A Life Cycle Model with Housing, Portfolio Allocation, and Mortgage Financing.*

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Abstract

This paper presents a model developed to explain the life-cycle patterns in both homeownership and portfolio allocation, and the relationship between them, using a model of rational agents. Two key innovations are incorporated into this model. First, housing is explicitly modeled as both a consumption and investment good, as opposed to examining just one aspect in isolation from the other. Second, traditional mortgage contracts are also explicitly introduced into the model. A finite horizon life-cycle model including both of these innovations is then solved and calibrated using data from the Health and Retirement Survey. As a result, the model proves that the "over-investment" of housing is not inconsistent with the behavior of rational, forward-looking agents. The model is also able to capture the negative correlation between the housing share and risky asset share of portfolio. The results show how the desire to consume large homes contributes both to an increase in the portfolio share of housing and a decrease in the portfolio share of risky assets. This model demonstrates the close link between the housing "over-investment" and the stock "under-investment" puzzles.

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1. Introduction

This paper develops a model to explain the life-cycle patterns in both homeownership and portfolio allocation using a model of rational agents. Two key innovations are incorporated into this model. First, housing is explicitly modeled as both a consumption and investment good, as opposed to examining just one aspect in isolation from the other. Second, traditional mortgage contracts are explicitly introduced into the model. A finite horizon life-cycle model is solved and calibrated using data from the Health and Retirement Survey (HRS). The calibrated model is then used to explore a range of possible causes for the large share of household portfolios held in owner-occupied homes.

The purchase of a home is one of the most significant and complex economic transactions in which the average household is ever involved. For most households, the equity in their home makes up the majority, if not almost all, of their wealth.[1] This lack of portfolio diversification or “over-investment” in housing, despite the presence of such instruments as home equity loans and cash-out refinancing, poses a significant empirical puzzle. The unique role and nature of housing in a household's economic decision-making may explain the persistence of this puzzle. A unique aspect of owner-occupied housing is its dual role as a consumption good as well as an investment vehicle. Financial assets are fairly homogenous, can be bought and sold incrementally, and with relatively low transaction costs. However, owner-occupied housing can only be bought and sold in large non-divisible chunks (i.e. a house) and only after the household has gone through a long and costly transaction process. Another difference is the large amount of heterogeneity across owner-occupied housing, both in terms of location, amenities, and rates of appreciation. Finally, the purchase of a home is often the most significant example of a household borrowing against its future income, and thus relaxing a binding liquidity constraint. Both the institutional characteristics of the mortgage industry, in particular the downpayment, income, and credit constraints it imposes, and the tax treatment of mortgage interest and realized capital gains from home sales also play a key role in influencing a household's decision as to when, where, and how much house to buy.

The impact of housing on the allocation of total household wealth between real and financial assets is not the end of the story. Recent work by Flavin and Yashmita (2002) has demonstrated that the life-cycle path of ownership in equities can be explained in part by the effect of housing. The authors argue that young homeowners "over-invest" in owner-occupied housing. That is, the amount of housing purchased may be optimal for the consumption decision, but not in terms of portfolio diversification. Due to this lack of diversification, the young homeowners then prefer less risky liquid assets, until their overall portfolio has grown to a point where they are no longer "over-investing" in owner-occupied housing. As a result, the shares of stocks and bonds held exhibit a life-cycle pattern. This work indicates that housing may not only

[1] Flavin and Yamashita (2002) calculate from the Panel Study of Income Dynamics that among households that are homeowners, and with a head between 18 and 30 years old, 67.8% of their portfolio is in their home. The authors find that this ratio is still over 60% for households with a head over 71 years old.

result in a lack of diversification between real and financial assets, but also distorts the portfolio allocation decision between different types of financial assets.

The goal of this paper is to examine first solve and calibrate a model that replicates the large share of household portfolios held in owner-occupied homes. The calibrated model is then used to examine the relationship between the “over-investment” in housing and the composition of the financial portfolio. A companion paper, Nichols (2003), uses a version of the model containing only a risk free financial asset to explore possible explanations of the "over-investment" in housing. The focus of this paper is to take the "over-investment" in housing as a given and explore the effect of this decision on the financial portfolio allocation of the household.

In the model, households chose their current consumption, their savings, the share of their savings allocated to risky assets, and which type of housing to occupy. The housing tenure choice includes a rental unit, a small home, and a large home. Large homes cost more and provide greater utility than small homes, which in turn provide greater utility than a rental unit. Households face uncertainty in the return on risky assets and the different types of homes, the probability of survival, and a transitory shock to income, which is otherwise a deterministic function of age. The model includes moving, maintenance, and transaction costs. Both the ability to and the costs of defaulting on a mortgage are also included in the model.

The model is solved given the terms of a traditional mortgage contract. The value of non-structural parameters, such as returns on different types of assets, the survival probability, mortgage terms, and income process are taken from historical data. Values of structural parameters are chosen based on a calibration of the model results using data from the Health and Retirement Survey (HRS). One goal of the model is to replicate the negative relationship between the portfolio share of housing and of risky assets observed by Flavin and Yamashita (2002). The relationship between the "over-investment" in housing and the financial portfolio allocation decision is explored by changing the risk structure, preferences, tax structure, or mortgage contract. Under each alternative set of assumptions, the model is resolved and new sets of simulations are generated. The goal of this exercise is to demonstrate how the relationship between the share of wealth held in owner-occupied housing and in risky financial assets is effected by the role of housing as a consumption good, the tax treatment of housing, and the role of the mortgage contract.

The remaining of this section lays out the structure of the paper. Section 2 contains a review of both the relevant housing and macroeconomic literature. Special attention is given to the work on the interaction between the housing tenure decision and portfolio allocation. Section 3 describes the model, including the decisions facing the household, the sources of uncertainty, and the budget constraints of the household. Section 4 describes the calibration approach and provides the resulting values of the structural parameters. In Section 6 alternate assumptions are used to explore how the unique role and nature of the housing asset impact the portfolio allocation decision. Section 7 concludes the paper.

2. Literature Review

Previous work on this question tended to focus on different factors behind housing demand in isolation. Often housing was treated as either an investment good or a consumption good. The models that explicitly captured housing's dual role as an investment good and a consumption good did include mortgage financing. The contribution of the current paper is to model housing as both an investment good and a consumption while explicitly including mortgage financing. The resulting model can then be used to explore all the possible explanations for the "over-investment" in housing previously discussed in the literature.

An early and significant paper on tenure choice and the life cycle was Henderson and Ioannides (1989). In this paper the authors estimated models of joint tenure, length of stay, and consumption level of families in the housing market. They conceded the need for a dynamic framework for modeling housing tenure choice, but argued that such a problem is too complicated for closed form solutions to be found analytically. Instead they estimated several reduced-form models of tenure choice, length of stay, and level of housing consumption using PSID data from 1971-1981. They found that differences in tenure choice could be largely explained by varying stages of the life cycle. This paper offers one explanation for the "over-investment" in housing. Households that wish to consume more owner-occupied housing than they wish to invest in are consumption constrained and hence over-invest. The effects of consumption constraints will be one of the possible explanations explored in the current paper.

A range of other work has also studied the housing tenure decision from a variety of angles. Haurin (1991) explored the interaction of income volatility and home-ownership. The author estimated a reduced-form model using data from the National Longitudinal Study of Youth (NLSY) to show that households with less volatile income were more likely to own, reflecting their reduced probability of defaulting on their mortgage. LaFayette, Haurin, and Hendershott (1995) showed that households might choose different mortgage products in an effort to relax liquidity constraints and purchase a home. Hoyt and Rosenthal (1990) explored the effect of the treatment of capital gains from selling owner-occupied housing on the demand for housing. The effect of changes in tax policy will be another possible explanations explored in the current paper.

Shiller and Weiss (1998) explored the possibility of using alternative mortgage contracts to reduce the diversifiable risk born by homeowners. They assessed the moral hazard associated with a range of alternative mortgage contracts, including; (1) reverse mortgages, (2) home equity insurance, (3) shared appreciation mortgages, (4) housing partnerships, (5) shared equity mortgages (SEM), and (6) sale of remainder interest. Another possible explanation for the "over-investment" in housing is the leveraging benefit associated with a mortgage. This benefit is absent for SEM. The effects of the leveraging benefit will be explored in the current paper through the introduction of a SEM.

In Fernández-Villaverde and Krueger (2001) the authors observed that young consumers have portfolios with little liquid assets but a significant amount invested in durables.

The authors hypothesized that young consumers can only borrow against future income by using their durable assets as collateral for loans. They then developed a structural life-cycle model with endogenous borrowing constraints and interest rates. They did not explicitly model other types of liquidity constraints or directly address the issue of tenure choice. Martin (2001) argued that consumers have an inaction region in the purchase of durable goods caused by transaction costs. Martin then argued that the inaction region in durable goods induces variation in the consumption of non-durable goods. Below we discuss a paper, Grossman and Larqoue (1990), which made a similar argument about the effect of the inaction region on portfolio allocation. The role of transaction costs in the share of wealth held in housing will be explored in this paper's model.

Hurst and Stafford (2002) examined the use of housing equity to smooth consumption. Specifically, they explored refinancing behavior in a life-cycle model. The authors both solved a structural version of their model and estimated a reduced-form version using PSID data. However, they did not include renting as an alternative in the model or explicitly included realistic liquidity constraints. The ability to refinance is not currently included as an option in the current paper, but presents a likely extension.

There has been a set of papers that explicitly explores the link between the housing "over-investment" puzzle and the stock "under-investment", or equity premium, puzzle. For the most part these papers take the housing "over-investment" puzzle, and use it to explain the equity premium puzzle. The major paper in this branch of research is Flavin and Yamashita (2002). Their hypothesis was that young homeowners "over-invest" in owner-occupied housing from the point of view of the optimally diversified portfolio. They then prefer less risky liquid assets, until their overall portfolio has grown to a point where they are not "over-invested" in owner-occupied housing. As a result the shares of stocks and bonds exhibit a life-cycle pattern. The authors used data from the PSID to show that among households where the age of the head was between 18 and 30, the portfolio share for home equity was 67.8% while for stocks it was 5.6%. In comparison, the same ratios for households where the age of the head was between 51 and 60 were 58.6% and 11.3% respectively.

Fratantoni (1997) took a similar approach to explain the equity premium puzzle. The author solved a finite-horizon model with exogenous housing consumption and showed that the introduction of housing in the model reduced the share of risky assets held by households. The author then calibrated his model using Survey of Consumer Finance (SCF) data. In neither of the two previous papers did the authors include endogenous tenure choice. Both of the previous papers took the amount of housing consumed as a given, an assumption that was relaxed in the next paper. Hu (2002) developed a similar model where housing is endogenous. The author solved a finite-horizon model that allowed for households to hold a risk free asset, a risky asset, or risky owner-occupied housing. The approach taken in Hu's paper is used as the basis for much of the work in this paper. As does Hu, this paper allows for income variability but imposes a life-cycle pattern to the income process. While the current paper is focused on the housing "over-investment" puzzle, it also attempts to replicate the findings of these papers on the connection between "over-investment" in housing and "under-investment" in stocks.

The paper that is the main source of the methodology used in this project is Rust and Phelan (1996). In this paper the authors set up and solved a dynamic programming problem of labor supply with incomplete markets, Social Security, and Medicare. They then estimated their structural parameters using data from the Retirement History Survey (RHS). The dynamic programming problem in this paper is solved by discretizing the continuous state spaces and then using backward recursion to solve for the optimal value of the continuous choice variable at each point on the state space grid. When the optimal value of the continuous choice variable takes the problem off the state space grid, numerical interpolation is used. Numerical integration over the sources of uncertainty is used to find the expected value function for the next period. The detailed rules governing the Social Security and Medicare application processes and benefits are imbedded in the income transition matrix. The model developed in this paper has a similar structure, but instead imbeds the detailed characteristics of the mortgage market contract in the income transition matrix.

3. Model

This section defines the structure of the finite-horizon life-cycle model used to describe a household's savings, investment, and housing decisions, and is structured as follows: (1) a description of the household's preferences; (2) the household's endowments; (3) the savings, mortgage, and investment technology facing the household; (4) and finally the budget constraints imposed by the savings, mortgage, and investment technology. The section concludes with a discussion of the method used to solve the household's optimization problem.

Households receive utility from the consumption of both a non-durable good and the stock of housing that they own. Their optimization problem is to maximize their lifetime utility, defined as;

$$E\sum_{t=20}^{80}\beta^{t} * surv_{t} * U(c_{t,} h_{i_C}, mv) + \beta^{t} * (1 - surv_{t}) * \theta * U_{B}(b_{t}) \quad (3.1)$$
$$c_{t} > 0, \forall t$$

$$U(c_t, h_{ic}) = \frac{(c_t^{\phi} h_{ic}^{1-\phi} mv)^{1-\lambda}}{1-\lambda} \quad (3.2)$$

$$U_B(b_t) = \frac{b_t^{1-\lambda}}{1-\lambda} \quad (3.3),$$

where c_t represents the consumption of non-durables, h_{ic} represents the utility associated with the housing tenure choice i_c, and b_t is the combined value of financial and housing wealth left as a bequest. The parameter β represents the discount rate, $surv_t$ represents the probability of surviving one more period, θ represents a bequest parameter, ϕ represents the measure of preference between of housing and consumption, and λ represents a measure of risk aversion. A household lives at most 80 years and face uncertainty on their own survival, temporary income shocks, and the return on both housing and risky assets. Households are also not allowed to consume negative amounts of non-durable goods.

In the model the consumption of the non-durable good is continuous, but the choices for housing consumption are discrete. Three different alternatives for housing are presented in the model: a rental unit, a small home, and a large home, $i_c \in \{i_r = rent, i_s = small, i_l = l\arg e\}$. Many other factors in the model are conditional on current housing tenure, including rent or mortgage payments, maintenance costs, level of utility derived from housing, and the rate of appreciation in home value. One difference between this model and many other models of housing demand is the incorporation of discrete housing choice. Many models of housing allow agents to purchase whatever number desired of homogenous housing units, for a given price per unit. This more abstract approach is appropriate from the theoretical viewpoint. However, one of the goals of this paper is to model the demand for the different types of owner-occupied housing. Therefore, only a discrete number of housing tenure choices are used, with each assigned a given level of utility associated with that certain housing tenure choice. The values of these structural parameters are among those chosen in the calibration described in the next section.

A household is "born" at age 20 with zero financial and housing wealth. They start off as renters with no savings. In each period they receive a draw from an age-dependent income process. Many papers on this topic model income as consisting of a persistent and transient component. The persistent component represents the history of permanent income shocks and is included as another state variable. In this paper, there is no permanent income shock, only transitory shocks. The income process is defined as a deterministic function of age, adjusted by some percent up or down due to a transitory shock, as shown below in log form;

$$\log(w_t) = \psi_0 + \psi_1 * t + \psi_2 * t^2 + \sigma_i \quad (3.4).$$

In retirement the level of income is set to 60% the level of income in last year of working, prior to any adjustment for the transitory shock experienced in that year. Income during retirement is still subject to transitory shocks.

Households can store their wealth in two different classes of assets, financial and real. The household's financial assets are held in a portfolio of risk free and risky assets. The household can, at no cost, rebalance their financial portfolio between risk free and risky assets every period. Household's facing binding liquidity constraint for financial assets, that is they cannot borrow against their future income. Household also cannot purchase leveraged portfolios, where they borrow the risk free asset to invest in the risky asset. The transition rule for the level of financial wealth is defined as;

$$\begin{aligned} A_{t+1} &= E((1-tax)*(\alpha r_s + (1-\alpha)r) + 1)*(A_t - c_t - x_t(i_c, age_t)) + (1-tax)*w_t \\ &+ tax * mipay_t(i_c, age_t)) + (1 + (1-tax)*(\alpha r_s + (1-\alpha)r)) * gain_t \end{aligned} \quad (3.5)$$

$$A_{t+1} \geq 0 \quad (3.6)$$

$$0 \leq \alpha \leq 1 \quad (3.7)$$

$$r_s = \overline{r_s} + \sigma_s \quad (3.8),$$

where $\alpha$is the share invest in risky assets, $\overline{r_s}$ is expected return on risky assets, σ_s is the standard deviation of that return, *r* is the return on risk-free assets, W_t is the level of financial assets in period t, w_t is the wage received in period t, x_t is the housing costs incurred in period t, $gain_t$ is the net gain any housing transaction taken in period t, *tax* is the tax rate on income and capital gains, and $mipay_t$ is the mortgage interest paid in period t.. Note that the net gain from home sale is tax-free and that the mortgage interest paid is deducted from taxable income. Both the housing expenses and the amount of the mortgage interest deduction are functions of the current housing choice and age of mortgage. The age of a mortgage for a rental unit or a mortgage older than the term in the mortgage contract is defined by default to be 0. The timing is such that households receive their wages at the same time they realize the returns on their investment from the previous period. The results in the state variable A_t representing all available cash on hand, consisting of previous financial wealth and current income.

