

# Optimal Contracts for Teams of Money Managers

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# Optimal Contracts for Teams of Money Managers

## Abstract

The optimal organizational form and optimal incentive contract are characterized for a team of money managers, assuming that the investor (principal) is risk averse and that each manager's (agent's) actions affect both that manager's expected return and the correlation of returns between managers. If the managers are risk tolerant, then a noncooperative team organization and a strictly competitive contract, in which each manager is rewarded both for doing well and for doing better than the team, is the most efficient way to discourage herding within the team. This is despite the fact that, in such a contract total wages paid are a concave function of total returns, and so using the contract to discourage herding (and thus achieve lower risk) is in direct conflict with the investor's objective of using the contract to transfer risk onto the managers. As the risk aversion of both the investor and the managers increases, cooperation among managers becomes the optimal way to organize the team. For some parameter values, if everyone is risk averse, first-best can be achieved under cooperation. First-best without herding can never be achieved if the managers are risk tolerant, or if cooperation is infeasible.

Keywords: money managers, moral hazard, limited liability

JEL codes: G23, C72, D82, L23, G11

# 1 Introduction

This paper examines the portfolio management problem from the perspective of moral hazard in teams. As argued by Sharpe (1981) and Barry and Starks (1984), an investor may optimally prefer that her money be managed by a team of managers rather than a single manager. Sharpe's argument was based on two points. First, different managers may have expertise in different styles of investing or different assets. Second, there is a diversification benefit in that any single manager may make a serious error.<sup>1</sup> Barry and Starks (1984) argued that risk sharing concerns provide a further reason for employing multiple money managers. In support of this argument Lakonishok, Shleifer, Thaler and Vishny (1992) in a study of pension funds pointed out that: "Some plans manage their money internally, but more commonly they hire several money managers and split the pension plan's money among them."

In order for a risk averse investor, who prefers both higher expected returns and lower risk, to take advantage of employing multiple money managers these managers must diversify in their efforts. That is, they must have incentives not to herd. One method to ensure diversification might be to simply employ multiple information gatherers and a single manager, rather than multiple managers. However, as pointed out by Sharpe (1981), this may not work because those who gather the information may wish to control its use and because it may be difficult to attribute value to individual pieces of information.

Alternatively, the investor may simply divide her money among multiple managers who advertise different investment styles. While this will achieve some diversification objectives, it requires a level of management on the part of the investor herself that may go well beyond what she is willing and able to exert. For example, there are many dimensions to investment styles and much room for individual judgement when assigning potential investments to particular styles. In addition, it may be very difficult, if not impossible for an investor to observe whether a manager is engaging in the stated style of investment. Musto (1999), for example, provides evidence that money fund managers engage in "window dressing" with the

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<sup>1</sup>American Funds at their website ([www.americanfunds.com](http://www.americanfunds.com)) posts the following argument for managing their funds with teams: "Because each investment professional offers unique expertise and acts on different convictions, this approach furthers the benefits of diversification."

purpose of misrepresenting the risk of their holdings at the time of disclosure. The nature of money management is such that trading can occur easily and frequently. Thus, in order for an investor to truly know what a manager is doing, the investor would need to monitor the portfolio on a constant basis, or at the very least on a frequent and random basis. The problem is that such intense monitoring is likely to be costly. Lakonishok, Shleifer, Thaler and Vishny (1992) argue that while pension fund managers can monitor frequently, any in depth monitoring of the portfolio composition is apt to occur only at the year end, rather than throughout the year.

The objective of this paper is to propose a method for employing multiple money managers that provides them with incentives to diversify, and does not require direct monitoring of the compositions of their portfolios. The proposals put forth in this paper involve two parts: the optimal organizational form of the team and the optimal contract between the investor and managers. In terms of organizational form I will consider a “noncooperative” form in which managers are not able to monitor each other or sign side contracts and a “cooperative” form in which they can monitor and enter into side contracts. Within each form I will determine the optimal contract, as a function of the parameters, and then will determine the optimal organizational form. The contracts proposed in this paper require only that the returns of individual managers, and in some cases only that the total return of the team of managers, be observed. In order to determine these optimal contracts I design a model of moral hazard in teams in which the correlation between individual agents’ outcomes is endogenous. The contracts proposed always provide sufficient incentives for each agent to exert effort in order to achieve higher expected returns for the investor. In the cases where the investor optimally prefers the agents to diversify in their efforts (this depends on the investor’s risk attitude), the contract also provides sufficient incentives to encourage this.

A main result of the paper is that the optimal organizational form of the team and the structure of the optimal wage contract depend on the risk attitudes of both the investor and the money managers. This makes intuitive sense because diversification affects the riskiness of the total portfolio return. Thus, the most efficient way in which to discourage herding (encourage diversification) will depend very much on the risk attitudes of the managers.

In addition, the contract between the investor and managers serves not only to provide incentives for the managers to take particular actions, but also is a vehicle for risk sharing between the investor and managers. It is for this reason that the investor's risk attitude also affects the design of the optimal contract.

If the money managers are relatively risk tolerant, then the most efficient way to provide incentives for the managers to not herd, is with a strictly competitive contract (and noncooperative organizational form). Under such a contract a manager is rewarded for doing strictly better than other managers, something that is less likely to occur if that manager herds. The downside of such a competitive contract is that the total wages paid by the investor are a concave function of the total returns, and as such, the contract does not optimally transfer risk to the managers. Thus, when managers are risk tolerant there is a tension between the objective of providing incentives for the managers to lower the portfolio risk by not herding, and the objective of transferring risk to the managers.<sup>2</sup> Despite this tension, if diversification across managers is valuable (for decreasing portfolio risk), then the competitive contract will be optimal when managers are risk tolerant.

Even though competitive contracts do not optimally transfer risk from the investor to the managers as a group, such contracts do impose risk on individual managers. This is because the nature of the contract is such that some managers are winners and some are losers. If the managers are risk averse, then they must be compensated for taking on this risk, thus adding to the expense of the competitive contract. Fortunately, as the managers become more risk averse, their incentives with regard to diversification can be more easily aligned with those of the investor. As is shown in the paper, the first-best contract calls for paying wages that are based only on the team performance, not on individual managers' performances. If the managers are risk averse enough, then this is also second best. In this case, the cooperative organizational form is optimal and the investor optimally offers a single team contract, rather than individual competitive contracts. Related to this, it is also shown that if the managers are risk averse enough, for some parameter values, first best

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<sup>2</sup>If the managers are risk tolerant one might consider transferring the risk entirely to them so that the investor receives a guaranteed return. The problem with this is that it would require the managers in some states to pay out to the investor more than they earn in their investments. This is not a reasonable assumption for money management, and so I instead make a limited liability assumption that rules out this solution.

can be achieved. First best with nonherding cannot be achieved if the managers are risk tolerant.

In practice, we observe fund companies that are organized on different models. Fidelity, for example, is organized around a “star” system in which each fund has only one manager. American Funds, on the other hand, uses a team organization for their funds. In terms of compensation contracts we don’t know how these are structured.<sup>3</sup> There is one study, however, that provides some intriguing related evidence. Chevalier and Ellison (1999) examine career movements of fund managers and their investment behavior. They provide evidence on the relation between nonherding, success or failure, and termination or promotion. They find that, in terms of the probability of termination, the cost of nonherding (deviating from other managers) and failing is significantly greater than the benefit of nonherding and succeeding. But, in terms of the probability of promotion the opposite holds: the benefit of nonherding and succeeding is significantly greater than the cost of nonherding and failing. Putting these two results together, whether a manager wants to herd will depend on the relative strength of each of these incentives and on the risk attitude of the manager. If the manager is risk averse enough, then such a reward structure will certainly provide incentives to herd. The results presented in this paper are consistent with this result in that, if managers are highly risk averse, then a competitive incentive structure does not optimally discourage herding.

This paper contributes both to the literature on money management and to the moral hazard in teams literature. Sharpe (1981) and Barry and Starks (1984) provide arguments for why multiple money managers may be optimal. Barry and Starks (1984) approach this problem as a moral hazard problem in that they show that providing risk averse money managers with the proper incentives may be less expensive with multiple managers, than with a single manager. Starks (1987), Grinblatt and Titman (1989), Stoughton (1993), Admati and Pfleiderer (1997) and Dybvig, Farnsworth and Carpenter (2001) all examine compensation contracts for single money managers. Brown, Harlow and Starks (1996) find evidence of risk shifting of money managers, perhaps due to the tournament (competitive)

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<sup>3</sup>Deli (2002) studies contracts between funds and firms that provide advisory services to the funds. He finds that 93% of his sample have advisory contracts based solely on a percent of assets. However, these advisors are not the managers of the funds.

nature of the money management industry in which money managers are rewarded for doing better than competitors. None of these papers consider the diversification/herding problem that is modelled here.

Holmström (1982) brought the moral hazard in teams problem to the forefront by showing that when the results of individual agents' efforts cannot be observed, so that the fruits of these efforts must be shared with the entire team, then this causes difficulties in the design of the contract. In the model developed here, even though the returns on individual investments may be observed by the investor, individual contributions to the overall riskiness of the portfolio cannot be observed. Thus, the moral hazard in teams problem arises with regard to discouraging herding within the team.

This is not the first paper to provide conditions under which a competitive contract is optimal. However, optimality of a competitive contract arises in this paper for different reasons than in earlier literature. Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), Mookherjee (1984) and Ramakrishnan and Thakor (1991) all show that a competitive contract may be optimal for encouraging effort when individual returns are highly correlated. Green and Stokey (1983), Nalebuff and Stiglitz (1983) and Ramakrishnan and Thakor (1991) show further that if individual returns are uncorrelated then competition does not add value. In this paper, however, if the investor optimally wishes to encourage diversification (low correlation among individual returns), then the optimal contract may be competitive. That is, competition may be an essential element of a contract that discourages money managers from herding in order to achieve low correlation between their returns.

Of the papers cited above, this paper shares most in common with Ramakrishnan and Thakor (1991) in that both examine forms of contracts with multiple agents under different assumptions regarding competition or cooperation.<sup>4</sup> This paper differs from Ramakrishnan and Thakor (1991) in two very important dimensions. First, the principal (investor) here is risk averse while Ramakrishnan and Thakor's principal is risk neutral. Second, each of the money managers makes two decisions: whether to herd and whether to exert effort, whereas in Ramakrishnan and Thakor there are two tasks that may be performed individually or shared by two agents.

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<sup>4</sup>Macho-Stadler and Pérez-Castrillo (1993) also consider cooperation, but with a different approach.

Most of the earlier multiple agent literature assumes that the principal is risk neutral. Two exceptions to this are Barry and Starks (1984) and Stoughton and Zechner (1999).<sup>5</sup> The latter paper examines optimal compensation schemes for division managers who can choose the variance of their divisional investments. They, however, take the correlation between divisional returns as exogenously given, whereas this paper treats correlation as endogenous. Also, in their mechanism design problems the wage contract offered by the principal must satisfy incentive compatibility for the agents on one dimension: either choice of standard deviation, or truthful revelation of information. In the problem posed here the wage contract must satisfy incentive compatibility for the agents on two dimensions: nonherding and effort choice. The extra dimension affects the contract design.<sup>6</sup>

The following section describes the model. Section 3 provides some groundwork by presenting the optimal contract, given that the investor is risk neutral and does not optimally discourage herding. The results of the paper are presented in Sections 4 through 6 in which it is assumed that the investor is risk averse. Section 4 examines the optimal contract with a noncooperative organizational form. Section 5 examines the optimal contract with a cooperative organizational form. In Section 6 the different types of organizational forms and contracts are compared. It is this section that presents the main general results of the paper. All derivations and proofs are in the Appendix. The Appendix also includes a list of variable definitions.

## 2 The Model

The model presented here is a very stylized model of money management. Because solving for optimal contracts with multiple risk-averse agents can be very difficult, the components of the model are kept as simple as possible. There is exactly one investor in the model and two money managers. Each money manager will choose an asset class exactly once, and will

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<sup>5</sup>Stoughton and Zechner do not specifically refer to their principal as risk averse. However, their principal's objective function is decreasing in the variance of total returns across agents.

<sup>6</sup>There are also a number of related papers that allow agents to affect the distributions of outcomes. Diamond (1998) and Palomino and Prat (2002) allow a single agent to choose both an effort level and the distribution of the outcome. In Khanna (1998) there are multiple principal/agent (one principal and one agent) relationships in which the agents choose whether to herd. In all of these papers the principal and agent are risk neutral.



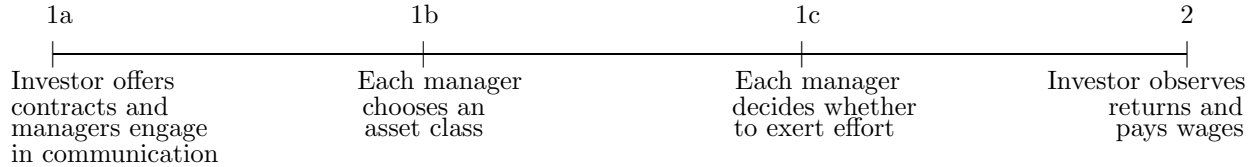


Figure 1: Timeline

not change that choice. As discussed in the introduction, if each manager really only chose her asset class once, and then stayed with that choice, then the investor could monitor this choice. The assumption of a one-time choice is made here only in the interest of modelling tractability. It is assumed in this model that monitoring is infeasible.

Figure 1 presents the timeline. I assume that the organizational form is chosen before time 1a. At time 1a the principal/investor offers a wage contract either individually to each money manager, or to the team as a whole. Individual contracts may be functions of individual performance, team performance, or a combination of the two. Each money manager may accept or reject the contract. It will be assumed that all of the bargaining power lies with the investor, so a contract offer is a take-it-or-leave-it offer. As such, a manager will accept as long as she is not made worse off by the contract.

After accepting the contract each manager chooses an “asset class” in which to invest. Before making this choice managers in the team talk with each other, so that they may coordinate their choices.<sup>7</sup> Neither these conversations, nor the subsequent actions taken by the managers, can be observed by the investor. Under the noncooperative organizational form, the managers are also unable to monitor each other. Thus, under this organizational form the communication is “cheap talk”, i.e. “communication ...that is costless but which nonetheless might be useful in coordinating actions” (Kreps (1990), page 388). Thus, the agreed upon actions must be self enforcing in that each manager must have no incentives to deviate from these actions.

Each manager decides, at time 1c, whether to exert effort in order to obtain information about the possible investments within her asset class. Doing so will increase the expected return of her investment. The manager then makes an investment. At time 2, returns on

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<sup>7</sup>This is based on the very realistic assumption that money managers do talk to each other, especially if they are in the same firm.

investments are realized. The investor observes neither the asset class nor effort choices, but can observe individual returns. The wage(s) paid at this time may be individual wages or a single wage paid to the team, in which case the managers will then divide the wage.

