

rbiprobit: Recursive bivariate probit estimation and decomposition of marginal effects

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Motivation

Effects of interest

1. What we want

- ▶ Estimate: Effect of binary or treatment variable on binary outcome variable
- ▶ Treatment variable itself is endogenous
- ▶ Unobservables may correlate with treatment and outcome equation
- ▶ Compute average treatment effect
- ▶ Compute average marginal effect of independent variables

2. What doesn't work:

- ▶ `margins` gives incorrect treatment effect using `biprobit`
- ▶ `margins` gives incorrect average marginal effect using `biprobit`
- ▶ `ivprobit` inappropriate; treatment variable is binary

3. What we need

- ▶ Correct Estimation of a recursive bivariate probit model (RBPM)
- ▶ Considering recursive nature of the model for postestimation commands

Contribution

A new Stata package

- ▶ `rbiprobit` estimates RBPMs like `biprobit` or `cmp`
 - ▶ allows weights (`pw`, `fw`, `iw`)
 - ▶ provides various variance estimators (`vce`)
 - ▶ `bootstrap`, `jackknife`, and `svy` prefix are allowed
- ▶ `rbiprobit` accounts for recursive nature in postestimation
- ▶ Postestimation commands enable
 - ▶ Correct predictions
 - ▶ Computation of different treatment effects
 - ▶ Decomposition of average marginal effects of independent variables
 - ▶ Standard errors using the delta method or bootstrapping

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Recursive bivariate probit model

The Model

A structural model with endogenous explanatory treatment variable y_2

$$y_1^* = x'\beta + \alpha y_2 + u_1 \quad , y_1 = 1 \left[y_1^* > 0 \right] \quad (1)$$

$$y_2^* = z'\gamma + u_2 \quad , y_2 = 1 \left[y_2^* > 0 \right] \quad (2)$$

$$\text{with } \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \sim \mathcal{N} \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right]$$

- ▶ correlation between u_1 and u_2 induces endogeneity
- ▶ parametric distribution assumption is bivariate normality
- ▶ x' and z' can share some or all independent variables
- ▶ Greene (2018) notes that endogenous nature of y_2 can be ignored
- ▶ Han and Lee (2019): estimates are at best weakly identified if $x = z$

Recursive bivariate probit model

Treatment Effects: ATE and ATET

1. Average treatment effect (ATE)

$$ATE = \Phi(x'\beta + \alpha) - \Phi(x'\beta)$$

- ▶ Ceteris-paribus scenario over full sample
- ▶ Effect of discrete change in treatment holding all other observed and unobserved variables constant

Recursive bivariate probit model

Treatment Effects: ATEC

2. Average treatment effect on conditional probability of outcome success (ATEC)

$$\text{ATEC} = \frac{\Phi_2(x'\beta + \alpha, z'\gamma, \rho)}{\Phi(z'\gamma)} - \frac{\Phi_2(x'\beta, -z'\gamma, -\rho)}{\Phi(-z'\gamma)}$$

- ▶ Accounts for selection on unobservables
- ▶ Utilizes conditional probabilities of the outcome $\Pr(y_{1i} = 1 | y_{2i} = s)$ for $s = 0, 1$ over full sample.
- ▶ Effect of a discrete change in treatment, holding only the observed variables constant
- ▶ Imposing no constraint on ρ to account for changes in unobserved variables as a consequence of the treatment
- ▶ ATEC collapses to ATE if $\rho = 0$

Decomposition of Marginal Effects

Joint and Conditional Probabilities

- ▶ Independent variable d appears in both x' and z'
- ▶ Decomposition of total marginal effects on the probabilities (except marginal probabilities) are then
 1. Continuous Variables (see Greene, 2018)

$$ME = \frac{\partial \Pr}{\partial \begin{pmatrix} x_d \\ z_d \end{pmatrix}} = \underbrace{\frac{\partial \Pr}{\partial x_d}}_{\text{direct effect}} + \underbrace{\frac{\partial \Pr}{\partial z_d}}_{\text{indirect effect}}$$

2. Discrete Variables (see Hasebe, 2013; Edwards et al., 2019)

$$ME = \underbrace{[\Pr|_{x_d=1} - \Pr|_{x_d=0}]}_{\text{direct effect}} + \underbrace{[\Pr|_{z_d=1} - \Pr|_{z_d=0}]}_{\text{indirect effect}}$$

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Basic Syntax

```
rbiprobit depvar [=] [indepvars] [if] [in] [weight]  
      , endogenous(depvar_en [=] [indepvars_en] [, enopts]) [options]
```

