3$  and  $\hat{\alpha}(c) = .7c$  so Proposition 1b still applies. Unlike Example 4, gains to higher types are now weighted more than gains to lower types. Shifting surplus to higher types now requires trading less than the first-best quantity as Figure 3 illustrates.

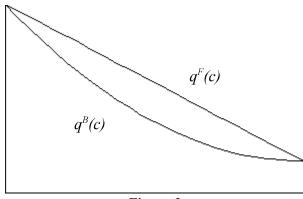


Figure 3

## 3. Partial Delegation.

Consider now an alternative game in which the corporation delegates the responsibility for choosing its output level to a wholly-owned subsidiary. Subsidiary payoffs and choices will be denoted by *a* because the subsidiary is a formal agent of the corporation and informal agent (arising through any contract incentives) of the supplier. In this section, we consider a partial delegation scenario in which the corporation makes a type report to the subsidiary that is public. We use the term "partial delegation" because the corporation delegates the responsibility for deciding how much to purchase to its subsidiary but retains control of how information is released to the supplier.

Decisions are made in the context of a two-stage game. In the first stage, the corporation announces a cost report  $\kappa$  and a contract  $T^c(q,\kappa)$  and the supplier simultaneously announces a contract  $T^s(q,\kappa)$ . By assuming the contracts are offered simultaneously, we eliminate the bias towards delegation that might be due to a first-mover advantage. In the second stage, the subsidiary chooses q. Since technically, our game is one of common agency it is now necessary that we consider competition in nonlinear contracts.<sup>14</sup> We will focus on equilibria of this game for which the contracts are differentiable and

<sup>&</sup>lt;sup>14</sup>Martimort and Stole (1997) show that in this common agency setting, non-linear price competition induces no loss of generality while direct mechanism competition does. That is, restricting attention to competition in direct mechanisms is not the appropriate way to apply the Revelation

we will refer to these equilibria as "differentiable partial delegation" equilibria.<sup>15</sup>

One can interpret  $T^{c}(q,\kappa)$  as the "dividend" paid to the corporation and  $\kappa$  as the transfer price for the intermediate good produced by the corporation. As a distinct economic unit, the subsidiary earns profit of

$$a(q,\kappa) = V(q,\kappa) - T^{c}(q,\kappa) - T^{s}(q,\kappa).$$
<sup>(17)</sup>

If  $\kappa$  is zero, then the subsidiary is only concerned with its revenues and ignores the cost of the intermediate good, except as it is may be reflected in  $T^c(\cdot,0)$ . If  $\kappa$  equals  $\underline{c}$ , then the subsidiary is directly responsible for the minimum cost of the intermediate good and any excess costs would have to be reflected in  $T^c(\cdot,\underline{c})$ . Alternatively, if  $\kappa$  equals  $\overline{c}$ , the subsidiary is directly responsible for the maximal cost of the intermediate good and  $T^c(\cdot,\overline{c})$  can specify any "rebates" available to the subsidiary. Notice that the corporation's contract depends on q and  $\kappa$  but not on c as a contract that depends on private information is not verifiable and hence not enforceable. Corporation profit is also affected by this form of delegation. Now

$$\pi^{p}(q,\kappa,c) = H(q) + (\kappa - c)q + T^{c}(q,\kappa).$$
<sup>(18)</sup>

The term  $(\kappa - c)q$  represents the corporation's transfer price derived profit. The definition of supplier profit remains the same as in (1). The "p" superscript is used to help remind us that we are dealing with partial delegation.

We begin by analyzing the second stage problem, where the subsidiary chooses its optimal output given the contracts offered by the supplier and the corporation. Assuming differentiable contracts, any strictly profit maximizing quantity chosen by the subsidiary will satisfy

$$\mathbf{v}_{q}(q) = \kappa + T_{q}^{c}(q,\kappa) + T_{q}^{s}(q,\kappa).$$
<sup>(19)</sup>

Principle in common agency games and thus invalidates any normative analysis. On the other hand, because the agent has quasi-linear preferences, one can restrict attention to competition in non-linear contracts and not miss any allocations that could be generated by the equilibrium of some more complicated Nash game between the principals.

<sup>&</sup>lt;sup>15</sup>When both principals are informed, this differentiability condition will imply that  $q(c) = q^{F}(c)$ in any Nash equilibrium with an interior solution for the agent's action. However, interior solutions that are not first best could arise if schedules are not differentiable. Our assumption thus rules out this source of multiplicity of equilibria from the full information game.