The investor wishes to maximize the following objective function:

$$E[R - W] - \psi var[R - W] \tag{1}$$

where  $R$  is the total return across the two managers,  $W$  is the total wage paid and  $\psi$  is an exogenously given nonnegative constant. This function takes the form of a certainty equivalent resulting from a negative exponential utility function, with the coefficient of absolute risk aversion equal to  $2\psi$ :  $-exp(-2\psi(R - W))$ , where  $R - W$  is Normally distributed. As described below, the variables here are not assumed to be Normally distributed, so equation (1) may be viewed simply as a reduced form representation that captures the risk–return tradeoff.

Each manager wishes to maximize the following:

$$E[w|x, e] - c_x(e) - \alpha var[w|x, e] \tag{2}$$

where  $w$  is the individual wage,  $x$  is an index for the asset class chosen by the manager,  $e \in \{0, 1\}$  is the level of effort exerted,  $c_x(e)$  = cost of exerting effort  $e$  in class  $x$  ( $c_x(0) = 0, \forall x$ ), and  $\alpha$  is an exogenously given nonnegative constant.

The asset classes differ in their costs of exerting effort and in their cross-correlations of returns. It is assumed that there is a single asset class, asset class one, which has a lower cost of effort than all other classes. Optimal portfolio diversification, however, will require that the managers invest in different classes and that each of these classes be something other than class one. The assumption of one asset class that is different from all other classes is made largely for reasons of modelling tractability, in that this assumption will make it easier to write the incentive compatibility constraints. This assumption does not drive the results. But, it is also not far from reality in that this asset class may be thought of as something like an index. From this point forward I will say that a manager chooses not to herd when that manager chooses an asset class other than class 1.

To further simplify and clarify the model, the following assumptions are made:

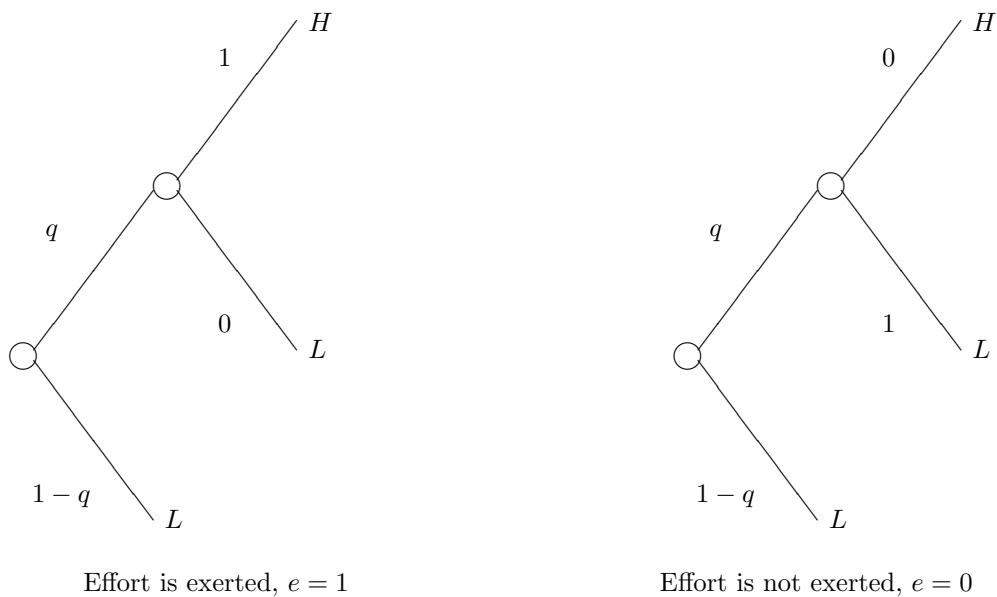


Figure 2: Probability Trees

- Each manager's investment return can take on one of two values: 0 or 1. Thus, the total return  $R \in \{0, 1, 2\}$ .
- The unconditional distribution on return for each manager's investment (not conditioned on the other manager's return) is as follows: If the manager exerts effort ( $e = 1$ ), then 1 is realized with probability  $p$  and 0 with probability  $1 - p$ . If the manager does not exert effort ( $e = 0$ ), then 1 is realized with probability zero. These probabilities are identical across asset classes.
- Asset class one has cost of effort =  $c_1$ .  $\forall x \neq 1, c_x = c_2$ .  $c_1 < c_2$ .  $c_x$  is the cost that each manager must expend in order to stochastically improve the expected return of her investments, if she is investing in assets of type  $x$ . The per manager cost of diversification is the extra cost of effort,  $c_2 - c_1$ . The idea behind this is that if a manager does something different from other money managers, then she will need to work a bit harder in order to do well.
- Cross correlation coefficients are defined assuming that all money managers exert effort. The interaction between these correlation coefficients and effort levels is developed below. If the two managers both invest in asset class one, then the correlation coefficient of their returns is 1. If they invest in different classes, neither of which is class 1, then the correlation coefficient of their returns is zero:  $\rho_{yx} = 0, \forall x \neq y$  and  $y, x \neq 1$ . If they invest in different classes, one of which is class 1, then the correlation coefficient of their returns is:  $\rho_{1x} = \rho, \forall x \neq 1, \rho > 0$ . Given a two-person team, these assumptions place the following restriction on  $\rho$ :  $0 < \rho < \sqrt{.5}$ .
- Attention will be restricted to parameter values such that  $p \geq c_2$  so that the benefit to exerting effort is at least as large as the cost.<sup>8</sup>

<sup>8</sup>Two additional parameter restrictions are discussed at the beginning of the Appendix:  $\psi, \alpha \leq 1/2$  and  $p \geq 1/4$ .

The probability trees in Figure 2 help to illustrate the interaction between the correlation coefficients and effort levels.  $H = 1$  is a good outcome;  $L = 0$  is a bad outcome. The first node in each of the trees represents the outcome for the asset class. Each asset class experiences a good outcome with probability  $q$  and a bad outcome with probability  $1 - q$ . The second node represents the outcome for a given money manager who has either exerted effort or has not. If effort is exerted ( $e = 1$ ), then a good outcome ( $H$ ) is achieved with probability  $q$ . If effort is not exerted ( $e = 0$ ), then a good outcome is achieved with probability 0. Any dependence between managers' outcomes occurs through the correlation of their asset classes. Unconditioned on the other manager's performance,  $q = p$ . Conditioned on the other manager's asset class experiencing a good outcome,  $q = p + \rho(1 - p)$ .

There are four possible outcomes for team returns:  $HH \equiv$  both managers get good outcomes;  $LL \equiv$  both managers get bad outcomes;  $HL \equiv$  the first manager gets a good outcome and the second a bad;  $LH \equiv$  the first manager gets a bad outcome and the second a good. If both managers exert effort, then:

$$\begin{aligned} \text{prob both get } H = p_{HH} &= p^2 + \rho p(1 - p) \\ \text{prob both get } L = p_{LL} &= (1 - p)^2 + \rho p(1 - p) \\ p_{HL} = p_{LH} &= (1 - \rho)p(1 - p) \end{aligned}$$

If one manager exerts effort, while the other doesn't:

$$\begin{aligned} \text{prob both get } H = p'_{HH} &= 0 \\ \text{prob both get } L = p'_{LL} &= p_{LL} + p_{HL} = 1 - p \\ \text{prob shirker gets } L \text{ while worker gets } H = p'_{LH} &= p_{HL} + p_{HH} = p \\ \text{prob shirker gets } H \text{ while worker gets } L = p'_{HL} &= 0 \end{aligned}$$

## 2.1 Team Organization

The problem is solved under two different assumptions regarding the organizational form of the team of managers. The first organizational style considered is "noncooperative". With this style of organization the team members are either unable to observe each others' actions, or if they can observe them, are unable to verify their observations in a way that

would enable them to write contracts with each other based on such observations. The role of the team under this assumption is two-fold: i) Managers can engage in “cheap talk” prior to selecting their asset classes. ii) Wages paid to individuals may be based on the performance of the team as a whole. The reason that the pre-play communication is referred to as cheap talk is that the communication is costless and managers are unable to monitor each other. It is also assumed that the investor is unable to monitor team members’ actions. Thus, coordination through cheap talk is beneficial to the investor only if it is incentive compatible for each manager to agree (in the cheap-talk communication) to take actions preferred by the investor, and for each manager to then follow the agreed upon strategy. It is assumed that the investor can observe individual outcomes and so can pay wages to individuals that are based either on individual or team performance, or a combination of both.

The second organizational style considered is “cooperative”. Under this type of organization, team members can monitor each others’ actions and can sign side contracts to share wages. Managers can thus coinsure. It is again assumed that the investor cannot observe any actions, either of the individuals or the team as a whole. Because the managers can make side contracts, the investor will pay one wage to the team as a whole.

In Sections 4 and 5 the optimal contract is characterized for each of these types of team organization. In Sections 5 and 6 conditions are determined such that one or the other organizational form is optimal. In all cases the optimum is determined from the perspective of the investor.<sup>9</sup> In the following subsection the constraints for the noncooperative team organization are developed. The corresponding constraints for the cooperative organization are developed in Section 5.

## **2.2 Constraints under Noncooperation**

Because of the symmetry of the problem (all type 2 asset classes have equal costs of exerting effort and identical distributions on outcomes), it is not necessary to model the communication stage of the game. What is necessary is to determine the optimal contract such that it is incentive compatible both for the managers to agree in their communication to engage

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<sup>9</sup>The managers’ preferences regarding organizational form are discussed in Section 6.

in activities that the investor prefers, and for the managers to then follow the agreed upon actions.

The contract that is offered to each manager is defined as a vector of values:  $\{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}$ , where  $w_{ab}$  is the wage paid to a money manager whose investments return  $a$  while the other manager realizes a return of  $b$ . The optimal contract will result in the largest possible value of equation (1), subject to satisfying the following constraints:

Individual Rationality (IR) for each manager:

$$E[w|x, e] - \alpha var[w|x, e] \geq c_x(e) \quad (3)$$

Satisfying this constraint ensures that the manager will be willing to participate.

Incentive compatibility for exertion of effort (effort-IC):

$$\begin{aligned} E[w|class_x, e = 1] - \alpha var[w|class_x, e = 1] - c_x &\geq \\ E[w|class_x, e = 0] - \alpha var[w|class_x, e = 0] \end{aligned} \quad (4)$$

where  $c_x$  is shorthand for  $c_x(1)$ .

If the investor optimally wishes that the managers not herd (each chooses an asset class of type 2), then the following incentive compatibility constraint (nonherding-IC) must be satisfied:

$$E[w|class_2, e] - \alpha var[w|class_2, e] - c_2 \geq E[w|cl_1, cl_2, e] - \alpha var[w|cl_1, cl_2, e] - c_1 \quad (5)$$

where  $class_2$  means that both managers invest in assets of type 2 and  $cl_1, cl_2$  means that one manager invests in an asset of type 1 and the other in an asset of type 2.<sup>10</sup>

It is assumed that money managers have limited liability so that the wage in each state must be nonnegative:

$$w_{HH}, w_{HL}, w_{LH}, w_{LL} \geq 0. \quad (6)$$

When characterizing contracts we can say that *the wage contract is flat* if  $w_{HH} = w_{HL} = w_{LH} = w_{LL}$ . If the wage contract is not flat, then *the wage contract is based only on individual*

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<sup>10</sup>In general, it is also necessary that both constraints be jointly satisfied. I.e., it must be incentive compatible for each manager to choose a type 2 asset class and exert effort rather than choose asset class one and not exert effort. However, for the parameter assumptions made here (zero probability of good outcome with no effort) this joint constraint is redundant.

*performance* if  $w_{HH} = w_{HL}$  and  $w_{LH} = w_{LL}$  and *the wage contract is based only on team performance* if  $w_{HL} = w_{LH}$ .

Because it is assumed that a manager who exerts no effort adds no value (the probability of a good outcome with zero effort is zero), the investor will always wish that managers exert effort. We must consider, however, the possibility that the cost of satisfying the nonherding constraint may be higher than the benefit, in which case the investor will optimally offer a wage contract that does not satisfy this constraint. Most of the analysis of the paper will focus on parameter values such that the investor does optimally want the managers not to herd. But, when determining the optimal contract as a function of risk attitudes, it will be necessary to allow the investor to choose an undiversified portfolio. The following section provides the necessary groundwork to do this.

### 3 Optimal Contracts with Herding

In this section it is assumed that  $\psi = 0$  so that the investor cares only about maximizing expected net return:  $E[R - W]$ . Thus, the investor receives no benefit from portfolio diversification and optimally prefers the money managers to both invest in asset class one. As such, this section covers the smaller, more standard, problem in which the agents have only one dimension to their action space: effort choice.

In a first-best world the investor will be able to observe whether each manager has taken the desired actions. Thus, the optimal first-best wage contract need only satisfy the individual rationality and limited liability constraints: (3) and (6). If the managers are also risk neutral, so that  $\alpha = 0$ , then the first-best contract can take on any form such that  $E[w] = c_1$ . If the managers are risk averse ( $\alpha > 0$ ), then the first-best contract is a flat wage. If the wage contract is not flat, then the investor must pay an expected wage that is greater than  $c_1$  in order to compensate the managers for taking on risk.

If the investor cannot observe the money managers' actions, then the investor is faced with a moral hazard problem and must design a wage contract that satisfies the effort constraint. If the managers herd and exert effort, then the probabilities of the various outcomes are:  $p_{HL} = p_{LH} = 0$ ,  $p_{HH} = p$  and  $p_{LL} = 1 - p$ . Because the state  $HL$  cannot occur if

both managers exert effort, the cooperative organizational form is not relevant when the investor does not wish to encourage diversification. Thus, the optimal contract need only be determined assuming noncooperation.

The effort constraint, given that the managers herd, is<sup>11</sup>

$$w_{HH} - w_{LH} + \alpha(1 - p) \left( w_{LH}^2 - w_{HH}^2 + 2w_{LL}(w_{HH} - w_{LH}) \right) \geq \frac{c_1}{p}. \quad (7)$$

It is clear that if the managers are risk neutral ( $\alpha = 0$ ), then the optimal contract will have  $w_{HH} = \frac{c_1}{p}$  and  $w_{HL} = w_{LH} = w_{LL} = 0$ . First-best can be achieved because we have assumed that the probability of success, given no effort, is zero.