Households can also store their wealth in real assets through purchasing a house. It is only through the purchase of a house, and the acquisition of a mortgage loan, that households can borrow against their future income. This aspect of the model reflects the idea from Fernández-Villaverde and Krueger of durable goods functioning as collateral. The only mortgage contract available to the household in this model requires a 20% down payment, has a term of 30 years, and requires mortgage payments based on a fixed interest rate and the size of the original mortgage. Households are also required to pay a transaction cost equal to 10% of the value of the home that they are purchasing. This represents realtors' fees, credit checks, and other expenses associated with the purchase. The seller of the home pays half of these costs and the buyer pays the other half.

The real price of housing has a positive trend over the life cycle. The purchase price of either a small or large home increases non-stochastically by the average market price increase in each period. The value of homes that have already been purchased changes based on a stochastic process, with the expected increase equal to the non-stochastic market price increase. A household who has had a series of excellent draws in home price appreciation will own a home worth relatively more than a comparable home on the market. A household who has had a series of poor draws in home price appreciation will own a home worth relatively less than a comparable home on the market. The formulas for the home price appreciation are as follows;

$$\begin{aligned} P_t^{i_c} &= \overline{r_h^t} P_o^{i_c}, i_c \in \{i_s, i_l\} \\ P_t^{i_c} &= 0, i_c = i_r \end{aligned} \qquad (3.9)$$

$$\begin{aligned} H_{t+1}^{i_c} &= Er_h H_t^{i_c}, i_c \in \{i_s, i_l\} \\ H_{t+1}^{i_c} &= 0, i_c = i_r \end{aligned} \qquad (3.10)$$

$$r_h = \bar{r}_h + \sigma_h \qquad (3.11)$$

where $P_t^{i_c}$ is the market price of a house of type i_c in period t and $H_{t+1}^{i_c}$ is the value of a house of type i_c in period t+1 which was worth $H_t^{i_c}$ in period t. The average return on housing is $\overline{r_h}$ and the standard deviation on that return is σ_h.

The mortgage payment is based on the home price when purchased, and does not change over the life of the mortgage. The formula for a mortgage payment at time *t* after age_t years on a house of type i_c is;

$$paym(i_c, t, age_t) = \frac{(mrat) * (1 - down) * P_{t-age_t}^{i_c}}{(1 - mrat)^{-term}} \qquad (3.12),$$

where *mrat* is the mortgage interest rate, down is the requires down payment, and *term* is the term of the mortgage. Rent is defined to always be equal to 80% of the mortgage payment on a small house purchased in that year. This captures the ability of households to lock in their housing payments at a fixed level and avoid future real increases in the cost of housing services. The cost of housing services also reflects the maintenance costs paid by homeowners. As a result, the formula for the cost of housing services is;

$$x_t(i_c, age_t) = paym(i_c, t, age_t) + main * H_t^{i_c}, i_c \in \{i_s, i_l\}$$
$$x_t(i_r, age_t) = 0.8 * paym(i_s, t, 0) \quad (3.13),$$

where *main* is the percent of current home value required in maintenance costs.

The present value of the household's home equity is the current value of the house minus the amount of the outstanding mortgage balance. While the value of the house increases or decreases according to the stochastic return on housing, the outstanding mortgage balance is a monotonically declining function of the age of the mortgage. The formula for the mortgage balance at time *t* after age_t years on a house of type i_c is;

$$mort_t(i_c, age_t) = paym(i_c, t, age_t) * \frac{1-(1+mrat)^{age_t - term}}{mrat}, i_c \in \{i_s, i_l\}, age_t \le term$$
$$mort_t(i_c, age_t) = 0, i_c \in \{i_s, i_l\}, age_t > term \quad (3.14).$$
$$mort_t(i_r, age_t) = 0$$

The formulas for the remaining mortgage balance and the mortgage payment are used to calculate the amount of mortgage interest paid for tax purposes. The formula for the mortgage interest deduction is;

$$mrpay_t(i_c, age_t) = paym(i_c, t, age_t) - (mort_t(i_c, age_t) - mort_{t+1}(i_c, age_t + 1)) \quad (3.15)$$

The net gain, after paying transaction costs and down payments, for a household moving from housing type i_c to housing type $i_m \in \{i_r = rent, i_s = small, i_l = l\arg e, i_n = stay\}$ is;

$$gain_t = H_t^{i_c} - mort_t(i_c, age_t) - down * P_t^{i_m} -$$
$$0.5 * trans * P_t^{i_m} - 0.5 * trans * H_t^{i_c} - move, i_m \in \{i_r, i_s, i_l\} \quad (3.16),$$
$$gain_t = 0, i_m = i_n$$

where *move* is a fixed moving cost paid regardless of which type of housing is being purchased. When the household choose not to move, i_m=i_n, there is zero net gain.

The model also contains a default penalty. In any period the household must be able to cover their housing expenses. If they fail to do so, they must move the next period in to rental housing, forfeiting all their home equity and all their financial equity above some small nominal amount. Households who can cover their expenses by selling their current house and extracting their home equity are allowed to do so, however they still must move into rental housing for at least one period. The advantage of this for the household is the ability to keep their housing equity. Their current consumption is also constrained to that same small nominal amount. The default constraints are;

$$(A_t - x_t < 0) and (A_t - x_t + gain_t > 0) \Rightarrow i_m \ne i_n \quad (3.17)$$

$$(A_t - x_t < 0) and (A_t - x_t + gain_t < 0) \Rightarrow \{i_m = i_r, c_t = \underline{c}, A_{t+1} = \underline{c}\} \quad (3.18),$$

where $\underline{c}$ is the amount of consumption and wealth protected from creditors in default. The first constraint affects those households who are forced to move, but can avoid defaulting and the second constraint affects those households that default. The restriction that A_{t+1} may not be negative, with the definitions of x_t and $gain_t$, and the budget constraint create an upper bound on possible levels of non-durable consumption. It also rules out some possible changes in housing tenure. If the household cannot afford the down payment for a large home without incurring negative wealth, they are not allowed to move to such a home.

Table 3.1 Choice and State Variables

Variable Name	Symbol	Definition
State Variables		
Age	t	Increments of one year, 20 to 80
Financial Assets	A_t	Continuous, discretized
House Value	H_t	Continuous, discretized
Age of Mortgage	age_t	Increments of one year, 0 to 30
Current Tenure	i_c	i_r = renter, i_s = small home, i_l = large home
Choice Variables		
Consumption	c_t	Continuous
Share of Financial Assets Invested in Risky Assets	α	Continuous, discretized (0% to 100% in increments of 10%)
New Tenure	i_m	i_r = renter, i_s = small home, i_l = large home, i_n = stay in current home

The household's problem is to choose a series of choice variables $\{c_t, \alpha, i_m\}$ over a series of state variables $\{t, age_t, i_c, A_t, H_t\}$ to optimize equation (3.1) given equations (3.2) to (3.18). Table 3.1 below summarizes the different state and choice variables. Note that the share of financial assets invested in risky assets has been converted into 5 discrete values.

Table 3.2 above summarizes the structural and non-structural parameters, many of which have already been discussed. In the calibration of the model, the values of the non-structural parameters are derived from a wide range of historical data sources. The values of the structural parameters are generated through the a calibration with data from the Health and Retirement Survey.

The value function of the household is the maximum, subject to the default constraints of the value functions for the households who decide to move to tenure type $i_m \in \{i_r = rent, i_s = small, i_l = large, i_n = stay\}$, V^{i_m}.

$$\begin{aligned}&(A_t - x_t < 0) and (A_t - x_t + gain_t > 0) \Rightarrow \\ &V_t(i_c, A_t, H_t, age_t) = \max_{i_m, c_t, \alpha, (i_m \neq i_n)} \{V_t^{i_m}(i_m, A_t, H_t, age_t)\}\end{aligned} \quad (3.19)$$

$$(A_t - x_t < 0) and (A_t - x_t + gain_t < 0) \Rightarrow$$
$$V_t(i_c, A_t, H_t, age_t) = U(\underline{c}, h_{i_c}) + \beta * surv_t * V_t(i_r, \underline{c}, 0, 0) + \beta * (1 - surv_t) * U_B(\underline{c}) \quad (3.20)$$

$$(A_t - x_t > 0) \Rightarrow$$
$$V_t(i_c, A_t, H_t, age_t) = \max_{i_m, c_t, \alpha}\{V_t^{i_m}(i_m, A_t, H_t, age_t)\} \quad (3.21),$$

where the three different possible default states result in three different definitions of the value function.

Table 3.2 Structural and Non-Structural Parameters

Parameter Name and Definition	**Symbol**
Structural Parameters	
Coefficient of relative risk aversion	λ
Discount rate	β
Measure of preference between of housing and consumption	ϕ
Disutility associated with moving	*mv*
Utility associated with tenure choice i_c	h_{ic}
Bequest parameter	θ
Non-Structural Parameters	
Return on risk free assets	*r*
Return and standard deviation on risky assets (stocks)	$\overline{r_s}, \sigma_{s_c}$
Return and standard deviation on housing	$\overline{r_h}, \sigma_h$
Initial price of home type i_c	$P_o^{i_c}$
Mortgage interest rate	*mrat*
Percent required as downpayment	*down*
Percent of home price lost to transaction costs	*tran*
Percent of home price paid in maintenance costs	*main*
Tax rate on income and capital gain	*tax*
Moving costs	*move*
Wage coefficients	$\psi_0, \psi_1, \psi_2, \sigma_i$
Probability of survival at age t*	$surv_t$

The value function conditional on choosing new tenure choice i_m is;

$$V_t^{i_m}(i_c, A_t, H_t, age_t) = \max_{c_t, \alpha}\{U(c_t, h_{i_c}) + \beta * surv_t * V_t(i_m, A_{t+1}, P_{t+1}, 1) +$$
$$\beta * (1 - surv_t) * U_B(A_{t+1} + P_{t+1} - mort_t(i_m, 1))\} \quad (3.22)$$

such that equations (3.2) to (3.18) hold.

The structure of this problem contains several significant sources of non-continuity. The first of course is the discrete nature of housing tenure, which functions as both a choice and a state variable. The second main source is the structure of the value function, which is defined as the maximum over forty-four different value functions, one for each possible combination of four tenure choices and eleven portfolio allocations. This non-continuity of the model prevents the use of analytical methods to derive a solution. It also prevents the derivation of Euler equations.

The model is solve by discretized the two continuous state spaces into grids with thirty value for financial wealth plus income and ten values for home value. The stochastic processes for the return on housing, the return on stocks, and the transitory shocks for income is modeled by numerical integration using the Gaussian-Legendre quadrature method with five, five, and ten grid points respectively. The stochastic processes also include the probability of a crash in the stock market (a loss of 100%) and a single period of unemployment (income equals zero). Both of these shocks are independent and exogenous and have no effect on future stock returns or income. The model is solved recursively for each year, from age 80 to age 20, for each of the three different tenure choices. The resulting problem has 1.7 million different combinations of state spaces. For each state space, the optimal level of consumption is determined using a bracketing method with bilinear interpolation for eleven different possible portfolios combinations in four different possible tenure states, the fourth being no change in current tenure status. The numerical integration required at each for each of these 44 different combinations involved calculating the next period value function over 330 different possible combinations of income, risky asset return, and house price shocks. The tenure choice-portfolio combination with the highest value determines the solution to the problem at that combination of state spaces.

The code used to solve this problem is in C. One solution of the problem initially took roughly two weeks on a dual processor Pentium Xeon 1.8GHz with 512K L2 cache and 1GB of RAM running Linux. In order to improve the run-time, the code was re-written to take advantage of parallel processing, using the Message Passing Interface (MPI) standard. In this version of the code one processor is designated the master while a pool of other processors are designated slaves. As the model is solved recursively by year, the master distributes the current value function for all previous years to the slaves. Each slave then solves for the optimal value function for a sub-set of state spaces for the given year. The slaves then return the new value function values to the master. The master then combines the new values with the value function for the previous year, completing the recursion for one year. The problem was solved using 61 high-performance Digital Alpha 64-bit microprocessors running at 450MHz each on a scalable parallel Cray T3E at the Pittsburgh Supercomputing Center. One solution involved roughly 1.3 billion evaluations of the value function and tool roughly eight and a half hours. A future version of this paper will explore an alternative solution method using parametric value function approximation that should further reduce the run time.

The solution of the model consists of vectors for the value function and policy functions for each of the 1.7 million different state spaces. Naturally, the solution in this form is very difficult to interpret. The results of the model are presented in two different ways, graphs of the policy functions for specific sub-sets of the state space and graphs of simulation results across the life cycle. The simulation results are used to calibrate the model to data from the Health and Retirement Survey.

4. Calibration

This section presents the results of the model calibration. The section begins with a description of the characteristics of the stochastic processes in the model. The values chosen for the non-structural parameters and the data source from where these values were taken are also presented. Next, the method used to choose the structural parameters is discussed. The results from these calibrations, based on HRS data from 1992 to 2000, for a several income tracks are then presented. A series of graphs of the policy functions, from one of the calibrated models, across select sub-sets of the state space are then presented, in order to illuminate the factors driving the economic decisions of the household. Finally, some results from simulations based on the calibrated model are given, and compared with what is observed empirically.

As was discussed in the previous section, the income process consists of a deterministic and a transitory factor. The income process is based on the results of regressions of age and age-squared on restricted Social Security earnings data from the HRS. The dependent variable is the log of the wage in constant 1990 dollars. The transitory factor of wage is reflected in the estimated standard error of the regression. The wage is converted from log to level terms in the model. The implicit assumption is that the transitory shock is a multiplicative shock as opposed to an additive shock. At age 65 the level of the deterministic wage falls to a flat level equal to 60% of the last period's income before any transitory shocks, representing a system of forced retirement and a defined benefit pension plan. The coefficients and standard deviation used in this version of the model are shown in Table 4.2 below.

Table 4.1 Log Income Regression Results

Constant	ψ_0	7.28626
Coefficient Age	ψ_1	0.10278
Coefficient of Age^2	ψ_2	-0.00098
Std. Dev.	σ_i	0.80778
R^2		15.5%
Probability of Unemployment	*fired*	1%

The remaining non-structural parameters are taken from a variety of sources. The market prices small and large homes are the result of initializing the deterministic home prices series at age 60 with the National Association of Realtors 1990 median home price. The price of a small home is defined as 80% of the median price and the price of a large home is defined as 120% of the median. The home prices are converted to constant 1990 dollars and the deterministic home prices series are calculated using the average return. The average and standard deviation of the return on housing is taken from the national OFHEO repeat sales price index from 1975 to 2001, after using the CPI to adjust the nominal returns. The mortgage interest rate used is the average rate on loans with 80% loan-to-value ratios as reported by Freddie Mac from 1969 to 2001, adjusting for the interest rate. The mortgage payments are calculated using the mortgage rate, initial mortgage balance, and a term of 30 years. The percent required for downpayment represents the minimum needed to avoid paying mortgage insurance. The transaction, maintenance, and moving costs are based on survey data provided by the National Association of Realtors. The values chosen for the current version of the model are presented in Table 4.3 below. The return on risky assets is based on the real return on the Standard & Poor's index from 1969 to 2001. The index data was taken from the Yahoo Finance web page and the nominal returns was deflated by the change in the CPI for all urban consumers.