We denote strictly positive solutions to (19) as  $q^{a}(\kappa)$ .<sup>16</sup>

We now turn to the choice of the contracts to be offered by the principals. First consider the case of the supplier, who selects a contract,  $T^{s}(q,\kappa)$ , that can depend on both the quantity selected by the agent and the cost parameter that is reported to the subsidiary by the corporation. Although the supplier does not know the true cost, c, this is still a full information problem because the supplier knows the transfer price that is being used by the subsidiary to make decisions. Since  $\kappa$  is known, the supplier will choose  $T^{s}(q,\kappa)$  to drive the subsidiary to zero profit. Using this fact in (17), the supplier's optimization problem can be expressed as choosing q to maximize

$$s^{p}(q,\mathbf{\kappa}) = V(q,\mathbf{\kappa}) - C(q) - T^{c}(q,\mathbf{\kappa})$$
<sup>(20)</sup>

The supplier's best response,  $T^{s}(q, \kappa)$ , can then be chosen to induce the selection of the preferred quantity, using (11), and zero profits for the subsidiary. The preferred quantity satisfies  $v_q(q^{a}) - \kappa - C'(q^{a}) = T_q^{c}(q^{a}, \kappa)$ . Combining these necessary conditions with (19), it follows that the supplier's contract must satisfy

$$T_q^s(q^a,\kappa) = C'(q^a).$$
<sup>(21)</sup>

The supplier's non-linear price schedule has the "local truthfulness" property (in the sense of Bernheim and Whinston (1986)), because the marginal transfer required from the subsidiary at the optimal choice will reflect the marginal cost of an increment in quantity to the supplier.

The parent corporation will choose  $\kappa$  and  $T^{c}(q,\kappa)$  to maximize the payoff in (18). Since the corporation's schedule can be chosen to drive the subsidiary to zero profit, given  $T^{s}(q,\kappa)$ , its payoff can be expressed as

$$\pi^{p}(q,\kappa,c) = H(q) + (\kappa - c)q - T^{s}(q,\kappa)$$
<sup>(22)</sup>

The corporation will choose q and  $\kappa$  to maximize (22). The necessary condition for the corporation's preferred quantity is  $H'(q^a) + v_q(q^a) - c = T_q^s(q^a, \kappa)$ . Combining this with (19), the corporation's non-linear schedule must satisfy

$$T_{q}^{c}(q^{a},\kappa) = -H'(q^{a}) + c - \kappa.$$
(23)

<sup>&</sup>lt;sup>16</sup>All common agency games have zero activity equilibria in which both principals offer contracts that charge exorbitant prices and the agent chooses to buy nothing. Our analysis will focus on positive activity equilibria.

in order to induce the preferred choice of quantity by the subsidiary. As in the case of the supplier, the corporation's non-linear schedule will be locally truthful because it reflects the marginal cost of output to the corporation (when evaluated at the chosen  $\kappa$ ). The necessary condition for choice of  $\kappa$  requires that  $T_{\kappa}^{s}(q^{a}(\kappa),\kappa) = 0$  at an interior solution. The transfer price is used by the corporation to minimize the transfer that must be paid to the supplier, because the transfer price does not play a critical role in determining the output level. This happens because the corporation can influence the subsidiary's choice of output through both  $\kappa$  and  $T_{q}^{c}(q^{a},k)$ . Any change in these parameters that keeps their sum constant will leave subsidiary output unaffected.

The profits of the corporation in the best response will be

$$\mathbf{I}^{p}(c) = \max_{q,\kappa} \pi^{p}(q,\kappa,c) \tag{24}$$

which must satisfy  $\mathbf{IP}(c) \ge 0$  in order for the type *c* corporation to participate. Using the envelope theorem, this yields

$$\mathbf{\Pi}_{c}^{p} = -q^{a}(\mathbf{\kappa}(c)) \tag{25}$$

where  $\kappa(c)$  is the corporation's choice of transfer price. The participation constraint then simplifies to  $\mathbf{IP}(\vec{c}) \ge \mathbf{0}$ . If a quantity schedule  $q^{p}(c)$  represents an equilibrium to the partial delegation game, it will generate information rents for the corporation that are governed by (25). Note that these information rents are governed by the same incentive compatibility condition as in the bargaining game in (5).

We can now use the results from the best response functions of the respective players to characterize the quantity schedules that are equilibria in the partial delegation game. In order for a schedule  $q^{p}(c)$  to be an equilibrium, it must satisfy  $q^{p}(c) = q^{a}(\kappa^{*}(c))$ , where  $\kappa^{*}(c)$  is the equilibrium transfer price schedule chosen by the corporation. Substituting (21) and (23) into (19) yields the requirement that  $W_{q}(q^{a}(\kappa),c) = 0$ , so that the only quantity schedule that satisfies the necessary conditions for the partial delegation equilibrium is the first best quantity schedule,  $q^{p}(c) = q^{F}(c)$ .