If the managers are risk averse, then first best cannot be achieved. In the optimal second-best contract the investor takes advantage of the fact that the high correlation between the managers' returns acts somewhat as a monitoring device and the investor optimally sets  $w_{LH} = 0$ . (As can be seen from the above equation, the value of  $w_{HL}$  is irrelevant.) The difference between the wages in the states  $HH$  and  $LL$  depends on the level of the managers' risk aversion. If  $\alpha$  is low enough, then  $w_{LL}$  is set to zero.

In general, the second-best contract with a risk-neutral investor will be based on a combination of individual and team performance. The risk-neutral investor's objective function is an increasing function only of individual performance. However, in a world with moral hazard, team performance provides additional information as to whether an agent has exerted effort. For this reason, the second-best contract has a team component. In fact, consistent with results in Ramakrishnan and Thakor (1991), even if diversification costs nothing directly ( $c_2 = c_1$ ), a risk-neutral investor will prefer that money managers not diversify. This is because of the monitoring value of having the managers' outcomes highly correlated. If the investor is risk neutral, then it is to the investor's advantage to have managers engaging in highly correlated tasks. In the remaining sections it is assumed that the investor is risk averse so that the investor may wish to give up this monitoring device in the interest of portfolio diversification.

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<sup>11</sup>See the Appendix for derivations.



## 4 Optimal Nonherding Contracts, with a Noncooperative Organization

A risk-averse investor cares about portfolio diversification. The investor will want the money managers to avoid herding, if the cost is not too high relative to the investor's aversion to risk. The cost to not herding takes two forms: a higher cost of effort ( $c_2 > c_1$ ) for which the managers must be compensated, and the investor's decreased ability to monitor effort by observing outcomes.

In addition to encouraging diversification, a risk-averse investor will wish to use the wage contract to pass some of the risk onto the money managers. This is done by paying total wages that are an increasing, convex function of total returns. The first-best wage contract when the investor is risk averse and the managers are risk neutral takes a very simple form: Nothing is paid in the state  $LL$ . If both managers invest in asset classes of type 2, then  $w_{HH} = \frac{c_2}{p_{HH}}$  and  $w_{HL} = w_{LH} = 0$ , as long as  $2c_2 \leq p_{HH}$ . If  $2c_2 > p_{HH}$ , then wages are spread across the states  $HH$  and  $HL$ , but  $w_{HH} > w_{HL} + w_{LH}$ , so that the contract is convex.<sup>12</sup> In a first-best world the investor need not worry about providing incentives for not herding. Thus, as is shown in the Appendix, as long as  $c_2$  is not too much higher than  $c_1$ , a risk-averse investor strictly prefers that the managers diversify.

A contract which pays strictly higher wages for higher outcomes is consistent with encouraging effort on the part of the money managers, but it may be inconsistent with encouraging diversification. The next section examines the optimal contract in the presence of moral hazard when the investor is risk averse and the money managers are risk neutral.

### 4.1 Second Best when Managers are Risk Neutral

A risk-averse investor will strictly prefer that the managers not herd, as long as  $\psi$  (the investor's risk aversion coefficient) is large enough relative to  $c_2 - c_1$ . In this case, the nonherding incentive compatibility constraint is strictly binding.<sup>13</sup> Satisfying this constraint when the managers are risk neutral will require that total wages be strictly concave in total returns:  $w_{HL}^* + w_{LH}^* > w_{HH}^*$ . This is clearly not first best. As shown above, the first-

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<sup>12</sup>The precise contract is derived in the Appendix.

<sup>13</sup>Given that the managers are risk neutral.

best contract when the managers are risk neutral is strictly convex. Providing incentives for money managers to diversify is thus in conflict with the investor’s desire to use the wage contract to directly reduce risk by passing the risk onto the managers. Thus, we obtain the first proposition.

**Proposition 1.** *First best cannot be achieved if nonherding is first-best optimal and the money managers are risk neutral.*

The first-best contract when only the investor is risk averse may be based entirely on team performance. The second-best contract specifies, however, that  $w_{HL} > w_{LH}$  and that  $w_{HL} > w_{HH}$ . Thus, the contract that is offered in the presence of moral hazard is a function both of team and individual performance. Each manager is rewarded for achieving a good outcome and for achieving a better outcome than the other manager.

**Proposition 2.** *If the money managers are risk neutral, then the second-best contract that discourages herding is a “competitive” contract, in that each manager’s compensation is decreasing in the other manager’s performance ( $w_{HH} < w_{HL}$ ). Total wage paid in this contract is a strictly concave function of total returns earned by the team of managers.*

The contract takes the form of a competitive contract for a different reason than in the existing literature.<sup>14</sup> Earlier literature has pointed out that if agents’ outcomes are highly correlated with each other, then a competitive contract can efficiently encourage effort, and if agents’ outcomes are independent then a competitive contract provides no benefit in encouraging effort. In this model, if the managers optimally diversify across their tasks, then their outcomes are independent. The strictly competitive aspect of the contract is used to encourage managers to diversify, so that their outcomes will be independent.

## 4.2 First Best when Managers are Risk Averse

The first-best contract is the contract that optimally allocates risk between the investor and managers. The investor lowers her own risk by designing a contract with a positive covariance between total wages and total investment returns. The problem is that such a contract increases the risk for the managers, and the investor must compensate the managers

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<sup>14</sup>See the Introduction for a discussion of the literature.

for taking on risk by offering them a higher expected wage. The optimal first-best contract balances these two concerns. Such a contract is based entirely on team performance ( $w_{HL} = w_{LH}$ ), because adding an individual component to the contract ( $w_{HL} \neq w_{LH}$ ), increases the variance of individual wages, without affecting the covariance of total wages with total returns. Thus, *when money managers are risk averse, it is strictly suboptimal in a first-best world to base wages even partially on individual performance.*<sup>15</sup>

The first-best risk-sharing contract will be linear in total returns, unless the limited liability constraints, which prevent the investor from paying a negative wage in the event of the worst outcome, are strictly binding. If the limited liability constraints are nonbinding, then the optimal risk-sharing contract takes the form:<sup>16</sup>

$$w_{HL} = w_{LH} = x \quad w_{HH} = x + \left( \frac{\psi}{\alpha + 2\psi} \right) \quad w_{LL} = x - \left( \frac{\psi}{\alpha + 2\psi} \right) \quad (8)$$

where  $x$  is such that Individual Rationality is exactly satisfied. If limited liability is binding then the optimal contract takes the form:

$$w_{HL} = w_{LH} = x' \quad w_{HH} = \frac{2}{1+p} \left( \frac{\psi}{\alpha + 2\psi} + px' \right) \quad w_{LL} = 0 \quad (9)$$

where  $x'$  is such that Individual Rationality is exactly satisfied. It is shown in the Appendix that in this case  $x' < \frac{\psi}{\alpha + 2\psi}$ , so that  $w_{HH} > 2w_{HL}$ . The first-best contract is thus either linear or convex in total returns.

### 4.3 Second Best when Managers are Risk Averse

A risk-averse investor wishes to align wages with returns, so that the total wage paid is strictly increasing in total return. This objective is consistent with providing managers with incentives to exert effort. However, as shown in section 4.1, if the managers are risk neutral, then it is inconsistent with providing incentives for managers to diversify. When managers are risk neutral, satisfaction of the nonherding-IC constraint requires that the investor pay strictly more in states where the managers have different outcomes:  $w_{HL} + w_{LH} > w_{HH} + w_{LL}$ . This is in conflict with the investor's objective of minimizing her own risk.

<sup>15</sup>The first-best contract is derived in the Appendix and this statement is proved.

<sup>16</sup>Limited liability is nonbinding iff  $\frac{c\alpha}{p} \geq 2 \left( \frac{\psi}{\alpha + 2\psi} \right) \left( 1 - (1-p) \left( \frac{\alpha\psi}{\alpha + 2\psi} \right) \right)$ . The contract presented here for limited liability nonbinding takes the same form as the risk-sharing contract presented in Barry and Starks (1984).

In this section the optimal contract is characterized only for the case such that the investor optimally prefers that the managers not herd.<sup>17</sup> When managers are risk averse, the investor's and the managers' incentives become better aligned in this regard. This is because, under the first-best contract, a manager who deviates and herds causes (for most parameter values) the variance of her wages to increase as a result of herding. This is because under the first-best contract wages are based only on team outcome:  $w_{HH} > w_{HL} = w_{LH} \geq w_{LL}$ . If a manager deviates and herds, then some probability weight shifts from the set of outcomes within which the manager receives a constant wage ( $\{HL, LH\}$ ) to the set of outcomes within which wage is not constant ( $\{HH, LL\}$ ), thus increasing the variance of individual wage.<sup>18</sup> This increase in variance, together with the managers' aversion to risk, enables the first-best contract to satisfy the nonherding incentive compatibility constraint for some parameter values. The following proposition states this result, but also points out that this is not sufficient for first best to be attained.

**Proposition 3.** *If both the investor and the money managers are risk averse, and the managers do not cooperate, then:*

- *there exist parameter values such that the first-best wage contract provides sufficient incentives for the managers not to herd.*
- *However, first best still cannot be achieved.*

It is only when the managers are risk averse that it is possible for the first-best contract to provide sufficient incentives not to herd. A first-best contract that is not highly convex will satisfy the nonherding incentive compatibility constraint, if the managers are risk averse and the incremental cost of exerting effort when not herding is not too high. However, any contract, such as the first-best contract, that is based only on team performance and that just satisfies the participation (incentive rationality) constraint will not satisfy the effort

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<sup>17</sup>In Section 6 numerical examples are presented that include cases where the investor optimally prefers not to pay the extra cost of diversification.

<sup>18</sup>It is shown in the proof of Proposition 3 that a deviation from the investor's first-best asset choice causes the managers' variance of wage to increase for all but very extreme parameter values. It is only in the case that the wage contract is highly convex (very little wage is paid in any state other than  $HH$ ) and the probability of state  $HH$  is quite high, that such a deviation will lower the managers' wage variance. A sufficient, but not necessary, condition for the managers' variance of wage to increase is  $p \leq 0.6642$ . Even if this condition does not hold, the variance will increase, unless in addition  $c_2$  is very small and/or the investor is much more risk averse than the managers.

incentive compatibility constraint. For this reason, even if incentives are aligned with regard to asset choice, first best cannot be achieved.

Finally, we can investigate the form of the second-best contract when both the investor and money managers are risk averse. We know that the first-best contract is based entirely on team performance. In the presence of moral hazard the optimal contract may have an individual component, but it will never be based entirely on individual performance. The reason is that a contract based only on individual performance cannot provide incentives for not herding.

**Proposition 4.** *If both the investor and money managers are risk averse, and the team is noncooperative, then the second-best contract that encourages diversification:*

- a) cannot be based entirely on individual performance.*
- b) is based entirely on team performance if and only if the effort constraint is nonbinding.*

*However, in this case total wage paid is still a strictly concave function of total returns.*

The second-best contract will generally be based on both individual and team performance. It is necessary to have a team component to the contract in order to discourage herding. If money managers are risk neutral, then in order to discourage herding the second best contract must include a “competitive” element. If the managers are risk averse, the second-best contract may be based only on team performance, without a competitive element:  $w_{HH} \geq w_{HL} = w_{LH} \geq w_{LL}$ . But, even if the contract is noncompetitive, it will be similar to the competitive contract in that total wages paid to the team are strictly concave in total returns. (Keep in mind that the first-best risk-sharing contract is convex.) In a concave contract,  $w_{LH} - w_{LL} > w_{HH} - w_{HL}$ . Thus, a contract that is both noncompetitive ( $w_{HH} - w_{HL} \geq 0$ ) and concave induces the managers to take the desired actions mainly by punishing the team in states such that both managers do poorly (low value for  $w_{LL}$ ), rather than rewarding individuals.

## 5 Cooperation between Managers

Up to this point it has been assumed either that the managers cannot observe each others' actions, or if they can, such observations are unverifiable so that the managers are unable to write contracts among themselves based on such observations. Under this assumption the only means that the managers have for coordinating their actions is through "cheap talk" communication. In this section an alternative team organization is investigated in which the managers may "cooperate" among themselves. The notion of cooperation employed here is a somewhat limited version of that employed by Ramakrishnan and Thakor (1991). In that paper agents can not only monitor each other's efforts and share payments, but they can also share in the completion of tasks. Each manager here must still complete her own task separately, but the managers can monitor each other and can sign side contracts to share wages. I.e., managers can coinsure. As in the rest of the paper, it is still assumed here that the investor is unable to observe any actions taken by the managers. The purpose of this section is to determine whether, and under what conditions, the investor is better off with this alternative organizational form.

Because the managers can sign side contracts, the investor pays a single wage to the team and the money managers agree among themselves as to how they will share the wage. The managers can also make agreements regarding who will take which actions. For this reason, the moral hazard problem becomes a bit more complicated in that the contract must now satisfy two nonherding incentive compatibility constraints and two effort incentive compatibility constraints. This is because it must be incentive compatible for both managers to take each desired action versus only one doing so *and* versus neither doing so.<sup>19</sup> The precise constraints are presented in the proof of Proposition 5 in the Appendix.

Allowing for cooperation does not change the first-best contract. This contract is based only on total team performance. It is only in the second-best world, where the investor cannot observe actions, that the organizational style of the team, cooperation versus noncooperation, matters. Thus, the nature of the second-best contract, and the value of the investor's

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<sup>19</sup>The effort incentive compatibility constraints also differ in that if the managers agree that only one manager will exert effort, then that manager will exert effort in class one, rather than class two.

objective function in a second-best world, may change.

The case investigated by Ramakrishnan and Thakor (1991) is that of a risk-neutral principal and risk-averse agents. They showed that if agents' outcomes are highly correlated, then allowing for cooperation between the agents worsens the principal's outcome. This occurs for two reasons. First, if the agents' outcomes are highly correlated then the principal can best encourage effort with a competitive contract. Second, because of the high correlation, there is no (or little) coinsurance benefit for the agents. The Ramakrishnan and Thakor result carries over to this model. If the investor's risk attitude, relative to the incremental cost of diversification, is such that she prefers that the managers do not diversify (so that their outcomes are highly correlated), then she will optimally encourage the managers to exert effort by offering them each a competitive contract.

If the investor is risk averse and the managers risk neutral, then cooperation makes it less expensive for the investor to induce the managers not to herd. (The reason for this is discussed below following Proposition 6.) However, it is found that satisfying the nonherding-incentive-compatibility constraints still requires, when the managers are risk neutral, that total wages are a concave function of total returns. Thus, with risk-neutral managers cooperation does not enable first best to be achieved. However, if the managers are risk averse, then there is an added benefit to cooperation in that it permits the managers to coinsure. This added benefit, together with the lower cost of discouraging herding, can enable first best to be achieved.