Table 4.2 Values of Non-Structural Parameters

Parameter Name and Definition	Symbol	Value
Real risk free rate of return	r	1%
Price of small homes, at age 60	$P_{60}^{i_s}$	8.024
Price of large homes, at age 60	$P_{60}^{i_l}$	12.036
Real return on housing	$\overline{r_h}$	0.97%
Standard deviation of housing return	σ_h	3.2%
Real return on risky asset	$\overline{r_s}$	3.94%
Standard deviation of risky asset	σ_{s_c}	15.37%
Probability of 100% loss on risky asset	*bust*	1%
Mortgage interest rate	r_m	6%
Percent required as downpayment	*down*	20%
Percent of home price lost to transaction costs	*trans*	10%
Maintenance costs	*main*	0.7%
Moving costs	*move*	0.3

Note: Units are in $10,000s or percent.

The model is calibrated using data from the HRS from 1992 to 2002. The sample consists of 7,607 households that were present in the sample in 1992. For each household, the share of their portfolio invested in their home and the share invested in the stock market is calculated for each year. The data identifies renters, but does not identify whether the household lives in a large or an small home. The households are sorted based on the proximity of the value of their home to the prices for large and small homes used in the model. Table 4.4 below contains some of the descriptive statistics found in this data sample. The data supports the stylized fact from Flavin and Yashmita that homeowners who are “over-invested” in housing and react by shifting their financial portfolio towards risk free assets. The correlation coefficient between the share of the portfolio invested in the home and the share invested in stocks is –0.291. Flavin and Yashmita argue that as households grow their overall portfolio, they then start to increase their relative holdings in stock and decrease their relative holdings in their home. This stylized fact is also reflected in the data. The correlation coefficient between the level of financial wealth and the share invested in the home is –0.130 while the same correlation coefficient for the share invested in stocks is 0.124. The goal of the calibration will be to replicate these stylized facts.

Table 4.4 Historical data

	Total	Rental Units	Small Homes	Large Homes
Percent	100%	21.8%	42.7%	35.4%
Financial Assets	113.192 (530.078)	45.635 (298.586)	106.535 (495.424)	162.854 (660.891)
Risky Asset Share*	7.4% (19.3%)	7.1% (23.2%)	5.9% (17.4%)	9.4% (19.9%)
Net Equity in Home			5.337 (3.217)	15.911 (0.000)
Home Equity Share*			65.1% (99.5%)	56.0% (83.0%)

Note: The standard deviations are presented in parentheses. Units are in $10,000s.

Table 4.5 Values of Structural Parameters in Calibrated Model

Income Track	λ	β	ϕ	h_{ir}	h_{is}	h_{il}	**mv**	θ
Total	2	0.99	0.6	5	10	20	0.75	0.75

Figures 4.1 through 4.3 report sample policy functions for a range of households. The households have been arbitrarily chosen to demonstrate the behavior of a representative household in the model. Figure 4.1 shows the policy functions of a young, 25 year old, renter contemplating the purchase of their first home. Figure 4.2 shows a slightly older household, at age 35, who already owns a small home and is considering trading up to a larger one. Finally Figure 4.3 shows the decisions of an older homeowner, age 55, living in a larger home and looking towards retirement. In each of these last two charts, the policy functions of the homeowners are shown after 5, 15, and 25 years of owning their home, in order to demonstrate how the policy functions evolve over time. For each of these time periods, the three different policy functions are shown. The first corresponding with a homeowner who has received the worst possible return on housing in each period, the second representing the homeowner who has experienced average returns in each period, and the third policy function reflects the decisions of the homeowner who has received the best possible returns in each period. This is done to show the range of policy functions across all possible realizations of home value.

The two panels in Figure 4.1 shows the tenure choice, consumption, and portfolio allocation as a function of wealth for a 25-year-old renter. The consumption function is reported, as it is for all of the charts, as a share of total wealth. Households with zero wealth are unable to pay their rent and must move to a new rental unit. As their wealth increases, they are able to avoid defaulting on their rent and do not move. However the do not yet have the funds available to increase their consumption of housing and purchase a house. As a result, they compensate by increasing their consumption of non-durables, and the share of total wealth consumed is a fairly high 20%. This pattern of increasing non-durable consumption while in an inaction region regarding the consumption of durables mirrors the results of Martin (2001). As wealth continues to increase, the household is able to first move into a small home, and then a larger home. For levels of wealth near the time of the home purchase, consumption of non-durables dip as the household spends much of their wealth on the downpayment and transaction costs associated with the home purchase.

The third panel in Figure 4.1 shows how changes in housing tenure decisions can impact the portfolio allocation decision. Households tend to shift their financial portfolios out of risky assets when they decide to first purchase a house, much as the reduce consumption. Given that households are required to provide a 20% downpayment, it is natural to assume that such first time home buyers have a fairly large share of their total portfolio tied up in owner occupied housing. Following the logic of Flavin and Yamashita (2002), the household responds by investing in fewer risky financial assets. As total wealth increases, the portfolio share of housing for these new home owners fall, and the share of the portfolio held in stocks increases. As can be seen in this figure the portfolio allocation policy function is not particularly smooth for higher wealth values. This is a result of the combination of the discrete nature of the portfolio allocation choice and the lack of separation in the value functions for different portfolio allocations. This second issue is explored in detail later in this section.

Figure 4.2 reports a range of policy functions for a household who purchased a small home when they were 35 years old. The first row of charts shows their policy functions 5 years after the purchase of their home under three different possible levels of home price appreciation. The pattern for tenure choice is the same for all three possible realization of home value. Poor households either default on their mortgages and become renters or sell their homes and become renters in order to avoid a default. Households who have experienced above average homre price appreciation have enough equity to finance a move to another small home. As wealth increases, the household can avoid having to move. Finally, as wealth increases they are able to trade-up to a larger home. This pattern is, in general, consistent for all types of households in this model. The better the history of home price appreciation, the sooner the household can trade-up.

As can be seen in the consumption functions, consumption as a share of total wealth is higher for those households not moving. This finding is also consistent across all types of households in this model. The portfolio allocation functions again show that households reduce their investment in risky assets in periods when they first purchase a new home. Interestingly, the sharp dip in the portfolio allocation function for those households who have experienced the best home appreciation seems to occur in each case when the total wealth of the household is in the neighborhood of $300,000. The consumption functions also show a slight jump at this point. This pattern is also seen in other cases. The implication is that there exists a level of wealth sufficiently high that once the household has exceeded it, it both increases significantly its consumption and becomes more aggressive in investing.

Figure 4.3 shows the policy functions for a household who purchased a larger home at age 50. The tenure choice functions here show households no financial wealth having to sell their homes and become either renters or purchase another house, depending on the level of home equity. As the level of financial wealth increases slightly, the households have cover their mortgage payments and avoid moving. As can be seen in the graphs of the consumption function these households save aggressively in order to finance a move to another home, preferably a large one. This allows those households with above average returns on housing to rebalance their portfolios. As wealth increases, this motivation to rebalance their portfolio between housing and financial wealth wanes. Note that both consumption and portfolio allocation drops significantly in periods in which the household decides to move and the rebalance their portfolios.

The preceding figures provide some insight into the behavior of a certain types of the households. However, given the complicated nature of the model and the choices facing the household, it is worthwhile to dig a bit deeper. Figure 4.4 shows a set of value and policy function for one specific household, those of a 65 year old who purchased a large home at age 50 and has not moved since and who has experience the best possible returns on housing. This figure shows the value and policy functions contingent on each separate tenure choice facing the household. The household's actual optimal choice can be seen in the middle row of Figure 4.3 in the dotted-dashed line. The first panel demonstrates that the value functions are quite smooth and monotonically increasing in wealth. The choice to move to a rental or a small home is everywhere dominated by the decision to either stay or move to another large home. For lower levels of wealth the decision to stay dominates while for higher levels of wealth the decision to move dominates. The second panel shows that consumption is higher for movers at almost any level of wealth, reflecting the impact of realized capital gains from the home sale. However the consumption function for those who stay is steadily increasing in wealth, while those for movers level off. This results in the slope of the value function of those who stay being higher than for those who move, as was seen in the first panel. The third panel shows the portfolio allocation policy functions for the different tenure choices. The portfolio allocation for those who stay is very conservative for low levels of wealth, when the risk of mortgage default is still high. As wealth increases those who stay move all of their financial wealth into the risky asset. Households who move all invest less in the risky asset than those who stay. Again this represents the negative correlation between the housing and risky asset share of portfolio. The sharp dip in portfolio allocation commented on above is present across all possible decisions to move.

The next two charts explore the differences in value and policy functions across the different levels of portfolio allocation. Figure 4.5 displays the functions for those households staying and for those moving to rental units while Figure 4.6 displays the functions for those households moving to small or large homes. As can be seen, there is not much separation in the value function for the different levels of portfolio allocations. This results in fairly ragged portfolio allocation policy functions, as was mentioned above. The consumption functions, at least those for the movers, do provide an interesting pattern. For extremely low or extremely high levels of wealth, households investing in few risk asset consume more of the non-durable good. For an intermediate range of wealth the opposite is true, with more aggressive investors consuming more.

In order to better explore the implications of the model 500 simulations are generated using the calibrated model. The results manage to match several patterns seen in the empirical data. The table and figures below contain the results from these simulations. Households begin at age 20 as renters with no assets. Households retire at age 65 and live to at most 80 years of age. The simulations track their accumulation of housing and financial wealth over their lifetime. Figures 4.7 through 4.11 presents the simulation results across the life cycle and compares them with the data from the HRS. These figures show the role of housing over the life cycle, and how consumption and investment decisions are linked to housing decisions.

Figure 4.7 shows a fairly typical pattern of life-cycle consumption in the presence of liquidity constraints. While consumption is much smoother than income, it still retains a humped shape. In particular, consumption is low in the early part of the life cycle as households save for the down payment. This can be seen both in the level of consumption in Figure 4.7 and in the consumption as a share of total wealth in Figure 4.9a. As households age consumption increases, reaching a peak around age 60 and then declining in anticipation of retirement. Consumption as a share of total wealth to income peaks even earlier, at age 40. Consumption as a share of wealth reaches a minimum at age 65 then begins to increase for the rest of the lifecycle as households draw down their wealth. Figure 4.5 reflects the increase mobility in retirement as households extract home equity, showing how average tenure length increases over the lifecycle until decreasing in retirement.

The importance of housing wealth in retirement is emphasized by the next set of figures. Figure 4.10a shows that housing wealth has a sharp hump over the life cycle, reaching a peak at 65. Financial wealth is not quite so sharply humped and peaks earlier at age 60. Neither housing nor financial wealth approaches the levels seen in the HRS. These differences may be the result a lack of heterogeneity in the income processes, which may be too low. In addition, the HRS data does not show any sharp decline in housing wealth in retirement. This failing may reflect the disutility associated with moving increasing with age or a different bequest motive attributed to owner-occupied housing. The lower, flatter, and earlier hump in financial wealth reflects the strategy of first drawing down the more liquid financial wealth before paying the high transaction costs associated with extracting home equity. The flat tails seen in the financial wealth chart show that both the young and the old desire a certain level of financial wealth to act as a buffer stock to smooth transitory income shocks and avoid mortgage defaults.

Figures 4.10c and 4.10d provides the most significant result of the model. The simulated share of assets held in housing is consistently over 50% and is extremely close to that seen in the HRS. The housing share is high among young households who must invest a large portion of their savings in a downpayment. As financial wealth grows faster than housing wealth, this share falls in middle age, only to increase again in retirement as households draw down financial wealth prior to extracting home equity. This matches the "over-investment" in housing seen in the empirical data, including the HRS, using a model of rational, forward looking agents. The implication is that the "over-investment" in housing is the result of something innate in the nature of the housing good or the mortgage contract used to purchase it and not the result of sub-optimal behavior by non-rational consumers. Possible explanations for the high level of home investment seen in the model are discussed in a companion paper. The second figure shows the pattern of portfolio allocation over the life cycle, which is a mirror image of the housing share graph. Young households who have large shares of their wealth tied up in downpayments invest less in the risky asset as does older households who have drawn down their financial wealth. The risky portfolio share peaks at age 50, just when the housing portfolio share reaches its low point. These two figures show that this model accomplishes the goal of matching the stylized facts of Flavin and Yamashita (2002), the negative correlation between the share invested in housing and the share invested in risky assets.

The final set of figures from the simulations document the role of housing over the life cycle. Figure 4.11a shows home-ownership increasing rapidly for younger households, peaking between ages 50 and 65, and declining in retirement. The share of homeowners living in larger homes has a similar hump, as seen in Figure 4.11b. Both of these charts document the strategy of trading down in retirement in order to access housing wealth to finance consumption. Figure 4.11c documents an interesting pattern. Younger households are more likely to move into smaller homes. As the age, and increase both their housing and financial wealth, they are able trade up into larger more expensive homes. In retirement, they begin to migrate back to rental units, extracting their home equity. However, many households have accumulated enough wealth to sell their large homes in retirement and move into other large homes.

Table 4.6 Simulation Results – Traditional Mortgage

	Total	Rental Units	Small Homes	Large Homes
Percent	100%	40.0%	20.2%	39.8%
Consumption	0.849	0.449	0.694	1.322
	(0.608)	(0.295)	(0.347)	(0.619)
Financial Assets	2.504	1.014	0.597	4.966
	(3.893)	(0.732)	(0.516)	(5.221)
Risky Asset	78.9%	69.5%	78.3%	88.4%
Share	(23.5%)	(25.4%)	(24.6%)	(16.0%)
Expected Value	-8.755	-12.187	-8.557	-5.482
	(4.905)	(5.531)	(2.126)	(2.292)
Tenure Length	5.5	4.6	1.9	8.1
	(6.3)	(3.6)	(1.6)	(8.4)
Net Equity in			1.5	4.1
Home			(0.3)	(2.7)
Home Equity			74.8%	54.8%
Share			(17.4%)	(24.9%)

Note: The standard deviations are presented in parentheses.

Table 4.6 above shows some of the sample statistics from the simulation results. The simulation also does a fairly decent job matching up with the HRS data in terms of the share of portfolio held in the home. In the simulation the share of the total portfolio held in home equity for large homes is 54.8% and for small homeowners is 74.8%, compared to 56.0% and 65.1% in the HRS data. These numbers show that the model does very well in capturing the "over-investment" in housing seen in the data. The model also captures how wealth, better diversified, households tend to own larger homes. The simulation does seem to significantly under predict the amount of home equity held by homeowners. This may reflect the existence of some very expensive homes in the data, while by definition the simulation includes only the average priced small and large homes. These results also show how renters, who are aggressively saving for a down payment, have the smallest risky asset portfolio share. Large homeowners, with the smallest housing asset portfolio share have the largest risky asset portfolio share, again reflect the stylized facts from Flavin and Yamashita (2002).

Table 4.7 Tenure Transitions – Base Case

		Don't Move	Move to Rental	Move to Small Home	Move to Large Home
Current Renter	Transition Probability	80.5%	2.1%	9.8%	4.2%
	Wealth	0.964	0.023	1.411	1.312
	Risky Portfolio Share	72.0%	0%	65.2%	65.1%
	Age	32.3	25.7	30.0	30.9
	Tenure Length	4.5	1.9	5.0	5.3
Current Small Home Owner	Transition Probability	44.8%	12.8%	16.1%	25.5%
	Wealth	0.593	0.217	0.680	0.741
	Risky Portfolio Share	75.6%	83.6%	85.7%	75.7%
	Age	31.1	36.0	33.6	31.7
	Tenure Length	2.1	1.3	1.7	2.2
	Portfolio Share of Housing	73.0%	89.2%	74.4%	71.1%
Current Large Home Owner	Transition Probability	61.9%	6.1%	9.9%	21.0%
	Wealth	7.048	0.249	0.731	2.157
	Risky Portfolio Share	92.2%	89.4%	88.3%	76.8%
	Age	36.7	34.0	33.9	35.4
	Tenure Length	11.7	1.1	1.1	2.5
	Portfolio Share of Housing	45.0%	90.6%	78.9%	61.9%

Table 4.7 more fully explores the role of housing tenure decisions in the model. As can be seen in both Table 4.7 and Figure 4.11d, there is quite a bit of mobility in this model. Small homeowners are the most likely to move, with the change of moving 55.2%. Most of these small homeowners, (25.5%) are trading up to large homes. The table also shows that renters are twice as likely to move to small homes than directly to large homes. This reflects the role of the small

home as a transition state between rentals and large homes. Renters will only move to other rentals when they are evicted for missing their rent payments. This explains both the low level of wealth and lack of investment in risky assets for that transition.