In order to establish that the first best quantity schedule is an equilibrium to this game, we must construct non-linear contracts that satisfy the corporation's and the supplier's best response functions. Suppose we consider the following contracts:

$$T^{c}(q,\kappa) = -H(q) + \phi^{c}(\kappa) \quad \text{and} \quad T^{s}(q) = C'(q) + \phi^{s}.$$
(26)

It is straightforward to verify that with these contracts,  $\kappa(c) = c$  will be a (weak) best response for the

corporation, and this will yield  $q^a(c) = q^F(c)$  as the best response for the subsidiary.<sup>17</sup> Since the best responses must drive the subsidiary to zero profits, the parameters in the contracts in (26) must satisfy  $W(q^F(c),c) = \Phi^s + \Phi^c(c)$ . In addition, the participation constraints require that  $\Phi^s$ ,  $\Phi^c(\kappa) \ge 0$ . These conditions yield the requirement that  $\Phi^s \in [0, W(q^F(c),c)]$  and  $\Phi^c(c) = W(q^F(c),c) - \Phi^s$ . It then follows

from (25) that 
$$\phi^c(c) = \phi^c(\overline{c}) + \int_c^c q^F(t) dt$$
.

**Proposition 3.** There exist a continuum of (positive production) differentiable partial delegation equilibria. They all induce first-best equilibrium quantities. These equilibria generate payoffs to the corporation of  $\mathbf{IP}(c) = \mathbf{IP}(\overline{c}) + \int_{t=c}^{\overline{c}} q^F(t) dt$  where  $\mathbf{IP}(\overline{c}) \in [0, W(q^F(\overline{c}), \overline{c})]$  and to the supplier of  $W(q^F(\overline{c}), \overline{c}) - \mathbf{IP}(\overline{c})$ .

Note that the equilibria in the partial delegation game generate payoffs that are equivalent to those in the solutions to the bargaining problem that yield first best quantities, as identified in the Corollary 2. The participation constraint for the supplier in the partial delegation case requires that  $s^{p}(q^{p}(c),\kappa^{*}(c)) \ge 0$  for all c. This condition is more stringent than the participation constraint for the bargaining game, which only requires this condition to hold in expected value terms. However, the more stringent participation condition does not constrain the payoffs that can be achieved with the first best quantities because (25) requires that the corporation capture any surplus generated by lower values of c. Thus, the payoff to the supplier must be independent of the realization of c with first best quantities.

Partial delegation cannot yield a Pareto improvement over direct bargaining, because the equilibria to the partial delegation game are a proper subset of the set of incentive efficient equilibria identified in Proposition 1. However, the corporation might choose partial delegation if it results in an outcome with a higher payoff than under direct bargaining.

*Corollary 4.* If  $\hat{\alpha}(c) < \gamma$  and (14) is satisfied, then the corporation prefers partial delegation to direct

<sup>&</sup>lt;sup>17</sup> Although there will be a unique equilibrium quantity schedule  $q^{p}(c) = q^{F}(c)$ , the equilibrium mapping  $\kappa^{*}(c)$  is not unique. For example, let  $T^{c}(q,\kappa) = -H(q) + \phi^{c}(\kappa) + (b(\kappa) - \kappa)q$ , where  $b(\kappa)$  is a strictly monotone function mapping [c,c] onto itself. In this case,  $\kappa^{*}(c) = b^{-1}(c)$  would be a best response for the corporation and would result in a first best choice of quantities by the subsidiary. Since the subsidiary profit in (17) does not depend on c, only strictly monotonic  $\kappa^{*}(\cdot)$  can arise in equilibrium.

### bargaining.

**Proof.** Corporation profit under direct bargaining equals  $\int_{c}^{\overline{c}} q^{B}(t)dt$  where  $q^{B}(t) < q^{F}(t)$  for all  $t \in (\underline{c}, \overline{c}]$ . The lowest possible equilibrium payoff under partial delegation equals  $\int_{c}^{\overline{c}} q^{F}(t)dt$ . Q.E.D.

One example satisfying the conditions of Corollary 4 arises if under direct bargaining the supplier can make a take-it-or-leave-it offer to the corporation, as in the principal-agent formulation (Example 2 above). Delegation benefits the corporation in this case by preventing the supplier from distorting the quantity schedule to engage in rent extraction. This suggests that partial delegation should be attractive in situations where the corporation has low bargaining power in the bargaining game. Note that this benefit of delegation will hold even if the bargaining power of the subsidiary is lower than that of the corporation, because even the worst partial delegation equilibrium payoff of the corporation is higher than the corporation's direct bargaining payoff.

A second way in which partial delegation could be more attractive is if the structure of the game gives the corporation more bargaining power in the selection of the equilibrium. For example, suppose that in the partial delegation game the corporation is able to move first and commit to the schedule it offers to the subsidiary. This would allow the corporation to earn the maximum profit by setting  $\mathbf{IP}(\vec{c}) = W(q^F(\vec{c}),\vec{c})$ , which would leave the supplier with zero surplus. This effect is in the spirit of Fershtmann and Judd (1987) because delegation allows the corporation to commit to a more aggressive play in interactions with the supplier.

