KU LEUVEN

Fitting mixed random regret minimization models using `mixrandregret`.

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- Martina Vandebroek

Research Centre for Operations Research and Statistics (ORSTAT)

1. Random Regret Minimization Models
2. Differences between RUM and RRM models.
3. Mixed Random Regret Minimization Models
4. Individual Level Parameters
5. Implementation
6. Conclusions
7. Bibliography

1 Outline

1. Random Regret Minimization Models
   - Random Utility vs Random Regret
   - Classical Regret Function
2. Differences between RUM and RRM models.
3. Mixed Random Regret Minimization Models
4. Individual Level Parameters
5. Implementation
6. Conclusions

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$\Rightarrow$ Regret models will (formalize and) minimize this notion of regret!

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- $x_{ins,m}$: value of attribute m of alternative i for individual n in choice situation s. (yes, 4 sub-indexes; I am sorry...)
- y_{ins}: response variable (**choice**). It takes the value of 1 when alternative i is chosen by individual n in choice situation s; 0 otherwise..

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  - R_{ins} is described in terms of ***attribute level regret*** $(R_{i \leftrightarrow jns,m})$.

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$(x_{jns,m} - x_{ins,m})$	Attribute \ Route	$j=1$	$j=2$	$j=3$
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$(x_{jns,m} - x_{2ns,C})$	Travel Cost	2	0	-1
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- Takeaway: We will define $R_{i\leftrightarrow jns,m}$ in terms of the attribute differences.

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- (Chorus, 2010) proposed the following attribute level regret:

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- However, they have drastically different interpretation(more on that later).

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- In our example: $M = 2$ (Time and Cost) and $J = 3$.
- Regret of alternative 1 (R_{1ns}) will be given by:

$$\begin{aligned} R_{1ns} &= \sum_{j \neq i}^{3} \sum_{m=1}^{M} \ln\left[1 + \exp\left\{\beta_{n,m}(x_{jns,m} - x_{ins,m})\right\}\right] \\ &= \ln\left[1 + \exp\left\{\beta_{n,T}\left(x_{2ns,T} - x_{1ns,T}\right)\right\}\right] + \ln\left[1 + \exp\left\{\beta_{n,c}\left(x_{2ns,C} - x_{1ns,C}\right)\right\}\right] \\ &+ \ln\left[1 + \exp\left\{\beta_{n,T}\left(x_{3ns,T} - x_{1ns,T}\right)\right\}\right] + \ln\left[1 + \exp\left\{\beta_{n,C}\left(x_{3ns,C} - x_{1ns,C}\right)\right\}\right] \end{aligned}$$

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2. Acknowledging that the minimization of the random regret is mathematically equivalent to maximizing the negative of the regret.
3. Hence, the probabilities can be derived using the Multinomial Logit:

$$P_{ins} = \frac{\exp\left(-R_{ins}\right)}{\sum_{j=1}^{J} \exp\left(-R_{jns}\right)} \qquad \text{for } i = 1, \ldots, J \tag{1}$$

1 Classical RRM (Chorus, 2010):Towards the log-likelihood.

1. Defining $RR_{ins} = R_{ins} + \varepsilon_{ins}$, where ε_{ins} is a type I Extreme Value i.i.d. error.
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4. Consequently, the log-likelihood will be given by:

$$\ln L = \sum_{n=1}^{N} \sum_{s=1}^{S} \sum_{i=1}^{J} y_{in} \times \ln\left(P_{ins}\right)$$

2 Outline

1. Random Regret Minimization Models
2. Differences between RUM and RRM models.
   Taste Parameter Interpretation in RRM models
3. Mixed Random Regret Minimization Models
4. Individual Level Parameters
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  - For instance if $\widehat{\beta}_{n,m} < 0$ suggests that regret decreases as the level of that attribute increases **in a non-chosen alternative**, in comparison to the level of the same attribute in the chosen alternative (e.g: Total Time).
- All in all, the parameters in RUM and RRM, are expected to have the same sign, even though their interpretation is drastically different.

3 Outline

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$$\ln L(\boldsymbol{\beta}) = \sum_{n=1}^{N}\ln\left[\int_{\boldsymbol{\beta}} P_n(\boldsymbol{\beta}) f\left(\boldsymbol{\beta}|\varphi\right) d\boldsymbol{\beta}\right] \tag{3}$$

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- R is the number of draws and r is the r-th draw from $f(\boldsymbol{\beta}|\varphi)$.

4 Outline

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- For this estimation we will use the command `mixrbeta` after estimating the population parameters using `mixrandregret` (Zhu, 2022).