The risky share of portfolio for renters is higher for those not moving and lower for those moving, as households reduce their share invested in risky assets in the period in which they purchase a home. This pattern was seen previously in the policy functions. Households are required to invest a sizeable amount of their wealth in housing in order to meet downpayment requirements. This forces "over-investment" in housing results in a reduction in demand for risky assets, following the logic of Flavin and Yamashits (2002). The larger the home purchased, the larger the required downpayment, and the smaller the share invested in risky assets. Interestingly, those small or large home owners purchasing a large home tend to have both greater wealth and a small share of wealth held in housing than those moving to small homes or rental units. In general, the housing share of portfolio is lower for those households not moving that for those households moving. This reflects the fact that the desire to rebalance the portfolio is driving many of these moves. The one exception is small homeowners trading up to large homeowners, who have a housing share of portfolio similar to that of non-movers. For these households the motive is not rebalance the portfolio but to increase housing consumption by moving to a larger home.

This section has established the two significant accomplishments of the model; (1) the ability to match the "over-investment" in housing seen in the data, and (2) the ability to capture the negative correlation between the housing share and risky asset share of portfolio. The fact that this was accomplished within a framework of rational, forward-looking agents is not insignificant. However, due to the complexity of the model, many of the implications are not intuitive. The next section explores how the role of housing as a consumption good, the risk structure of the model, the tax treatment of housing, and the role of the mortgage contract effect the relationship between the housing share and risky asset share of portfolio.

5. The Relationship Between the Housing Share and Risky Asset Share of Portfolio

The chief accomplishments of the calibrated model are to match the "over-investment" in housing seen in the data and to capture the negative correlation between the housing share and risky asset share of portfolio. This was done by including a large number of factors, from mortgage contracts to tax policy, in order to make the model as realistic as possible. The disadvantage is the complexity of the model clouds the causal effects. It is able to replicate the relationship between the housing share and risky asset share of portfolio. This section addresses the issue through a series of comparative static exercises. A list of the possible factors affecting the relationship between the housing share and risky asset share of portfolio are developed and one-by-one they are excluded from the model creating a model with an alternative set of assumptions. Each alternative model is then re-solved and the simulations regenerated. The levels of wealth accumulation, housing demand, and portfolio allocation under each alternative assumption are compared to the base case.

Table 5.1 Explanations for the "Over-Investment" in Housing

Alternative Assumption	**Model Effects**
Alternate Risk Structures	
More Frequent Stock Market Crashes	*bust*=5%
More Frequent Unemployment	*fired*=5%
Alternate Preferences	
Consumption Constrained	$h_{ir}=h_{is}=h_{il}$
Alternate Taxes	
Repeal Mortgage Interest Tax Deduction	*mrpay*=0 for all i_c
Tax Net Gain on Housing	Replace (3.5) with (5.1)
Alternate Mortgage Contracts	
Low Transaction Costs	*down*=0.01, *tran*=0.01
Fixed Payments	Replace (3.12) with (5.2)

Table 5.1 above summarizes seven possible factors affecting the relationship between the housing share and risky asset share of portfolio, along with how they are reflected in the model. The first two alternate assumptions explore how the probability of two "worse case scenarios" effects the household's portfolio decisions. The simulation results for these alternate assumptions are shown on Figure 5.1 and 5.2. An increase in the possibility of a stock market crash and 100% of risky assets from 1% to 5% results in no household holding any risky assets. They restrict themselves only to housing and the risk free asset as investments. Under this alternate assumption, households naturally hold less financial wealth but also hold less housing wealth. The portfolio share of housing increases slightly for households in middle age and early retirement. While there is no real change in the rate or composition of homeownership,

household mobility does increase just prior to retirement. The increase in risk associated with risky assets drives the households away from the equity market. The lower overall returns prevent the households from accumulating as much wealth. This prevents them from rebalancing their portfolios as frequently as they would like, and forces them to extract their home equity more quickly.

The second assumption simulated on these two figures shows the effect of an increase in the probability periods of unemployment from 1% to 5%. As in the main model, these periods are unemployment are not persistent. Naturally this change has reduced the present-value of the income stream to the households, resulting in less housing and financial wealth. Households also hold a slightly smaller share of portfolio in housing when young and when old. Middle aged households slightly increase the share of portfolio in housing. The risky share of portfolio is reduced across the board. This reflects the "crowding-out" of demand for risky assets by an increase in the background risk associated with income, similar to the intuition of Cocco (2000). The homeownership rate and demand for large homes are reduced under this assumption while mobility is increase significantly among working households. Households with more volatile income are less likely to own, to own large homes, and must move more often to avoid default.

The next alternate assumption explores the role of housing as a consumption good. Under this assumption both small and large homes generate the same utility as rentals. The results, as seen on Figures 5.3 and 5.4, are quite dramatic. Both housing wealth and financial wealth increase significantly. The housing share of portfolio declines and the risky share of portfolio increases. Homeownership is lower among the young, but higher across the rest of the life-cycle. No households purchase large homes, and mobility is much lower with the exception of early in retirement. The absence of a consumption motive to trade up to larger more expensive homes allows households to spend significantly less, both on transaction costs and higher mortgage payments.

There is no consumption motive behind the demand for small homes, which generate the same utility as rentals. When this assumption is simulated in the companion paper, in the absence of a risky financial asset, almost all households remain renters. The only motive to purchase housing is the desire to diversify by owning an asset with stochastic returns uncorrelated with the risky asset returns. The broader implication is that role of housing as a consumption good encourages homeowners to "over-invest" in housing. This in turns drives down the demand for risky financial assets. These model results provide support for the intuition behind Flavin and Yamashita (2000).

The next set of assumptions examines the reaction of household’s behavior to tax changes. The results are presented in Figures 5.5 and 5.6. The first assumption tested is the repeal of the mortgage interest tax deduction. As can be seen, this change has little effect on the behavior of households, except for a slight reduction in the demand for larger homes and a equally slight reduction in housing and financial wealth. The base model also does not tax the net gain on a home sale. In order to tax the capital gain on housing equation (3.5) is replaced with;

$$\begin{aligned}A_{t+1} &= E((1-tax)*(\alpha r_s + (1-\alpha)r)+1)*(A_t - c_t - x_t(i_c, age_t)) + (1-tax)*w_t \\ &+ tax*mipay_t(i_c, age_t)) + ((1-tax)*(\alpha r_s + (1-\alpha)r)+1)*(1-tax)*gain_t\end{aligned} \quad (5.1).$$

Note that the tax on capital gain on housing is actually a benefit to households that are selling their home at a loss. They are able to deduct that loss from their taxes. Households react to this change by reducing their housing wealth but significantly increasing their financial wealth. The housing share and risky asset share of portfolio both decreases, significantly in the case of the former. Homeownership declines slightly among working households, while declining in retirement. The demand for large homes shifts in the opposite direction increasing among workers and decreasing among retirees.

This shift towards larger homes reflects the risk-sharing aspect of the capital gains tax. Now that households can write off their losses on their home sales, they are willing to purchase larger homes and place more of their equity at risk. However they also want to decummulate their assets more quickly in retirement, while they still have large reserves of financial assets to pay the capital gains taxes. The high tax rate decreases the portfolio share of the asset being taxed, i.e. housing, as well as the portfolio share of the risky asset. However overall demand for these assets do not decline, as households increase their position in the risk free asset.
The last set of assumptions demonstrates one of the key accomplishments of the model. The model explicitly models the mortgage contract facing the household. The intuition is that the terms of the mortgage contract have direct and significant effects on the economic decisions of the households. The purchase of a house requires two different sets of expenditures. The first is a larger initial payment to cover the down payment and transaction costs. The second is a recurring monthly charge for the life of the mortgage. In the case of a traditional fixed-rate mortgage, this recurring monthly charge is fixed for the life of the mortgage. Both of these types of payments are increasing functions of the price of a home. The more expensive a home is, the high the monthly mortgage payments. Given that the market price of homes are increasing steadily in the model, the mortgage payment on a home purchased in year 60 will be significantly high than the mortgage payment on a home purchased in year 40. This aspect of the mortgage contract provides the household a way to insulate themselves from the effects of rising home prices and rents, much as the households in Sinai and Souleles (2003) use owner-occupied housing to insulate themselves against rent risk.

The impact of the fixed nature of the mortgage payment is tested in the following way. The mortgage payments are held constant for all ages, regardless of the original purchase price of the home. This is accomplished by replacing equation (3.12) with;

$$paym(i_c, t, age_t) = \frac{(mrat)*(1-down)*P_{60}^{i_c}}{(1-mrat)^{-term}} \quad (5.2).$$

Under this assumption the mortgage payment is no longer indexed to the home price in the year the house was purchased. Instead it is constant, indexed to the home price in year 60, regardless of the year in which the home was purchased. As Figures 5.7 and 5.8 shows, under this assumption both the level of housing wealth, financial wealth, the homeownership rate, and the demand for larger homes all drops. There is no real change in the either the housing or risky asset share of portfolio. These changes can be largely explained by the relative increase in the cost of housing under this assumption, that is younger households now face higher fixed mortgage payments.

The last alternate assumption provides some of the most counter-intuitive results in the paper. Under this assumption both the transaction costs and downpayment required are reduced significantly, from 10% and 20% respectively to 1% each. In the companion paper without the risk asset this results, not unexpectantly, in an increase in demand for larger homes and in mobility in retirement. The results in this model, with risky assets, are far more striking. Homeownership, housing wealth, mobility, and the housing share of portfolio all drop like a rock. What few households do purchase homes only purchase large homes. Households accumulate much more financial wealth, buy hold a smaller share in the risky asset.

One possible explanation of this pattern involves the combination of large transaction costs and deterministically increasing home prices. As was seen in some of the policy functions, it becomes more difficult to trade up to larger homes as time passes and the market price of homes increases. In response to this, household in the base case try to purchase their homes as early as possible as insurance against increasing home prices. They fear that if they do not acquire a position in the housing market early, they might be priced out of the market later. In order to finance the significant downpayment and transaction costs they save aggressively but invest conservatively. Much of their savings then is lost on housing transaction costs or invested in housing. The continuing need to finance new moves, either to trade-up or rebalance the portfolio, continue to place a drag on financial wealth in the form of transaction costs and downpayments. Households accept these costs in order to enter the housing market early in the life-cycle and insure affordable access to the housing market later in the life-cycle.

Once the downpayment and transaction costs are reduced, households no longer have to acquire early positions in the housing market. They are free to invest in the financial market and reap higher returns, safe in the knowledge that they retain the option to enter the housing market at will. However, in higher wealth levels the household now achieves allow them to increase consumption of non-durables and reduce the demand for housing. They still retain the utility associated with the option of being homeowner whenever they choose. The lack of a second asset with uncorrelated stochastic returns in housing reduces the level of diversification and households reduce their risky asset share of portfolio.

6. Conclusion

One of the goals of this research was to explain the life-cycle patterns in both homeownership and portfolio allocation using a model of rational agents. Two key innovations are incorporated in the model. First, housing is explicitly modeled as both a consumption and investment good, as opposed to examining just one aspect in isolation from the other. Second, mortgage contracts are explicitly introduced into the model. The result is a more realistic treatment of the role of housing in an agent's economic decision-making over its lifetime. The model is then used to explore the relationship between the housing share and risky asset share of portfolio.

The model does manage to match the share of home equity as a percent of the entire portfolio seen in the HRS data. As a result, the model proves that the “over-investment” of housing is not inconsistent with the behavior of rational, forward-looking agents. The model is also able to capture the negative correlation between the housing share and risky asset share of portfolio. This provides a theoretical basis for the intuition behind Flavin and Yamashita (2002).

A series of alternate assumptions are used to explore how the unique role and nature of the housing asset impact the portfolio allocation decision. The results show how the desire to consume large homes contributes both to an increase in the portfolio share of housing and a decrease in the portfolio share of risky assets. The imposition of taxes on the capital gains of housing reduces both the housing and risky asset portfolio shares. Finally a significant reduction in transaction costs actually reduces the demand from housing significantly, by allowing households to hold more wealth in financial assets with higher returns while retaining the option to become homeowners at any time. This experiment demonstrates the close link between the housing “over-investment” and the stock “under-investment” puzzles.

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Figure 4.1a
Tenure Choice
25 Year Old Renter