# 4. Full Delegation.

To emphasize the importance of separating the output decisions from the information reporting, we now consider a two-stage game in which the corporation delegates both responsibilities to the subsidiary. We are assuming that the corporation truthfully reveals its private information to its subsidiary, but not to the supplier, so that the subsidiary knows the true value of c when it makes its output decision.<sup>18</sup> We maintain the same order of moves as in the partial delegation case. However, the difference in informational assumptions will change the form of the contracts that are offered by the corporation and the supplier.

In the first stage, the corporation and the supplier simultaneously offer non-linear contracts  $T^{c}(q,c)$  and  $T^{s}(q)$  that specify transfers to be made from the subsidiary to the corporation and the supplier

<sup>&</sup>lt;sup>18</sup>It turns out that in equilibrium the corporation will not have an incentive to misrepresent its information to the subsidiary.

as a function of the subsidiary's choice of q. The corporation's contract can be conditioned on the realization of c because it knows the value at the time of contracting. Since there is not a public report of the supplier's marginal cost in this case, the contract offered by the supplier can depend only on the quantity. As in the previous section, the analysis is limited to the case in which the contracts are differentiable. In the second stage, the subsidiary chooses q to maximize

$$a^{f}(q,c) \equiv V(q,c) - T^{c}(q,c) - T^{s}(q)$$
(27)

and makes the required transfers. We will use an "f" superscript, as above, to denote profits in this full delegation setting. We will refer to equilibria as "full delegation" equilibria.

Our main result in this section is to show that the full delegation game has a continuum of equilibria that include the incentive efficient equilibria identified in Proposition 1. In addition, we show that there are other inefficient equilibria to the full delegation game. In the full delegation game, the supplier does not know the true cost of the corporation and thus has an incentive to engage in rent extraction from the subsidiary. As a result, we obtain equilibria with quantity schedules that are not first best. This contrasts with the case of partial delegation, where the public report of the transfer cost created a full information problem for the supplier and eliminated the rent extraction incentive.

Given the differentiable contracts chosen by the corporation and the supplier, the subsidiary's choice of q at an interior solution will satisfy

$$V_q(q,c) - T_q^s(q) - T_q^c(q,c) = 0$$
 and  $a_{qq}^f(q(c),c) \le 0$ 

Let  $q^{f}(c) = argmax_{q} a^{f}(q,c)$  and let  $A(c) \equiv a^{f}(q^{f}(c),c)$ . Analogous to (3) and (4),  $q^{f}(c)$  must satisfy  $dA(c)/dc = -q^{f}(c) - T_{c}^{c}(q(c),c)$  and  $-(1 + T_{qc}^{c})q^{f}(c) \ge 0$ . (28)

Due to (28), the corporation's contract affects the subsidiary's incentive compatibility conditions in two ways. The first is through the relationship between the subsidiary's type and its net return. For instance, low cost types must earn lower profits than high cost types if the corporation chooses to extract significantly more rents from low cost types (i.e.,  $T_c^c(q^f(c),c) < V_c(q^f(c),c) = -q^f(c))$ ). The second effect is through the marginal valuation of q. If  $T_{qc}^c > -1$ , then a low cost subsidiary places a higher marginal valuation on q and q'(c) < 0 is required for implementation. However, if  $T_{qc}^c < -1$  the schedule must be non-decreasing in c.

In the first stage, corporation profit equals

$$\pi^{t}(q,c) = H(q) + T^{c}(q,c) = H(q) + V(q,c) - T^{s}(q) - a^{t}(q,c)$$
<sup>(29)</sup>

where the second equality follows by substitution using (27). Thus, the corporation's problem is to design  $T^{c}(q,c)$  so that the subsidiary chooses the value of q that maximizes (29) subject to the constraint

that  $a^{f}(q,c) \ge 0$ . The corporation's optimal choice of q will satisfy  $H'(q) + v_{q}(q,c) = T_{q}^{s}(q)$  and  $H''(q) + v_{qq}(q,c) - T_{qq}^{s}(q) \le 0$ , with  $T^{c}(\cdot,c)$  chosen such that A(c) = 0. Note that, from the second order conditions, the corporation's optimal quantity schedule will satisfy  $q'(c) \le 0$ . Combining the corporation's and the subsidiary's necessary conditions implies  $T_{q}^{c}(q(c),c) = -H'(q(c))$ . Consistent with this requirement, let  $q^{*}(\cdot) > 0$  denote the corporation's desired output schedule. We will investigate the existence of equilibria that generate  $q^{*}(\cdot)$  and in which the contract of the corporation takes the form

$$T^{c}(q,c) = \Pi'(c) - H(q).$$
<sup>(30)</sup>

The contract in (30) has three immediate implications. First,  $T_q^c(q^*(c),c) = -H'(q^*(c))$ . Second, since  $T_{qc}^c = 0$ , the subsidiary's second-order incentive compatibility condition implies  $q^{*'}(c) \le 0$ . Third, using (18) and the fact that subsidiary profit is zero with any corporation best response to  $T^s(\cdot)$ , (30) implies  $\Pi_c^f(c) = -q^*(c)$ .