- ▶ *depvar* and *depvar_en* have to be 0/1 variables
- ▶ *depvar_en* automatically added to outcome equation as factor-variable
- ▶ Factor variables and time-series operators allowed
- ▶ *bootstrap*, *jackknife*, and *svy* prefix are allowed
- ▶ Variance estimators: *robust*, *cluster robust*, *bootstrap*, ...
- ▶ Linear constraints are applicable

Postestimation Commands

probability functions

Predictions

```
predict [type] newvar [if] [in] [, statistic]
```

statistic

p11	Pr(depvar = 1, depvar_en = 1); the default
p10	Pr(depvar = 1, depvar_en = 0)
p01	Pr(depvar = 0, depvar_en = 1)
p00	Pr(depvar = 0, depvar_en = 0)
pmarg1	Pr(depvar = 1); marginal success probability for outcome eq.
pmarg2	Pr(depvar_en = 1); marginal success probability for treatment eq.
pcond1	Pr(depvar = 1 depvar_en = 1)
pcond2	Pr(depvar_en = 1 depvar = 1)
xb1	linear prediction for outcome eq.
xb2	linear prediction for treatment eq.
...	

Postestimation Commands

Margins and Treatment Effects

```
rbiprobit margdec [if] [in] [weight] [, response_options options]
```

```
rbiprobit tmeffects [if] [in] [weight] [, tmeffect(effecttype) options]
```

rbiprobit margdec options

`effect(effecttype)` specify type of effect; *effecttype* may be total, direct, or indirect; default is total

`predict(pred_opt)` estimate margins for predict, *pred_opt* ; multiple predict not applicable

`dydx(varlist)` estimate marginal effect of variables in *varlist*

...

rbiprobit tmeffects options

`tmeffect(effecttype)` specify type of effect; *effecttype* may be ate, atet, or atec; default is ate

...

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An empirical application

independent variables

Preference for redistribution and hazard of national culture by immigrants

1. Research question

Does the perception of immigrants as a hazard of national culture effect natives' preference for redistribution?

2. Data

- ▶ European Social Survey (Wave 7, 2014)
- ▶ Individual Data from the United Kingdom
- ▶ Data adjusted for demonstration purposes
- ▶ Sample restricted to respondents with no migration background

3. The Model

- ▶ Binary outcome variable: `redist`
Should the government reduce difference in income levels?
(Agree = 1, Disagree = 0)
- ▶ Binary treatment variable: `imcult`
Do immigrants undermine or enrich country's cultural life?
(Undermine = 1, Enrich = 0)

rbiprobit output table

```
. use "https://cobanomics.github.io/rbiprobit/data/ess7_uk.dta", clear
(Modified excerpt from European Social Survey Wave 7 for United Kingdom)

. global indeplist      c.age##c.age i.female i.urban educyrs rignore i.lbf

. rbiprobit redist = $indeplist hhincdec hhmemb ///
> , endog(imcult = $indeplist i.pareduc imcont) nolog
```

```
Recursive Bivariate Probit Regression      Number of obs      =      1,256
Log likelihood = -1120.1179                Wald chi2(19)      =      334.05
                                           Prob > chi2        =      0.0000
```

		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	

redist							
	imcult						
Culture	undermined	-1.093271	.3153386	-3.47	0.001	-1.711323	-.4752184
	age	.0415339	.0155237	2.68	0.007	.011108	.0719598
	c.age#c.age	-.0003429	.0001435	-2.39	0.017	-.0006243	-.0000616
	female						
	Female	-.0292688	.0874852	-0.33	0.738	-.2007366	.1421991
	urban						
[1]	(Sub)Urban	.0833317	.0967068	0.86	0.389	-.1062102	.2728736
	educyrs	.0060353	.0124718	0.48	0.628	-.0184091	.0304796
	rignore	.1570994	.0275802	5.70	0.000	.1030432	.2111557
	lbf						
	Employed	-.0546754	.1064449	-0.51	0.607	-.2633036	.1539527
	hhincdec	-.0636975	.0172709	-3.69	0.000	-.0975479	-.0298471
	hhmemb	-.0008141	.0409355	-0.02	0.984	-.0810461	.079418
	_cons	-1.732201	.6640082	-2.61	0.009	-3.033633	-.4307688

rbiprobit output table (con't)

imcult							
	age	.0075091	.0164422	0.46	0.648	-.0247169	.0397352
	c.age#c.age	-.0000784	.0001574	-0.50	0.619	-.0003869	.0002302
	female						
	Female	.3339391	.0884987	3.77	0.000	.1604849	.5073933
	urban						
[1]	(Sub)Urban	-.1769626	.100775	-1.76	0.079	-.374478	.0205528
	educyrs	-.0586824	.0114027	-5.15	0.000	-.0810314	-.0363335
	rightleft	-.1342447	.0219501	-6.12	0.000	-.1772661	-.0912234
	lbf						
	Employed	-.1640761	.1136411	-1.44	0.149	-.3868085	.0586563
	pareduc						
	Academic parent	-.1822516	.0965664	-1.89	0.059	-.3715182	.0070151
	imcont	-.0794116	.0255358	-3.11	0.002	-.1294609	-.0293623
	_cons	2.854024	.4839785	5.90	0.000	1.905444	3.802604