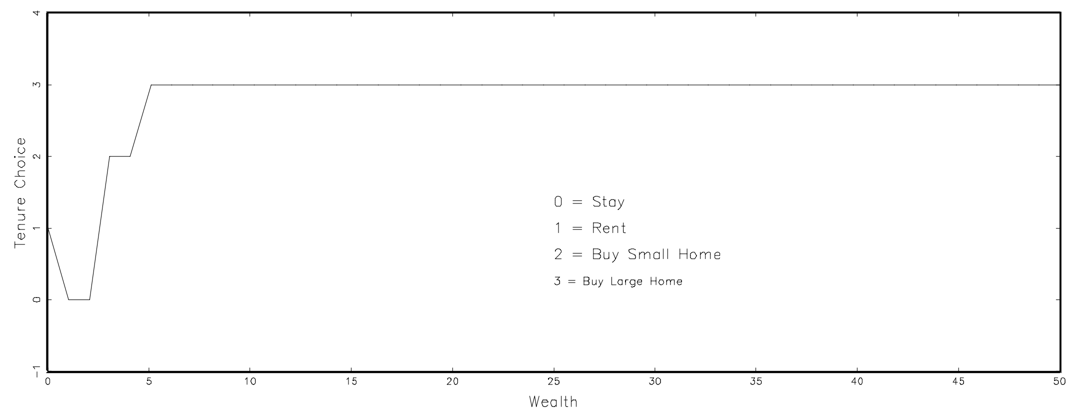


Figure 4.1b
Consumption
25 Year Old Renter

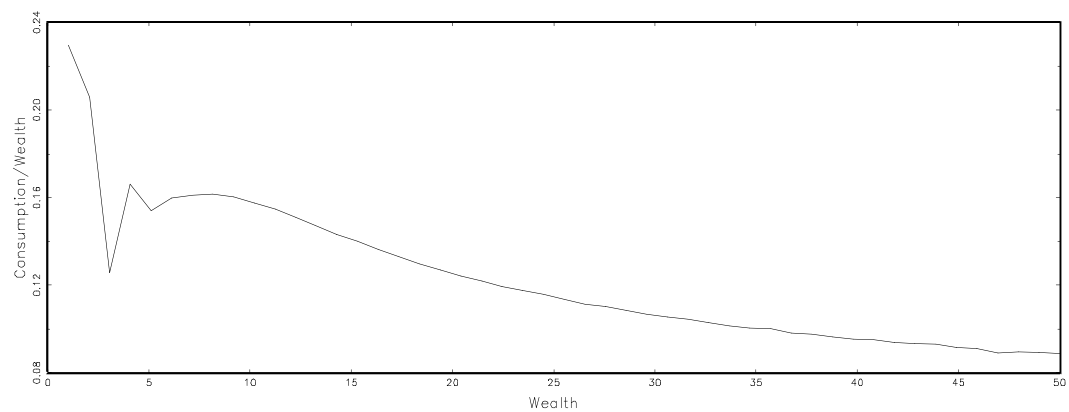


Figure 4.1c
Consumption
25 Year Old Renter

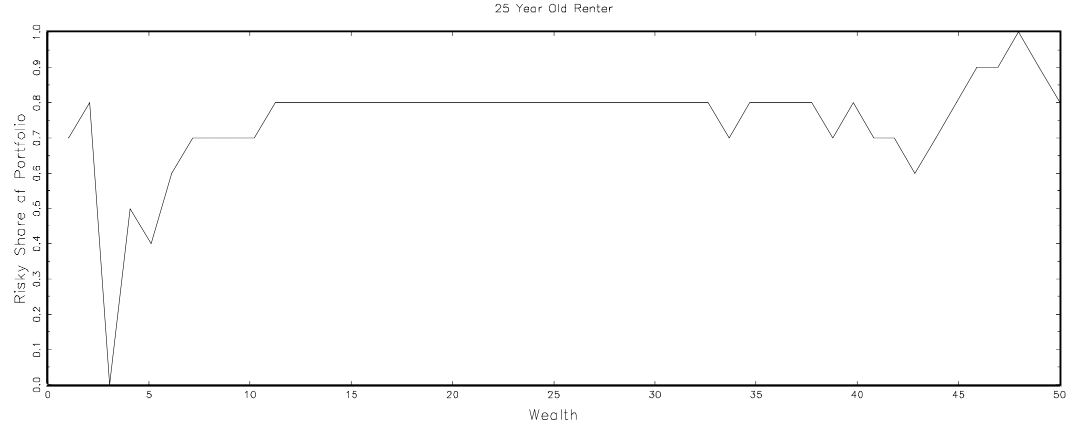
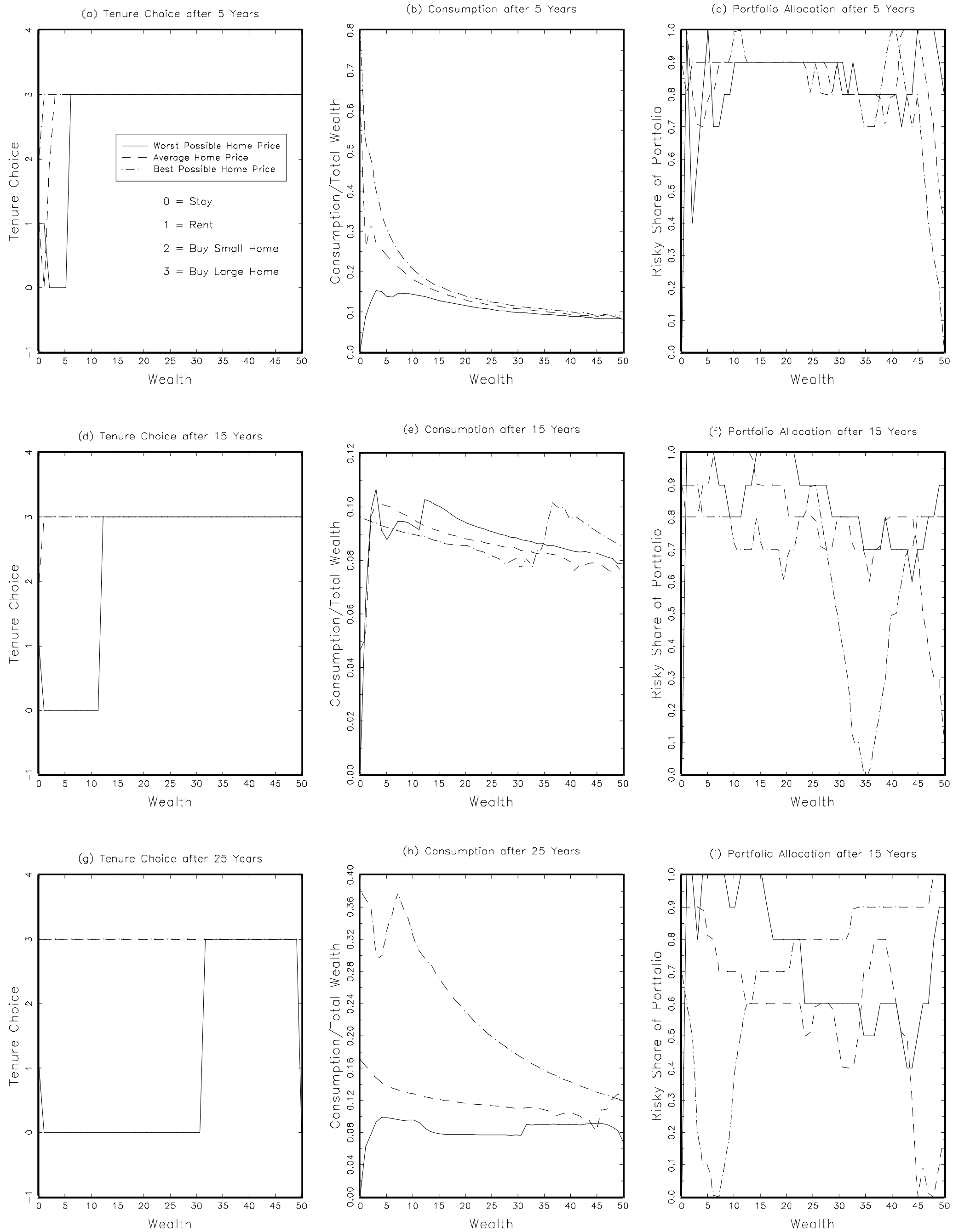


Figure 4.2 Policy Functions for a 35 Year Old Who Has Just Purchased a Small Home



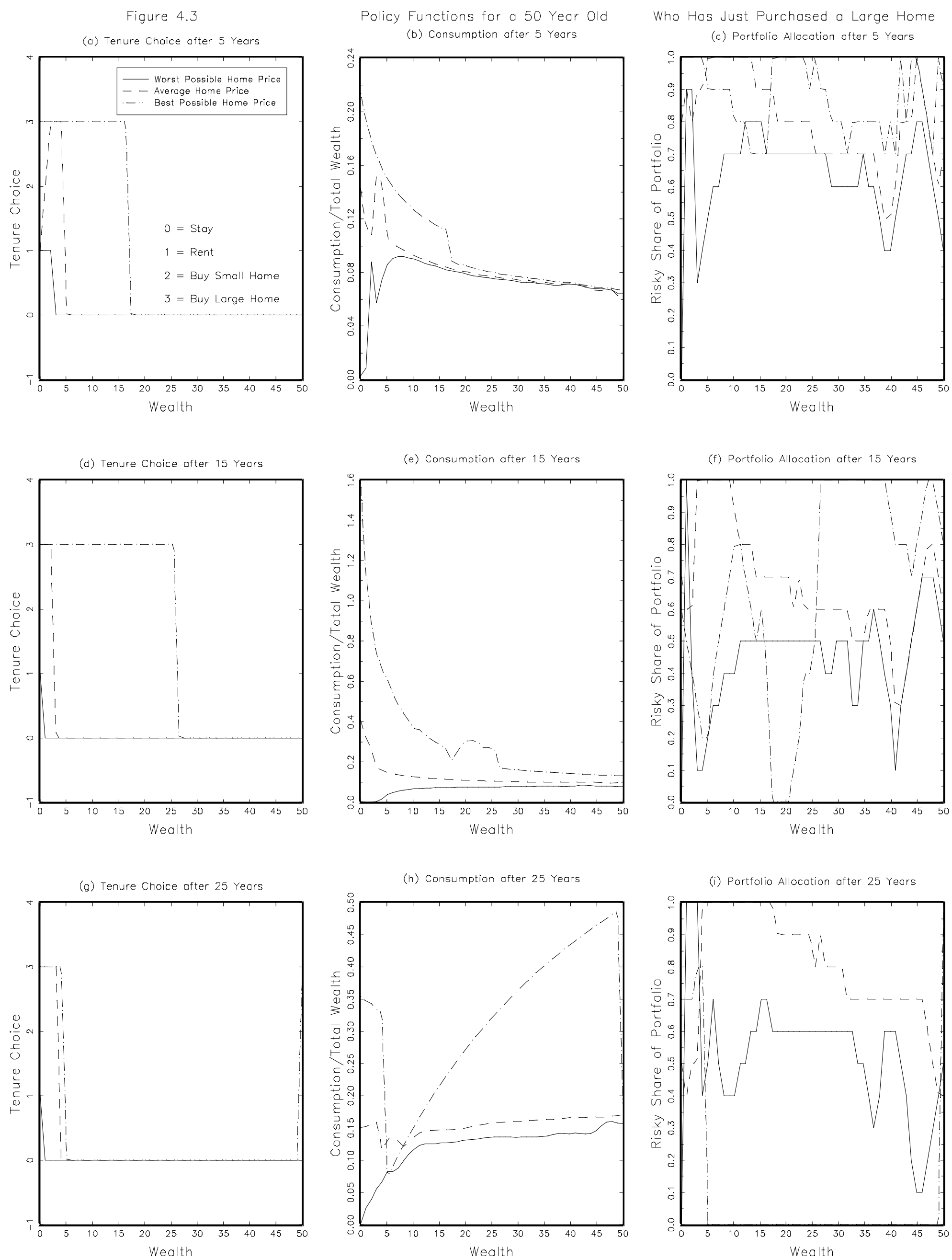


Figure 4.3
Policy Functions for a 50 Year Old
Who Has Just Purchased a Large Home
(a) Tenure Choice after 5 Years
(b) Consumption after 5 Years
(c) Portfolio Allocation after 5 Years
Worst Possible Home Price
Average Home Price
Best Possible Home Price
0 = Stay
1 = Rent
2 = Buy Small Home
3 = Buy Large Home
Tenure Choice
Consumption/Total Wealth
Risky Share of Portfolio
Wealth
(d) Tenure Choice after 15 Years
(e) Consumption after 15 Years
(f) Portfolio Allocation after 15 Years
(g) Tenure Choice after 25 Years
(h) Consumption after 25 Years
(i) Portfolio Allocation after 25 Years

Figure 4.4 Policy and Value Functions for a 50 Year Old 15 Years after Purchasing a Large Home

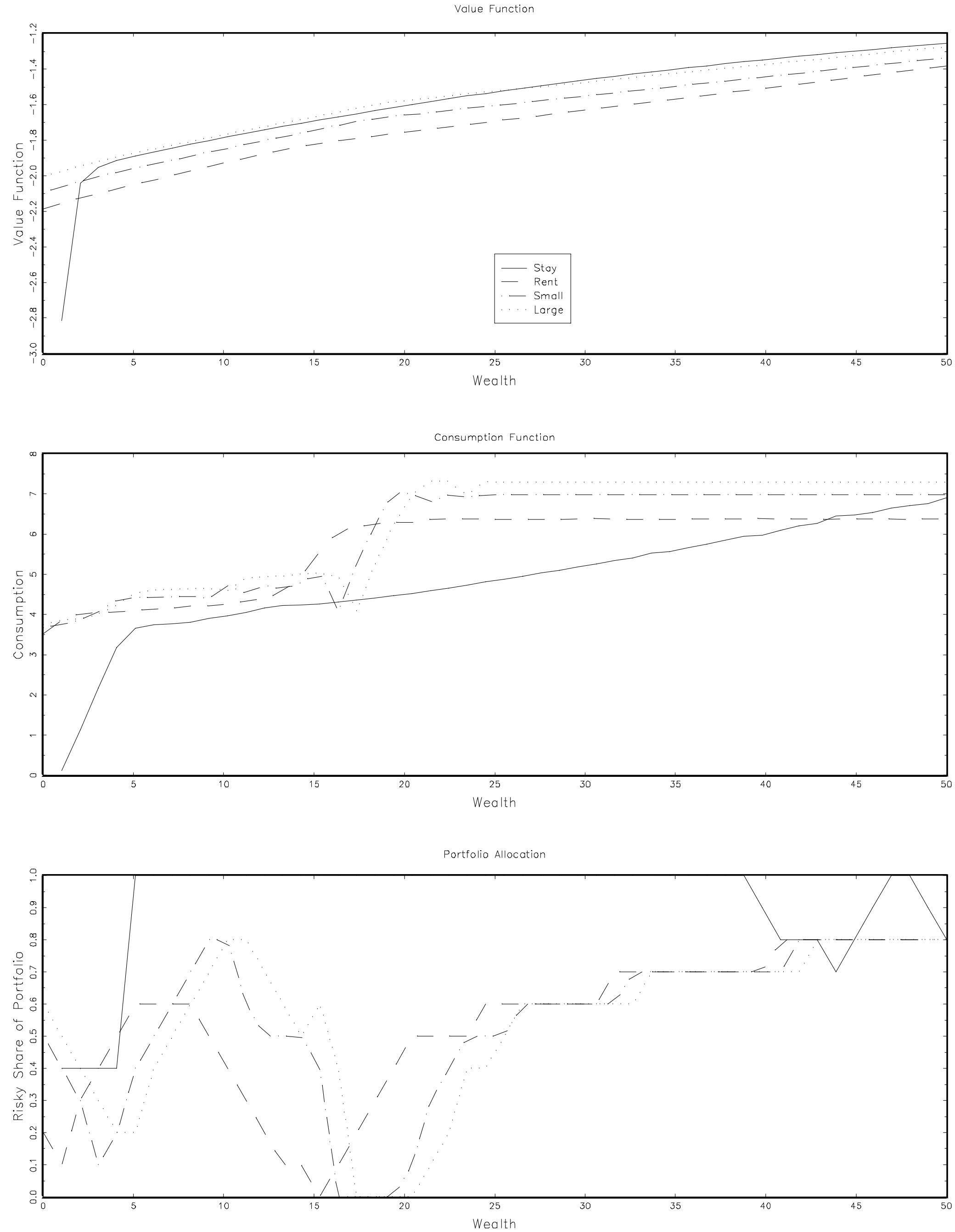


Figure 4.5
Value and Policy Functions for a 50 Year Old, 15 Years after Purchasing a Large Home

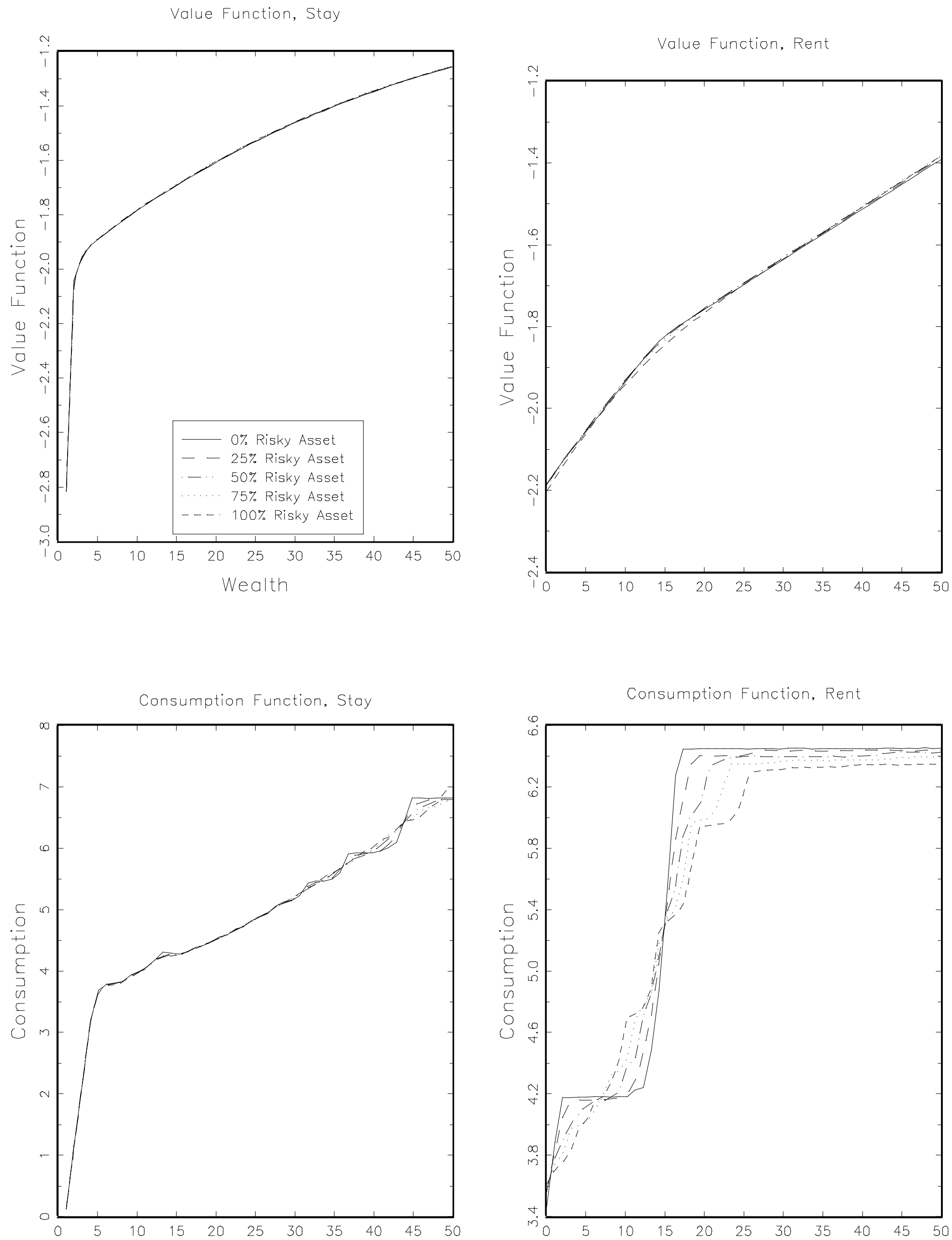


Figure 4.6
Value and Policy Functions for a 50 Year Old, 15 Years after Purchasing a Large Home