**Proposition 5.** *If the money managers are able to cooperate in their decisions and in their sharing of wages, and if they are risk averse, then there exist parameter values such that the investor can achieve first best. Cooperation does not enable the investor to achieve first best if the managers are risk neutral.*

This proposition is proved in the Appendix by first showing that first best cannot be achieved if the money managers are risk neutral. Next, an example is found in which first best can be achieved. In this example, both the investor and the managers are risk averse, and the parameter values are such that the first-best contract is linear, but limited liability is just binding:  $w_{LL} = 0, w_{HL} + w_{LH} = w_{HH}$ . In addition, the incremental cost of diversification

is low enough relative to the benefit of diversification, so that the nonherding constraints are satisfied by the first-best contract, with and without cooperation. In this example, if the managers are unable to cooperate, then the effort constraint is not satisfied by the first-best contract. With cooperation, however, the effort constraints are satisfied and first best is achieved.

It is clear from Proposition 5 that cooperation will improve the investor's second-best outcome for at least some parameter values. To further understand the potential benefits and costs of switching to a cooperative organization we next examine how such a switch would affect the cost of satisfying each set of incentive compatibility constraints: nonherding and effort. Proposition 6 summarizes these results. An intuitive explanation of the reasoning follows the proposition.

**Proposition 6.**

- a) *Allowing for cooperation among money managers will, for most parameter values, decrease the cost of discouraging herding.*
- b) *If the managers are risk neutral (risk averse), then enabling cooperation cannot (can) decrease the cost of encouraging effort.*

For most parameter values allowing for cooperation decreases the cost of discouraging herding. This occurs because, under cooperation, the managers recognize that herding behavior by one manager affects both of them. From a manager's perspective, the potential benefit of herding is the same as with noncooperation (lower cost of exerting effort), but the cost is higher, because under cooperation a herder recognizes the total team's wage loss (and/or increase in variance). It is shown in the Appendix that a sufficient (but not necessary) condition for cooperation to decrease the cost of discouraging herding is that the noncooperative contract be either linear or concave in total returns. It is only if the noncooperative contract is convex *and* the relative incremental cost of diversifying is very small,<sup>20</sup> that allowing for cooperation may increase the cost of discouraging herding. Thus, if discouraging herding under noncooperation is costly, then switching to a cooperative team organization will reduce this cost.

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<sup>20</sup> $(c_2 - c_1)/(pp(1 - p))$  very small



In the problem posed here, however, the contract must satisfy not only a nonherding constraint, but also an effort constraint. These two constraints interact in a such a way that, even though discouraging herding may be cheaper with cooperation, a noncooperative organizational form may still dominate cooperation. This is easiest to understand by considering the case of risk-neutral money managers. Under both cooperation and noncooperation, if the managers are risk neutral, then the nonherding–IC constraint requires that total wages be a concave function of total returns:  $w_{HL} + w_{LH} > w_{HH} + w_{LL}$ . (Even though the cost of discouraging herding generally decreases with cooperation, the cost still remains strictly positive.) This functional form causes the effort constraint to become more difficult to satisfy under cooperation. The reason is that, under cooperation the managers can increase the probability of achieving different outcomes ( $HL$ ) by coordinating so that one works and the other doesn't.<sup>21</sup> As such, cooperation never decreases, and generally increases, the cost of inducing effort when the managers are risk neutral. While this is most easily seen in the extreme case of risk neutral managers, the result will also hold for risk averse managers, if the risk aversion is relatively low. In contrast, if the managers are risk averse enough, then the coinsurance benefit of cooperation, together with the lower cost of inducing nonherding, will cause cooperation to be the optimal form of organization.

These results highlight the importance of having modelled this problem with two dimensions to the incentive compatibility problem: nonherding *and* exertion of effort. If I had instead modelled the problem only with a nonherding incentive compatibility constraint, then I would have incorrectly concluded that cooperation is for most parameter values the optimal form of organization. Instead, the requirements of encouraging both nonherding and exertion of effort interact in such a way that the optimal organizational form depends on the parameter values. Because of this interaction I have up to this point been able only to state conditions such that allowing for cooperation is *not* beneficial and conditions such that it *may* be beneficial to the investor. It is expected, however, that cooperation will be more beneficial if the managers, and possibly also the investor, are more risk averse. This intuition is confirmed by the results of the following section.

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<sup>21</sup>There is also the effect that when cooperating, if exactly one manager works, then that manager invests in class 1, so that the team saves an amount  $2c_2 - c_1$  in effort costs, rather than saving just  $c_2$ .

## 6 Cooperation versus Competition

In the proofs of Propositions 4 through 6, algorithms are derived for determining the optimal contracts under noncooperation and cooperation when everyone is risk averse. These algorithms are complex enough so that in the previous sections it was only possible to characterize the optimal contracts, rather than present general analytical solutions. (If the managers are risk neutral, then simple analytical solutions are obtained.) It is possible, however, to use these algorithms to determine numerically the optimal contract, for any given set of parameters. In this section, the choices of organizational form and optimal contract are further explored through the use of numerical calculations.

Based on the results of the previous section it is expected that an increase in the risk aversion of the money managers will make cooperation more advantageous relative to competition. This result is illustrated in Figure 3 which shows how the optimal organizational form and contract style vary with risk attitudes. The optimal form is that form that results in the highest expected utility for the investor. The investor's risk aversion coefficient ( $\psi$ ) is plotted along the horizontal axis and the managers' risk aversion coefficient ( $\alpha$ ) is plotted along the vertical axis.<sup>22</sup>

When the investor is not very risk averse she optimally chooses not to pay the extra cost of diversification and so offers a wage contract that motivates effort, but does not discourage herding. This is indicated in Figure 3 by the "Nondiversified" region. As discussed in Section 3, without diversification the choice of organizational form is not relevant. As the investor becomes more risk averse, providing incentives to diversify becomes optimal. In both the "Competitive" and "Cooperative" regions of Figure 3 the contract offered to the managers provides incentives both to exert effort and not to herd. As the managers become more risk averse cooperation dominates noncooperation. For all parameter values tested, it was found that if noncooperation dominates cooperation, then the optimal contract is strictly

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<sup>22</sup>In Figures 3 and 4,  $p = 0.7$ ,  $\rho = 0.7$ ,  $c_1 = 0.300$  and  $c_2 = 0.30875$ . At each point sampled in Figure 3, first the optimal contract was determined for each of the three styles: nondiversification, diversification without cooperation, diversification with cooperation. Next, the style with the highest expected utility for the investor was selected as the optimal style. The same method was used to create the graphs of Figure 4, except that in the left-hand (right-hand) graph only nondiversification and diversification without cooperation (with cooperation) were considered. The "jaggedness" of the boundaries occurs for two reasons: i) The sampling is done at the intersection points of a  $50 \times 50$  grid. ii) In the neighborhoods of the boundaries the value functions for the different styles are close to equal. Any apparent nonmonotonicities along the boundaries are due to limitations in the precision of the optimization algorithm, rather than any economic phenomenon.

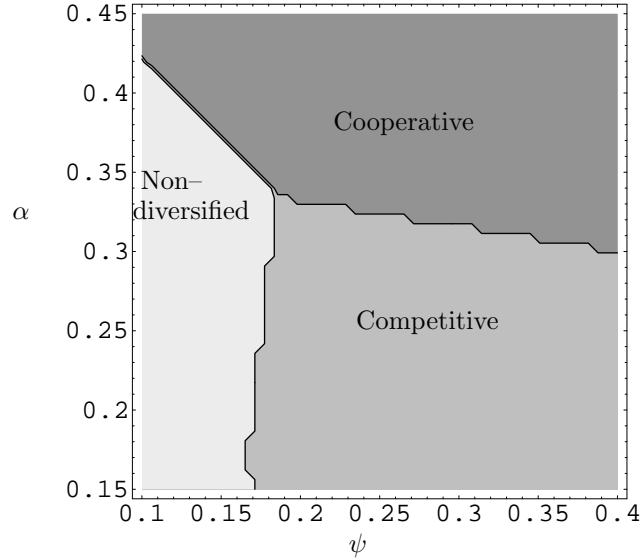


Figure 3: Optimal Contract Style

$\psi$  = investor's risk aversion coefficient  
 $\alpha$  = money managers' risk aversion coefficient

competitive.

If the money managers are highly risk averse, then even when the investor is relatively risk tolerant, it is optimal to organize the team to allow for cooperation and to offer a single team contract that discourages herding. This is because the risk aversion of the managers makes it relatively low cost for the investor to motivate nonherding. This is illustrated in the graphs of Figure 4 which show the tradeoff between diversifying or not, where just one organizational form is considered for motivating nonherding. As Figure 4a illustrates, if only a noncooperative organization is considered, then the decision of whether to discourage herding is driven almost entirely by the investor's risk attitude. Figure 4b shows that if cooperation is considered, then the risk attitude of the managers also matters.

Figure 4 is also important in that up to this point the discussion of the "optimal" organizational form has been entirely from the perspective of the investor. Figure 3 was drawn assuming that the investor is able to determine the organizational form. This is not a concern in the cooperative form. In this organizational form, the team is paid just one wage and so the team members do not have incentives to subvert the investor's chosen organizational form. In the noncooperative form, however, it may be possible for the team members to

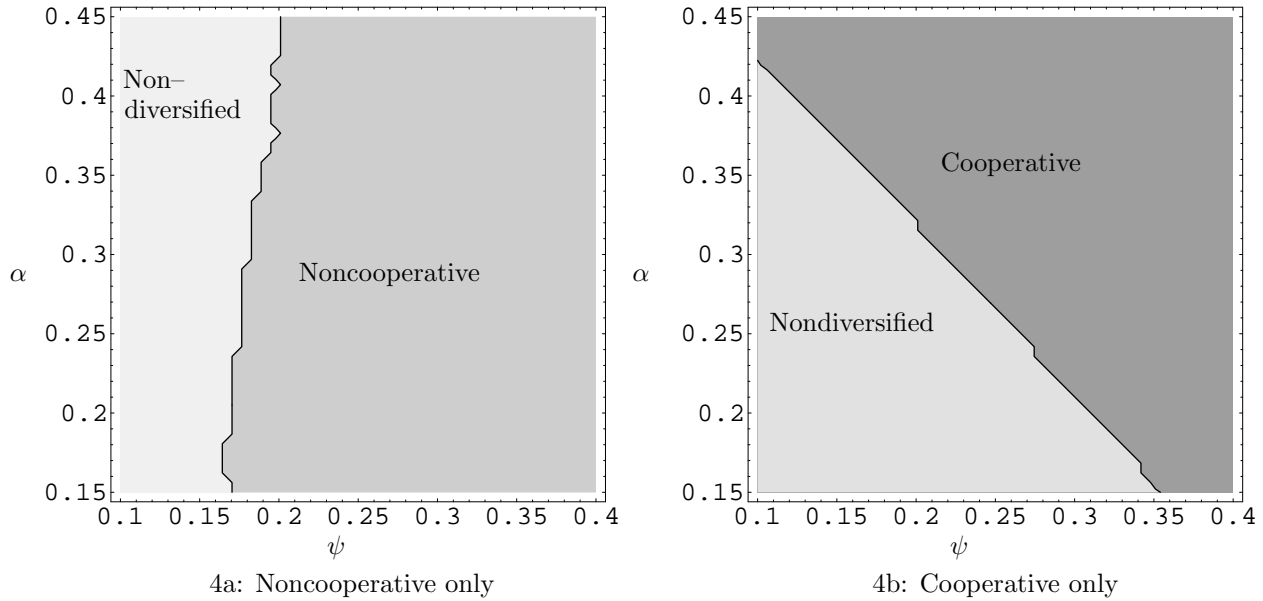


Figure 4: Optimal Contract Style

$\psi$  = investor's risk aversion coefficient  
 $\alpha$  = money managers' risk aversion coefficient

subvert the investor's wishes and sign side contracts to cooperate with each other. Given that the optimal contract under noncooperation is strictly competitive, i.e.  $w_{HL} > w_{HH}$ , the managers will have incentives to coordinate so that only one of them works, and they split both the salary and the cost of working. If a noncooperative environment cannot be maintained and managers are able to sign side contracts, then Figure 4b, rather than Figure 3, will illustrate the tradeoff between optimal contract styles.<sup>23</sup> However, this is likely not to be a significant concern, because a manager who does well will want ex post to subvert such a side contract. Given that such side contracts are not sanctioned by the investor such subversion will likely work, making side contracting infeasible.

It is also found that the organizational form affects the relationship between the investor's expected utility and the managers' risk attitudes. Under noncooperation, the investor's expected utility is strictly decreasing in the risk aversion of the managers. This is because the investor must compensate the managers for taking on risk. Under cooperation, however, the investor's expected utility may be increasing or decreasing in the managers' risk aversion.

<sup>23</sup>For a similar result in a somewhat different setting, the reader is referred to Slezak and Khanna (2000).