Since the supplier does not know the corporation/subsidiary's type, the supplier chooses a contract  $T^{s}(q)$  to maximize  $\mathscr{C}_{c}[T^{s}(q^{f}(c)) - C(q^{f}(c))]$ . Using (27) and (30), the supplier's optimization problem can be expressed as the choice of a quantity schedule to maximize the difference between social surplus and the aggregate rents earned by the subsidiary and the corporation, subject to the incentive compatibility conditions in (28).

$$S^{f} = \max_{q(\cdot)} \mathscr{E}_{c}[W(q,c) - \Pi'(c) - A(c)]$$
s.t. a)  $A'(c) = q^{*}(c) - q(c)$ 
b)  $A(c) \ge 0$ 
c)  $q'(c) \le 0$ .
(31)

If we denote a solution to (31) by  $\hat{q}(\cdot)$ , then the supplier's best response will be the contract that induces the subsidiary to choose  $\hat{q}(\cdot)$ . Since  $\hat{q}(\cdot)$  is monotonic by (31c), the Taxation Principle allows us to write the supplier's best response to  $T^{c}(q,c)$  as

$$T^{s}(q) = V(q,\hat{q}^{-1}(q)) + H(q) - A(\bar{c}) - \prod^{c}(\bar{c}) - \int^{\bar{c}}_{t=\hat{q}^{-1}(q)} \hat{q}(t) dt$$
(32)

where the last two terms come from integrating constraint (31a). In equilibrium,  $\hat{q}(c)$  must equal  $q^*(c)$ .

**Proposition 5.** Let  $q^*(c)$  be a strictly decreasing, differentiable function satisfying<sup>19</sup>

$$W_{q}(q^{*},c) = \frac{F(c) - \beta^{*}(c)}{f(c)}$$
(33)

where  $\beta^*(c)$  is a non-decreasing function such that  $0 \le \beta^*(c) \le 1$ . For any

$$II^{f}(c) \in \left[0, \int_{c=\underline{c}}^{c} (W(q^{*}(c), c)f(c) - q^{*}(c)F(c))dc\right]$$
(34)

there will be a Nash equilibrium in non-linear contracts in which the payoff to a type c corporation from its subsidiary will be

$$H^{f}(c) = H^{f}(c) + \int_{t=c}^{c} q^{*}(t) dt.$$
(35)

The equilibrium schedules  $(T^{c}(q,c),T^{s}(q))$  are obtained by substituting (35) into (30) and by substituting  $q^{*}(c)$  for  $\hat{q}(c)$  in (32).

Proposition 5 provides a partial characterization of equilibria because we have restricted attention to non-linear corporation contracts of the form specified in (20). Corollary 6 shows that this set of equilibria is large enough to make important comparisons with Proposition 1.

*Corollary 6.* All fully separating incentive efficient bargaining allocations are delegation equilibrium allocations.

**Proof.** For those allocations defined by Proposition 1a, (12) implies  $\beta^*(c) = \hat{\alpha}(c)F(c)/(\gamma + \lambda^*)$ . For those allocations defined by Proposition 1b, (13) implies  $\beta^*(c) = \hat{\alpha}(c)F(c)/(\hat{\alpha}(c) + \lambda^{**})$ . Simple inspection of both cases reveals that  $\beta^*(\cdot)$  is non-negative, non-decreasing, and always less than or equal to one. *Q.E.D.* 

A few examples illustrate that the delegation game with this class of schedules can result in a range of quantity schedules including the first-best.

<sup>&</sup>lt;sup>19</sup>Solutions to (33) are robust to any changes in the definition of the corporation's direct and indirect benefits that do not change v(q,c) + H(q) for all q.

*Example 6.* Define  $\beta^*(c) = ((1-\gamma)/\gamma) \cdot F(c)$ . The solution defined by (33) corresponds to the incentive efficient allocation from Proposition 1a associated with  $\alpha(c) \equiv \alpha = 1 - \gamma \leq \frac{1}{2}$ .

*Example 7.* Define  $\beta^*(c) = x \in [0,1]$ . If we choose x = 0, then (33) implies  $q^*(c) = q^F(c)$  and  $q^*(c) < q^F(c)$  for c > c. This is the same quantity schedule as in Proposition 1a when  $\gamma=1$ . In contrast, if we choose x = 1, then  $q^*(c) = q^F(c)$  and  $q^*(c) > q^F(c)$  for  $c < c^{20}$ . This quantity schedule cannot be incentive efficient as it corresponds to  $\hat{\alpha}(c) \propto 1/F(c)$  which in turn requires  $\alpha(\cdot) = 0$ . From the first part of this example,  $\alpha(\cdot) = 0$  generates a very different incentive efficient quantity schedule.