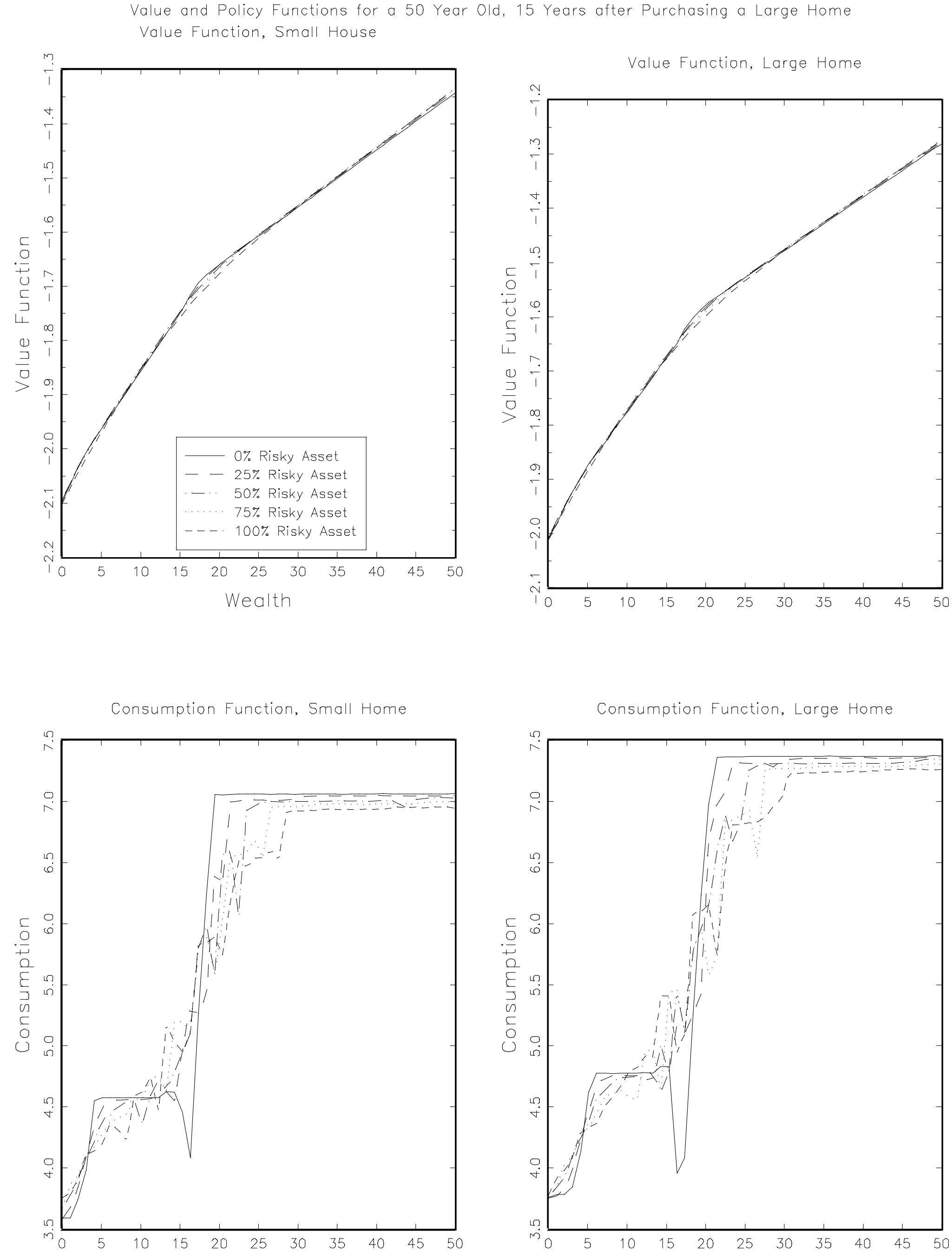


Figure 4.7
Wages and Consumption

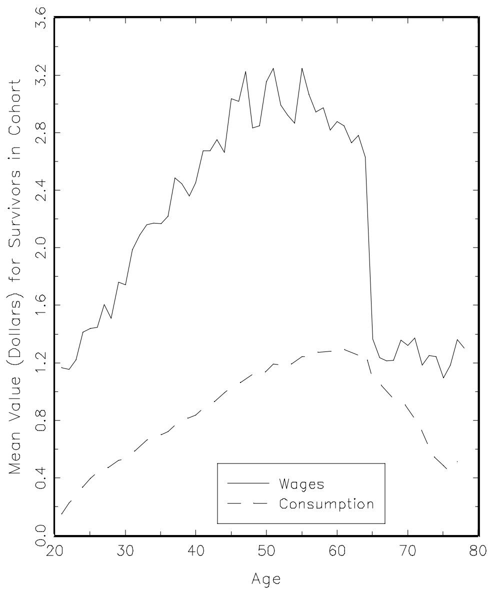


Figure 4.8
Simulated Average Tenure Length

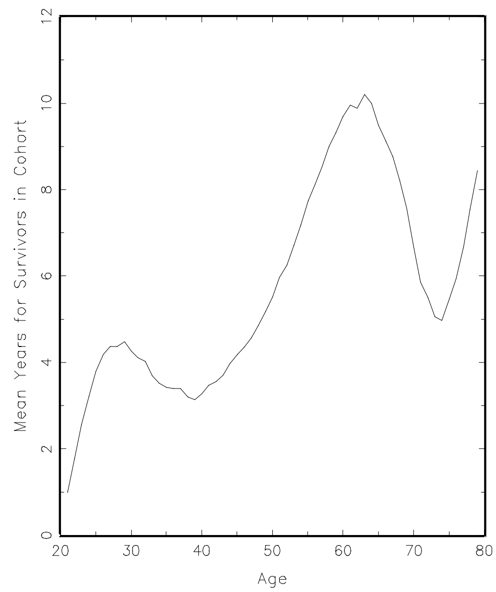


Figure 4.9a
Consumption as Share of Total Wealth

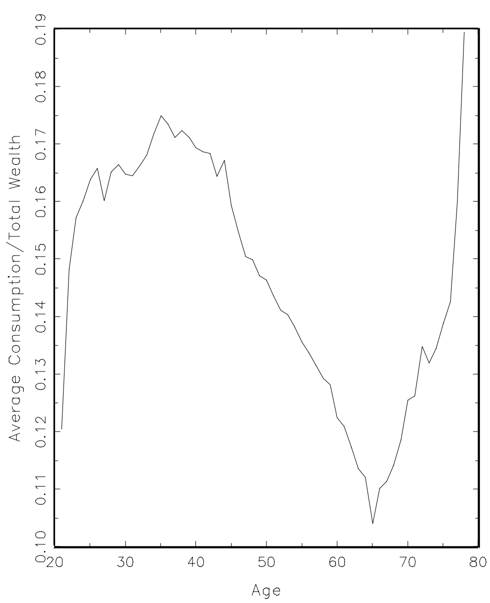


Figure 4.9b
Ration of Total Wealth to Income

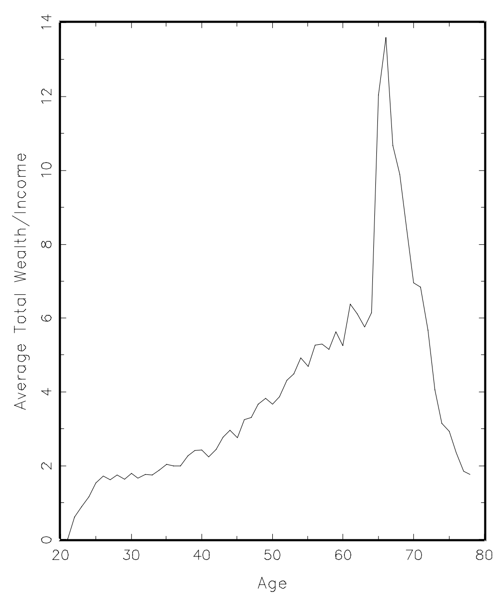


Figure 4.10a
Simulated vs. Actual Housing Wealth

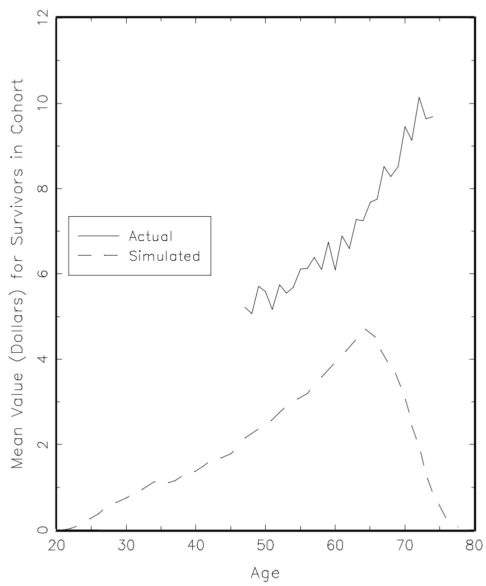


Figure 4.10b
Simulated vs. Actual Financial Wealth

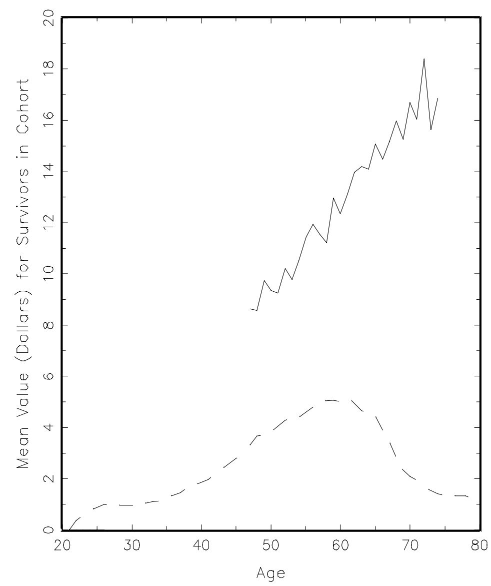


Figure 4.10c
Actual vs. Simulated Share of Housing Assets

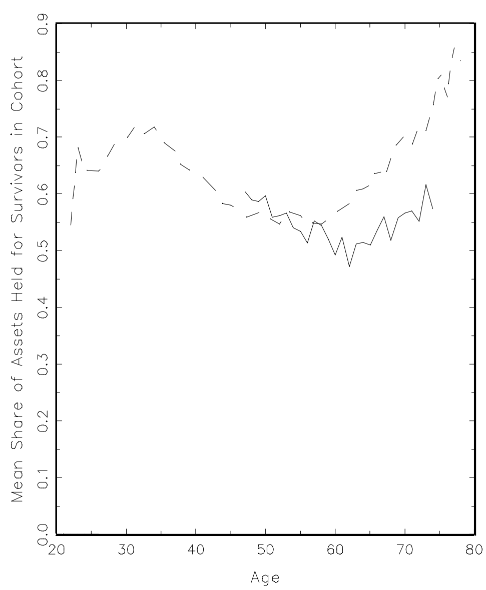


Figure 4.10d
Simulated Risky Share of Financial Assets

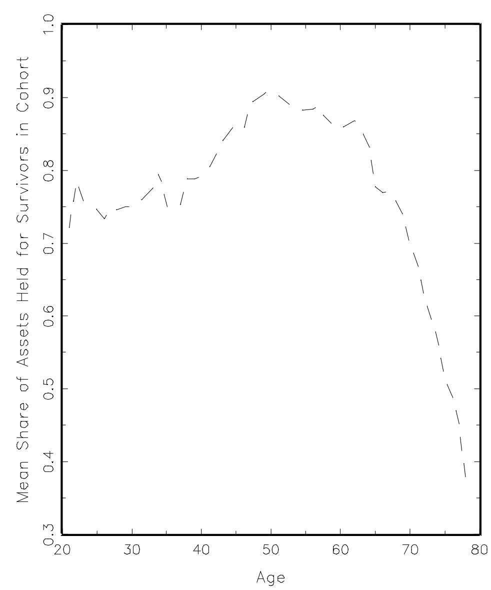


Figure 4.11a
Actual vs. Simulated Homeownership Rate

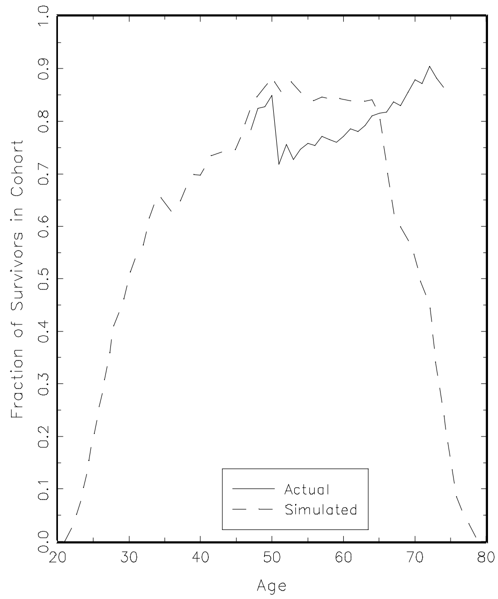


Figure 4.11b
Actual vs. Simulated Tenure Choice of Homeowners
