Testing Distributional Assumptions: A GMM Approach*

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First version: February 2002

This version: April 1, 2003

Abstract

In this paper, we consider testing marginal distributional assumptions. Special cases that we consider are the Pearson's family like the Gaussian, Student, Gamma, Beta and uniform distributions. The test statistics we consider are based on the first moment conditions derived by Hansen and Scheinkman (1995) when one considers a continuous time model. These moment conditions are valid even if the observations are not a sample of a continuous time model. We treat in detail the parameter uncertainty problem when the considered process is not observed but depends on estimators of unknown parameters. We also consider the time series case and adopt a HAC approach for this purpose. This is a generalization of Bontemps and Meddahi (2002) who considered this approach for the Normal case.

Keywords: GMM, Hansen-Scheinkman moment conditions, parameter uncertainty, serial correlation.

JEL codes: C12, C15.

*The second author acknowledges financial support from CRSNG, IFM2, and MITACS.

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1 Introduction

In many econometric models, distributional assumptions play an important role in the estimation, inference and forecasting procedures. Robust inference methods against distributional assumption are available, such as the Quasi-Maximum-Likelihood of White (1982, QML) and Generalized Method of Moments of Hansen (1982, GMM). However, knowing the true distribution of the considered random variable may be useful for improving the inference. This is the case in the stochastic volatility case where several studies showed that simulation and Bayesian methods outperform the QML method and the GMM (Jacquier, Polson and Rossi, 1994; Kim, Shephard and Chib, 1998; Andersen, Chung and Sorensen, 1999; Gallant and Tauchen, 1999). Moreover, knowing the distribution is also crucial when one forecasts non linear variables like the volatility in the EGARCH model of Nelson (1991) or the high frequency realized volatility model of Andersen, Bollerslev, Diebold and Labys (2001a, ABDL). This is also important when one evaluates density forecasts as in Diebold, Gunther and Tay (1998). This is also important in the risk management literature since the most popular measure of risk, that is the Value at Risk (VaR), is based on quantiles and, hence, on distributional assumptions. In continuous time modeling, Chen, Hansen and Scheinkman (2000) argue that an interesting approach is to first specify the unconditional distribution of the process, then specify the diffusion term. Therefore, developing tests procedure for distributional assumption diagnostics in both cross-sectional and time-series settings is of particular interest. This is the main objective of our paper.

In this paper, we consider testing distributions whose special examples are the Gaussian, Student, Gamma, Beta and uniform distribution. We already considered testing Gaussianity in Bontemps and Meddahi (2002). Those examples are chosen for their importance in the financial literature.

It is well known that exchange rates returns are conditionally heteroskedastic and that a good specification for the standardized residuals is a Student distribution (see Kim, Shephard and Chib, 1998). Therefore, Chib, Nardari and Shephard (2000) and Jacquier, Polson and Rossi (2000) used Bayesian methods to estimate such models for efficiency purposes. However, such methods are inconsistent if the Student assumption is not valid. Therefore, developing simple testing procedures is of interest.

The Gamma distribution is important in the interest rate literature. The most popular continuous time model is the square-root model of Cox, Ingersoll and Ross (1984). It turns out that the marginal distribution of this model is a Gamma distribution. Moreover, a recent paper by Ahn and Gao (1999) showed that the best scalar diffusion model for the short term interest rate is a square-root process for the reciprocal of the short term interest rate. In other words, in that case, the marginal distribution of the reciprocal of the short term interest rate is Gamma.

The test functions we consider are based on the first moments conditions given by Hansen and Scheinkman (1995). For a continuous time model, these authors gave two sets of moment conditions related to the marginal and conditional distributions respectively. Thus we consider the first class of moment conditions. Note that these moment conditions hold also when the considered random variable is not a sample of a continuous time process. When the marginal distribution is Gaussian, Hansen and Scheinkman (1995) moment conditions are known as the Stein (1972) equation; see Schoutens (2000). Bontemps and Meddahi (2002) considered this equation to test Gaussianity. Hence, this paper is a generalization of Bontemps and Meddahi (2002).

Given the tests functions, we test them by using the GMM method of Hansen (1982). We do this for two reasons. The first one is that by using the results of Newey (1985) and Tauchen (1985), the GMM framework is well suited for taking into account the parameter uncertainty problem since in general the random variable of interest is not observable. This is the case for exchange rates since the Student distribution is made on the standardized residuals. Thus, one has to first estimate the volatility model and then test the Student assumption by using the fitted residuals. An important result we derive is that in many cases, like the Gamma distribution, it is possible to characterize tests functions that are robust against the parameter uncertainty of some parameters.

The second reason of using the GMM is that this setting is appropriate for taking into account the serial correlation when the variable of interest is a times series like interest rate or high frequency realized volatility (ABDL 2001b; Barndorff-Nielsen and Shephard, 2001). We use a HAC approach in order to take into account this serial correlation. Note that this approach is related to one considered by Ait-Sahalia (1996), Conley et al. (1997) and Conley, Hansen and Liu (1997). The difference with this works is that we do not necessarily assume that the considered process is a sample of a continuous time model. Moreover, we make more attention in choosing the tests functions, for parameter uncertainty purposes.

The paper is organized as follows. Section 2 describes the methodology, the way the moment conditions are derived for testing distributional assumptions. Section 3 derives the asymptotic distribution of the tests and treat the problem of parameter uncertainty. Section 4 treats the special case of the pearson's distributions, sections 5, 6 an T treats in detail the Student, Gamma and Beta case (for which the Uniform distribution is some particular case).