*Example 8.* Define  $\beta^*(c) = F(c)$ . In this case, (33a) yields the first-best quantity schedule,  $q^F(c)$ .

Example 7 is significant because it establishes the existence of incentive inefficient equilibria. Together Corollary 6 and Example 7 also illustrates the role of commitment. By setting  $\mathbf{II}(c) = \int_{t=c}^{c} q^{F}(t)dt$ , the corporation imposes a strong individual rationality constraint on the subsidiary, a constraint the supplier must respect. Constraint (31a) now requires that  $A'(c) = q^{F}(c) - q(c)$ . If for some value of  $c^*$ ,  $A(c^*)=0$ , then (31a) and (31b) imply that, for c close to but below  $c^*$ , only schedules with  $q(c) \ge q^{F}(c)$  are feasible as lower quantities would imply negative subsidiary profit for those lower types.

Note that the benefits of delegation to the corporation will depend upon which equilibrium the principals coordinate. The structure of the contracts in (30) and (32) show that part of this coordination involves implicit agreement on the corporation's equilibrium rent. This was not the case under partial delegation as all differentiable equilibria induce first-best output. If full delegation does not change the corporation's bargaining power, then only changes in the timing of moves, (e.g. a Stackelberg game)) would enable the corporation to guarantee high equilibrium payoffs relative to direct bargaining. Finally, Laffont and Martimort (1998) point out that collusion between the subsidiary and the supplier might undermine the corporation's effort to increase its rents. This possibility is absent in our partial delegation game. However, even in our full delegation game this is not an issue. Any rents the subsidiary might earn from a side contract with the supplier cannot be retained in equilibrium because the corporation has full information. To be effective, incentive compatibility of the side-contract would require the supplier to pay the subsidiary information rents that the corporation cannot anticipate in

<sup>&</sup>lt;sup>20</sup>Standard hazard rate conditions on  $F(\cdot)$  exist that ensure  $q^*(\cdot)$  is strictly decreasing in these examples as well as examples in which only an interior type produces a first-best quantity.

equilibrium.<sup>21</sup>

#### 5. Concluding Remarks.

As an alternative to recent papers that focus on the potential for delegation to mitigate the impact of collusion among privately-informed agents, this paper considers the potential for delegation by an informed party in influencing bargaining outcomes. We believe our paper offers three new results. First, our characterization of the set of incentive efficient direct bargaining allocations extends the work of Spulber (1988). Our results cover a larger set of welfare weights and identify allocations that can involve over production as well as the standard underproduction. Second, our analysis of our full delegation game extends the work of Fershtman and Kalai (1997) to incorporate the effects of private information. We show that the commitment effect induced by the introduction of a delegate, allows an informed principal to induce favorable rent shifting. Third, our partial delegation game identifies what we believe is a new and intriguing property of delegation: partial delegation creates the opportunity for an informed principal to earn information rents without creating a quantity distortion.<sup>22</sup>

Our paper suggests a number of extensions. The literature on category management certainly suggests the need to include a moral hazard component to the agent's utility as well as private supplier information. Also, in our full delegation game, renegotiation between the corporation and the subsidiary after information is shared may give the corporation an incentive to strategically manage what it tells its subsidiary. Once the subsidiary learns the corporation's private information, it may have an incentive to use that information to bargain for positive surplus.<sup>23</sup> Obviously, one way to respond to this possibility is by only partially delegating. However, we do not at this time rule out alternative responses to agent

<sup>23</sup>We thank Matt Mitchell for this suggestion.

<sup>&</sup>lt;sup>21</sup>The way in which Laffont and Martimort (1998) introduce communication limits does not have any effect in our model. Thus, our argument is equivalent to their Theorem 1 asserting no equilibrium loss from side-contracting.

<sup>&</sup>lt;sup>22</sup>In an earlier version of this paper, we considered a partial equilibrium game in which the corporation and the supplier first simultaneously announce their contracts and then the corporation reports *c* to its subsidiary and  $\kappa$  to the supplier. In addition, we assumed that  $c \in \{L,H\}$  with L < H. The key property that persists with a continuum of possible type values is that the equilibrium quantity will be distorted away from the first-best level. Because the subsidiary's profit depends on *c* (as well as  $\kappa$ ), the supplier must pay the subsidiary an information rent. Even though this rent will be appropriated by the corporation, it will nonetheless introduce a quantity distortion.

renegotiation in a full delegation setting as there may be important economic settings in which partial delegation is not feasible. We hope to address these and other extensions in future work.