	/atanrho	.6042669	.2333532	2.59	0.010	.1469031	1.061631

	rho	.5400788	.1652875			.1458554	.7862872

Wald test of rho=0: chi2(1) = 6.70549				Prob > chi2 = 0.0096			

- ▶ In ML estimation ρ is not directly estimated, but $\operatorname{atanh} \rho$
- ▶ estimated correlation between error terms is positive and significantly different from zero
- ▶ Applying `lrmodel` would give the LR model test instead of the Wald test

The correlation parameter

Comparison: biprobit vs. rbiprobit

```
. biprobit (redist = $indeplist hhincdec i.hhincmsc)
> (imcult = $indeplist i.pareduc imcont)
```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
/athrho	-.0592815	.0641919	-0.92	0.356	-.1850952	.0665323
rho	-.0592121	.0639668			-.18301	.0664343

```
. rbiprobit redist = $indeplist hhincdec i.hhincmsc
> , endog(imcult = $indeplist i.pareduc imcont)
```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
imcult						
Culture undermined	-1.093271	.3153386	-3.47	0.001	-1.711323	-.4752184
/atanrho	.6042669	.2333532	2.59	0.010	.1469031	1.061631
rho	.5400788	.1652875			.1458554	.7862872

Explanation according to Filippini et al. (2018)

The correlation parameter in `biprobit` is a weighted average of the correlation parameter from `rbiprobit` and the coefficient on treatment variable `imcult`

Postestimation: Predictions

biprobit workaround

Comparison: biprobit vs. rbiprobit

```
. qui: rbiprobit redist = $indeplist hhincdec hhmemb ///
>           , endog(imcult = $indeplist i.pareduc imcont)

. predict p11_rbp, p11

. qui: biprobit (redist = $indeplist hhincdec hhmemb i.imcult) ///
>           (imcult = $indeplist i.pareduc imcont), nolog

. predict p11_bp, p11

. compare p11_rbp p11_bp
```

	count	----- minimum	difference average	----- maximum
p11_rbp<p11_bp	230	-.3982587	-.2817366	-.0239254
p11_rbp=p11_bp	1025			
p11_rbp>p11_bp	1	1.49e-08	1.49e-08	1.49e-08
jointly defined	1256	-.3982587	-.0515919	1.49e-08
total	1256			

Incorrect predictions after biprobit

biprobit doesn't account for recursive nature of the model, e.g. takes observed y_{2i} instead of $y_{2i} = 1 \forall i$ in $\Pr(y_{1i} = 1, y_{2i} = 1)$

Postestimation: Treatment effects

rbiprobit tmeffects: Average treatment effects

```
. rbiprobit tmeffects, tmeffect(ate)
```

```
Treatment effect          Number of obs    =      1,256
Model VCE      : OIM
```

```
Expression  : Pr(redist=1), predict(pmarg1)
Effect      : Average treatment effect
dydx w.r.t. : 1.imcult
```

```
-----+-----
          |              Delta-method
          |              dy/dx   Std. Err.      z    P>|z|    [95% Conf. Interval]
-----+-----
       ate |    -0.355684   .1156057   -3.08   0.002   -0.582267   -0.129101
-----+-----
```

```
. rbiprobit tmeffects, tmeffect(atec)
```

```
Treatment effect          Number of obs    =      1,256
Model VCE      : OIM
```

```
Expression  : Pr(redist=1|imcult=1)-Pr(redist=1|imcult=0), predict(pcond1)-predict(pc
> ondl0)
Effect      : Average treatment effect on conditional probability
dydx w.r.t. : 1.imcult
```

```
-----+-----
          |              Delta-method
          |              dy/dx   Std. Err.      z    P>|z|    [95% Conf. Interval]
-----+-----
       atec |    -0.017399   .0300759   -0.58   0.563   -0.0763467   .0415488
-----+-----
```