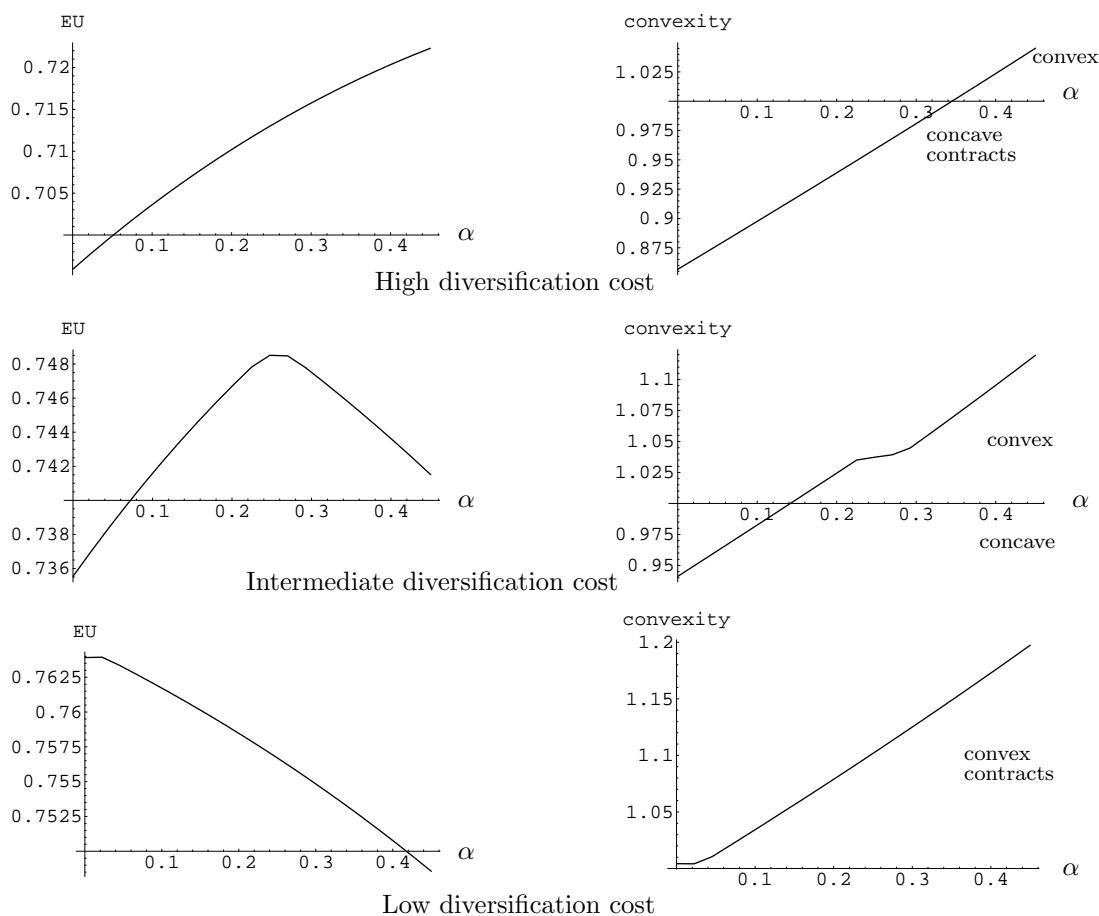


Figure 5: Cooperative Contract

Expected utility (EU) and contract convexity versus managers' risk aversion coefficient ( $\alpha$ ), for three different levels of incremental diversification cost ( $c_2 - c_1$ ). Convexity is less (greater) than one for a contract in which the total wages are a concave (convex) function of the total return.

The difference occurs for two reasons. First, under cooperation the managers can coinsure so that the investor need not compensate them as much for taking on risk. Second, as discussed in the previous section, under cooperation each manager recognizes the full cost imposed on the team by herding. Part of this cost is due to an increased variance of wages. Thus, risk aversion on the part of the managers is more effective, under cooperation than under noncooperation, in helping to align incentives.

Figure 5 presents the results of a number of calculations of the optimal contract under cooperation. The left-hand graphs show the investor's expected utility, and the right-hand graphs show the convexity of the wage contract, versus the managers' risk aversion coefficient,  $\alpha$ . The variable "convexity" is  $(2w_{HH} - w_{HL} - w_{LH}) / (w_{HL} + w_{LH} - 2w_{LL})$ . Thus, values of "convexity" less than one indicate that the contract is concave; values greater than one

indicate it is convex. The top two graphs are for the same example as is illustrated in Figures 3 and 4, with  $\psi = 0.25$ . In this example the cooperative contract is concave for all but very high values of the managers' risk aversion, and the investor's expected utility is increasing in the managers' risk aversion. In the middle and lower graphs, the parameter values are the same except that the incremental cost of exerting effort when not herding ( $c_2 - c_1$ ) is smaller.<sup>24</sup>

As is indicated by Figure 5, the shape of the optimal contract depends on the incremental cost of diversification. As this cost decreases, the optimal contract becomes more convex. If this cost is high enough so that optimal contract is concave or only slightly convex, then it is also the case that the investor's expected utility is increasing in the managers' risk aversion. This is due to the fact that the risk aversion of the money managers helps to align the incentives of the investor and managers with regard to herding. For low enough values of the incremental cost of diversification, the incentive-alignment benefit of higher risk aversion on the part of the managers is overwhelmed by the extra amount that must be paid to the managers for taking on risk. Thus, when the incremental cost of diversification is low, the investor's expected utility is decreasing in the managers' risk aversion.

Figure 5 illustrates further that as the managers become more risk averse, the optimal cooperative contract becomes more convex (less concave). As discussed earlier, a concave contract may be necessary in order to motivate the managers not to herd. This is costly to the investor who would prefer a convex contract in order to transfer risk to the managers. As the managers become more risk averse, it is possible to discourage herding with a less concave contract.

In summary, as the managers' risk aversion increases and/or the incremental cost of diversification decreases, cooperation becomes optimal and the contract becomes more convex (less concave). If the cost of diversification is significant, then increased risk aversion on the part of the managers can increase the investor's expected utility, despite the extra amount that the investor must pay to compensate the managers for taking on risk.

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<sup>24</sup>In Figure 5,  $\psi = 0.25$ ,  $p = 0.7$ ,  $\rho = 0.7$ ,  $c_1 = 0.300$ . In the top graph  $c_2 = 0.30875$ ; in the middle  $c_2 = 0.304$ ; in the bottom  $c_2 = 0.3005$ . Although not shown here, if the equivalent graphs to Figure 3 were shown for the latter two examples, cooperation would be optimal for most values of risk aversion.

## 7 Conclusion

This paper characterizes the optimal wage contract and optimal organizational form for a team of money managers, given a risk-averse investor. The problem examined here is unique in that the investor wishes to provide incentives for the managers to both exert effort and to choose tasks that are not highly correlated with each other so as to minimize the investor's risk. In all cases examined, the optimal contract is performance based. But, the performance that is relevant may be that of individual managers, of the team as a whole or of the two combined.

It is shown that the optimal structure of the contract and the optimal organizational form of the team depend on the risk attitudes of the managers and the investor. If the managers are relatively risk tolerant, then a noncooperative team organization and a strictly competitive contract, in which each manager is rewarded both for doing well and for doing better than the team, is the most efficient way to discourage herding within the team. This is despite the fact that, in such a contract total wages paid are a concave function of total returns, and so using the contract to discourage herding (and thus lower risk) is in direct conflict with the investor's objective of using the contract to transfer risk onto the managers. As the managers become more risk averse, it becomes less necessary to offer a competitive contract in order to discourage herding. As the risk aversion of both the investor and the managers increases, cooperation among managers becomes the optimal way to organize the team. Under cooperation managers are paid as a team, rather than as individuals, thus enabling them to co-insure among themselves. For some parameter values, if everyone is risk averse, first best can be achieved under cooperation. First best can never be achieved if only the investor or only the managers are risk averse, or if cooperation is not possible.

## 8 Appendix

### Notation:

- $\psi$  = investor's risk aversion coefficient
- $\alpha$  = money managers' risk aversion coefficient
- $p$  = probability that single manager's return is 1, given that effort is exerted
- $\rho$  = cross correlation between assets in class one and assets in any other classes
- $c_k$  = cost of exerting effort in class  $k$ ,  $c_2 > c_1$
- $p_{ab}$  = probability that manager  $i$ 's investment returns a and  $j$ 's returns b, given that both exert effort
- $p'_{ab}$  = probability that manager  $i$ 's investment returns a and  $j$ 's returns b, given that  $i$  shirks and  $j$  exerts effort
- $w_{ab}$  = wage paid to manager whose investment returns  $a$  when other manager's returns  $b$
- $W$  = sum of wages paid to two managers
- $r$  = realized return for individual manager
- $R$  = sum of returns across two managers

### Parameter restrictions:

1.  $c_1 < c_2 < p$
2.  $\psi \leq \frac{1}{2}$   $\alpha \leq \frac{1}{2}$  (The first of these ensures that the investor never finds it profitable to give money away in order to decrease risk.)
3.  $p \geq .25$  (This assumption is made purely to ease the proof of Proposition 3. It is a sufficient, but not necessary, condition for the result.)

**Derivation of effort–IC, risk-averse managers herd.**  $p_{HL} = 0$ ,  $p_{HH} = p$ ,  $p_{LL} = 1 - p$ .

$$\begin{aligned}
 \text{var}[w|e = 1] &= p(1-p)(w_{HH} - w_{LL})^2 \\
 \text{var}[w|e = 0] &= p(1-p)(w_{LL}^2 + w_{LH}^2 - 2w_{LL}w_{LH}) \\
 \text{var}[w|e = 0] - \text{var}[w|e = 1] &= p(1-p)\left(w_{LH}^2 - w_{HH}^2 + 2w_{LL}(w_{HH} - w_{LH})\right) \\
 E[w|e = 1] - E[w|e = 0] &= p(w_{HH} - w_{LH})
 \end{aligned}$$

The effort–IC constraint (7) follows directly from applying these to equation (4).

**First best, risk-neutral managers.** The investor's objective function is:

$$\begin{aligned}
 \max_{\vec{w}} Z &= 2E[r - w] - \psi \text{var}[R - W] \\
 &= 2E[r] - 2E[w] - \psi \text{var}[R] - \psi \text{var}[W] + 2\psi (E[RW] - 4ErEw)
 \end{aligned}$$

In the first-best contract, IR is exactly satisfied:  $Ew = c$ . (It is assumed that  $\psi$  is not so high that the investor will wish to give away cash to decrease variance.)

$$\begin{aligned}
 \text{var}[R] &= \text{var}[r_1 + r_2] = 2(1 + \rho)p(1 - p) \\
 \text{var}[W] &= 2(2p_{HH}w_{HH}^2 + p_{HL}(w_{HL} + w_{LH})^2 + 2p_{LL}w_{LL}^2) - 4c^2
 \end{aligned}$$



With no herding ( $\rho = 0$ ):

$$\begin{aligned} \text{var}[R] &= 2\text{var}[r] = 2p(1-p) \\ E[RW] &= 4p^2w_{HH} + 2p(1-p)(w_{HL} + w_{LH}) \\ \text{var}[W] &= 2(2p^2w_{HH}^2 + p(1-p)(w_{HL} + w_{LH})^2 + 2(1-p)^2w_{LL}^2) - 4c_2^2 \\ Z &= 2(p - c_2) - 2\psi p(1-p) - 8\psi pc_2 + \psi(2E[RW] - \text{var}[W]) \end{aligned}$$

Only the last term depends on the form of the wage contract. Dividing this term by the constant  $2\psi$  and dropping the constant term  $4c_2^2$ , the investor wishes to:

$$\max_w V = 2p^2w_{HH}(2 - w_{HH}) + p(1-p)(w_{HL} + w_{LH})(2 - (w_{HL} + w_{LH})) - 2(1-p)^2w_{LL}^2$$

subject to:  $p^2w_{HH} + p(1-p)(w_{HL} + w_{LH}) + (1-p)^2w_{LL} = c_2$ . Clearly  $w_{LL}^* = 0$ . Letting  $w_b \equiv (w_{HL} + w_{LH})$ ,  $w_b = \frac{c_2 - p^2w_{HH}}{p(1-p)}$ . Substituting this into  $V$  above and taking the partial derivative wrt  $w_{HH}$  it is found that

$$w_{HL} + w_{LH} = \max \left[ 0, \frac{2c_2 - p^2}{p(2-p)} \right] \quad \text{and} \quad w_{HH} = \begin{cases} \frac{c_2}{p^2} & \text{if } c_2 \leq \frac{p^2}{2} \\ \frac{c_2 + p(1-p)}{p(2-p)} & \text{otherwise} \end{cases}$$

Because we have assumed that  $p > c_2$ ,  $w_{HH}$  is always  $> w_{HL} + w_{LH}$ . Note also that setting  $w_{HL} = w_{LH}$ , so that the contract is based only on team performance, is first-best optimal.

**Proof of Propositions 1 and 2.** If  $c_2 - c_1$  is so high that the investor prefers, in a first-best world, that the managers herd, then first best can be attained (because the probability of a good outcome with zero effort is zero). The focus here is on parameter values such that nonherding is first-best optimal.

effort-IC:

$$p(w_{HH} - w_{LH}) + (1-p)(w_{HL} - w_{LL}) \geq \frac{c_2}{p} \quad (12)$$

nonherding-IC:

$$\begin{aligned} p^2w_{HH} + p(1-p)(w_{HL} + w_{LH}) + (1-p)^2w_{LL} - c_2 &\geq \\ (p^2 + \delta_\rho)w_{HH} + (p(1-p) - \delta_\rho)(w_{HL} + w_{LH}) + ((1-p)^2 + \delta_\rho)w_{LL} - c_1 & \\ \implies w_{HL} + w_{LH} - w_{HH} - w_{LL} &\geq \frac{c_2 - c_1}{\rho p(1-p)} = \frac{c_2 - c_1}{\delta_\rho} \end{aligned} \quad (13)$$

where  $\delta_\rho = \rho p(1-p)$ . Limited liability:  $w_{HH}, w_{HL}, w_{LH}, w_{LL} \geq 0$

Incentive rationality, given effort-IC and limited liability, is nonbinding.

Based on these constraints and the first best derivation above, it is clear that when money managers are risk neutral,  $w_{LL}^* = w_{LH}^* = 0$ . ( $w_{LH}$  either appears in negative form on the LHS of a constraint, or appears in additive form with  $w_{HL}$ .) The constraints are thus:

$$pw_{HH} + (1-p)w_{HL} \geq \frac{c_2}{p} \quad \text{and} \quad w_{HL} - w_{HH} \geq \frac{c_2 - c_1}{\rho p(1-p)} \quad (14)$$

The first constraint can be satisfied with a first-best contract, but the second prevents the investor from achieving first best. The second constraint requires that  $w_{HL} > w_{HH}$ . ■

**First best, risk-averse money managers.** This contract is derived for later comparison with the 2nd-best contract. With no herding  $\rho = 0$ :

$$\begin{aligned}
\max_{\vec{w}} Z &= 2(p - c_2) - 2\psi p(1 - p) - 2\alpha \text{var}[w] - \psi \text{var}[W] + 2\psi (E[RW] - 4ErEw) \\
E[RW] &= 4p^2 w_{HH} + 2p(1 - p)(w_{HL} + w_{LH}) \\
Ew &= p^2 w_{HH} + p(1 - p)(w_{HL} + w_{LH}) + (1 - p)^2 w_{LL} \\
E[RW] - 4ErEw &= 4(1 - p)p^2 w_{HH} + 2(1 - 2p)p(1 - p)(w_{HL} + w_{LH}) - 4p(1 - p)^2 w_{LL} \\
\text{var}[W] &= 2\text{var}[w] + 2\text{cov}(w_1, w_2) = 4\text{var}[w] - 2p(1 - p)(w_{HL} - w_{LH})^2 \\
\text{cov}(w_1, w_2 | \rho = 0) &= p(1 - p) \left( 2(w_{HL}w_{LH} - w_{HH}w_{LL}) \right. \\
&\quad \left. - 2(pw_{HH} + (1 - p)w_{LL})(w_{HL} + w_{LH} - w_{HH} - w_{LL}) \right. \\
&\quad \left. - p(1 - p)(w_{HL} + w_{LH} - w_{HH} - w_{LL})^2 \right)
\end{aligned}$$

The investor wishes to:

$$\begin{aligned}
\min_{\vec{w}} V &= (\alpha + 2\psi)\text{var}[w] - \psi p(1 - p)(w_{HL} - w_{LH})^2 \\
&\quad - 2\psi p(1 - p)(2pw_{HH} + (1 - 2p)(w_{HL} + w_{LH}) - 2(1 - p)w_{LL})
\end{aligned} \tag{15}$$

subject to: (IR is just satisfied in first best.)

$$Ew = c_2 + \alpha \text{var}[w] \tag{16}$$

If the wage is constant ( $w_{HH} = w_{HL} = w_{LH} = w_{LL}$ ), then  $\text{var}[w] = 0$  and  $V = 0$ . Otherwise:

$$\begin{aligned}
\text{var}[w] &= p(1 - p) \left( p(1 + p)w_{HH}^2 + (1 - p)(2 - p)w_{LL}^2 + w_{HL}^2 + w_{LH}^2 \right. \\
&\quad \left. - p(1 - p)(w_{HL} + w_{LH})^2 - 2p^2 w_{HH}(w_{HL} + w_{LH}) \right. \\
&\quad \left. - 2(1 - p)^2 w_{LL}(w_{HL} + w_{LH}) - 2p(1 - p)w_{HH}w_{LL} \right)
\end{aligned}$$

Can the investor benefit by setting  $w_{HL} \neq w_{LH}$ ? Suppose that  $w_{HH} = w_{HL} - \varepsilon = w_{LH} + \varepsilon = w_{LL}$ .  $V$  in equation (15) becomes  $V(\varepsilon) = 2\alpha p(1 - p)\varepsilon^2 > 0$ , so that the value of the objective function is worse than with a constant wage. Thus, in the first-best contract wages are based entirely on team performance.