Share Large Homes

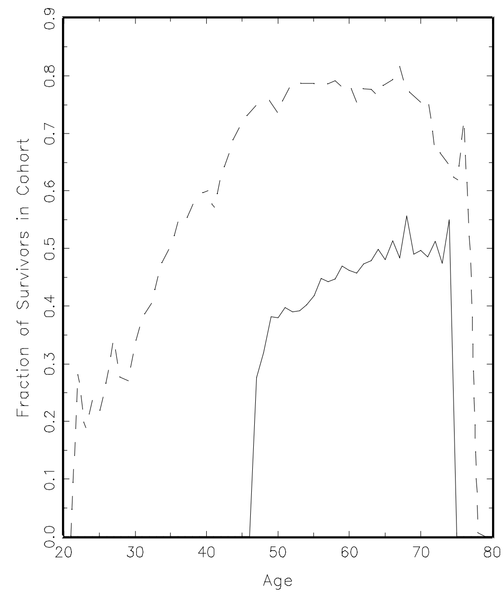


Figure 4.11c
Tenure Choice of Movers

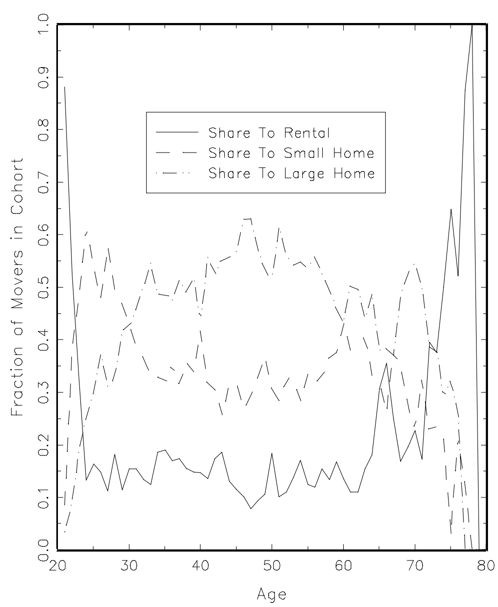


Figure 4.11d
Simulated Mobility

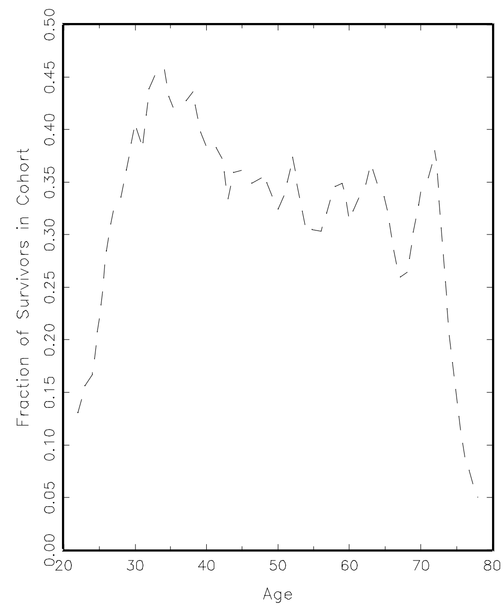


Figure 5.1a
Housing Wealth
Under Alternate Risk Strutures

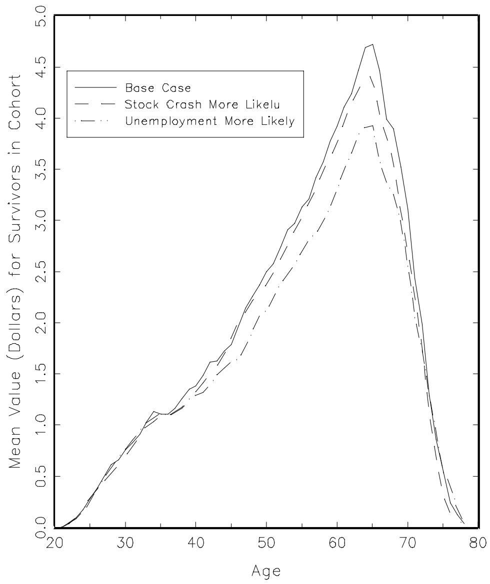


Figure 5.1b
Portfolio Share of Housing Assets
Under Alternate Risk Strutures

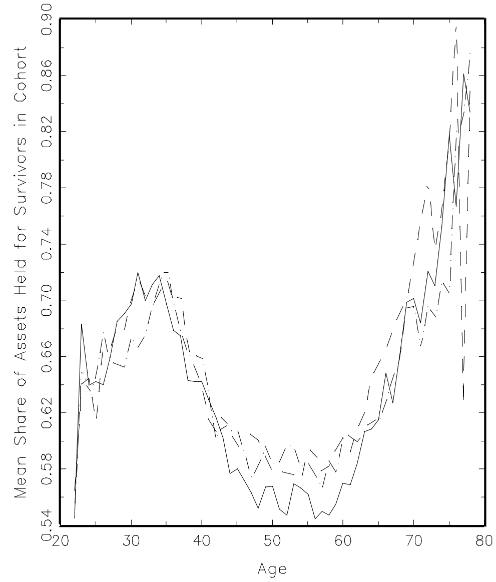


Figure 5.1c
Financial Wealth
Under Alternate Risk Strutures

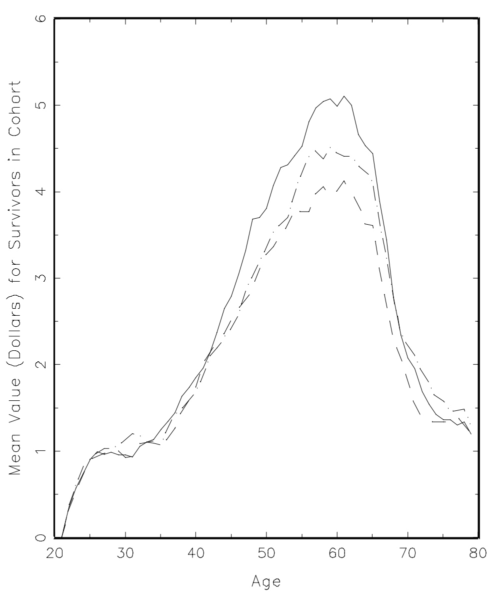


Figure 5.1d
Risky Share of Financial Portfolio
Under Alternate Risk Strutures

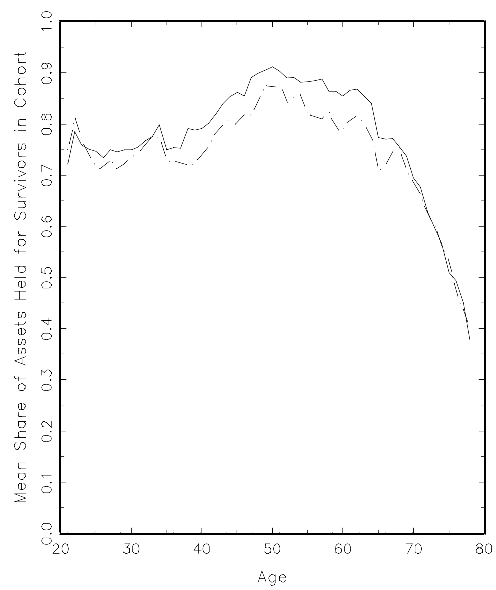


Figure 5.2a
Homeownership Rate
Under Alternate Risk Strutures

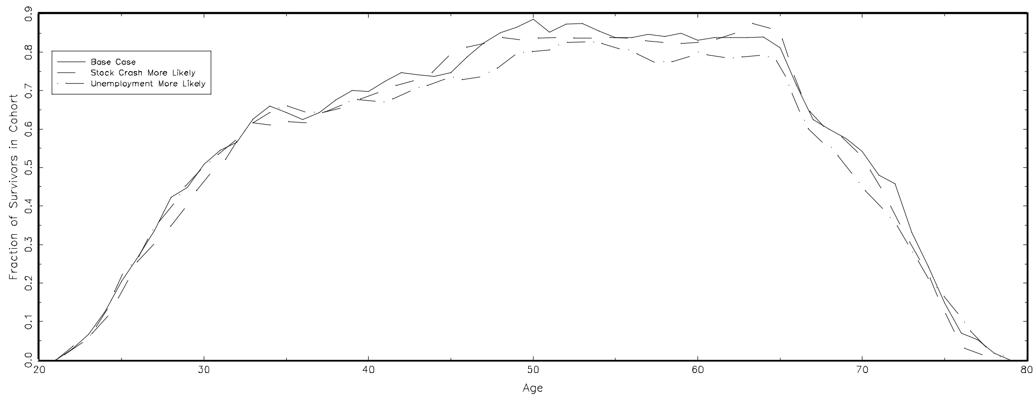


Figure 5.2b
Share Large Homes
Under Alternate Risk Strutures

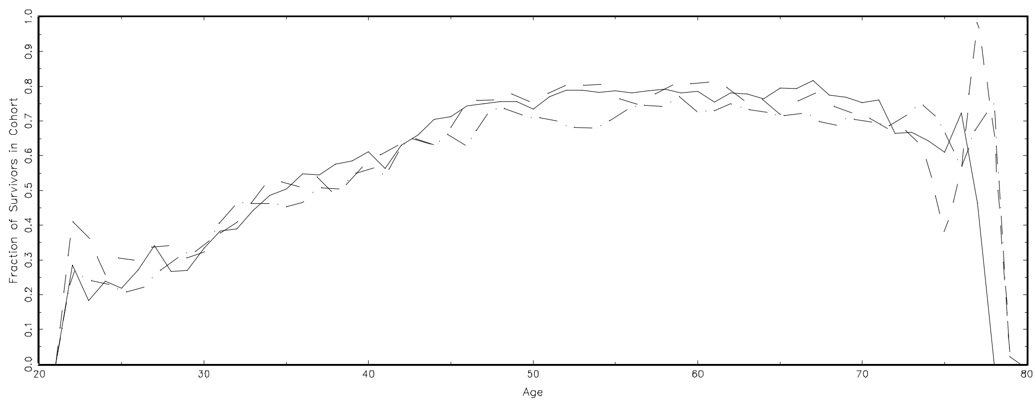
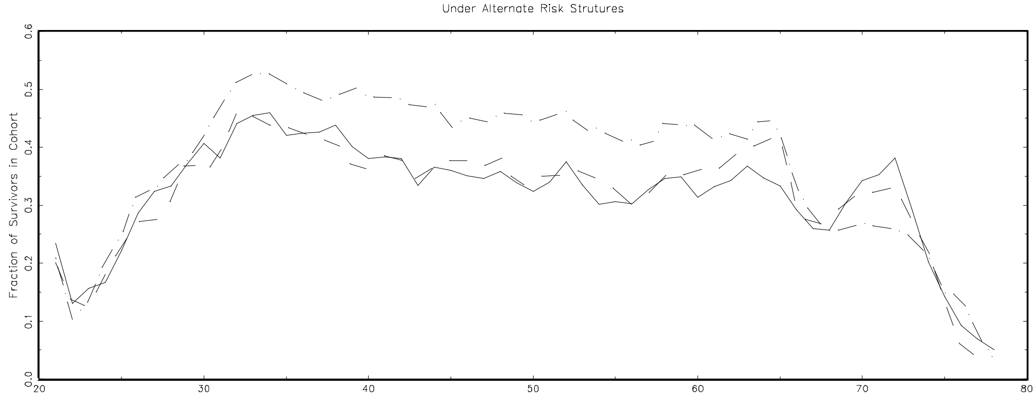