Postestimation: Marginal effects

rbiprobit margdec: Average marginal effects (continuous independent variable)

```
. rbiprobit margdec, dydx(rigleft) predict(p11) effect(direct)
```

```
Average marginal effects      Number of obs      =      1,256
Model VCE      : OIM
```

```
Expression      : Pr(redist=1,imcult=1), predict(p11)
dy/dx w.r.t.   : rigleft
```

```
-----+-----
```

	dy/dx	Delta-method Std. Err.	z	P> z	[95% Conf. Interval]	
rigleft	.0341355	.0053522	6.38	0.000	.0236452	.0446257

```
-----+-----
```

```
. rbiprobit margdec, dydx(rigleft) predict(p11) effect(indirect)
```

```
Average marginal effects      Number of obs      =      1,256
Model VCE      : OIM
```

```
Expression      : Pr(redist=1,imcult=1), predict(p11)
dy/dx w.r.t.   : rigleft
```

```
-----+-----
```

	dy/dx	Delta-method Std. Err.	z	P> z	[95% Conf. Interval]	
rigleft	-.0025357	.001461	-1.74	0.083	-.0053992	.0003278

```
-----+-----
```

Postestimation: Marginal effects

don't use margins

`rbiprobit margdec`: Average marginal effects (continuous independent variable)

```
. rbiprobit margdec, dydx(rigleft) predict(p11) effect(total)
```

```
Average marginal effects          Number of obs      =          1,256
Model VCE      : OIM
```

```
Expression      : Pr(redist=1,imcult=1), predict(p11)
dy/dx w.r.t.    : rigleft
```

```
-----+-----
```

	dy/dx	Delta-method Std. Err.	z	P> z	[95% Conf. Interval]
rigleft	.0315997	.0047834	6.61	0.000	.0222244 .0409751

```
-----+-----
```

- ▶ Direct effect of `rigleft` is positive
- ▶ Indirect effect of `rigleft` is negative
- ▶ Indirect effect doesn't offset direct effect entirely

Postestimation: Plots

`rbiprobit margdec` and `rbiprobit tmeffects`: `Marginsplot`

► Marginsplot of total average marginal effects

```
. rbiprobit margdec, dydx(rigleft hhincdec lbf) pr(p11) eff(total)
. marginsplot
```

► Marginsplot of indirect average marginal effects

```
. rbiprobit margdec, dydx(female pareduc) pr(p10) eff(indirect)
. marginsplot
```

► Marginsplot of average treatment effect

```
. rbiprobit tmeffects, tmeffect(ate)
. marginsplot
```

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Conclusion

- ▶ `rbiprobit` is a suitable alternative for `ivprobit`
- ▶ `rbiprobit` identified even without IV (theoretically)
- ▶ Without IV: identification of `rbiprobit` decisively based on bivariate normality assumption
- ▶ `rbiprobit` takes into account recursive nature of the model in contrast to `biprobit` or `cmp`
- ▶ Three different treatment effects computable
- ▶ Decomposition of marginal effects gives insight about insignificant total marginal effects


Thank you

Version 1.1.0 available

```
ssc install rbiprobit
```

In the future, minor updates available via

```
net install rbiprobit, from("https://cobanomics.github.io/rbiprobit/")
```

 github.com/cobanomics

 [@cobanomics](https://twitter.com/cobanomics)

 mustafa.coban@iab.de

 [mustafacoban.de](https://www.mustafacoban.de)

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1. Joint Probabilities

$$\Pr(y_1 = 1, y_2 = 1|x, z) = \Phi_2(x'\beta + \alpha, z'\gamma, \rho)$$

$$\Pr(y_1 = 1, y_2 = 0|x, z) = \Phi_2(x'\beta, -z'\gamma, -\rho)$$

$$\Pr(y_1 = 0, y_2 = 1|x, z) = \Phi_2(-x'\beta - \alpha, z'\gamma, -\rho)$$

$$\Pr(y_1 = 0, y_2 = 0|x, z) = \Phi_2(-x'\beta, -z'\gamma, \rho)$$

2. Conditional Probabilities

$$\Pr(y_1 = 1|y_2 = 1, x, z) = \frac{\Phi_2(x'\beta + \alpha, z'\gamma, \rho)}{\Phi(z'\gamma)}$$

$$\Pr(y_2 = 1|y_1 = 1, x, z) = \frac{\Phi_2(x'\beta + \alpha, z'\gamma, \rho)}{\Phi(x'\beta + \alpha)}$$