The first-best contract will be:  $w_{HL} = w_{LH} = x$ ,  $w_{HH} = x + \varepsilon_H$  and  $w_{LL} = x - \varepsilon_L$ , where  $x$  is determined so as to satisfy equation (16) and  $\varepsilon_H$  and  $\varepsilon_L$  are chosen to minimize  $V$ . (We will also need to check that the limited liability is satisfied:  $x \geq 0$ ,  $\varepsilon_L \leq x$  and  $\varepsilon_H \geq -x$ .)

$$\begin{aligned}
V(\varepsilon_H, \varepsilon_L) &= p(1 - p) \left( (\alpha + 2\psi)((p\varepsilon_H + (1 - p)\varepsilon_L)^2 + p\varepsilon_H^2 + (1 - p)\varepsilon_L^2) \right. \\
&\quad \left. - 4\psi(p\varepsilon_H + (1 - p)\varepsilon_L) \right) \\
\frac{\partial V(\varepsilon_H, \varepsilon_L)}{\partial \varepsilon_H} &= 2p^2(1 - p) \left( (\alpha + 2\psi)((1 + p)\varepsilon_H + (1 - p)\varepsilon_L) - 2\psi \right) \\
\frac{\partial V(\varepsilon_H, \varepsilon_L)}{\partial \varepsilon_L} &= 2p(1 - p)^2 \left( (\alpha + 2\psi)(p\varepsilon_H + (2 - p)\varepsilon_L) - 2\psi \right) \\
\varepsilon_H^* &= \varepsilon_L^* = \varepsilon^* = \frac{\psi}{\alpha + 2\psi}
\end{aligned}$$

Because  $\varepsilon^* > 0$ , limited liability requires us only to check that  $\varepsilon_L^* \leq x$ . Solving for  $x$ :

$$\begin{aligned}
E[\vec{w}|\varepsilon_H, \varepsilon_L] &= c_2 + \alpha \text{var}[\vec{w}|\varepsilon_H, \varepsilon_L] \\
x + p^2\varepsilon - (1-p)^2\varepsilon &= c_2 + 2\alpha p(1-p)\varepsilon^2 \\
x &= c_2 + \left(\frac{\psi}{\alpha + 2\psi}\right) \left(2p(1-p) \left(\frac{\alpha\psi}{\alpha + 2\psi}\right) + 1 - 2p\right) \\
x - \varepsilon_L^* \geq 0 \implies \frac{c_2}{p} &\geq \left(\frac{2\psi}{\alpha + 2\psi}\right) \left(1 - (1-p) \left(\frac{\alpha\psi}{\alpha + 2\psi}\right)\right) \tag{17}
\end{aligned}$$

If condition (17) doesn't hold ( $\alpha$  and  $c_2$  are too small), then  $\varepsilon_L = x'$  and  $w_{LL} = 0$ :

$$\begin{aligned}
\frac{\partial V(\varepsilon_H, \varepsilon_L)}{\partial \varepsilon_H} &= 2p^2(1-p) \left( (\alpha + 2\psi)((1+p)\varepsilon_H + (1-p)x') - 2\psi \right) \\
\varepsilon_H^* &= \frac{1}{1+p} \left( \frac{2\psi}{\alpha + 2\psi} - (1-p)x' \right)
\end{aligned}$$

Solving for  $x'$ :

$$\begin{aligned}
E[\vec{w}|\varepsilon_H, \varepsilon_L] &= c_2 + \alpha \text{var}[\vec{w}|\varepsilon_H, \varepsilon_L] \\
(2-p)px' + p^2\varepsilon_H &= c_2 + \alpha p(1-p) \left( p(1+p)\varepsilon_H^2 + (1-p)(2-p)x'^2 + 2p(1-p)\varepsilon_H x' \right)
\end{aligned}$$

$$\text{Let } r \equiv \frac{\psi}{\alpha + 2\psi} \quad \varepsilon_H^* = \frac{2r - (1-p)x'}{1+p}$$

$$\begin{aligned}
\left( (2-p)p - \frac{p^2(1-p)}{1+p} \right) x' + \frac{2p^2r}{1+p} - c_2 &= \\
\alpha p(1-p) \left( \frac{p}{1+p} (4r^2 - 4(1-p)rx' + (1-p)^2x'^2) + (1-p)(2-p)x'^2 \right. \\
&\quad \left. + \frac{2p(1-p)}{1+p} (2rx' - (1-p)x'^2) \right) \\
\left( \frac{2p}{1+p} \right) x' + \frac{2p^2r}{1+p} - c_2 &= \alpha p(1-p) \left( \frac{p}{1+p} (4r^2 - (1-p)^2x'^2) + (1-p)(2-p)x'^2 \right) \\
x' + pr - \left( \frac{1+p}{2p} \right) c_2 &= \alpha(1-p) \left( 2pr^2 + (1-p)x'^2 \right) \\
x' - \alpha(1-p)^2x'^2 &= \frac{(1+p)c_2}{2p} - pr(1 - 2\alpha(1-p)r)
\end{aligned}$$

We are considering the case such that condition (17) doesn't hold. Thus:

$$\begin{aligned}
x' - \alpha(1-p)^2x'^2 &< (1+p)r(1 - \alpha(1-p)r) - pr(1 - 2\alpha(1-p)r) \\
&= r - \alpha(1-p)^2r^2 \\
x' - r &< \alpha(1-p)^2(x' + r)(x' - r) \\
x' > r &\iff 1 < \alpha(1-p)^2(x' + r) \\
x' < r &\iff 1 > \alpha(1-p)^2(x' + r)
\end{aligned}$$

We know that  $x' < 1$  (the investor does not pay more than he receives),  $r < \frac{1}{2}$  and  $\alpha < \frac{1}{2}$ . Thus, the last inequality holds ( $x' < r$ ), so that  $\varepsilon_H^* > x' = \varepsilon_L^*$  and the contract is convex. (Note:  $x'$  must  $\geq 0$ , so if  $c_2$  is very small, then  $x' = 0$  and a positive wage is paid only in the state  $w_{HH}$ . Again, the contract is convex in total returns.)

### Proof of Proposition 3.

$$\begin{aligned} \text{var}[w|e = 1] &= p_{HH}(1 - p_{HH})w_{HH}^2 + p_{LL}(1 - p_{LL})w_{LL}^2 - p_{HL}^2(w_{HL} + w_{LH})^2 + p_{HL}(w_{HL}^2 + w_{LH}^2) \\ &\quad - 2p_{HH}p_{HL}w_{HH}(w_{HL} + w_{LH}) - 2p_{LL}p_{HL}w_{LL}(w_{HL} + w_{LH}) - 2p_{HH}p_{LL}w_{HH}w_{LL} \end{aligned}$$

If a manager deviates and herds, then probability weight shifts from outcomes  $HL$  and  $LH$  to  $HH$  and  $LL$ . If both invest in type 2 assets and exert effort:

$$p_{HH} = p^2 \quad p_{LL} = (1 - p)^2 \quad p_{HL} = p_{LH} = p(1 - p)$$

If one invests in type 2 and the other in type 1 and both exert effort:

$$p''_{HH} = p^2 + \delta_\rho \quad p''_{LL} = (1 - p)^2 + \delta_\rho \quad p''_{HL} = p''_{LH} = p(1 - p) - \delta_\rho$$

where  $\delta_\rho = \rho p(1 - p)$  and  $0 < \rho < \sqrt{.5}$

$$\begin{aligned} p''_{HH}(1 - p''_{HH}) &= (p^2 + \delta_\rho)(1 - p^2 - \delta_\rho) = p_{HH}(1 - p_{HH}) + \delta_\rho(1 - 2p^2 - \delta_\rho) \\ p''_{LL}(1 - p''_{LL}) &= ((1 - p)^2 + \delta_\rho)(1 - (1 - p)^2 - \delta_\rho) = p_{LL}(1 - p_{LL}) + \delta_\rho(1 - 2(1 - p)^2 - \delta_\rho) \\ p''_{HL} &= (p_{HL} - \delta_\rho)^2 = p_{HL}^2 - \delta_\rho(2p(1 - p) - \delta_\rho) \\ p''_{HH}p''_{HL} &= (p_{HH} + \delta_\rho)(p_{HL} - \delta_\rho) = p_{HH}p_{HL} - \delta_\rho(p_{HH} - p_{HL} + \delta_\rho) = p_{HH}p_{HL} - \delta_\rho(2p^2 - p + \delta_\rho) \\ p''_{LL}p''_{HL} &= (p_{LL} + \delta_\rho)(p_{HL} - \delta_\rho) = p_{LL}p_{HL} - \delta_\rho(p_{LL} - p_{HL} + \delta_\rho) = p_{LL}p_{HL} - \delta_\rho(1 - 3p + 2p^2 + \delta_\rho) \\ p''_{HH}p''_{LL} &= (p_{HH} + \delta_\rho)(p_{LL} + \delta_\rho) = p_{HH}p_{LL} + \delta_\rho(p_{HH} + p_{LL} + \delta_\rho) = p_{HH}p_{LL} + \delta_\rho(1 - 2p + 2p^2 + \delta_\rho) \end{aligned}$$

If a manager invests in asset class 1, while the other invests in an asset class of type 2 (while both exert effort), then the variance of wage for each increases by  $\phi$ , where

$$\begin{aligned} \phi &= \delta_\rho \left( (1 - 2p^2 - \delta_\rho)w_{HH}^2 - (1 - 4p + 2p^2 + \delta_\rho)w_{LL}^2 - 2(1 - 2p + 2p^2 + \delta_\rho)w_{HH}w_{LL} \right. \\ &\quad \left. + (2p - 2p^2 - \delta_\rho)(w_{HL} + w_{LH})^2 - w_{HL}^2 - w_{LH}^2 \right. \\ &\quad \left. + 2(2p^2 - p + \delta_\rho)w_{HH}(w_{HL} + w_{LH}) + 2(1 - 3p + 2p^2 + \delta_\rho)w_{LL}(w_{HL} + w_{LH}) \right) \end{aligned}$$

Under the first-best contract w/o limited liability binding:

$$\begin{aligned} \frac{\phi}{\delta_\rho} &= (1 - 2p^2 - \delta_\rho)(2\varepsilon x + \varepsilon^2) - (1 - 4p + 2p^2 + \delta_\rho)(-2\varepsilon x + \varepsilon^2) + 2(1 - 2p + 2p^2 + \delta_\rho)\varepsilon^2 \\ &\quad + 4(2p^2 - p + \delta_\rho - 1 + 3p - 2p^2 - \delta_\rho)\varepsilon x \\ &= 2\varepsilon^2 > 0 \end{aligned}$$

Under the first-best contract with limited liability binding:  $\varepsilon_H - x' = \frac{2(r-x')}{1+p} > 0$

$$\frac{\phi}{\delta_\rho} = (1 - 2p^2 - \delta_\rho)(2\varepsilon_H x' + \varepsilon_H^2) - (1 - 4p + 2p^2 + \delta_\rho)(-x'^2)$$

$$\begin{aligned}
& +4(2p^2 - p + \delta_\rho)\varepsilon_H x' - 4(1 - 3p + 2p^2 + \delta_\rho)x'^2 + 2(1 - 2p + 2p^2 + \delta_\rho)x'^2 \\
= & 2(1 - 2p + 2p^2)\varepsilon_H x' + (1 - 2p^2)\varepsilon_H^2 - (1 - 4p + 2p^2)x'^2 - \delta_\rho(\varepsilon_H - x')^2 \\
= & 2\varepsilon_H^2 - 4p(\varepsilon_H - x')x' - (1 + 2p^2 + \delta_\rho)(\varepsilon_H - x')^2 \\
= & \frac{2}{(1+p)^2} \left( (2(r-x') + (1+p)x')^2 - 4p(1+p)(r-x')x' - (1+2p^2 + \delta_\rho)2(r-x')^2 \right) \\
= & \frac{2}{(1+p)^2} \left( (1+p)^2 x'^2 + 4(1-p)(1+p)(r-x')x' + 2(1-2p^2 - \delta_\rho)(r-x')^2 \right)
\end{aligned}$$

$x' < r$ , so the middle term is positive. A sufficient, but not necessary, condition for  $\phi$  to be positive is  $p \leq 0.6642$ . Even if this condition does not hold,  $\phi$  will still be positive unless  $x'$  is also very small (very small  $c_2$ ) and/or  $r$  is large (the investor is much more risk averse than the managers).