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# Appendix

Proof of Propositions 1a and 1b: Our proof consists of three steps. First, substituting (3) into (10) yields

$$\widehat{W} = (\widehat{\alpha}(\overrightarrow{c}) - \gamma) \prod(\overrightarrow{c}) + \int_{c=\underline{c}}^{\overline{c}} [(\widehat{\alpha}(c) - \gamma)q(c)F(c)/f(c) + \gamma W(q(c),c)]f(c)dc.$$
(A.1)

Second, constraint (6) is equivalent to

$$\int_{c=\underline{c}}^{\overline{c}} \left[ W(q(c),c) - (\Pi(c) - \Pi(\overline{c})) \right] dF(c) \ge \Pi(\overline{c}).$$
(A.2)

For any  $\Pi(\cdot)$  defined by (3), integrating the left-hand side of (A.2) by parts produces

$$W(q(\overline{c}),\overline{c}) - \int_{c=\varrho}^{\overline{c}} W_q(q(c),c)q'(c)F(c)dc.$$
(A.3)

Since

$$W(q(\bar{c}),\bar{c}) = \int_{c=c}^{\bar{c}} dW(q(c),c)F(c),$$

(A.2) is equivalent to

$$\mathbf{II}(\vec{c}) \leq \int_{c=c}^{c} [W(q(c),c)f(c) + W_{c}(q(c),c)F(c)]dc.$$
(A.4)

Since  $\Pi(\overline{c}) \ge 0$ , we must also ensure that

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$$\int_{c=\underline{c}}^{\overline{c}} [W(q(c),c)f(c) + W_c(q(c),c)F(c)]dc \ge 0.$$
(A.5)

As a result, incentive efficient bargaining allocations can be derived by choosing  $q(\cdot)$  and  $\Pi(\vec{c})$  to maximize (A.1) subject to (A.4),  $\Pi(\vec{c}) \ge 0$ , (A.5), and  $q'(\cdot) \le 0$ .

As is common in the literature, the third step in the proof is to solve the optimization problem without the last constraint and then verify that it is satisfied. From this point on, we will also use Assumption (a) and replace  $W_c(q(c),c)$  with -q(c).

**Case 1:**  $\hat{\alpha}(\vec{c}) \leq \gamma$ . Given this assumption,  $\Pi(\vec{c}) = 0$  when  $\hat{\alpha}(\vec{c}) < \gamma$  and

$$\mathbf{I}(\overline{c}) \in [0, \int_{c=\underline{c}}^{c} [W(q(c),c)f(c) - q(c)F(c)]dc] \text{ when } \hat{\alpha}(\overline{c}) = \gamma. \text{ In either case, the Lagrangian is}$$

$$\mathcal{L} = (\hat{\alpha}(c) - \gamma + \lambda)(F(c)/f(c))q(c) + (\gamma + \lambda)W(q(c),c)$$
(A.6)

where  $\lambda \ge 0$  is the multiplier associated with (A.5). It is strictly concave in  $q(\cdot)$ . Define the quantity schedule  $q^{B}(c,\lambda)$  from the first-order condition with respect to  $q(\cdot)$  from (A.6). Thus,  $q^{B}(c,\lambda)$  is the solution to

$$W_q(q(c),c) = \left(1 - \frac{\hat{\alpha}(c)}{\gamma + \lambda}\right) \frac{F(c)}{f(c)}.$$
(A.7)

Define  $\lambda^*$  to be the smallest value of  $\lambda \ge 0$  such that (A.5) is satisfied by  $q^*(\cdot,\lambda)$ . Because the integrand in (A.5) is continuous in  $q(\cdot)$ ,  $\lambda^*$  is well-defined. If  $\partial q^*(c,\lambda^*)/\partial c \le 0$ , then  $q^B(\cdot) = q^*(\cdot,\lambda^*)$  is the incentive efficient quantity schedule. Since W(q,c) is strictly concave, totally differentiating (A.7) implies  $\partial q^*(c,\lambda^*)/\partial c \le 0$  if, and only if,

$$\left(\left(1-\frac{\hat{\alpha}(c)}{\gamma+\lambda^*}\right)\frac{F(c)}{f(c)}\right)'+1\geq 0.$$
(A.8)

Case 2:  $\hat{\alpha}(\vec{c}) > \gamma$ . For this case, (A.4) will bind,

$$\hat{W} = \int_{c=\underline{c}}^{\overline{c}} [\hat{\alpha}(\overline{c})W(q(c),c) + (\hat{\alpha}(c) - \hat{\alpha}(\overline{c}))q(c)F(c)/f(c)]f(c)dc, \qquad (A.9)$$

and

$$\mathcal{L} = (\hat{\alpha}(c) - \hat{\alpha}(\overline{c}) - \lambda)q(c)F(c)/f(c) + (\hat{\alpha}(\overline{c}) + \lambda)W(q(c),c).$$
(A.10)