3. Marginal Probabilities

$$\Pr(y_1 = 1|x) = \Phi(x'\beta + \alpha y_2)$$

$$\Pr(y_2 = 1|z) = \Phi(z'\gamma)$$

4. Unconditional Mean Function (see Blasch et al., 2019; Alrasheed, 2019)

$$\begin{aligned} E[y_1|x, z] &= \Pr(y_2 = 1|z) \cdot E[y_1|y_2 = 1, x, z] \\ &+ \Pr(y_2 = 0|z) \cdot E[y_1|y_2 = 0, x, z] \\ &= \Pr(y_1 = 1, y_2 = 1|x, z) + \Pr(y_1 = 1, y_2 = 0|x, z) \\ &= \Phi_2(x'\beta + \alpha, z'\gamma, \rho) + \Phi_2(x'\beta, -z'\gamma, -\rho) \end{aligned}$$

An empirical application

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Varlist of independent variables

- ▶ Independent variables common to both equations
 - ▶ Age (`age`)
 - ▶ Gender (`female`)
 - ▶ Place of residence (`urban`)
 - ▶ Years of education (`educyrs`)
 - ▶ Main activity, last 7 days (`lbf`)
 - ▶ Self-placement on political left-right scale (`right`)
- ▶ Independent Variables only in treatment equation
 - ▶ At least one parent is academic (`pareduc`)
 - ▶ Frequency of contact with immigrants beyond workplace and friendships (`imcont`)
- ▶ Independent Variables only in outcome equation
 - ▶ Household income (`hhincdec`)
 - ▶ Number of household members (`hhmemb`)

Postestimation: Predictions

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biprobit workaround for joint/conditional probabilities

```
. qui: rbiprobit redist = $indeplist hhincdec hhmemb ///
>           , endog(imcult = $indeplist i.pareduc imcont)

. predict p11_rbp, p11

. qui: biprobit (redist = $indeplist hhincdec hhmemb i.imcult) ///
>           (imcult = $indeplist i.pareduc imcont), nolog

. replace redist = 1
(982 real changes made)

. replace imcult = 1
(230 real changes made)

. predict p11_bp, p11

. compare p11_rbp p11_bp
```

	count	----- minimum	difference average	----- maximum
p11_rbp=p11_bp	1255			
p11_rbp>p11_bp	1	1.49e-08	1.49e-08	1.49e-08
jointly defined	1256	0	1.19e-11	1.49e-08
total	1256			

Postestimation: Treatment effects

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`rbiprobit tmeffects`: Average treatment effect on the treated

$$\text{ATET} = \Phi\left(\frac{x_i'\beta + \alpha - \rho z_i'\gamma}{\sqrt{1 - \rho^2}}\right) - \Phi\left(\frac{x_i'\beta - \rho z_i'\gamma}{\sqrt{1 - \rho^2}}\right) \quad \forall y_{2i} = 1$$

- ▶ Ceteris-paribus scenario over sub-sample of treated
- ▶ Effect of discrete change in treatment on the conditional probability

$$\Pr(y_{1i} = 1 | y_{2i} = 1)$$

```
. rbiprobit tmeffects, tmeffect(atet)
```

```
Treatment effect          Number of obs   =       1,026
Model VCE      : OIM

Expression      : normal(redist=1|imcult=1) - normal(redist=1|imcult=0)
Effect          : Average treatment effect on the treated
dydx w.r.t.    : 1.imcult
```

```
-----+-----
          |              Delta-method
          |              dy/dx   Std. Err.      z    P>|z|    [95% Conf. Interval]
-----+-----
      atet |   -.2459238   .0662172   -3.71   0.000   - .3757072   -.1161404
-----+-----
```


Postestimation: Marginal effects

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Incorrect standard errors using margins

```
. margins, dydx(rigleft) predict(p11)
```

```
Average marginal effects      Number of obs      =      1,256
Model VCE      : OIM
```

```
Expression      : Pr(redist=1,imcult=1), predict(p11)
dy/dx w.r.t.    : rigleft
```

```
-----+-----
          |              Delta-method
          |              dy/dx   Std. Err.      z    P>|z|    [95% Conf. Interval]
-----+-----
    rigleft |   .0315997   .0045339    6.97   0.000    .0227135    .040486
-----+-----
```

```
. rbiprobit margdec, dydx(rigleft) predict(p11) effect(total)
```

```
Average marginal effects      Number of obs      =      1,256
Model VCE      : OIM
```

```
Expression      : Pr(redist=1,imcult=1), predict(p11)
dy/dx w.r.t.    : rigleft
```

```
-----+-----
          |              Delta-method
          |              dy/dx   Std. Err.      z    P>|z|    [95% Conf. Interval]
-----+-----
    rigleft |   .0315997   .0047834    6.61   0.000    .0222244    .0409751
-----+-----
```