The next step is to check if the IC constraints can be satisfied with the first-best contract. Let  $Y \equiv Ew - \alpha var[w]$ . nonherding-IC requires that  $\Delta_2 \geq \frac{c_2 - c_1}{\rho p(1-p)}$  where

$$\Delta_2 \equiv \frac{Y|_{class2} - Y|_{class1}}{\rho p(1-p)} = \varepsilon_L - \varepsilon_H + \frac{\alpha\phi}{\delta_\rho}$$

nonherding-IC is clearly satisfied with a linear first-best contract if  $2\alpha\varepsilon^2 \geq c_2 - c_1$ . effort-IC requires that  $\Delta_1 \geq \frac{c_2}{p}$ .  $var[w|e=0] = p(1-p)(w_{LH} - w_{LL})^2 = p(1-p)\varepsilon_L^2$ . Using the derivations from the first best calculations:

$$\begin{aligned}
\Delta_1 & \equiv \frac{Y|_{effort} - Y|_{noeffort}}{p} \\
& = p\varepsilon_H + (1-p)\varepsilon_L - \alpha(1-p) \left( p(1+p)\varepsilon_H^2 + (1-3p+p^2)\varepsilon_L^2 + 2p(1-p)\varepsilon_H\varepsilon_L \right)
\end{aligned}$$

Individual rationality requires that  $\Delta_3 = (Ew - \alpha var[w])/p \geq \frac{c_2}{p}$ . In the first-best contract this is satisfied with equality, so a necessary and sufficient condition for the first best contract to satisfy effort-IC is that  $\Delta_3 \leq \Delta_1$ .

$$\begin{aligned}
\Delta_3 & = \frac{x}{p} + p\varepsilon_H - \frac{(1-p)^2}{p}\varepsilon_L \\
& \quad - \alpha(1-p) \left( p(1+p)\varepsilon_H^2 + (2-3p+p^2)\varepsilon_L^2 + 2p(1-p)\varepsilon_H\varepsilon_L \right)
\end{aligned}$$

Because of limited liability,  $x \geq \varepsilon_L$  and  $\Delta_3 - \Delta_1 \geq \varepsilon_L - \alpha(1-p)\varepsilon_L^2$ . Sufficient for this to be strictly positive is  $\varepsilon_L < 2$ , which is certainly satisfied. Thus, the first-best contract does not satisfy the effort constraint.  $\blacksquare$

**Proof of Proposition 4. Second best, managers are risk averse.** The derivation of the first-best contract and the derivations from the proof of Proposition 3 are applied, with the difference that it may be optimal here to have  $w_{HL} > w_{LH}$ . Let  $w_{HH} = x + \varepsilon_H$ ,  $w_{HL} = x + \varepsilon_M$ ,  $w_{LH} = x - \varepsilon_M$  and  $w_{LL} = x - \varepsilon_L$ . This proof answers the following questions: Can the optimal contract: a) be based only on individual performance:  $\varepsilon_H = \varepsilon_M = \varepsilon_L$ ? b) be based only on team performance:  $\varepsilon_M = 0$ ? The investor wishes to:

$$\begin{aligned}
\max_{\bar{w}} Z & = 2E[r - w] - \psi var[R - W] \\
& = 2p - 2Ew - \psi var[R] - \psi var[W] + 2\psi (E[RW] - 4pEw)
\end{aligned}$$

If managers don't herd ( $\rho = 0$ ) and exert effort, then

$$\begin{aligned}
Ew &= x + p^2\varepsilon_H - (1-p)^2\varepsilon_L \\
var[R] &= 2p(1-p) \\
E[RW] &= 4p^2w_{HH} + 2p(1-p)(w_{HL} + w_{LH}) = 4px + 4p^2\varepsilon_H \\
var[W] &= 4var[w] - 2p(1-p)(w_{HL} - w_{LH})^2 = 4var[w] - 8p(1-p)\varepsilon_M^2 \\
var[w] &= p(1-p)\left((p\varepsilon_H + (1-p)\varepsilon_L)^2 + p\varepsilon_H^2 + (1-p)\varepsilon_L^2 + 2\varepsilon_M^2\right) \\
Z &= 2p - 2\psi p(1-p) - 2x - 2p^2\varepsilon_H + 2(1-p)^2\varepsilon_L \\
&\quad - 4\psi p(1-p)\left((p\varepsilon_H + (1-p)\varepsilon_L)^2 + p\varepsilon_H^2 + (1-p)\varepsilon_L^2\right) \\
&\quad + 8\psi p^2(1-p)\varepsilon_H + 8\psi p(1-p)^2\varepsilon_L
\end{aligned}$$

$$\begin{aligned}
\Delta_1 &= p\varepsilon_H + (1-p)\varepsilon_L + \varepsilon \\
&\quad - \alpha(1-p)\left((p\varepsilon_H + (1-p)\varepsilon_L)^2 + p\varepsilon_H^2 - p\varepsilon_L^2 + \varepsilon_M^2 + 2\varepsilon_L\varepsilon_M\right)
\end{aligned} \tag{19}$$

$$\begin{aligned}
\Delta_2 &= \varepsilon_L - \varepsilon_H \\
&\quad + \alpha\left((1-2p^2 - \delta_\rho)\varepsilon_H^2 - (1-4p + 2p^2 + \delta_\rho)\varepsilon_L^2 - 2\varepsilon_M^2 + 2(1-2p + 2p^2 + \delta_\rho)\varepsilon_H\varepsilon_L\right)
\end{aligned} \tag{20}$$

$$\begin{aligned}
\Delta_3 &= (Ew - \alpha var[w])/p \\
&= \frac{x}{p} + p\varepsilon_H - \frac{(1-p)^2}{p}\varepsilon_L \\
&\quad - \alpha(1-p)\left((p\varepsilon_H + (1-p)\varepsilon_L)^2 + p\varepsilon_H^2 + (1-p)\varepsilon_L^2 + 2\varepsilon_M^2\right)
\end{aligned} \tag{21}$$

The problem can now be written as

$$\begin{aligned}
\min_{x, \varepsilon_H, \varepsilon_L, \varepsilon_M} V &= x + 2\psi p(1-p)\left((p\varepsilon_H + (1-p)\varepsilon_L)^2 + p\varepsilon_H^2 + (1-p)\varepsilon_L^2\right) \\
&\quad + p^2(1-4\psi(1-p))\varepsilon_H - (1-p)^2(1+4\psi p)\varepsilon_L
\end{aligned}$$

subject to

$$\Delta_1 \geq \frac{c_2}{p}, \quad \Delta_2 \geq \frac{c_2 - c_1}{\delta_\rho}, \quad \Delta_3 \geq \frac{c_2}{p}, \quad \varepsilon_L, \varepsilon_M \leq x, \quad x + \varepsilon_H, x + \varepsilon_M \geq 0$$

( $x$  will be as small as possible, subject to satisfying the individual rationality and limited liability constraints.)

a) If  $\varepsilon_H = \varepsilon_M = \varepsilon_L = \varepsilon$ , then  $\Delta_2 = 0$  and the nonherding-IC constraint is not satisfied. Thus, the second best contract cannot be based only on individual performance.

b) Try  $\varepsilon_M = 0$ : (Contract is based only on team performance.)

Lagrangian: We know from the proof of Proposition 3 that individual rationality is non-binding ( $\lambda_3 = 0$ ) if  $\varepsilon_M = 0$ . (Also, the limited liability constraints that contain  $\varepsilon_M$  are

nonbinding.)

$$\begin{aligned}
L[x, \varepsilon_H, \varepsilon_L, \varepsilon_M, \lambda_1, \lambda_2] &= -V - \lambda_1 \left( \frac{c_2}{p} - \Delta_1 \right) - \lambda_2 \left( \frac{c_2 - c_1}{\delta_\rho} - \Delta_2 \right) \\
\frac{\partial L}{\partial \varepsilon_M} &= \lambda_1 \left( 1 - 2\alpha(1-p)(\varepsilon_M + \varepsilon_L) \right) - 4\lambda_2 \alpha \varepsilon_M \\
\frac{\partial L}{\partial \varepsilon_M} \Big|_{\varepsilon_M=0} = 0 &\implies \varepsilon_L = \frac{1}{2\alpha(1-p)} \quad \text{or} \quad \lambda_1 = 0
\end{aligned}$$

If  $\lambda_1 > 0$  (effort constraint is binding), then  $\varepsilon_L = \frac{1}{2\alpha(1-p)} \geq \frac{1}{1-p} \implies w_{HL} + w_{LH} \geq \frac{2}{1-p} \implies EW \geq 2ER$ . This is clearly suboptimal; the investor would prefer to pay no salary and have no effort exerted. If the effort constraint is strictly binding (and the investor optimally prefers the managers to choose assets of type 2 and exert effort), then it must be the case that  $\varepsilon_M^* > 0$ .

Suppose  $\lambda_1 = 0$  (effort constraint is nonbinding). Because first-best cannot be achieved, nonherding-IC must be binding ( $\lambda_2 > 0$ ). It is clear from looking at the optimization problem that if the nonherding-IC constraint is the only binding constraint (apart from the limited liability constraints), then it is optimal for  $\varepsilon_M$  to be set to zero. Also, if constraints 1 and 3 are nonbinding, then  $\varepsilon_L^* > \varepsilon_H^*$ , so that the contract is strictly concave. The contract induces nonherding and effort exertion by the threat of punishment to the team. ■

### Proof of Proposition 5.

*Risk-neutral managers:* With cooperation, the wage contract must satisfy two effort constraints and two nonherding constraints: The managers must prefer both of them working to only one working and both working to neither working. They must prefer both choosing type 2 asset classes to one investing in class one and to both investing in class one. Only the total wage in state  $HL$ ,  $w_{HL} + w_{LH}$ , matters. Let  $w_b \equiv w_{HL} + w_{LH}$ . The nonherding-IC (assuming effort is exerted) constraint (13) becomes:

$$w_b - w_{HH} - w_{LL} \geq \frac{c_2 - c_1}{2\rho p(1-p)} = \frac{c_2 - c_1}{2\delta_\rho} \quad (22)$$

$$\text{and} \quad w_b - w_{HH} - w_{LL} \geq \frac{c_2 - c_1}{p(1-p)} = \frac{\rho(c_2 - c_1)}{\delta_\rho} \quad (23)$$

The nonherding constraint is easier to satisfy with cooperation, but still requires that  $w_b > w_{HH}$  so that first-best cannot be attained.

*Risk-averse managers:* To determine effort-IC:

$$\begin{aligned}
E[W|\text{both work in class 2}] &= 2 \left( p^2 w_{HH} + p(1-p)w_b + (1-p)^2 w_{LL} \right) \\
E[W|\text{one works in class 1}] &= p w_b + 2(1-p)w_{LL} \quad E[W|\text{neither works}] = 2w_{LL} \\
\text{var}[W|\text{both work in class 2}] &= 2p(1-p) \left( 2p(1+p)w_{HH}^2 + 2(1-p)(2-p)w_{LL}^2 \right. \\
&\quad \left. + (1-2p(1-p))w_b^2 - 4p^2 w_{HH}w_b - 4(1-p)^2 w_{LL}w_b \right. \\
&\quad \left. - 4p(1-p)w_{HH}w_{LL} \right) \\
\text{var}[W|\text{one works in class 1}] &= p(1-p) \left( w_b^2 + 4w_{LL}^2 - 4w_{LL}w_b \right)
\end{aligned}$$

With risk-averse managers the constraints depend on the sharing rule between managers because this affects the variance of wages. I assume that the team wage is shared equally when they take identical actions. When they take different actions, rather than specify one rule, I introduce a parameter  $\beta \in [\frac{1}{2}, 1]$ . If the wages are shared equally for all outcomes, then  $\beta = \frac{1}{2}$ . If the total wage risk is carried by just one manager, then  $\beta = 1$ . The two effort constraints require that  $\Delta_{1a} \geq \frac{c_2}{p}$  and  $\Delta_{1b} \geq \frac{c_2}{p}$ , where the  $\Delta_1$  functions are calculated as follows:

$$E[W|\text{both work}] - \frac{\alpha}{2} \text{var}[W|\text{both work}] \geq E[W|\text{one works}] - \beta \alpha \text{var}[W|\text{one works}] + 2c_2 - c_1$$

$$\Delta_{1a} = 2pw_{HH} + (1-2p)w_b - 2(1-p)w_{LL}$$

$$- \alpha(1-p) \left( 2p(1+p)w_{HH}^2 - 4p(1-p)w_{HH}w_{LL} - 4p^2w_{HH}w_b \right. \\ \left. + (1-\beta-2p(1-p))w_b^2 + 4(\beta-1+p(2-p))w_{LL}w_b \right. \\ \left. + 2((1-p)(2-p)-2\beta)w_{LL}^2 \right) - \frac{c_2-c_1}{p}$$

$$\text{and } E[W|\text{both work}] - \frac{\alpha}{2} \text{var}[W|\text{both work}] \geq 2w_{LL} + 2c_2$$

$$\Delta_{1b} = pw_{HH} + (1-p)w_b - (2-p)w_{LL} - \frac{\alpha}{2}(1-p) \left( 2p(1+p)w_{HH}^2 + 2(1-p)(2-p)w_{LL}^2 \right. \\ \left. + (1-2p(1-p))w_b^2 - 4p^2w_{HH}w_b - 4(1-p)^2w_{LL}w_b - 4p(1-p)w_{HH}w_{LL} \right)$$

Following the method of the proof of Proposition 3, a necessary and sufficient condition for the first-best contract to satisfy the effort constraints is  $\Delta_3 \leq \Delta_{1a}$  and  $\Delta_3 \leq \Delta_{1b}$ , where

$$\Delta_{1a}(\varepsilon_H, \varepsilon_L) = 2p\varepsilon_H + 2(1-p)\varepsilon_L$$

$$- 2\alpha(1-p) \left( (p\varepsilon_H + (1-p)\varepsilon_L)^2 + p\varepsilon_H^2 + (1-p)\varepsilon_L^2 \right) + 4\alpha\beta(1-p)\varepsilon_L^2$$

$$- \frac{c_2-c_1}{p}$$

$$\Delta_{1b}(\varepsilon_H, \varepsilon_L) = p\varepsilon_H + (2-p)\varepsilon_L - \alpha(1-p) \left( (p\varepsilon_H + (1-p)\varepsilon_L)^2 + p\varepsilon_H^2 + (1-p)\varepsilon_L^2 \right)$$

and  $\Delta_3(\varepsilon_H, \varepsilon_L)$  is from the proof of Proposition 3:

$$\Delta_3(\varepsilon_H, \varepsilon_L) = \frac{x}{p} + p\varepsilon_H - \frac{(1-p)^2}{p}\varepsilon_L$$

$$- \alpha(1-p) \left( (p\varepsilon_H + (1-p)\varepsilon_L)^2 + p\varepsilon_H^2 + (1-p)\varepsilon_L^2 \right) = \frac{c_2}{p}$$

$\Delta_3 - \Delta_{1b} = \frac{x}{p} - \frac{\varepsilon_L}{p}$  which is strictly positive when limited liability is nonbinding in first best, and zero otherwise. Thus, limited liability binding in first best, so that  $\varepsilon_L = x$ , is a necessary condition for  $\Delta_3 \leq \Delta_{1b}$ . In this case:

$$\Delta_3 - \Delta_{1a} = p(x - \varepsilon_H) + \alpha(1-p) \left( (p\varepsilon_H + (1-p)x)^2 + p\varepsilon_H^2 + (1-p-4\beta)x^2 \right) + \frac{c_2-c_1}{p} \quad (24)$$

With limited liability binding,  $\varepsilon_H \geq x$ . If  $x = \varepsilon_L = \varepsilon_H$ , then the above is equal to  $\frac{c_2-c_1}{p}$  if  $\beta = \frac{1}{2}$  and less than  $\frac{c_2-c_1}{p}$  for  $\beta > \frac{1}{2}$ .