Again,  $\mathcal{D}$  is strictly concave in  $q(\cdot)$ . Define  $q^{**}(\cdot,\lambda)$  from the first-order condition with respect to  $q(\cdot)$  from (A.10). Thus,  $q^{**}(\cdot,\lambda)$  is the solution to

$$W_q(q(c),c) = \left(1 - \frac{\hat{\alpha}(c)}{\hat{\alpha}(\bar{c}) + \lambda}\right) \frac{F(c)}{f(c)}.$$
(A.11)

Define  $\lambda^{**}$  to be the smallest value of  $\lambda \ge 0$  such that (A.5) is satisfied by  $q^{**}(\cdot,\lambda)$ . As with  $\lambda^*$  above,  $\lambda^{**}$  is well-defined. If  $\partial q^{**}(c,\lambda^{**})/\partial c \le 0$ , then  $q^B(\cdot) = q^{**}(\cdot,\lambda^{**})$  is the incentive efficient quantity schedule for this case. Totally differentiating (A.11) shows that  $\partial q^{**}(c,\lambda^{**})/\partial c \le 0$  if, and only if,

$$\left(\left(1-\frac{\hat{\alpha}(c)}{\hat{\alpha}(\bar{c})+\lambda^{**}}\right)\frac{F(c)}{f(c)}\right)'+1\geq 0.$$
(A.12)

*Q.E.D.* 

**Proof of Proposition 5:** In contrast to the standard principal agent problem in which (31b) binds only for the least cost type, the solution to the corporation's problem dictates that in equilibrium A(c) will equal zero for all c and thus can bind on open sets of types. Therefore, we formulate the supplier's problem as an optimal control problem with a pure state constraint. Following Seierstad and Sydsaeter, we form the Hamiltonian

 $\mathfrak{H}(q,A,\mu,c) = (W(q,c) - \Pi(c) - A(c))f(c) + \mu(c)[q^{*}(c) - q(c)]$ 

and the associated Lagrangian

$$\mathfrak{L}(q,\mathcal{A},\boldsymbol{\mu},\boldsymbol{\lambda},c) = \mathfrak{H}(q,\mathcal{A},\boldsymbol{\mu},c) + \lambda(c)A.$$

Theorem 5.1 in Seierstad and Sydsaeter shows that  $(q^*(c), A(c) = 0)$  will maximize (31) if there exists a piecewise continuous function  $\lambda(c) \ge 0$  and a continuous and piecewise continuously differentiable function  $\mu(c)$  satisfying the following conditions: (a)  $W_q(q^*,c)f(c) = \mu(c)$ , (b)  $\dot{\mu}(c) = f(c) - \lambda(c)$ , and (c)  $\mu(c) \le 0$ ,  $\mu(c) \ge 0$ . Condition (a) establishes that the quantities will depart from the first best if  $\mu(c) \ne 0$ . This will occur if it is optimal for the supplier to distort the quantity schedule in order to reduce the information rents it must pay the subsidiary.

Consider a candidate equilibrium quantity schedule  $q^*(c)$  and corporation payoff schedule  $\mathbf{II}^{f*}(c) \ge \mathbf{0}$ , which can be substituted into (30) and (32) to yield candidate contracts  $T^{c*}(q,c)$  and  $T^{s*}(q)$ . These contracts define an equilibrium if (i)  $q^*(c)$  solves (27) given  $T^{c*}(q,c)$  and  $T^{s*}(q)$ , (ii) A(c) = 0 for all c, (iii)  $q^*(c)$  solves (29) subject to  $\mathbf{II}(c) \ge \mathbf{0}$ , for all c, and given  $T^{s*}(q)$ , and (iv)  $q^*(c)$  solves (31) given  $T^{c*}(q,c)$ .

For  $q^*(c)$  strictly decreasing and differentiable, all four conditions are satisfied. (i): Given  $(T^{c*}(q,c),T^{s*}(q))$  and  $(q^*)'(c) < 0$ ,  $V_{qc} = -1$  implies that  $a^f(q,c)$  is concave in q and maximized at  $q^*(c)$ . (ii) and (iii): Straightforward calculations show that  $(T^{c*}(q,c),T^{s*}(q))$  implies  $A^f(c) \equiv 0$  and  $q^*(c) \in argmax_q \pi^f(q,c)$  s.t.  $A^f(c) \ge 0$ . (iv): Define  $\beta^*(c) = \int_{-\infty}^{c} \lambda(t) dt + \beta^*(c)$ . Since  $\lambda(t) \ge 0$ ,  $\beta^*(\cdot)$  is non-decreasing. Using condition (b) from above, this definition also implies that  $\mu(c) = F(c) - \beta^*(c) + k$ . Without loss of generality, we can set k = 0. Condition (c) from above will then be satisfied as long as  $0 \le \beta^*(c) \le 1$ . Appealing to Seierstad and Sydsæter (1987, Chapter 5, Theorem 3), any such function  $\beta^*(\cdot)$ yields a solution to (33),  $q^*(c)$ , that solves (31). Condition (34) guarantees that  $S^f \ge 0$ . Q.E.D.