5 Outline

1. Random Regret Minimization Models
2. Differences between RUM and RRM models.
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4. Individual Level Parameters
5. Implementation
   - Syntax
   - Outputs
6. Conclusions

5 Syntax

`mixrandregret` (Zhu, 2022) is implemented as a Mata-based `gf-0 ml` evaluator. The command allows the inclusion of normally and log-normally distributed random parameters.

`mixrandregret` *depvar* [*indepvars*] [*if*] [*in*] *group(varname)* *alternative(varname)* *rand(varlist)* [, `id`(*varname*) `basealternative`(*string*) `noconstant` `ln`(*string*) `nrep`(*string*) `burn`(*string*) `robust` `cluster`(*varname*) `level`(#) *maximize_options*]

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The command `mixrbeta` can be used after `mixrandregret` to calculate individual-level parameters corresponding to the variables in the specified *varlist* using equation (6).

`mixrbeta` *varlist* `saving`(filename) [, `plot` `nrep`(#) `burn`(#)]

5 The Data

- Data from van Cranenburgh (2018): Stated Choice (SC) experiment.

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1	1	First	23	6	0
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  - Three unlabeled route alternatives ($J = 3$).

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- Variables **choice** and **altern** allows us to identify each choice.

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- First we estimate a fixed parameters RRM model.

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. randregret choice total_time total_cost , gr(cs) alt(altern) rrmfn(classic) ///
> nocons cluster(id)  nolog
------------------------------------------------------------------------------
Fitting Classic RRM Model
------------------------------------------------------------------------------

RRM: Classic Random Regret Minimization Model
Case ID variable: cs                              Number of cases    =      1060
Alternative variable: altern                      Number of obs      =      3180
                                                  Wald chi2(2)       =     40.41
Log likelihood = -1118.4784                       Prob > chi2        =    0.0000
                                 (Std. Err. adjusted for  106 clusters in id)
------------------------------------------------------------------------------
             |               Robust
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
RRM          |
  total_time |   -.102813   .0182526    -5.63   0.000    -.1385874   -.0670386
  total_cost |   -.417101    .068059    -6.13   0.000    -.5504943   -.2837078
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. matrix b_rrm = e(b)
```

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- As expected, both parameter estimates are negative.

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. mixrandregret choice  total_cost , gr(cs) alt(altern) rand(total_time) id(id) ///
> nocons cluster(id)  nrep(500) from(init_mix_rrm) tech(bhhh)  nolog
Case ID variable: cs                               Number of cases    =       1060
Alternative variable: altern
Random variable(s): total_time
                                 (Std. Err. adjusted for  106 clusters in id)
Mixed random regret model                           Number of obs     =      3,180
                                                    Wald chi2(2)      =     606.11
Log likelihood = -771.05731                         Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |               OPG
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Mean         |
  total_cost |  -1.102136   .0449727   -24.51   0.000    -1.190281   -1.013991
  total_time |  -.3580736   .0581449    -6.16   0.000    -.4720355   -.2441117
-------------+----------------------------------------------------------------
SD           |
  total_time |   .5068268    .041366    12.25   0.000      .425751    .5879027
------------------------------------------------------------------------------

The sign of the estimated standard deviations is irrelevant: interpret them as
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- The mean of **total_time** is negative, as expected.

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```
. preserve

. /* Computing Individual Level Parameters */
. qui mixrbeta total_time , nrep(500)  replace saving("${graphs_route}\mixRRM_normal_idl")

. use "${graphs_route}\mixRRM_normal_idl" , replace

. list id  total_time  in 1/5
```

	id	total_time
1.	1	.37640482
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3.	3	.37672848
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- We observe that some of the individuals has a positive coefficient for Total Time (`total_time`).

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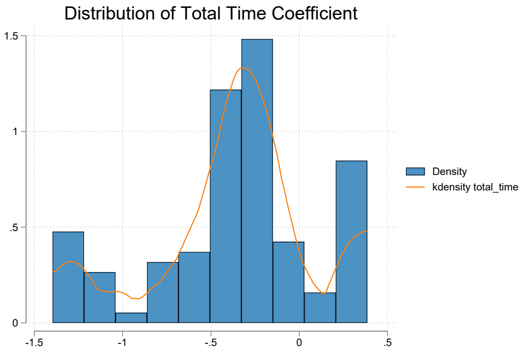

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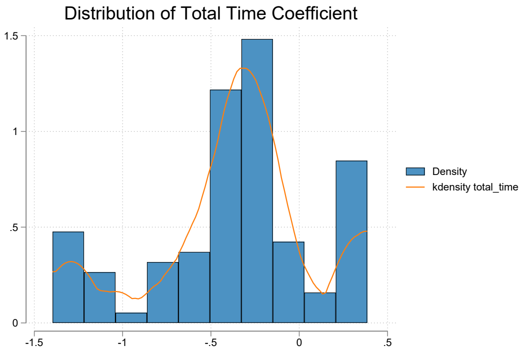