The next step is to identify conditions such that the nonherding constraints are satisfied by the first-best contract in this region. The two nonherding constraints are  $\Delta_{2a} \geq \frac{c_2 - c_1}{\delta_\rho}$  and  $\Delta_{2b} \geq \frac{c_2 - c_1}{\delta_\rho}$ , where the  $\Delta_2$  functions are calculated as follows: (It is assumed here that risk is shared equally between managers, even when they agree that exactly one will deviate. This assumption only makes it more difficult to satisfy nonherding-IC and thus, does not alter the results.)

$$\begin{aligned}
& E[W|class_2 \& e = 1] - \frac{\alpha}{2} \text{var}[W|class_2 \& e = 1] - 2c_2 \geq \\
& \qquad E[W|cl_1, cl_2 \& e = 1] - \frac{\alpha}{2} \text{var}[W|cl_1, cl_2 \& e = 1] - c_2 - c_1 \\
& \text{var}[W|cl_1, cl_2 \& e = 1] - \text{var}[W|class_2 \& e = 1] \\
& = 4\delta_\rho \left( (1 - 2p^2 - \delta_\rho)\varepsilon_H^2 - (1 - 4p + 2p^2 + \delta_\rho)\varepsilon_L^2 + 2(1 - 2p + 2p^2 + \delta_\rho)\varepsilon_H\varepsilon_L \right) \\
\Delta_{2a} & = 2(\varepsilon_L - \varepsilon_H) + \\
& \quad 2\alpha \left( (1 - 2p^2 - \delta_\rho)\varepsilon_H^2 - (1 - 4p + 2p^2 + \delta_\rho)\varepsilon_L^2 + 2(1 - 2p + 2p^2 + \delta_\rho)\varepsilon_H\varepsilon_L \right) \\
& = 2\Delta_2
\end{aligned}$$

where  $\Delta_2$  is defined in the proof of Proposition 4.

$$\begin{aligned}
& E[W|class_2 \& e = 1] - \frac{\alpha}{2} \text{var}[W|class_2 \& e = 1] - 2c_2 \geq \\
& \qquad E[W|class_1 \& e = 1] - \frac{\alpha}{2} \text{var}[W|class_1 \& e = 1] - 2c_1 \\
& \text{var}[W|class_1 \& e = 1] - \text{var}[W|class_2 \& e = 1] \\
& = 2p(1 - p) \left( 2(1 - p - p^2)\varepsilon_H^2 - 2(1 - 3p + p^2)\varepsilon_L^2 + 4(1 - p + p^2)\varepsilon_H\varepsilon_L \right) \\
\Delta_{2b} & = \frac{1}{\rho}(\varepsilon_L - \varepsilon_H) + \\
& \quad \frac{\alpha}{\rho} \left( (1 - p - p^2)\varepsilon_H^2 - (1 - 3p + p^2)\varepsilon_L^2 + 2(1 - p + p^2)\varepsilon_H\varepsilon_L \right) \\
& = \frac{1}{\rho} \left( \Delta_2(\varepsilon_H, \varepsilon_L) - \alpha p(1 - p)(1 - \rho)(\varepsilon_H - \varepsilon_L)^2 \right)
\end{aligned}$$

Both  $\Delta_{2a}$  and  $\Delta_{2b}$  are strictly positive when  $x = \varepsilon_L = \varepsilon_H$ . To show that there exist parameter values such that first best can be achieved under cooperation it is sufficient to show that there exist parameter values such that: i) in the first-best contract  $\varepsilon_L = \varepsilon_H = r$ , where  $r \equiv \psi/(\alpha + 2\psi)$ ; ii)  $c_2 - c_1 \leq \min[1/\rho, 2]\delta_\rho\Delta_2$ ; and iii) (from equation (24))  $c_2 - c_1 \leq 4(\beta - 1/2)\alpha r^2 p(1 - p)$ . From equation (17) in the derivation of the first-best contract (before the proof of Proposition 3) we know that part i) requires that  $c_2 = 2rp(1 - (1 - p)\alpha r)$ . Given part i),  $\Delta_2 = 2\alpha r^2$ . Thus, sufficient for parts ii) and iii) is:  $c_2 - c_1 \leq 2 \min[2\beta - 1, 2\rho]\alpha r^2 p(1 - p)$ . Assuming that, if only one manager works, then the manager who works takes on more than one-half of the wage risk ( $\beta > 1/2$ ), then there do exist parameter values such that these conditions are satisfied. ■

**Proof of Proposition 6.** Let  $w_b \equiv w_{HL} + w_{LH}$ .

*Risk-neutral managers:* As shown in the proof of Proposition 5, the nonherding constraints

are easier to satisfy with cooperation. Under cooperation, the effort–IC constraint (12) becomes:

$$2pw_{HH} + (1 - 2p)w_b - 2(1 - p)w_{LL} \geq \frac{c_2}{p} + \frac{c_2 - c_1}{p} \quad (25)$$

$$\text{and} \quad pw_{HH} + (1 - p)w_b - (2 - p)w_{LL} \geq \frac{c_2}{p} \quad (26)$$

With noncooperation  $w_{LH}$  is optimally set to zero and  $w_{LL}$  is optimally set to zero under both regimes, so constraint (26) is identical to the effort constraint in the noncooperative game, constraint (12). However, because nonherding–IC requires that  $w_b > w_{HH}$ , and because under cooperation, if one manager shirks, then the working manager will expend only  $c_1$  instead of  $c_2$ , constraint (25) is harder to satisfy than constraint (12).

*Risk-averse managers:* The nonherding–IC constraint under noncooperation is  $\Delta_2 \geq \frac{c_2 - c_1}{\delta_\rho}$ , where  $\Delta_2$  is defined in equation (20) in the proof of Proposition 4. The cost of inducing diversification is lowered under cooperation if the following two differences are both positive:

$$\Delta_{2a} - \Delta_2 = \Delta_2 + 2\alpha\varepsilon_M^2 \quad (27)$$

$$\Delta_{2b} - \Delta_2 = \frac{(1 - \rho)\Delta_2}{\rho} + \frac{2\alpha\varepsilon_M^2}{\rho} - \frac{\alpha(1 - \rho)p(1 - p)(\varepsilon_H - \varepsilon_L)^2}{\rho} \quad (28)$$

The difference in (27) is clearly positive. Also, if risk is not shared equally when managers choose different class types, then the second term of  $\Delta_{2a}$  will be larger, so the assumption of equal risk sharing has no effect. To determine the sign of the difference in (28), we can multiply the RHS of (28) by  $\rho/(1 - \rho)$  to obtain

$$\Delta_2 + \frac{2\alpha\varepsilon_M^2}{1 - \rho} - \alpha p(1 - p)(\varepsilon_H - \varepsilon_L)^2 \geq \varepsilon_L - \varepsilon_H + \alpha\zeta$$

where

$$\zeta = \varepsilon_H^2 - \varepsilon_L^2 - 2p(p\varepsilon_H^2 + (p - 2)\varepsilon_L^2) + 2(1 - 2p(1 - p))\varepsilon_H\varepsilon_L - (1 + \rho)p(1 - p)(\varepsilon_H - \varepsilon_L)^2.$$

$\varepsilon_L - \varepsilon_H + \alpha\zeta$  is strictly positive if  $\varepsilon_L \geq \varepsilon_H$ . For  $\varepsilon_L - \varepsilon_H + \alpha\zeta$  to be negative it is necessary that  $\varepsilon_L < \varepsilon_H$  and that  $\alpha p(1 - p)(\varepsilon_H - \varepsilon_L)^2 > \frac{c_2 - c_1}{\delta_\rho}$ . Even in this case the difference in (28) may be positive.

From the proof the proof of Proposition 4, the effort–IC constraint under noncooperation is  $\Delta_1 \geq \frac{c_2}{p}$ , where  $\Delta_1$  is defined in equation (19) in that proof. Continuing from the proof of Proposition 5, if both of the following differences are strictly positive, then allowing for cooperation lowers the cost of inducing effort.

$$\begin{aligned} \Delta_{1a} - \Delta_1 &= p\varepsilon_H + (1 - p)\varepsilon_L - \varepsilon_M + \alpha(1 - p)\left(\varepsilon_M^2 + 2\varepsilon_L\varepsilon_M\right) + 4\alpha\beta(1 - p)\varepsilon_L^2 \\ &\quad - \alpha(1 - p)\left((p\varepsilon_H + (1 - p)\varepsilon_L)^2 + p\varepsilon_H^2 + (2 - p)\varepsilon_L^2\right) - \frac{c_2 - c_1}{p} \\ &= \Delta_1 - 2\left(\varepsilon_M - \alpha(1 - p)\left(\varepsilon_M^2 + 2\varepsilon_L\varepsilon_M - \varepsilon_L^2\right)\right) + 4\alpha\beta(1 - p)\varepsilon_L^2 - \frac{c_2 - c_1}{p} \end{aligned}$$

$$\begin{aligned}
&\geq \Delta_1 - 2\varepsilon_M \left(1 - \alpha(1-p)(\varepsilon_M + 2\varepsilon_L)\right) - \frac{c_2 - c_1}{p} \\
\Delta_{1b} - \Delta_1 &= \varepsilon_L - \varepsilon_M - \alpha(1-p)(\varepsilon_L^2 - \varepsilon_M^2 - 2\varepsilon_L\varepsilon_M) \\
&= \varepsilon_L(1 - \alpha(1-p)\varepsilon_L) - \varepsilon_M \left(1 - \alpha(1-p)(\varepsilon_M + 2\varepsilon_L)\right)
\end{aligned}$$

If the individual component of the noncooperative contract is significant ( $\varepsilon_M$  large), and if  $c_2 - c_1$  is large, then allowing for cooperation can make it more expensive to provide incentives for exerting effort. Otherwise, allowing for cooperation lowers the cost of inducing effort. ■

## References

- [1] Admati, Anat R. and Paul Pfleiderer, 1997, Does it all add up? Benchmarks and the compensation of active portfolio managers, *Journal of Business* 70, 323–350.
- [2] Barry, Christopher B and Laura T Starks, 1984, Investment management and risk sharing with multiple managers, *Journal of Finance* 39, 477–491.
- [3] Brown, Keith C., W. V. Harlow and Laura T. Starks, 1996, Of tournaments and temptations: an analysis of managerial incentives in the mutual fund industry, *Journal of Finance* 51, 86–110.
- [4] Chevalier, Judith and Glenn Ellison, 1999, Career concerns of mutual fund managers, *Quarterly Journal of Economics*, 389–432.
- [5] Deli, Daniel N., 2002, Mutual fund advisory contracts: An empirical investigation, *Journal of Finance* 57, 109–133.
- [6] Diamond, Peter, 1998, Managerial incentives: on the near linearity of optimal compensation, *Journal of Political Economy* 106, 931–957.
- [7] Dybvig, Philip H., Heber K. Farnsworth and Jennifer N. Carpenter, 2001, Portfolio performance and agency, working paper, Washington University.
- [8] Green, Jerry R. and Nancy L. Stokey, 1983, A comparison of tournaments and contracts, *Journal of Political Economy* 91, 349–364.
- [9] Grinblatt, Mark and Sheridan Titman, 1989, Adverse risk incentives and the design of performance-based contracts, *Management Science* 35, 807–822.
- [10] Holmström, B., 1982, Moral hazard in teams, *Bell Journal of Economics*, 13, 324–340.
- [11] Khanna, Naveen, 1998, Optimal contracting with moral hazard and cascading, *Review of Financial Studies* 11, 559–596.
- [12] Kreps, David M., 1990, *A Course in Microeconomic Theory*, Princeton University Press, Princeton, New Jersey.
- [13] Lakonishok, Josef, Andrei Shleifer, Richard Thaler and Robert Vishny, 1992, Window dressing by pension fund managers, *American Economic Review* 81, 227–231.
- [14] Lazear, Edward P., and Sherwin Rosen, 1981, Rank-order tournaments as optimum labor contracts, *Journal of Political Economy* 89, 841–864.
- [15] Macho-Stadler, Inés and J. David Pérez-Castrillo, 1993, Moral hazard with several agents: the gains from cooperation, *International Journal of Industrial Organization* 11, 73–100.
- [16] Mookherjee, Dilip, 1984, Optimal incentive schemes with many agents, *Review of Economic Studies* 51, 433–446.
- [17] Musto, David K., 1999, Investment decisions depend on portfolio disclosures, *Journal of Finance* 54, 935–952.

- [18] Nalebuff, Barry J. and Joseph E. Stiglitz, 1983, Prizes and incentives: towards a general theory of compensation and competition, *Bell Journal of Economics* 14, 21–43.
- [19] Palomino, Frédéric and Andrea Prat, 2002, Risk taking and optimal contracts for money managers, *The Rand Journal of Economics* forthcoming.
- [20] Ramakrishnan, Ram T. S. and Anjan V. Thakor, 1991, Cooperation versus competition in agency, *Journal of Law, Economics and Organization* 7, 248–283.
- [21] Sharpe, W. F., 1981, Decentralized investment management, *Journal of Finance* 36, 217–234.
- [22] Slezak, Steve L. and Naveen Khanna, 2000, The effect of organizational form on information flow and decision quality: informational cascades in group decision making, *Journal of Economics and Management Strategy* 9, 115–156.
- [23] Starks, Laura T., 1987, Performance incentive fees: an agency theoretic approach, *Journal of Financial and Quantitative Analysis* 22, 17–32.
- [24] Stoughton, Neal M., 1993, Moral hazard and the portfolio management problem, *The Journal of Finance* 48 (5), 2009–2028.
- [25] Stoughton, Neal M. and Josef Zechner, 1999, Optimal capital allocation using RAROC and EVA, working paper, UC Irvine and University of Vienna.