Figure 5.2c
Simulated Mobility
Under Alternate Risk Strutures



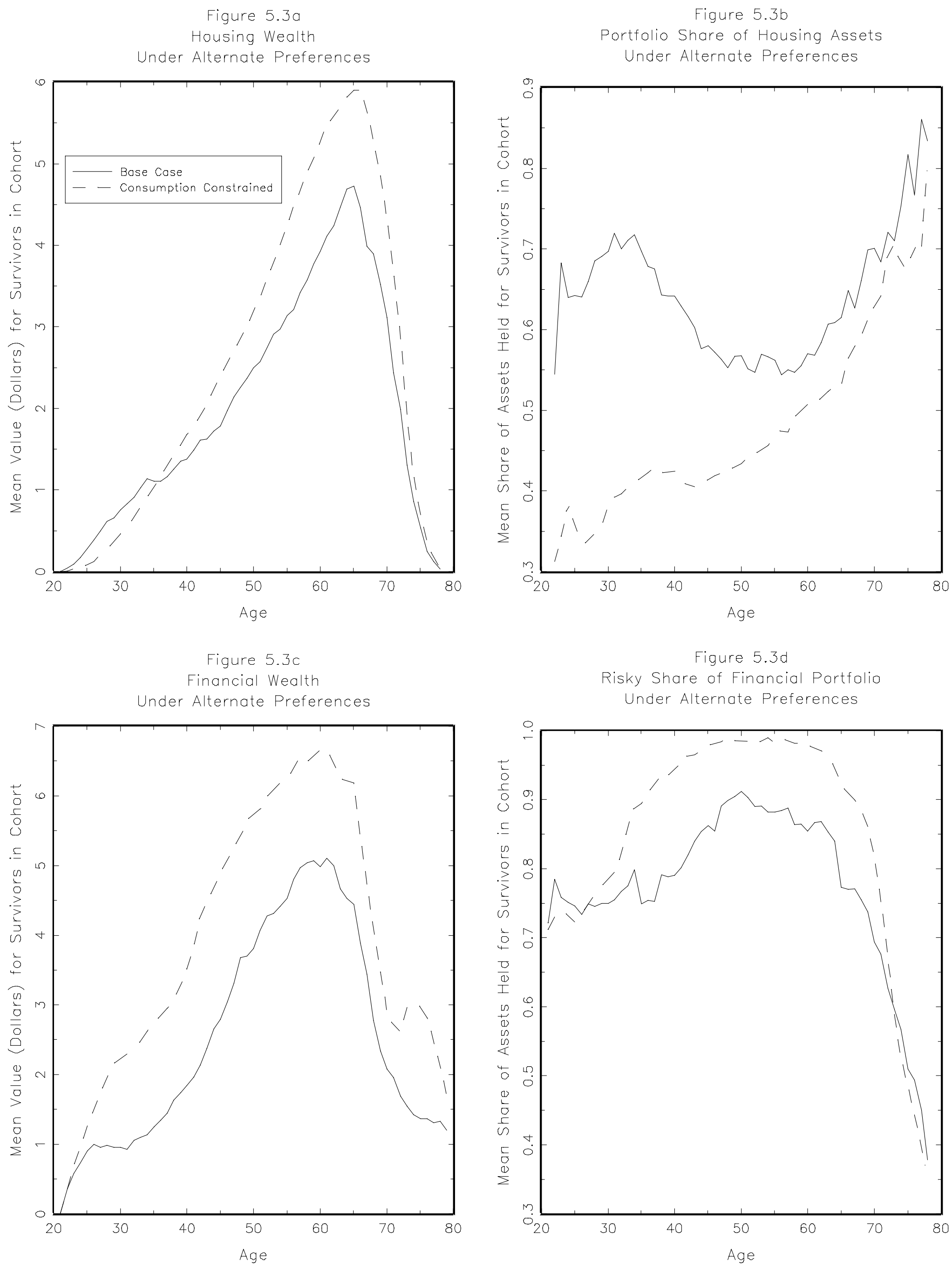


Figure 5.3a
Housing Wealth
Under Alternate Preferences

Figure 5.3b
Portfolio Share of Housing Assets
Under Alternate Preferences

Figure 5.3c
Financial Wealth
Under Alternate Preferences

Figure 5.3d
Risky Share of Financial Portfolio
Under Alternate Preferences

Figure 5.4a
Homeownership Rate
Under Alternate Preferences

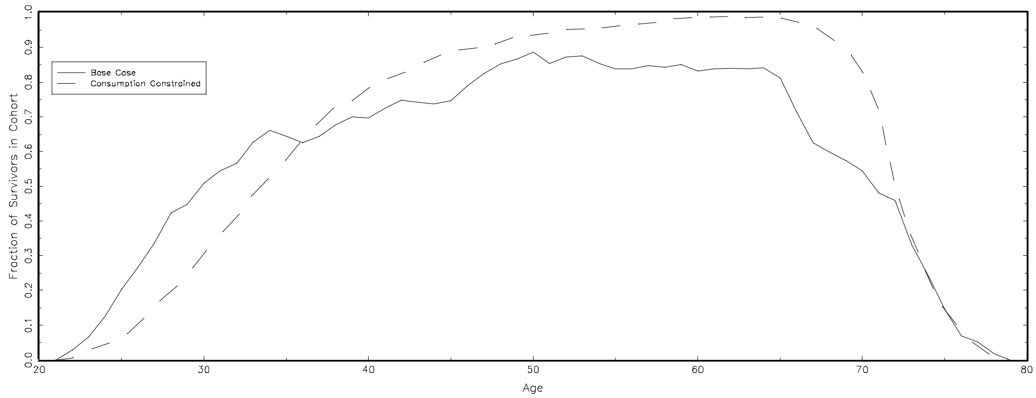


Figure 5.4b
Share Large Homes
Under Alternate Preferences

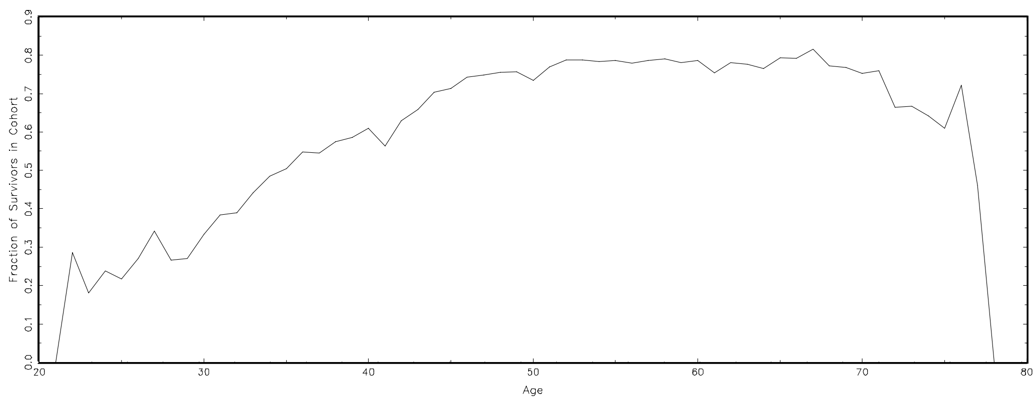


Figure 5.4c
Simulated Mobility
Under Alternate Risk Preferences

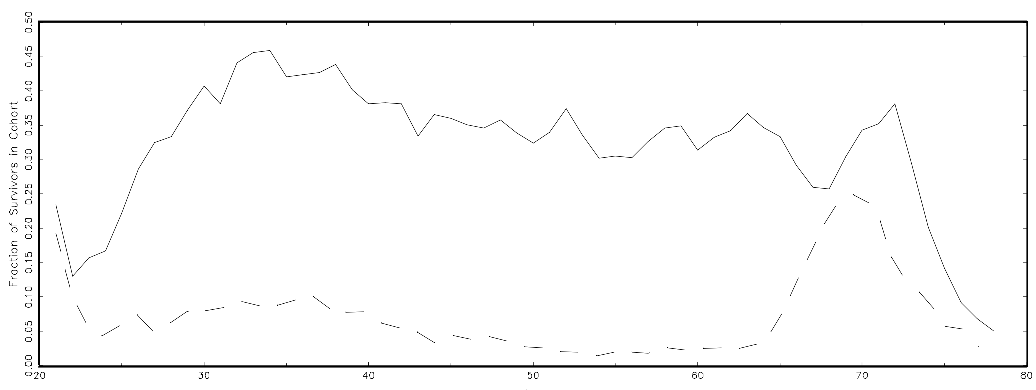


Figure 5.5a
Housing Wealth
Under Alternate Taxes

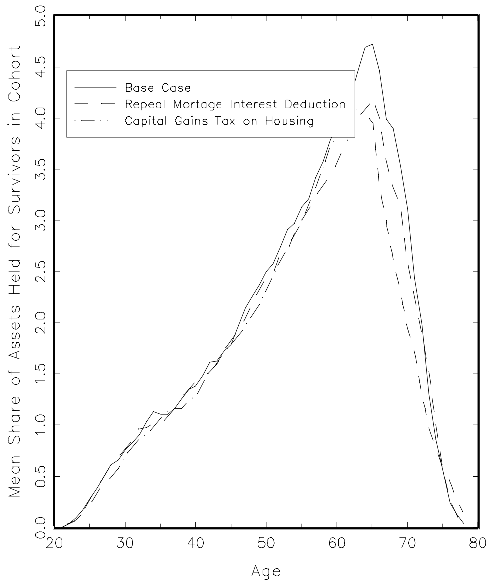


Figure 5.5b
Portfolio Share of Housing Assets
Under Alternate Taxes

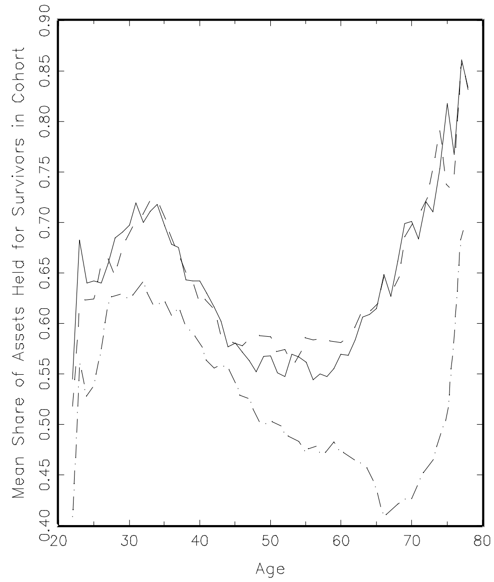


Figure 5.5c
Financial Wealth
Under Alternate Taxes

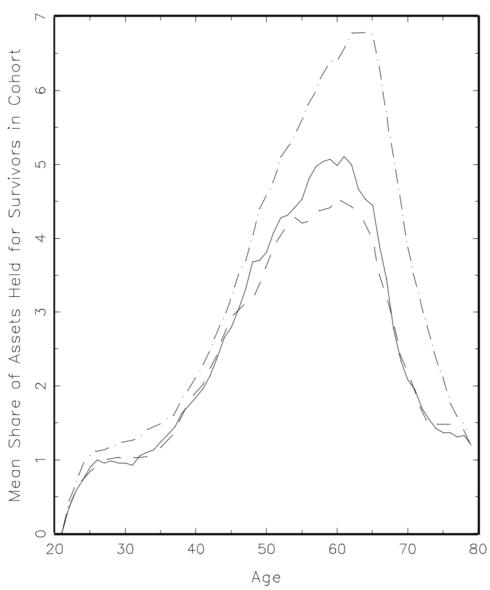


Figure 5.5d
Risky Share of Financial Portfolio
Under Alternate Taxes

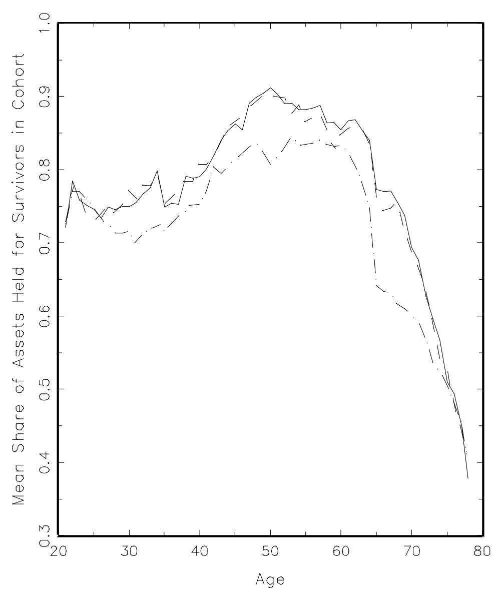


Figure 5.6a
Homeownership Rate
Under Alternate Taxes

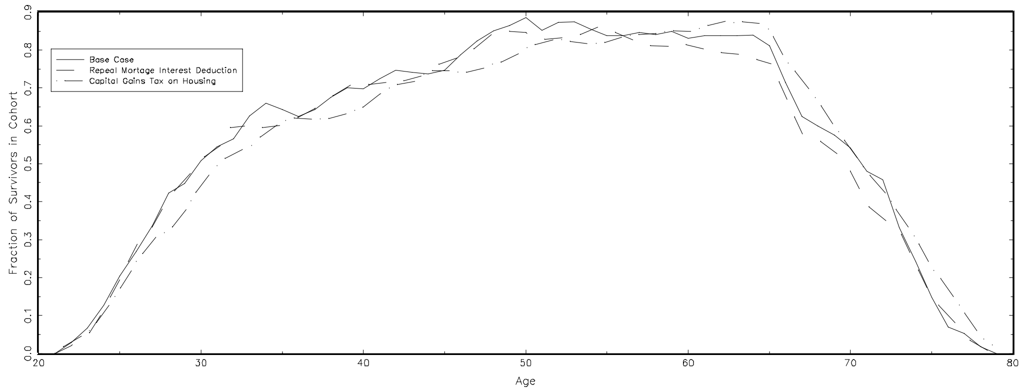


Figure 5.6b
Share Large Homes
Under Alternate Taxes

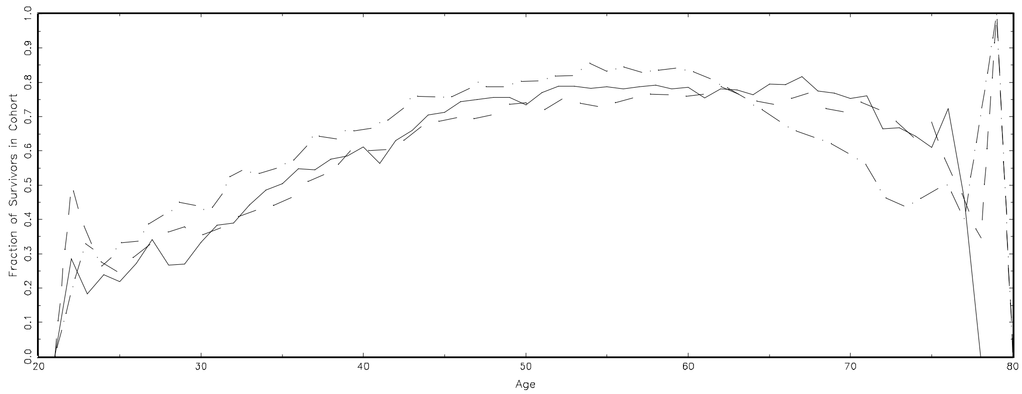


Figure 5.6c
Simulated Mobility
Under Alternate Taxes

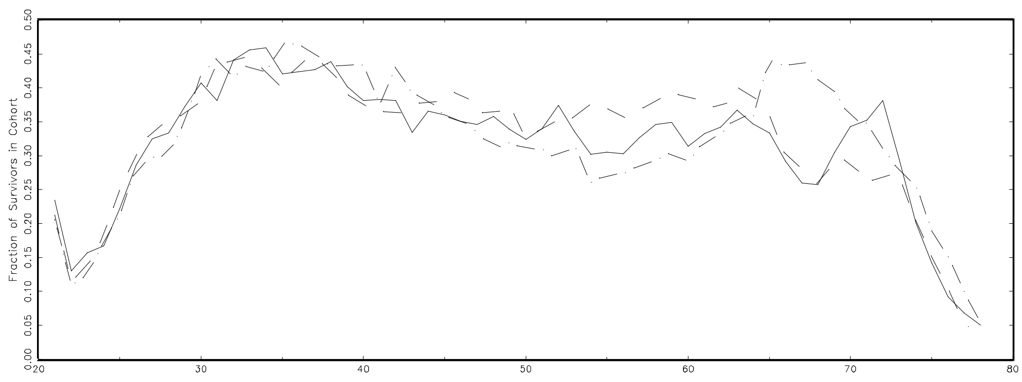


Figure 5.7a
Housing Wealth
Under Alternate Mortgage Contracts

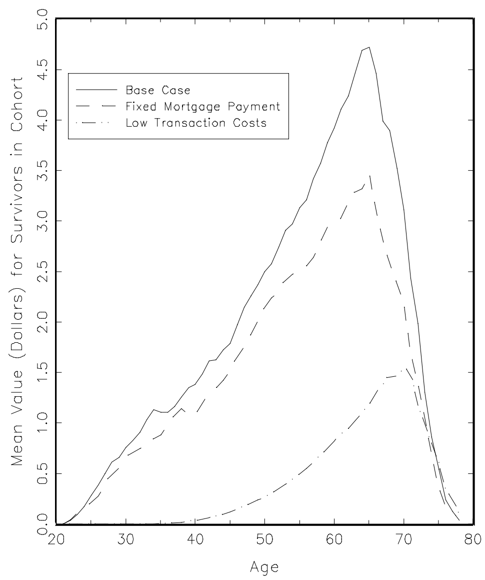


Figure 5.7b
Portfolio Share of Housing Assets
Under Alternate Mortgage Contracts

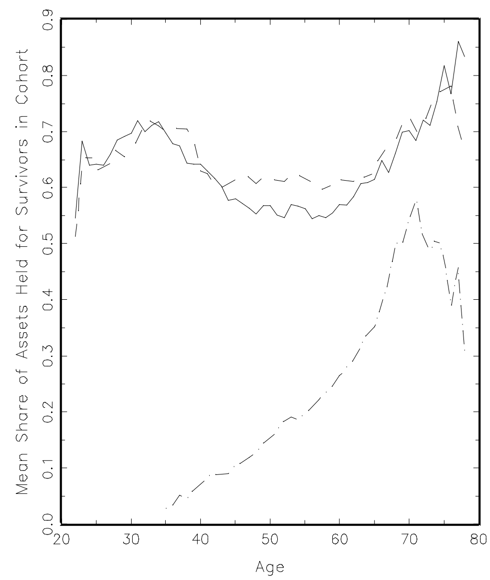


Figure 5.7c
Financial Wealth
Under Alternate Mortgage Contracts

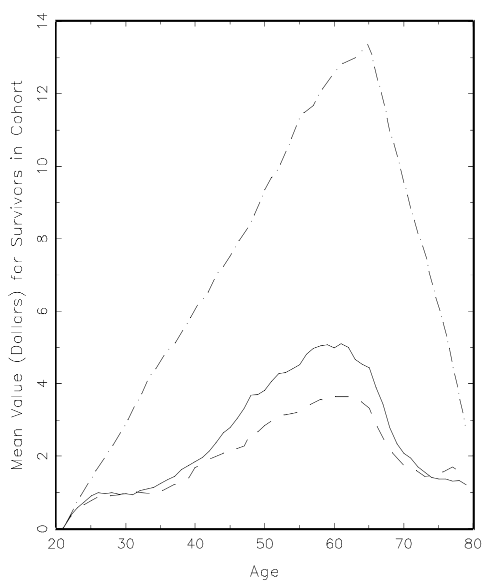


Figure 5.7d
Risky Share of Financial Portfolio
Under Alternate Mortgage Contracts

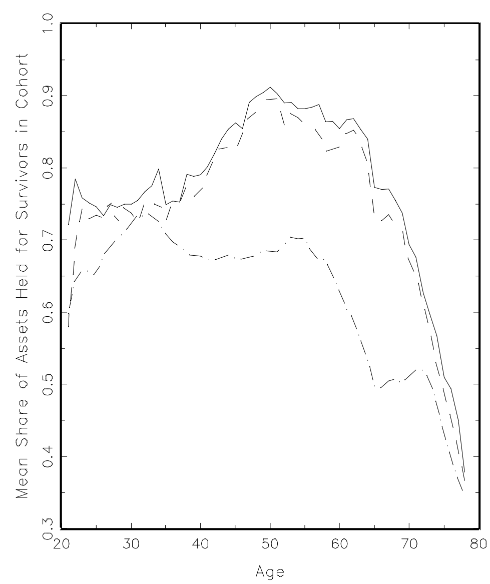


Figure 5.8a
Homeownership Rate
Under Alternate Mortgage Contracts

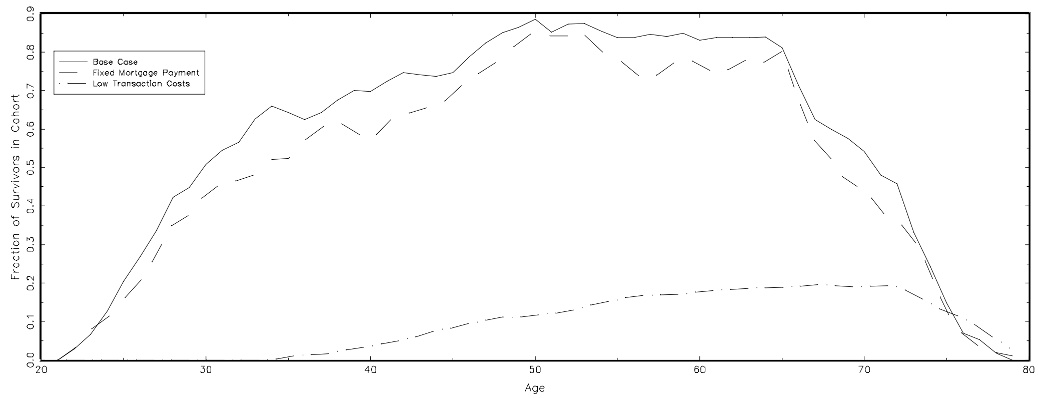


Figure 5.8b
Share Large Homes
Under Alternate Mortgage Contracts

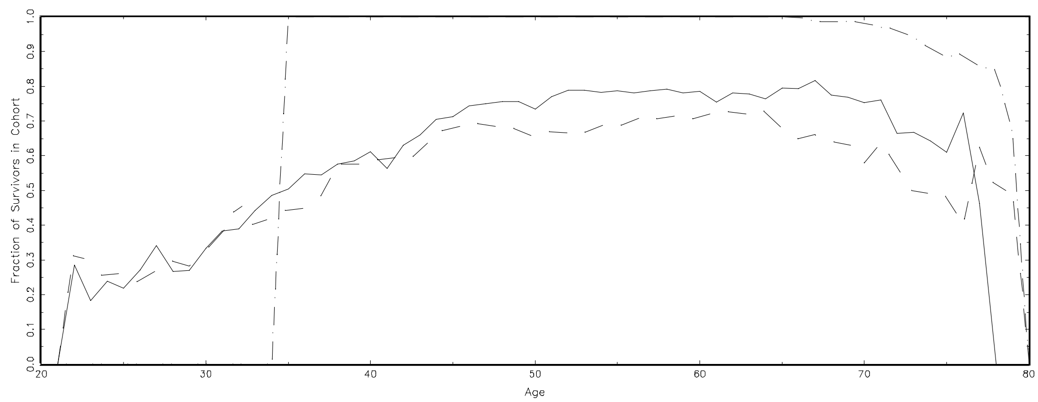


Figure 5.8c
Simulated Mobility
Under Alternate Mortgage Contracts