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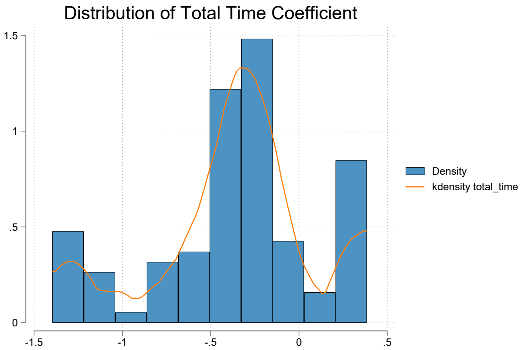

- We see some individuals with positive estimates.
- To prevent this from happening we can use a bounded distribution...

5 Mixed RRM model: Log-normal Distribution

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```
. gen ntt = -1 * total_time

. mixrandregret choice  total_cost , gr(cs) alt(altern) rand(ntt ) ln(1) id(id) ///
> nocons cluster(id)  nrep(500) tech(bhhh) from(b_mixrrm) nolog

Case ID variable: cs                         Number of cases    =       1060
Alternative variable: altern
Random variable(s): ntt

                                  (Std. Err. adjusted for  106 clusters in id)

Mixed random regret model                      Number of obs     =      3,180
                                               Wald chi2(2)      =    1230.55
Log likelihood = -785.27671                    Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |                OPG
      choice |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Mean         |
  total_cost |  -1.217682   .0442047   -27.55   0.000    -1.304321   -1.131042
         ntt |  -1.312285   .1562202    -8.40   0.000    -1.618471   -1.006099
-------------+----------------------------------------------------------------
SD           |
         ntt |   1.363632   .1185994    11.50   0.000     1.131181    1.596082
------------------------------------------------------------------------------

The sign of the estimated standard deviations is irrelevant: interpret them as
being positive
```

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```
. /* Computing Individual Level Parameters */
. qui mixrbeta ntt , nrep(500)  replace saving("${graphs_route}\mixRRM_ln_idl")

. use "${graphs_route}\mixRRM_ln_idl" , replace

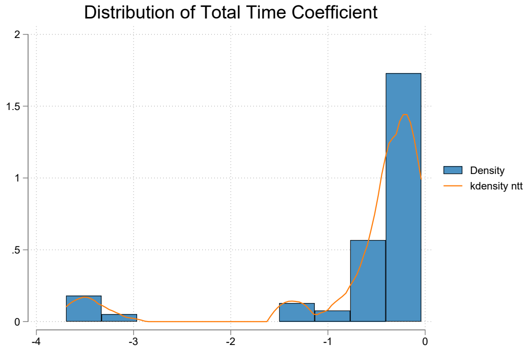
. replace ntt = -1 * ntt /*reverse sign for graph*/
(106 real changes made)

. list id  ntt  in 1/5
```

	id	ntt
1.	1	-.04032598
2.	2	-.08142616
3.	3	-.04047817
4.	4	-.04110615
5.	5	-.04025335

5 Mixed RRM model: Log-normal Distribution

- Individual Level Parameters when total time is assumed to be Log-normally distributed.



- Now we observe that the individual level parameters are all negative.

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- Finally, we can compute them using nlcom.

```
. nlcom ///
> (mean_time: -1*exp([Mean]_b[ntt]+0.5*[SD]_b[ntt]^2)) ///
> (med_time : -1*exp([Mean]_b[ntt])) ///
> (sd_time  :    exp([Mean]_b[ntt]+0.5*[SD]_b[ntt]^2)*sqrt(exp([SD]_b[ntt]^2)-1))

   mean_time:  -1*exp([Mean]_b[ntt]+0.5*[SD]_b[ntt]^2)
    med_time:  -1*exp([Mean]_b[ntt])
     sd_time:  exp([Mean]_b[ntt]+0.5*[SD]_b[ntt]^2)*sqrt(exp([SD]_b[ntt]^2)-1)
```

choice	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
mean_time	-.682127	.1587961	-4.30	0.000	-.9933616	-.3708923
med_time	-.2692041	.0420551	-6.40	0.000	-.3516307	-.1867776
sd_time	1.588122	.6295756	2.52	0.012	.3541763	2.822067

6 Outline

1. Random Regret Minimization Models
2. Differences between RUM and RRM models.
3. Mixed Random Regret Minimization Models
4. Individual Level Parameters
5. Implementation
6. Conclusions
7. Bibliography

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GitHub with Slides + Example code here:

Thanks ✌